Neutrino Pair Process from Nucleon-Nucleon Bremsstrahlung in SN Matter with Chiral Effective Potential

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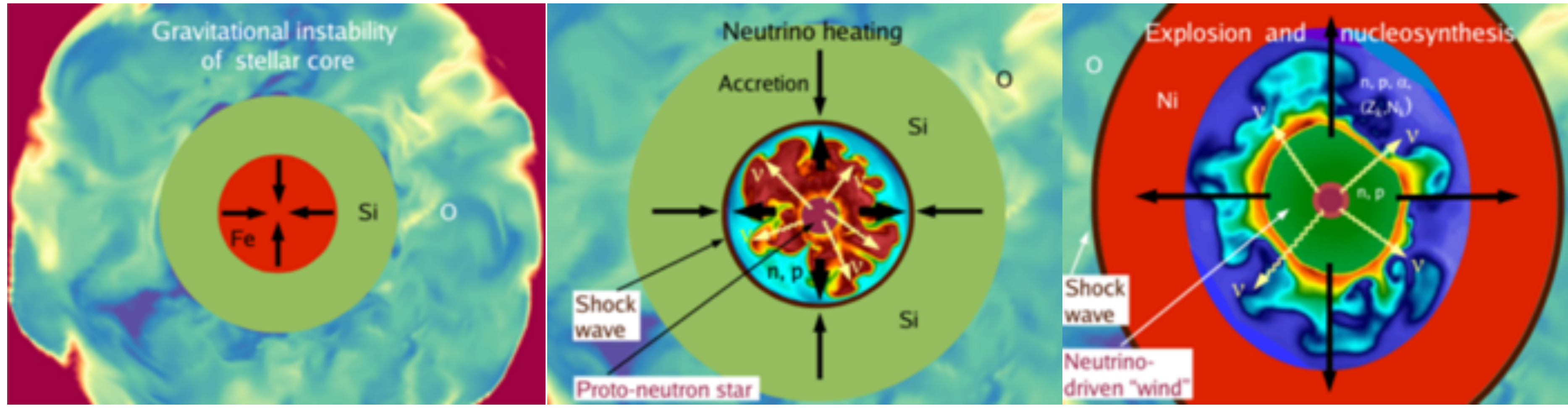
Guo & Martinez-Pinedo, ApJ 887, 58 (2019)

Outline

- ✦ Motivation
- ✦ Calculating the bremsstrahlung rates:
  nuclear potential/T-matrix, results & discussion
- ✦ Summary

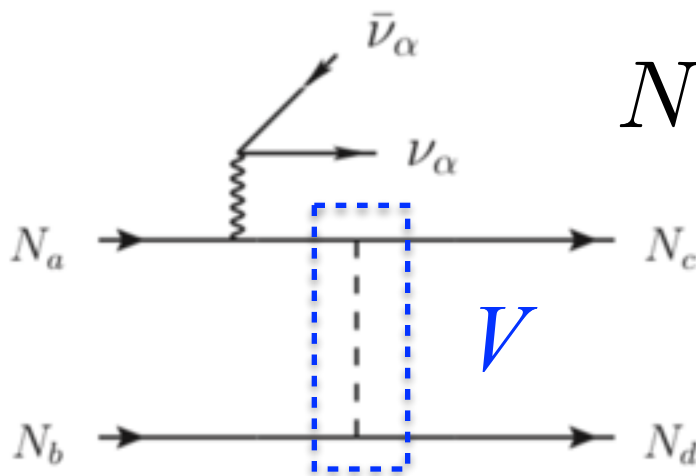
<u>Motivation</u>



Neutrino processes in hot/dense matter play important role in core-collapse supernova dynamics as well as neutrino signals and nucleosynthesis; **accurate calculations are highly demanded**.

We study neutrino pair-absorption/emission from NN bremsstrahlung which is important for neutrino production/transport in some conditions, especially for the heavy-flavor ones.

neutrino pair production: $N_a + N_b \to N_c + N_d + \nu_\alpha + \bar{\nu}_\alpha$

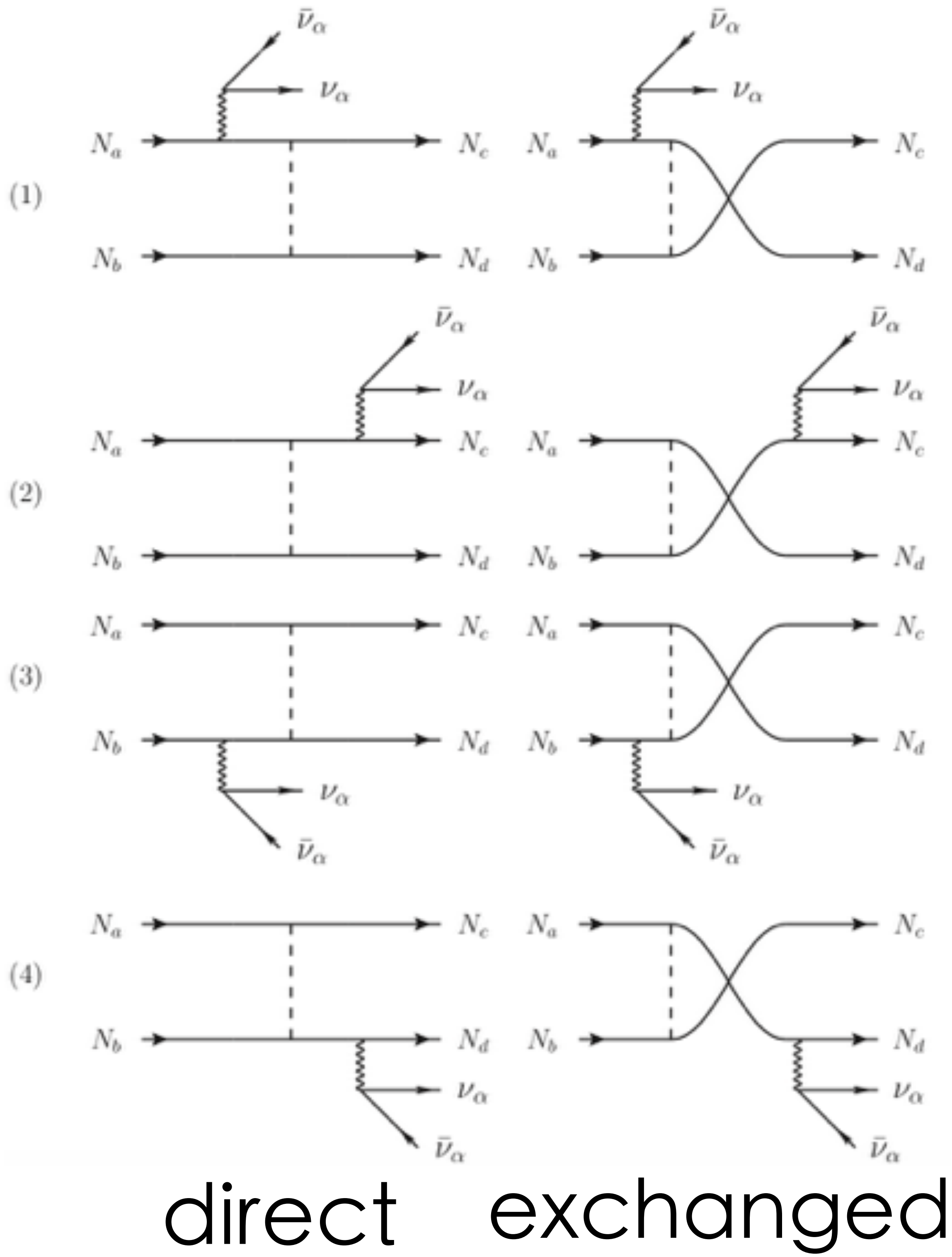


$N_{a,b,c,d}$: neutrons/protons

Axion, dark photon/scalar, or dark matter emissions from NN bremsstrahlung in SN/neutron stars.

diagrams considered and amplitudes

$$N_a + N_b \to N_c + N_d + \nu_\alpha + \bar{\nu}_\alpha$$



$$\mathcal{M}^{(1)} = \frac{G_F g_A^a}{2\sqrt{2}} \langle cd|V\sigma_i^{(a)}|ab\rangle \frac{l_i}{\omega}$$

$$\mathcal{M}^{(2)} = -\frac{G_F g_A^c}{2\sqrt{2}} \langle cd|\sigma_i^{(c)}V|ab\rangle \frac{l_i}{\omega},$$

$$\mathcal{M}^{(3)} = \frac{G_F g_A^b}{2\sqrt{2}} \langle cd|V\sigma_i^{(b)}|ab\rangle \frac{l_i}{\omega},$$

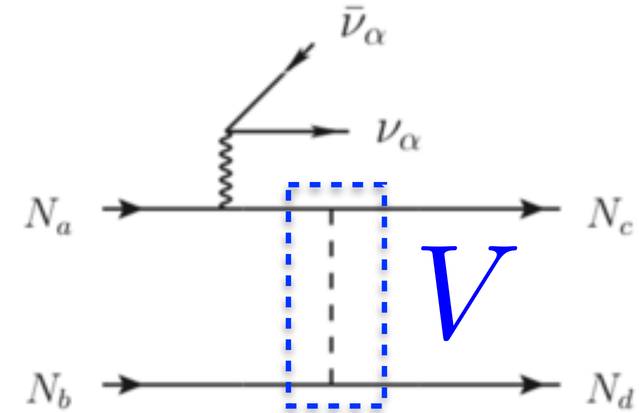
$$\mathcal{M}^{(4)} = -\frac{G_F g_A^d}{2\sqrt{2}} \langle cd|\sigma_i^{(d)}V|ab\rangle \frac{l_i}{\omega},$$

$$\mathcal{M}_{\rm tot} = \sum_{j=1}^{4} \mathcal{M}^{(j)} = \frac{G_F g_A}{2\sqrt{2}} \langle cd|[V, \sum_{r=1,2} \sigma_i^{(r)}\tau_z^{(r)}]|ab\rangle \frac{l_i}{\omega},$$

$$g_A^p = -g_A^n = g_A \simeq 1.27$$

- only axial-vector term contributes in non-relativistic limit
- ignoring many-body interactions
- what to take for V ?

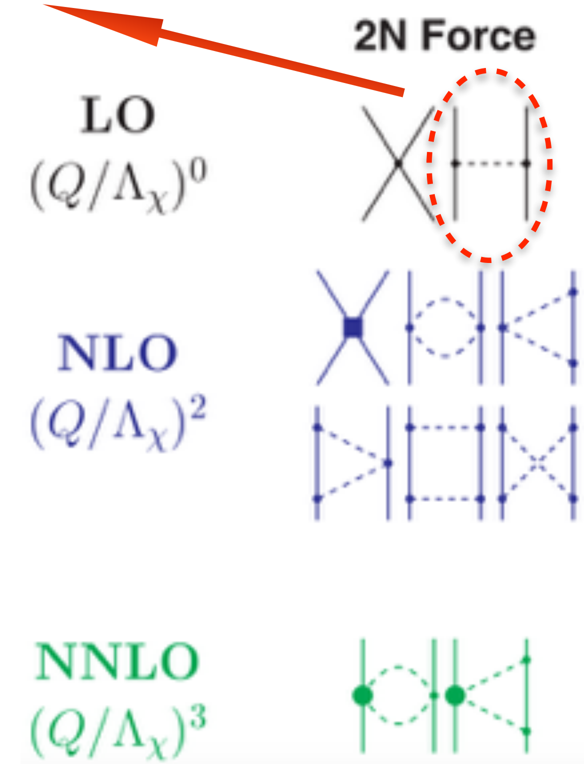
nucleon-nucleon potentials for



V could be (at Born level):

(i) One-Pion Exchange (OPE) potential (Hannestad & Raffelt 1998)

$$V_{\rm OPE}(\vec{p}\,',\vec{p}\,) = -\frac{g_A^2}{4f_\pi^2}\vec{\tau}_1\cdot\vec{\tau}_2\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{q^2+m_\pi^2}$$

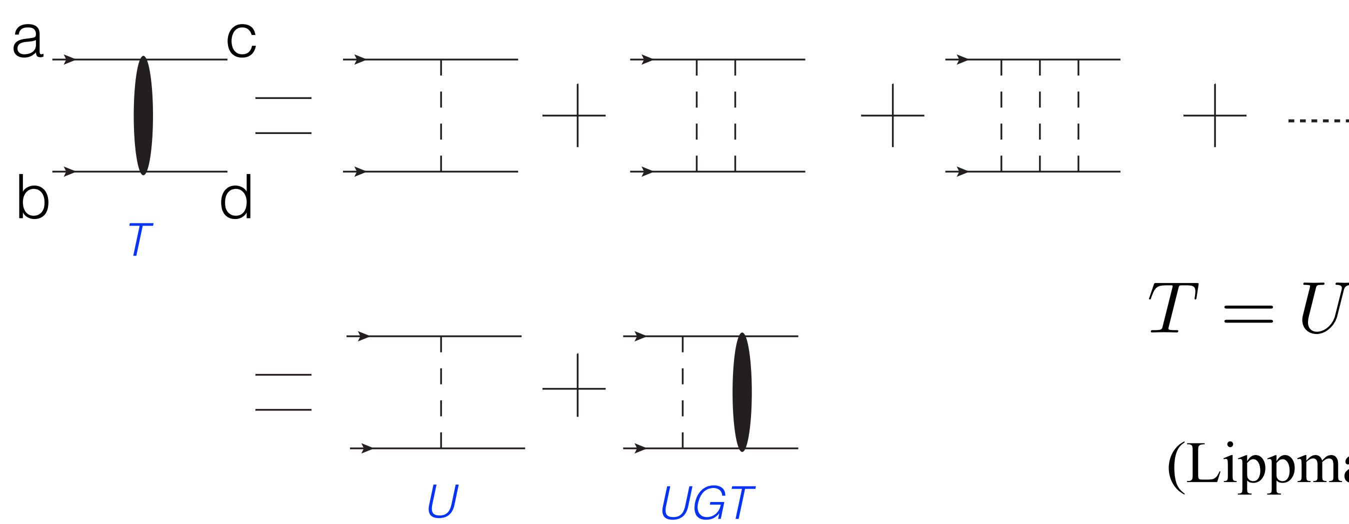


(ii) Chiral effective potential (Bacca *et al.* 2012)

- ✦ hamiltonian built from spontaneously broken chiral symmetry;
- ✦ an expansion in terms of Q/Λ_χ, coefficients fitted to scattering data, reach desired accuracy

figure from D. R. Entem *et al.* 2017

T-matrix formalism

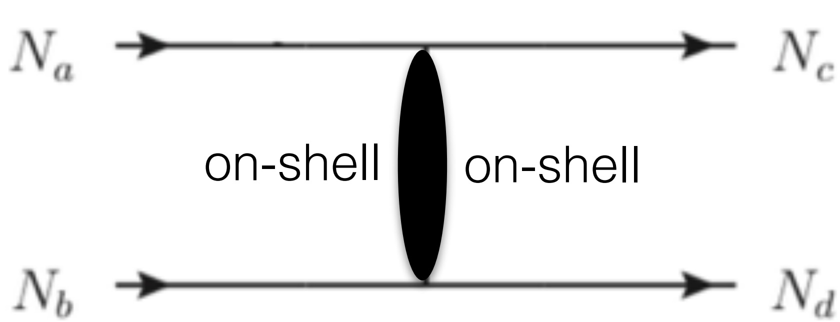


$$T = U + U\frac{1}{E - H_0 + i\epsilon}T$$

(Lippmann-Schwinger Equation)

<u>scattering amplitude is proportional to the T-matrix</u>

nucleon-nucleon scattering



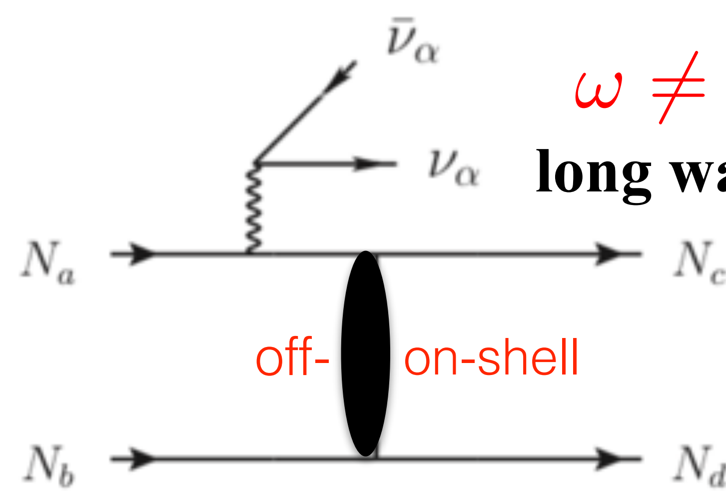
$$E = k^2/m_N = k'^2/m_N$$

on-shell T-matrix

nucleon-nucleon bremsstrahlung

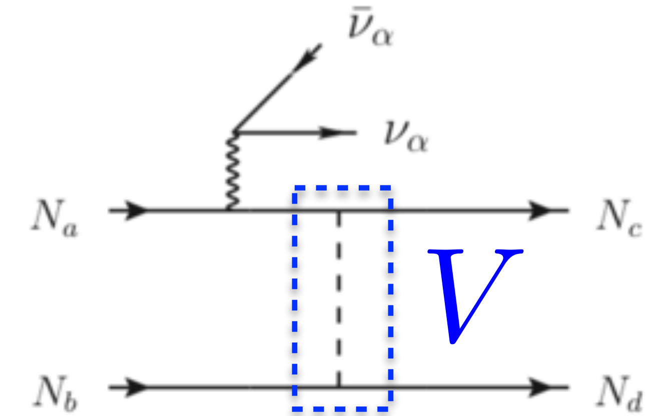
$\omega \neq 0, q \to 0$

long wavelength limit



$$\frac{k^2}{m_N} - \omega = E = \frac{k'^2}{m_N}$$

half-off-shell T-matrix



Take V to be:

vacuum/in-medium T-matrix based on chiral effective potential
(our work)

and for comparison,

OPE potential at Born level (Hannestad & Raffelt 1998)

'on-shell' T-matrix extracted from NN scattering data

(Bartl *et al.* 2014)

Typical conditions in SN

$$T_{SN}(\rho) = 3\ \mathrm{MeV}\ \left(\frac{\rho}{10^{11}\ \mathrm{g\ cm^{-3}}}\right)^{1/3}$$

$$\rho : 1.7 \times 10^{11-14}\ \mathrm{g/cm^3}$$

$$n_B : 10^{-4} - 10^{-1}\ \mathrm{fm}^{-3}$$

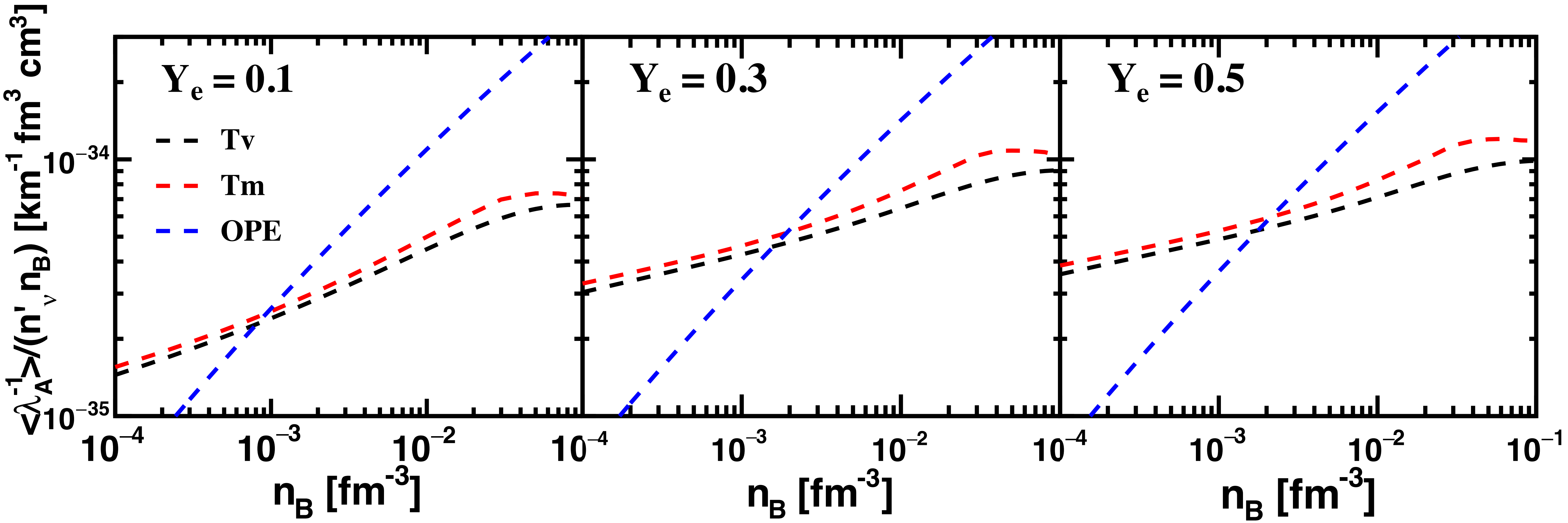
$$Y_e = \frac{n_p}{n_B} = \frac{n_p}{n_n + n_p} < 0.5$$

Opacity due to pair absorption $N + N + \nu + \bar{\nu} \to N + N$

$$\frac{\langle \lambda_A^{-1} \rangle}{n'_\nu} \equiv \frac{1}{n'_\nu} \frac{\int \lambda_A^{-1}(E_\nu) f(E_\nu) E_\nu^2 dE_\nu}{\int f(E_\nu) E_\nu^2 dE_\nu}$$

OPE at Born level, vacuum/in-medium T-matrix from chiral potential

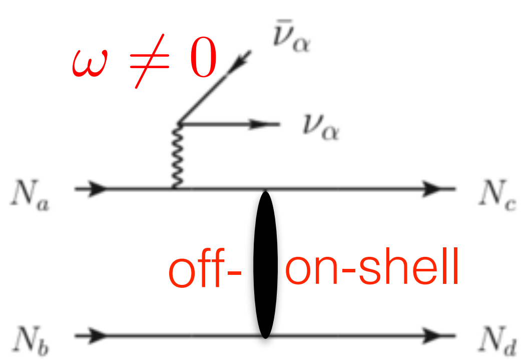
Born level is not enough



Compared to OPE results, using T-matrix:

- ✦ enhanced rates at low density due to resonant nuclear force at low energy (**Bartl *et al.* 2014**)
- ✦ suppressed rates at high density due to repulsive short-range forces & non-perturbative effects
- ✦ medium effects on T-matrix (in-medium T-matrix) enhance rates by ~10%

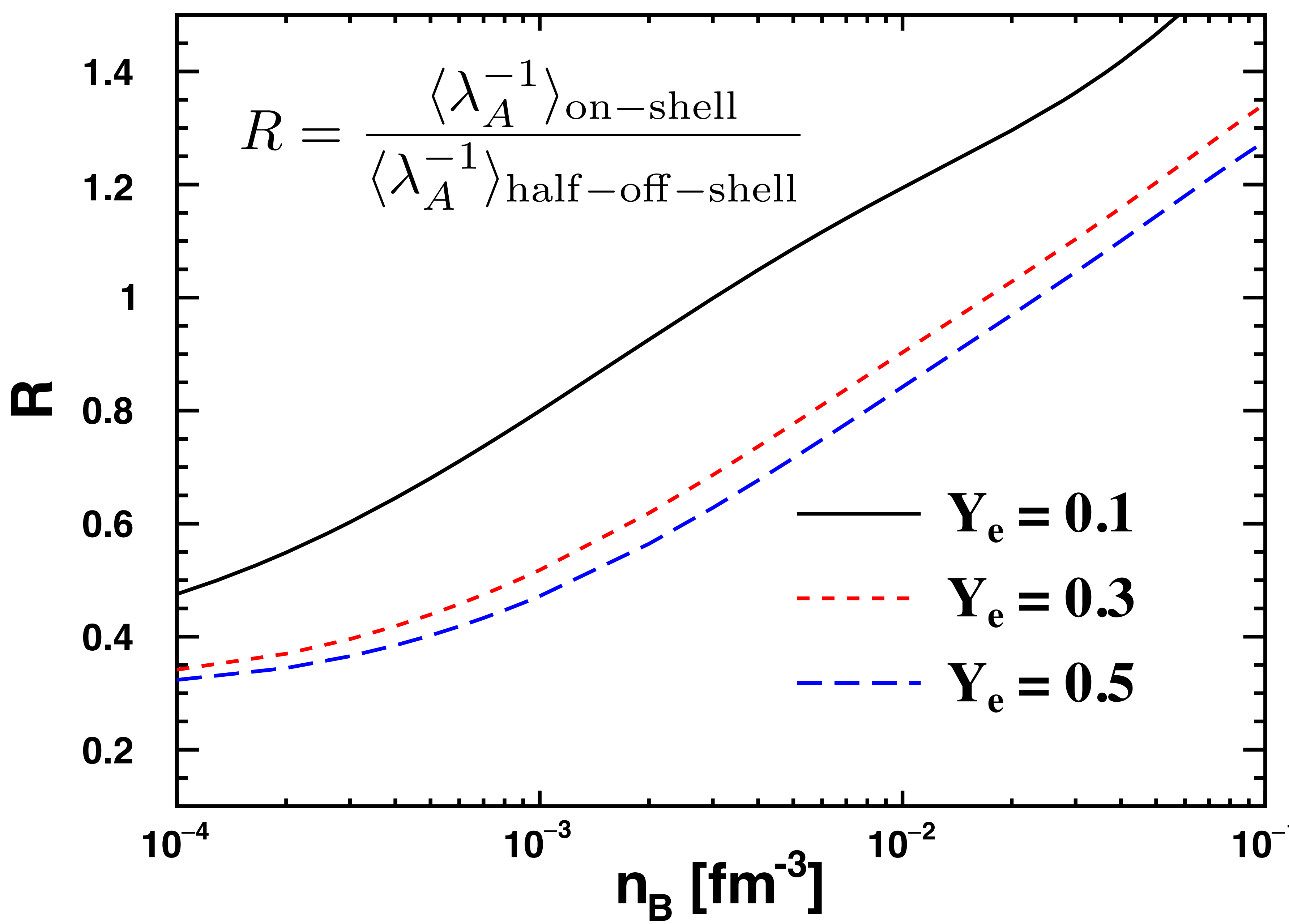
Using effective on-shell T-matrix is not enough



$$\langle k_f | \mathcal{T} | k_i \rangle \longrightarrow \langle \bar{k} | \mathcal{T} | \bar{k} \rangle, \quad \bar{k} = \sqrt{\frac{1}{2}(k_i^2 + k_f^2)}$$

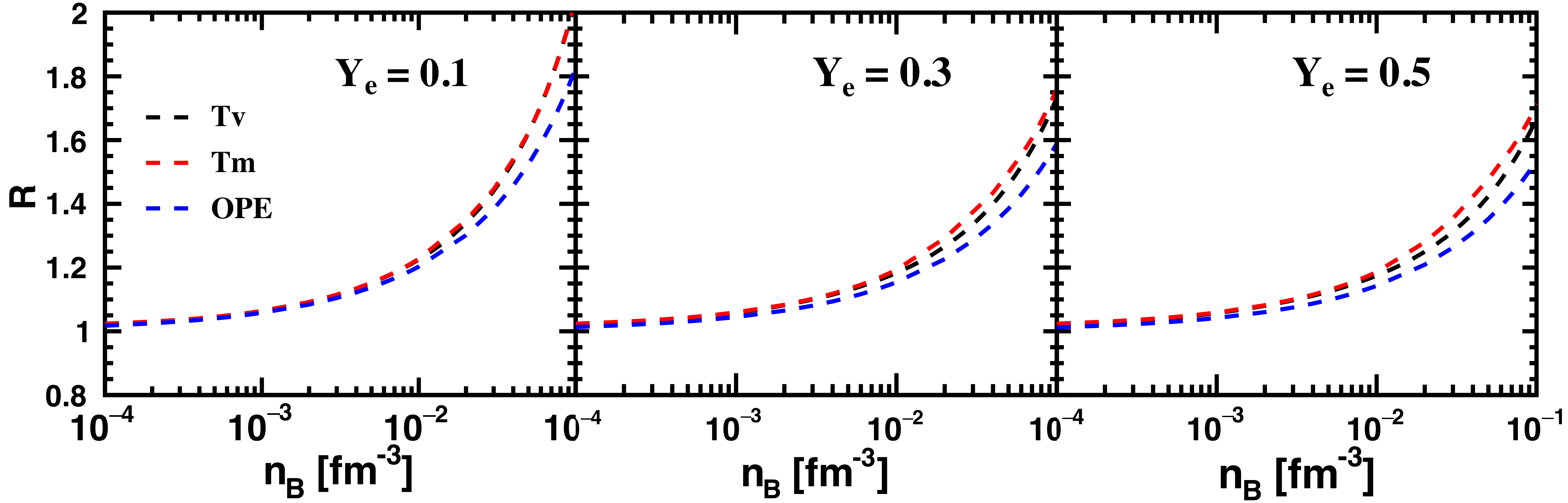
half-off-shell ‘on-shell’ Bartl *et al.* 2014

(can be simply extracted from NN scattering data)



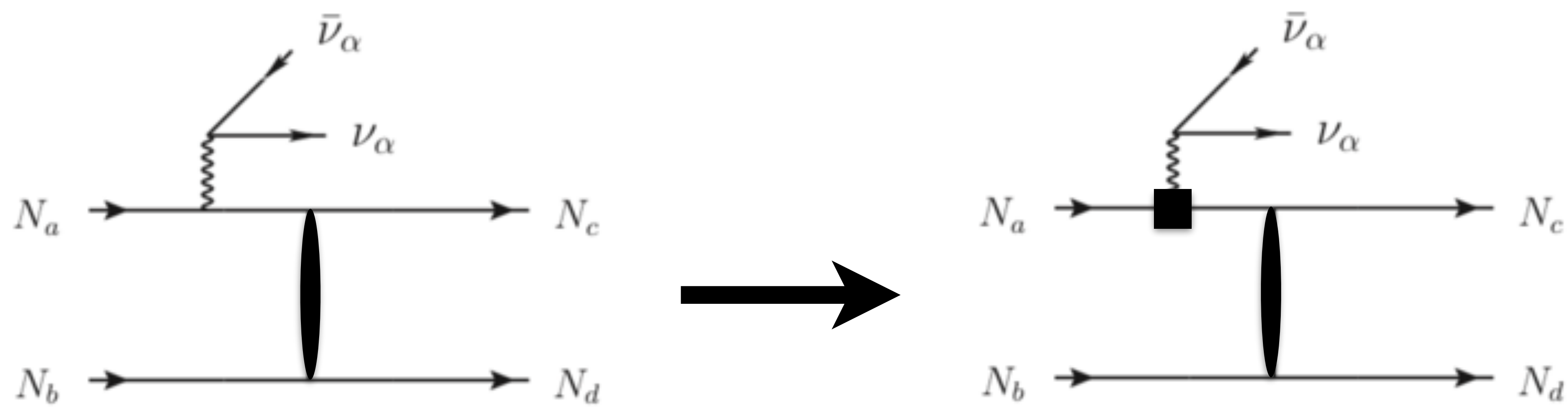
Ignoring the blocking of the final nucleon states

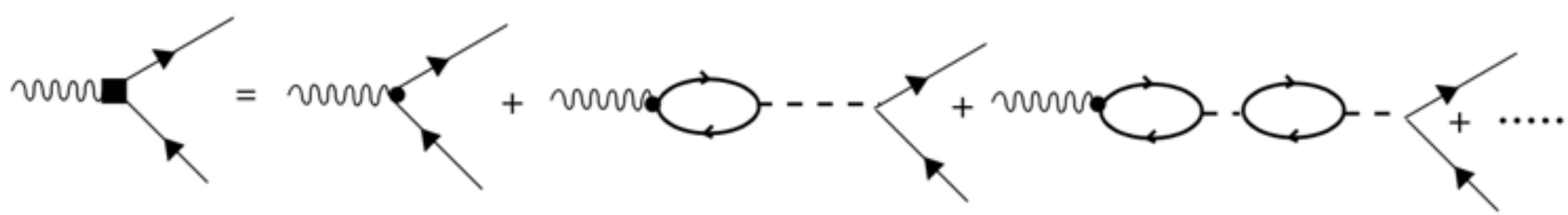
$$N_a + N_b + \nu_\alpha + \bar{\nu}_\alpha \to N_c + N_d$$



$$R = \frac{\langle \lambda_A^{-1} \rangle_{\text{no blocking}}}{\langle \lambda_A^{-1} \rangle_{\text{blocking}}}$$

Corrections due to Random Phase Approximation





dressed/screened vertex due to particle-hole excitation

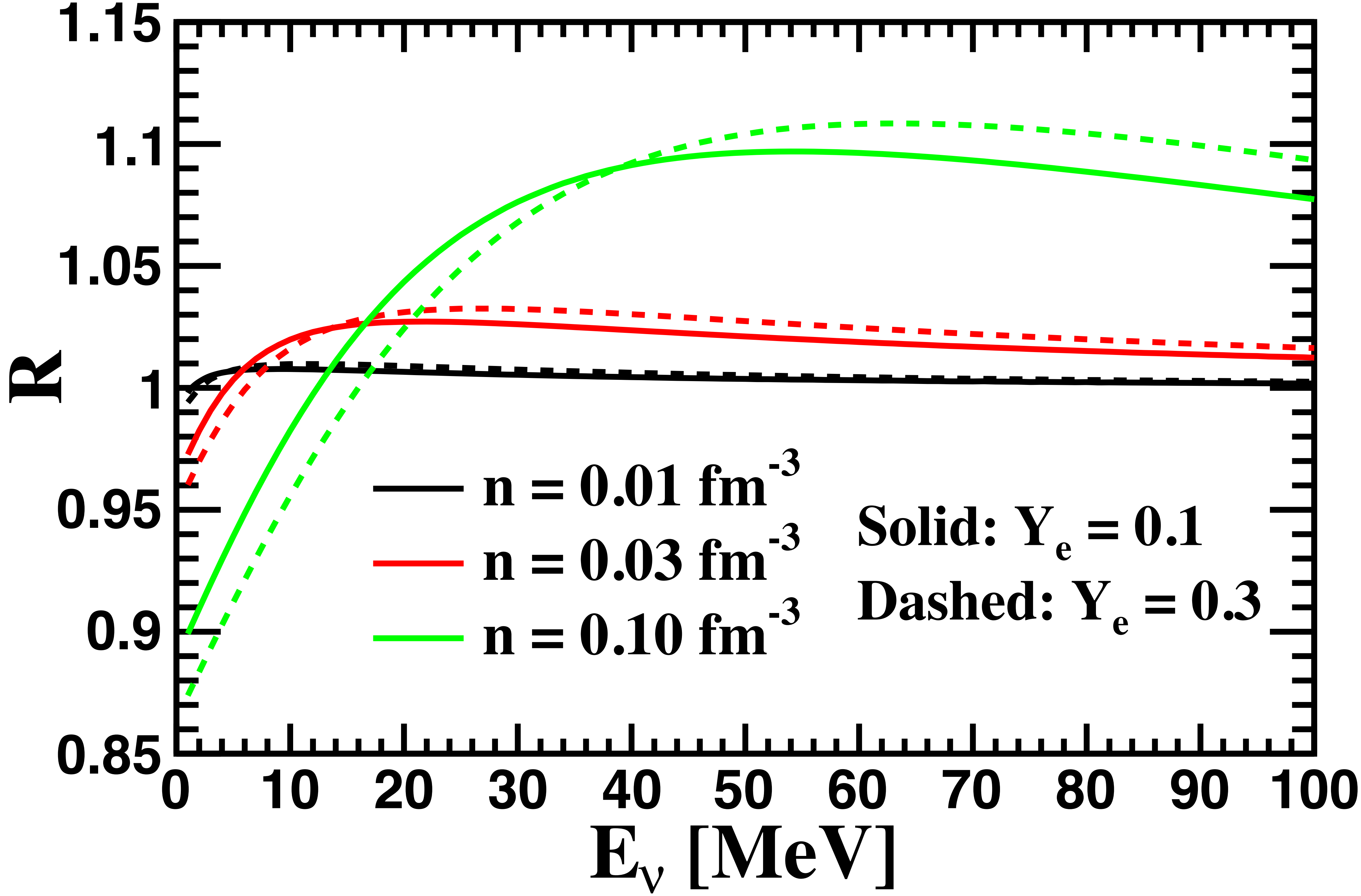
Burrows & Sawyer 1998

Reddy *et al.*, 1998

Burrows *et al.* 2006

Effect of RPA correlation is minor

$$R = \frac{\lambda^{-1}_{A,\mathrm{RPA}}(E_\nu)}{\lambda^{-1}_{A}(E_\nu)}$$



Conclusions

Compared to OPE, T-matrix gives rise to enhanced rates at low density and suppressed rates at high density relevant to SN by a factor of 2-3;

Compared to on-shell T-matrix, off-shell matrix gives rise to enhanced rates at low density by a factor of 2 and suppressed rates at high density relevant to SN by 20%;

Pauli blocking suppresses the rates by 20% at $10^{13}\ \mathrm{g/cm}^3$ and by 60% at $10^{14}\ \mathrm{g/cm}^3$;

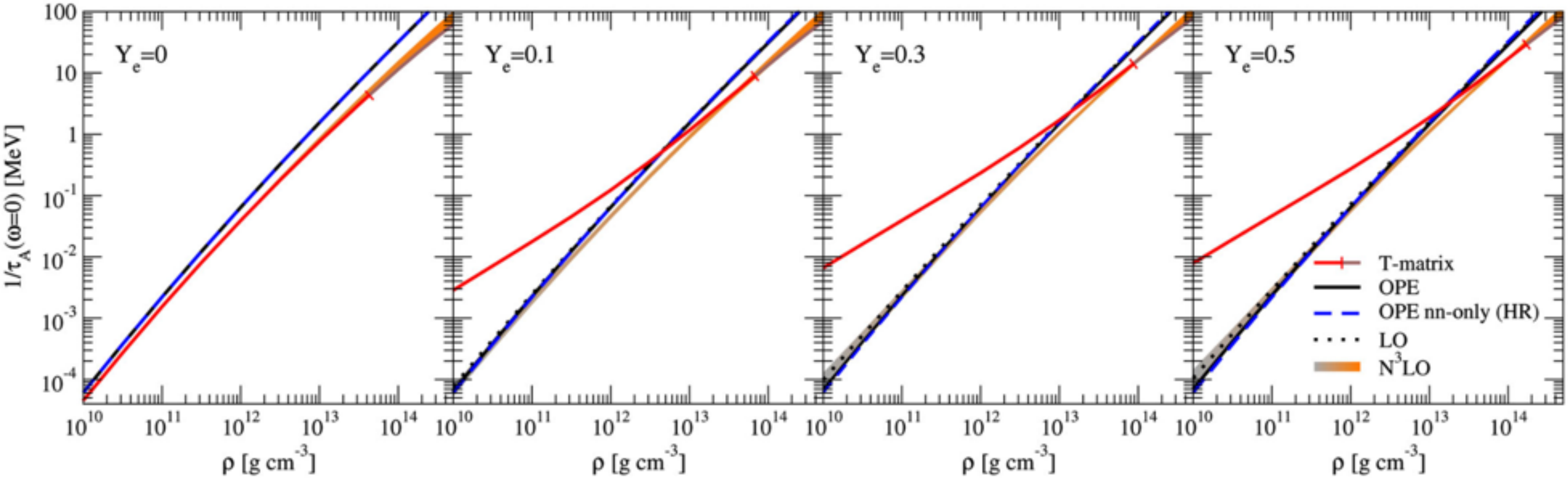
RPA correlation plays a negligible role in neutrino bremsstrahlung rates.

Effects of the our bremsstrahlung rates need to be tested in simulations (on-going).

Thanks

backup

Consistency between OPE & chiral force at Born level at low density



Bartl *et al.* 2014

amplitudes for bremsstrahlung in PW basis

$$\bar{H}(T_z) = \frac{1}{3\cdot 4(4\pi)^2} \sum_{\substack{J_iJ_fJ_i'J_f' \\ T_iT_fT_i'T_f' \\ l_il_fl_i'l_f' \\ S_iS_fL}} (-1)^{S_f+S_i+J_i+L+J_f'} [J_iJ_fJ_i'J_f'l_il_fl_i'l_f'][L]^2$$

$$\left\{\begin{array}{ccc} J_i & J_i' & L \\ l_i' & l_i & S_i \end{array}\right\} \left\{\begin{array}{ccc} J_f & J_f' & L \\ l_f' & l_f & S_f \end{array}\right\} \left\{\begin{array}{ccc} J_f & J_f' & L \\ J_i' & J_i & 1 \end{array}\right\} \begin{pmatrix} l_i & l_i' & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_f & l_f' & L \\ 0 & 0 & 0 \end{pmatrix} P_L(\hat{\boldsymbol{k}}_i \cdot \hat{\boldsymbol{k}}_f)$$

$$\langle k_f\, l_f S_f J_f\, T_f T_z || \boldsymbol{\mathcal{O}}(1-P_{12}) || k_i\, l_i S_i J_i\, T_i T_z\rangle \langle k_i\, l_i' S_i J_i'\, T_i' T_z || \tilde{\boldsymbol{\mathcal{O}}}(1-P_{12}) || k_f\, l_f' S_f J_f'\, T_f' T_z\rangle .$$

$$\frac{1}{2}\langle k_f\, l_f S_f J_f\, T_f T_z || \boldsymbol{\mathcal{O}}(1-P_{12}) || k_i\, l_i S_i J_i\, T_i T_z\rangle = \frac{1}{\omega}\Big\{ \mathcal{T}^{S_f J_f T_f}_{l_f l_i}(k_f, k_i; E_{k_f}) \langle k_i\, l_i S_f J_f\, T_f T_z || \boldsymbol{\mathcal{Y}} || k_i\, l_i S_i J_i\, T_i T_z\rangle$$
$$- \langle k_f\, l_f S_f J_f\, T_f T_z || \boldsymbol{\mathcal{Y}} || k_f\, l_f S_i J_i\, T_i T_z\rangle \mathcal{T}^{S_i J_i T_i}_{l_f l_i}(k_f, k_i; E_{k_i}) \Big\}$$

$$\frac{1}{2}\langle k_i\, l_i' S_i J_i'\, T_i' T_z || \tilde{\boldsymbol{\mathcal{O}}}(1-P_{12}) || k_f\, l_f' S_f J_f'\, T_f' T_z\rangle = \frac{1}{\omega}\Big\{ \big[\mathcal{T}^{S_i J_i' T_i'}_{l_f' l_i'}(k_f, k_i; E_{k_i})\big]^* \langle k_f\, l_f' S_i J_i'\, T_i' T_z || \boldsymbol{\mathcal{Y}} || k_f\, l_f' S_f J_f'\, T_f' T_z\rangle$$
$$- \langle k_i\, l_i' S_i J_i'\, T_i' T_z || \boldsymbol{\mathcal{Y}} || k_i\, l_i' S_f J_f'\, T_f' T_z\rangle \big[\mathcal{T}^{S_f J_f' T_f'}_{l_f' l_i'}(k_f, k_i; E_{k_f})\big]^* \Big\}$$

$$\langle k\, l S_f J_f\, T_f T_z || \boldsymbol{\mathcal{Y}} || k\, l S_i J_i\, T_i T_z\rangle = 6(-1)^{T_f - T_z + l + S_i + J_f + 1} [S_i S_f J_i J_f T_i T_f] \Big[(-1)^{S_f+T_f} + (-1)^{S_i+T_i}\Big]$$
$$\begin{pmatrix} T_f & 1 & T_i \\ -T_z & 0 & T_z \end{pmatrix} \left\{\begin{array}{ccc} S_f & S_i & 1 \\ J_i & J_f & l \end{array}\right\} \left\{\begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & 1 \\ S_i & S_f & \frac{1}{2} \end{array}\right\} \left\{\begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & 1 \\ T_i & T_f & \frac{1}{2} \end{array}\right\} .$$

T-matrix elements in partial wave basis

$$\mathcal{T}^{JST}_{LL'}(k',k;E) = U^{JST}_{LL'}(k',k) + \sum_{L''} \int \frac{k''^2 dk''}{(2\pi)^3} U^{JST}_{LL''}(k',k'') \frac{1}{E - \frac{k''^2}{m_N} + i\varepsilon} \mathcal{T}^{JST}_{LL''}(k'',k;E),$$

discretization of k, then

$$A_{ij}T_{jk} = U_{ik} \xrightarrow{\text{matrix inversion}} T = A^{-1}U$$

Bethe-Goldstone equation (in-medium version of LS)

$$\mathcal{T}^{JST}_{LL'}(k',k;K,\Omega) = V^{JST}_{LL'}(k',k) + \sum_{L''} \int \frac{k''^2 dk''}{(2\pi)^3} V^{JST}_{LL''}(k',k'') \bar{g}_{II}(K,\Omega,k'') \mathcal{T}^{JST}_{L''L}(k'',k;K,\Omega)$$

in-medium T-matrix

$$\bar{g}_{II}(K,\Omega,k) = \left\langle \frac{1 - f(\varepsilon(k_1)) - f(\varepsilon(k_2))}{\Omega - \varepsilon(k_1) - \varepsilon(k_2) + i\eta} \right\rangle_\theta$$

Including the blocking for the intermediate two-nucleon states

Taking *U* to be the chiral potential,

(D. R. Entem *et al.* 2017)

half-off-shell vacuum/in-medium *T*-matrix is computed

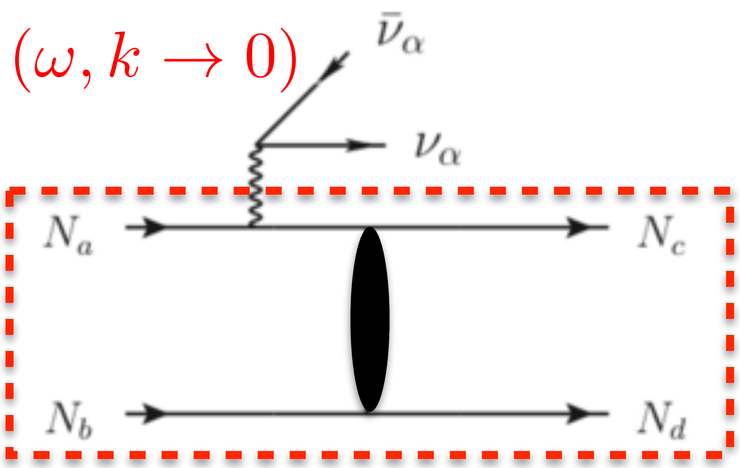
Rates and structure factor

$$\sum_{\text{spins}} |\mathcal{M}_{\text{tot}}|^2 \overset{\text{NR}}{=} \frac{C_A^2 G_F^2}{2} \sum_{ij} H_{ij} L^{ij} \xrightarrow{\text{isotropic}} 4C_A^2 G_F^2 \times \bar{H} \times E_\nu E'_\nu (3 - \cos\theta_{\nu\bar{\nu}})$$

where $$\bar{H} \equiv \frac{1}{3}\sum_i H_{ii} = \frac{1}{3}\sum_i \sum_{\text{spins}} |\langle cd|[V, \frac{1}{\omega}\sum_{r=1,2} \sigma_i^{(r)} \tau_z^{(r)}]|ab\rangle|^2.$$

Fermi-Golden's rule giving the rates: Integrating over the phase spaces of nucleons and neutrinos with Pauli blocking for the final states. ($N_a + N_b \to N_c + N_d + \nu_\alpha + \bar{\nu}_\alpha$)

Separating the hadronic and leptonic parts, one can introduce a structure factor for the nuclear medium, Hannestad & Raffelt 1998



$S_\sigma(\omega) \equiv S_\sigma(\omega, q \to 0)$

long wavelength limit

$$S_\sigma^{(\lambda\eta)}(\omega) = \frac{1}{n_B}\int \prod_{l=a,b,c,d} \frac{d^3 k_l}{(2\pi)^3} f_a f_b (1-f_c)(1-f_d)$$
$$\times \delta^{(3)}(\boldsymbol{k}_a + \boldsymbol{k}_b - \boldsymbol{k}_c - \boldsymbol{k}_d) \bar{H}^{(\lambda\eta)}(K, k_i, k_f, \cos\theta)$$
$$\times (2\pi)^4 \delta(E_a + E_b - E_c - E_d + \omega),$$

$$\lambda, \eta = n, p$$

$$S_\sigma = S_\sigma^{(nn)} + S_\sigma^{(np)} + S_\sigma^{(pp)}$$

Given the structure factor, integrating over the phase space of neutrinos gives the rates of neutrino bremsstrahlung:

$$N + N \to N + N + \nu + \bar{\nu}$$
$$\omega = -(E_\nu + E'_\nu) < 0$$

Emissivity from NN bremsstrahlung

$$\phi(E_\nu) = \frac{C_A^2 G_F^2 n_B E_\nu^2}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3}(3 - \cos\theta_{\nu\bar{\nu}}) \times S_\sigma(-E_\nu - E'_\nu)[1 - f'(E'_\nu)],$$

detailed balance

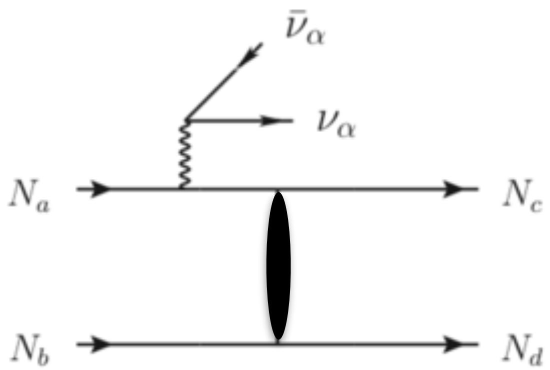
$$S_\sigma(-\omega) = S_\sigma(\omega) e^{-\omega/T}$$

Opacity due to neutrino-pair absorption

$$\lambda_A^{-1}(E_\nu) = C_A^2 G_F^2 n_B \int \frac{d^3k'}{(2\pi)^3}(3 - \cos\theta_{\nu\bar{\nu}}) f'(E'_\nu) \times S_\sigma(E_\nu + E'_\nu),$$

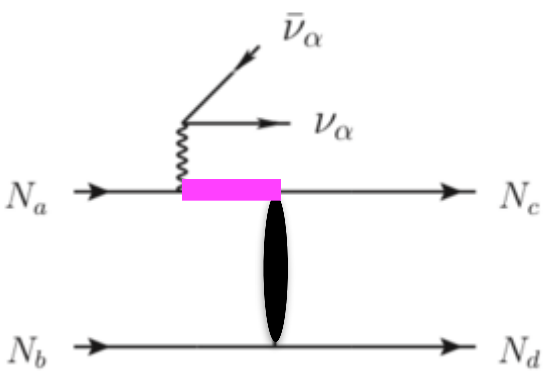
$$N + N + \nu + \bar{\nu} \to N + N$$
$$\omega = E_\nu + E'_\nu > 0$$

multi-scattering effects and normalized structure factor



$$\mathcal{M} \propto \frac{1}{\omega}, \quad S(\omega) \propto \frac{1}{\omega^2}$$

expected for bremsstrahlung in vacuum,
but not true in nuclear medium



the intermediate nucleon get constantly scattered
and thus obtain a width smearing the process

$$\mathcal{M} \propto \frac{1}{\omega + i\Gamma}, S_\sigma(\omega) \propto \frac{1}{\omega^2 + \Gamma^2}$$

Raffelt & Strobel 1997
Hannestad & Raffelt 1998

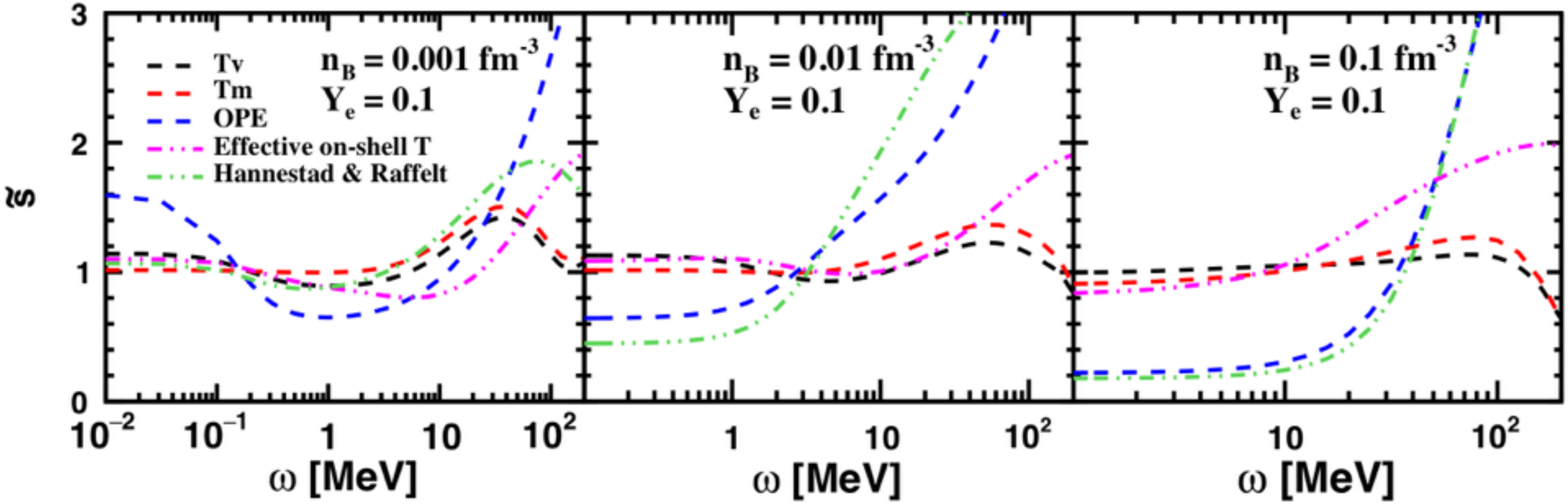
To determine Γ
[functions of T, n, Ye]

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_\sigma(\omega) = \int_0^{\infty} \frac{d\omega}{2\pi} S_\sigma(\omega)(1 + e^{-\omega/T})$$
$$= \frac{2}{n_B} \sum_{i=n,p} \int \frac{d^3p}{(2\pi)^3} f_i(\varepsilon(p))[1 - f_i(\varepsilon(p))]$$

exact for non-interacting system;
NN collision widen the spreading of $S_\sigma(\omega)$

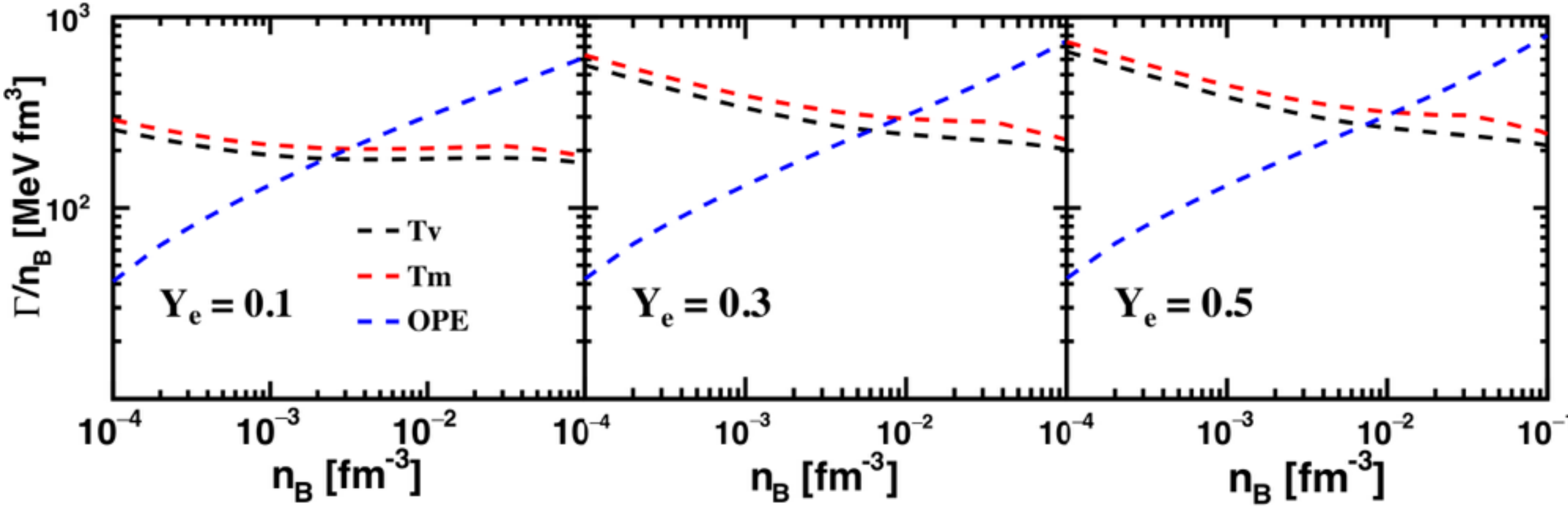
a finite Γ guarantees a smooth & physical structure factor,
otherwise $S_\sigma(\omega)$ diverges as $\omega \to 0$

comparisons of the structure factors



(Hannestad & Raffelt 1998; Bartl *et al.* 2014)

width/spin relaxation rate of nucleon



Formalisms for RPA correlation

$$S_\sigma(q,\omega) = \frac{2\mathrm{Im}\Pi(q,\omega)}{1-\exp(-\omega/T)},$$

fluctuation-dissipation theorem

see. e.g., Fetter & Walecka 1971

where

$$\Pi(q,\omega) \equiv \frac{-i}{Z}\int d^4x e^{-i\vec{q}\cdot\vec{x}} e^{i\omega t}\mathrm{Tr}\{e^{-\beta(H-\sum\mu_i N_i)} \times [n_n^{(3)} - n_p^{(3)}, n_n^{(3)} - n_p^{(3)}]\}\theta(t),$$

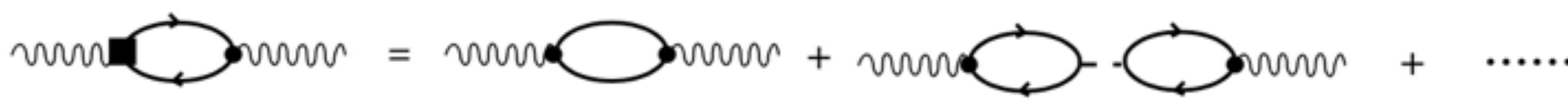
Including RPA

$$\mathrm{Im}[\Pi(\omega)] = \frac{1}{2}S_\sigma(\omega)[1-\exp(-\omega/T)],$$

$$\mathrm{Re}[\Pi(\omega)] = \frac{1}{\pi}\mathcal{P}\int d\omega' \frac{\mathrm{Im}[\Pi(\omega')]}{\omega-\omega'}.$$

$$S_\sigma^{\mathrm{RPA}}(\omega) = 2\mathrm{Im}[\Pi(\omega)][1-\exp(-\omega/T)]^{-1}\mathcal{C}_A^{-1},$$

where



ring approximation

$$\mathcal{C}_A = \{1 - v_{\mathrm{GT}}\mathrm{Re}[\Pi(\omega)]\}^2 + v_{\mathrm{GT}}^2\mathrm{Im}[\Pi(\omega)]^2,$$

with $v_{\mathrm{GT}}/n_B = 4.5\times10^{-5}\ \mathrm{MeV}^{-2}$

Burrows & Sawyer 1998
Reddy *et al.*, 1998

a general discussion of axial-structure factor

Dynamical structure function: correlation of spin-spin density operator

$$S_\sigma(\omega,\mathbf{k})=\frac{4}{3n_B V}\int_{-\infty}^{+\infty}dt e^{i\omega t}\langle \boldsymbol{\sigma}(t,\mathbf{k})\cdot\boldsymbol{\sigma}(0,-\mathbf{k})\rangle$$

$$S_\sigma(\omega,\mathbf{k})=\frac{1}{n_B}\frac{2\mathrm{Im}\Pi(q,\omega)}{1-\exp(-\omega/T)},$$

fluctuation-dissipation theorem

see, e.g., Fetter & Walecka 1971

where retarded polarization/response function is

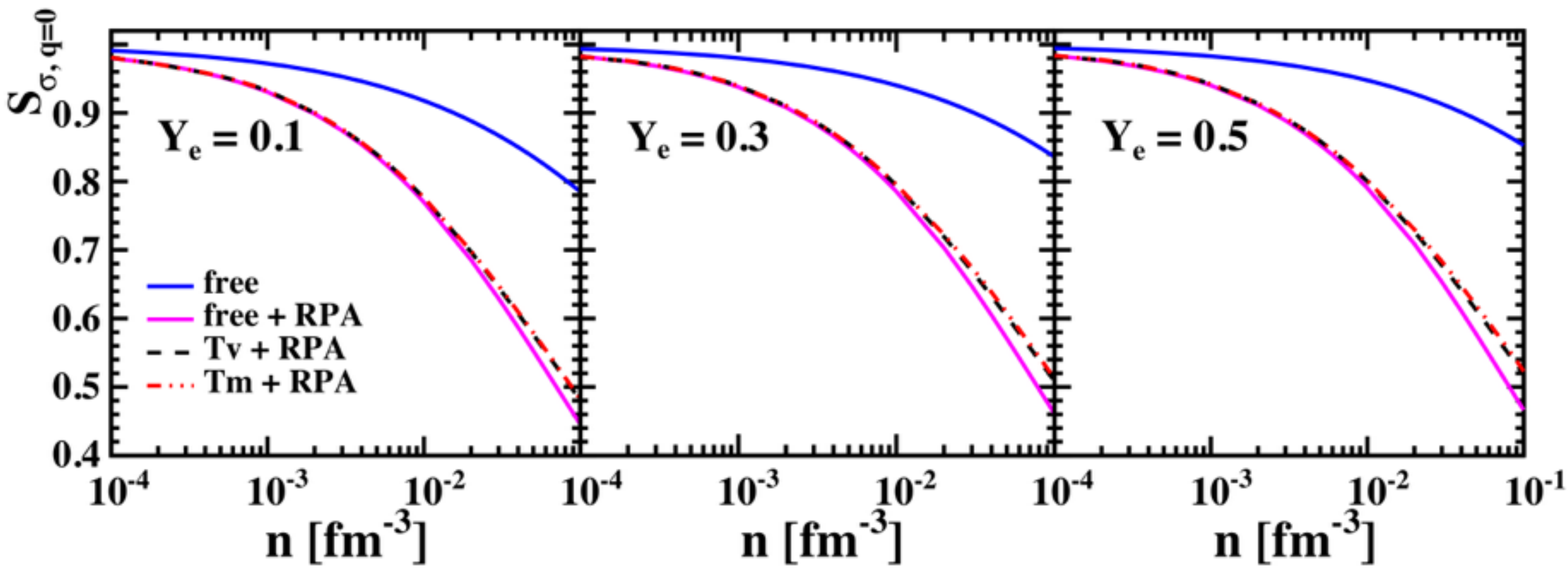
$$\Pi(q,\omega)\equiv\frac{-i}{Z}\int d^4x e^{-i\vec{q}\cdot\vec{x}}e^{i\omega t}\mathrm{Tr}\{e^{-\beta(H-\sum\mu_i N_i)}$$
$$\times[n_n^{(3)}-n_p^{(3)},n_n^{(3)}-n_p^{(3)}]\}\theta(t),$$

Cutting rules show the imaginary part can be approximated by the rates obtained from the Fermi-Golden's rule with statistical factors taken in account for the initial/final states.

Effects of RPA on the normalisation of $S_\sigma(\omega)$

$$S_{\sigma,q=0} \equiv \lim_{q\to 0} \int_{-\infty}^{\infty} \frac{S_\sigma(q,\omega)}{2\pi} d\omega = \int_{-\infty}^{\infty} \frac{S_\sigma(\omega)}{2\pi} d\omega.$$

[actually the static structure factor, constrained by EOS]



Extension to neutrino-nucleon scattering

$$N_a + N_b + \nu \to N_c + N_d + \nu$$

$$N_a + N_b + \nu \to N_c + N_d + \nu' : \quad |\omega| = |E_\nu - E'_\nu| < |\boldsymbol{q}|$$

$$\left.\begin{array}{l} N_a + N_b + \nu + \bar{\nu} \to N_c + N_d \\ N_a + N_b \to N_c + N_d + \nu + \bar{\nu} \end{array}\right\} |\omega| = |E_\nu + E'_\nu| > |\boldsymbol{q}|$$

ω and q the energy and momentum transferred to the nuclear medium

Long wavelength approx. ($q \to 0$) is only valid for pair-absorption/ emission from NN bremsstrahlung;

To describe neutrino-nucleon scattering, we need to keep the dependence on q.