Primordial Black Holes and Scalar Induced Gravitational Waves from Inflation with gravitationally enhanced friction

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Outline

01 Motivation

02 Enhanced curvature perturbations in inflation with gravitationally enhanced friction

03 Fraction of primordial black hole dark matter

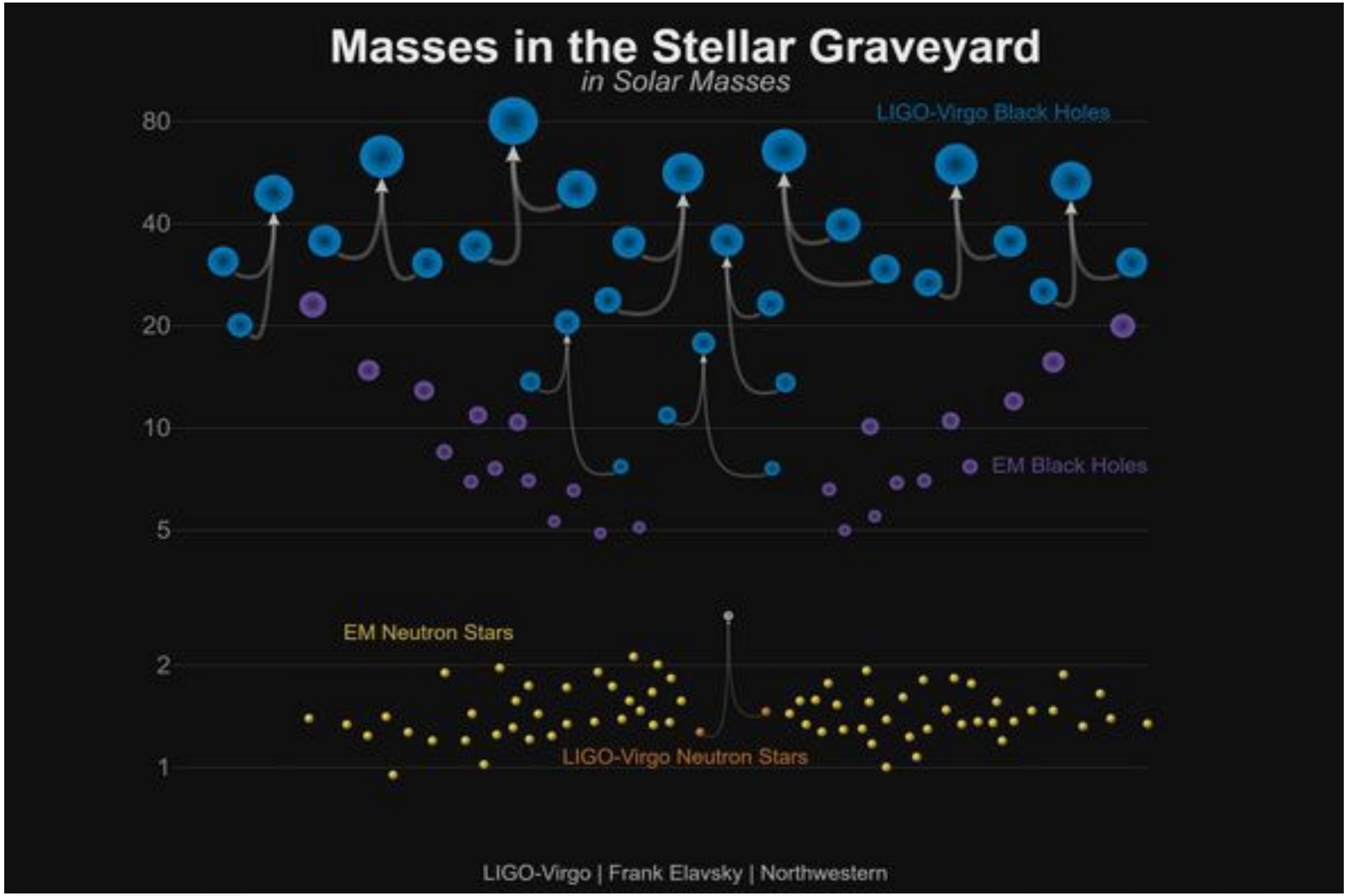
04 Scalar induced Gravitational waves

05 Conclusions

Outline

Motivation

LIGO/Virgo gravitational wave events



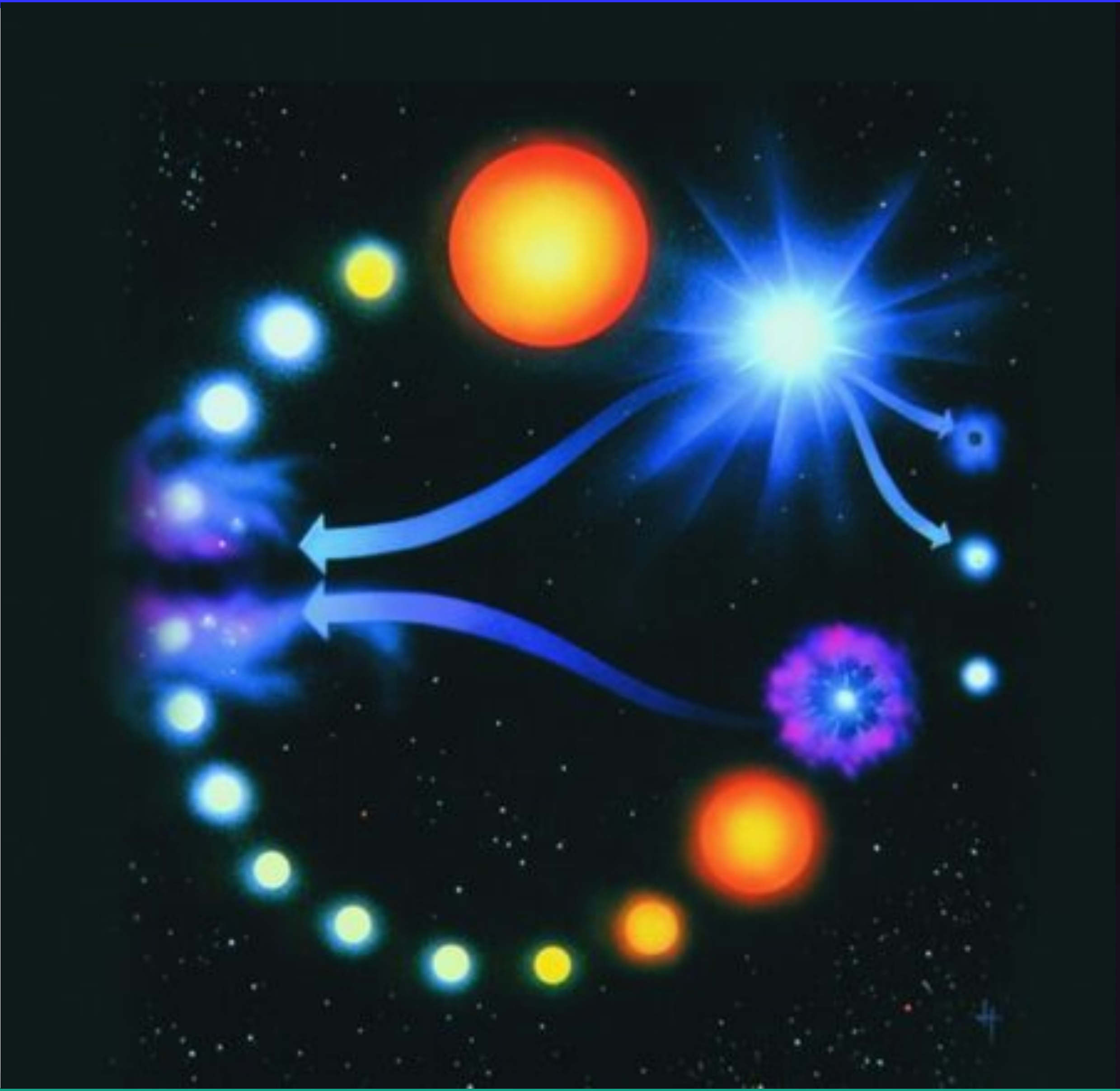
Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$
GW150914	$35.6^{+4.7}_{-3.1}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.7}_{-1.5}$
GW151012	$23.2^{+14.9}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.1}_{-1.2}$
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.5}$	$8.9^{+0.3}_{-0.3}$
GW170104	$30.8^{+7.3}_{-5.6}$	$20.0^{+4.9}_{-4.6}$	$21.4^{+2.2}_{-1.8}$
GW170608	$11.0^{+5.5}_{-1.7}$	$7.6^{+1.4}_{-2.2}$	$7.9^{+0.2}_{-0.2}$
GW170729	$50.2^{+16.2}_{-10.2}$	$34.0^{+9.1}_{-10.1}$	$35.4^{+6.5}_{-4.8}$
GW170809	$35.0^{+8.3}_{-5.9}$	$23.8^{+5.1}_{-5.2}$	$24.9^{+2.1}_{-1.7}$
GW170814	$30.6^{+5.6}_{-3.0}$	$25.2^{+2.8}_{-4.0}$	$24.1^{+1.4}_{-1.1}$
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$
GW170818	$35.4^{+7.5}_{-4.7}$	$26.7^{+4.3}_{-5.2}$	$26.5^{+2.1}_{-1.7}$
GW170823	$39.5^{+11.2}_{-6.7}$	$29.0^{+6.7}_{-7.8}$	$29.2^{+4.6}_{-3.6}$

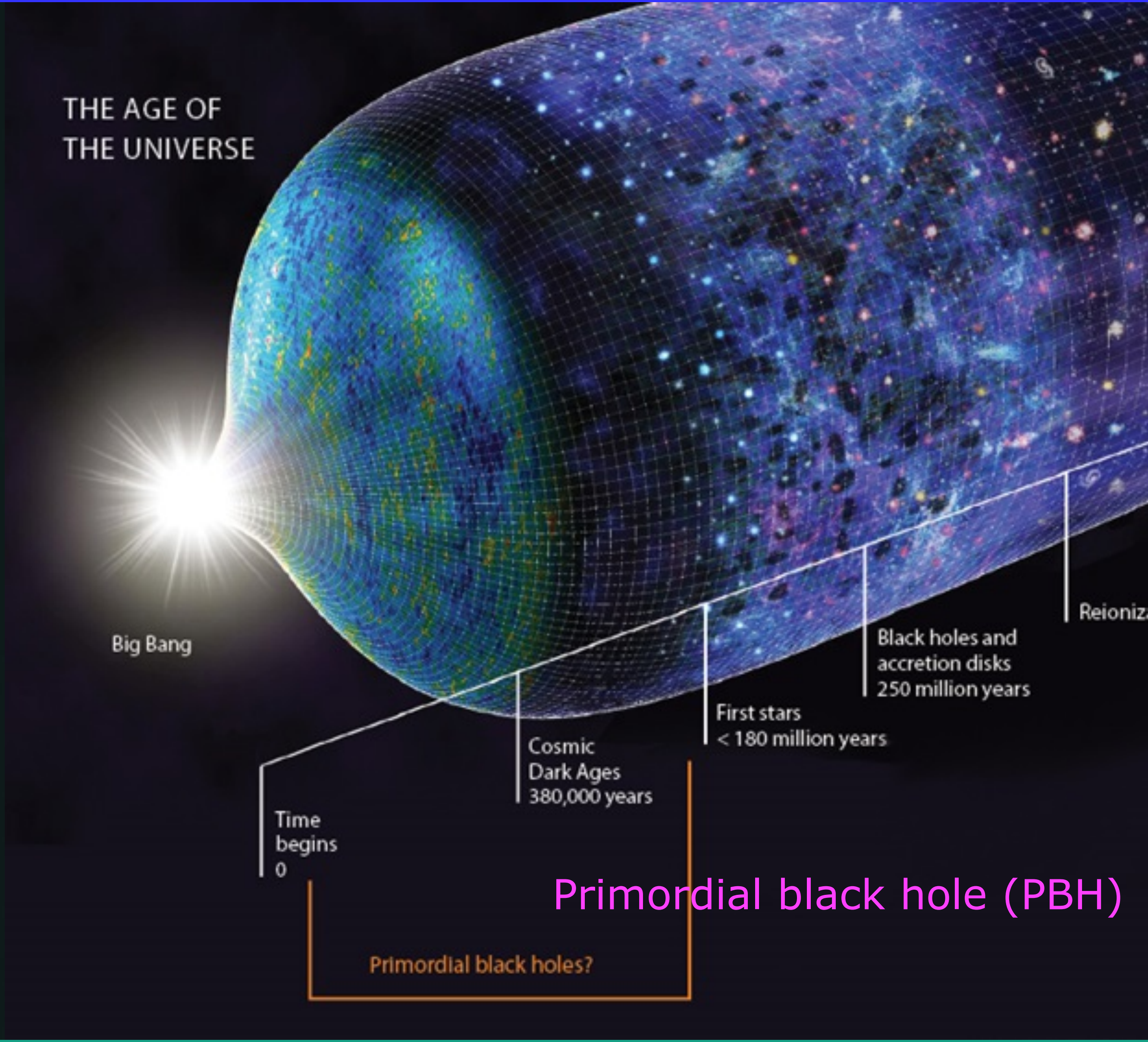
What is the origin of black hole (BH)?

B. P. Abbott *et al.* , Phys. Rev. X **9**, 031040 (2019)

Motivation

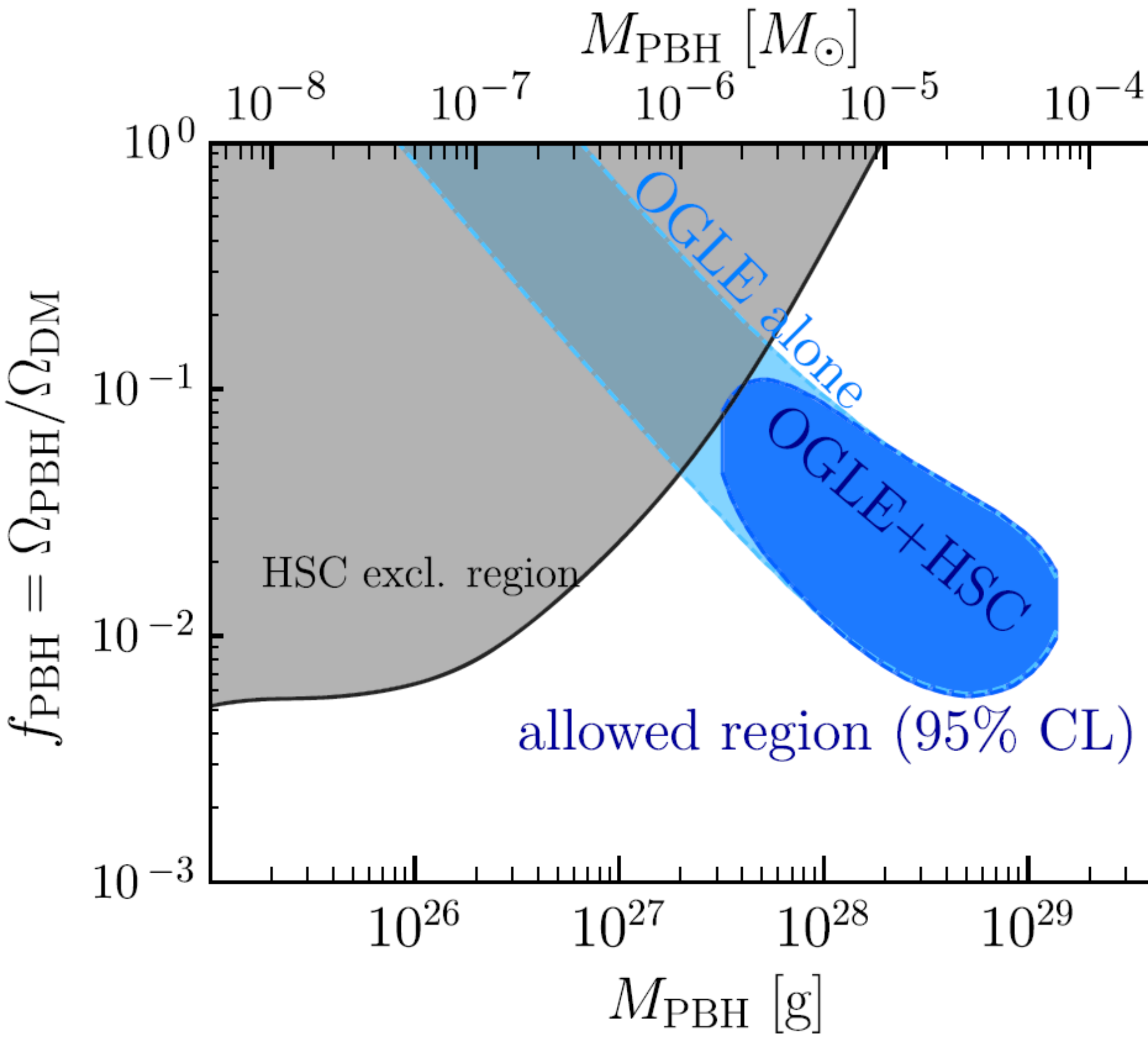
Formation of black hole





Primordial black hole (PBH)

Ultrashort-timescale microlensing events in the OGLE data

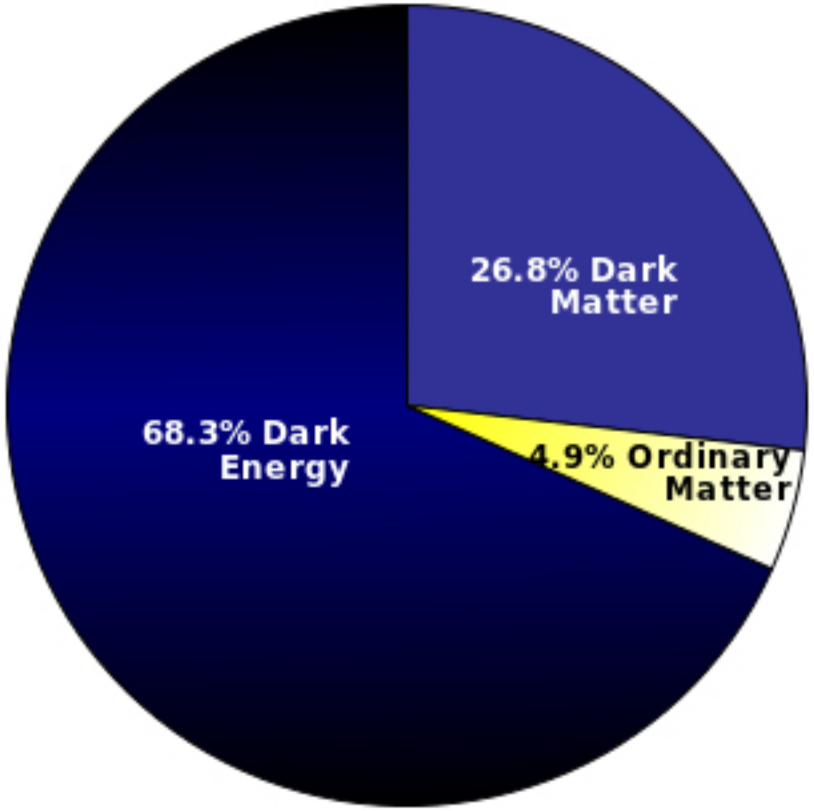


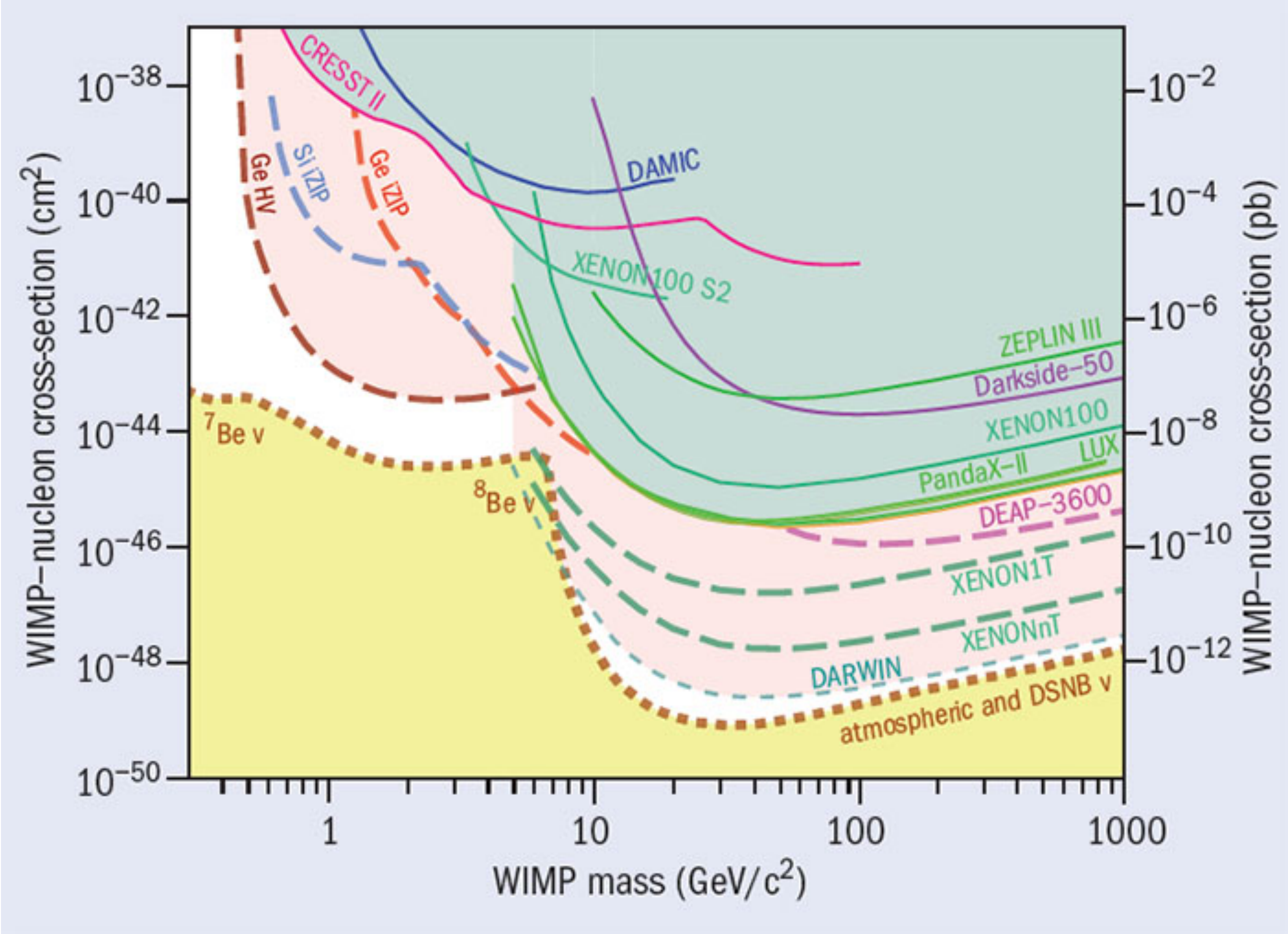
Shaded blue region is the 95% CL allowed region of PBH abundance, obtained by assuming that 6 ultrashort-timescale microlensing events in the OGLE data are due to PBHs

[1] P. Mróz *et al.* , Nature (London) **548**, 183 (2017) [2] H. Niikura *et al.* , Phys. Rev. D **99**, 083503 (2019)

Motivation

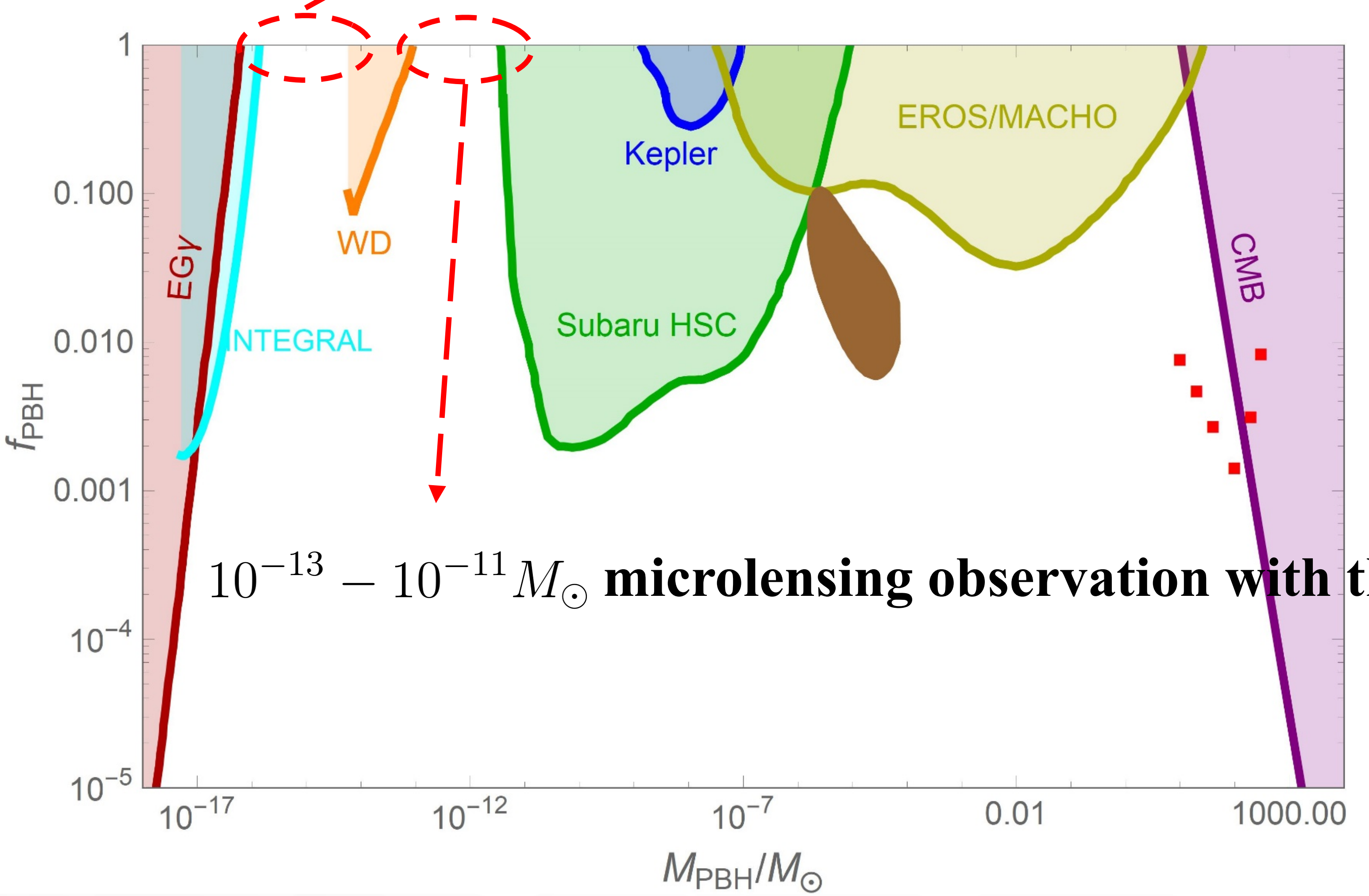
PBH is a possible candidate of dark matter





The constraints on the fraction of PBH dark matter

$10^{-16} - 10^{-14} M_\odot$ **gravitational femtolensing of gamma-ray bursts [1]**



$10^{-13} - 10^{-11} M_\odot$ **microlensing observation with the Subaru HSC [2]**

[1] A. Barnacka *et al.* ,Phys. Rev. D **86**, 043001 (2012); A. Katz *et al.* , JCAP 12 (2018) 005 [2] H. Niikura *et al.* , Nat. Astron. **3**, 524 (2019)

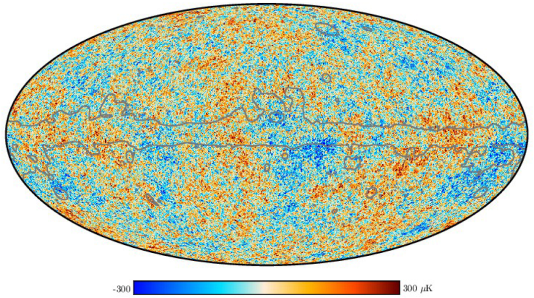
Motivation

Seed for PBHs: Primordial curvature perturbations

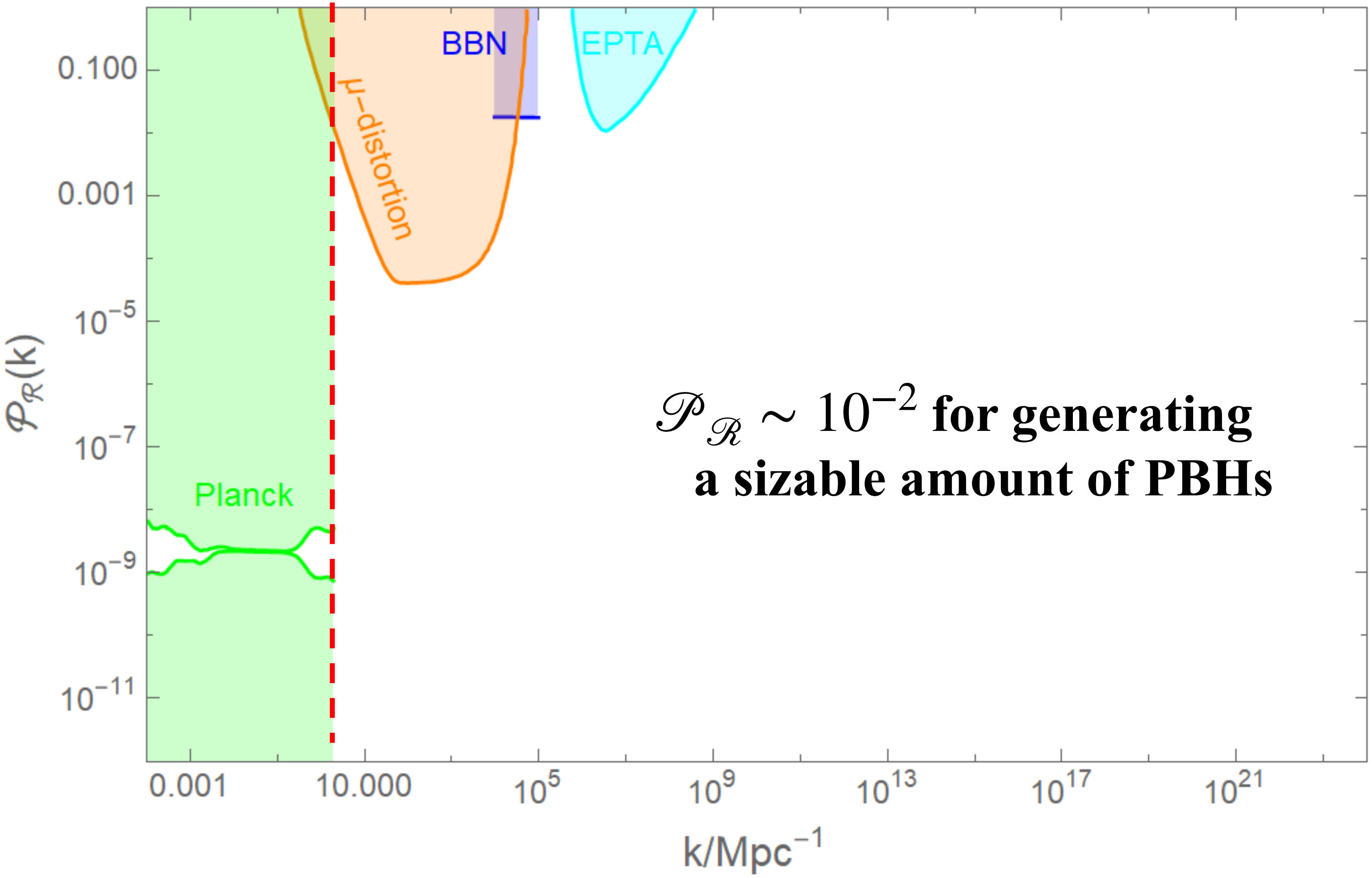
large scales

$k \lesssim 1\mathrm{Mpc}^{-1}$

$\mathcal{P}_{\mathcal{R}} \sim 10^{-9}$



small scales $k \gtrsim 1\mathrm{Mpc}^{-1}$



$\mathscr{P}_{\mathscr{R}} \sim 10^{-2}$ **for generating a sizable amount of PBHs**

Y. Akrami *et al.* , 1807.06211; D. J. Fixsen *et al.* , Astrophys. J. **473**, 576 (1996); K. Inomata *et al.* , PRD **94**, 043527 (2016); **99**, 043511 (2019)

Motivation

How to amplify the amplitude of power spectrum: Flatten potential

Simple single-field inflation model

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]$$

$$\kappa^2 = \frac{1}{M_{\rm pl}^2} = 8\pi G$$

Slow-roll approximation

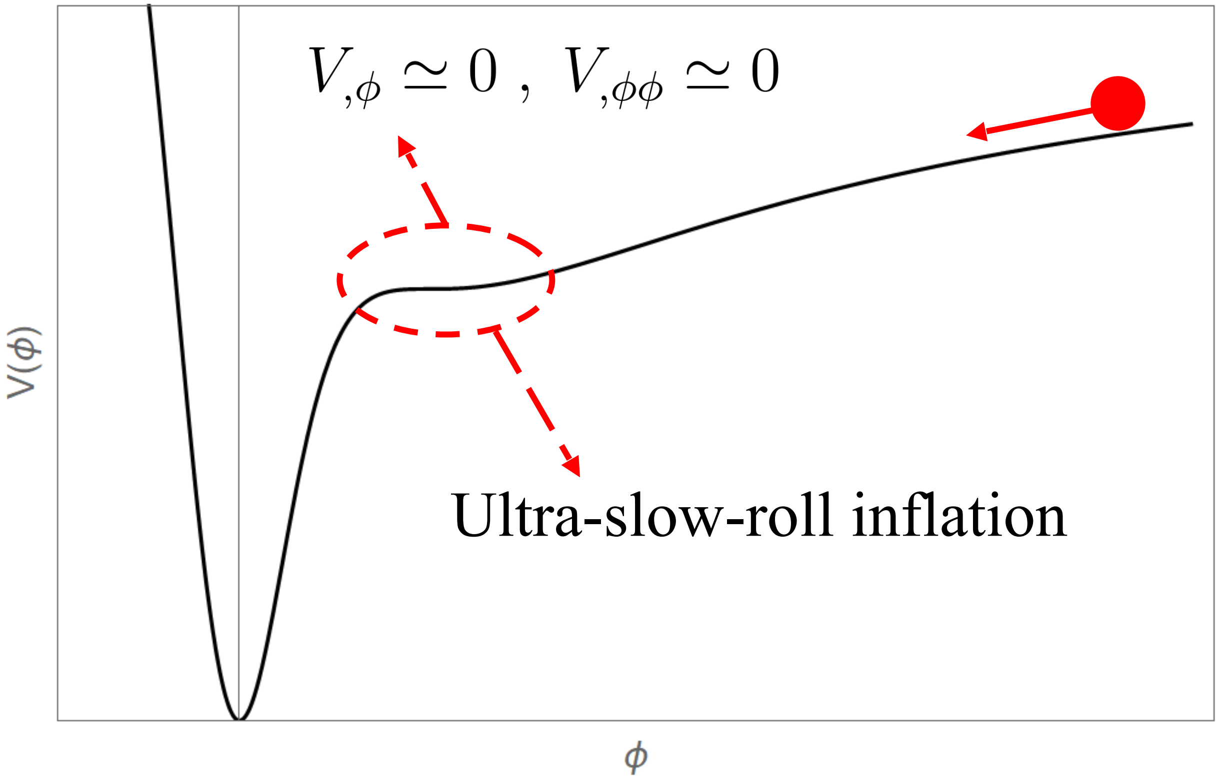
$$3H^2 \simeq \kappa^2 V(\phi) \qquad 3H\dot{\phi} \simeq -V_{,\phi}$$

Power spectrum

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{8\pi^2}\left(\frac{H}{M_{\rm pl}}\right)^2\frac{1}{\epsilon} \qquad \left(\epsilon \equiv -\frac{\dot{H}}{H^2}\right)$$

$$\epsilon \simeq \epsilon_V \quad \left(\epsilon_V = \frac{\kappa^2}{2}\frac{\dot{\phi}^2}{H^2} = \frac{1}{2\kappa^2}\left(\frac{V_{,\phi}}{V}\right)^2\right)$$

Near-inflection point



C. Germani and T. Prokopec, Phys. Dark Universe **18,** 6 (2017); Di and Gong, JCAP 07 (2018) 007.

Motivation

How to amplify the amplitude of power spectrum: Increase friction

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V(\phi)_{,\phi} = 0}$$

ultra-slow-roll inflation

flatten potential

increase friction

mechanism of gravitationally enhanced friction

nonminimal derivative coupling (NDC)

Outline

Basic equations

The action $$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}\left(g^{\mu\nu} - \kappa^2\xi G^{\mu\nu}\right)\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right]$$

$$\xi \equiv \theta(\phi) \qquad \Downarrow \qquad ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$$

Friedmann equation (FE)

$$3H^2 = \kappa^2\left[\frac{1}{2}\left(1 + 9\kappa^2\theta(\phi)H^2\right)\dot{\phi}^2 + V(\phi)\right]$$

Equation of motion for inflaton (EoM)

$$\left(1 + 3\kappa^2\theta(\phi)H^2\right)\ddot{\phi} + \left[1 + \kappa^2\theta(\phi)\left(2\dot{H} + 3H^2\right)\right]3H\dot{\phi} + \frac{3}{2}\kappa^2\theta_{,\phi}H^2\dot{\phi}^2 + V_{,\phi} = 0$$

Slow-roll inflation

Slow-roll parameters $\epsilon \equiv -\frac{\dot{H}}{H^2}$ $\delta_\phi \equiv \frac{\ddot{\phi}}{H\dot{\phi}}$ $\delta_X \equiv \frac{\kappa^2\dot{\phi}^2}{2H^2}$ $\delta_D \equiv \frac{\kappa^4\theta\dot{\phi}^2}{4}$

Slow-roll conditions $\{\epsilon, |\delta_\phi|, \delta_X, \delta_D\} \ll 1$

Auxilliary condition $|\kappa^2\theta_{,\phi}H\dot{\phi}| \ll \boxed{\mathcal{A} \equiv 1 + 3\kappa^2\theta(\phi)H^2}$ **for simplicity of calculation**

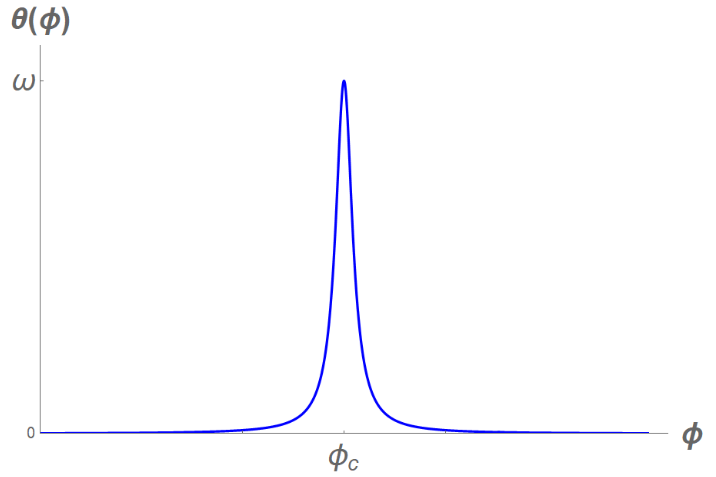
Approximate FE and EoM $3H^2 \simeq \kappa^2 V(\phi)$ $3H\mathcal{A}\dot{\phi} + V_{,\phi} \simeq 0$

$$\epsilon \simeq \frac{\epsilon_V}{\mathcal{A}} \quad \Rightarrow$$

If $\mathcal{A} \simeq 1$, $\epsilon \simeq \epsilon_{\rm V}$

If $\mathcal{A} \gg 1$, $\epsilon \ll \epsilon_{\rm V}$

How to achieve a large-amplitude curvature perturbations?



Consider the following special functional form

$$\boxed{\theta(\phi) = \frac{\omega}{\sqrt{\kappa^2 \left(\frac{\phi-\phi_c}{\sigma}\right)^2 + 1}}}$$

and a simple monomial potential which satisfies the CMB constraint

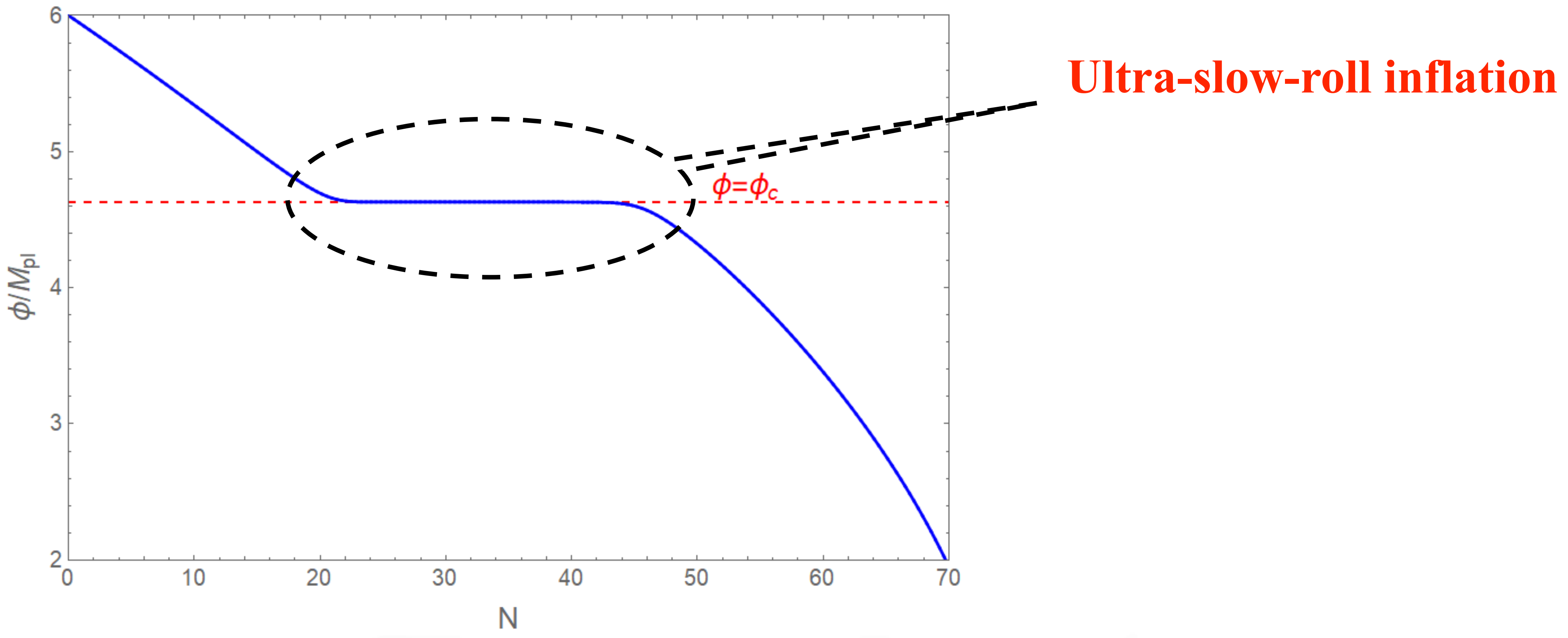
$$V(\phi) = \lambda M_{\rm pl}^{4-p} |\phi|^p \quad (p = 2/5)$$

Enhanced Power spectrum

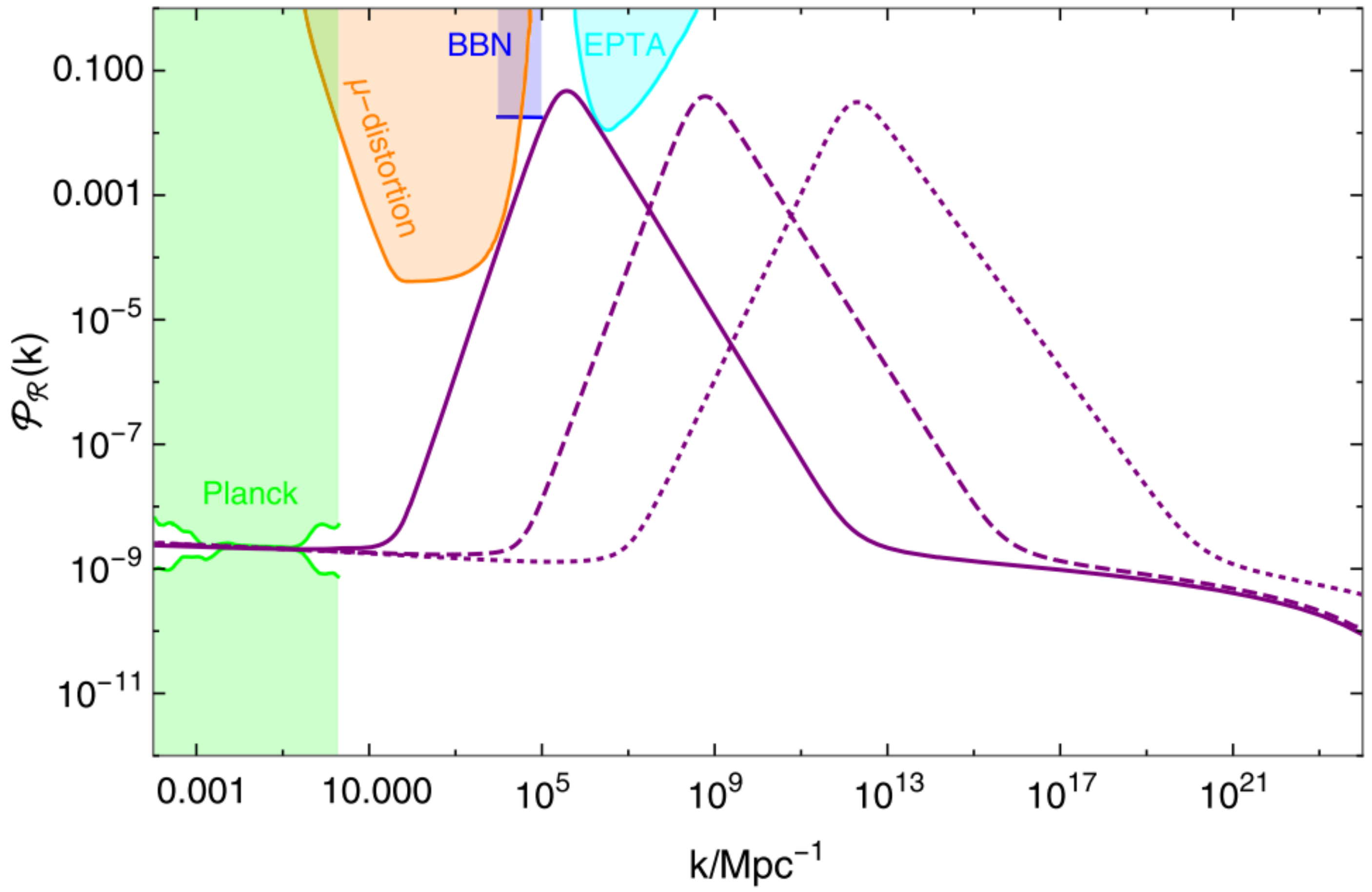
$$\mathscr{P}_{\mathscr{R}} \simeq \frac{\lambda}{12\pi^2 p^2} \left| \frac{\phi}{M_{\rm pl}} \right|^{2+p} \boxed{\cdot A}$$

Inflationary dynamics

Concrete case $\phi_c = 4.63 M_{\rm pl}\ ,\quad \sigma = 2.6 \times 10^{-9}\ ,\ \omega\lambda = 1.33 \times 10^{7}$



The enhanced power spectrum

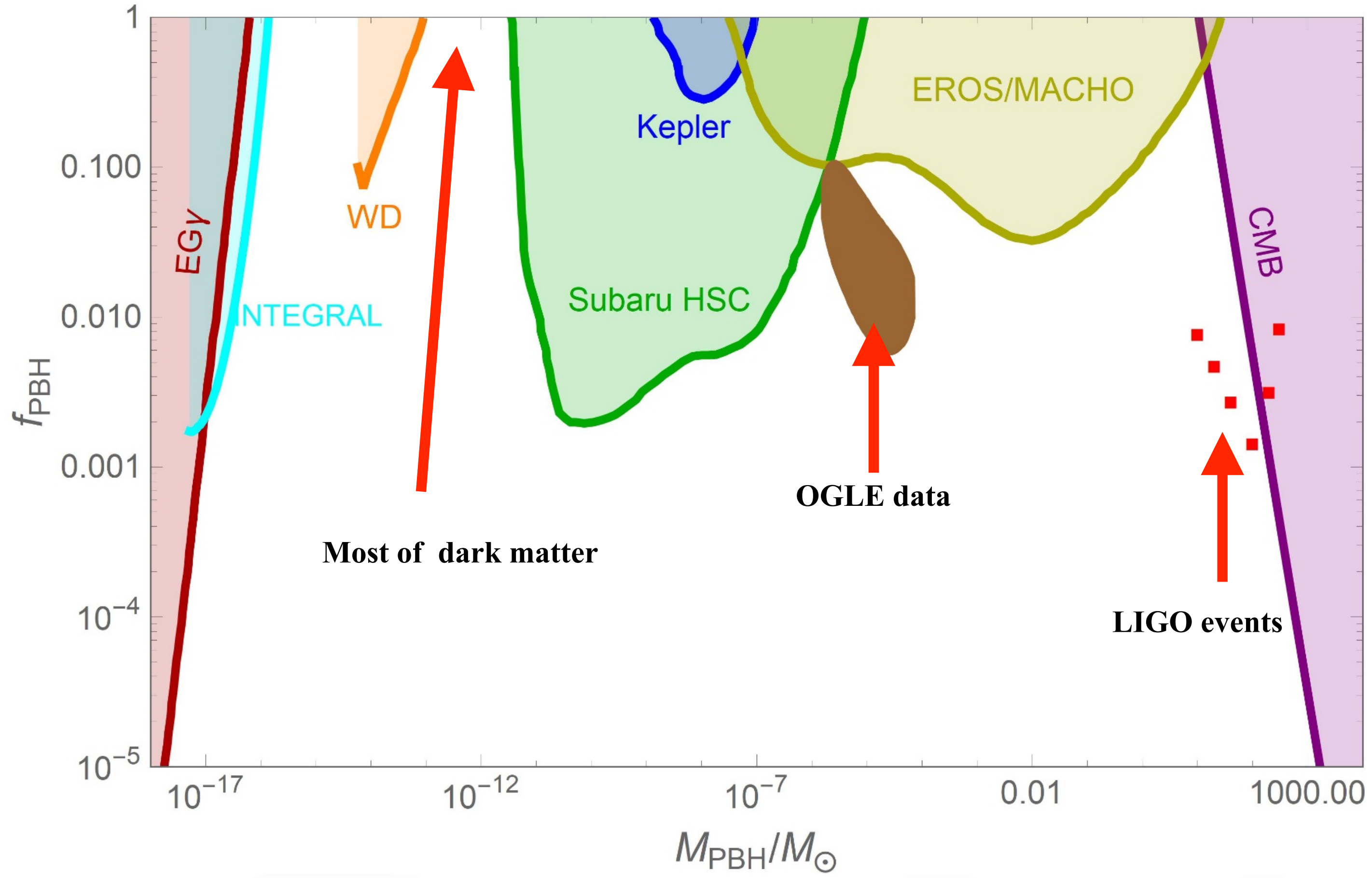


$\phi_c/M_{\rm pl}$	$\omega\lambda$	σ
4.63	1.33×10^7	2.6×10^{-9}
3.9	1.53×10^7	3×10^{-9}
3.3	1.978×10^7	3.4×10^{-9}

Outline

PBH

Can PBH explain the LIGO events, the ultrashort- timescale microlensing events in OGLE data, and the most of dark matter?



We need to investigate the fraction of PBH dark matter in different mass regions.

Basic formulas for PBH formation during radiation-dominated era

Under the assumption that the probability distribution function of perturbations is Gaussian, the production rate of PBHs with mass M based on the Press-Schechter theory is[1]

$$\beta(M) = \int_{\delta_c} \frac{d\delta}{\sqrt{2\pi\sigma^2(M)}} e^{-\frac{\delta^2}{2\sigma^2(M)}} = \frac{1}{2}\mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2\sigma^2(M)}}\right)$$

where δ_c ($\simeq 0.4$ [2]) is the threshold of the density perturbations for the PBH formation

The mass M of formed PBHs is related to the horizon mass at the horizon entry of the perturbations with the comoving wave number k

$$M(k) = \gamma \frac{4\pi}{\kappa^2 H}\bigg|_{k=aH} \simeq M_\odot \left(\frac{\gamma}{0.2}\right)\left(\frac{g_*}{10.75}\right)^{-\frac{1}{6}}\left(\frac{k}{1.9\times 10^6\ \mathrm{Mpc}^{-1}}\right)^{-2} \qquad (g_* \simeq 106.75)$$

where γ ($\simeq 0.2$)[3] is the ratio of the PBH mass to the horizon mass and indicates the efficiency of collapse

[1] S. Young *et al.* , JCAP 07 (2014) 045 [2] T. Harada *et al.* , PRD **88**, 084051 (2013); [3] B. J. Carr, APJ **201**, 1 (1975)

Basic formulas for PBH formation during radiation-dominated era

$\sigma^2(M)$ represents the variance of coarse-grained density contrast for the PBH mass M [1]

$$\sigma^2(M(k)) = \int d\ln q \; W^2(qk^{-1}) \frac{16}{81} (qk^{-1})^4 \mathcal{P}_{\mathcal{R}}(q)$$

where W is the window function, which is taken to be the Gaussian function $W(x) = e^{-x^2/2}$

The abundance of PBHs with mass M over logarithmic mass interval is estimated as

$$f_{\mathrm{PBH}}(M) \equiv \frac{1}{\Omega_{\mathrm{DM}}} \frac{d\Omega_{\mathrm{PBH}}}{d\ln M} \simeq \frac{\beta(M)}{1.84\times10^{-8}} \left(\frac{\gamma}{0.2}\right)^{\frac{3}{2}} \left(\frac{10.75}{g_*}\right)^{\frac{1}{4}} \left(\frac{0.12}{\Omega_{\mathrm{DM}}h^2}\right) \left(\frac{M}{M_\odot}\right)^{-\frac{1}{2}}$$

The fraction of PBHs

$$\frac{\Omega_{\mathrm{PBH}}}{\Omega_{\mathrm{DM}}} = \int \frac{dM}{M} f_{\mathrm{PBH}}(M) \qquad (\Omega_{\mathrm{DM}}h^2 \simeq 0.12^{[2]})$$

[1] S. Young *et al.* , JCAP 07 (2014) 045 [2] N. Aghanim *et al.* , 1807 .06209

Three parameter sets for three interesting masses of PBHs

- ➢ **stellar-mass** ($\sim \mathcal{O}(10) M_\odot$) **PBHs: LIGO/Virgo GW events**
- ➢ **earth-mass** ($\sim \mathcal{O}(10^{-5}) M_\odot$) **PBHs: OGLE microlensing events**
- ➢ **asteroid-mass** ($\sim \mathcal{O}(10^{-12}) M_\odot$) **PBHs: most of dark matter**

TABLE I: The successful parameter sets for producing the PBHs with mass around $\mathcal{O}(10)M_\odot$ (*Case 1*), $\mathcal{O}(10^{-5})M_\odot$ (*Case 2*) and $\mathcal{O}(10^{-12})M_\odot$ (*Case 3*).

#	$\phi_c/M_{\rm pl}$	$\omega\lambda$	σ
Case 1	4.63	1.33×10^7	2.6×10^{-9}
Case 2	3.9	1.53×10^7	3×10^{-9}
Case 3	3.3	1.978×10^7	3.4×10^{-9}

Observational constraints

Scalar spectral index and the tensor-to-scalar ratio

$$n_s \simeq 1 - \frac{1}{\mathcal{A}}\left[2\epsilon_V\left(4 - \frac{1}{\mathcal{A}}\right) - 2\eta_V\right] \qquad r \simeq \frac{16\epsilon_V}{\mathcal{A}} \qquad \left(\eta_V \equiv \frac{M_{\rm pl}^2}{V}\frac{d^2V}{d\phi^2}\right)$$

#	N_*	λ	n_s	r
Case 1	60	7.09×10^{-10}	0.9666	0.0431
Case 2	60	8.23×10^{-10}	0.9618	0.0497
Case 3	65	8.52×10^{-10}	0.9607	0.0512

Planck 2018 results [1] $(k_* = 0.05\mathrm{Mpc}^{-1})$

$$\ln(10^{10}\mathcal{P}_{\mathcal{R}}) = 3.044 \pm 0.014 \qquad (68\%\ \text{C.L.})$$

$$n_s = 0.9649 \pm 0.0042 \qquad (68\%\ \text{C.L.})$$

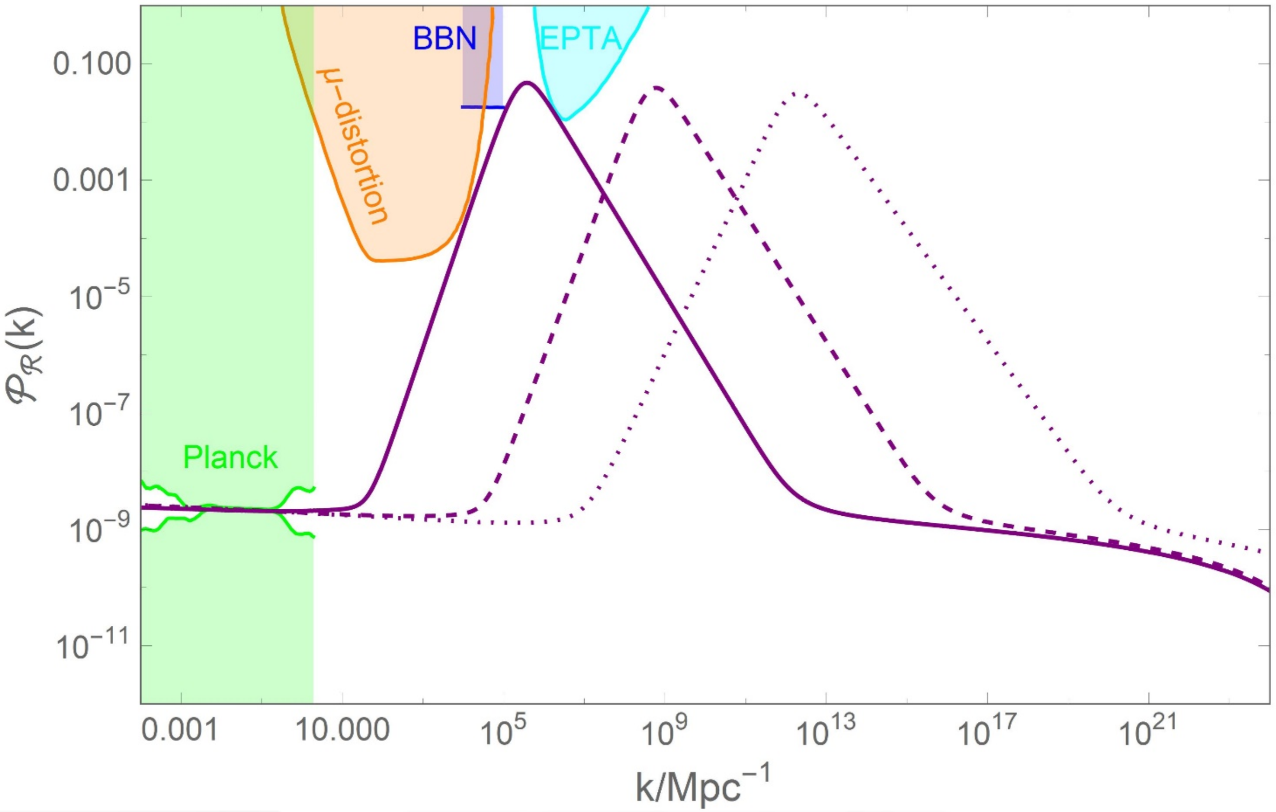
$$r < 0.07 \qquad (95\%\ \text{C.L.})$$

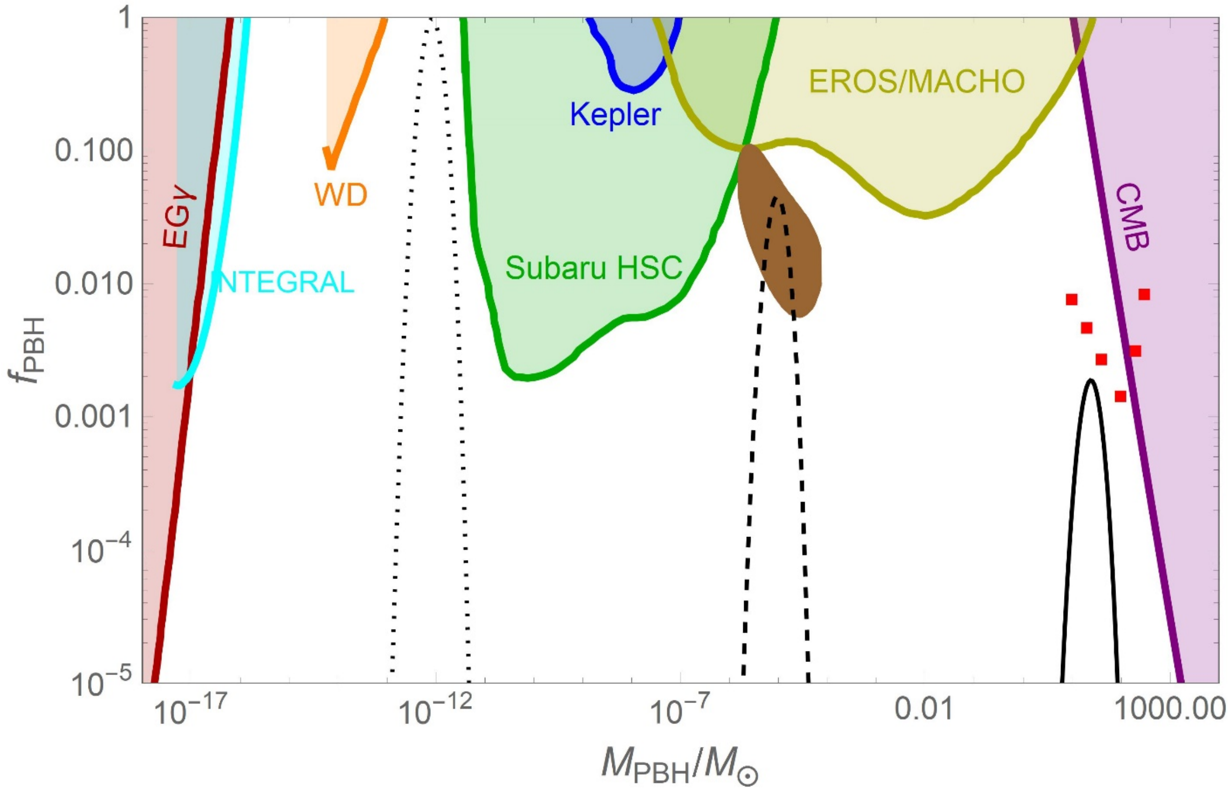
[1] Y. Akrami *et al.* , 1807.06211

Power spectra of curvature perturbations and mass spectra of PBHs

case 1 — solid line **case 2 — dashed line**

case 3 — dotted line

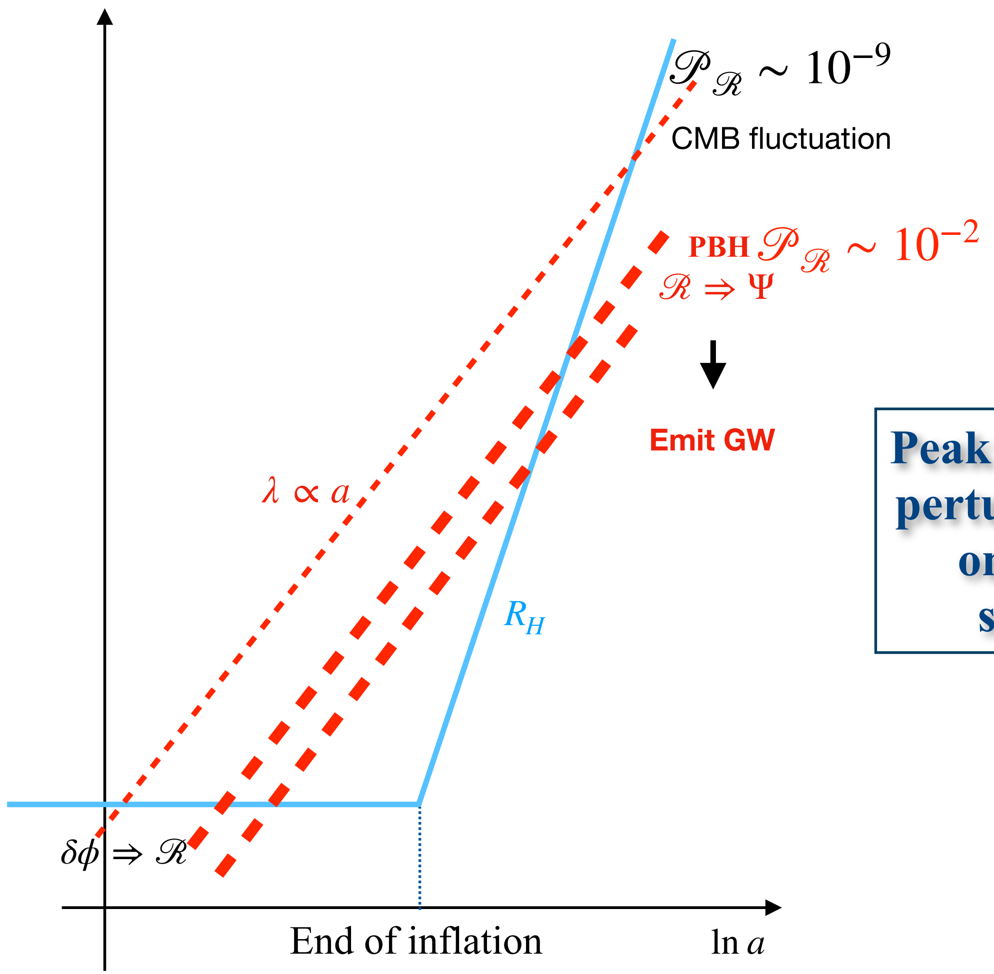




$M_{\mathrm{PBH}}^{\mathrm{peak}}/M_{\odot}$	$f_{\mathrm{PBH}}^{\mathrm{peak}}$	$\Omega_{\mathrm{PBH}}/\Omega_{\mathrm{DM}}$

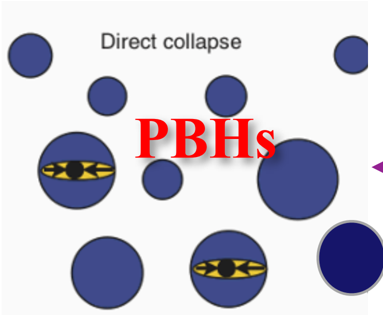
Outline

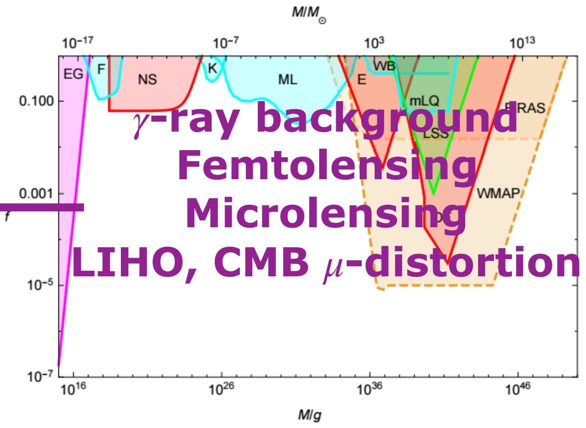
Scalar induced gravitational waves (SIGWs)



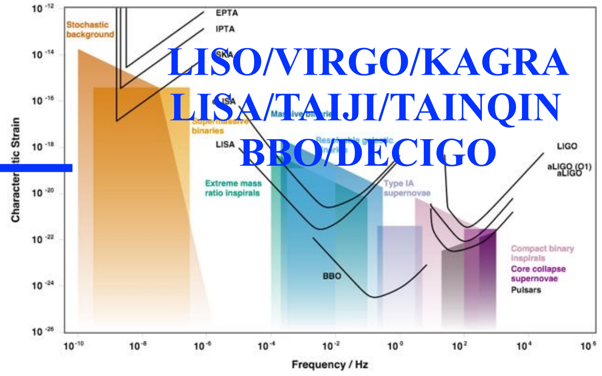
Peak of Scalar perturbations on small scales

Peak theory





Secondary coupling



Formalism of SIGWs

In the conformal Newtonian gauge, the perturbed FRW metric can be written as

$$ds^2 = a(\eta)^2\left\{-(1+2\Psi)d\eta^2 + \left[(1-2\Psi)\delta_{ij} + \frac{h_{ij}}{2}\right]dx^i dx^j\right\}$$

where $\eta \equiv \int a^{-1}dt$ is the conformal time, Ψ is the first-order scalar perturbation, and h_{ij} is the second-order transverse-traceless tensor perturbation

The equation of motion for second-order h_{ij} is given by

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\mathcal{T}_{ij}^{lm} S_{lm}$$

where $\mathcal{T}_{ij}^{lm}$ is the transverse-traceless projection operator and the source term has the form [1]

$$S_{ij}^{(2)} = 4\Psi\partial_i\partial_j\Psi + 2\partial_i\Psi\partial_j\Psi - \frac{1}{\mathcal{H}^2}\partial_i(\mathcal{H}\Psi + \Psi')\partial_j(\mathcal{H}\Psi + \Psi')$$

[1] K. N. Ananda *et al.* , Phys. Rev. D **75**, 123518 (2007)

Formalism of SIGWs

In the radiation-dominated era, the density parameter spectrum of GWs Ω_{GW} at η_c, which represents the time when Ω_{GW} stops growing, can be evaluated as [1]

$$\Omega_{\mathrm{GW}}(\eta_c,k)=\frac{1}{12}\int_0^\infty dv\int_{|1-v|}^{|1+v|}du\left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2\underline{\mathcal{P}_{\mathcal{R}}(ku)\mathcal{P}_{\mathcal{R}}(kv)}$$
$$\left(\frac{3}{4u^3v^3}\right)^2(u^2+v^2-3)^2$$
$$\left\{\left[-4uv+(u^2+v^2-3)\ln\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right]^2+\pi^2(u^2+v^2-3)^2\Theta(v+u-\sqrt{3})\right\}$$

The current energy parameter and frequency of GWs are given, respectively, by

$$\Omega_{\mathrm{GW},0}=0.83\left(\frac{g_c}{10.75}\right)^{-1/3}\Omega_{\mathrm{r},0}\Omega_{\mathrm{GW}}(\eta_c,k)\qquad f=1.546\times10^{-15}\frac{k}{1\mathrm{Mpc}^{-1}}\mathrm{Hz}$$

$$\boxed{(\Omega_{r,0}h^2\simeq4.2\times10^{-5}\ ,\ g_c\simeq106.75)}$$

[1] K. Kohri and T. Terada, Phys. Rev. D **97**, 123532 (2018)

Approximate spectra index of power spectrum

Through analytical calculations, we found that the power spectrum can be expressed approximately as the power law form

$$\mathcal{P}_{\mathcal{R}} \propto \begin{cases} k^{n_s^{(1)}} , & (k < k_p) \\ k^{n_s^{(2)}} , & (k > k_p) \end{cases}$$

where

$$n_s^{(1)} \simeq 3\left(1 - \sqrt{1 - \frac{4}{15}(\kappa\phi_c)^{-7/5}(\sigma\omega\lambda)^{-1}}\right)$$

$$n_s^{(2)} \simeq 3\left(1 - \sqrt{1 + \frac{4}{15}(\kappa\phi_c)^{-7/5}(\sigma\omega\lambda)^{-1}}\right)$$

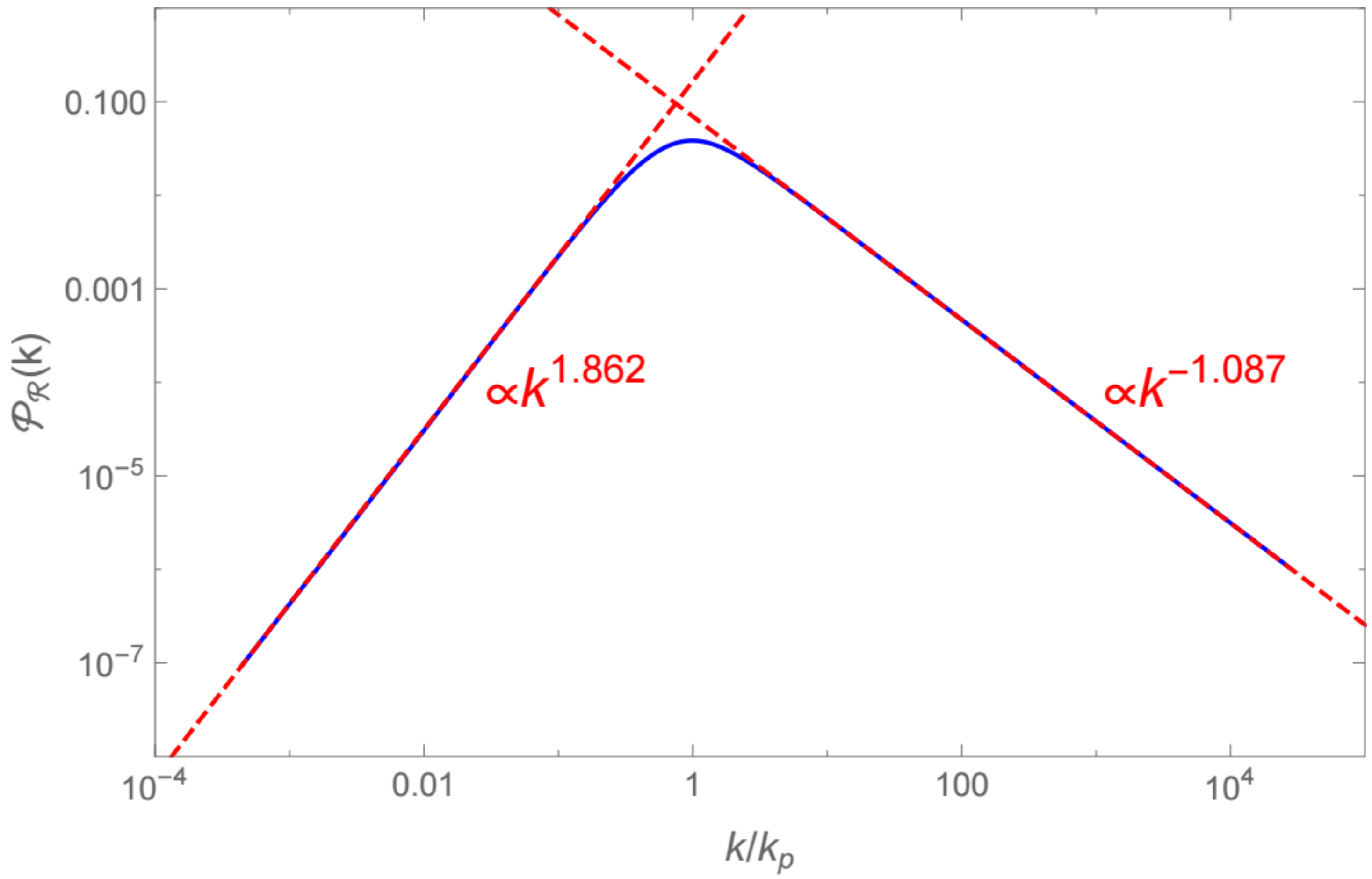
Approximate power-law power spectrum: an example

Earth-mass PBHs: $\phi_c = 3.82 M_{\rm pl}\ ,\ \sigma = 3\times 10^9\ ,\ \omega\lambda = 1.59\times 10^7$

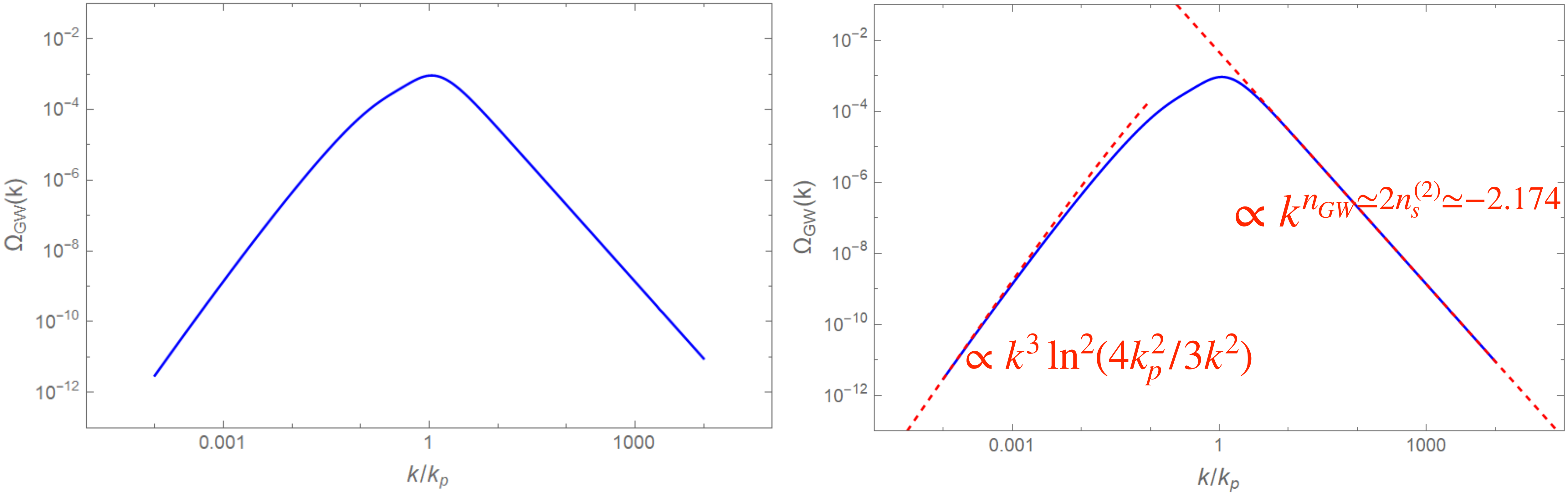
Analytic results

$$n_s^{(1)} \simeq 1.862$$

$$n_s^{(2)} \simeq -1.087$$



Density spectrum and scaling of SIGWs



The density spectrum (left panel) and the scaling (right one) of scalar induced GWs as a function of k

Scaling of SIGWs

In the ultraviolet regions $(k > k_p)$, if $\mathcal{P}_{\mathcal{R}} \propto k^{n_s}$ with $n_s > -4$, the density spectrum of SIGWs is approximated by a power-law function of k [1]

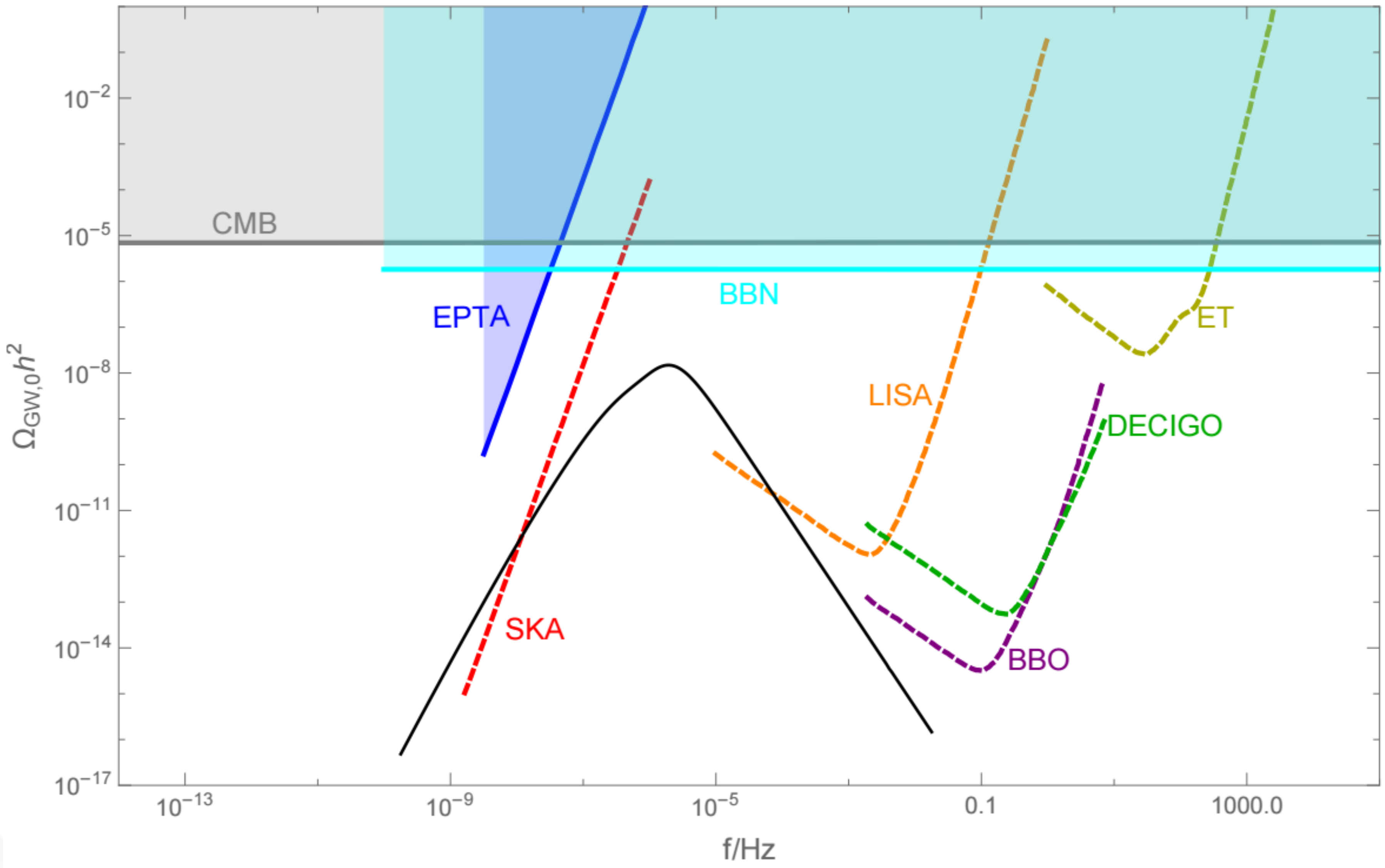
$$\Omega_{\mathrm{GW}}(k) \propto k^{n_{\mathrm{GW}}} \qquad n_{\mathrm{GW}} \simeq 2n_s$$

In the infrared regions $(k < k_p)$, the density spectrum of SIGWs has a log-dependent slope[2]

$$\Omega_{\mathrm{GW}}(k) \propto \left(\frac{k}{k_p}\right)^3 \ln^2\left(\frac{4k_p^2}{3k^2}\right) \qquad n_{\mathrm{GW}} \simeq 3 - \frac{4}{\ln \frac{4k_p^2}{3k^2}}$$

[1] Guo, *et al.* , 1907.05213 [2] Huang, *et al.* , 1910.09099

Density parameter spectrum of SIGWs



Outline

01 Motivation

02 Enhanced curvature perturbations in nonminimal derivative coupling inflation

03 Fraction of PBH dark matter

04 Scalar induced Gravitational waves

05 Conclusions

Conclusions

- The enhancement of the curvature perturbations can be realized in the nonminimal derivative coupling model with a coupling parameter related to the inflaton field.
- The obtained power spectrum of curvature perturbations has an enough large peak on the small scales and on the large scales satisfies the current observational constraints.
- The power spectrum in the vicinity of the peak can be well approximated by a power-law function of comoving wave number.
- By tuning three parameters, we can easily obtain a sharp mass spectrum of primordial black holes around specific mass such as , $\mathcal{O}(10)M$ $\mathcal{O}(10^{-5})M_\odot$ $\mathcal{O}(10^{-12})M_\odot$, which can explain the LIGO events, the ultrashort-timescale microlensing events in OGLE data, and the most of DM, respectively.
- The GW signal produced by scalar metric perturbations will be detected by SKA and LISA. Log-dependent slope of SIGWs in the infrared regions is confirmed, while in the ultraviolet regions a power-law scaling is obtained.

END

THANK YOU FOR ATTENTION

The power spectrum: approximate and exact solution