Entropy generation and decay of the cosmological constant in de Sitter space

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Introduction

- In 4D Einstein gravity with a cosmological constant $\Lambda = 3H^2$, de Sitter (dS) space is obtained as a classical solution:

$$\frac{1}{\kappa^2}\int d^4x\sqrt{-g}[R-6H^2] \qquad \kappa^2 = 16\pi G_N$$

$$\Rightarrow \quad ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad a(t) = e^{Ht}$$

- The dS entropy is proportional to the area of the horizon:

$$S = \frac{1}{4G_N}\cdot 4\pi(1/H)^2 = \frac{\pi}{G_N H^2}$$

'77 G. W. Gibbons, S. W. Hawking

- If $\Lambda \propto H^2$ and $S \propto H^{-2}$ anomalously scale with time, we may obtain an unified understanding of entropy generation and decay of the cosmological constant

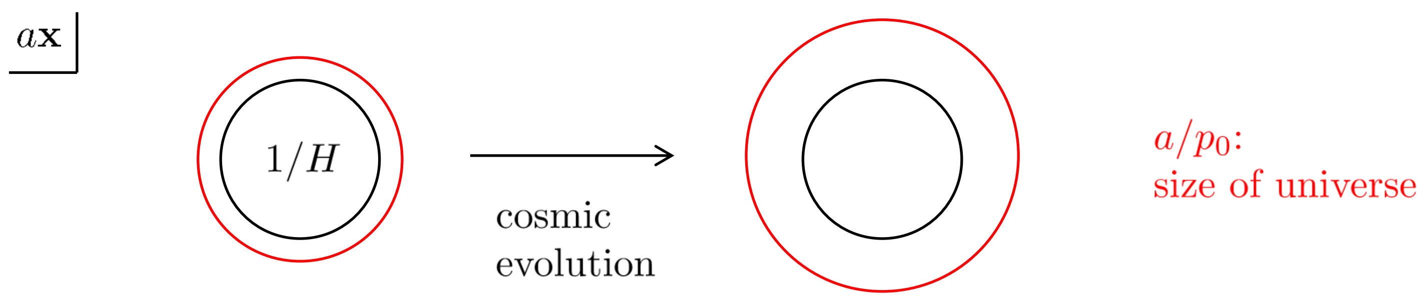
- What gives rise to such time evolutions? → Soft gravitons

'82 A. Vilenkin, L. H. Ford,
A. D. Linde,
A. A. Starobinsky

Quantum IR effects

- The propagator of a massless, minimally coupled scalar field has a secular growing term:

$$\langle\varphi^2(x)\rangle \simeq \int_{p=p_0}^{p=Ha} \frac{d^3p}{(2\pi)^3}\frac{H^2}{2p^3} = \frac{H^2}{4\pi^2}\underline{\log(a/a_0)} \qquad a_0 \equiv p_0/H$$



- In the presence of the light scalar field, physical quantities may acquire time dependent quantum effects through internal propagators

If $m^2 \sim H^2$, such IR effects are not induced as $\int d^3p/p^{2\nu},\ \nu = \sqrt{9/4 - m^2/H^2}$

Gravitational propagator

'94 N. C. Tsamis, R. P. Woodard

- Gravitational fluctuations induce quantum IR effects like a massless, minimally coupled scalar field φ:

$$g_{\mu\nu} = a^2 e^{2\omega} \eta_{\mu\rho} (e^h)^\rho{}_\nu, \;\; h^\mu{}_\mu = 0$$

$$\text{Adding} \quad \mathcal{L}_{\rm GF} = -\frac{1}{2\kappa^2} a^2 \eta^{\mu\nu} F_\mu F_\nu,$$

$$F_\mu = \partial_\rho h^\rho{}_\mu - 2\partial_\mu \omega + 2Hah^0{}_\mu + 4Ha\delta^0{}_\mu \omega$$

Quantum IR effects come from

$$h^{00} \simeq 2\omega, \quad \langle \omega(x)\omega(x') \rangle \simeq -\frac{3}{16} \kappa^2 \langle \varphi(x)\varphi(x') \rangle$$

$$\langle \tilde{h}^{ij}(x)\tilde{h}^{kl}(x') \rangle = (\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk} - \frac{2}{3}\delta^{ij}\delta^{kl})\kappa^2 \langle \varphi(x)\varphi(x') \rangle \qquad \tilde{h}^{ij}\text{: traceless}$$

$$\langle b^i(x)\bar{b}^j(x') \rangle = \delta^{ij}\kappa^2 \langle \varphi(x)\varphi(x') \rangle \qquad b^\mu\text{: FP ghost}$$

$h^{00} - 2\omega$, h^{0i}, b^0 with $m^2 \sim H^2$ are neglected

What we did

- Gravitational fluctuations on dS background may induce anomalous time evolutions of physical quantities (e.g. Λ). In contrast to scalar fields, the fine-tuning of the quadratic term is not necessary

→ dS space is unstable due to self-fluctuations?

'96 N. C. Tsamis, R. P. Woodard

- We derived the effective action up to the one-loop level and found that the dimensionless coupling $g = G_N H^2/\pi$ is dynamically screened by soft gravitons

- Furthermore, we found that the time evolution of the corresponding dS entropy $S(t) = 1/g(t)$ can be identified with that of the von Neumann entropy of the conformal IR mode

One-loop effective action

On a homogeneous, isotropic background $\hat{g}_{\mu\nu} = a^2\eta_{\mu\nu}$,

$$\frac{1}{\kappa^2}\int d^4x\sqrt{-\hat{g}}\big[\hat{R}(1+\langle 4\omega^2\rangle) - 6H^2(1+\langle 8\omega^2\rangle) - \hat{R}^{\mu}{}_{\nu}\langle 2h^{\nu}{}_{\mu}\omega\rangle\big]$$

$$\simeq \frac{1}{\kappa^2}\int d^4x\sqrt{-\hat{g}}\big[\hat{R}a_c^{-2\gamma} - 6H^2 a_c^{-4\gamma} - (\hat{R}^0{}_0 - \hat{R}^1{}_1)2\gamma\log a_c\big]$$

$$\because \langle\omega^2\rangle = -\tfrac{\gamma}{2}\log a_c, \gamma = \tfrac{3}{8}\tfrac{\kappa^2 H^2}{4\pi^2}$$

The $\sqrt{-\hat{g}}\hat{R}$, $\sqrt{-\hat{g}}$ terms indicate an anomalous scaling

$$\frac{1}{\kappa^2(t)} = \frac{1}{\kappa^2}a_c^{-2\gamma}, \quad \frac{H^2(t)}{\kappa^2(t)} = \frac{H^2}{\kappa^2}a_c^{-4\gamma} \quad \Rightarrow \quad H^2(t) = H^2 a_c^{-2\gamma}$$

$$\Rightarrow \quad a = a_c^{1+\gamma} \qquad\qquad a_c = \frac{1}{-H\tau}$$

However, if $a \neq a_c$, the covariance is not manifest: $\hat{R}^0{}_0 - \hat{R}^1{}_1 = -2\gamma H^2 a_c^{-2\gamma}$

Inflaton as a counter term

Introducing an inflaton f with $\frac{1}{\kappa^2}\left(\frac{V'}{V}\right)^2 = \gamma,$

$$\frac{1}{\kappa^2}\int d^4x\sqrt{-g}\left[R - 6H^2V(f) - \frac{\kappa^2}{2}g^{\mu\nu}\partial_\mu f\partial_\nu f\right]$$

$$V = 1 - \sqrt{\gamma}\kappa f = a_c^{-2\gamma}$$

The covariance is kept manifest: $\hat{R}^0{}_0 - \hat{R}^1{}_1 + \frac{\kappa^2}{2}a^{-2}\partial_0 f\partial_0 f = 0$

After a conformal transformation,

$$\frac{1}{\kappa^2}\int d^4x\sqrt{-\hat{g}}\left[\hat{R} - 6H^2a_c^{-2\gamma} - \frac{\kappa^2}{2}\hat{g}^{\mu\nu}\partial_\mu f\partial_\nu f\right]$$

$$\frac{1}{\kappa^2(t)} = \frac{1}{\kappa^2}, \quad H^2(t) = H^2a_c^{-2\gamma} \qquad\qquad \kappa^2 = 16\pi G_N$$

Soft gravitons screen the dimensionless coupling: $g(t) = G_NH^2a_c^{-2\gamma}/\pi$

dS entropy

The dS entropy increases with the decay of the coupling

$$S = \frac{1}{g(t)} = \frac{\pi}{G_N H^2} a_c^{2\gamma}$$

Considering the conformal IR mode, we can reproduce the same result

$$S = \log Z$$

$$\because \; F = \cancel{U} - TS = -T \log Z$$

$$= \frac{1}{16\pi G_N} \int d^4x \sqrt{\hat{g}} \left[\hat{R} \langle e^{2\omega} \rangle - 6H^2 \langle e^{4\omega} \rangle \right]$$

by rotating $dS_4 \to S_4$ (adiabatically)

$$= \frac{\pi}{G_N H^2} (1 - \langle 4\omega^2 \rangle) = \frac{\pi}{G_N H^2} a_c^{2\gamma}$$

This time evolution is local: $S = \frac{\pi}{G_N H^2}(1 + 2\gamma \log a_c) = \frac{\pi}{G_N H^2} + 3Ht$

von Neumann entropy

The distribution function of the conformal mode at the superhorizon scale follows the Fokker-Planck equation:

$$\frac{\partial}{\partial t}\rho(t,\omega) = \frac{\gamma}{2}\cdot\frac{H}{2}\frac{\partial^2}{\partial\omega^2}\rho(t,\omega)$$

Gaussian $\Leftarrow \gamma \ll 1$

$$\rho(t,\omega) = \sqrt{\frac{4\xi(t)}{\pi g}}\exp\Big\{-\frac{4\xi(t)}{g}\omega^2\Big\}, \quad \xi(t) = \frac{1}{1+6Ht}$$

From the solution, the von Neumann entropy can be constructed

$$\begin{aligned} S_N &= -\mathrm{tr}(\rho\log\rho) \\ &= \frac{1}{2}\Big\{1 + \log\pi g - \log 4\xi(t)\Big\} \\ &\to \frac{1}{2}\log(1+6Ht) \ \sim 3Ht \text{ at } Ht \ll 1 \end{aligned}$$

Consistent with the dS entropy

It can describe not only the local time evolution but also <u>the global one</u>

β function

From the time evolution of the von Neumann entropy,

$$S(t) = \frac{1}{g} - \frac{1}{2}\log \xi(t)$$

Equivalent to the action

Since the bare action $S_B = \frac{1}{g(t)} + \frac{1}{2}\log \xi(t)$ is time independent,

$$\beta(g(t)) = -\frac{1}{2}g^2(t), \quad \beta(g(t)) \equiv \frac{\partial}{\partial \log(1+6Ht)} g(t)$$

$g(t) = G_N H^2(t)/\pi$ is asymptotically free toward future

Solving it exactly, we obtain the global time evolution:

$$g(t) = \frac{2}{\log(1+6Ht) + \frac{2}{g}}$$

Liouville gravity

'19 H. Kitamoto, Y. Kitazawa

2D Lioville gravity with a large central charge $c = N_B + N_F/2 > 25$ resembles 4D Einstein gravity

$$S(t) = \frac{c-25}{6}\log\left(\frac{H^2}{H^2(t)}\right) \qquad \text{increases}$$

$$H^2(t) = H^2(1 + \langle 2\omega^2 \rangle) = H^2 a_c^{-\frac{12}{c-25}} \qquad \text{decreases}$$

$$\because \ \langle \omega^2 \rangle = -\frac{6}{c-25}\log a_c \qquad \text{negative norm}$$

The von Neumann entropy can be identified with the dS entropy and can describe the global time evolution

$$\frac{\partial}{\partial t}\rho(t,\omega) = \frac{6}{c-25}\cdot\frac{H}{2}\frac{\partial}{\partial \omega^2}\rho(t,\omega)$$

$$S_N = -\mathrm{tr}(\rho\log\rho)$$
$$\to \frac{1}{2}\log(1+4Ht) \ \sim 2Ht \text{ at } Ht \ll 1$$

Summary

- In order to obtain the covariance of the effective Einstein action, it is necessary to introduce an inflaton as a counter term

- The one-loop effective action shows that soft gravitons screen the dimensionless coupling $g = G_N H^2/\pi$ → Instability of dS space

- The dS entropy increases with the decay of the coupling, and it can be identified with the von Neumann entropy of the conformal mode at the superhorizon scale

- From the von Neumann entropy, we derived the one-loop β function. Solving the renormalization group equation exactly, we found that the coupling logarithmically decays with time: $g \propto 1/\log Ht$

- The screening of the cosmological constant occurs also in 2D Liouville gravity with $c > 25$ ← Negative norm of the conformal mode