D meson mixing based on dispersion relation

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charm

- **Charm quark mass is a unique scale.**

$$m_c = 1.3\ \text{GeV}$$

— too heavy for ChPT

— too light for $\Lambda_{\mathrm{QCD}}/m_c$ expansion?

Theoretically challenging

charm

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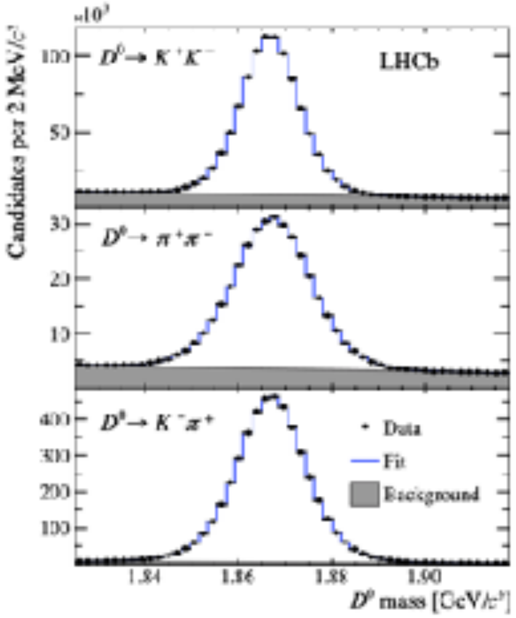
Theoretically challenging

- **High statistics data is provided.**

— in a good stage to test theories



LHCb [1810.06874]

charm

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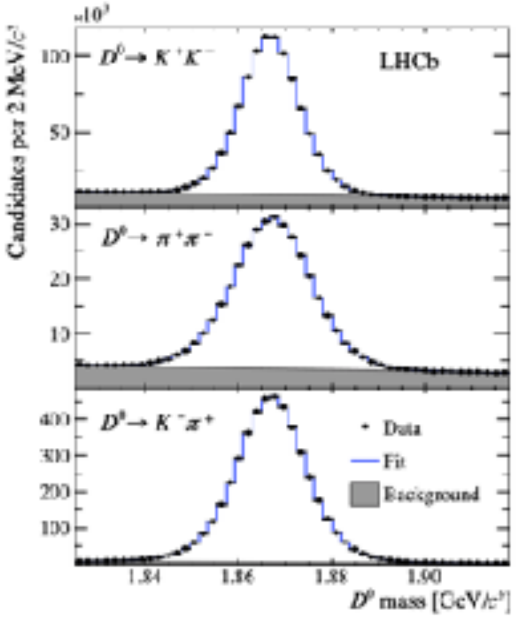


LHCb [1810.06874]

- **D meson mixing.**

— experimental data are not quantitatively reproduced yet

Outline

(A) **D meson mixing**

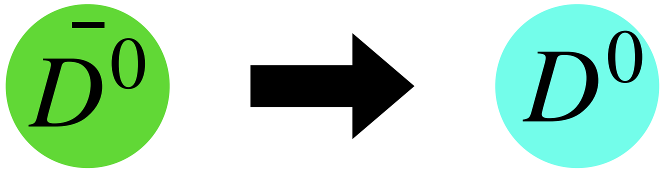
— experiment / theoretical methods

(B) **Dispersion relation**

— parametrization

$D^0 - \bar{D}^0$ mixing

time evolution



Time evolution Eq. $$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}$$

Mass eigenstate $$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$$

$q/p \neq$ unity

↕

CP violation

observables

(CP conserving limit)

$$x = (M_1 - M_2)/\Gamma = 2M_{12}/\Gamma$$ **mass difference**

$$y = (\Gamma_1 - \Gamma_2)/2\Gamma = 2\Gamma_{12}/\Gamma$$ **width difference**

Experiment

$x = (3.9^{+1.1}_{-1.2}) \times 10^{-3}$ $y = (6.51^{+0.63}_{-0.69}) \times 10^{-3}$ **non-zero** $> 11.5\sigma$

$|q/p| = 0.969^{+0.050}_{-0.045}$ $\arg(q/p)(^\circ) = -3.9^{+4.5}_{-4.6}$ **CP violation not measured**

HFLAV at Moriond2019

Γ **: total width**

D meson mixing: theory

Two methods

◉Exclusive

Hadronic-level analysis

Not calculable $\left\{\begin{array}{l}\Gamma[D \to \pi\pi] \\ \Gamma[D \to KK]\end{array}\right.$

↓

Data are used

(our study)

◉Inclusive

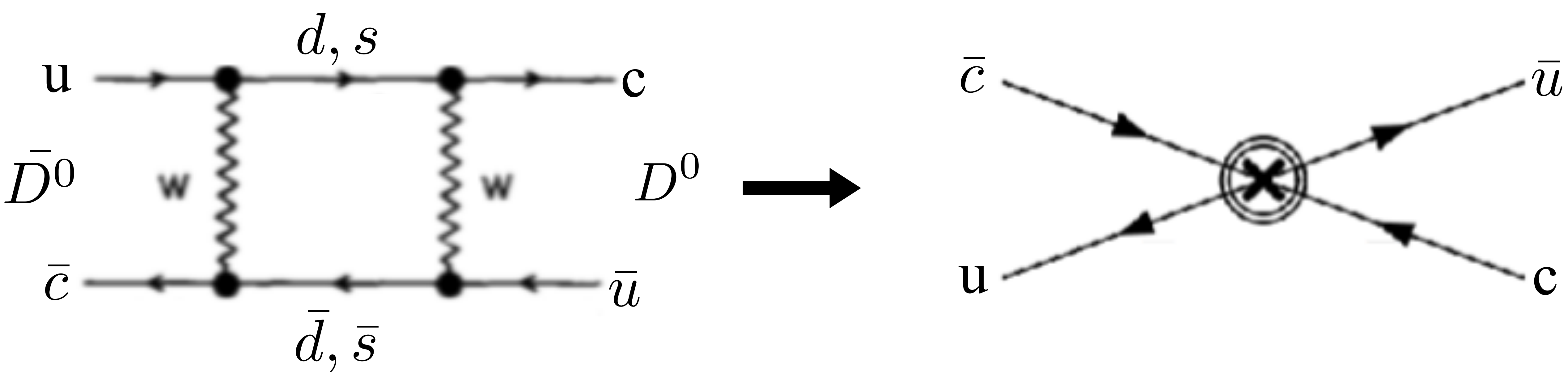
Quark-level analysis

without data

(quark-hadron duality)

Inclusive approach

Quark-level



box diagrams

4-fermi interaction

OPE/Heavy quark expansion (HQE)

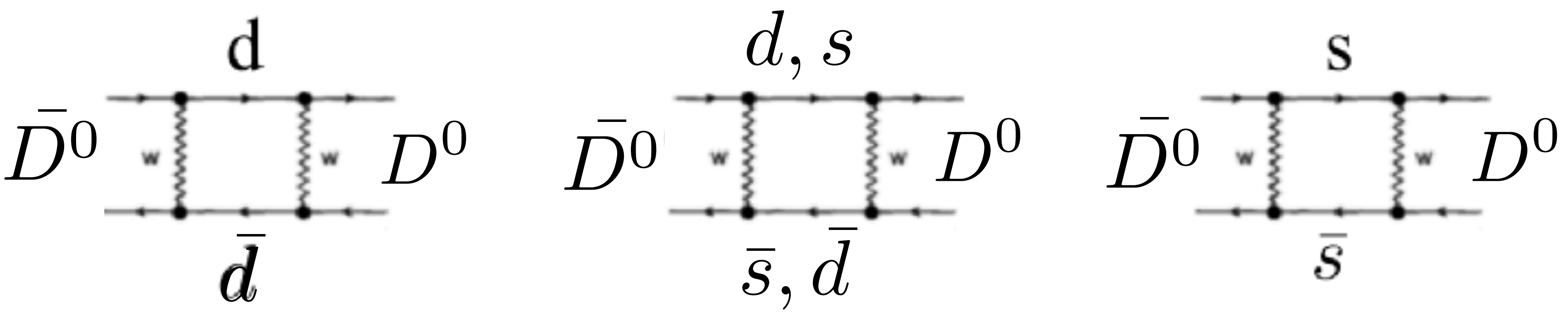
$$\Pi_{12} = M_{12} - \frac{i}{2}\Gamma_{12} \longrightarrow \Pi_{12} = \sum_n \frac{C_n}{\boxed{m_c^n}} \langle D^0 | \mathcal{O}_n^{\Delta C=2} | \bar{D}^0 \rangle$$

C_n **: Wilson coefficients**

n: **dimension of operator**

Contributions

$$\lambda_i = V_{ci} V_{ui}^*$$



CKM suppressed

~~**+ bottom loop**~~

neglect

For $m_s = m_d$

CKM unitarity

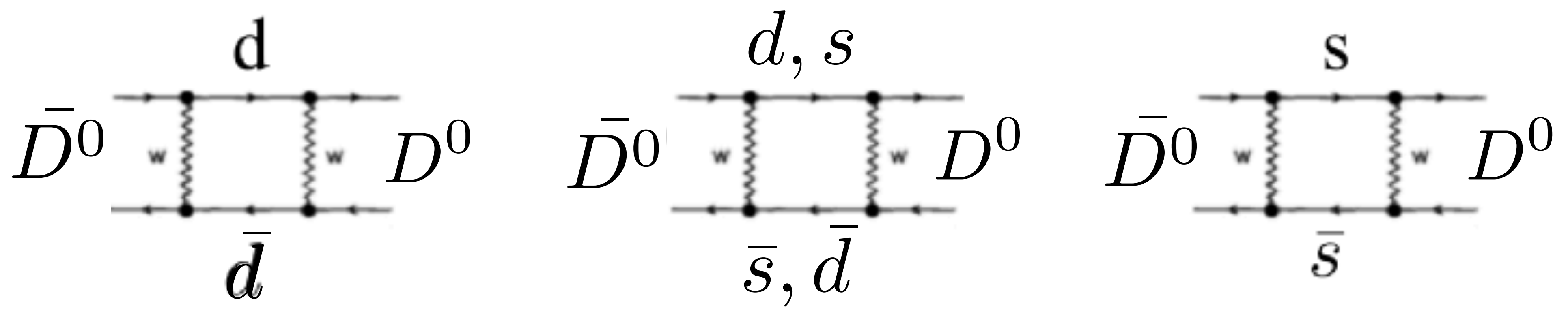
$$\lambda_d + \lambda_s + \cancel{\lambda_b} = 0$$

neglect

summation $\propto \lambda_d^2 + 2\lambda_d \lambda_s + \lambda_s^2$

Contributions

$\lambda_i = V_{ci} V_{ui}^*$



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neglect

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➡ **Suppressed by GIM mechanism.**

non-zero contributions to D mixing

$$\left\{ \begin{array}{l} \text{SU(3) breaking: } \dfrac{m_s^2 - m_d^2}{m_c^2} \\ \text{small CKM } : \lambda_b = \mathcal{O}(\lambda^5) \end{array} \right.$$

Theory / Exp. comparison (for inclusive)

D meson

Hagelin 1981, Cheng 1982

Buras, Slominski and Steger 1984

NLO QCD Golowich and Petrov 2005

SM $$\begin{cases} x \simeq 6 \times 10^{-7} \\ y \simeq 6 \times 10^{-7} \end{cases}$$

Suppressed by GIM

HFLAV at Moriond2019

Exp. $$\begin{cases} x = (3.9^{+1.1}_{-1.2}) \times 10^{-3} \\ y = (6.51^{+0.63}_{-0.69}) \times 10^{-3} \end{cases}$$

B_s meson

Artuso, Borissov and Lenz, 2016

SM $$\begin{cases} \Delta M_s = (18.3 \pm 2.7)\mathrm{ps}^{-1} \\ \Delta\Gamma_s = (0.088 \pm 0.020)\mathrm{ps}^{-1} \end{cases}$$

HFLAV

Exp. $$\begin{cases} \Delta M_s = (17.757 \pm 0.021)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_s = (0.082 \pm 0.006)\ \mathrm{ps}^{-1} \end{cases}$$

B_d meson

Artuso, Borissov and Lenz, 2016

SM $$\begin{cases} \Delta M_d = (0.528 \pm 0.078)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_d = (2.61 \pm 0.59) \cdot 10^{-3}\ \mathrm{ps}^{-1} \end{cases}$$

HFLAV

Exp. $$\begin{cases} \Delta M_d = (0.5055 \pm 0.0020)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_d = 0.66(1 \pm 10) \cdot 10^{-3}\ \mathrm{ps}^{-1} \end{cases}$$

- For B_s, B_d mesons, the data are reproduced within 1σ.
- For D meson, the order of magnitude is not reproduced within leading-power.

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$$\begin{cases} x \simeq 6 \times 10^{-7} \\ y \simeq 6 \times 10^{-7} \end{cases}$$

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HFLAV at Moriond2019

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SN

Ex

Possibilities discussed in the literature

HQE is not convergent for charm?

— Higher dimensional operators (D=9,12) avoid severe GIM cancellation?

$x, y \sim \mathcal{O}(10^{-3})$ Bigi and Uraltsev, 2001

$x \sim y \lesssim 10^{-3}$ Falk, Grossman, Ligeti and Petrov, 2001

(non-perturbative matrix element required)

Violation of quark-hadron duality?

— 20% violation explains the data. (based on model)

Jubb, Kirk, Lenz and Tetlalmatzi-Xolocotzi, 2017

Beyond the standard model?

— x is explainable.
Golowich, Hewett, Pakvasa and Petrov, 2009

— contribution to y is small.
Golowich, Pakvasa and Petrov, 2007

2016

$^{-1}$

^{3} ps^{-1}

ps^{-1}

^{3} ps^{-1}

- For B_s, B_d mesons, the data are reproduced within 1σ.
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Inclusive approach

Strategy in this study

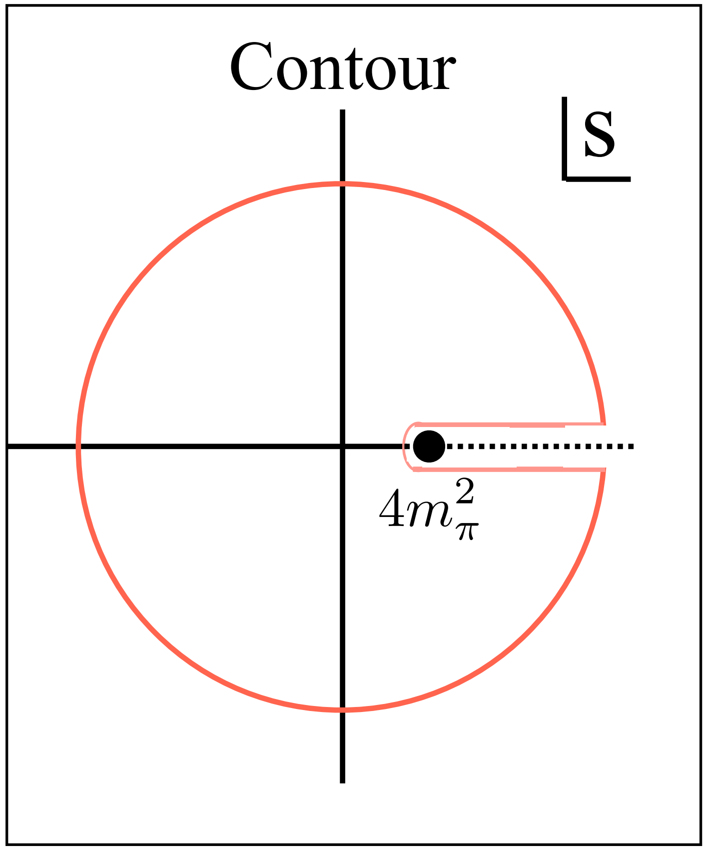
- We use the dispersion relation.
- HQE is applicable above Λ. $\Lambda \sim m_b$

→ $1/m_b$ expansion works better than $1/m_c$

Dispersion relation

$s = m_q^2$

$$x(m_D^2) = -\frac{1}{\pi} P \int_{4m_\pi^2}^{\infty} \frac{y(s)}{s - m_D^2} ds$$ for D meson



generalized

$$\Rightarrow x(m_q^2) = -\frac{1}{\pi} P \int_{4m_\pi^2}^{\infty} \frac{y(s)}{s - m_q^2} ds$$ for fictitious D meson

Dispersion relation

$$\pi x(s) = P\int_{4m_\pi^2}^{\infty} \frac{y(s')}{s-s'}ds'$$

$$= P\int_{4m_\pi^2}^{\textcircled{\Lambda}} \frac{y(s')}{s-s'}ds' + P\int_{\textcircled{\Lambda}}^{\infty} \frac{y(s')}{s-s'}ds'$$

devided into two pieces

Dispersion relation

$$\pi x(s) = P\int_{4m_\pi^2}^{\infty} \frac{y(s')}{s-s'}ds'$$

$$= P\int_{4m_\pi^2}^{\Lambda} \frac{y(s')}{s-s'}ds' + P\int_{\Lambda}^{\infty} \frac{y(s')}{s-s'}ds'$$

unknown function **calculable object**

$$\Rightarrow \int_{4m_\pi^2}^{\Lambda} \frac{y(s')}{s-s'}\mathrm{d}s' = \omega(s)$$

s: large

$s > \Lambda$

Fredholm equation: multiple solutions do exist.

$$\omega(s) = \pi x(s) - P\int_{\Lambda}^{\infty} \frac{y(s')}{s-s'}ds'$$

expansion of l.h.s.

Problem to solve

$$\int_{4m_\pi^2}^{\Lambda} \frac{y(s')}{s - s'} \mathrm{d}s' = \omega(s)$$

$$\frac{1}{s - s'} = \frac{1}{s - m^2 - (s' - m^2)}$$

m^2 : constant

$$= \frac{1}{s - m^2} \frac{1}{1 - \frac{s' - m^2}{s - m^2}}$$

(Taylor expansion)

expansion of l.h.s.

Problem to solve

$$\int_{4m_\pi^2}^{\Lambda} \frac{y(s')}{s - s'} \mathrm{d}s' = \omega(s)$$

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m^2 : constant

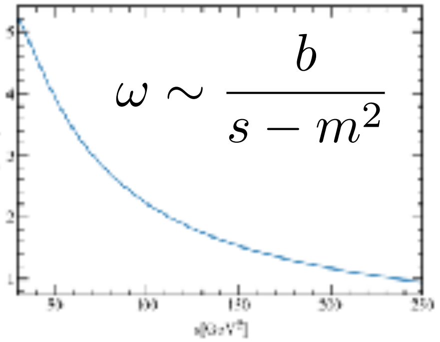
$$= \frac{1}{s - m^2} \frac{1}{1 - \frac{s' - m^2}{s - m^2}}$$

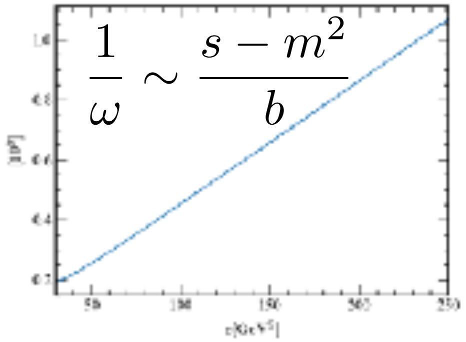
(Taylor expansion)

$$= \frac{1}{s - m^2} + \underbrace{\frac{(s' - m^2)}{(s - m^2)^2} + \frac{(s' - m^2)^2}{(s - m^2)^3} + \cdots}_{\text{subleading}}$$

This expansion is convergent since $s >> s'$

Mathematical consistency: $\omega(s) \simeq \dfrac{b}{s - m^2}$ for large s





Problem to solve

$$\int_{4m_\pi^2}^{\Lambda} \frac{y(s')}{\boxed{s - s'}} \mathrm{d}s' = \omega(s)$$

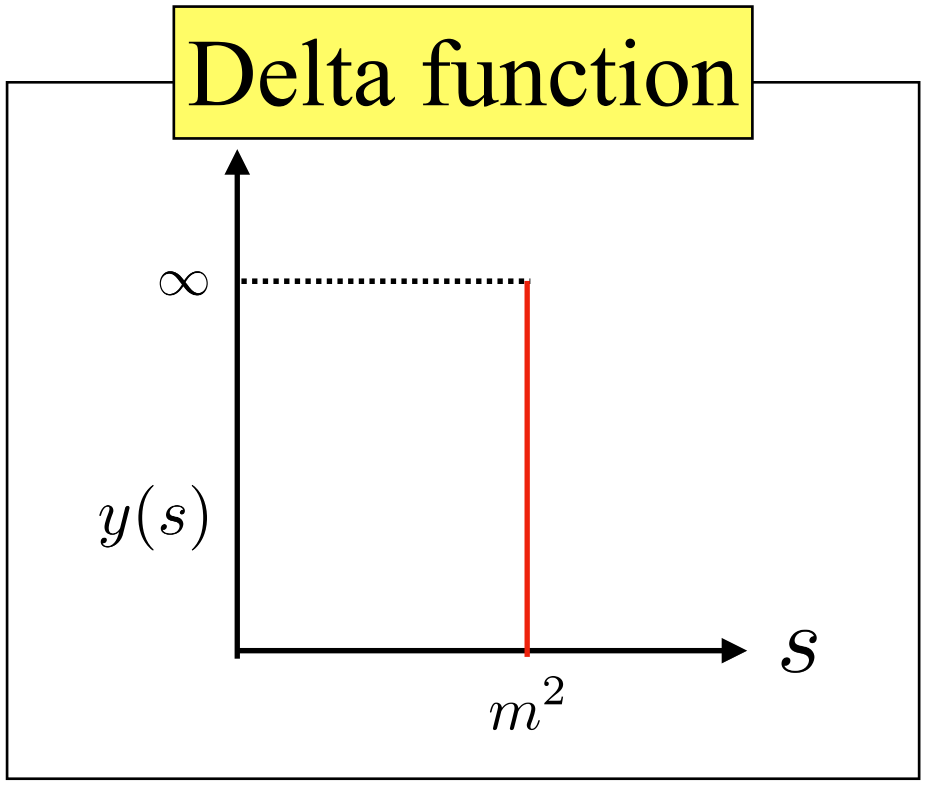
use of the approximation: $\omega(s) \simeq \dfrac{b}{s - m^2}$

The problem becomes →

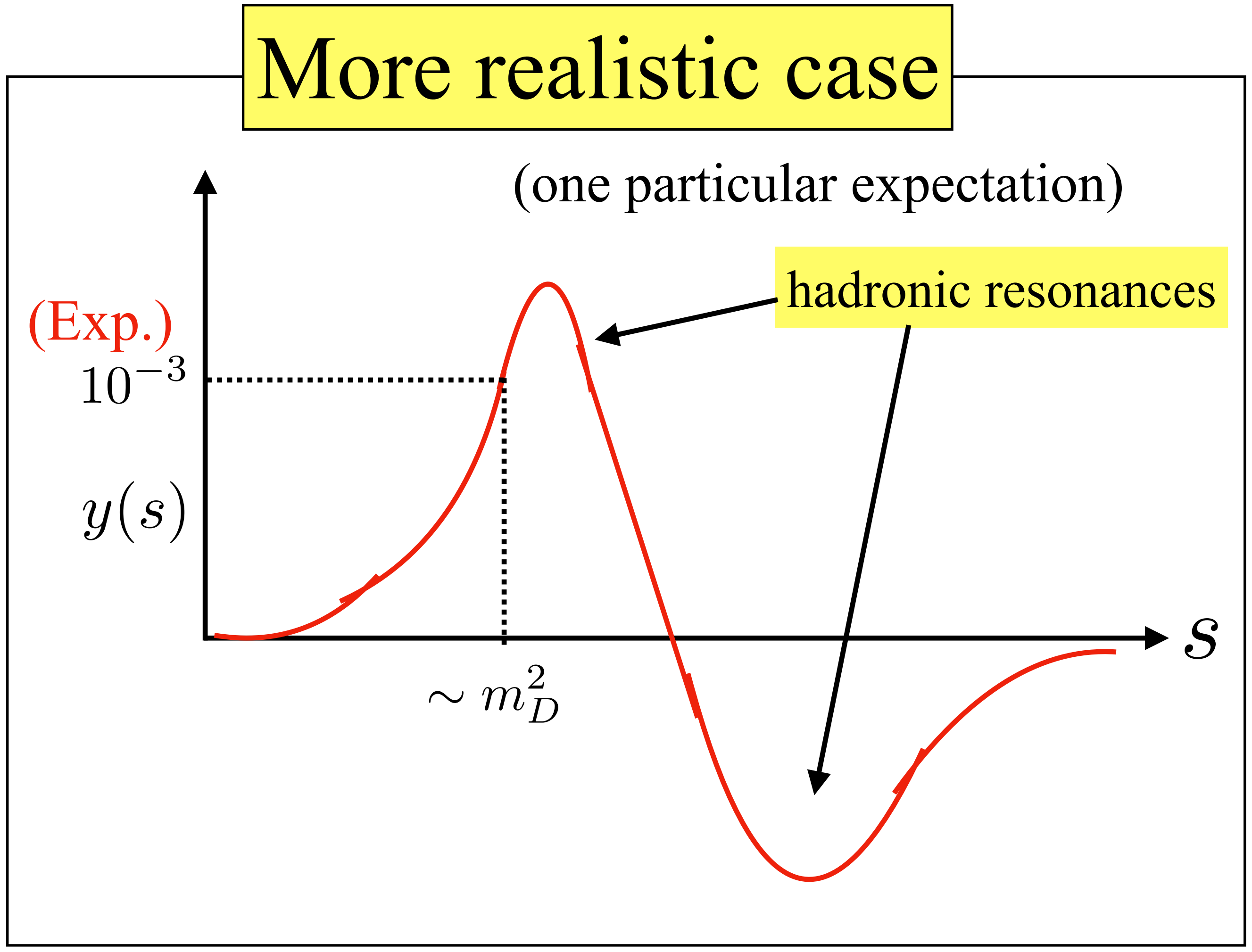
$$\int_{4m_\pi^2}^{\Lambda} \frac{y(s')}{s - s'} ds' = \frac{b}{s - m^2}$$

(one particular) solution: $y(s') = b\,\delta(s' - m^2)$

Delta function insufficient?



Delta function



More realistic case

(one particular expectation)

Points to improve

○ Delta function should be smeared to be finite.

○ Higher order terms in the expansion should be taken into account.

Parametrization

$$y(s) = Ns\frac{b_0 + b_1(s - m^2) + b_2(s - m^2)^2}{[(s - m^2)^2 + d^2]^2}$$

Strategy

① substitute theoretical input

$$\int_{4m_\pi^2}^{\Lambda} \frac{y(s')}{s - s'} ds' = \omega(s)$$

② Fit unknown constants so as to reproduce the theoretical input.

unknowns: (m^2, d, b_0, b_1, b_2)

③ Pick out a solution and check whether experimental data are reproduced.

Minimum structure

How to handle multiple solutions

(1) Fix (m^2, d)

(2) Fit (b_0, b_1, b_2)

via the least square method

Residual sum of squares (RSS)

$$\mathrm{RSS} = \sum_{i=1}^{200} \left(\omega(s_i) - \int_{\Lambda}^{\infty} \frac{y(s')}{s_i - s'} ds' \right)^2$$

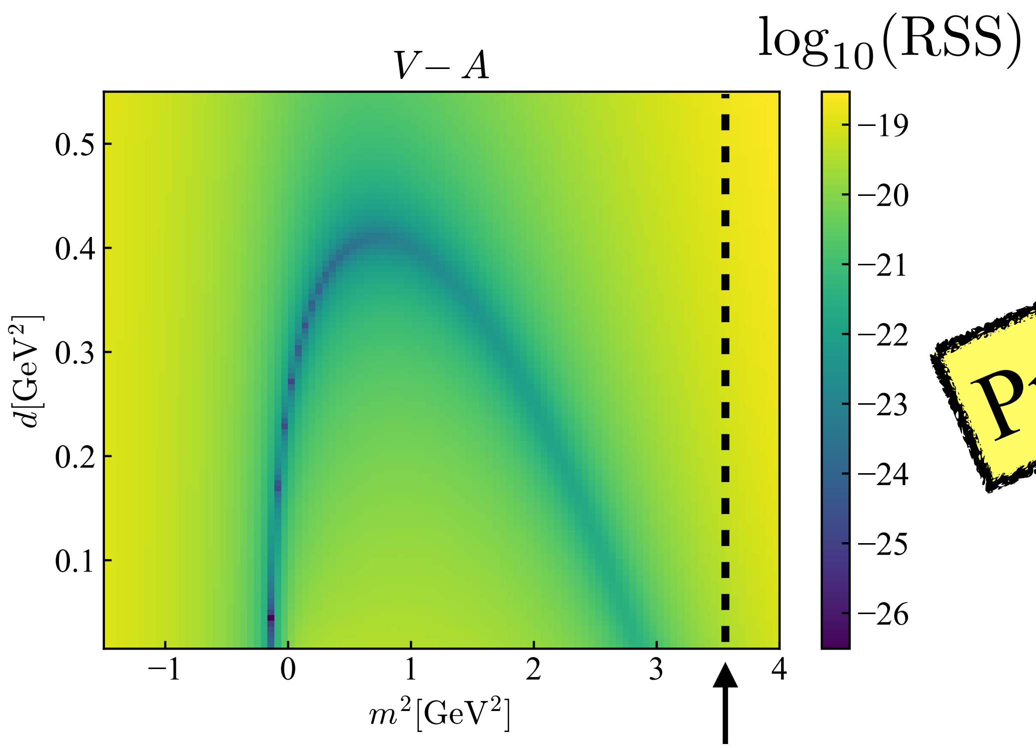
$s_i : [30\mathrm{GeV}^2, 250\mathrm{GeV}^2]$

$m_s = 96.6$ MeV

$\Lambda = 15\ \mathrm{GeV}^2$

$\mathcal{O}_1 = (\bar{c}u)_{\mathrm{V-A}}(\bar{c}u)_{\mathrm{V-A}}$

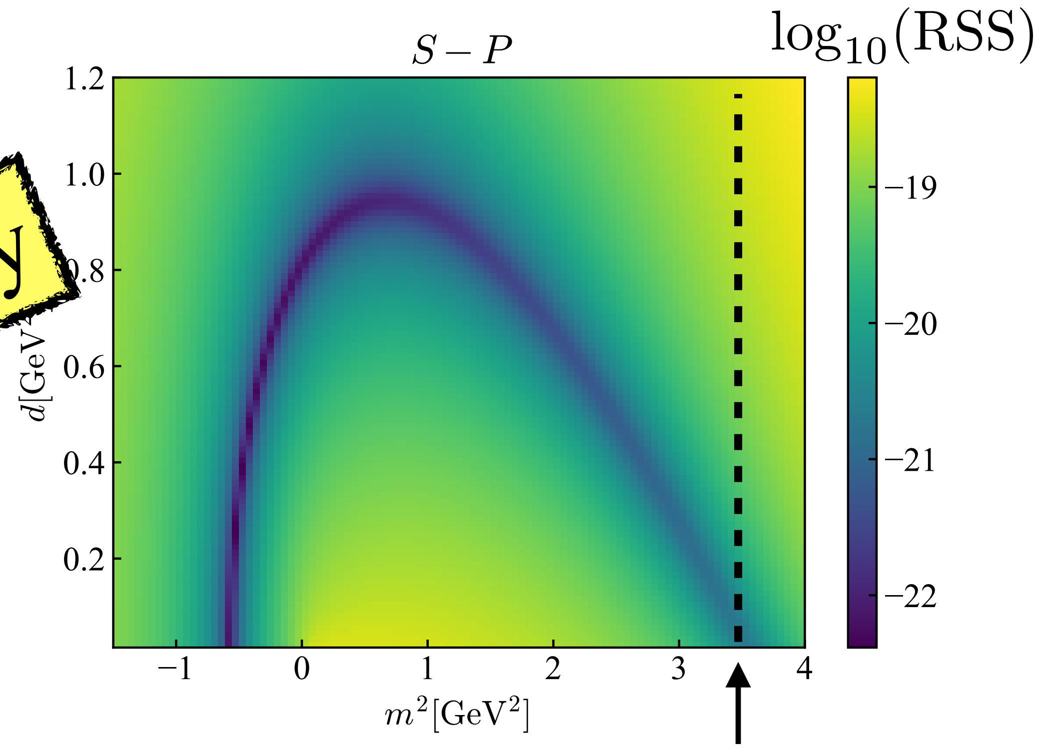
$\mathcal{O}_2 = (\bar{c}u)_{\mathrm{S-P}}(\bar{c}u)_{\mathrm{S-P}}$



$\sim m_D^2$

(enhancement not expected)

Preliminary



$\sim m_D^2$

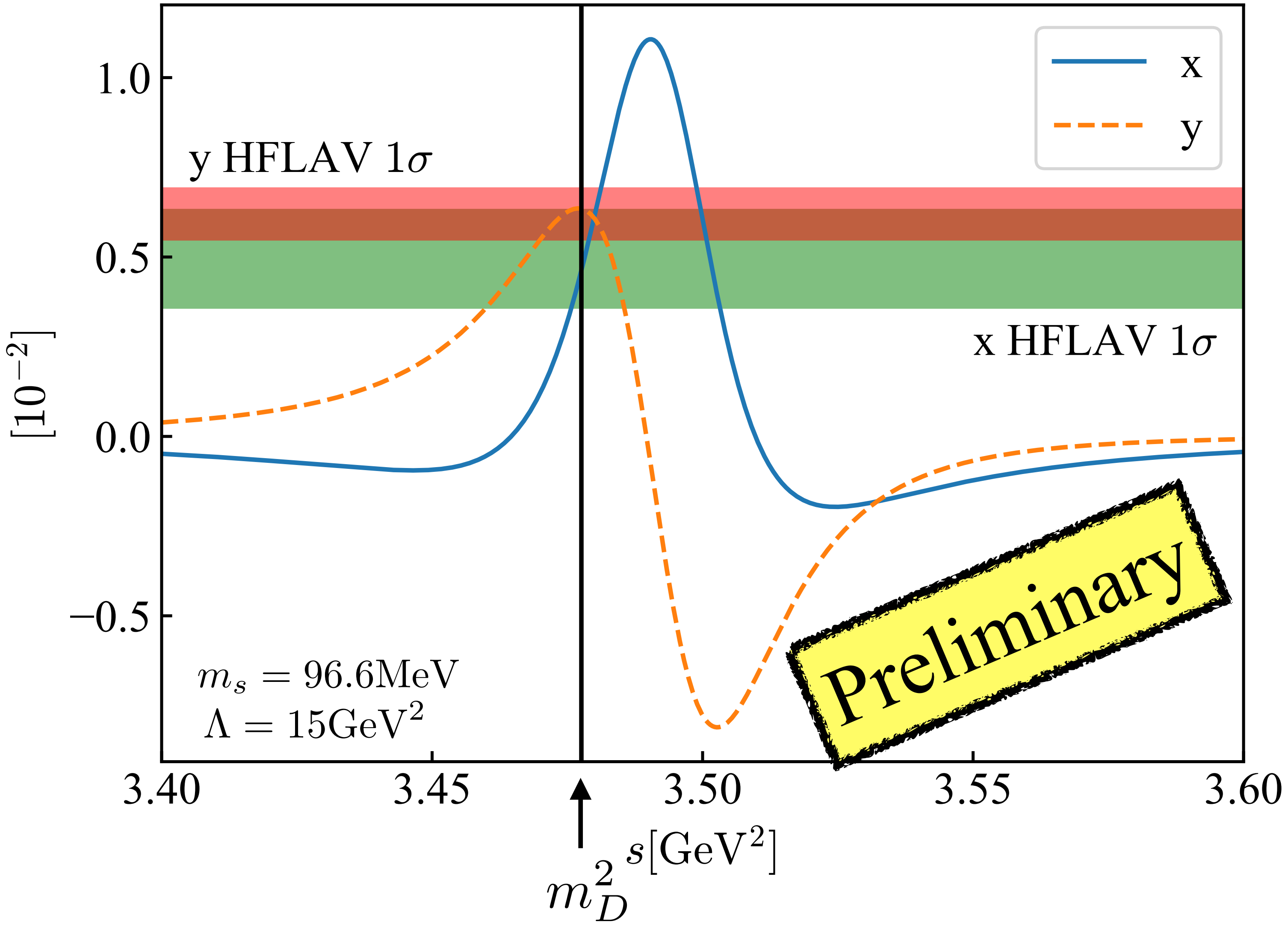
$m^2 = 3.49\ \mathrm{GeV}^2 \quad d = 0.022\ \mathrm{GeV}^2 \quad b_0 = -8.31 \times 10^{-5}\ \mathrm{GeV}^2$

$b_1 = -3.45 \times 10^{-2} \quad b_2 = 0.189\ \mathrm{GeV}^{-2}$

Result

HFLAV2019 Experimental data $\begin{cases} \text{Green band: x} \\ \text{Red band: y} \end{cases}$

Without CP violation



b_2 contribution is only 10% of total x and y.

$$y(s) = Ns\frac{b_0 + b_1(s-m^2) + b_2(s-m^2)^2}{[(s-m^2)^2 + d^2]^2}$$

Summary

◎ **Using the OPE, which is reliable above $\sim m_b$, the dispersion relation turns into an inverse problem.**

◎ **Parametrization of the solution is proposed so as to respect the argument of the multipole expantion.**

◎ **The specific solution in the SM can quantitatively explain the experimental data.**

Backup

Parametrization

$$y(s) = Ns \frac{b_0 + b_1(s - m^2) + b_2(s - m^2)^2}{[(s - m^2)^2 + d^2]^2}$$

(a) The narrow width limit reproduces delta function.

$$\lim_{d \to 0} y(s) = b\, \delta(s - m^2) + \cdots$$

(b) It parametrizes higher order terms in the Taylor expansion.

(c) The boundary condition, $\Gamma_{12}(4m_\pi^2) = 0$, is satisfied. $(4m_\pi^2 \sim 0)$

or $y(4m_\pi^2) = 0$

Expressions in the SM

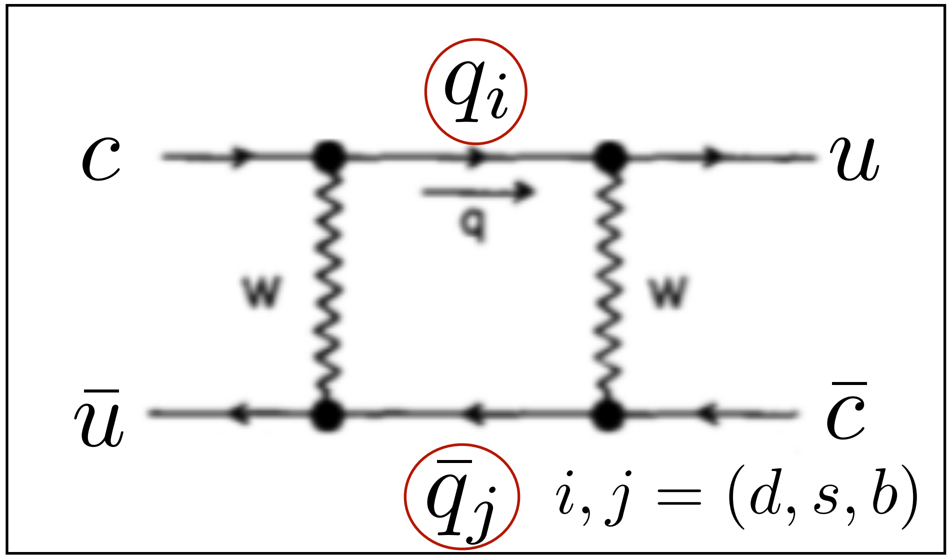
$$M_{12} - \frac{i}{2}\Gamma_{12} = B\,\langle D^0|\mathcal{O}_1|\bar{D}^0\rangle + C\,\langle D^0|\mathcal{O}_2|\bar{D}^0\rangle$$

Two contributions
$$\begin{cases} \mathcal{O}_1 = (\bar{c}u)_{\mathrm{V-A}}(\bar{c}u)_{\mathrm{V-A}} \\ \mathcal{O}_2 = (\bar{c}u)_{\mathrm{S-P}}(\bar{c}u)_{\mathrm{S-P}} \end{cases}$$

Flavor sum

$$\lambda_i = V_{ci}V_{ui}^*$$

Explicit formula
$$\begin{cases} M_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H \displaystyle\sum_{i,j}^{d,s,b} \lambda_i\lambda_j \left[\xi_1 B_1(\mu) B_{ij}^{(d)}(s) + \xi_2 R(s) B_2(\mu) C_{ij}^{(d)}(s)\right], \\ \Gamma_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H \displaystyle\sum_{i,j}^{d,s,b} \lambda_i\lambda_j \left[\xi_1 B_1(\mu) B_{ij}^{(a)}(s) + \xi_2 R(s) B_2(\mu) C_{ij}^{(a)}(s)\right], \end{cases}$$



CKM unitarity

$$\lambda_d + \lambda_s + \lambda_b = 0 \longleftrightarrow \lambda_d = -\lambda_s - \lambda_b$$

$$\begin{cases} M_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H (\lambda_s^2 U_{ss}^{(d)} + 2\lambda_s\lambda_b U_{sb}^{(d)} + \lambda_b^2 U_{bb}^{(d)}) \\ \Gamma_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H (\lambda_s^2 U_{ss}^{(a)} + 2\lambda_s\lambda_b U_{sb}^{(a)} + \lambda_b^2 U_{bb}^{(a)}) \end{cases}$$

U : loop function

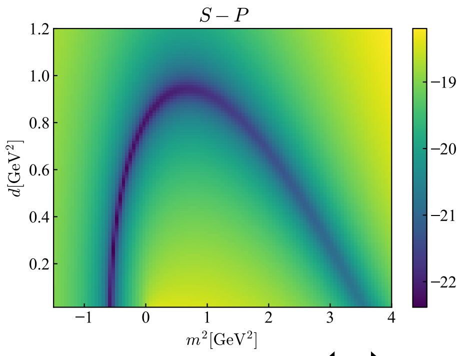
Buras, Slominski and Steger,
Nucl. Phys. B245, 369 (1984)

Solution of Fredholm equation

- ○ Solution is sensitive to the variation of an input.

$$\int_{4m_\pi^2}^{\Lambda^2}\frac{y(s')}{s'-s}\mathrm{d}s' = \overset{\text{input}}{\boxed{\omega(s)}}$$

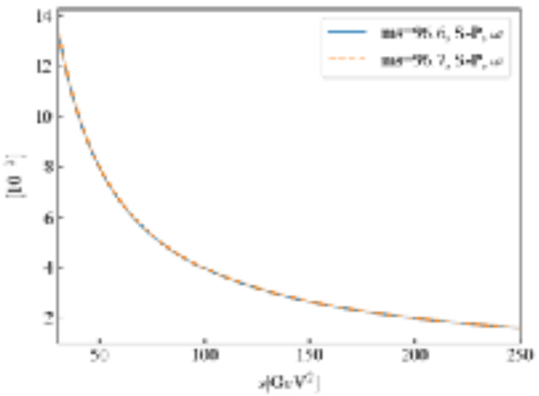
Example: $m_s = 96.6\text{MeV}$ vs $m_s = 96.7\text{MeV}$



$\sim m_D^2$

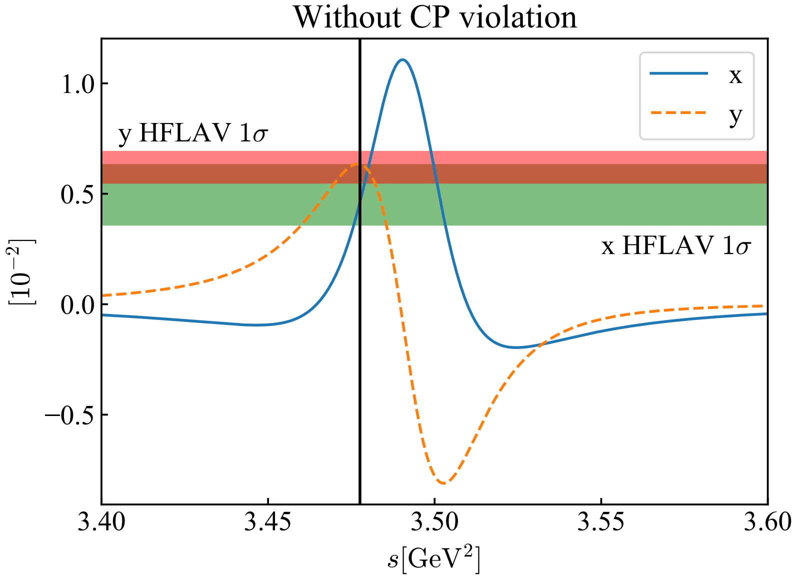
input: almost unchanged

0.3%



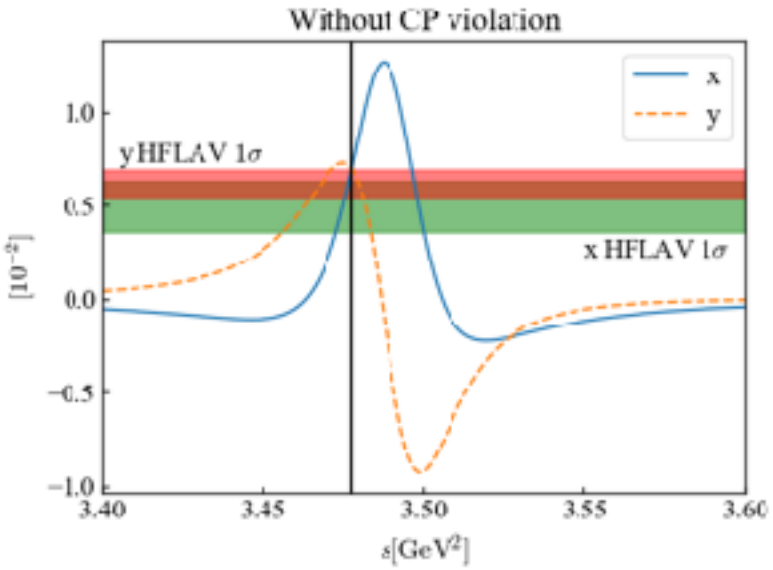
solution: modified

$m_s = 96.6$ MeV



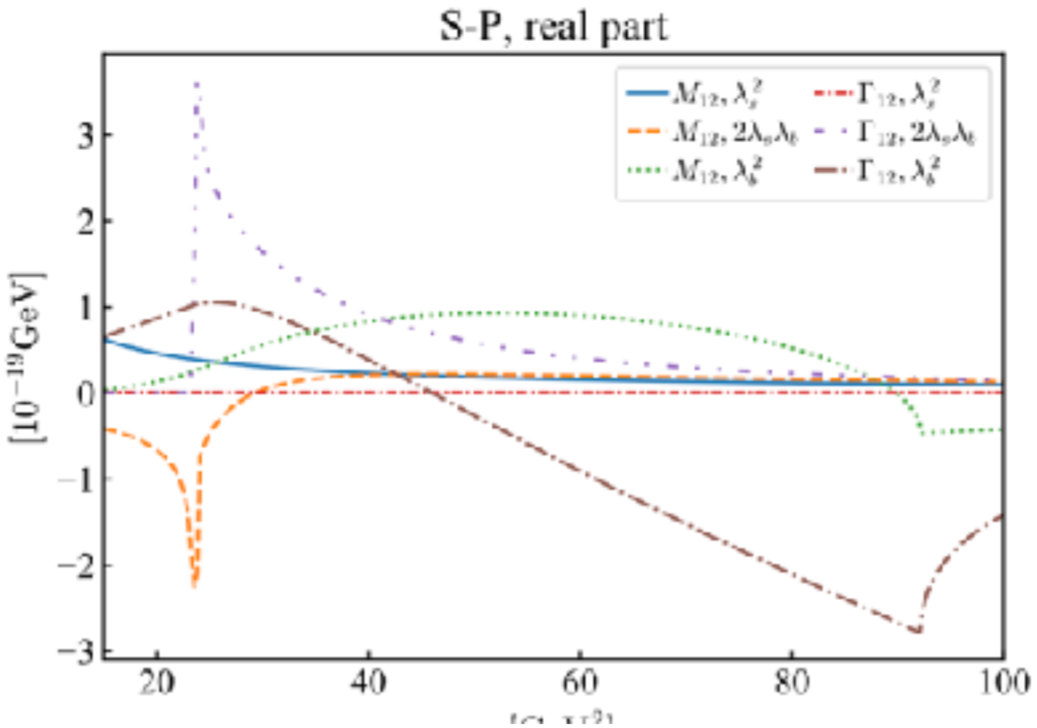
1σ explainable

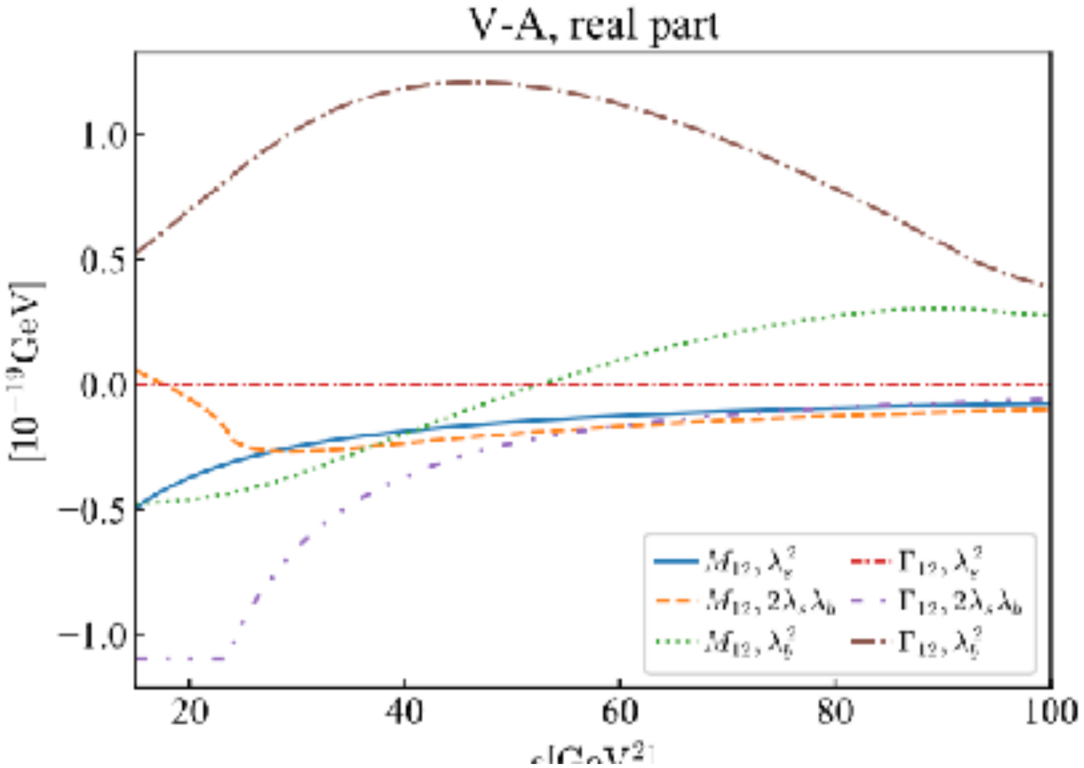
$m_s = 96.7$ MeV

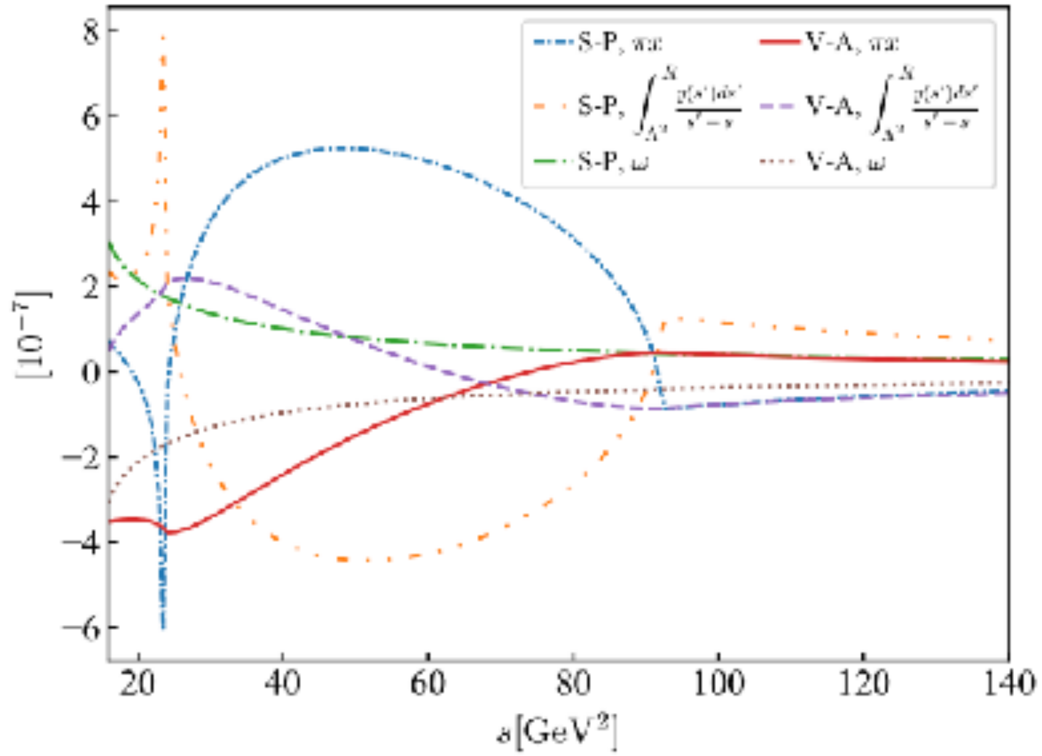


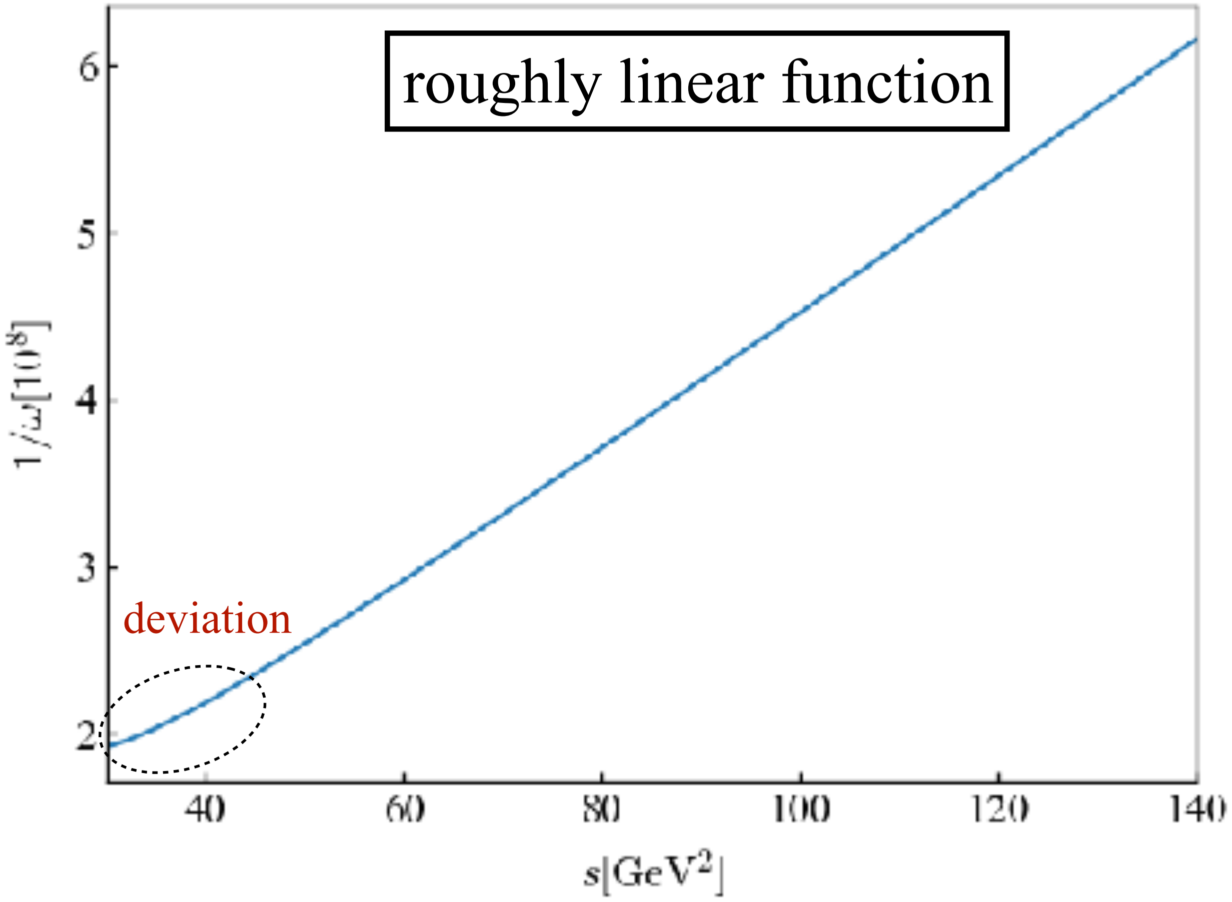
not explainable

Input of OPE









roughly linear function

deviation

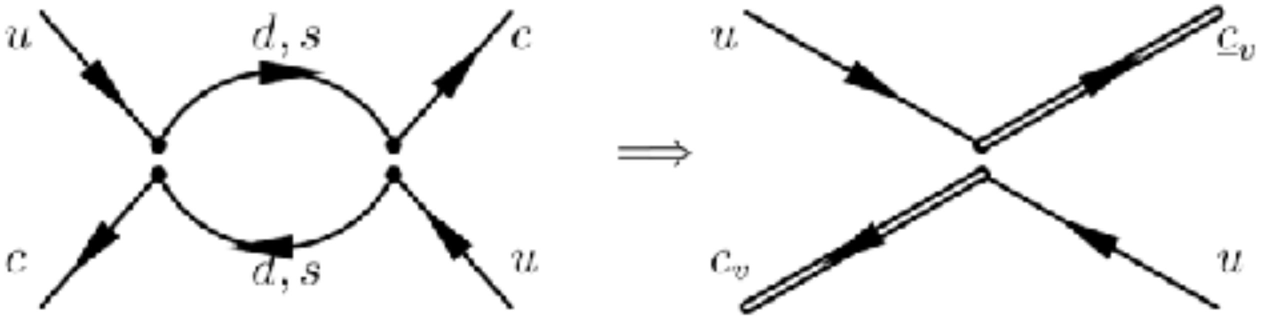
Structure of higher-dimensional operators

Ohl, Ricciardi and Simmons [9301212]

D=6

Leading in $1/m_q$

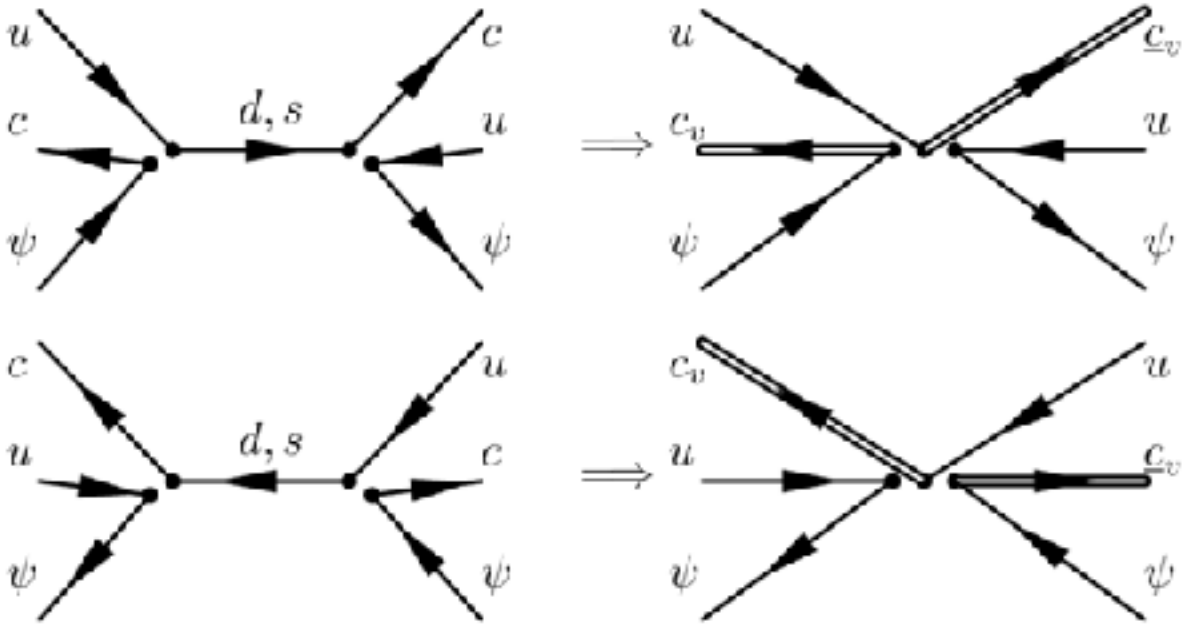
$(\bar{c}_v\Gamma_1 u)\,(\underline{c}_v\Gamma_2 u)$



D=9

Subleading in $1/m_q$

$\left(\bar{\psi}\Gamma_1 u\right)\,(\bar{c}_v\Gamma_2\psi)\,(\underline{c}_v\Gamma_3 u)$

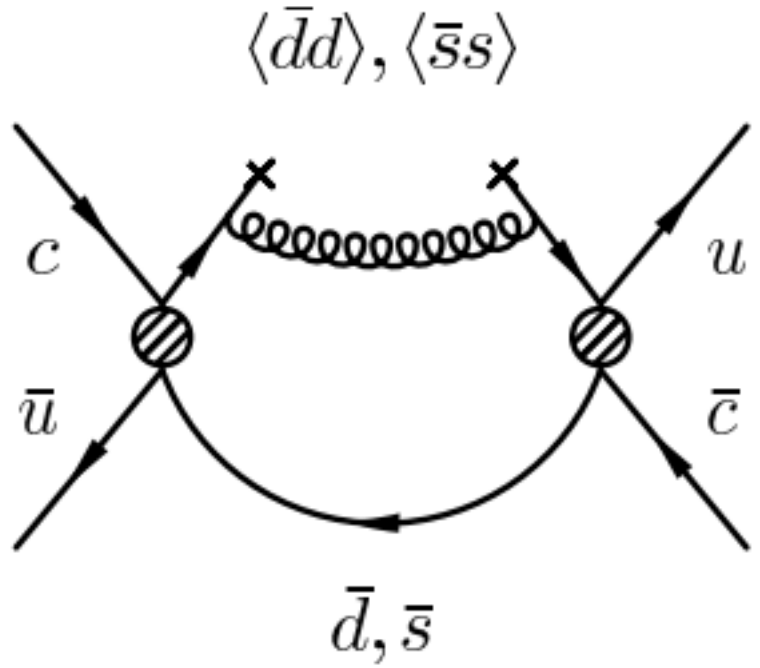


Γ_i : color/Dirac structure

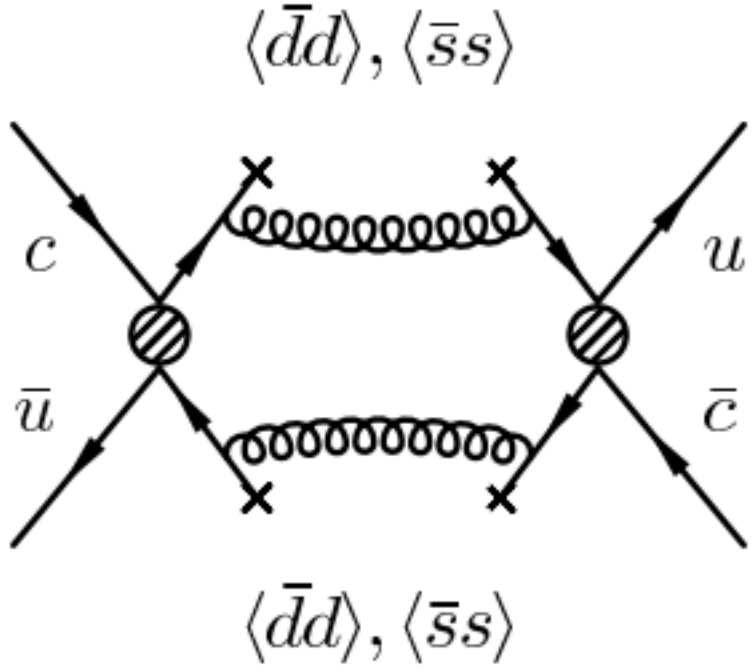
large m_q : leading term is dominant

Effect of higer-dimensional operators

Bobrowski, Lenz and Riedl [1002.4794]



D=9



D=12

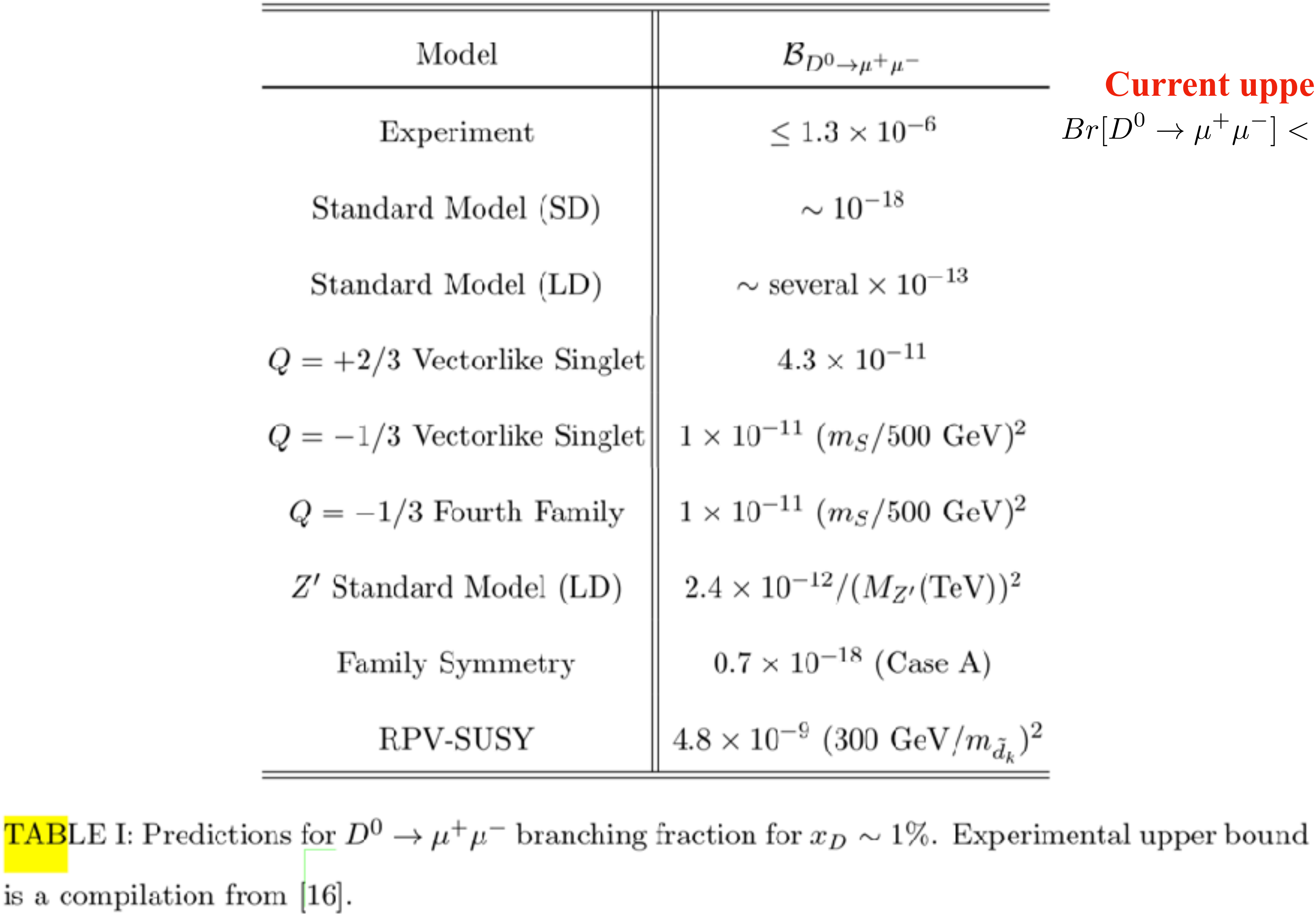
$\mathcal{O}(\alpha_s(4\pi)\langle\bar{q}q\rangle/m_c^3)$

$\mathcal{O}(\alpha_s^2(4\pi)^2\langle\bar{q}q\rangle^2/m_c^6)$

y	no GIM	with GIM
$D = 6, 7$	$2 \cdot 10^{-2}$	$5 \cdot 10^{-7}$
$D = 9$	$5 \cdot 10^{-4}$	?
$D = 12$	$2 \cdot 10^{-5}$	?

Constraint of $D^0 \to \mu^+\mu^-$ for $x \sim 1\%$

Golowich, Hewett, Pakvasa and Petrov, 2009



Model	$\mathcal{B}_{D^0\to\mu^+\mu^-}$
Experiment	$\leq 1.3 \times 10^{-6}$
Standard Model (SD)	$\sim 10^{-18}$
Standard Model (LD)	$\sim$ several $\times 10^{-13}$
$Q = +2/3$ Vectorlike Singlet	4.3×10^{-11}
$Q = -1/3$ Vectorlike Singlet	$1 \times 10^{-11}\ (m_S/500\ \text{GeV})^2$
$Q = -1/3$ Fourth Family	$1 \times 10^{-11}\ (m_S/500\ \text{GeV})^2$
Z' Standard Model (LD)	$2.4 \times 10^{-12}/(M_{Z'}(\text{TeV}))^2$
Family Symmetry	0.7×10^{-18} (Case A)
RPV-SUSY	$4.8 \times 10^{-9}\ (300\ \text{GeV}/m_{\tilde{d}_k})^2$

TABLE I: Predictions for $D^0 \to \mu^+\mu^-$ branching fraction for $x_D \sim 1\%$. Experimental upper bound is a compilation from [16].

Current upper limit

$Br[D^0 \to \mu^+\mu^-] < 6.2 \times 10^{-9}$

Exclusive approach

exclusive sum

$D_{\pm}$: CP eigenstate

(CP limit)

$$y \approx \frac{\Gamma_+ - \Gamma_-}{2\Gamma} = \frac{1}{2}\sum_n \left(\mathcal{B}(D_+ \to n) - \mathcal{B}(D_- \to n)\right)$$

For PP mode

$$\begin{aligned} y_{PP} = {} & \mathcal{B}(\pi^+\pi^-) + \mathcal{B}(\pi^0\pi^0) + \mathcal{B}(\pi^0\eta) + \mathcal{B}(\pi^0\eta') + \mathcal{B}(\eta\eta) + \mathcal{B}(\eta\eta') + \mathcal{B}(K^+K^-) + \mathcal{B}(K^0\bar{K}^0) \\ & -2\cos\delta_{K^+\pi^-}\sqrt{\mathcal{B}(K^-\pi^+)\mathcal{B}(K^+\pi^-)} - 2\cos\delta_{K^0\pi^0}\sqrt{\mathcal{B}(\bar{K}^0\pi^0)\mathcal{B}(K^0\pi^0)} \\ & -2\cos\delta_{K^0\eta}\sqrt{\mathcal{B}(\bar{K}^0\eta)\mathcal{B}(K^0\eta)} - 2\cos\delta_{K^0\eta'}\sqrt{\mathcal{B}(\bar{K}^0\eta')\mathcal{B}(K^0\eta')}\ . \end{aligned}$$

data is used to determine y

Exclusive approaches

Exp: $y = (0.651^{+0.063}_{-0.069})\%$ (CPV allowed)

Topological approach

H-Y Cheng, C-W Chiang, 2010

$y_{PP} = (0.086 \pm 0.041)\%.$

two solutions

$$y_{PV} = (0.269 \pm 0.253)\% \ (A, A1)$$
$$y_{PV} = (0.152 \pm 0.220)\% \ (S, S1)$$

H-Y Jiang, F-S Yu, Q. Qin, H-n. Li and C-D Lu, 2017

Factorization assisted topological (FAT) approach

$D \to PP \qquad D \to PV \qquad D \to VV$

VV :negligible

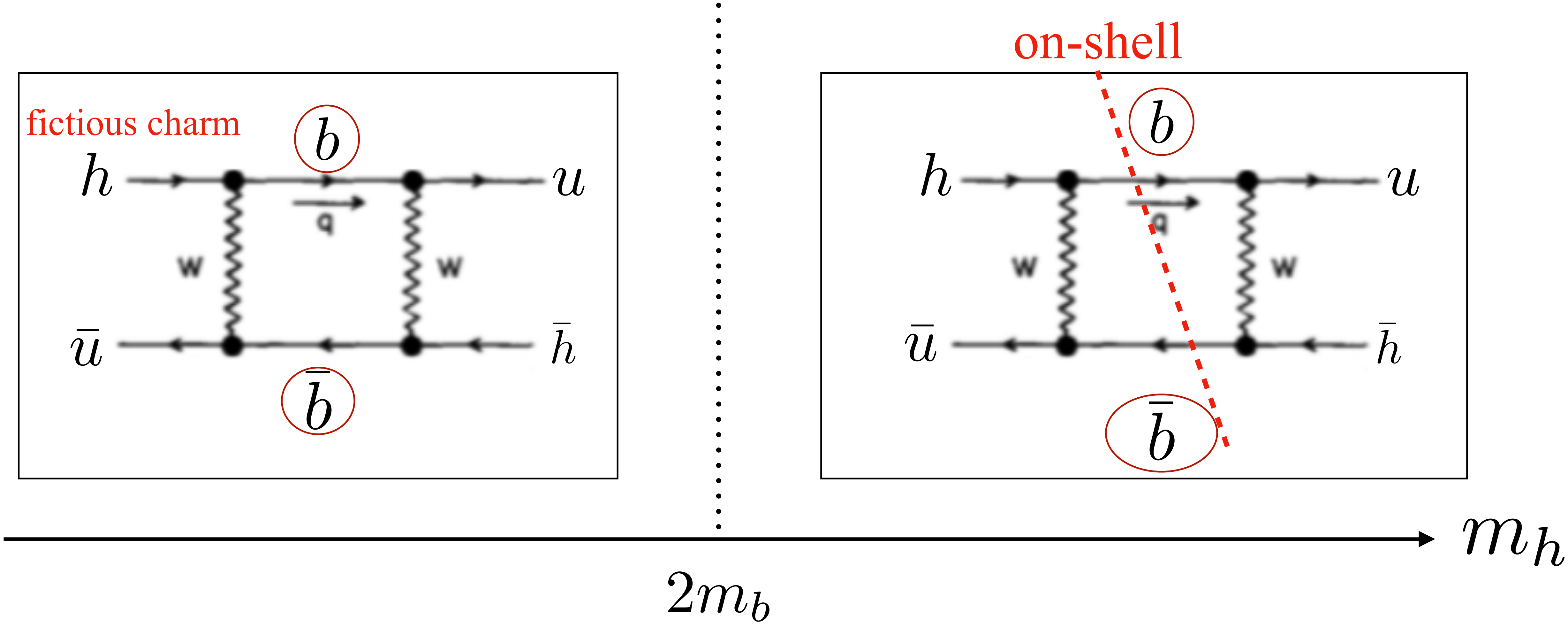
$y_{PP+PV} = (0.21 \pm 0.07)\%,$ below data

➡ imply other two/multi-body final states' contribution

Exclusive ➡ **half-value explainable**

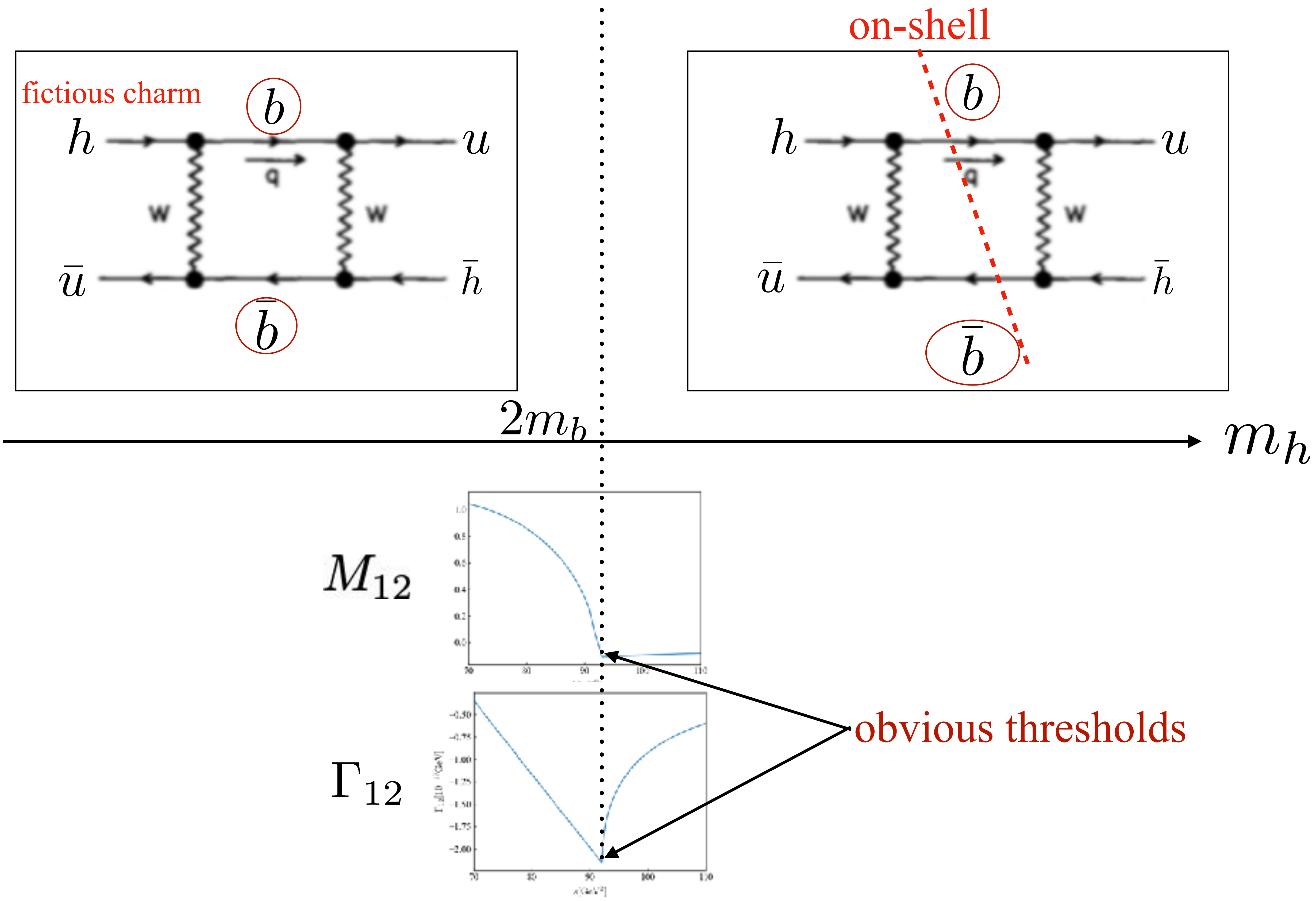
thresholds

(Example: two bottom quarks)



(Example: two bottom quarks)

thresholds



thresholds

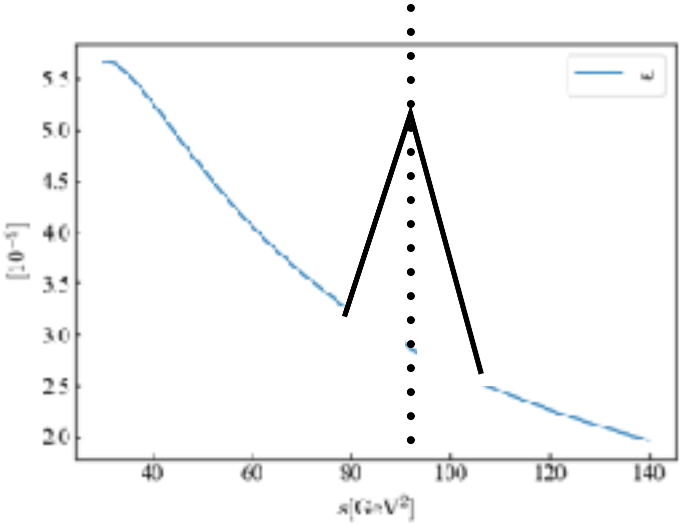
Problem to solve

$$\int_{4m_\pi^2}^{\Lambda} \frac{y(s')}{s-s'}ds' = \omega(s)$$

input (r.h.s.)

$$\omega(s) = \pi x(s) - P\int_{\Lambda}^{\infty} \frac{y(s')}{s-s'}ds'$$

$\omega(s)$



obvious peak?

thresholds

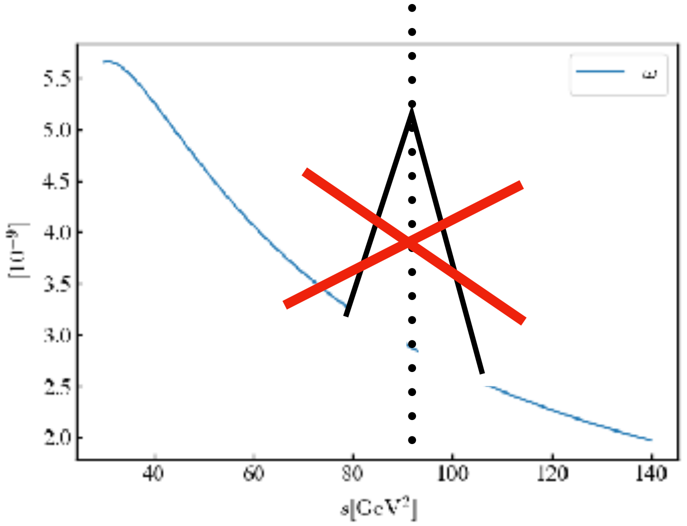
Problem to solve

$$\int_{4m_\pi^2}^{\Lambda} \frac{y(s')}{s-s'} ds' = \omega(s)$$

input (r.h.s.)

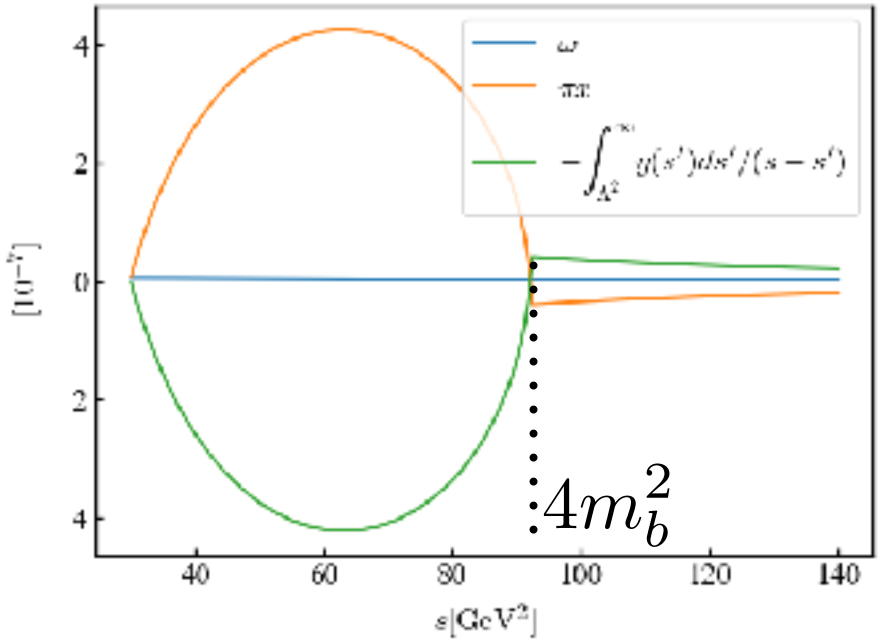
$$\omega(s) = \pi x(s) - P\int_{\Lambda}^{\infty} \frac{y(s')}{s-s'} ds'$$

$\omega(s)$

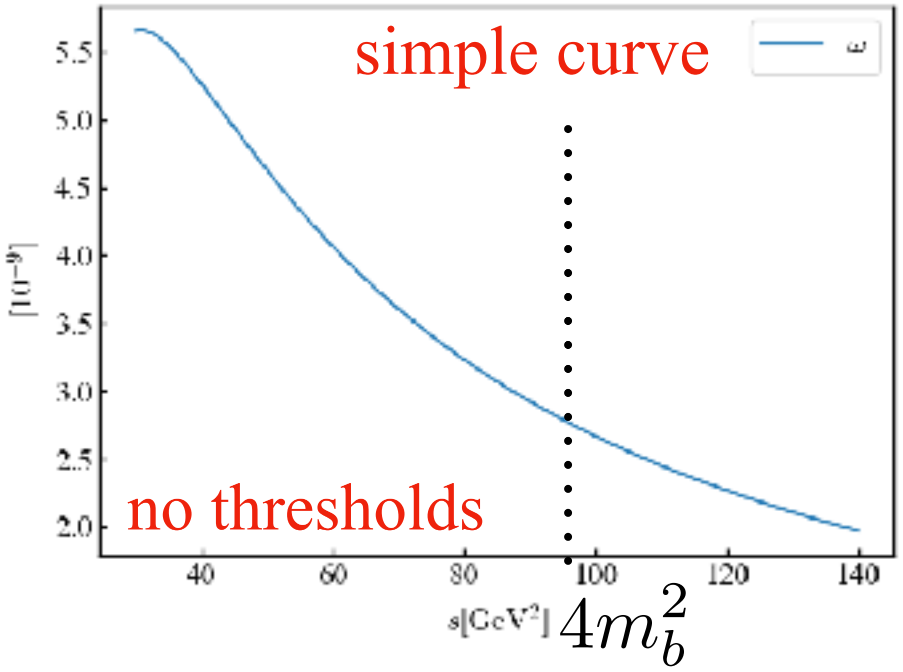


obvious peak?

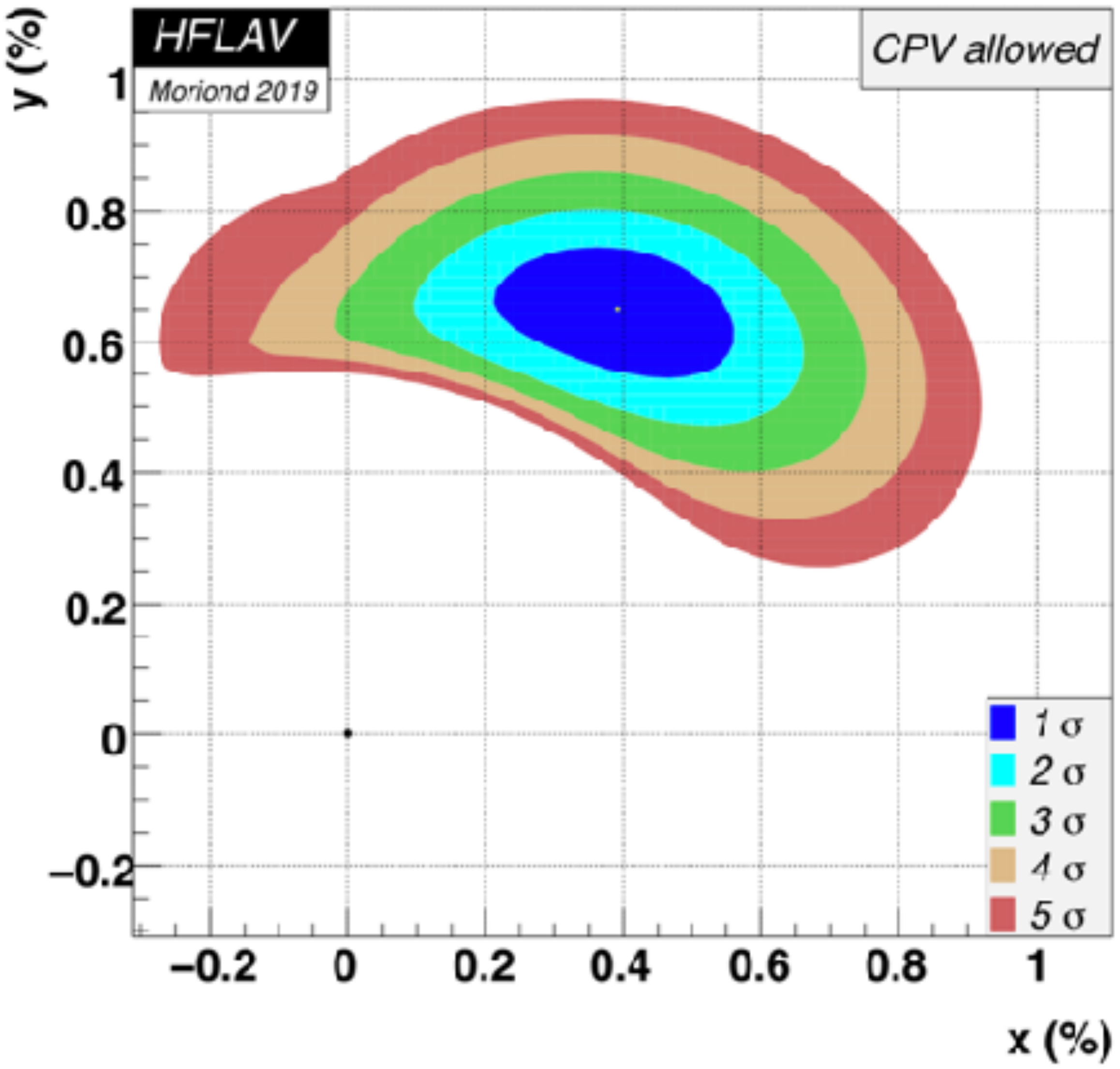
Cancellation

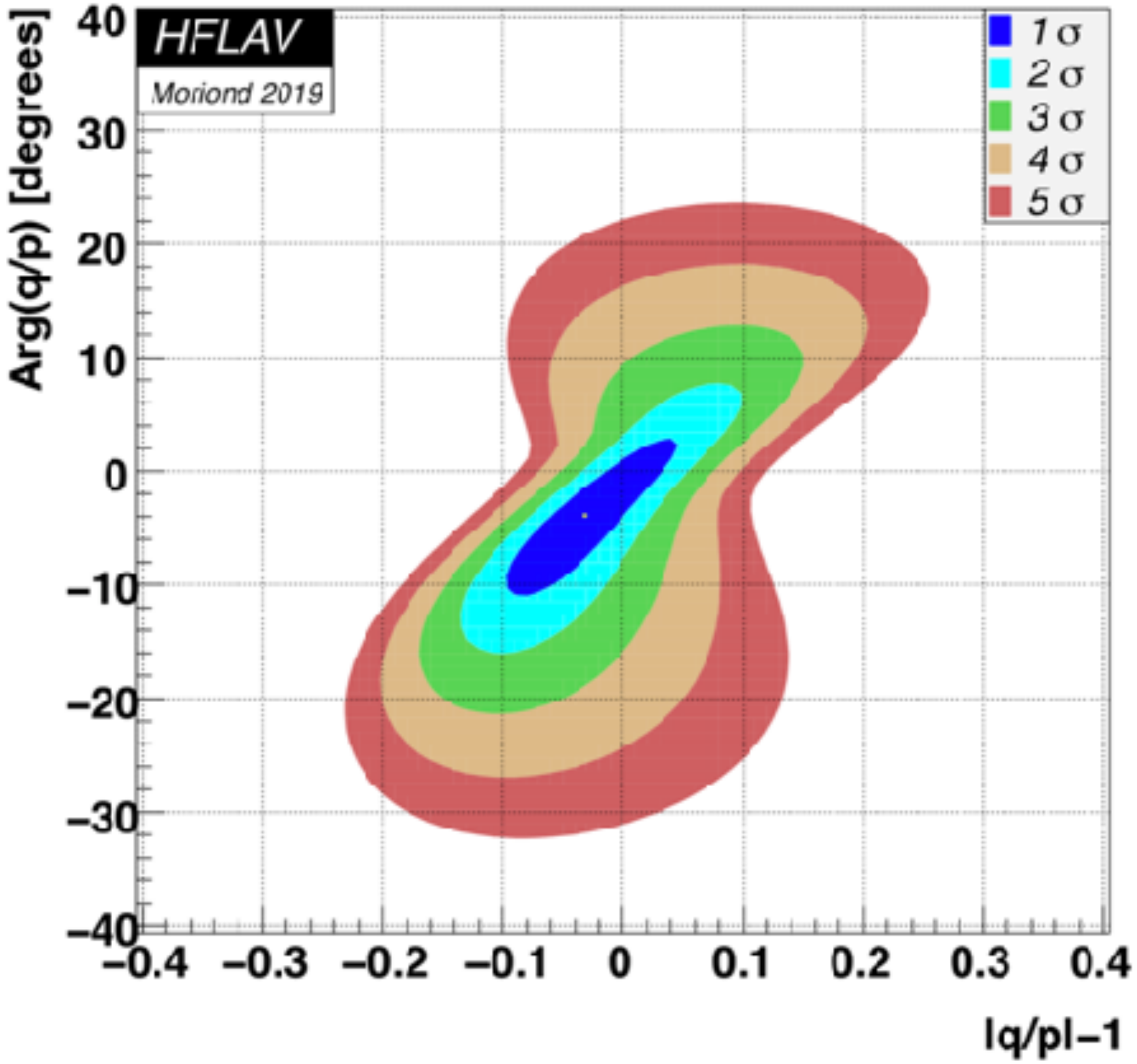


$\omega(s)$



$D^0 - \bar{D}^0$ mixing: experiment





$(x, y) = (0, 0)$ is excluded by >>11.5σ

➡ **mixing is verified**

$(|q/p| - 1, \mathrm{Arg}(q/p)) = (0, 0)$ is arrowed

➡ **No signal for CP violation**