# Applying Data Science to Cosmology and String Theory

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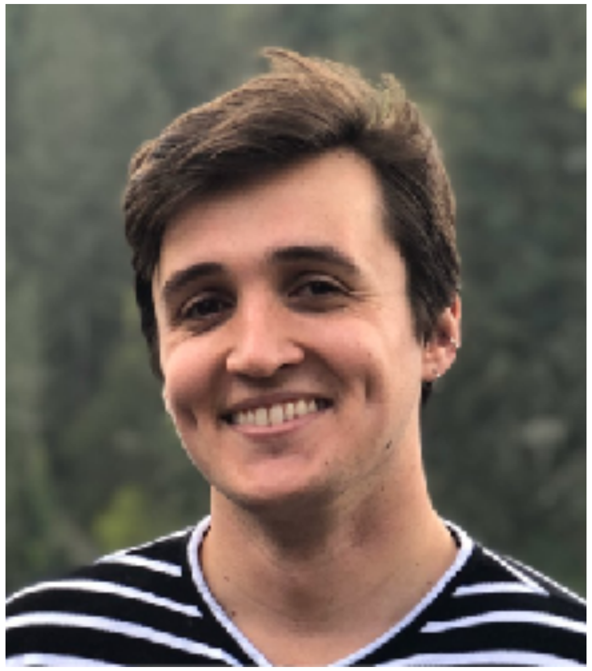
# Based on work with:

**Matteo Biagetti**



**Alex Cole**

**Andreas Schachner**

- “Persistent Homology and Non-Gaussianity”, A. Cole, GS, JCAP **1803**, 025 (2018) [arXiv:1712.08159 [astro-ph.CO]].
- “Topological Data Analysis for the String Landscape”, A. Cole, GS, JHEP **1903**, 054 (2019) [arXiv: 1812.06960 [hep-th]].
- “Searching the Landscape of Flux Vacua with Genetic Algorithms,” A. Cole, A. Schachner, GS, JHEP **1911**, 045 (2019) [arXiv:1907.10072 [hep-th]].
- “Persistent Homology and Large Scale Structure”, M. Biagetti, A. Cole, GS, in progress.

# Big Data in Big Sciences

**Cosmology** is marching into a big data era:

| **Experimental Data** | 2013 | 2020 | 2030+ |
| --- | --- | --- | --- |
| Storage | 1PB | 6PB | 100-1500PB |
| Cores | $10^3$ | 70K | 300+K |
| CPU hours | $3\text{x}10^6$ hrs | $2 \times 10^8$ hrs | $\sim 10^9$ hrs |
| **Simulations** | 2013 | 2020 | 2030+ |
| Storage | 1-10 PB | 10-100PB | $>$ 100PB - 1EB |
| Cores | 0.1-1M | 10-100M | $>$ 1G |
| CPU hours | 200M | >20G | $>$ 100G |

| | data volume | schedule |
| --- | --- | --- |
| SDSS | 40 TB | 2000-2020 |
| DESI | 2 PB | 2019-2027 |
| LSST | > 60 PB | 2020-2030 |
| Euclid | >10 PB | 2020-2027 |
| WFIRST | >2 PB | 2023-2030 |
| CMB-S4 | $10^4 \times$Planck | 2020-2027(?) |
| SKA | 4.6 EB | 2019-2030(?) |

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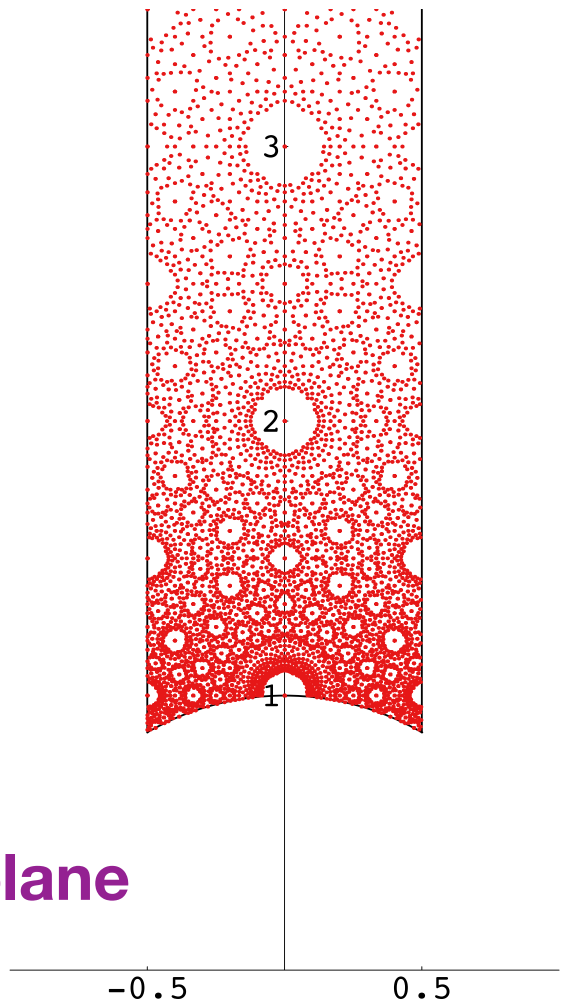
In terms of sheer volume, nothing trumps the volume of ***theoretical data of string vacua***. A rough estimate gives:

$$10^{500} \text{ (Type IIB flux vacua)} \qquad 10^{272,000} \text{ (F theory flux vacua)}$$

**[Ashok-Denef-Douglas]** **[Taylor-Wang]**

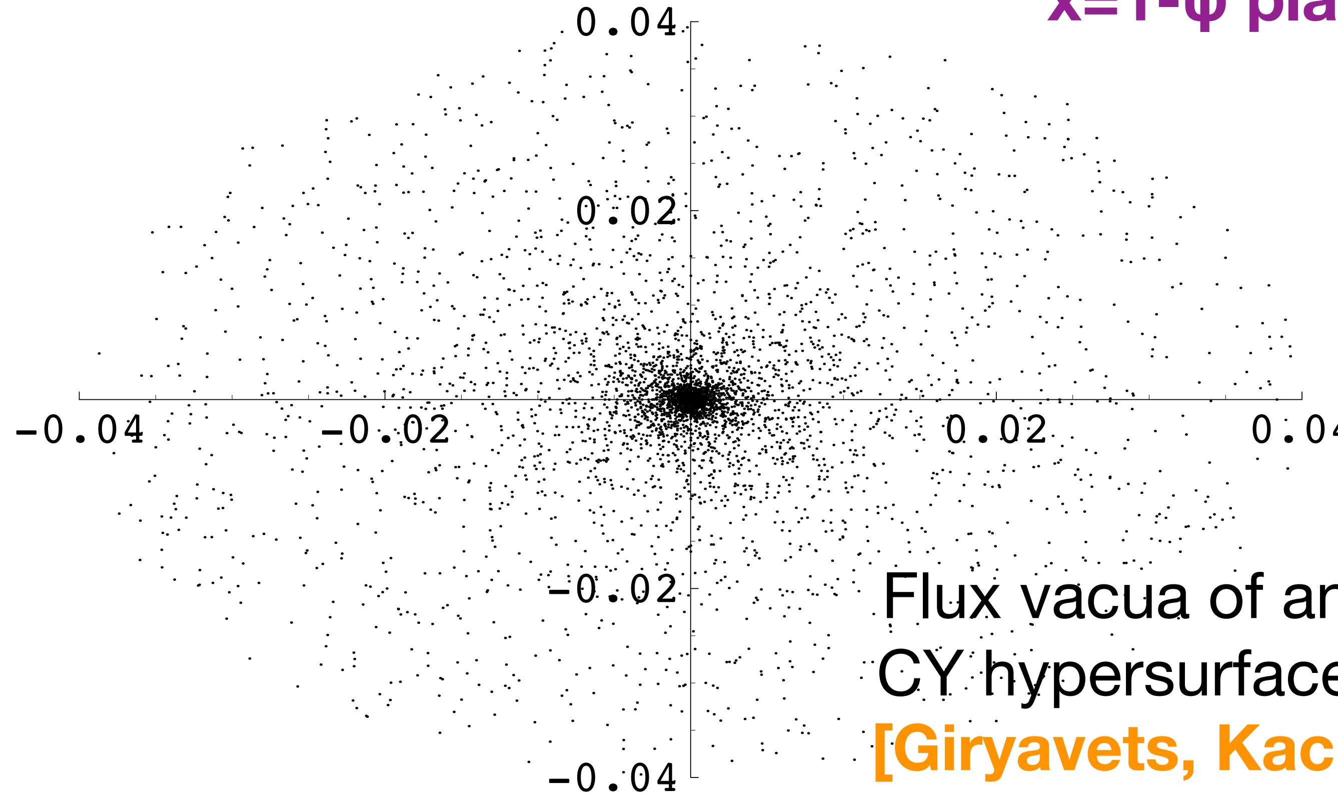
# Distribution of String Vacua

Flux vacua on rigid CY
**[Denef-Douglas]**

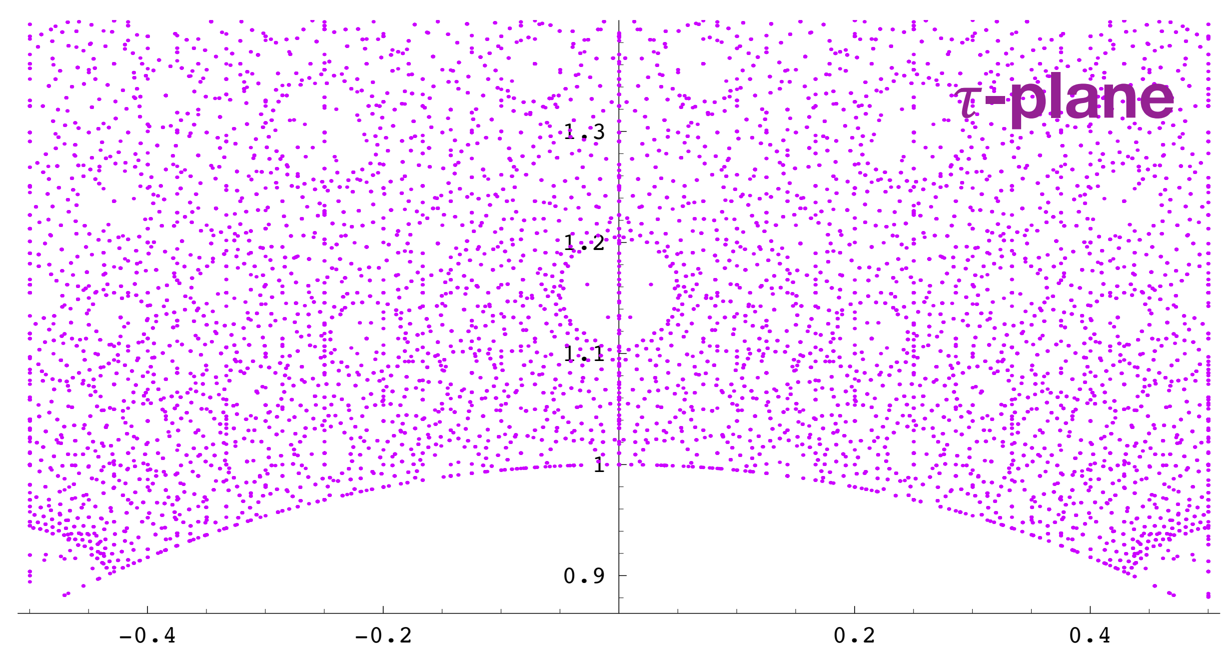



***τ*-plane**

**x=1-ψ plane**



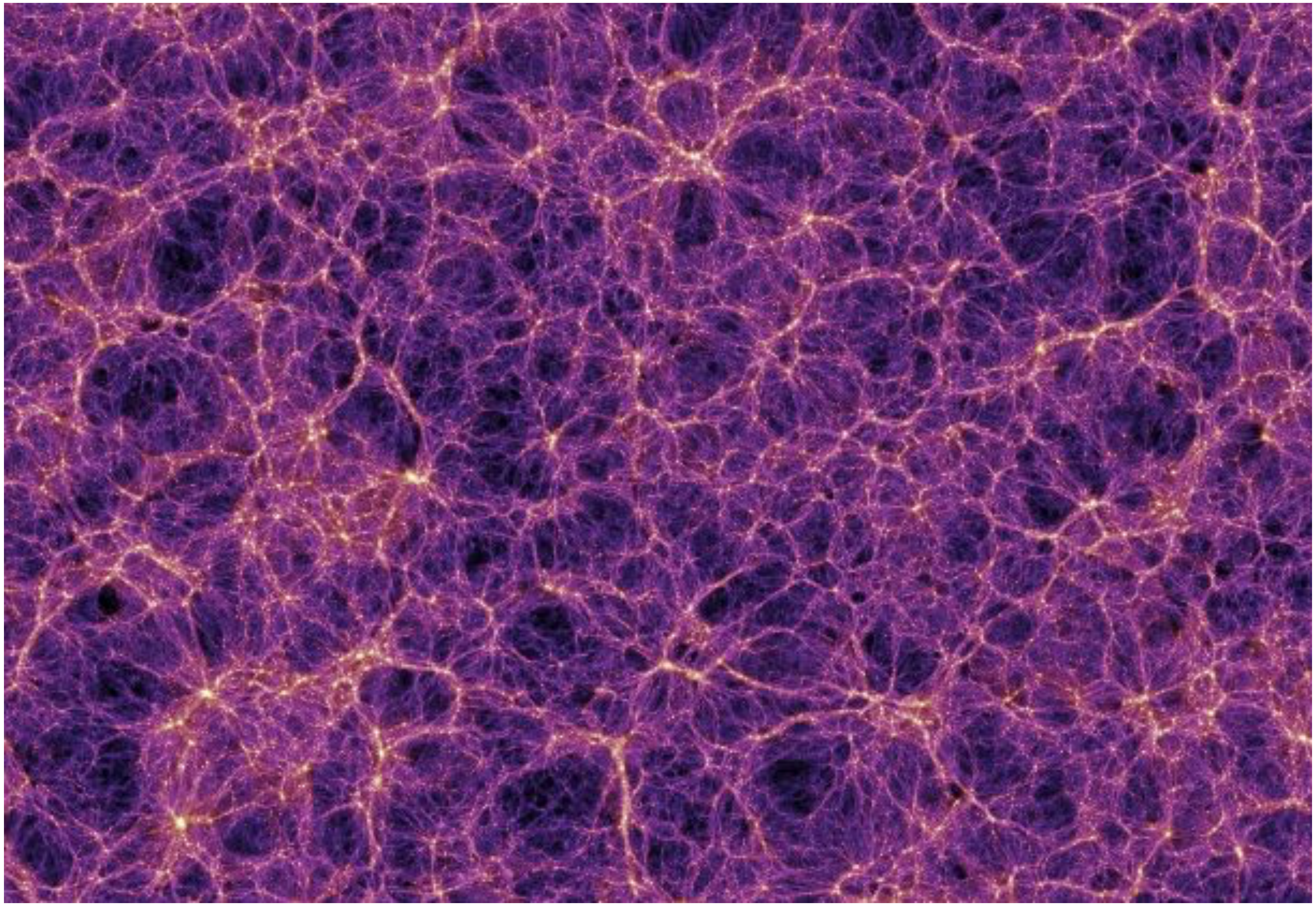

Flux vacua of an orientifold of CY hypersurface in $WP^4_{1,1,1,1,4}$
**[Giryavets, Kachru, Tripathy]**



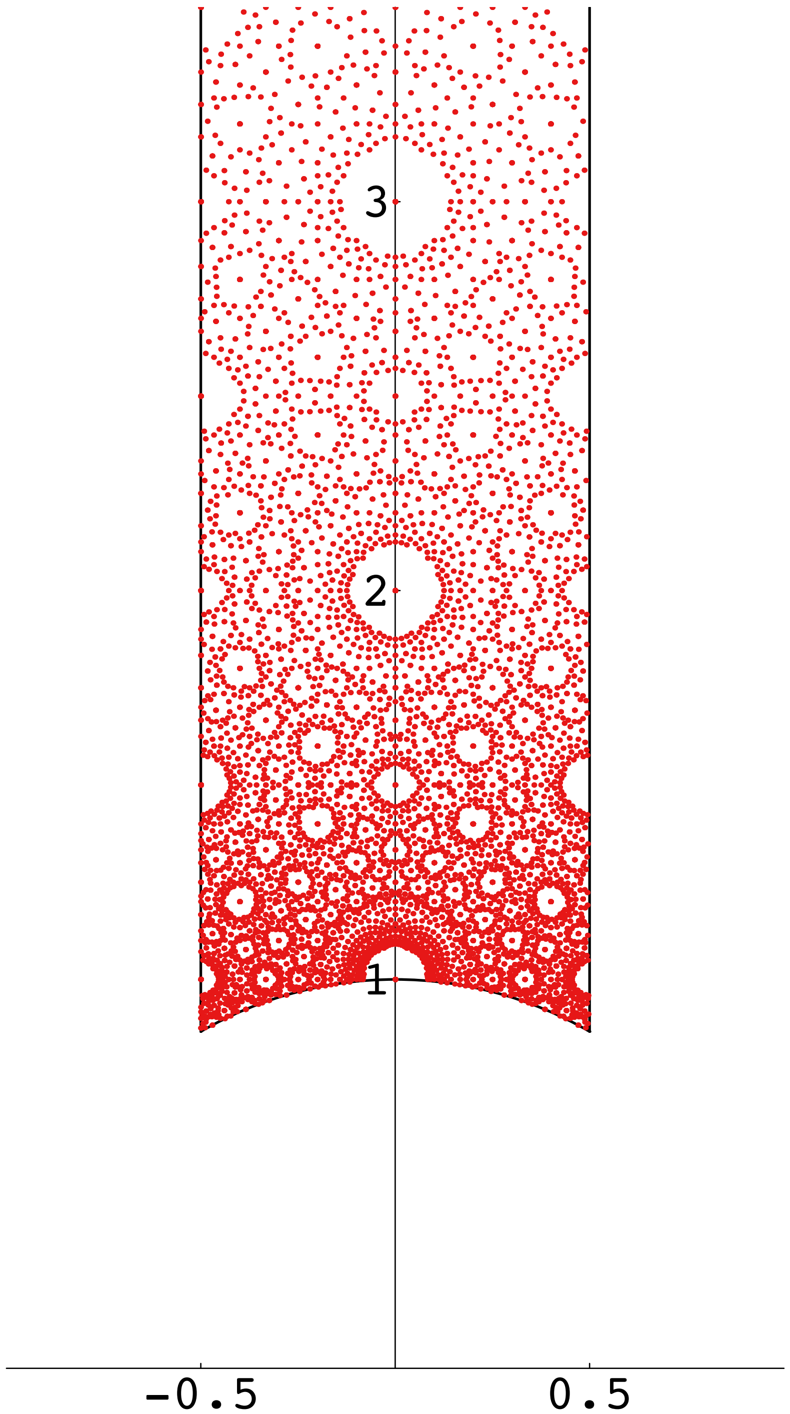

Toroidal Flux vacua with W=0
**[DeWolfe, Giryavets, Kachru, Taylor]**

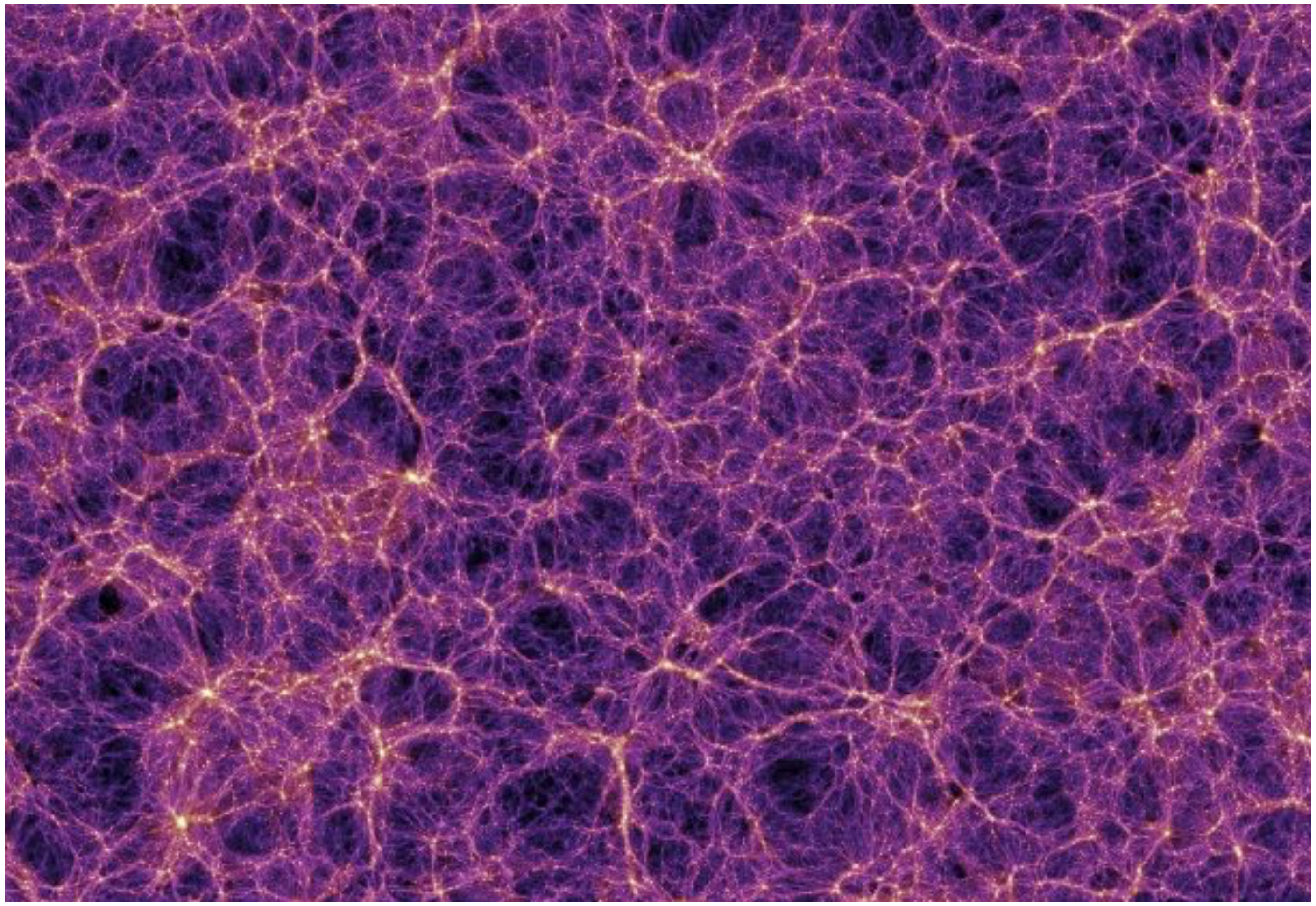
# Distribution of Large Scale Structure

Similar **clustering** and **void** features also appear in LSS:



# The Shape of Data

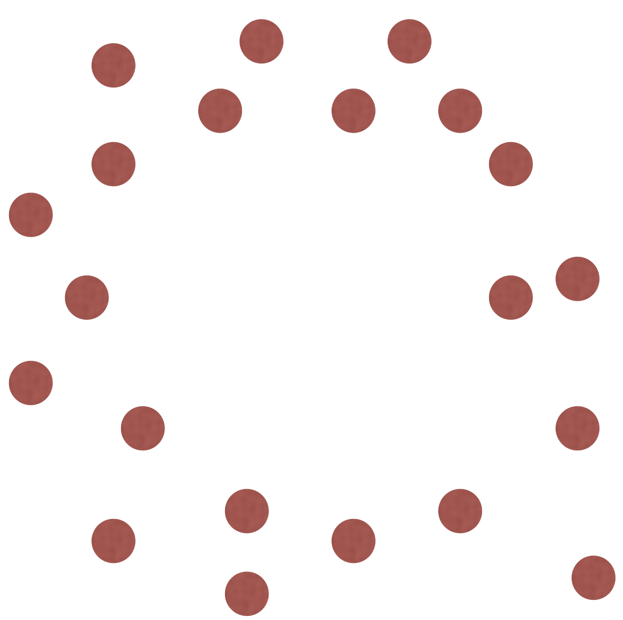


**This remarkable unity of physics suggests that we can use similar tools to analyze the structure of the cosmos [Cole, GS, '17]; [Biagetti, Cole, GS, '19] and the string landscape [Cole, GS, '18]**

# Topological Data Analysis

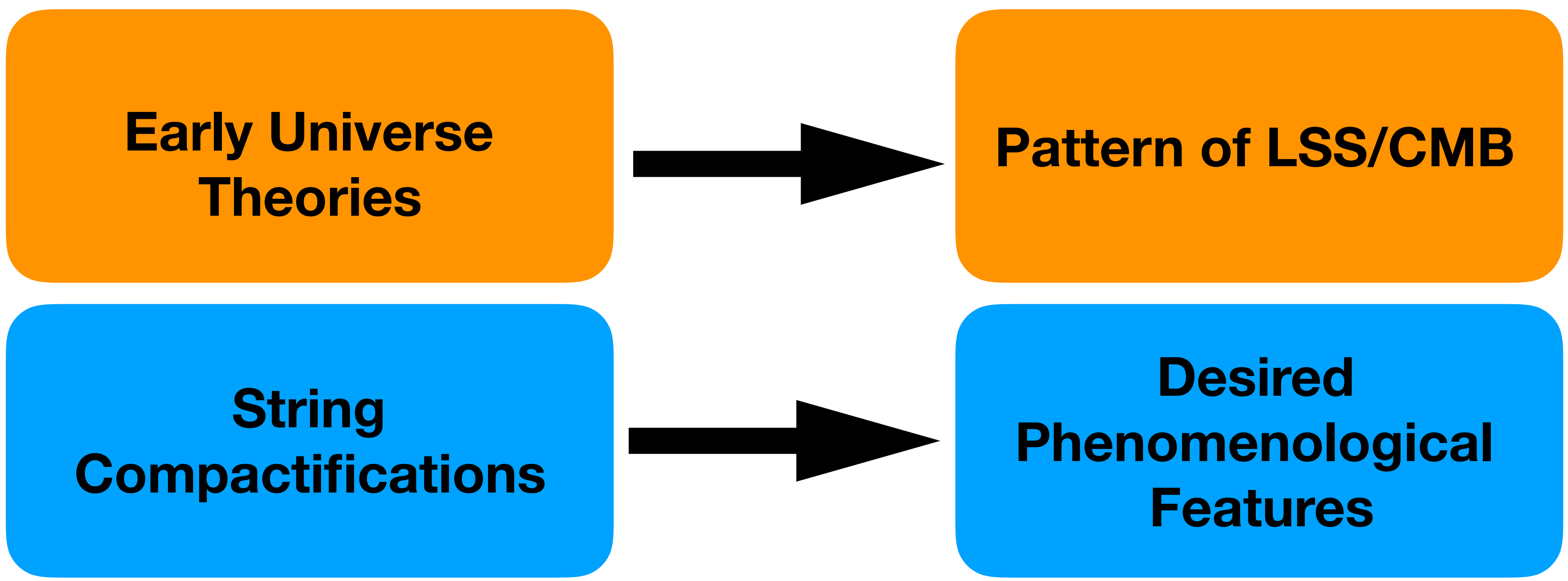
- When the space of data is huge, we cannot simply “visualize” the structure of data. We need a systematic diagnostic tool.
- Topological data analysis (TDA) is a systematic tool in applied topology to diagnose the “shape” of data.
- To compute the shape of a discrete set of data points (point cloud) with some stability, we need a notion of ***persistence***.



**Vary simplicial complexes formed by the point cloud with continuous parameters (filtration parameters)**

# Topological Data Analysis

- TDA is widely used in other fields, e.g., imaging, neuroscience, and drug design. It is well suited for machine learning.
- From the persistent homology of the point cloud, we can test e.g., the effectiveness of drugs. Similarly, we can test:

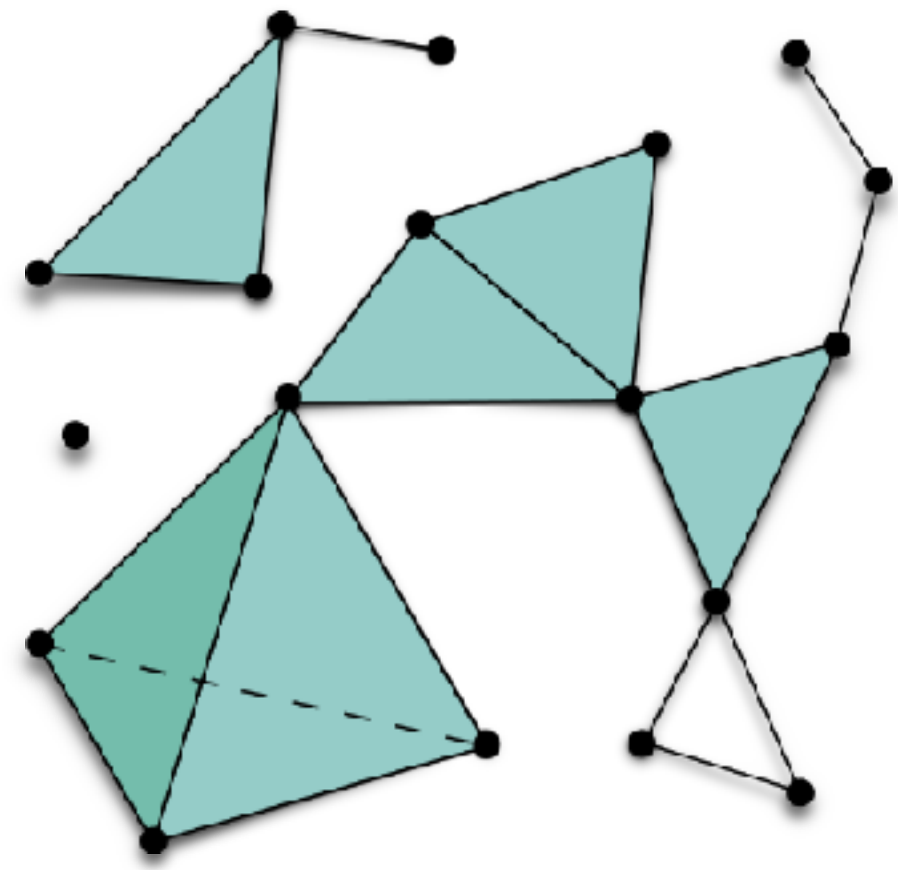



- A **selector algorithm** is often used due to the huge volume of data. We applied TDA + these algorithms on cosmological datasets **[Cole, GS, '17];[Biagetti, Cole, GS, '19]** and string data **[Cole, GS, '18].**

# Topological Data Analysis

# Simplicial Complexes

- In $\mathbb{R}^3$, simplices are vertices, edges, triangles, and tetrahedra
- Simplicial complexes are collections of simplices that are:
  - Closed under intersection of simplices
  - Closed under taking faces of simplices
- Combinatorial representations — easy calculations for computers



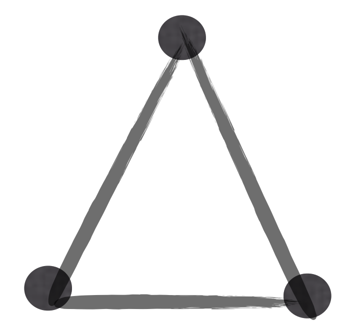
Source: Wikipedia, “Simplicial Complex”

# Simplicial Homology

- Given a simplicial complex, define a boundary operator $\partial_p$ that maps p-simplices to (p-1)-simplices
  - We want to count independent p-cycles (i.e. p-loops) that are not boundaries of higher-dimensional objects
- Group theoretic: $Z_p = \ker \partial_p$ , $B_p = \mathrm{im}\ \partial_{p+1}$,

$$H_p \equiv Z_p / B_p$$

- Betti numbers: $\beta_p \equiv \mathrm{rank} H_p$
  - 0-th Betti number is number of connected components
  - p-th Betti number is number of independent p-loops
- In practice, homology calculation is a matrix reduction



vs.

$\beta_0 = 1$ $\quad$ $\beta_0 = 1$

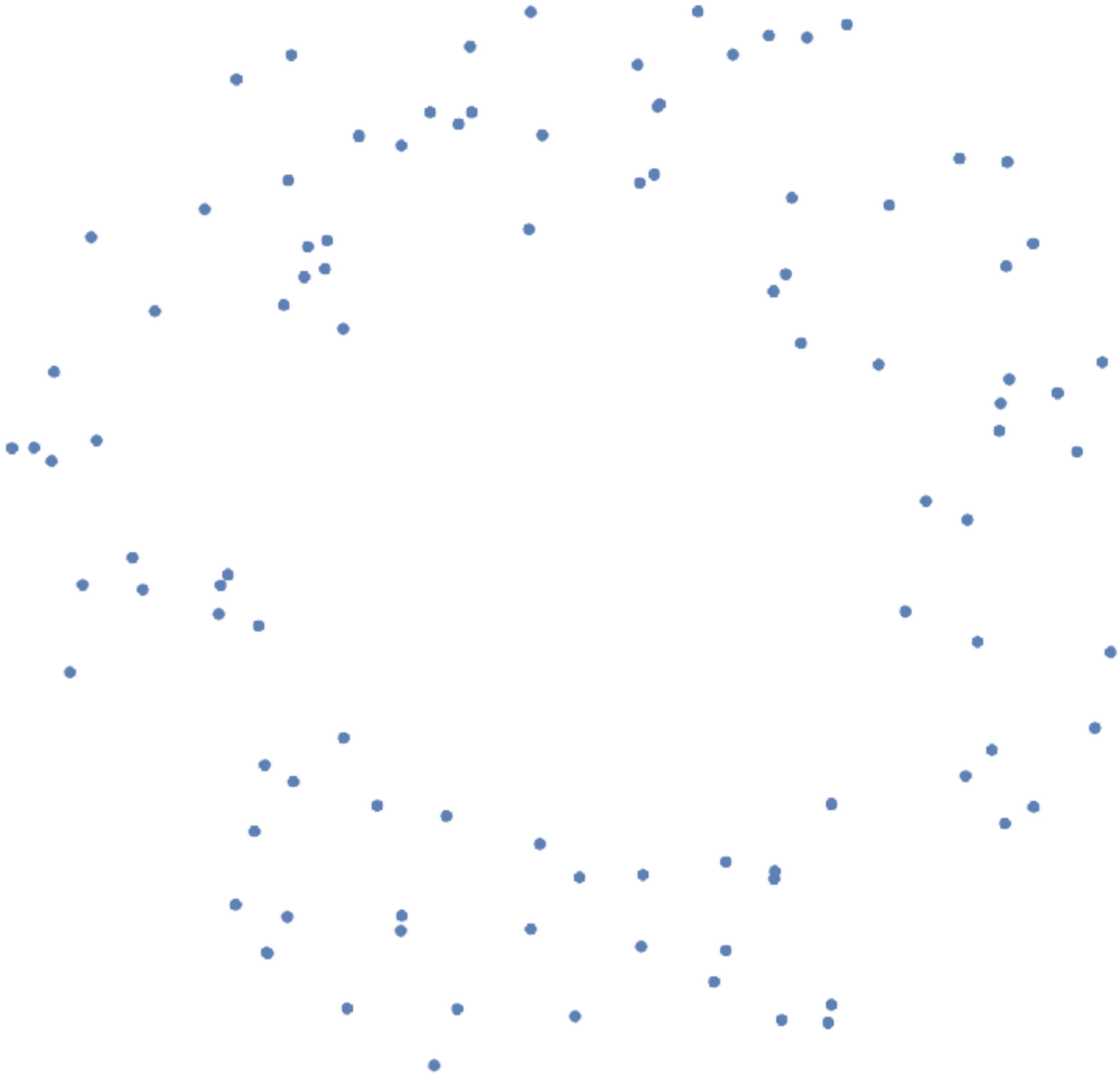
$\beta_1 = 1$ $\quad$ $\beta_1 = 0$

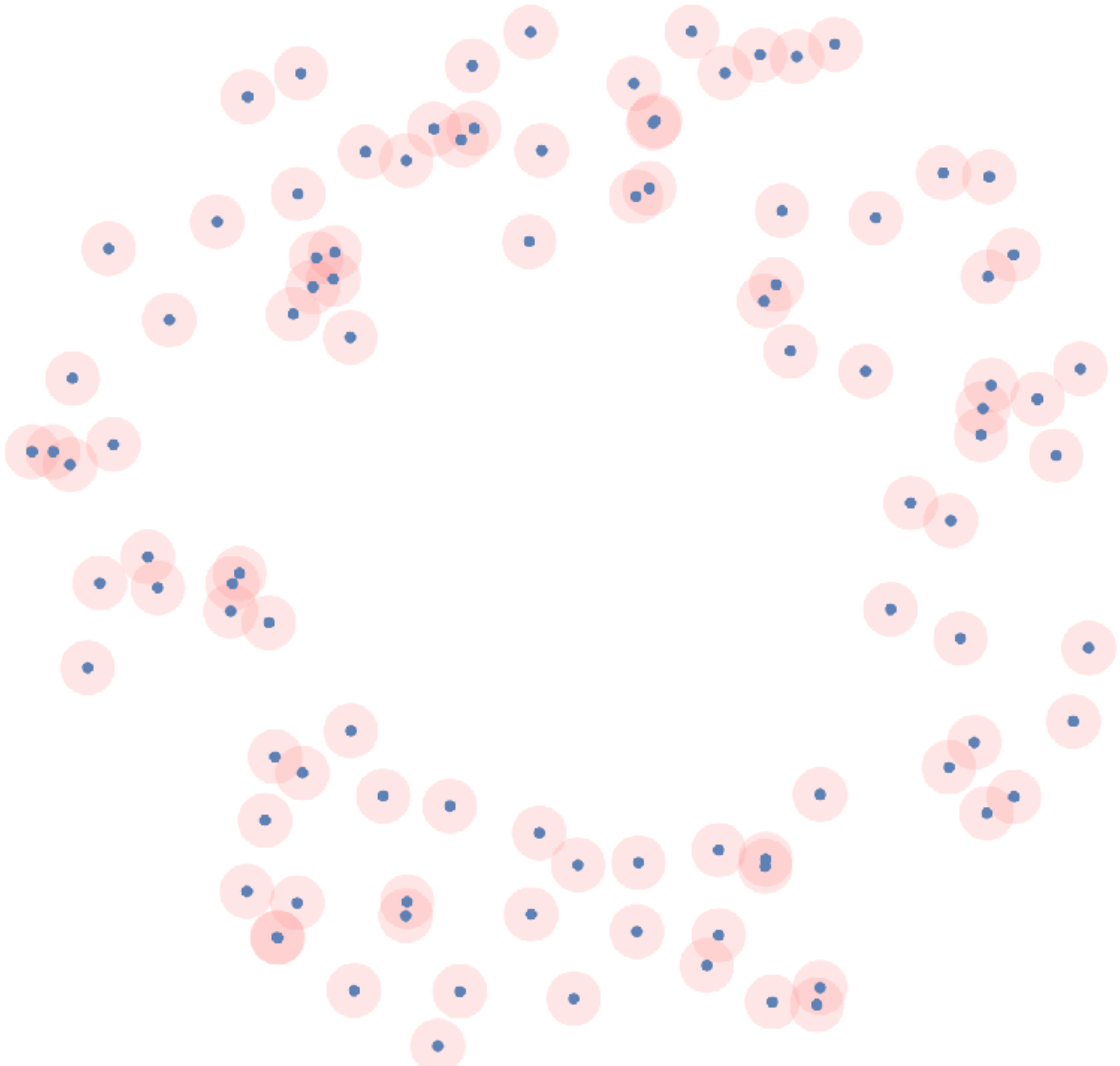
# Persistence

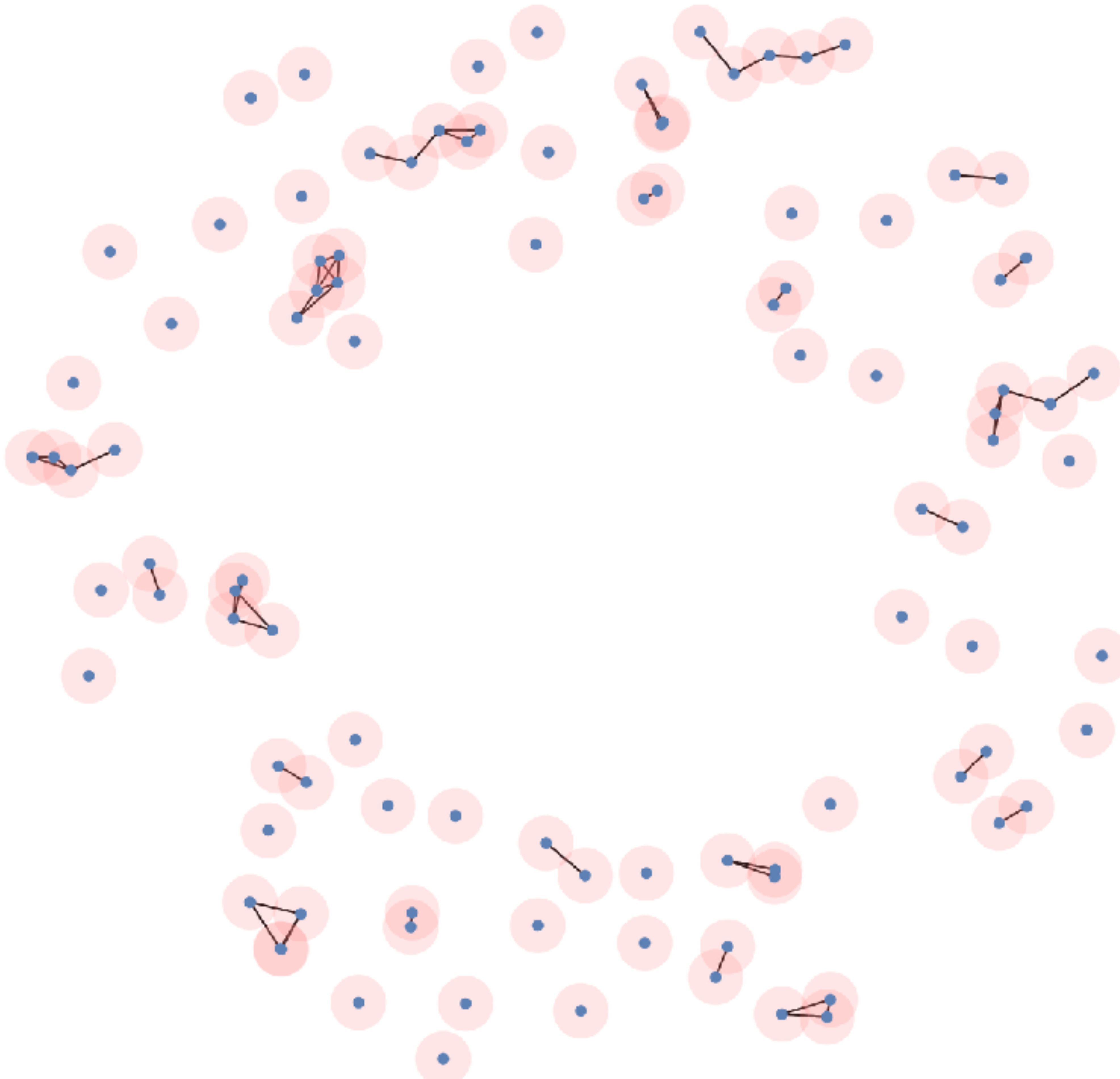
- How to choose simplicial representation of our data?
- *Persistent* homology: vary simplicial representation $\Sigma_\nu$ of data with some *filtration parameter* $\nu$ such that

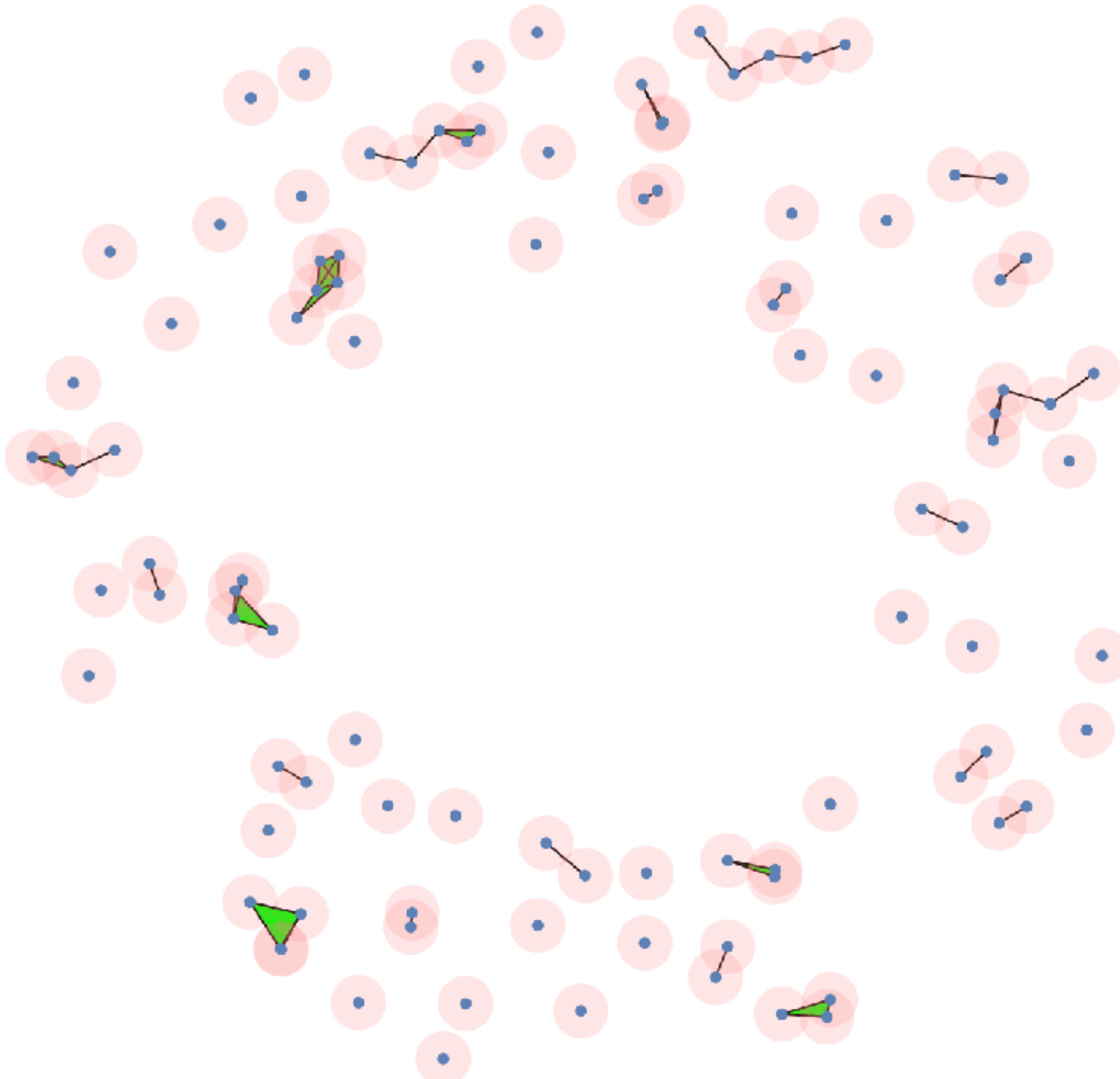
$$\nu_1 \leq \nu_2 \implies \Sigma_{\nu_1} \subseteq \Sigma_{\nu_2}$$

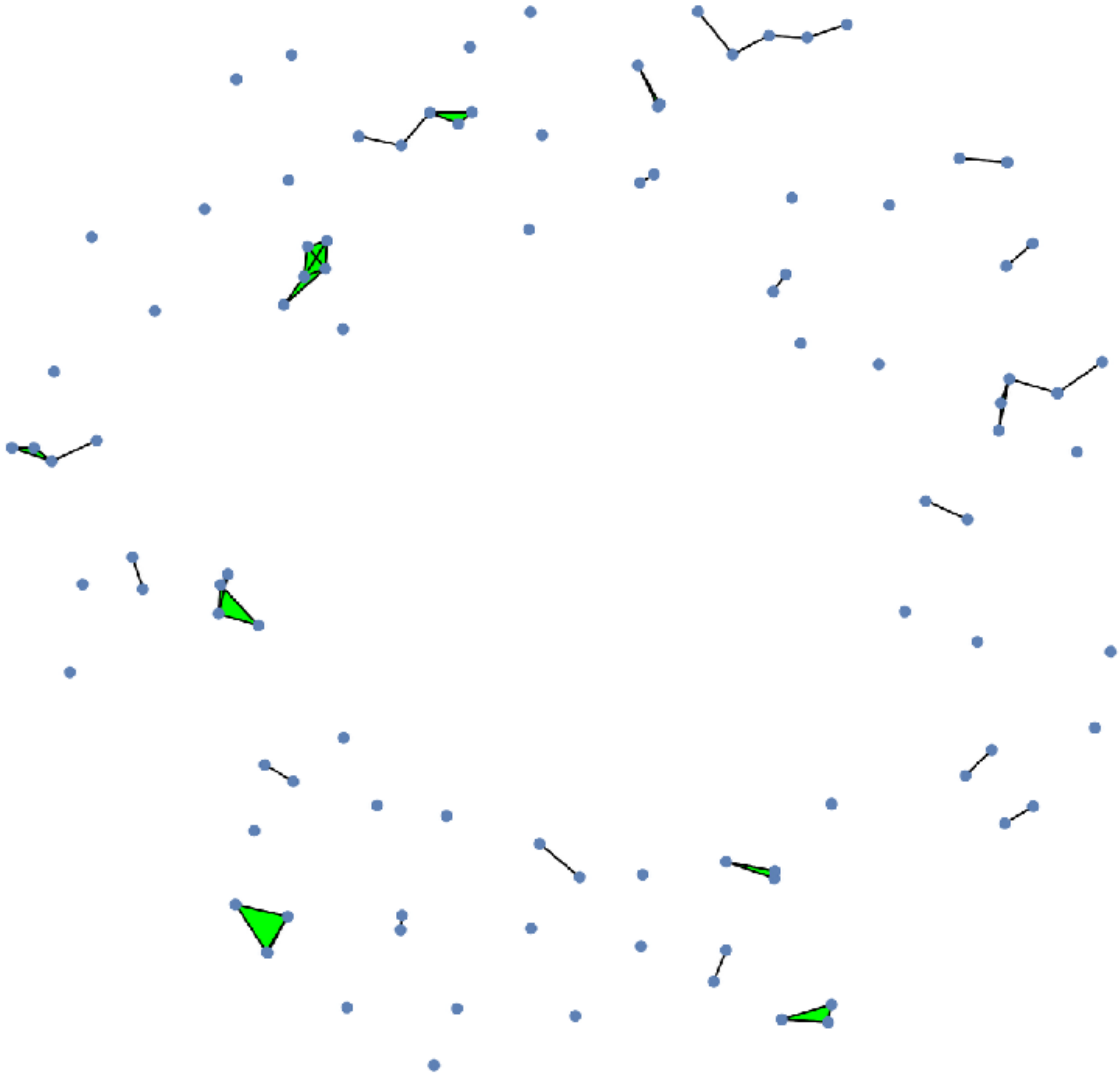
  - Track each distinct feature's lifetime (birth and death)
- Intuition: "real" topological features *persist*, short-lived features are noise
- Procedure is stable against perturbations to data **[Cohen-Steiner 2005]**

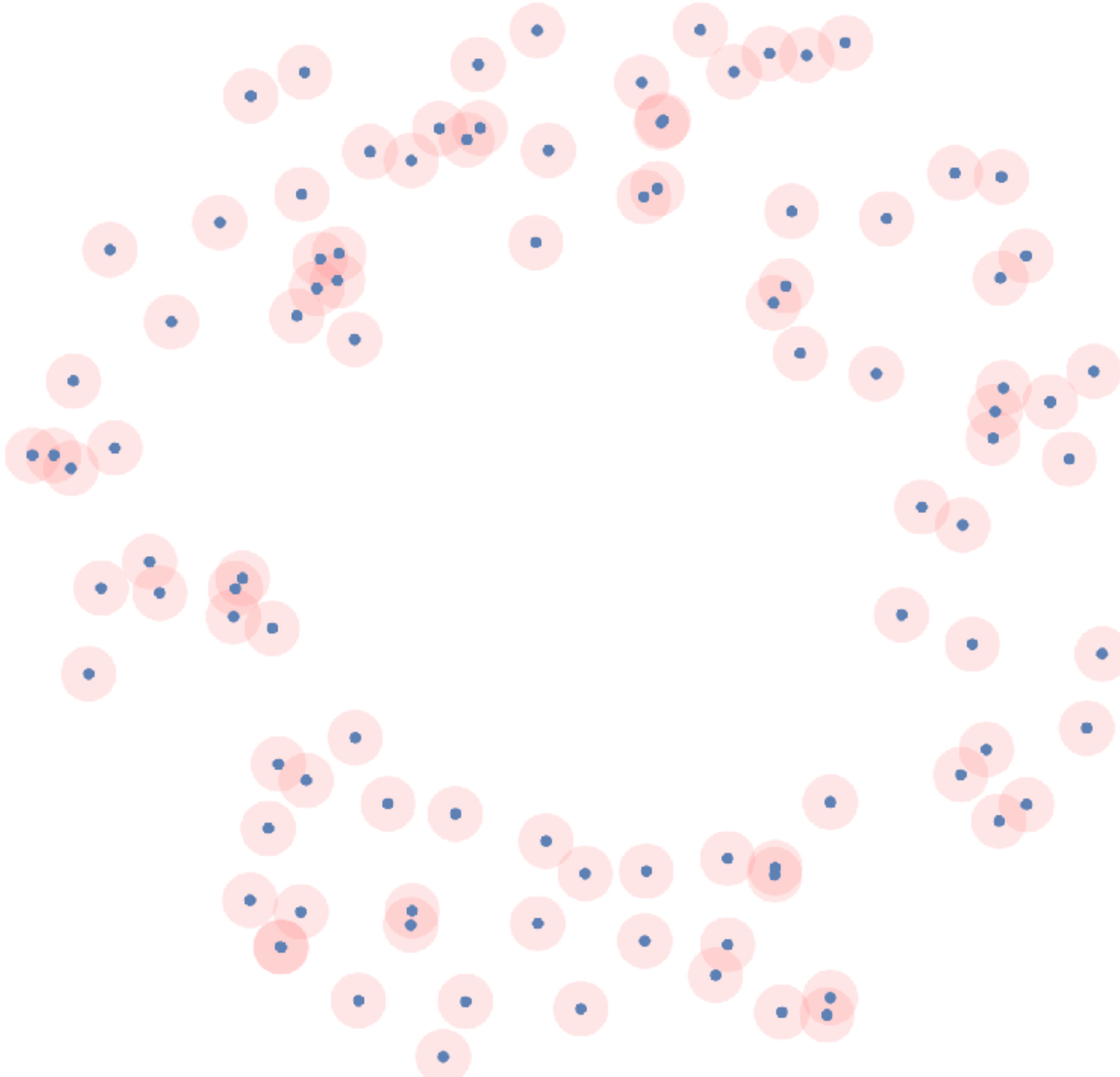


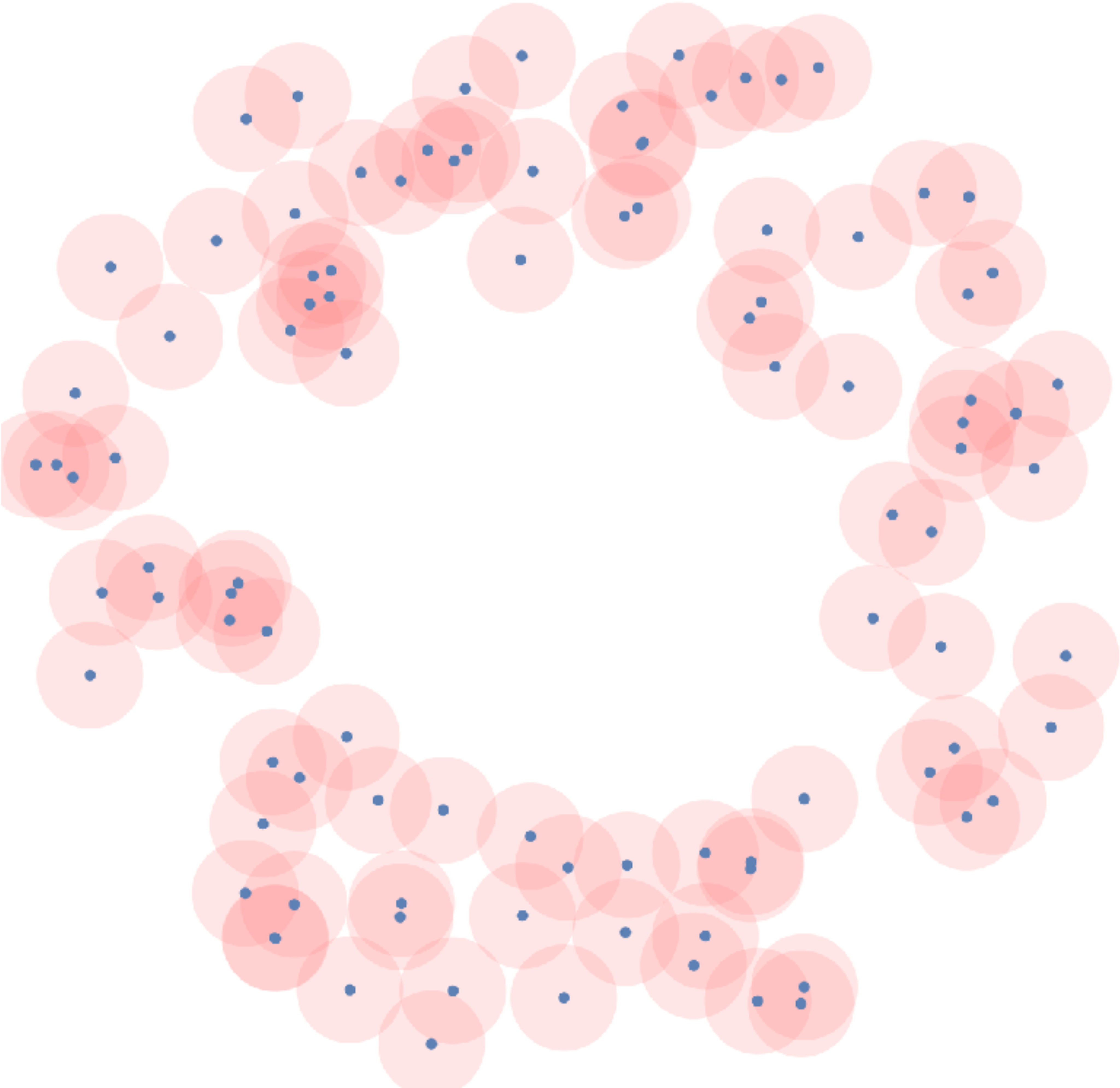


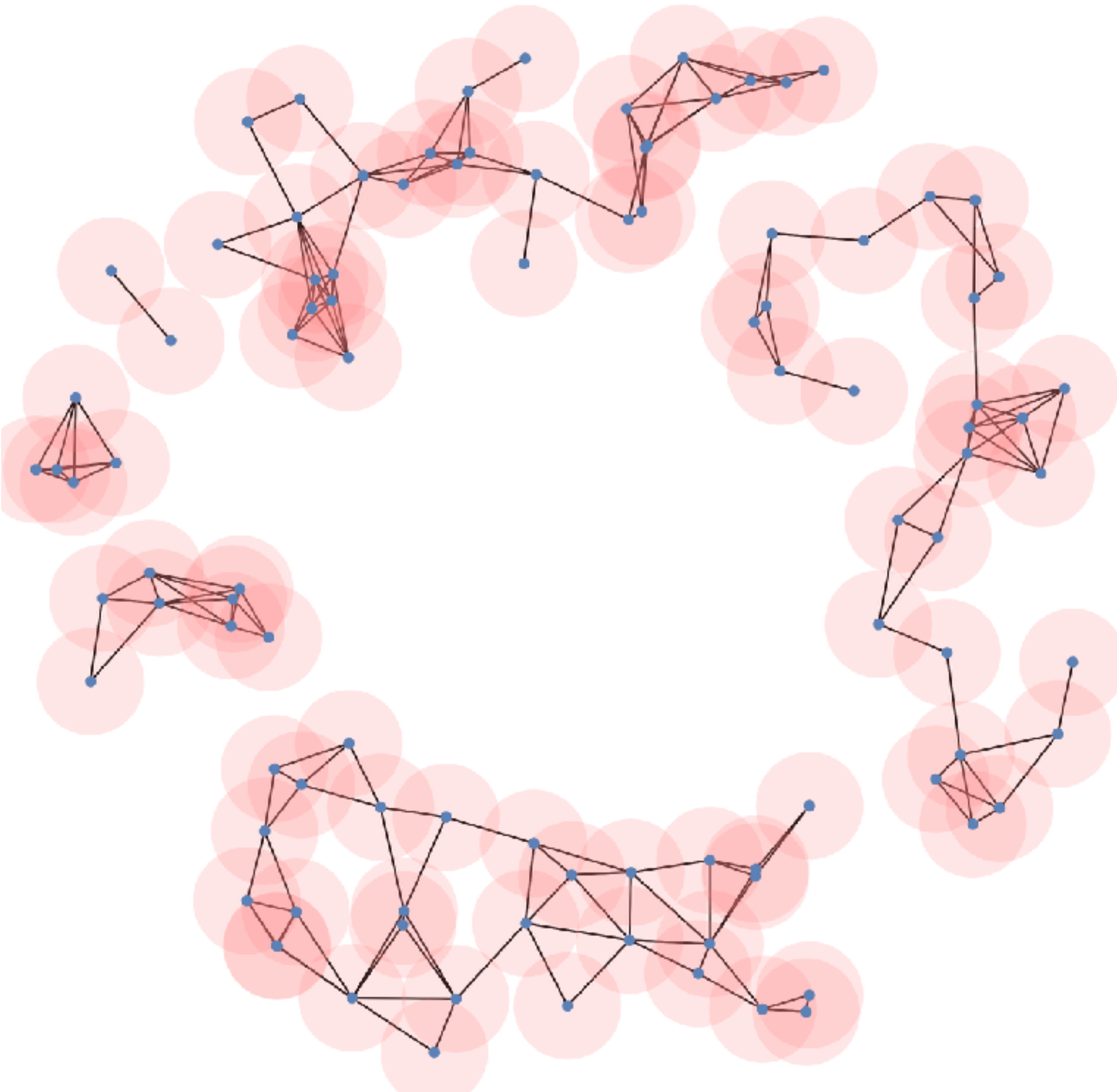


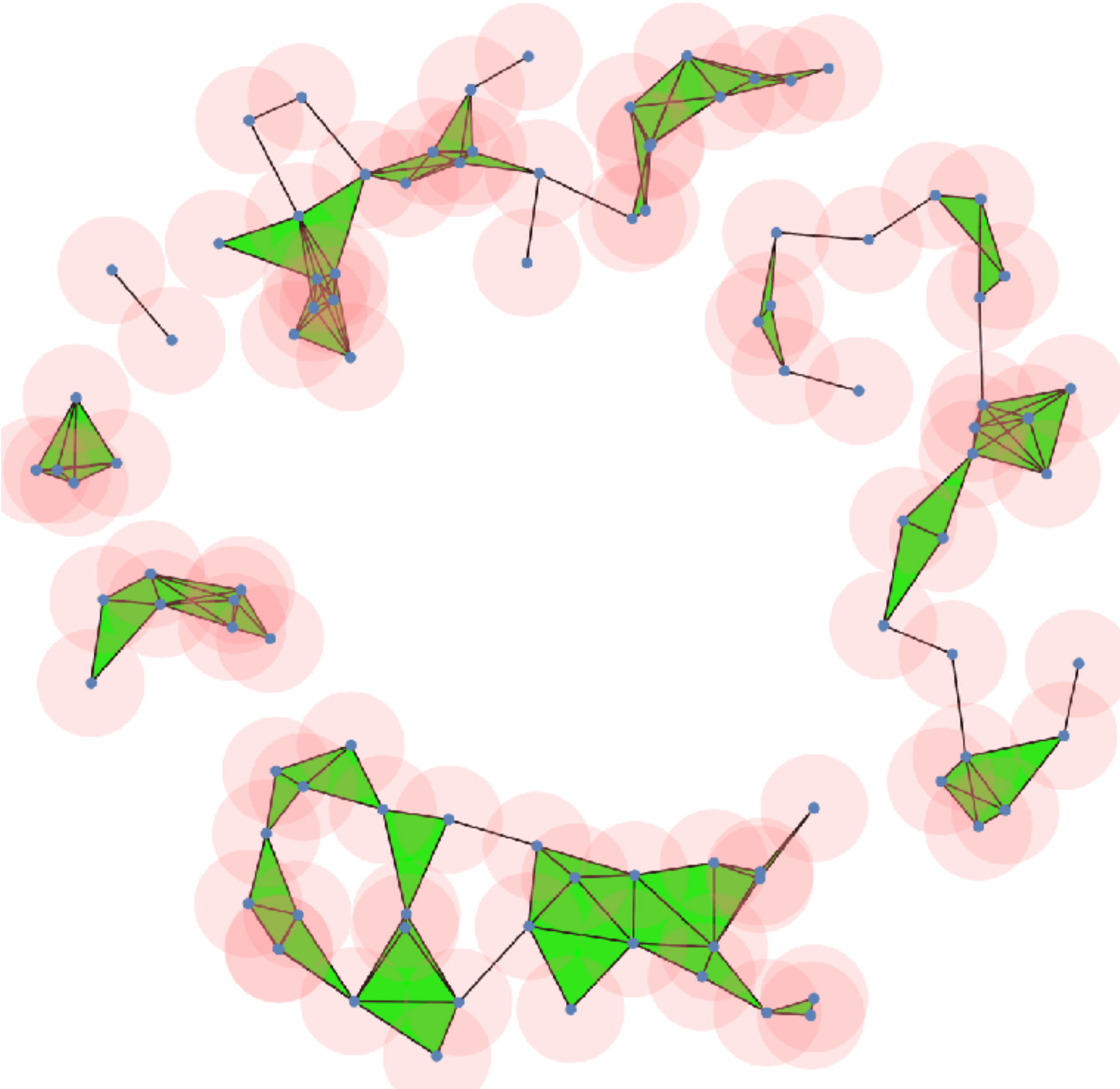


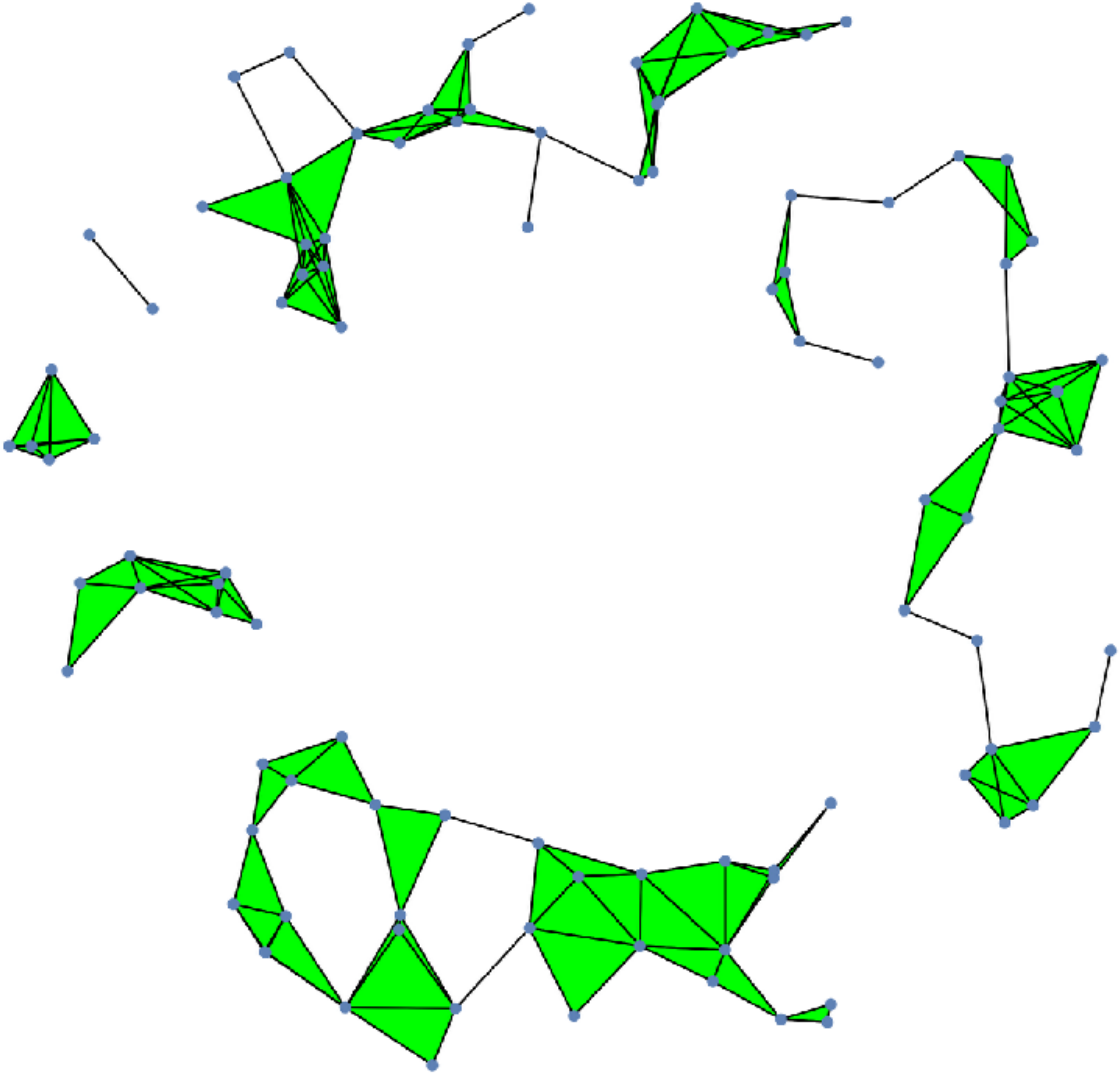


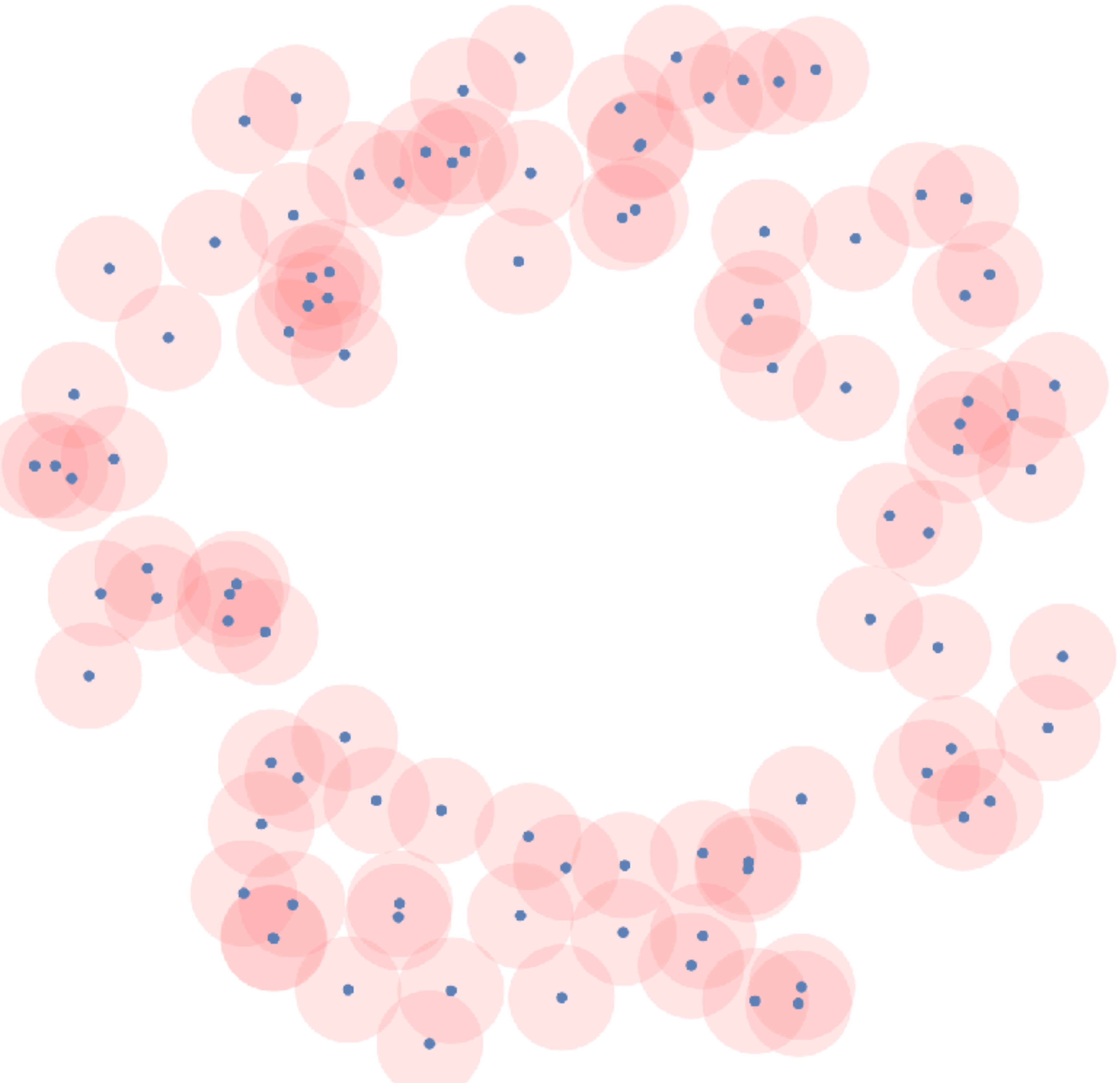


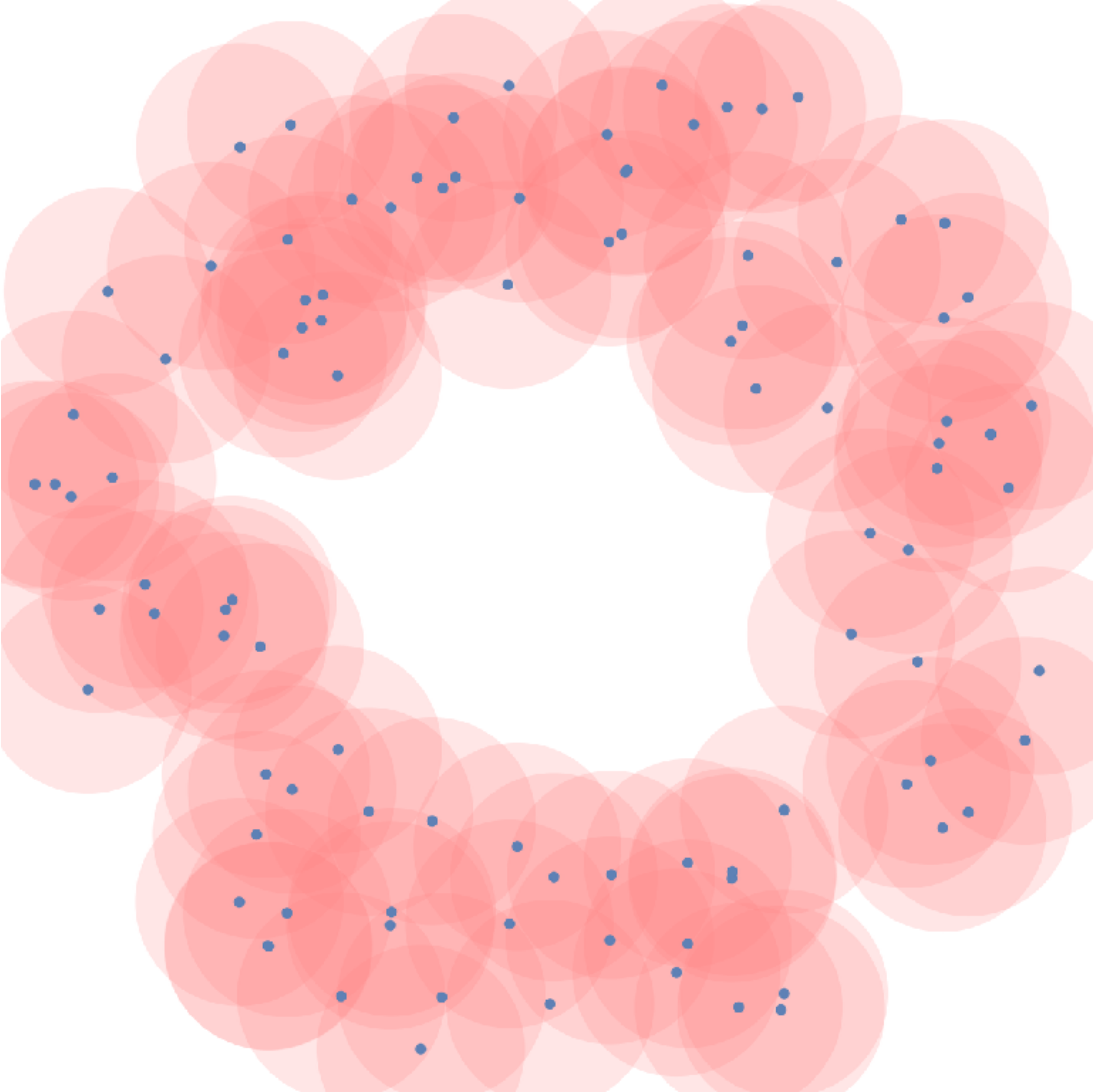


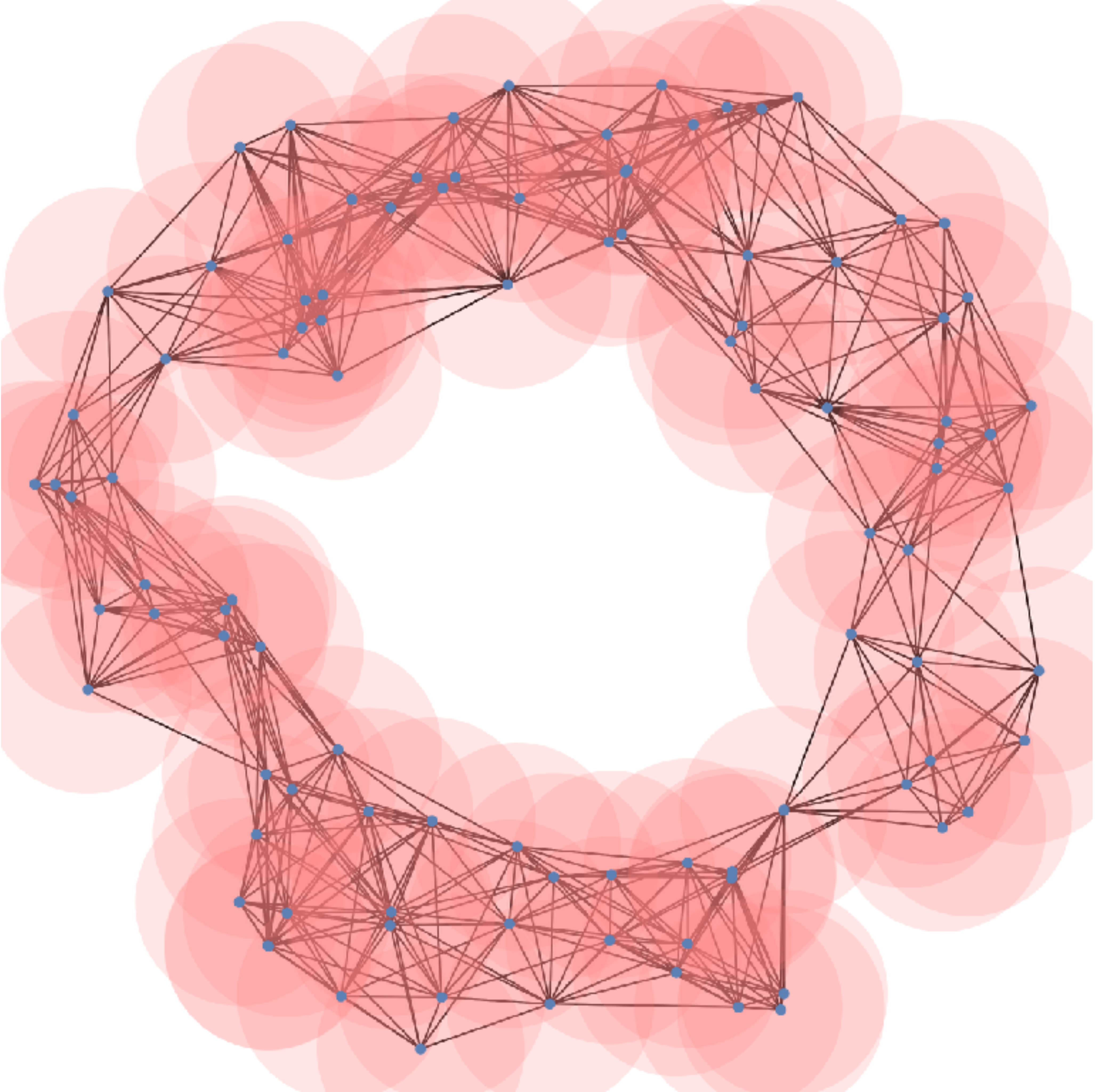


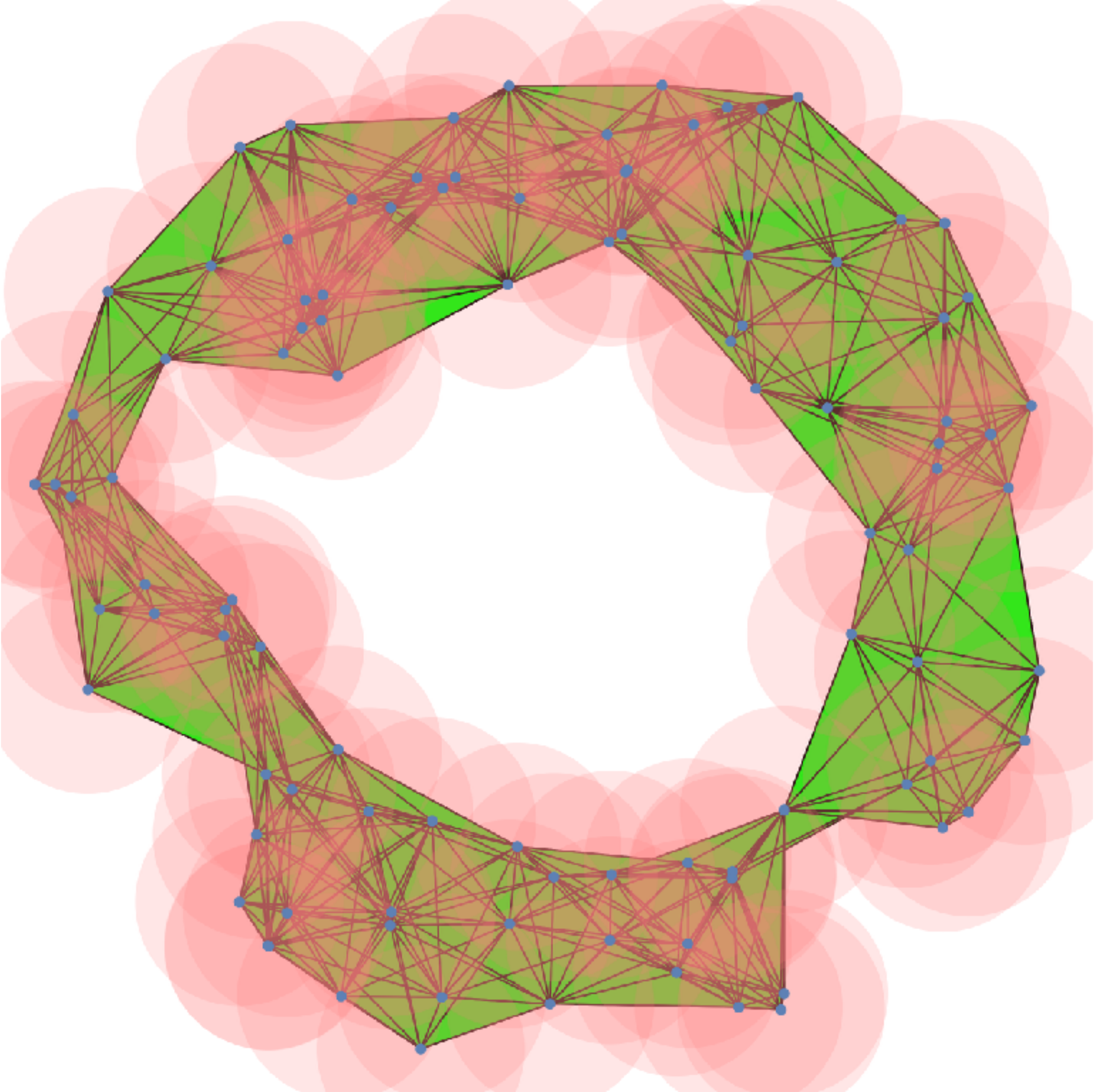


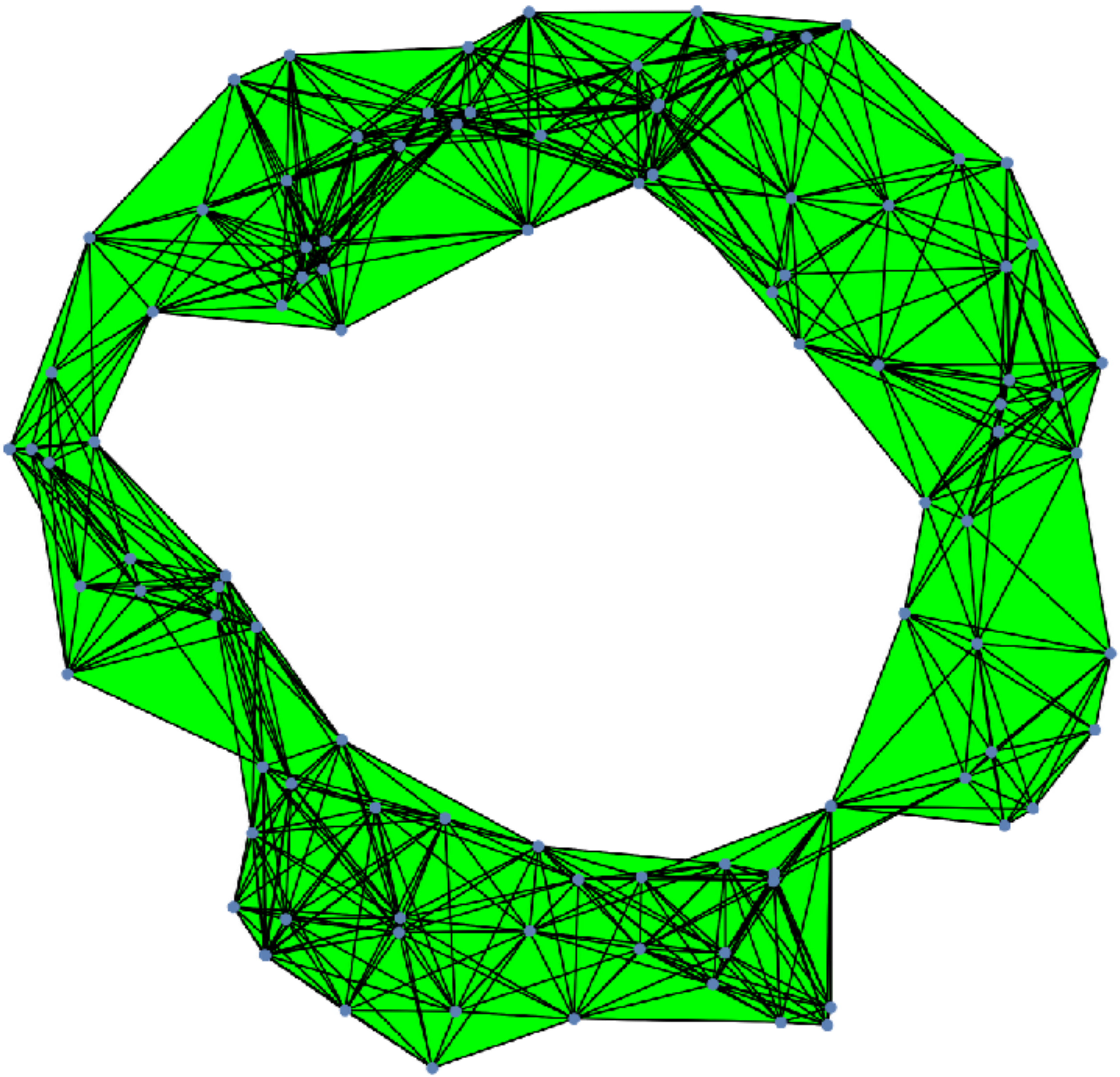






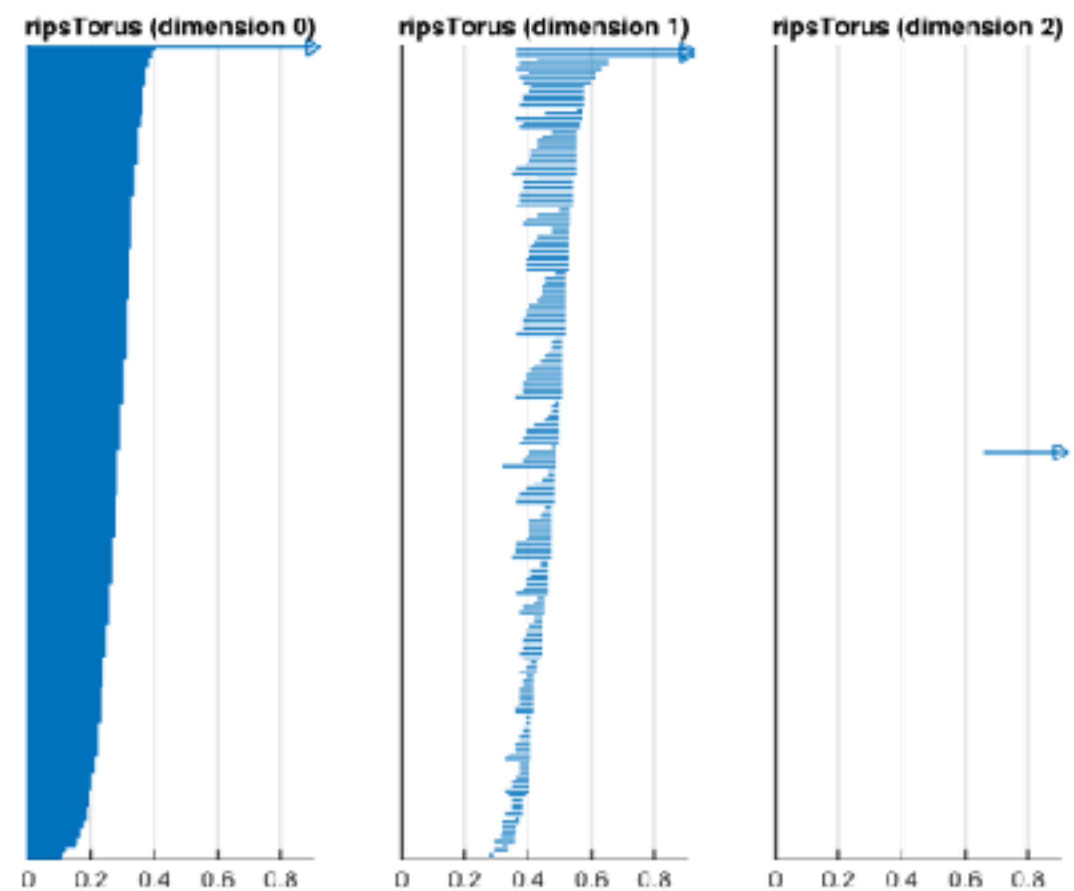


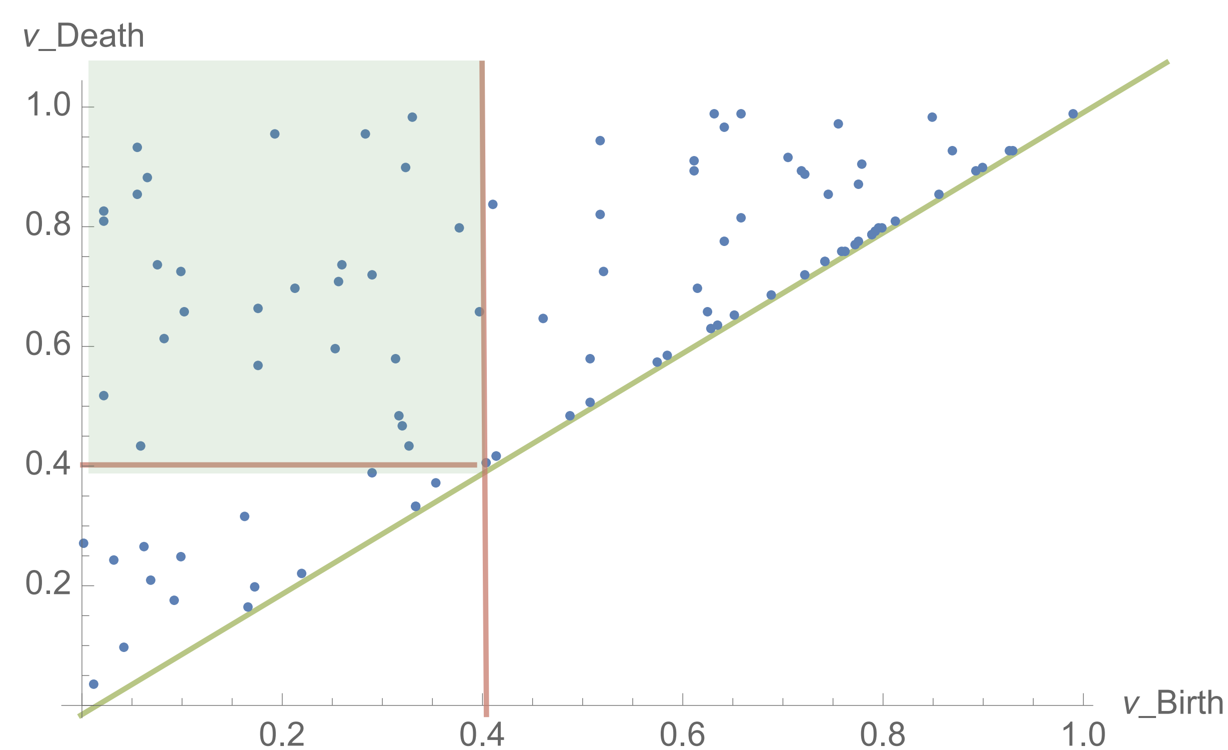




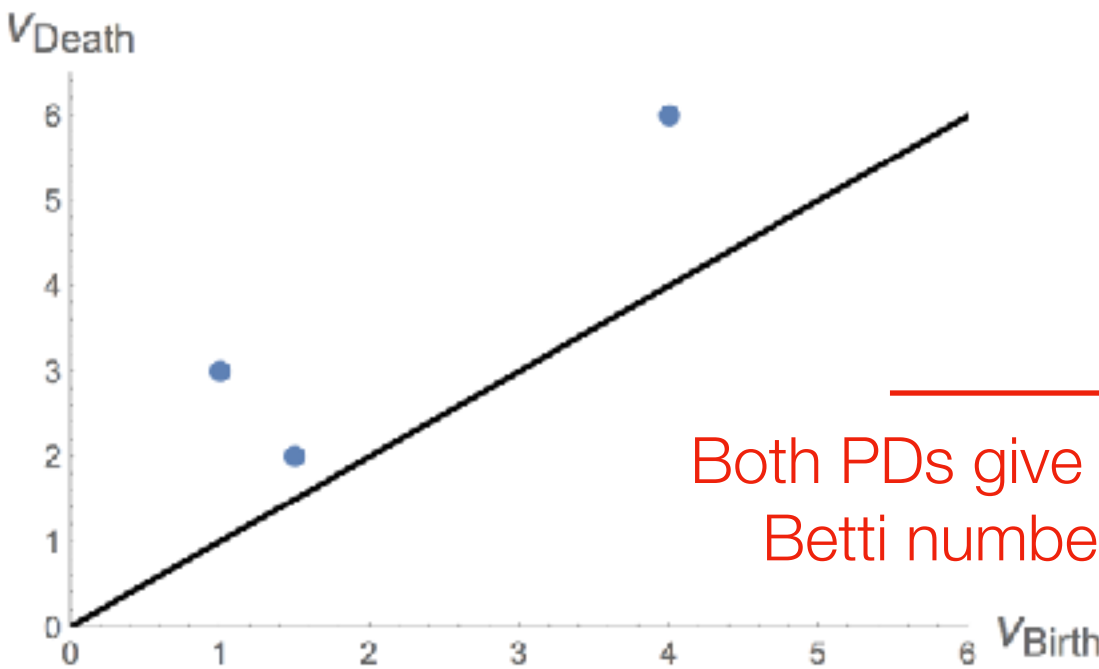
# Visualizing Persistent Homology

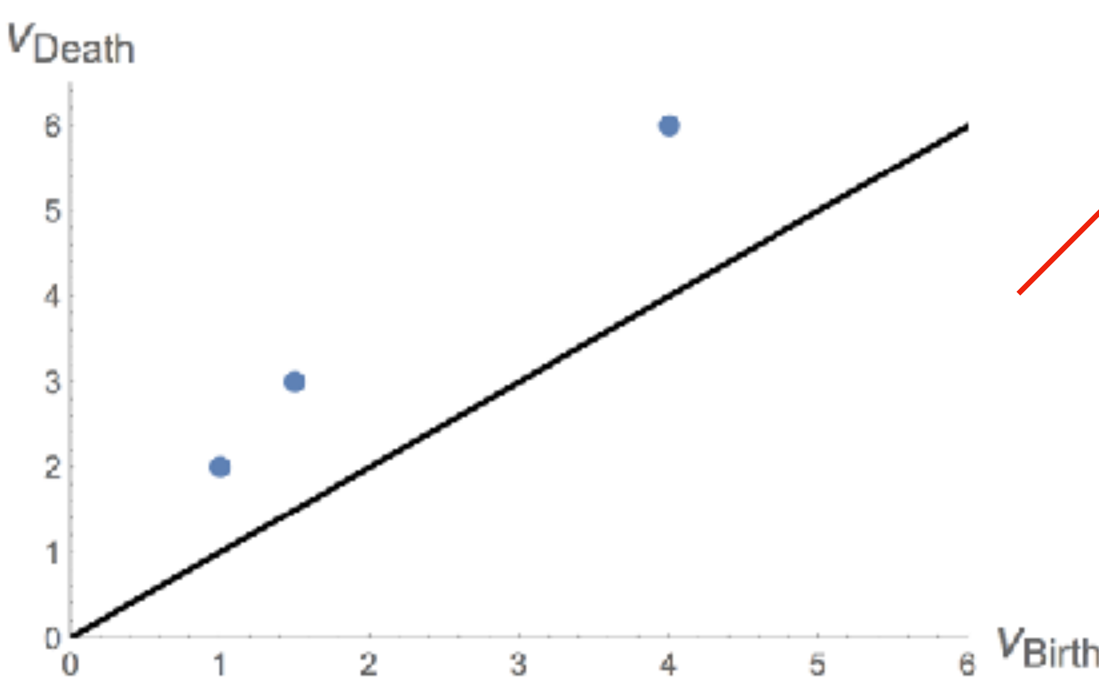
- **Barcodes**:
    - Each horizontal line represents an independent cycle contributing to a particular Betti number (i.e. a connected component, loop, void…)
    - Lines start at birth and end at death
    - To calculate Betti number, make vertical slice and count intersections
- **Persistence diagrams**:
    - Scatter plot, each point representing an independent cycle
    - Calculate Betti number by counting “living” cycles



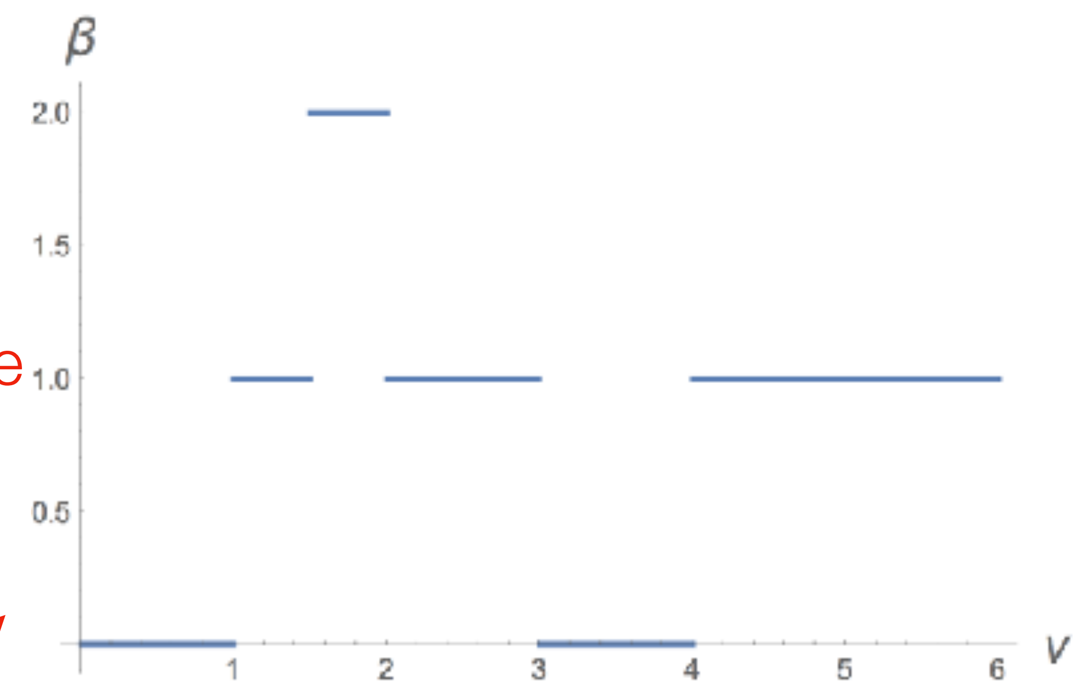



# Persistence diagrams contain more information than Betti number curves!




Both PDs give the same Betti number curve



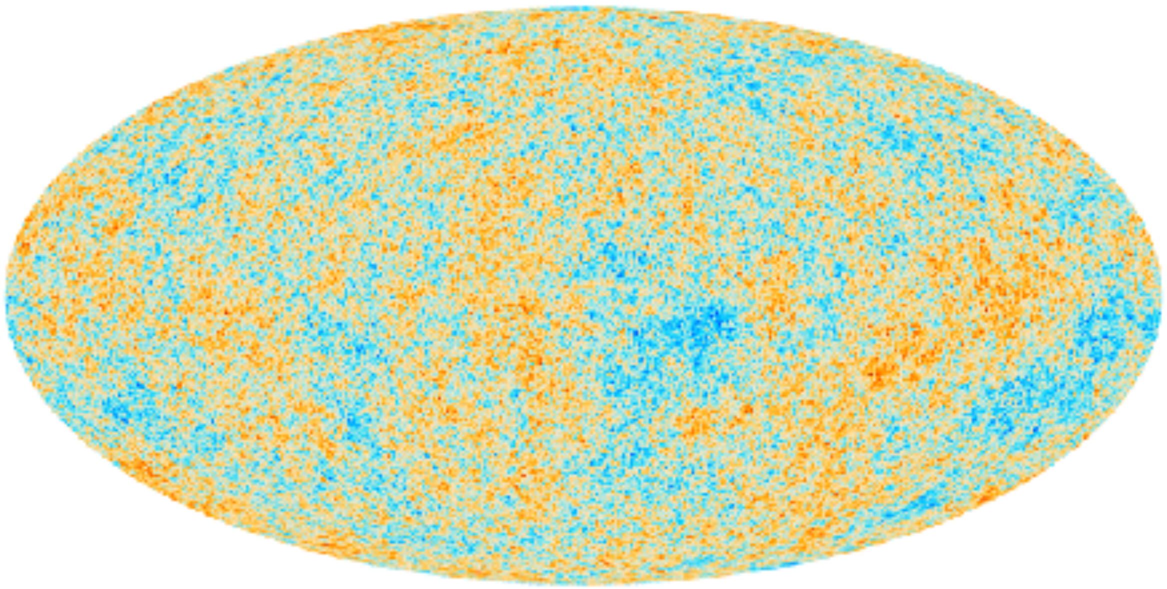

**We can exploit this to improve the data analysis of CMB** **[Cole, GS, '17]** **& LSS** **[Biagetti, Cole, GS, work in progress]**

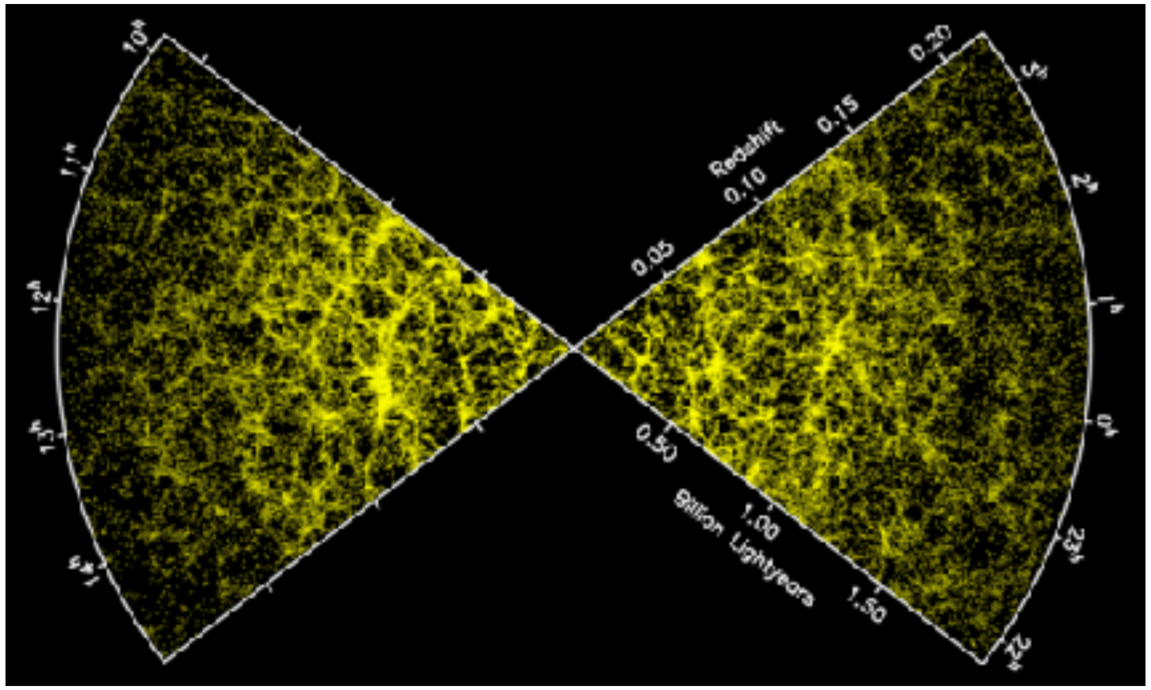
# Applying TDA to Cosmology

# Inflation

[Starobinsky];[Guth];[Linde];[Albrecht, Steinhardt];…

- Period of **accelerated expansion** in early universe
  - Solves flatness, horizon, and monopole problems
  - Predicts **nearly scale-invariant**, **Gaussian** curvature fluctuations
    - Source anisotropies in CMB, inhomogeneities in LSS
- A myriad of models. Taxonomy done mostly through their observables ($n_s$, $r$)





# Anisotropies

- The lowest order correlation we can extract from the anisotropies is the **power spectrum**

$$\left\langle 0 \left| \hat{\mathcal{R}}_{\mathbf{k_1}} \hat{\mathcal{R}}_{\mathbf{k_2}} \right| 0 \right\rangle = (2\pi)^3 P_{\mathcal{R}}(k_1) \delta(\mathbf{k}_1 + \mathbf{k}_2) \qquad \Delta^2_{\mathcal{R}} = \left(\frac{k^3}{2\pi^2}\right) P^2_{\mathcal{R}} \propto k^{n_s - 1}$$

- For a Gaussian theory, the power spectrum dictates all higher-pt correlations.
- However, the inflationary fluctuations are not perfectly Gaussian.
- The leading **non-Gaussianity** is the **bispectrum**:

$$\langle 0 | \, \hat{\mathcal{R}}_{\mathbf{k_1}} \hat{\mathcal{R}}_{\mathbf{k_2}} \hat{\mathcal{R}}_{\mathbf{k_3}} \, | 0 \rangle = (2\pi)^3 \, \delta^3 (\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) F(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k}_3)$$

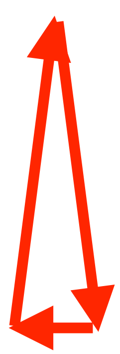
- Scaling and symmetries imply that F($\mathbf{k_1}$, $\mathbf{k_2}$, $\mathbf{k_3}$) is fixed by an overall **size** ~ $f_{NL}$ and its ‘’**shape”** F(1, $k_2/k_1$, $k_3/k_1$).
- More **powerful discriminator** of inflationary models.

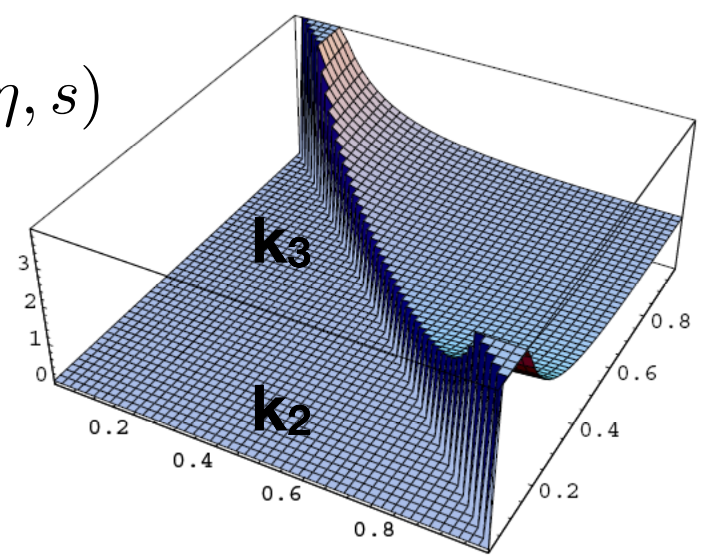
# Non-Gaussianities

- The bispectrum for **single field slow-roll** inflation was computed in **[Maldacena, '02];[Acquaviva et al, '02]**; its size is $f_{NL} \sim \mathcal{O}(\varepsilon, \boldsymbol{\eta})$:
- The bispectrum for **general single field inflation** was found to be parametrized by 5 parameters **[Chen, Huang, Kachru, GS, '06]**:

**Local shape**

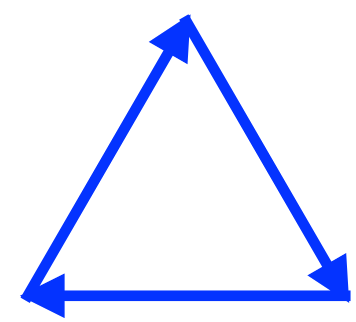
$$f_{NL}^{local} \sim \mathcal{O}(\epsilon, \eta, s)$$

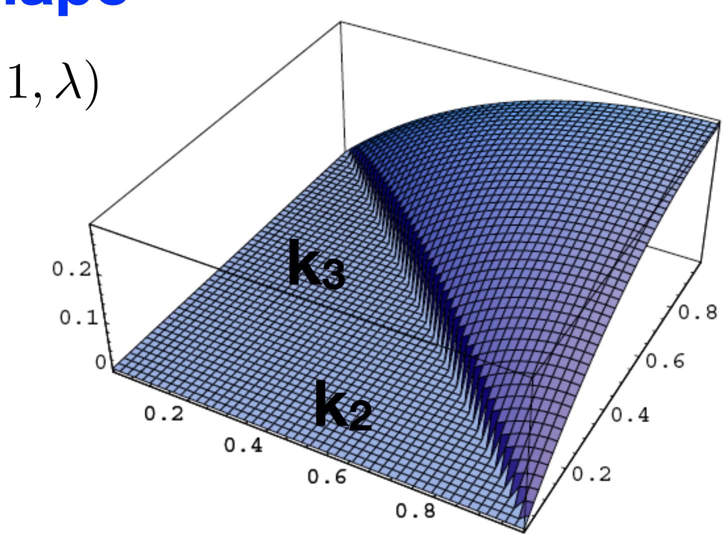





**Equilateral shape**

$$f_{NL}^{equil} \sim \mathcal{O}(\frac{1}{c_s^2} - 1, \lambda)$$



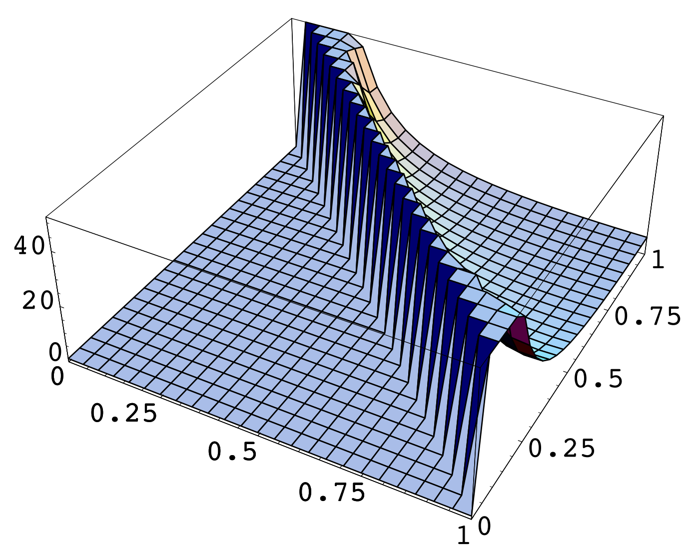



- There is also an **"orthogonal shape"** but it "looks" qualitatively like the equilateral shape (***challenge for machine learning?***).

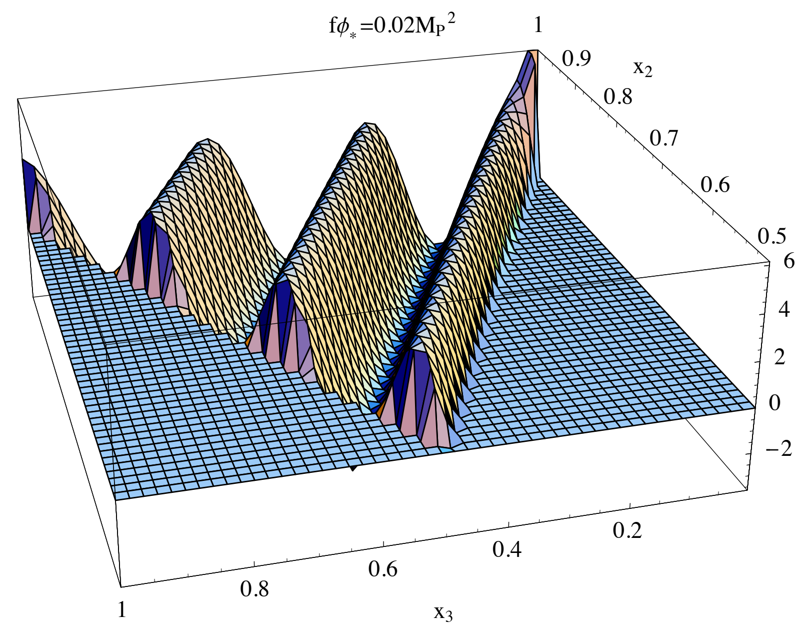
# Non-Gaussianities

- More complicated models which involve non-standard initial conditions, features in potential (e.g. axion monodromy), or multiple fields or quasi-single field can give rise to more shapes:

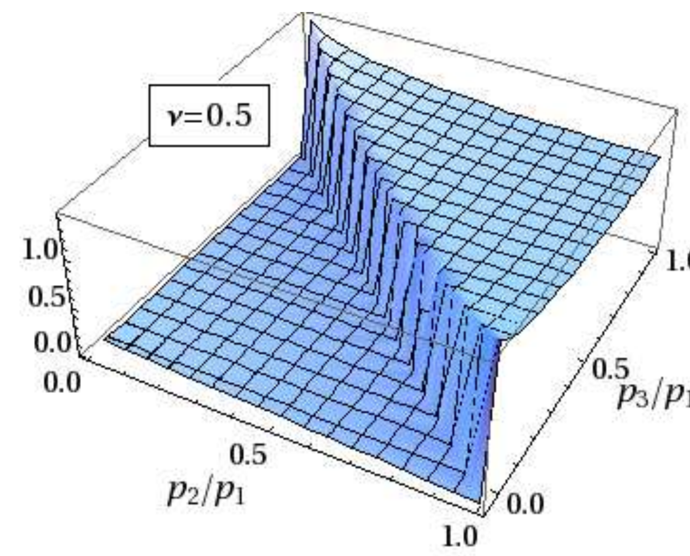
**Non Bunch-Davis**




**Axion Monodromy**




**Quasi-single field**




- Like scattering amplitudes in particle physics, non-Gaussianties can reveal interactions governing inflation: ***cosmological collider.***

- In collider physics: use ***different strategies*** for different particles.

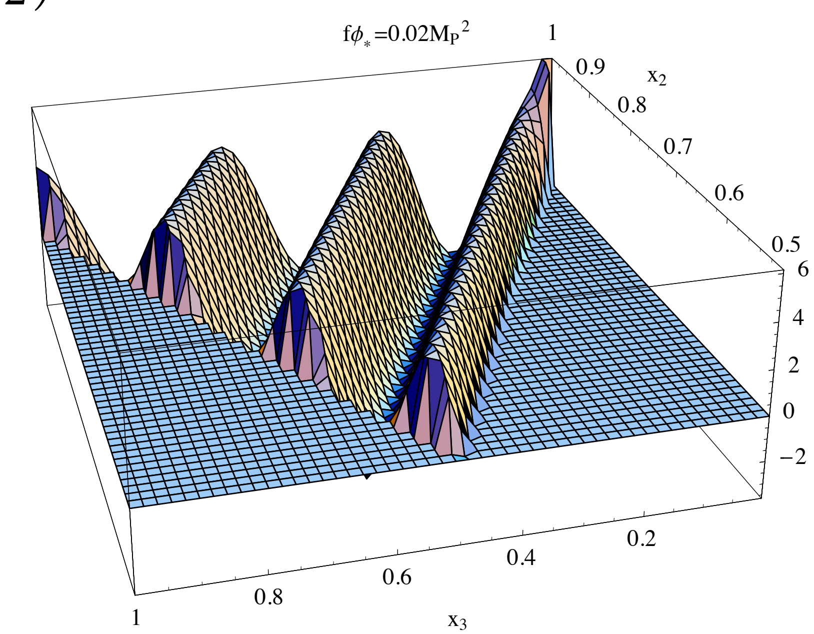
# Measuring Non-Gaussianity

- **Harmonic space:** fits with *templates* of bispectrum, trispectrum, etc. One can define a "cosine" between distributions:

$$\cos(F_1, F_2) = \frac{F_1 \cdot F_2}{(F_1 \cdot F_1)^{1/2} (F_2 \cdot F_2)^{1/2}}$$

- Some shapes are harder to find, e.g.,

**Resonant shape (axion monodromy)**



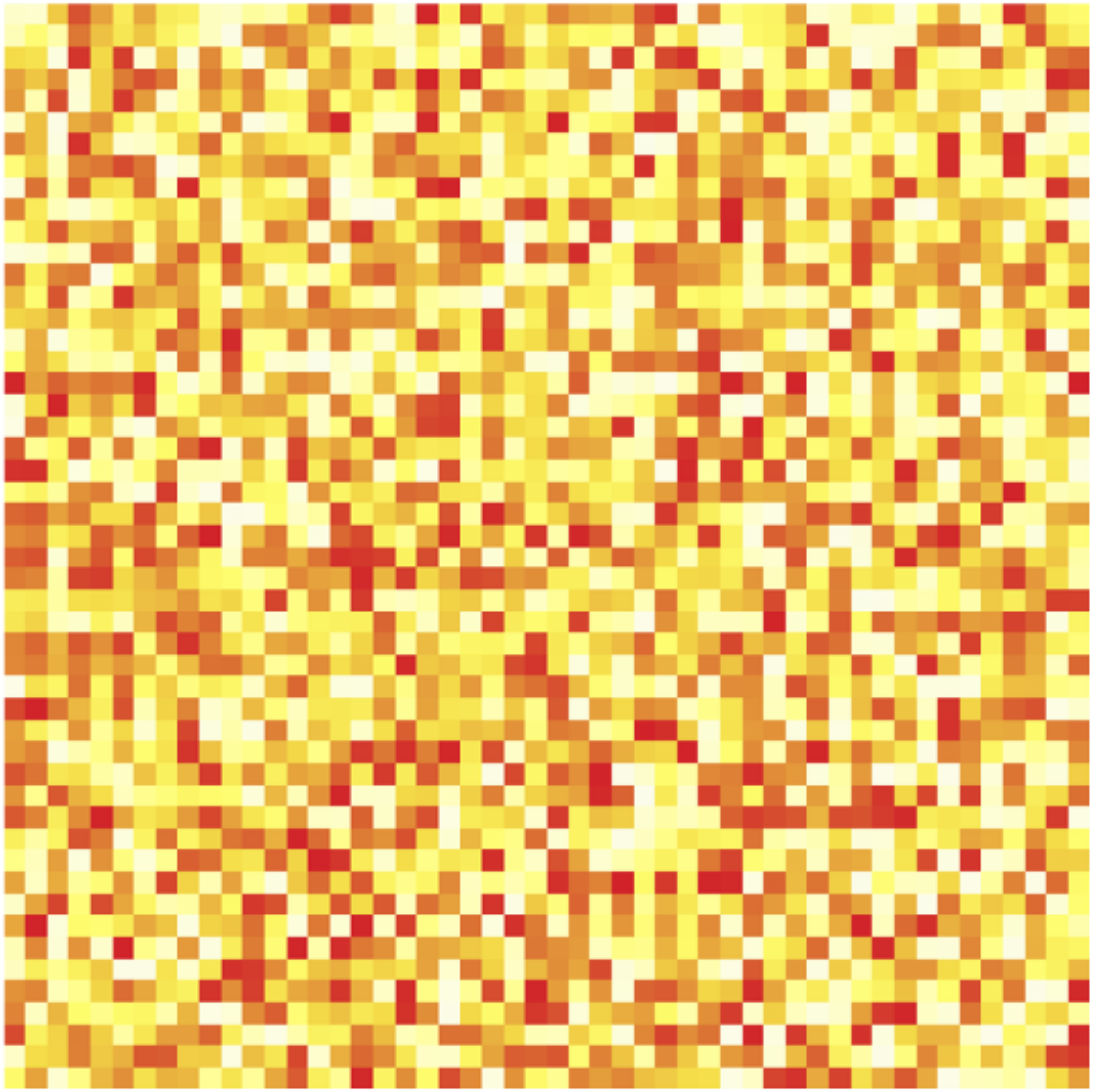

- Geometrical/topological: **Minkowski functionals** (for CMB: area fraction, length of boundaries, and genus of excursion sets)

- Current bound on non-Gaussianity (Planck '15):

$$f_{NL}^{local} = 2.5 \pm 5.7 \qquad\qquad f_{NL}^{equil} = -16 \pm 70$$

# Sublevel Filtration

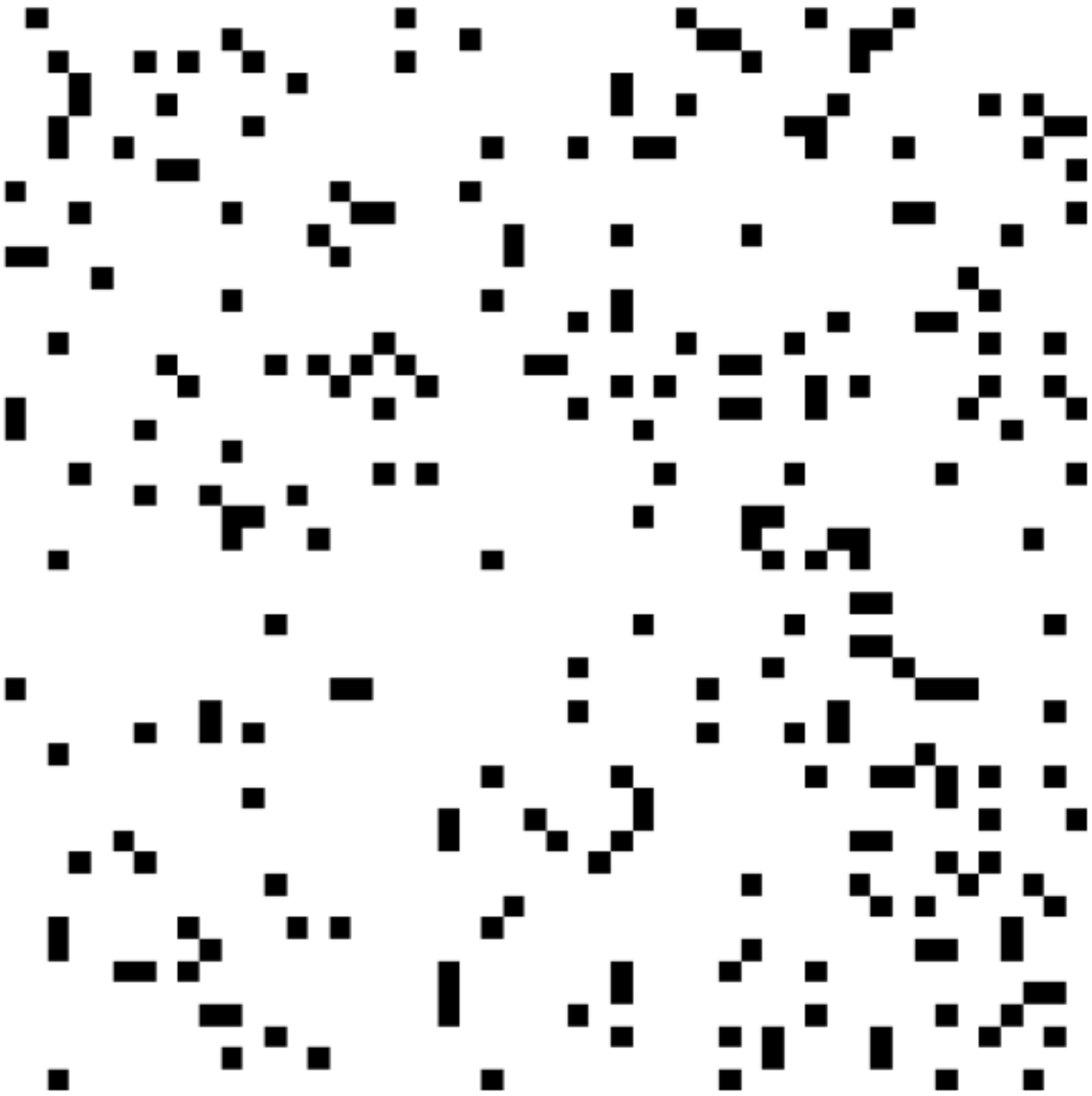
(Hotter points are deeper red)



# Sublevel Filtration

$\nu = -1$

Many distinct components, no loops



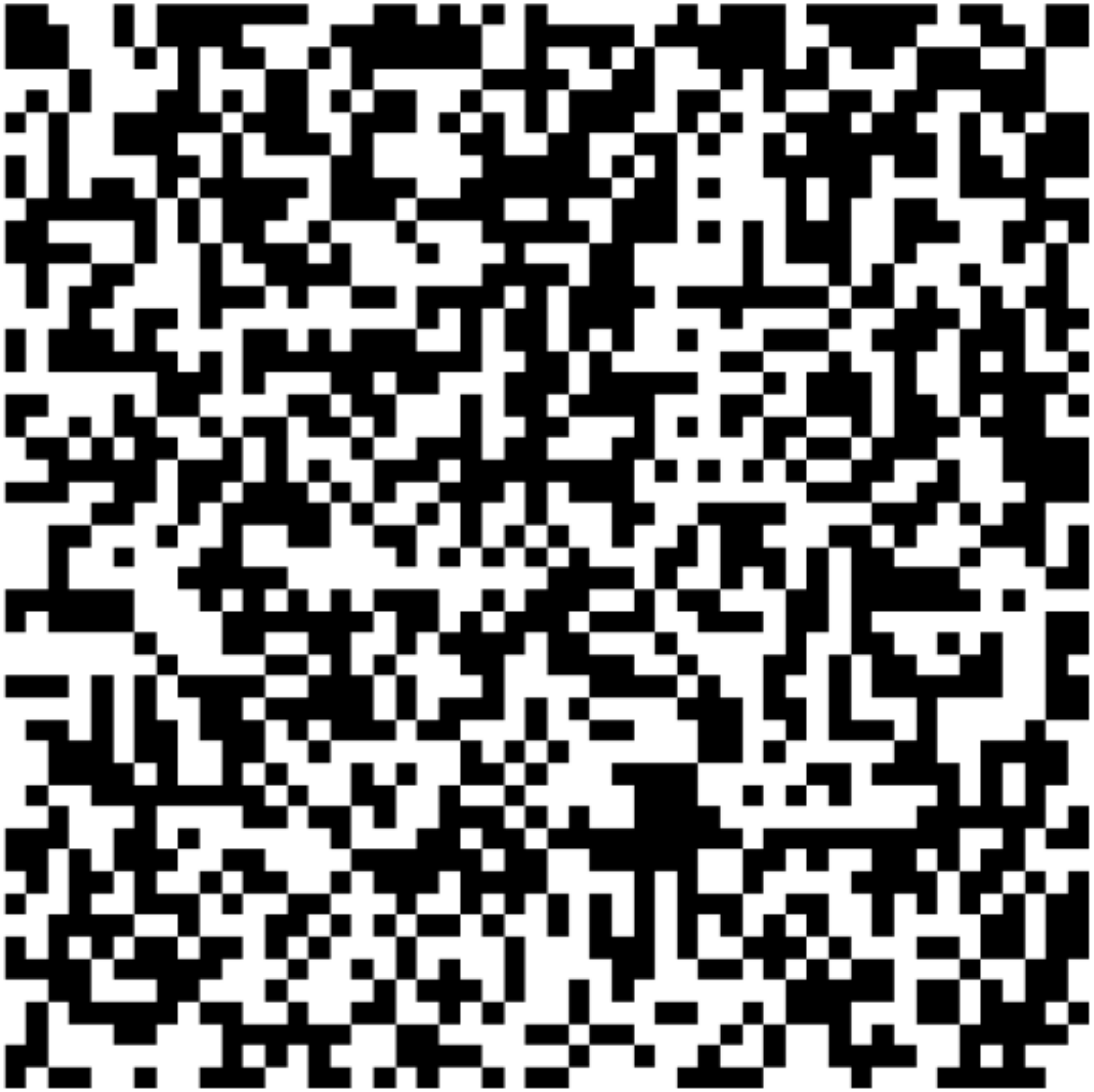
(Sublevel set in black)

# Sublevel Filtration

$\nu = 0$

Many loops, fewer distinct components

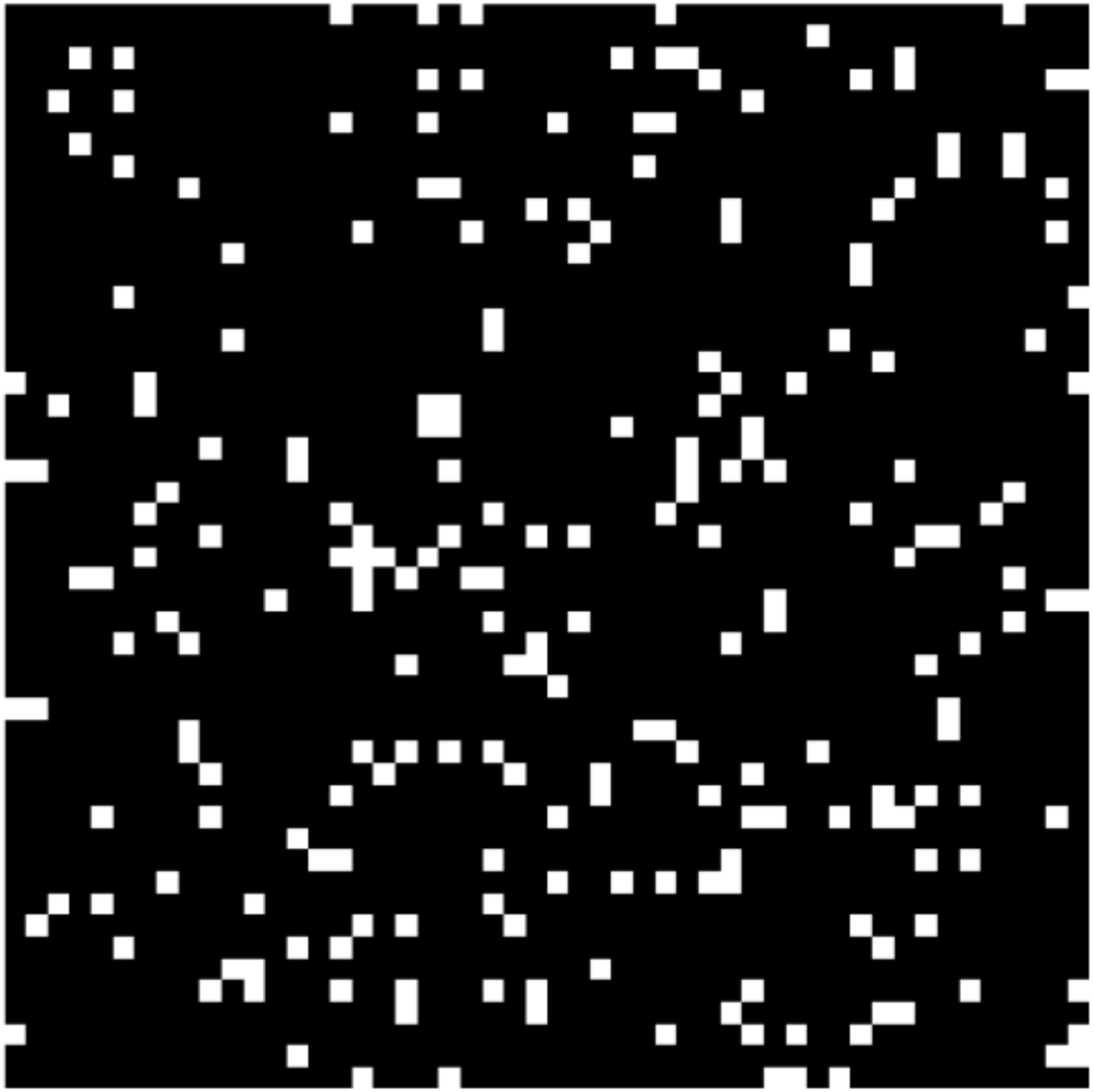
(Sublevel set in black)



# Sublevel Filtration

$\nu = 1$

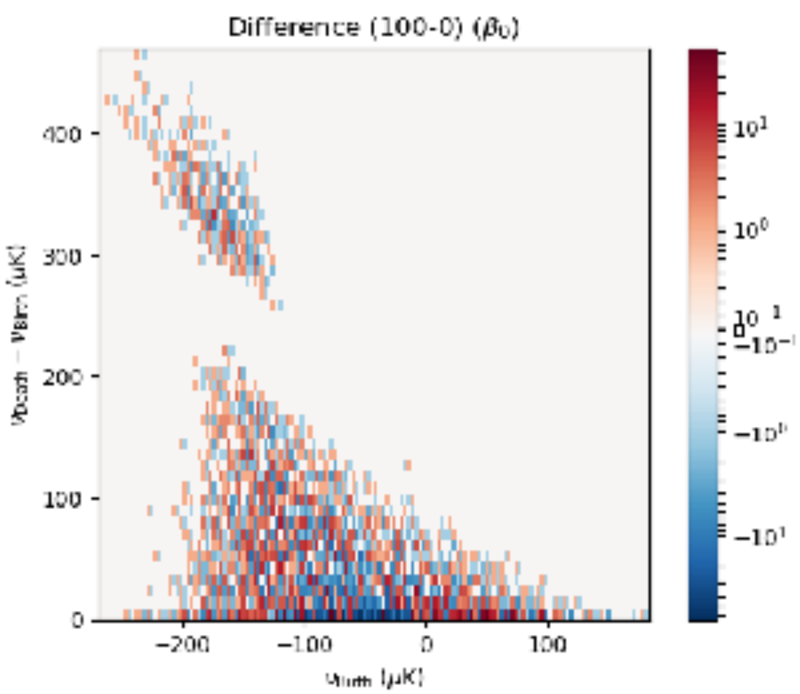
One connected component, many loops have been filled in

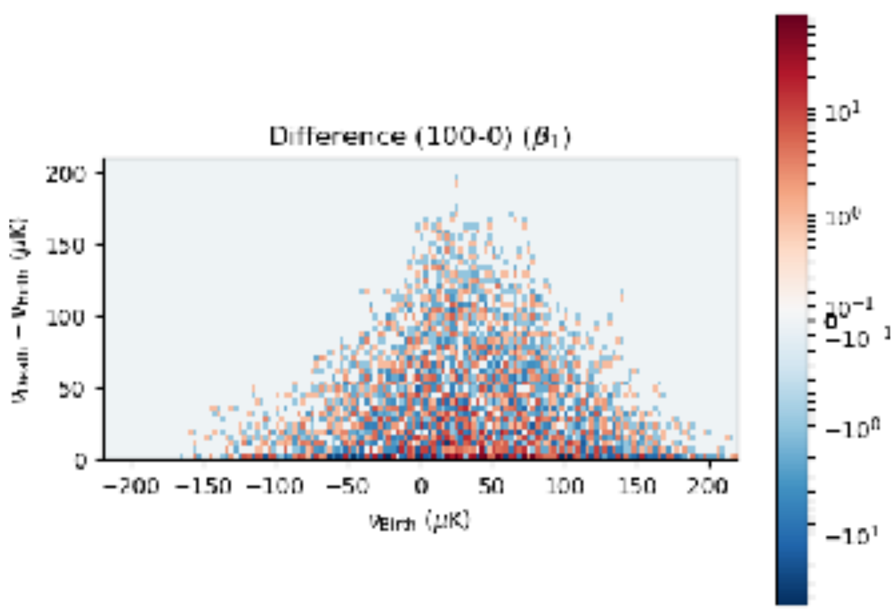


(Sublevel set in black)

# Sensitivity to Non-Gaussianity

- We first carried out TDA for local NG and with low-resolution maps ($\ell_{max}\sim$ 1024) as a warmup, more in our pipeline.
- We binned the persistence diagrams for different $f_{NL}$, & computed the likelihood function:



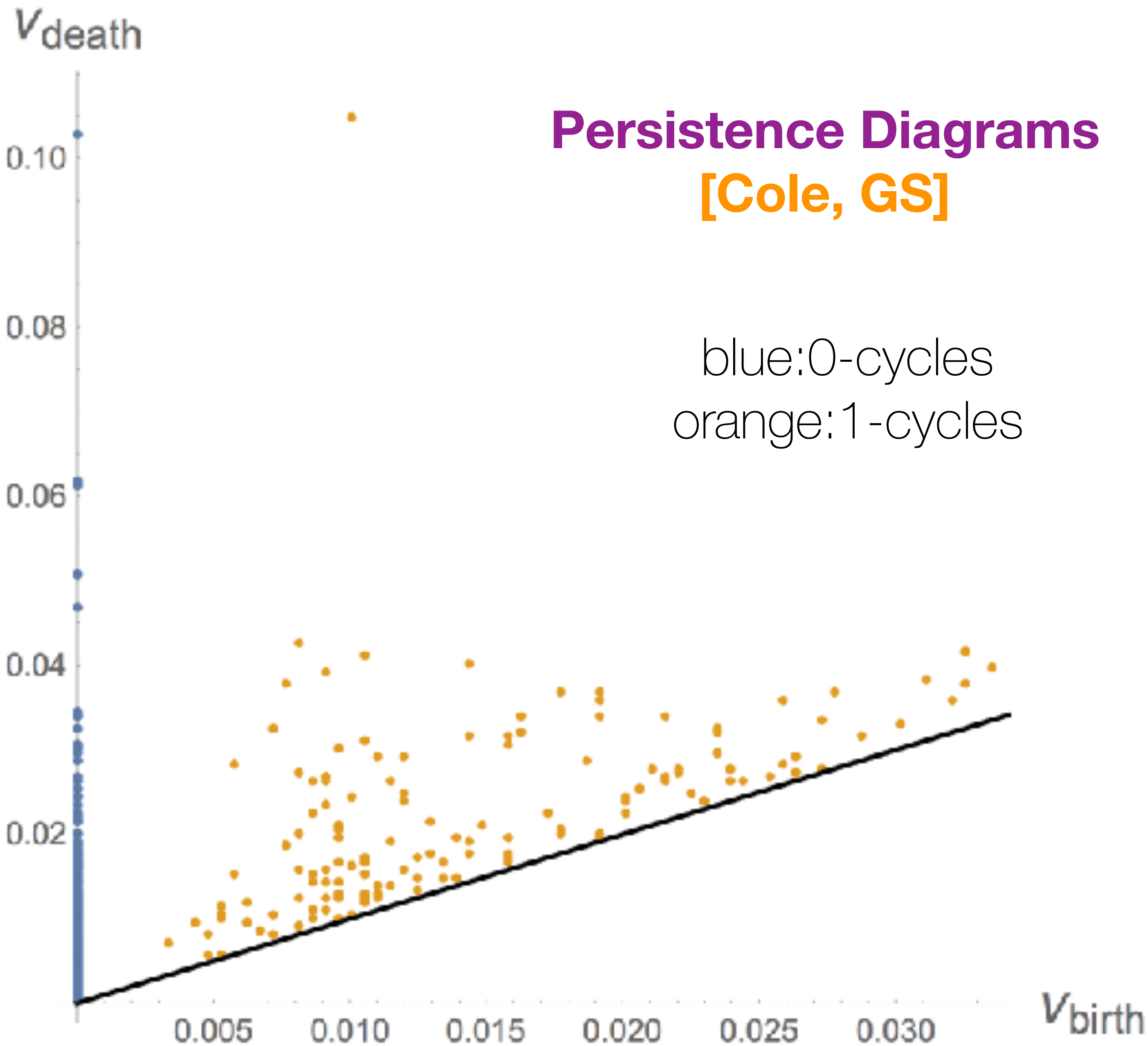

**1σ error bar**

| Statistic | $\Delta f_{NL}$ |
|---|---|
| $\beta_0$ | 67.4 |
| $\beta_1$ | 66.1 |
| $\beta_0 + \beta_1$ | 60.6 |
| $PD_0$ | 39.1 |
| $PD_1$ | 37.4 |
| $PD_0 + PD_1$ | 35.8 |

- More sensitive statistic than Minkowski functionals or Betti number curves, **PDs strengthens topological analysis significantly**.
- N.B. Lower resolution maps used here compared to Planck's.
- Potentially more powerful for **other shapes of NG**.

# Applying TDA to String Vacua

# TDA for String Vacua

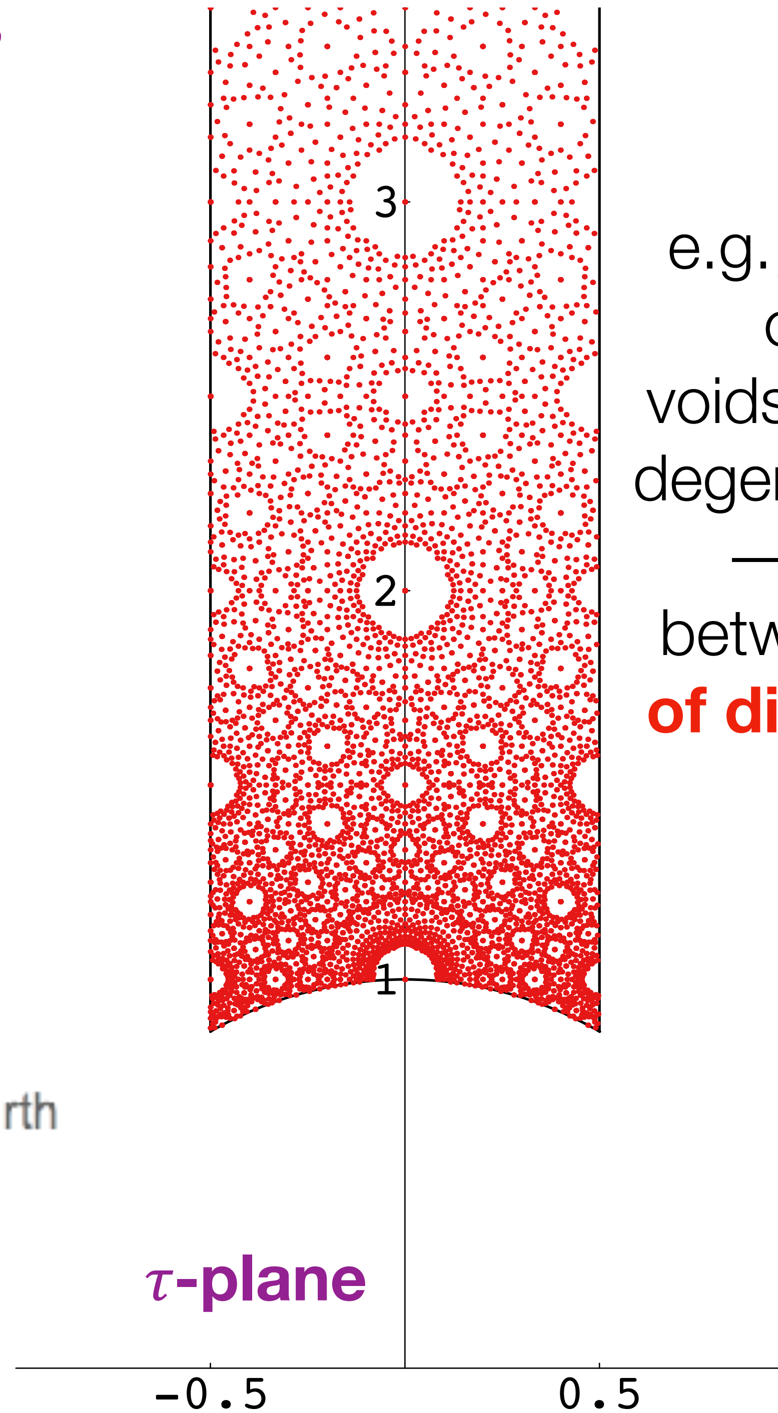



**Persistence Diagrams**
**[Cole, GS]**

blue:0-cycles
orange:1-cycles

See also **[Cirafici]** for the barcodes

**"Topological Complexity"**




e.g., for flux vacua on rigid CY, voids correspond to degeneracy of vacua — relationship between **topology of distribution** and **physics**

**$\tau$-plane**

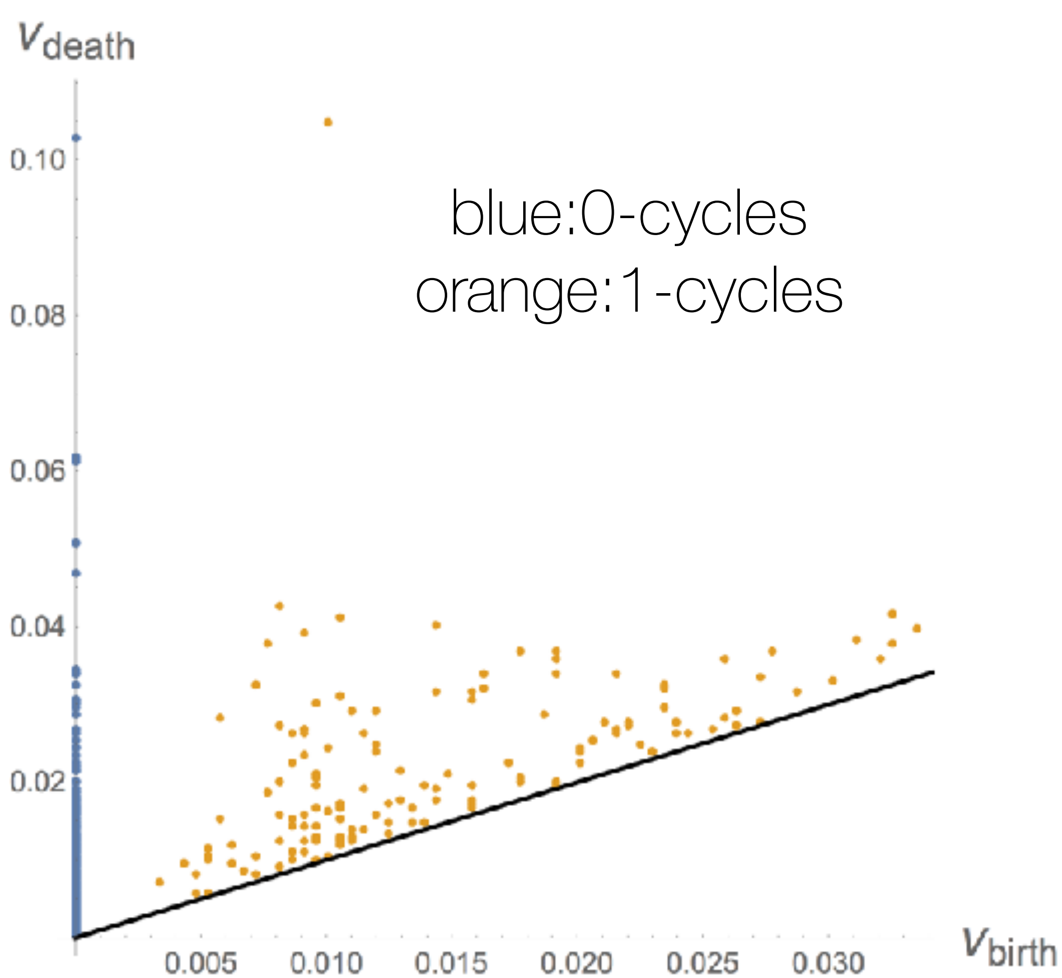
# Toy Example: IIB Flux Vacua on Rigid CY

- **Superpotential** W = A$\tau$+B where the flux quanta:

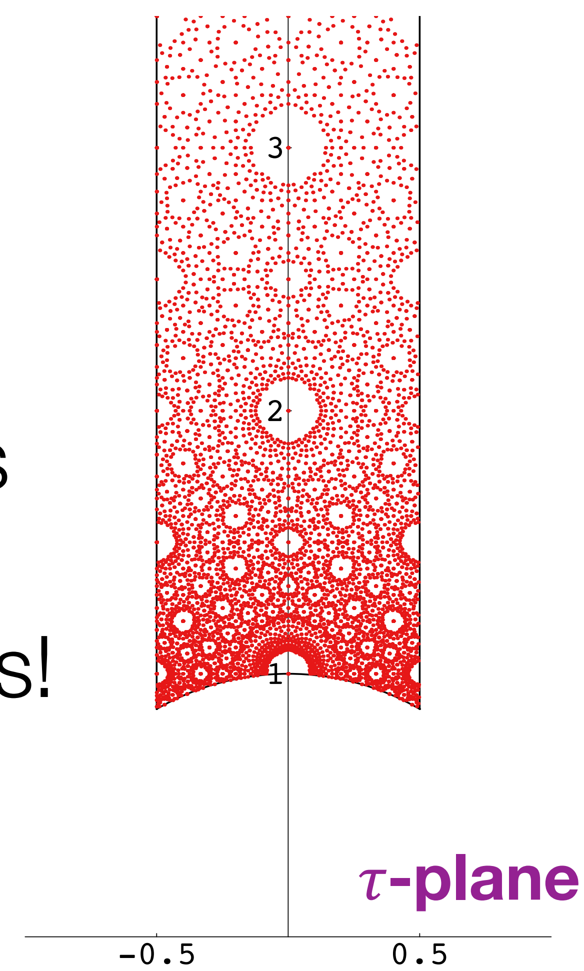
$$A = -h_1 - ih_2, \quad B = f_1 + if_2, \quad h_1, h_2, f_1, f_2 \in \mathbb{Z}$$

subject to **tadpole cancellation**: $N_{\text{flux}} = f_1 h_2 - h_1 f_2 \leq L_{\max}$

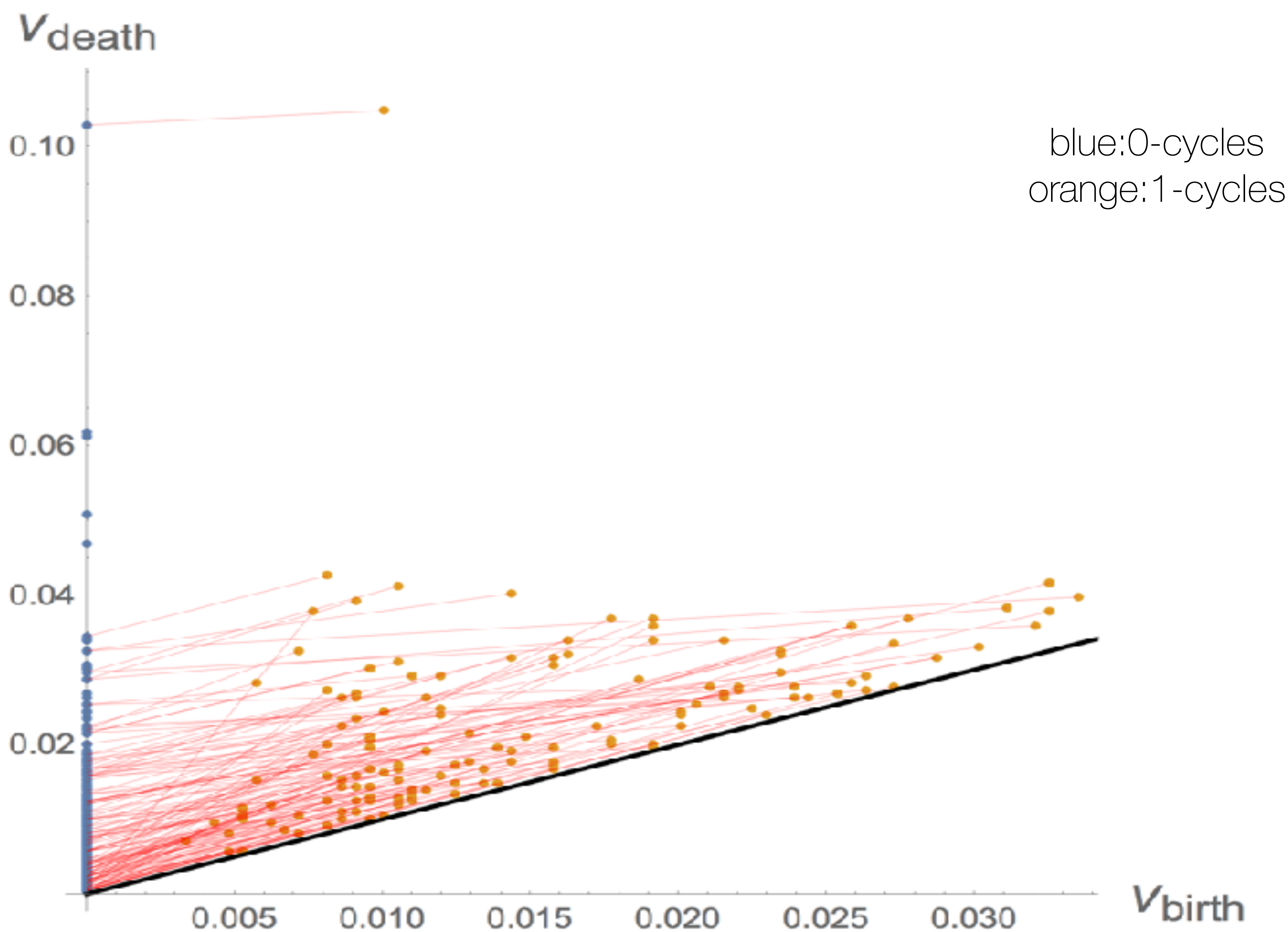
- Vacua are mapped to the **fundamental domain** using SL(2,Z).




**Short-lived**
but **correlated**
topological features
most apparent in
persistence diagrams!



# Persistence Pairing




[Cole,GS]

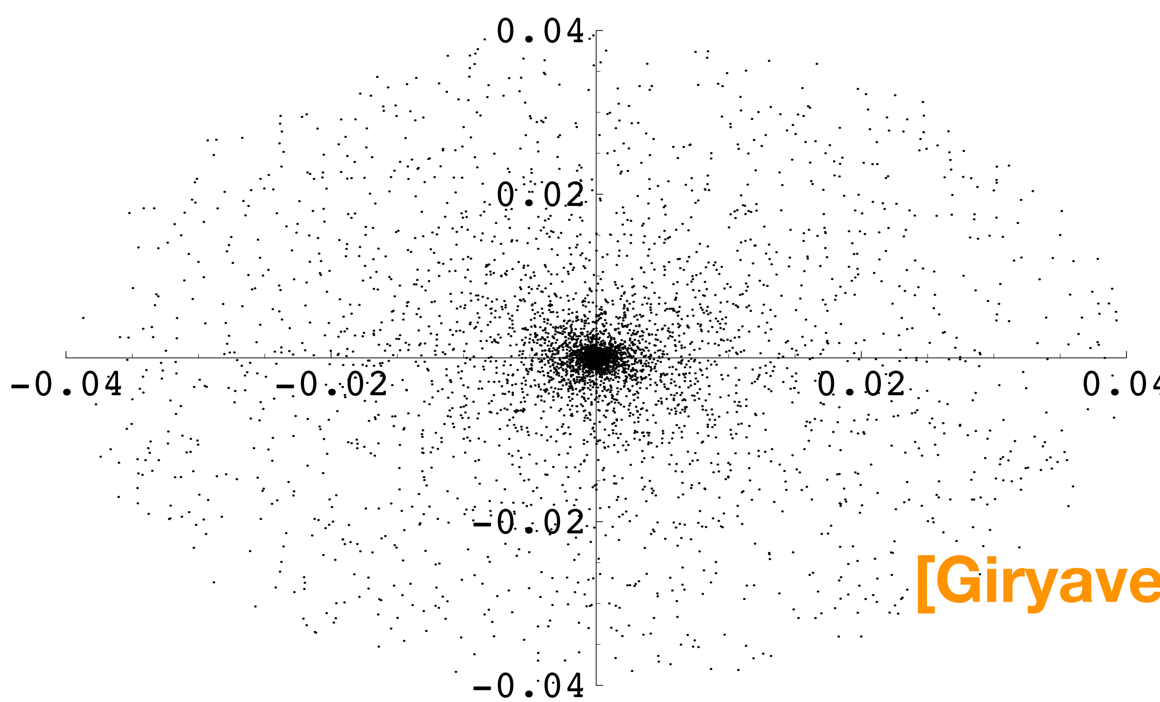
# Flux Vacua on CY Hypersurface

- In general, not possible to visualize a *higher dim.* data space.
- For example, flux vacua of IIB orientifold on CY hypersurface:

$$\sum_{i=1}^{4} x_i^8 + 4x_0^2 - 8\psi\, x_0x_1x_2x_3x_4 = 0\,, \quad x_i \in \mathbf{WP}^4{}_{1,1,1,1,4}$$

has $h^{1,1}=1$, $h^{2,1}=149$ and discrete symmetry $\Gamma=Z_8{}^2 \times Z_2$. The only $\Gamma$-invariant moduli: complex structure modulus $\psi$ & axio-dilaton $\tau$.

**Projecting onto the x=1-ψ plane**



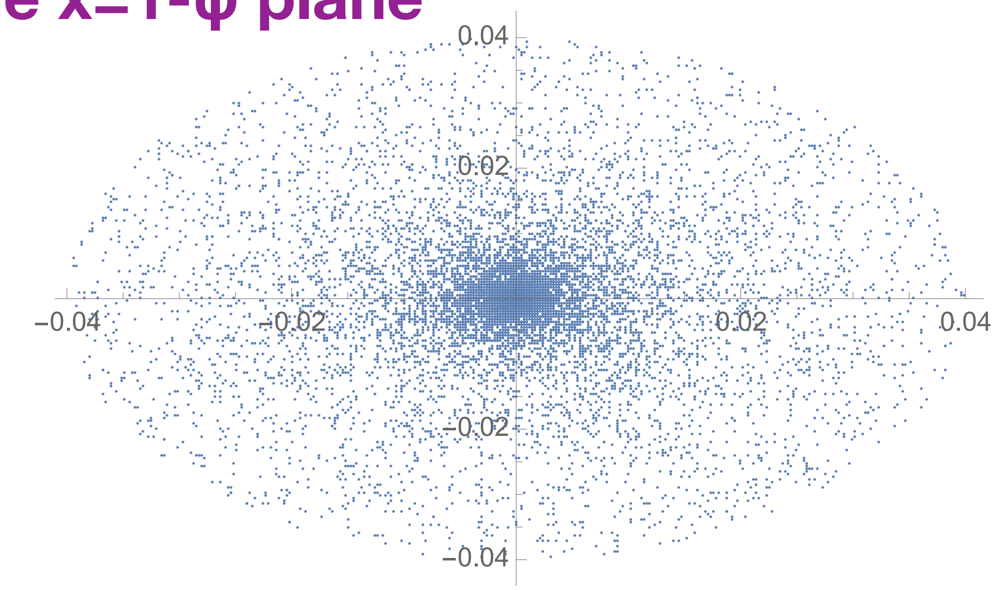

**[Giryavets, Kachru, Tripathy]**

- TDA can more systematically diagnose the vacuum structure.

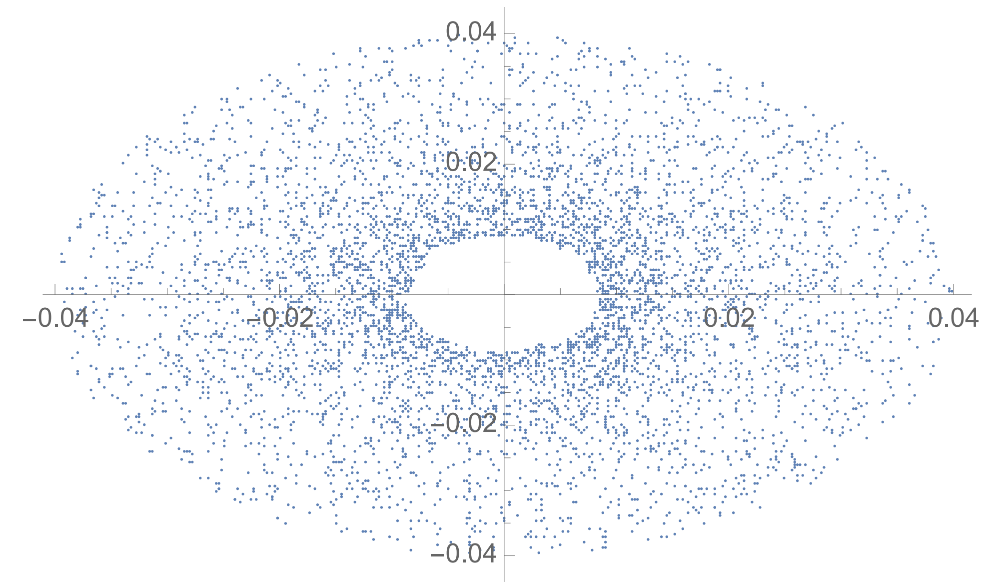
**[Cole,GS]**

# Flux Vacua on CY Hypersurface

**Projecting onto the x=1-ψ plane**




$$\rho < \rho_{\mathrm{th}}$$

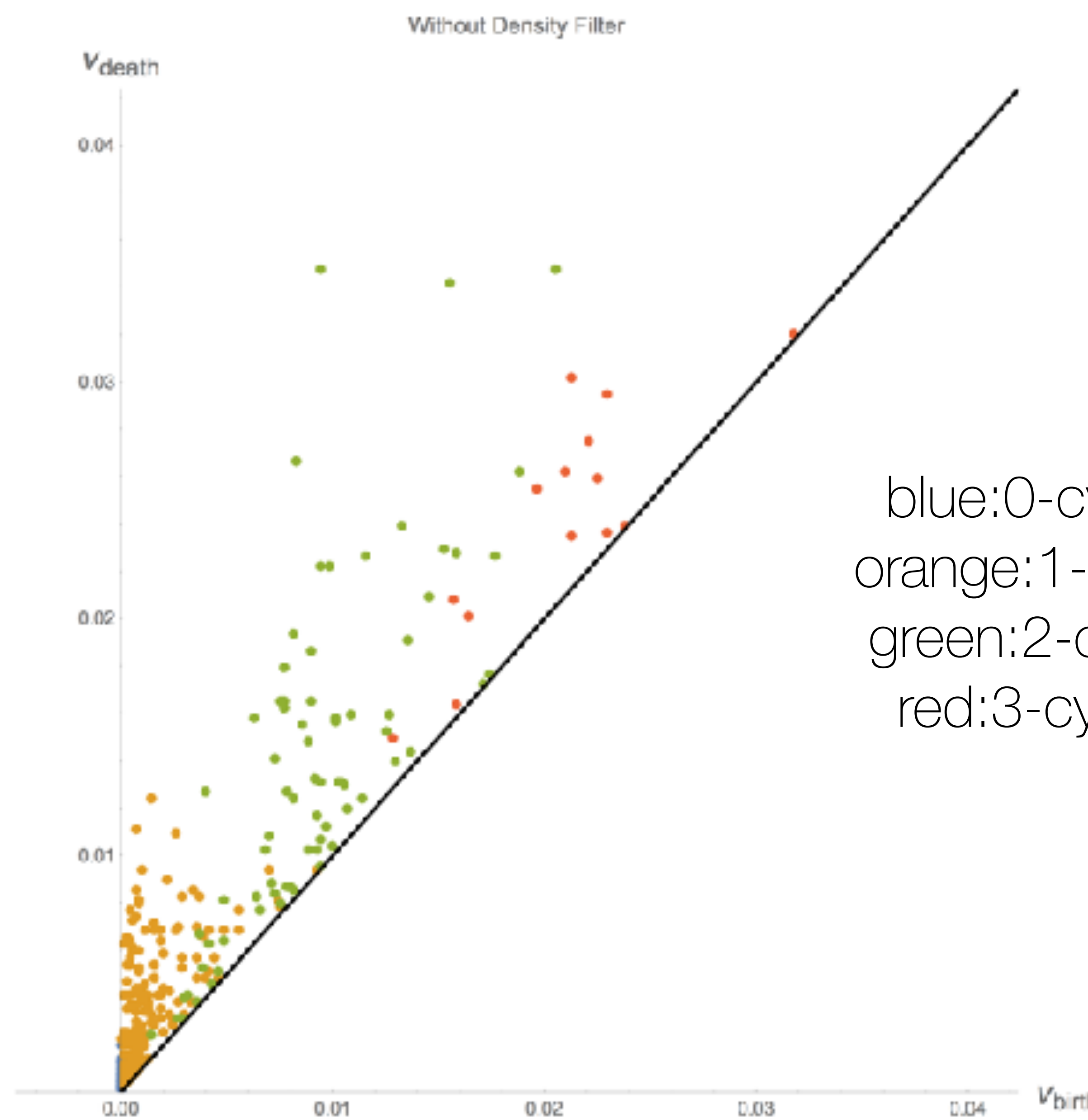



- To identify cluster, apply density cutoff (excises cluster, results in identifiable void)
- Does this cluster/void exist in the full four-dimensional space? (Might not if clustering correlates with structure in axiodilaton.) Are there significant higher dimensional features?
  - These questions can be answered with persistent homology

[Cole,GS]

# Flux Vacua on CY Hypersurface

- To identify cluster, apply density cutoff (excises cluster, results in identifiable void)
- We found a long-lived 1-cycle in the full four-dim. space and only observe short-lived higher dimension features (sampling noise)

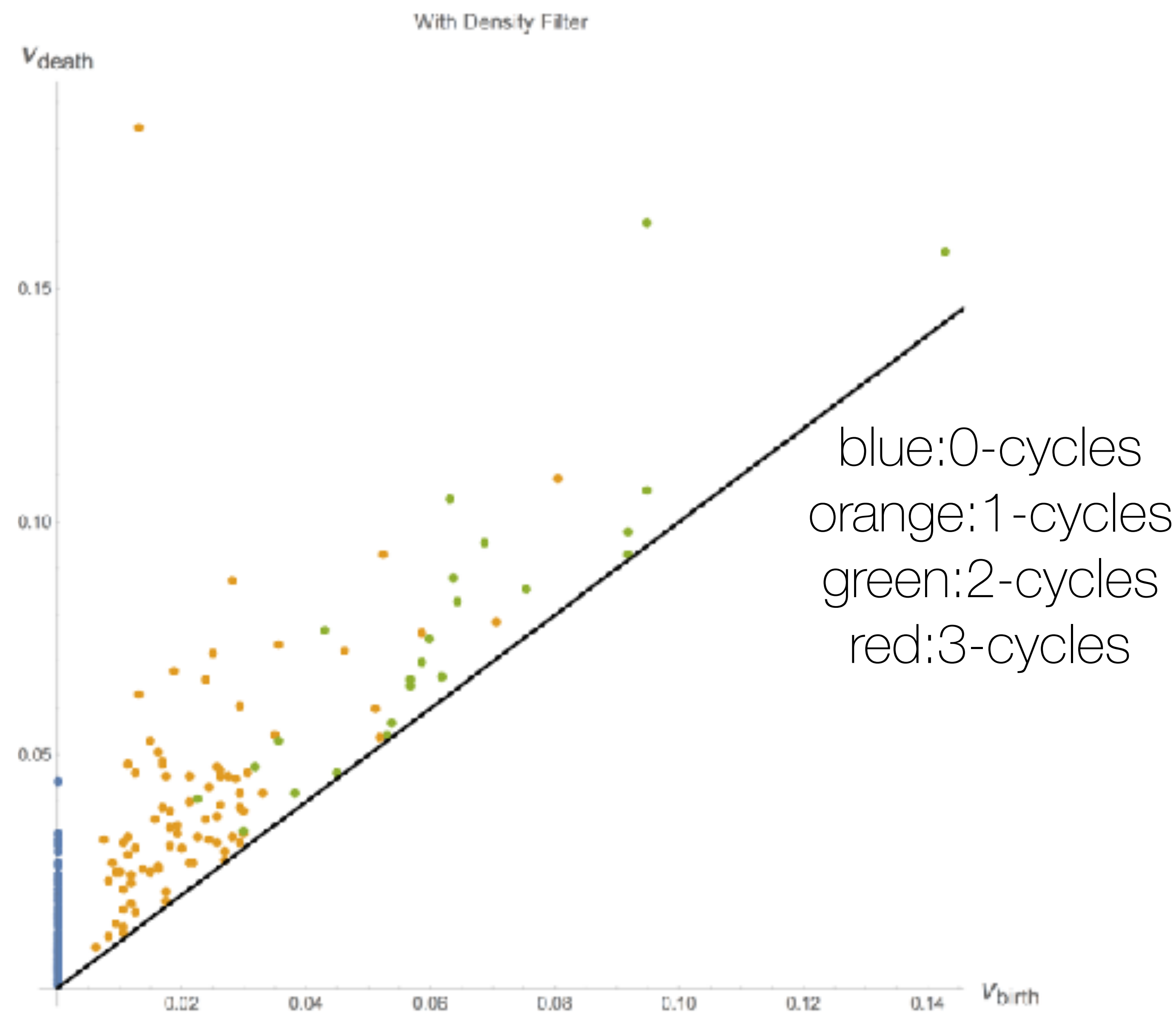



blue:0-cycles
orange:1-cycles
green:2-cycles
red:3-cycles

[Cole,GS]

# Flux Vacua on CY Hypersurface

- To identify cluster, apply density cutoff (excises cluster, results in identifiable void)
- We found a long-lived 1-cycle in the full four-dim. space and only observe short-lived higher dimension features (sampling noise)

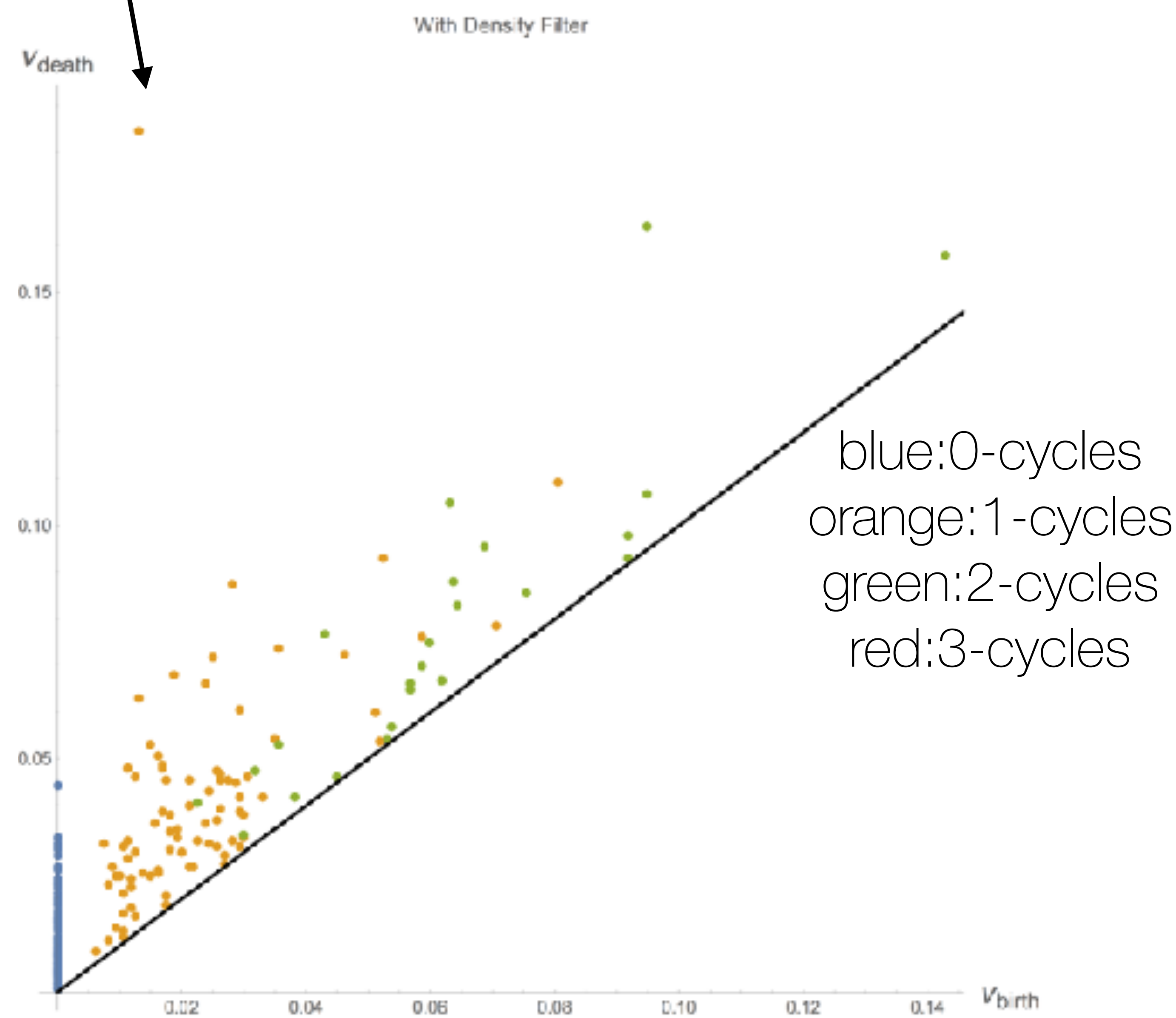



[Cole,GS]

# Flux Vacua on CY Hypersurface

- To identify cluster, apply density cutoff (excises cluster, results in identifiable void)
- We found a long-lived 1-cycle in the full four-dim. space and only observe short-lived higher dimension features (sampling noise)

**long-lived 1-cycle**

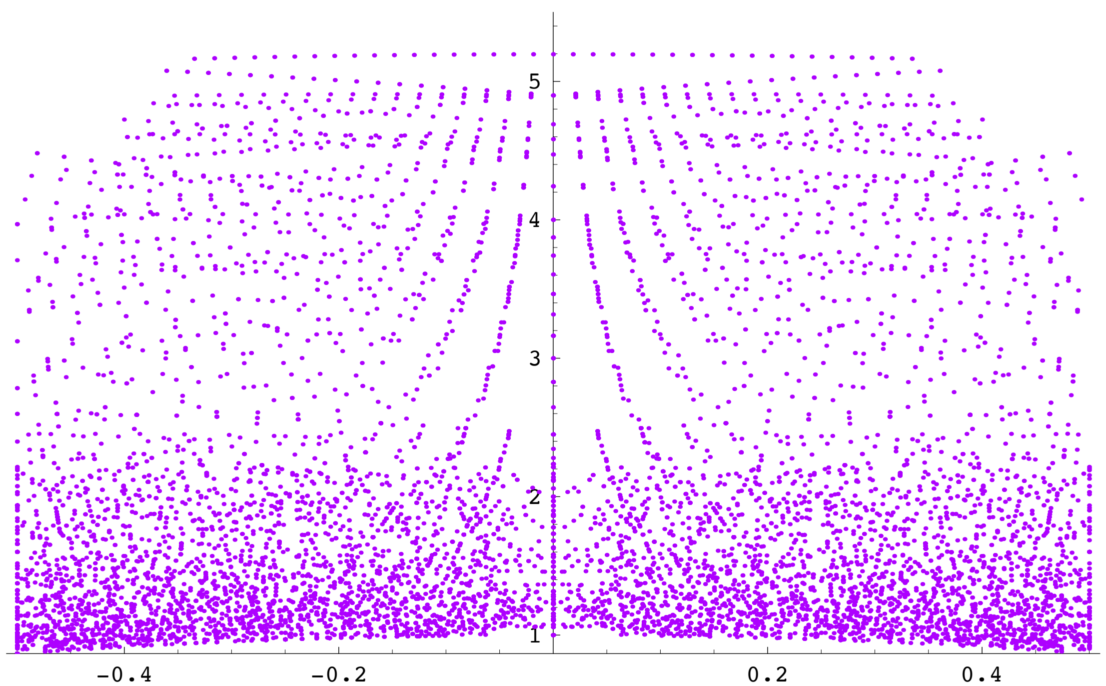



[Cole,GS]

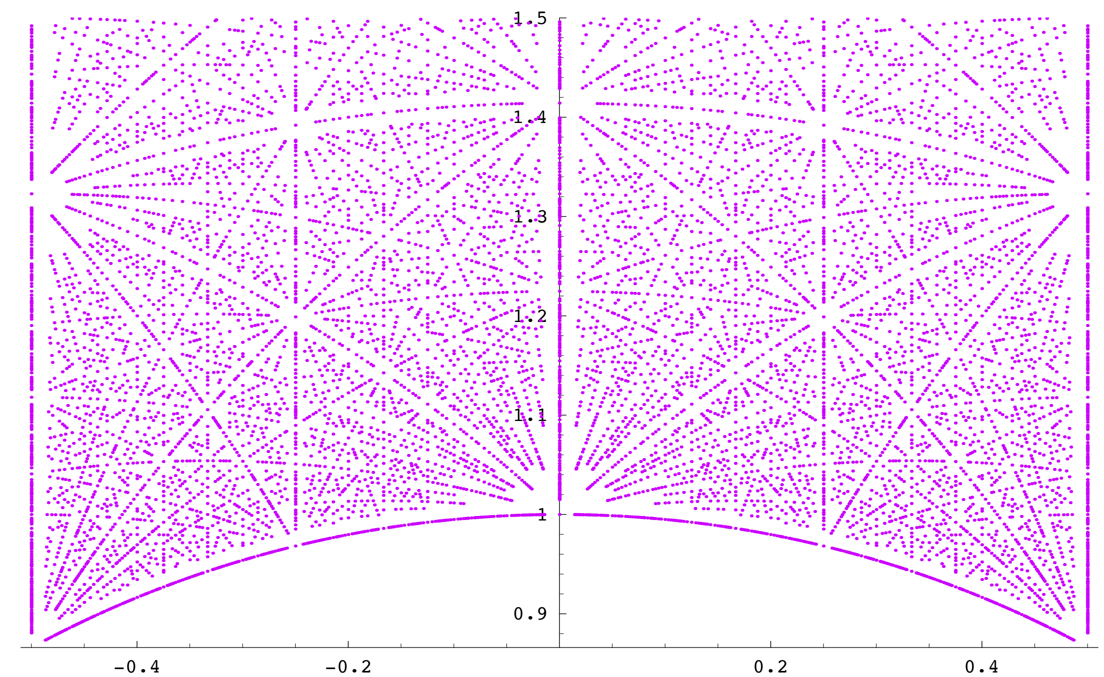
# Flux Vacua on Symmetric $T^6$

- Factorizable $T^6 = (T^2)^3$ with equal complex structure $z_1 = z_2 = z_3 = z$
- Two complex moduli: complex structure modulus z and axio-dilaton $\tau$.
- Number-theoretical methods were used to find distributions of vacua with W=0 and with discrete symmetries **[DeWolfe, Giryavets, Kachru,Taylor]**

**Generic vacua on z-plane**



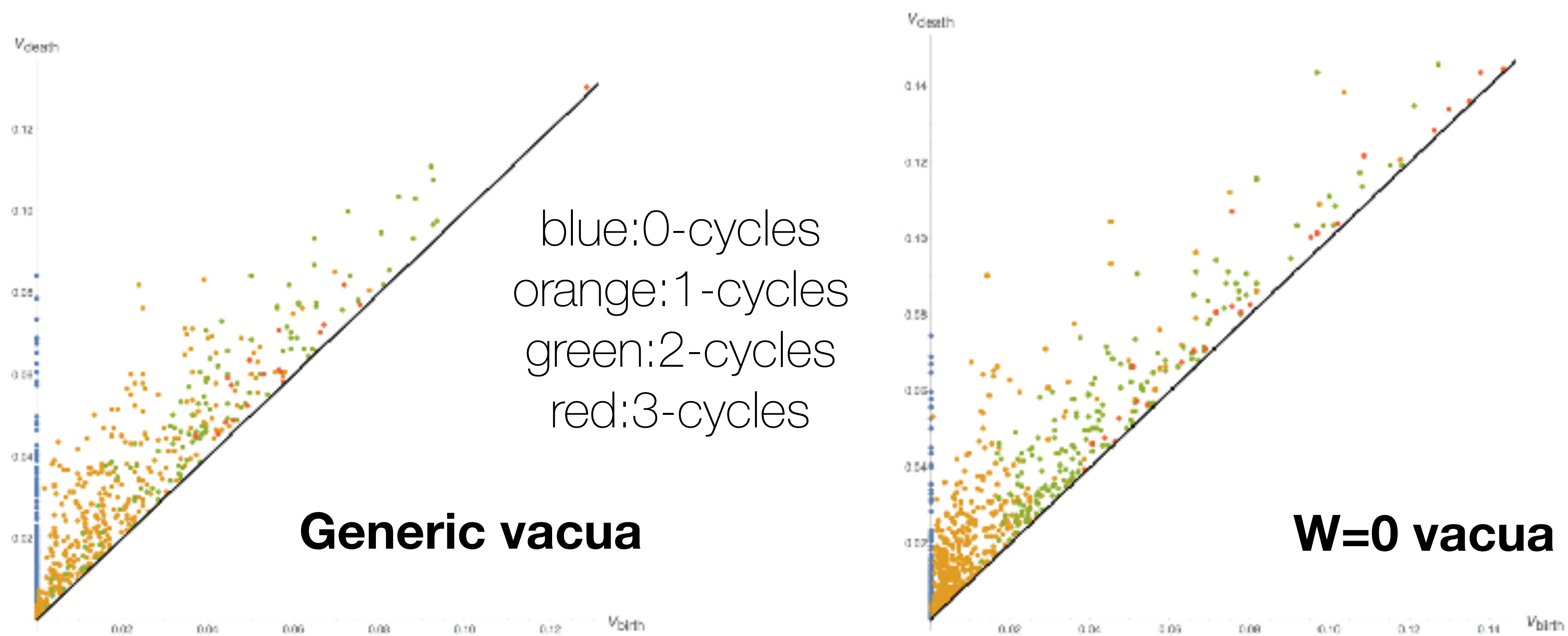

**W=0 vacua on z-plane**




- How do “cuts” like restricting to W=0 vacua (e.g., discrete R-symmetry, motivated by **[Nelson, Seiberg]**) change the topology of distribution?

# Flux Vacua on Symmetric $T^6$

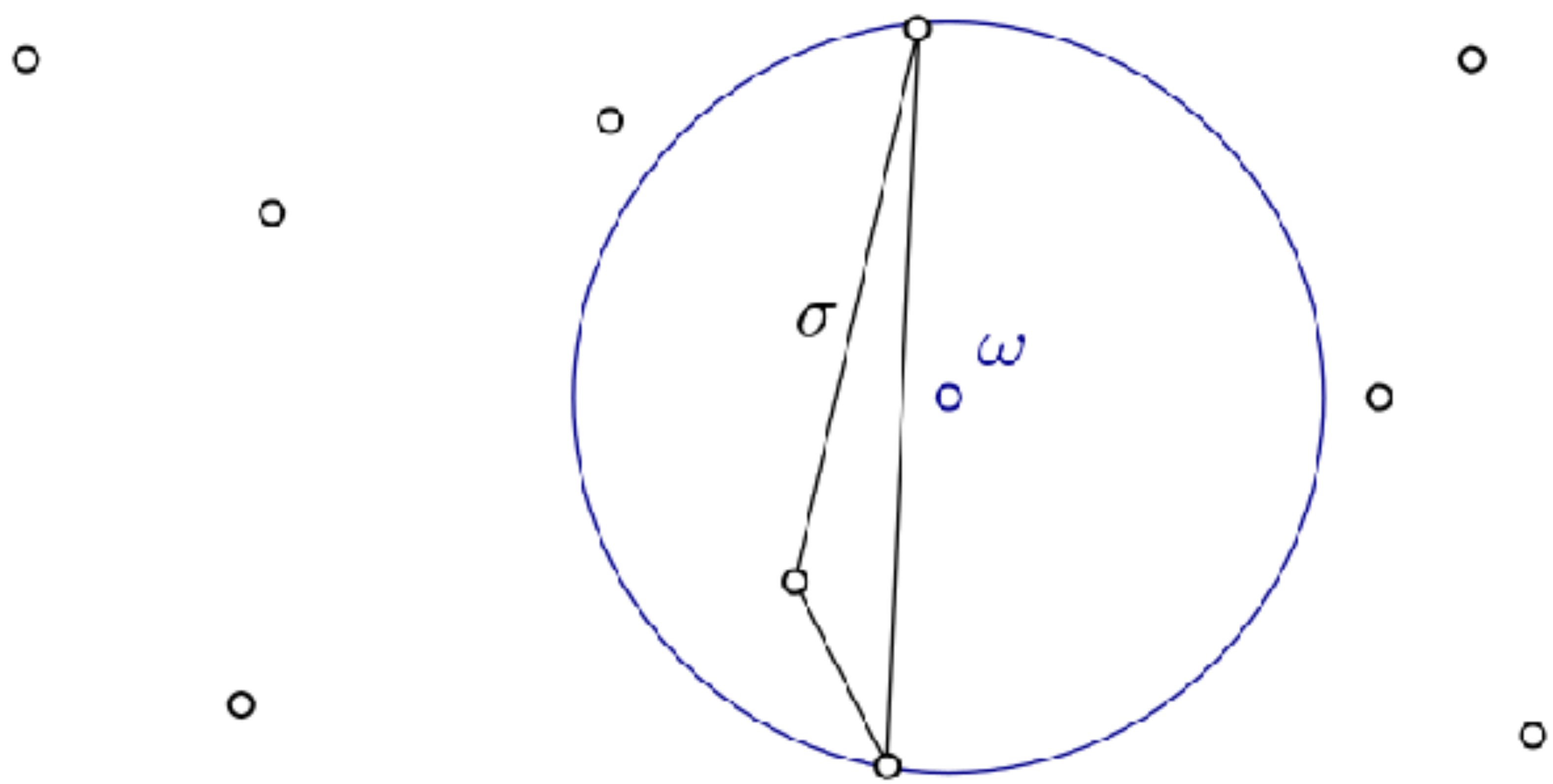
- Comparing persistent homology:

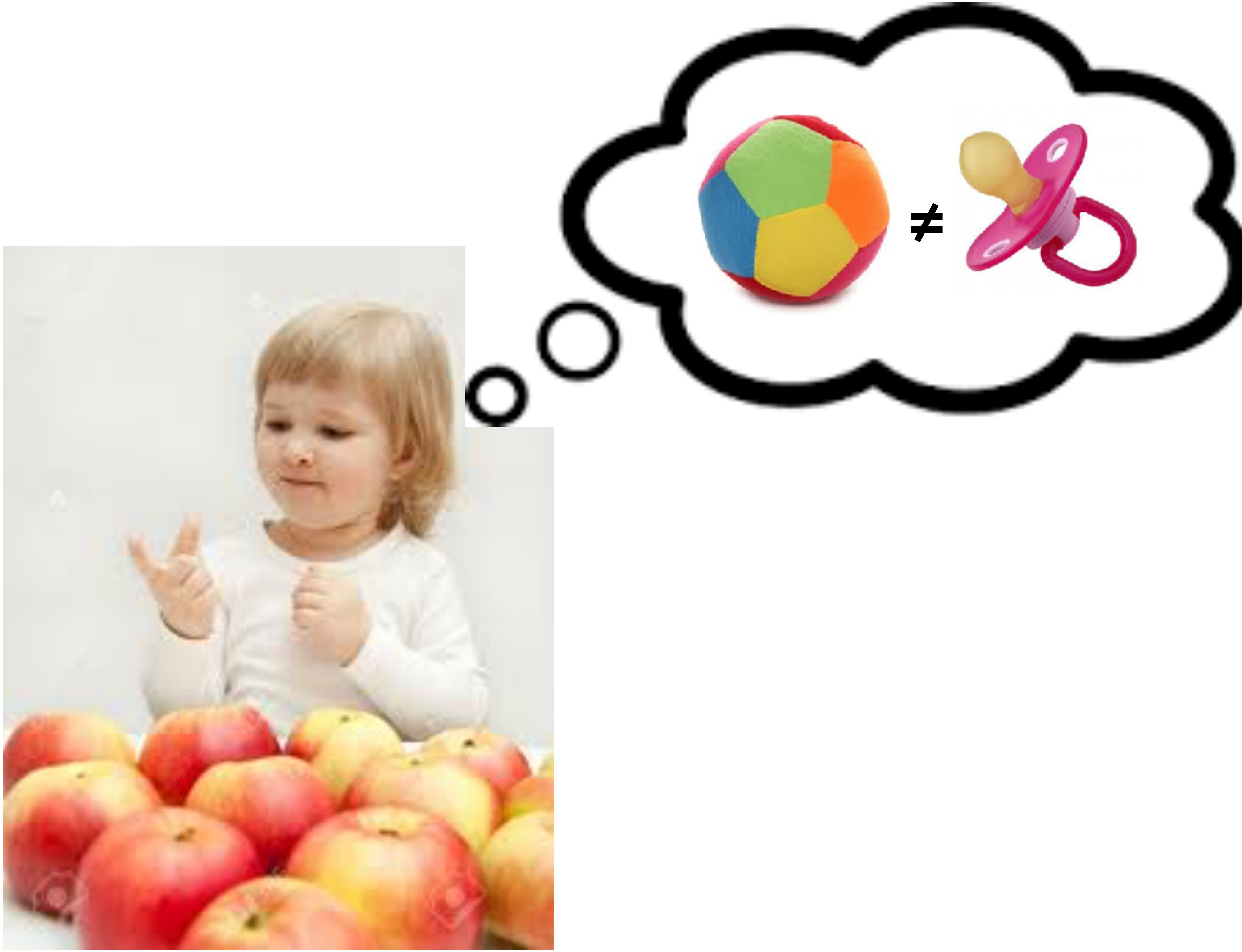


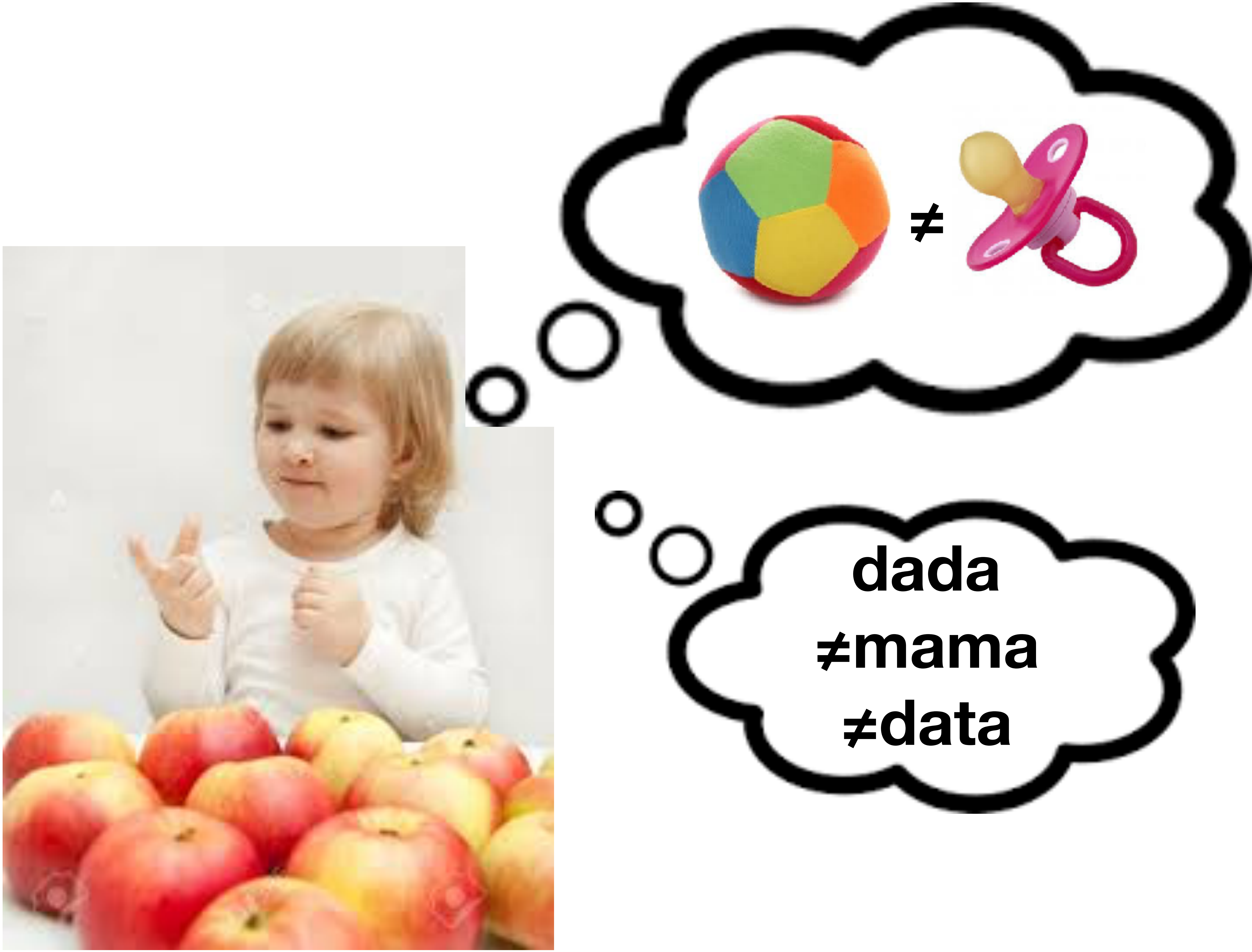

- W=0 cut adds complexity! Long-lived higher dimensional topological features differs from that for generic vacua.

# Sampling in TDA

- We can't realistically include all $10^{500}$ vacua as vertices
- Can sample the topology via the **witness complex**:
  - From the entire point cloud Z, choose a ***landmark set L*** as the complex's vertices. Often chosen randomly or via sequential maxmin algorithm
  - Let $m_k(z)$ be the distance from some z∈Z to the (k+1)-nearest landmark point. Then, given filtration parameter $\nu$, the simplex $[l_0 l_1 \dots l_k]$ is included in the witness complex if $\max\{d(l_0, z), d(l_1, z), \dots, d(l_k, z)\} \leq \nu + m_k(z)$



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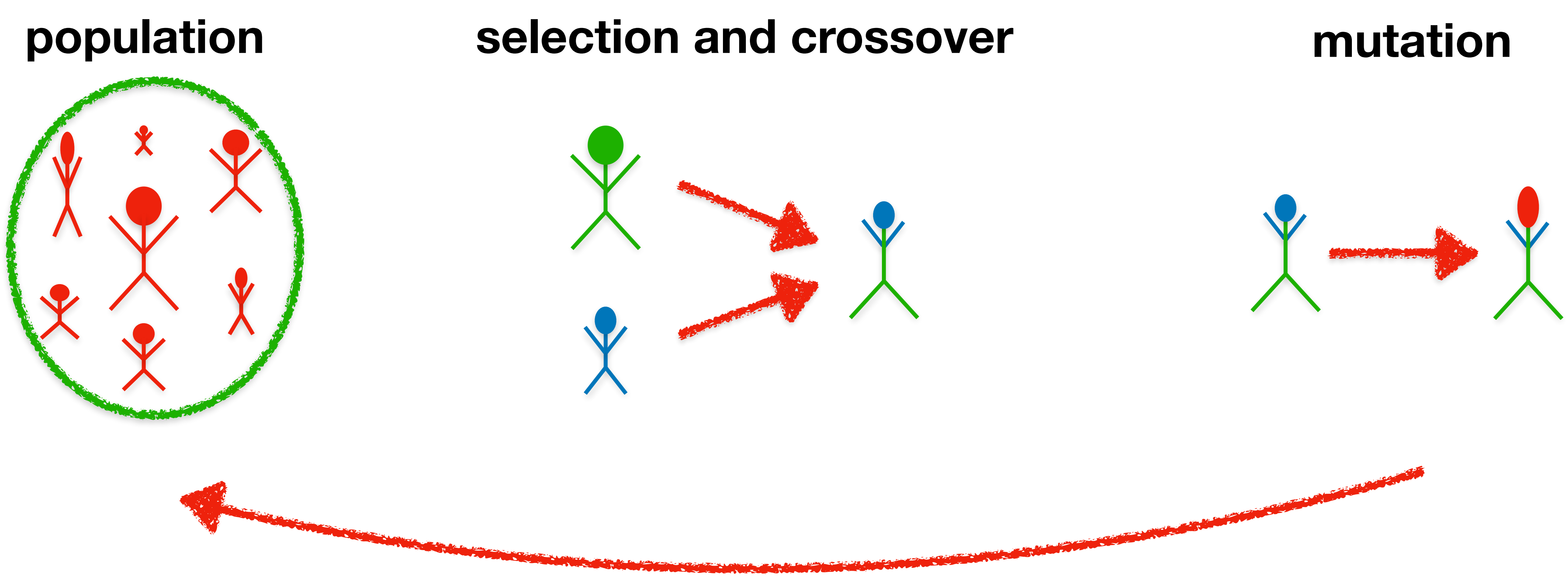
# Cherry Picking?

# Purposeful Search

- Is there a way to effectively search for string vacua with desired properties (e.g., small $\Lambda$, or large axion decay constant)?
- Nature has provided a solution: **evolution!**



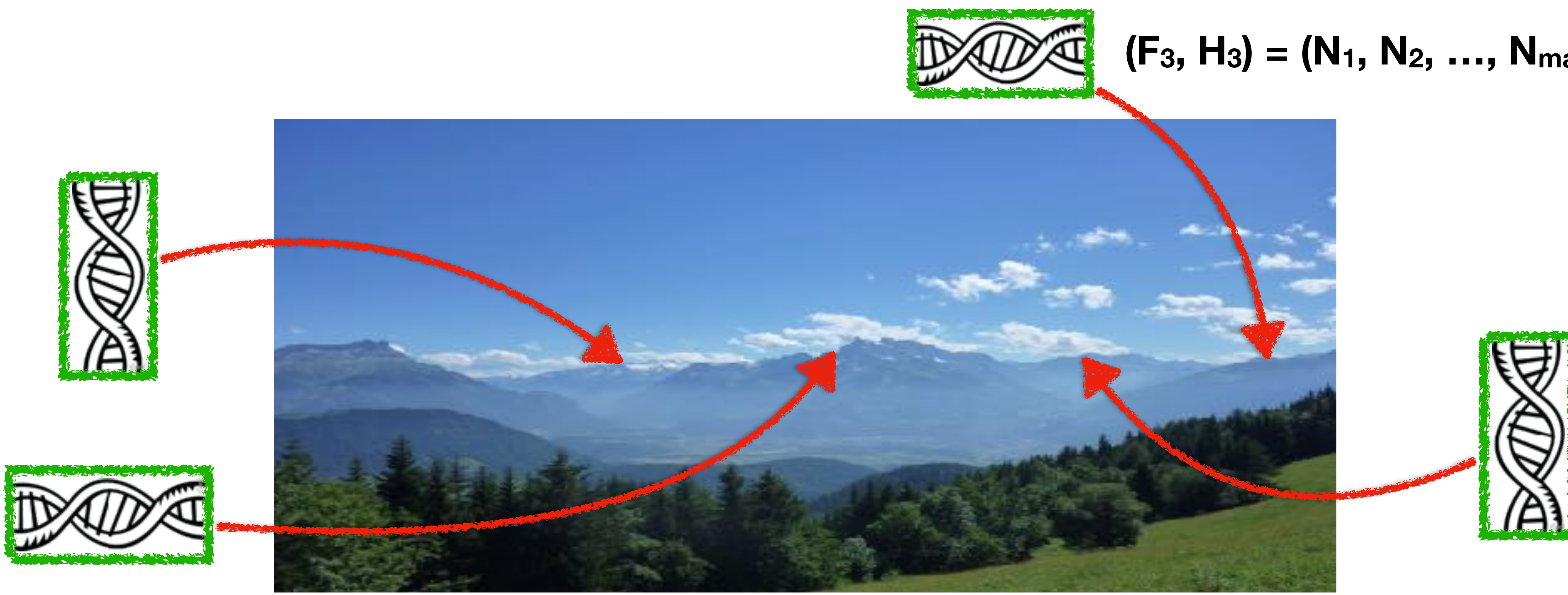

- Starting with a population of string vacua, we can “breed” them (allowing for mutation as in Nature) to get a fitter population.

# Searching the Landscape of Flux Vacua with Genetic Algorithms [Cole, Schachner, GS]

**General motivation:** **find vacua with phenomenologically interesting features**

**Idea:** **mimic biology by imitating evolution**

$(F_3, H_3) = (N_1, N_2, \ldots, N_{max})$



# Conclusions

# Conclusions

- Applications of TDA to **cosmological datasets** and **string vacua**.
- Persistence diagrams strengthen constraints on local **non-Gaussianities**, and potentially other shapes & other observables.
- Techniques we developed have been applied to analyze the structure of string vacua. We performed initial study of simple flux vacua.
- Next step is to examine the **topology** of string vacua point clouds with desired features, supplementing earlier work on ***statistics***:
  - Enhanced symmetries **[DeWolfe, Giryavets, Kachru, Taylor], …**
  - Particle physics features **[Marchesano, GS, Wang];[Dienes];[Gmeiner, Blumenhagen, Honecker, Lust, Weigand], [Douglas, Taylor], …**
- **Genetic Algorithms** can effectively search for vacua with desired properties (minimizing $g_s$, $W_0$, $\Lambda$, or maximizing $f_{axion, ..}$). They can potentially be used to test various conjectures of quantum gravity.