Randomness and chaos in qubit models

Pak Hang Chris Lau (NCTS)
with Chen-Te Ma (SCNU), Jeff Murugan (Cape Town U.) and Masaki Tezuka (Kyoto U)

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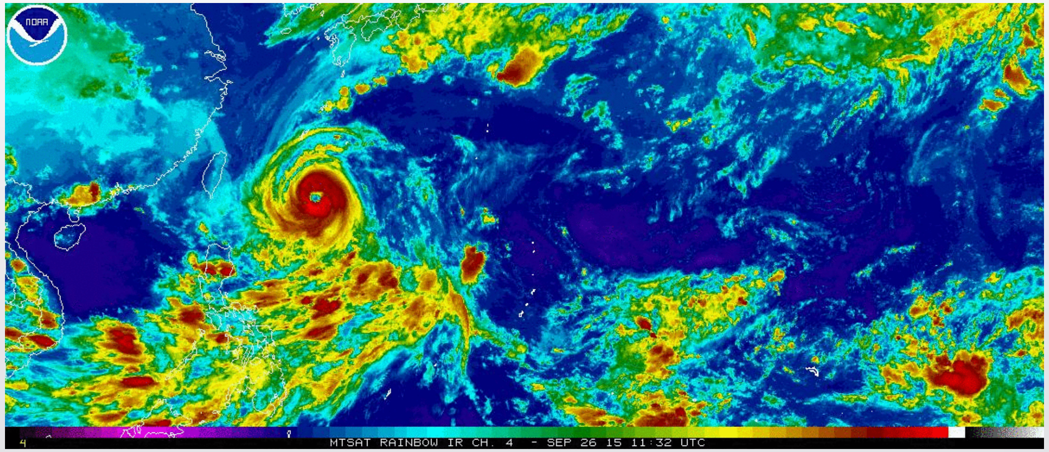
Classical chaos

- Non-linear system
  - Double rod pendulum
  - Weather
- Sensitivity to initial condition

$$\frac{\delta x(t)}{\delta x(0)} \sim e^{\lambda t}$$



https://giphy.com/gifs/weather-wZYbh9Peck75e

Quantum chaos

- Quantum ?→ Classical chaos
- Out of time-ordered correlator (OTOC)
- Random matrix theory
  - Spectral form factor
  - Level spacing distribution

Diagnostic 1

- Out of time ordered correlator (OTOC)

$$C_T = -\langle [W(t), U(0)]^2 \rangle_\beta \sim e^{2\lambda t}$$

- Quantum mechanics
  - Operators $\hat{x}, \hat{p}$

$$[\hat{x}(t), \hat{p}(0)] = i \frac{\delta x(t)}{\delta x(0)}$$

- Maximum bound

$$\lambda \leq \frac{2\pi k_B}{\beta \hbar}$$

Diagnostic 2

- Quantum → expectation value
  - Probabilistic
  - Statistic properties of the system

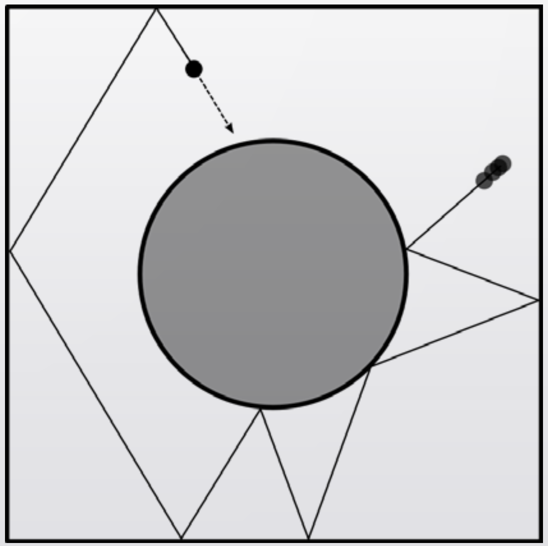
- Random matrix theory (RMT)
  - Spectra of heavy nuclei
    - Many-body
    - Many states
  - Wigner (1955)

E. P. Wigner, *Annals of Mathematics*, vol. 62, no. 3, 1955, pp. 548–564.

Classical chaos & RMT

- Sinai billiard
    - Yakov Sinai (1963)
        - Ergodic
    - Chaotic

- Quantum Sinai billiard
    - Spectrum
    - RMT



Y. G. Sinai, Soviet Math. Doklady 4 pp. 1818-1822 (1963)
K. Hashimoto, K. Murata, R. Yoshii JHEP 1710 (2017) 138

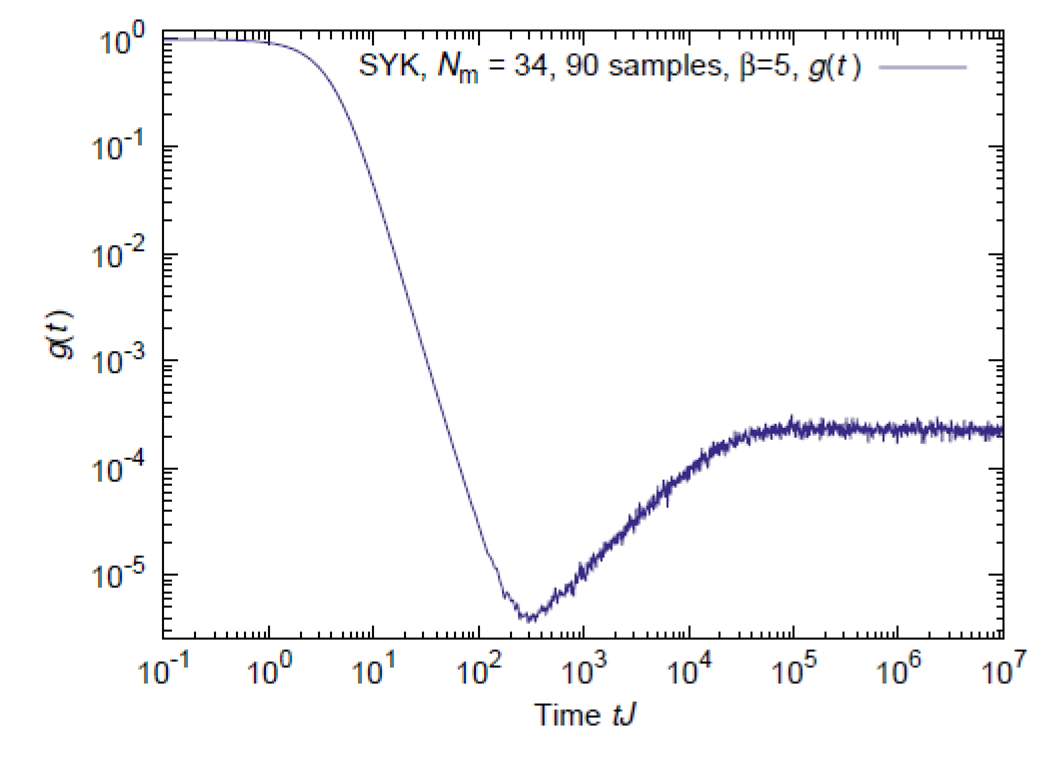
https://en.wikipedia.org/wiki/Dynamical_billiards

Diagnostic 2 - RMT

- Spectral form factor (SFF)
  - Correlation between eigenvalues

$$g(t) = \frac{|Z(\beta, t)|^2}{|Z(\beta, 0)|^2}$$

$$Z(\beta, t) = \mathrm{Tr}\left(e^{(-\beta + it)H}\right)$$



J.S. Cotler et.al JHEP (2017) 1705: 118.

Sachedev-Ye-Kitaev model

- Hamiltonian $$H = i^{q/2} \sum_{1 \leq i_1 < \ldots < i_q \leq N} J_{i_1 \ldots i_q} \psi_{i_1} \ldots \psi_{i_q}$$
- Majorana fermions $\{\psi_i, \psi_j\} = \delta_{ij}$
- Random coupling
  - Gaussian variance $(q-1)! \frac{J^2}{N^{q-1}}$
- Most studied case
  - $q = 4$ large N (chaotic)

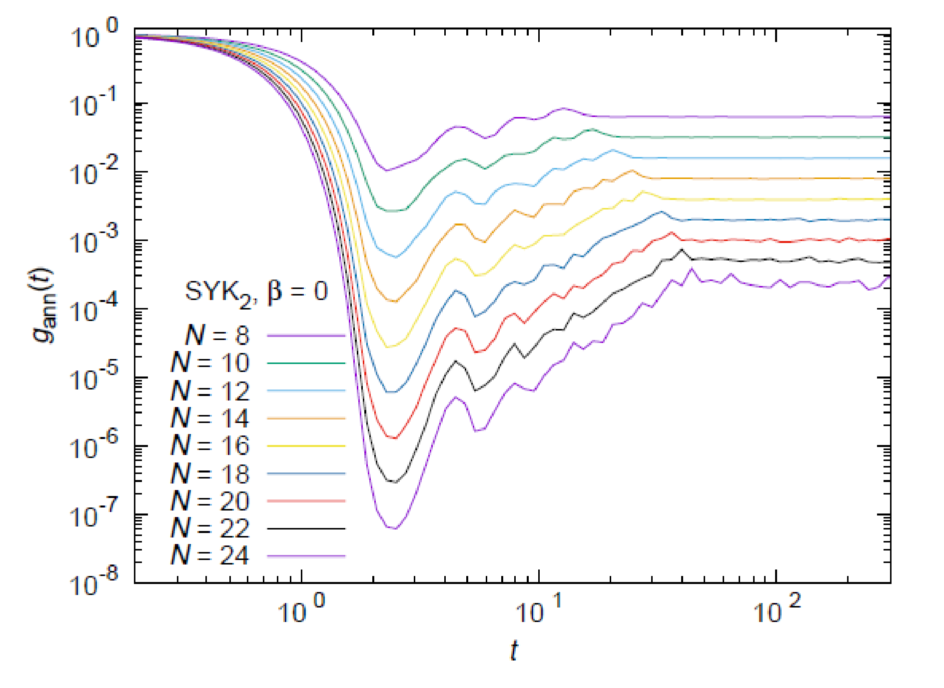
Model

- $q = 2$ Sachdev-Ye-Kitaev model (SYK)
  - Majorona fermions
  - Random coupling
    - Gaussian
  - Integrable

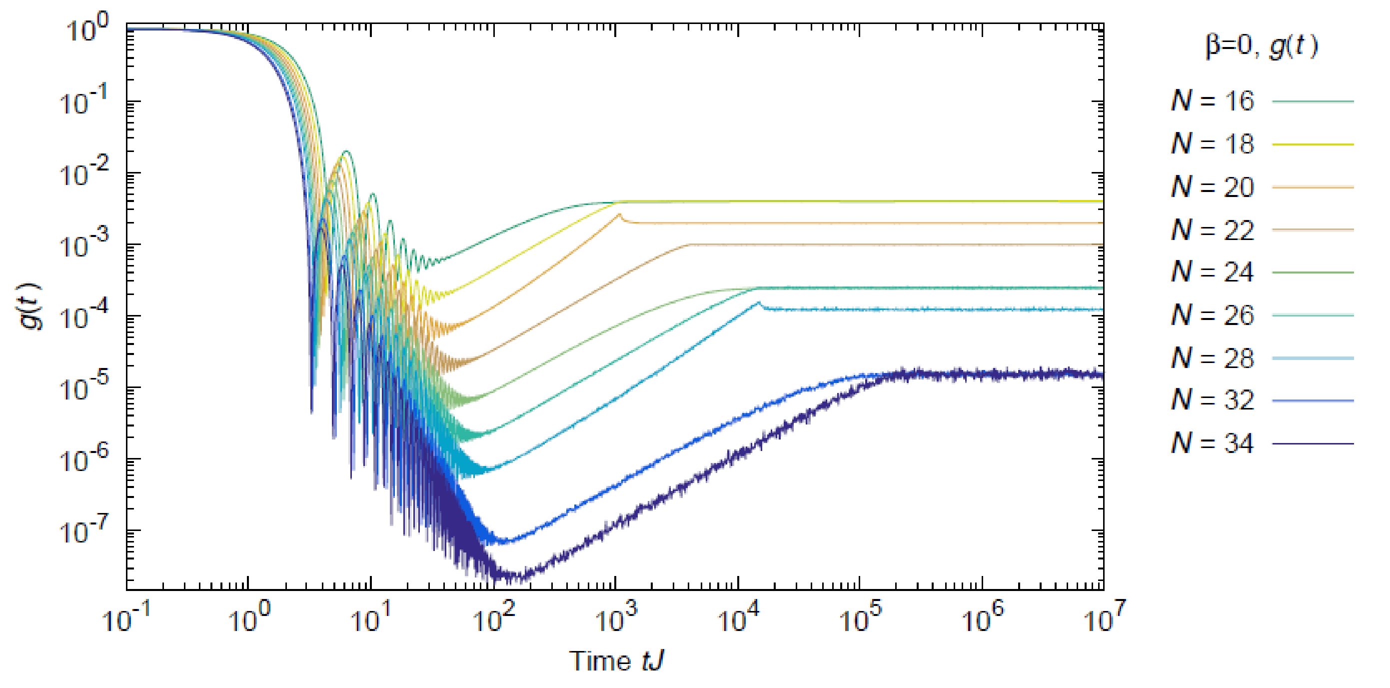
$$\mathcal{P} = \exp\left(- \sum_{j_1, j_2 = 1}^{N} \mathcal{J}_{j_1 j_2}^2 \frac{N}{4 J^2} \right)$$

$$H_{\mathrm{SYK2}} \equiv \frac{i}{2} \sum_{j_1, j_2 = 1}^{N} \mathcal{J}_{j_1 j_2} \psi_{j_1} \psi_{j_2} ,$$

SYK_2 - Spectral form factor



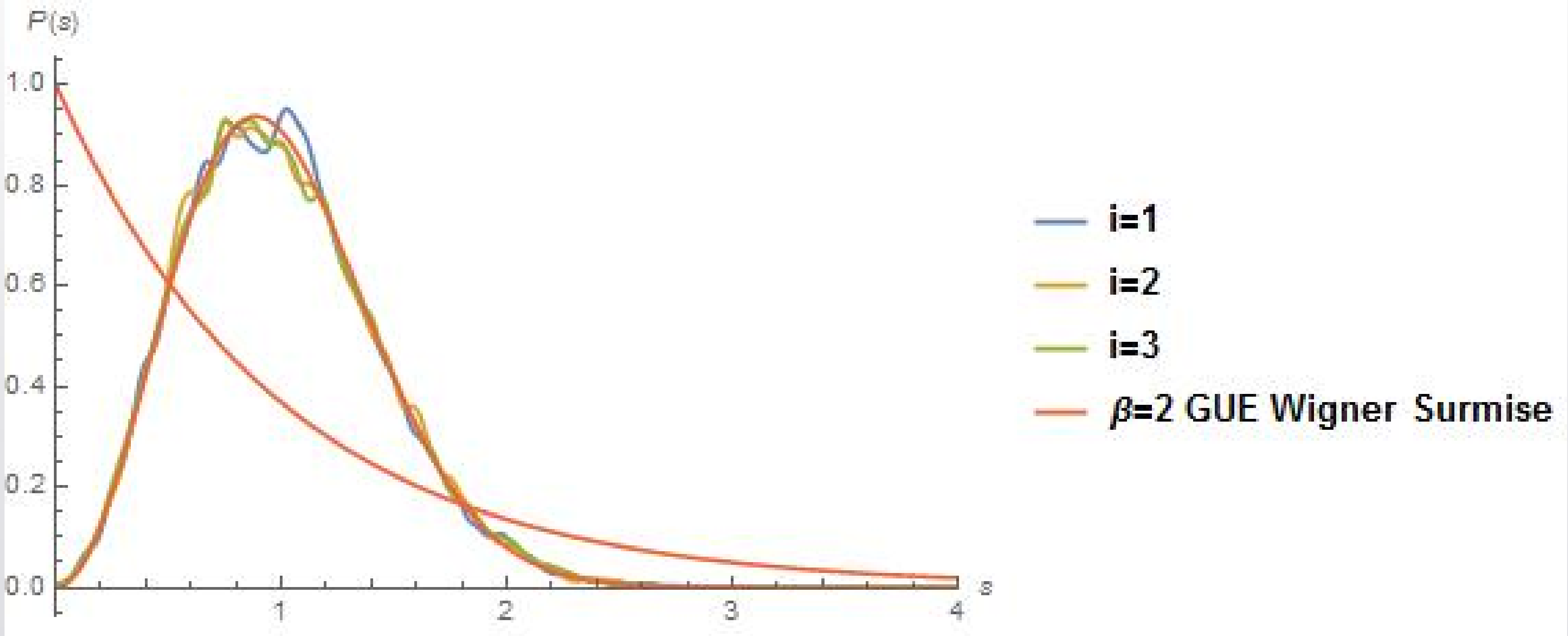
SYK_2



SYK_4

J.S. Cotler et.al JHEP (2017) 2017: 118.

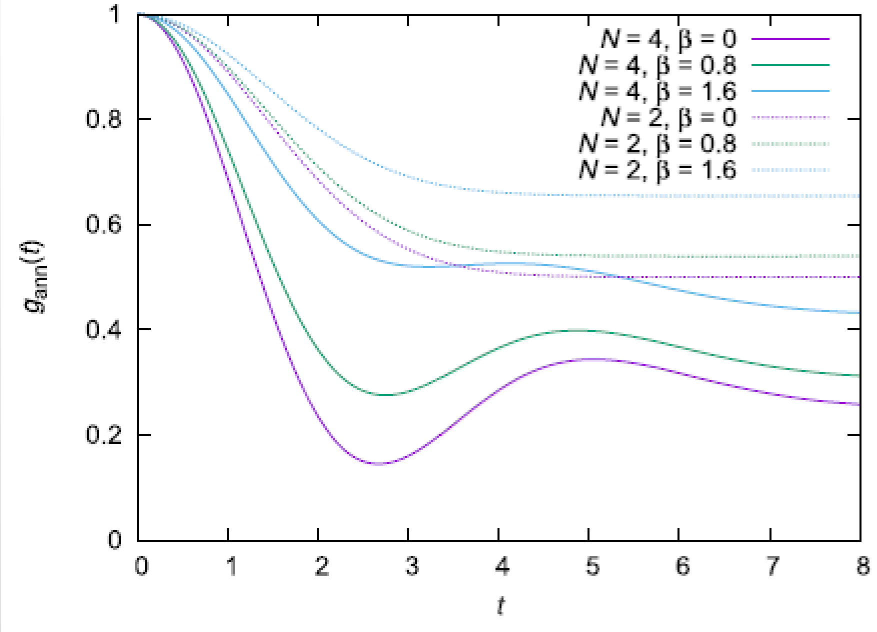
Level spacing distribution



N=6 SYK_2

Small N SFF

- Only $N = 2$
  - No dip-ramp-plateau
- $N = 2$ solvable analytically
- Eigenvectors
  - Independent of coupling
  - Origin of dip-ramp-plateau?



Comparison and summary

- Same behaviour as the SYK4 model (chaotic) for $N > 2$
  - Transverse Ising model gives consistent result
  - General integrable spin system?

- Dip-ramp-plateau
  - Generate from large set of eigenstates
  - $N = 2$ eigenvectors independent of couplings
  - Not sufficient condition to diagnose chaos