Generation of quasiparticles by particle-antiparticle mixing and limitations of quantum mechanics

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Contents

- Limitations of QM in the presence of heavy particle-antiparticle mixing
- QFT description of the mixing

QM of mixing

Effective Hamiltonian

$$i\frac{d}{dt}\begin{pmatrix} |\Phi(t)\rangle \\ |\Phi^*(t)\rangle \end{pmatrix} = H_{\text{eff}}\begin{pmatrix} |\Phi(t)\rangle \\ |\Phi^*(t)\rangle \end{pmatrix}$$

Self-energy

$$H_{\text{eff}} := m_\Phi\left[1 - \frac{1}{2}\Sigma'(m_\Phi^2)\right].$$

$$C_\Phi H_{\text{eff}} C_\Phi^{-1} = m_\Phi\left[1 - \frac{1}{2}(C\Sigma' C^{-1})(m_\Phi^2)\right] = m_\Phi\left[1 - \frac{1}{2}\widehat{\Sigma}'(m_\Phi^2)\right] = \begin{pmatrix} p_{\widehat{\Phi}_1} & 0 \\ 0 & p_{\widehat{\Phi}_2} \end{pmatrix}$$

Diagonalization

$$p^2_{\widehat{\Phi}_{\widehat{\alpha}}} = m^2_{\widehat{\Phi}_{\widehat{\alpha}}} - i m_{\widehat{\Phi}_{\widehat{\alpha}}}\Gamma_{\widehat{\Phi}_{\widehat{\alpha}}}$$

$$|\widehat{\Phi}_{\widehat{\alpha}}(t)\rangle = e^{-ip_{\widehat{\Phi}_{\widehat{\alpha}}}(t-t_0)}\,|\widehat{\Phi}_{\widehat{\alpha}}\rangle$$

Mass eigenstate: a fixed linear combination of basis states

Time evolution

$$e^{-ip_{\widehat{\Phi}_{\widehat{\alpha}}}t} = e^{-im_{\widehat{\Phi}_{\widehat{\alpha}}}t}e^{-(\Gamma_{\widehat{\Phi}_{\widehat{\alpha}}}/2)t}.$$

FT

$$\frac{i}{p^2 - p^2_{\widehat{\Phi}_{\widehat{\gamma}}}}$$

In QFT, obtain this structure by diagonalizing the resummed propagator.

Resummed propagator

Resummed propagator

$$
\begin{aligned}
i\Delta(p^2) &= i(p^2 - M_\Phi^2)^{-1} + i(p^2 - M_\Phi^2)^{-1}[i\Sigma(p^2)]i(p^2 - M_\Phi^2)^{-1} \\
&\qquad + i(p^2 - M_\Phi^2)^{-1}[i\Sigma(p^2)]i(p^2 - M_\Phi^2)^{-1}[i\Sigma(p^2)]i(p^2 - M_\Phi^2)^{-1} + \cdots \\
&= \sum_{n=0}^{\infty} i(p^2 - M_\Phi^2)^{-1}\left\{[i\Sigma(p^2)]i(p^2 - M_\Phi^2)^{-1}\right\}^n \\
&= i\left\{[1 + \Sigma'(p^2)]p^2 - M_\Phi^2\right\}^{-1},
\end{aligned}
\qquad M_\Phi := m_\Phi I_2 \qquad \Sigma(p^2) = p^2\Sigma'(p^2)
$$

Mass Self-energy

(a) $i\Delta_{11}(p^2)$ (b) $i\Delta_{22}(p^2)$ (c) $i\Delta_{12}(p^2)$ (d) $i\Delta_{21}(p^2)$

$\Phi_1 = \Phi,\ \Phi_2 = \Phi^\dagger (= \Phi^*)$

Diagonalization and pole expansion

Diagonalized

Mixing matrix

$$\boxed{\widehat{\Sigma}'(p^2) = C(p^2)\Sigma'(p^2)C^{-1}(p^2),}$$

$$\widehat{\Delta}(p^2) := C(p^2)\Delta(p^2)C^{-1}(p^2) = m_\Phi^{-2}P^2(p^2)[p^2 - P^2(p^2)]^{-1}$$

Unstable particle

$$P^2_{\widehat{\alpha}}(p^2) = m^2_\Phi[1 + \widehat{\Sigma}'_{\widehat{\alpha}}(p^2)]^{-1}$$

The mixing matrix is **non-unitary** because of the absorptive part in the self-energy.

After expansion around the pole

$$i\widehat{\Delta}_{\widehat{\alpha}}(p^2) = \frac{iR_{\widehat{\Phi}_{\widehat{\alpha}}}}{p^2 - p^2_{\widehat{\Phi}_{\widehat{\alpha}}}} + \cdots \longleftrightarrow e^{-ip_{\widehat{\Phi}_{\widehat{\alpha}}}t} = e^{-im_{\widehat{\Phi}_{\widehat{\alpha}}}t}e^{-(\Gamma_{\widehat{\Phi}_{\widehat{\alpha}}}/2)t}.$$

$$p^2_{\widehat{\Phi}_{\widehat{\alpha}}} = m^2_{\widehat{\Phi}_{\widehat{\alpha}}} - im_{\widehat{\Phi}_{\widehat{\alpha}}}\Gamma_{\widehat{\Phi}_{\widehat{\alpha}}}$$

$$\lim_{p^2\to p^2_{\widehat{\Phi}_{\widehat{\alpha}}}}(p^2 - p^2_{\widehat{\Phi}_{\widehat{\alpha}}})\widehat{\Delta}_{\widehat{\alpha}}(p^2) = R_{\widehat{\Phi}_{\widehat{\alpha}}}$$

Limitations of QM

$$i\Delta_{\beta\alpha}(p^2) = \sum_{\widehat{\gamma}} (C_\Phi^{-1})_{\beta\widehat{\gamma}} \frac{i}{p^2 - p^2_{\widehat{\Phi}_{\widehat{\gamma}}}} (C_\Phi)_{\widehat{\gamma}\alpha} + \cdots$$

$$C_\Phi = C(m_\Phi^2)$$

$$\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} i\Delta_{\beta\alpha}(p^2) = \langle\Omega|\Phi_\beta(x)\Phi_\alpha^\dagger(y)|\Omega\rangle, \ (x^0 > y^0)$$

Time-ordered correlation function

$$\Phi_1 = \Phi, \ \Phi_2 = \Phi^\dagger (= \Phi^*)$$

$$\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{i}{p^2 - p^2_{\widehat{\Phi}_{\widehat{\alpha}}}} + \cdots = \langle\Omega|\widehat{\Phi}^f_{\widehat{\alpha}}(x)\widehat{\Phi}^{i\dagger}_{\widehat{\alpha}}(y)|\Omega\rangle, \ (x^0 > y^0)$$

$$\widehat{\Phi}^i_{\widehat{\alpha}} = \sum_\beta (C_\Phi^{-1})^\dagger_{\widehat{\alpha}\beta}\Phi_\beta, \quad \widehat{\Phi}^f_{\widehat{\alpha}} = \sum_\beta (C_\Phi)_{\widehat{\alpha}\beta}\Phi_\beta$$

$$\widehat{\Phi}^f_{\widehat{\alpha}} \neq \widehat{\Phi}^i_{\widehat{\alpha}}$$

Non-unitary

$$e^{-ip_{\widehat{\Phi}_{\widehat{\alpha}}}(x^0-y^0)} \propto \langle\Omega|\widehat{\Phi}^f_{\widehat{\alpha}}(x)\widehat{\Phi}^{i\dagger}_{\widehat{\alpha}}(y)|\Omega\rangle, \ (x^0 > y^0)$$

Quasiparticle

The particle of $\widehat{\Phi}_{\widehat{\alpha}}$ in the interacting theory is the degree of freedom that emerges as an excitation of $\widehat{\Phi}^i_{\widehat{\alpha}}$ and ends as an excitation of $\widehat{\Phi}^f_{\widehat{\alpha}}$ ($\widehat{\Phi}^f_{\widehat{\alpha}} \neq \widehat{\Phi}^i_{\widehat{\alpha}}$).

Limitations of QM

$$i\frac{d}{dt}\begin{pmatrix}|\Phi(t)\rangle\\|\Phi^*(t)\rangle\end{pmatrix}=H_{\rm eff}\begin{pmatrix}|\Phi(t)\rangle\\|\Phi^*(t)\rangle\end{pmatrix}$$

After diagonalization

$$i(d/dt)|\widehat{\Phi}_{\widehat{\alpha}}(t)\rangle = p_{\widehat{\Phi}_{\widehat{\alpha}}}|\widehat{\Phi}_{\widehat{\alpha}}(t)\rangle$$

QM Solution

Mass eigenstate: a fixed linear combination of basis states

$$|\widehat{\Phi}_{\widehat{\alpha}}(t)\rangle = e^{-ip_{\widehat{\Phi}_{\widehat{\alpha}}}(t-t_0)}|\widehat{\Phi}_{\widehat{\alpha}}(t_0)\rangle$$

Quantum mechanics is not a proper non-relativistic limit of the quantum field theory in the presence of heavy particle-antiparticle mixing.

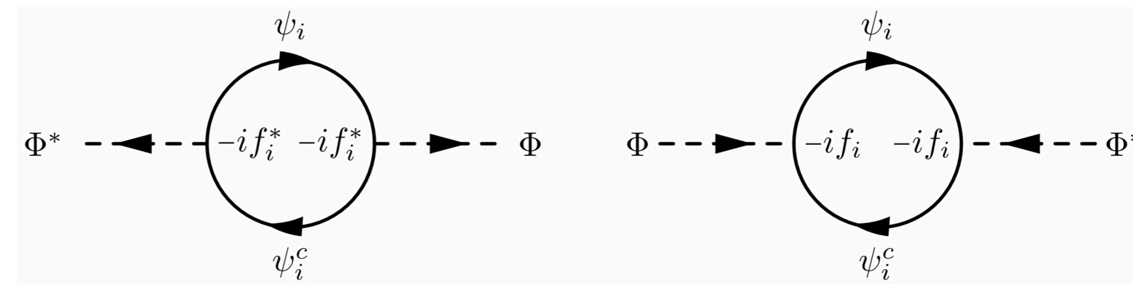
We must find a way outside QM to describe particle-antiparticle mixing.

Read the decay rates from scattering mediated by on-shell quasiparticles.

QFT of mixing

Mixing Tagging

$$\mathcal{L}_{\rm int} = -\sum_i f_i \overline{\psi_i}\psi_i\Phi - \sum_i h_i \overline{\chi_i}\xi\Phi + \text{H.c.},$$

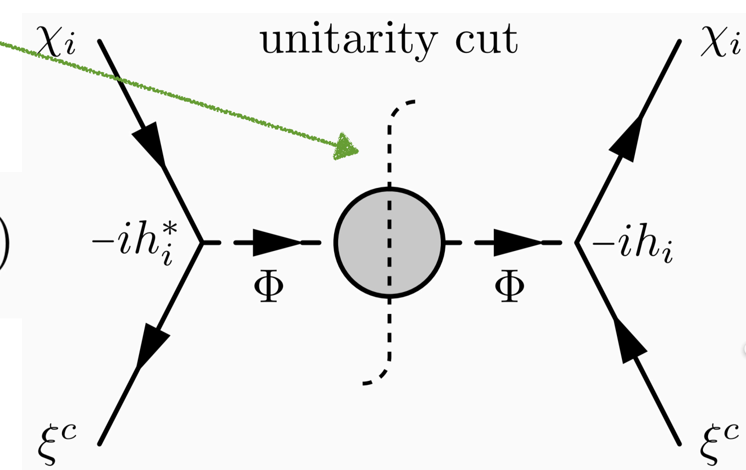


Initial states

Transition rate

$\chi_j\xi^c,\ \chi_j^c\xi,\ \psi_j\psi_j^c$

$\sum_{\widehat{\alpha}}\Gamma^{\rm scat}_{X\to\widehat{\Phi}_{\widehat{\alpha}}\to\chi_i\xi^c}$



$$\int d\Pi_{\chi_i}\int d\Pi_{\xi^c}\ (2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}_{\chi_i}-\mathbf{p}_{\xi^c})$$

$$d\Pi_X = \prod_{j\in X}\frac{d^3\mathbf{p}_j}{(2\pi)^3 2E_j}$$

$$\sum_{\widehat{\alpha}}\Gamma_{\widehat{\Phi}_{\widehat{\alpha}}} = \sum_{X,\widehat{\alpha},Y}\Gamma^{\rm scat}_{X\to\widehat{\Phi}_{\widehat{\alpha}}\to Y}$$

Numerically confirmed

Cannot define the transition rate for each quasiparticle.

From the imaginary part of the pole

QFT of mixing

Partial decay widths

$$\Gamma(\Phi \to \chi_i \xi^c) = \sum_{j,\widehat{\alpha}} \Gamma^{\text{scat}}_{\chi_j \xi^c \to \widehat{\Phi}_{\widehat{\alpha}} \to \chi_i \xi^c},$$

$$\Gamma(\Phi^* \to \chi_i \xi^c) = \sum_{j,\widehat{\alpha}} \Gamma^{\text{scat}}_{\chi_j^c \xi \to \widehat{\Phi}_{\widehat{\alpha}} \to \chi_i \xi^c}.$$

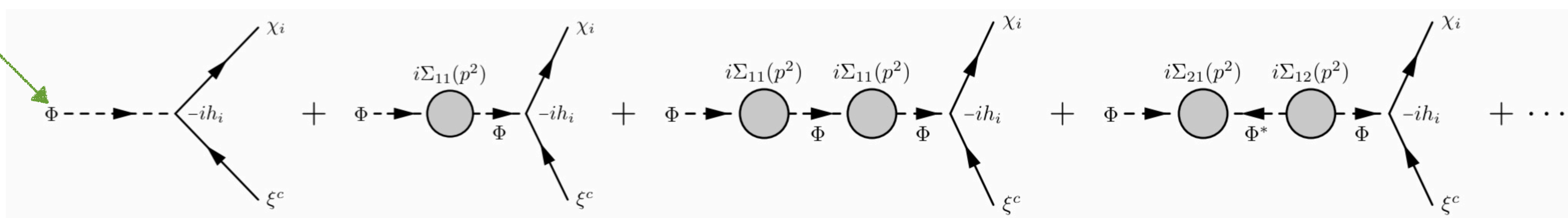
Off-shell

On-shell

Consistent with a well-known formula in the absence of mixing.

This sum vanishes.

On-shell



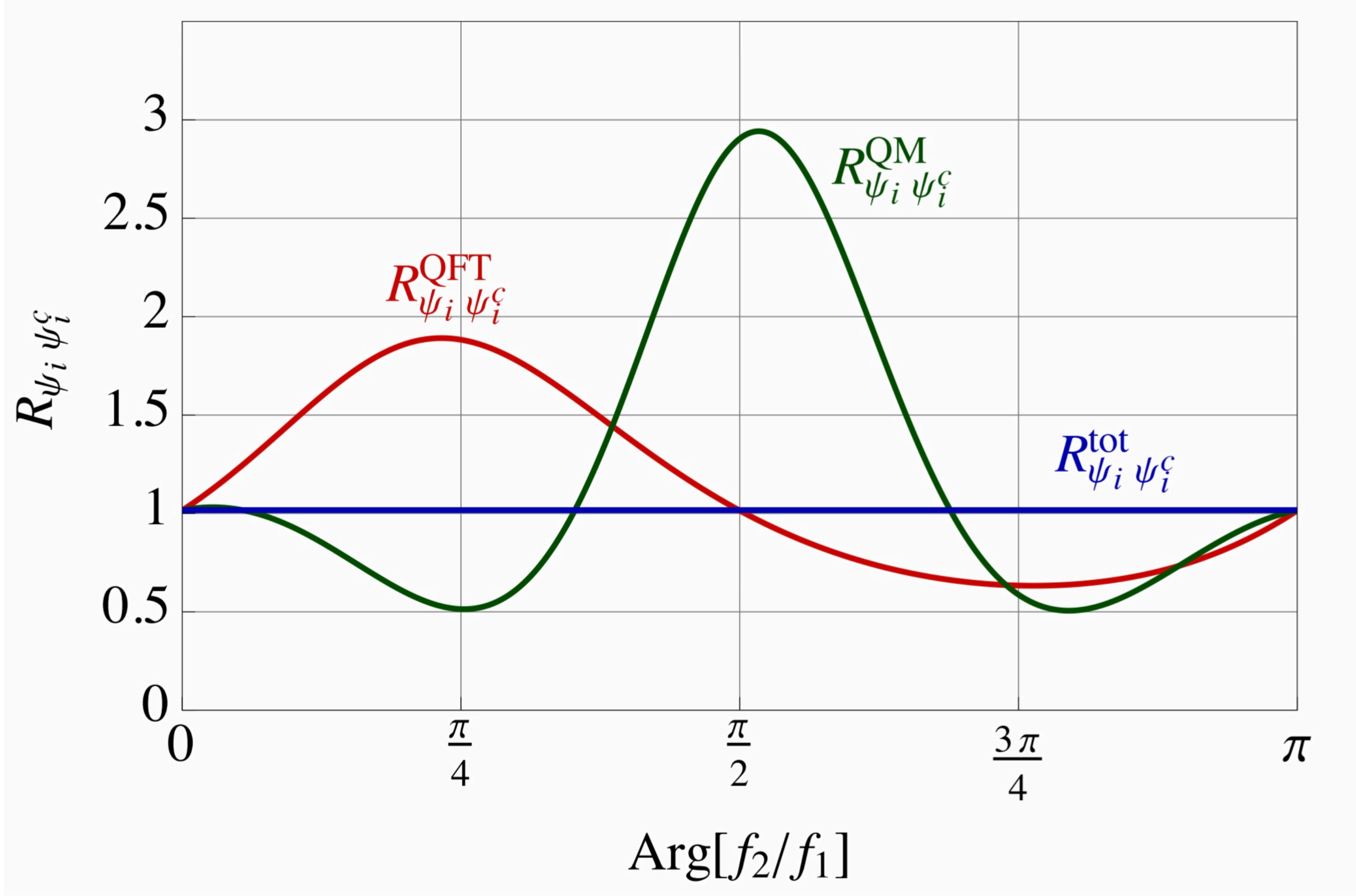
$$\Gamma(\Phi_\alpha \to \chi_i \xi^c) = \frac{1}{2m_\Phi} \int d\Pi_{\chi_i} \int d\Pi_{\xi^c} \; (2\pi)^4 \delta^4(p_{\chi_i} + p_{\xi^c} - p_{\Phi_\alpha}) |\mathcal{M}(\Phi_\alpha \to \chi_i \xi^c)|^2,$$

Cannot consider the mixing effect.

Comparison of QFT and QM

There can be a big difference in the presence of interferences.

$$R_{\psi_i \psi_i^c} = \frac{\Gamma(\Phi \to \psi_2 \psi_2^c)}{\Gamma(\Phi \to \psi_1 \psi_1^c)}.$$



$$\Phi \to \cdots \to \Phi \to \psi_i \psi_i^c$$

$$\Phi \to \cdots \to \Phi^* \to \psi_i \psi_i^c$$

Interference

Conclusion

- QM is not a proper non-relativistic limit of QFT in the presence of heavy particle-antiparticle mixing.
- A method in QFT is developed which correctly consider the mixing effect.
- The mixing of neutral mesons and CP violation should be reanalyzed.
- It might have an implication for the B-anomalies.
- The same method can be applied to the well-known problem in the CP asymmetry in the decays of Majorana particles.