Anisotropic Gravitational Wave Background from Non-Gaussian Perturbations

Kin-Wang Ng (吳建宏)

Institute of Physics, Academia Sinica

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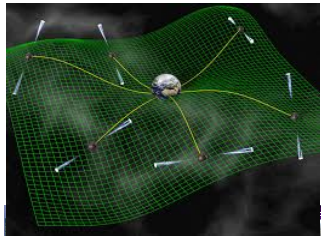
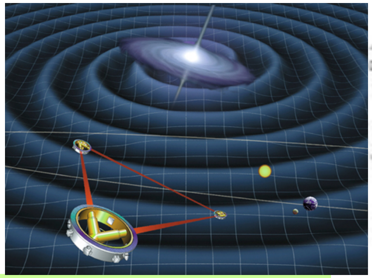
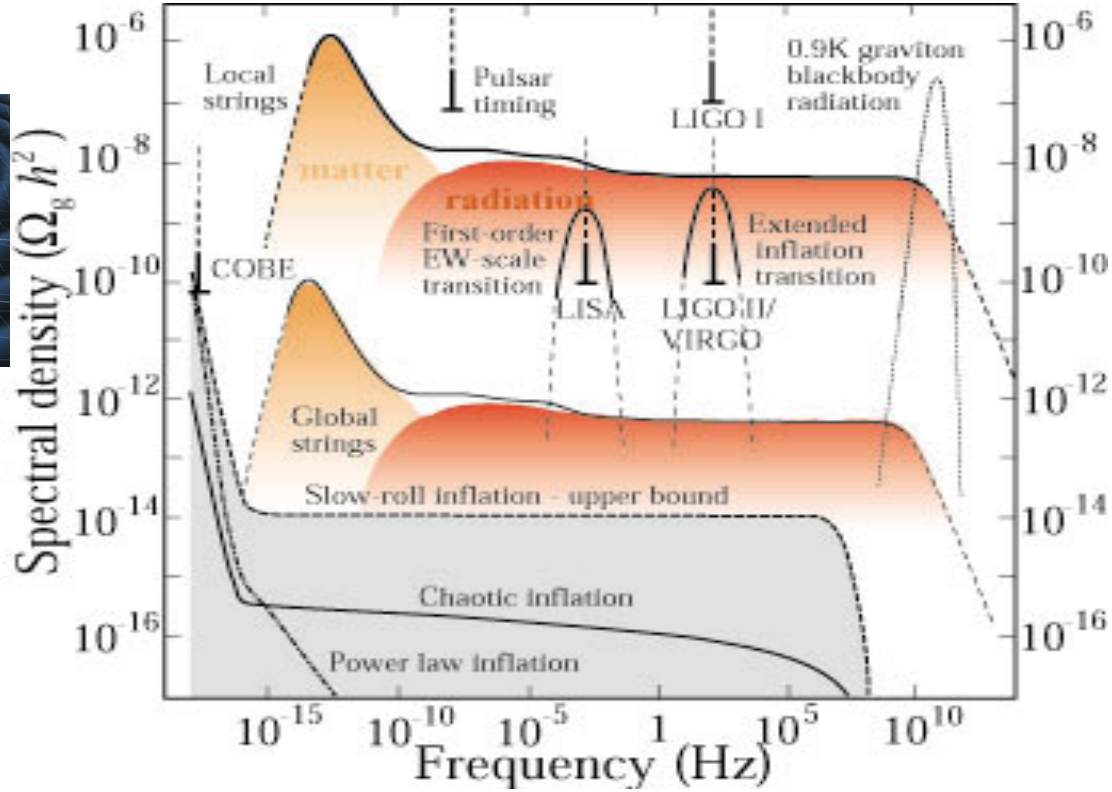
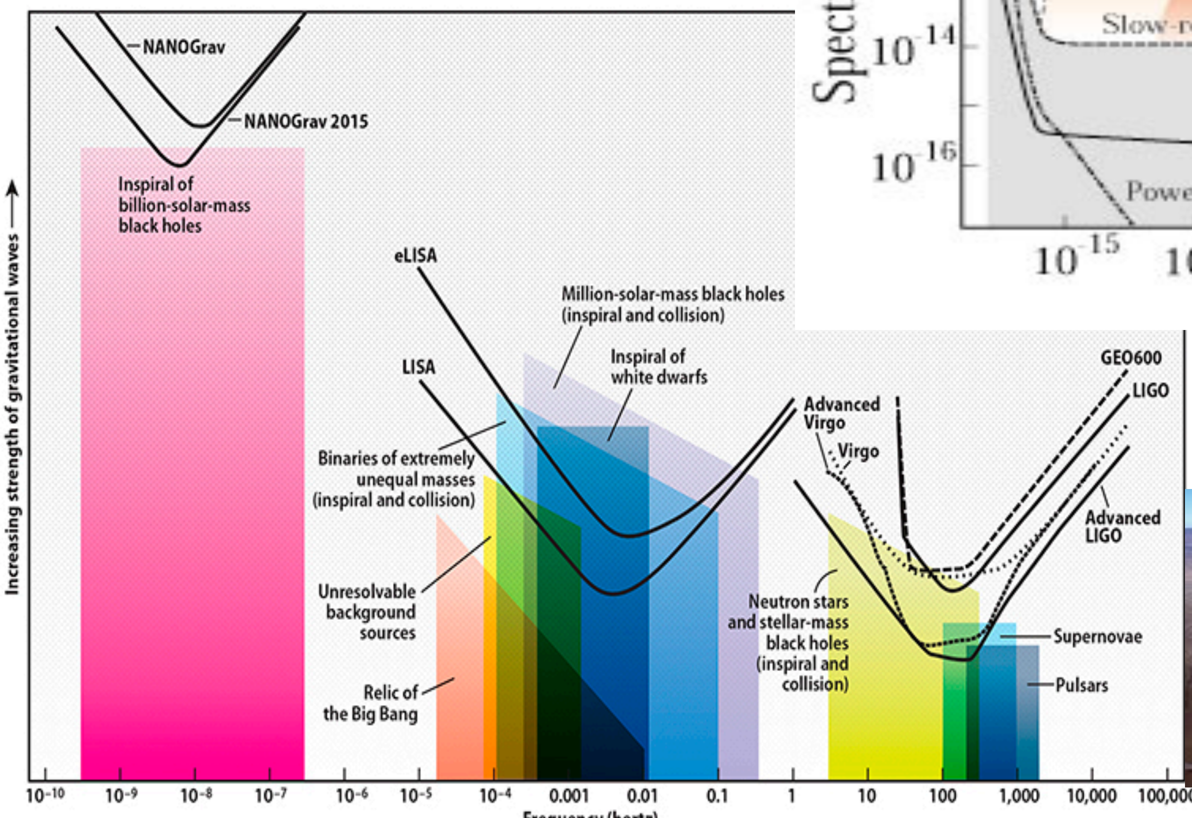
Cosmological sources for gravitational waves







Astrophysical sources



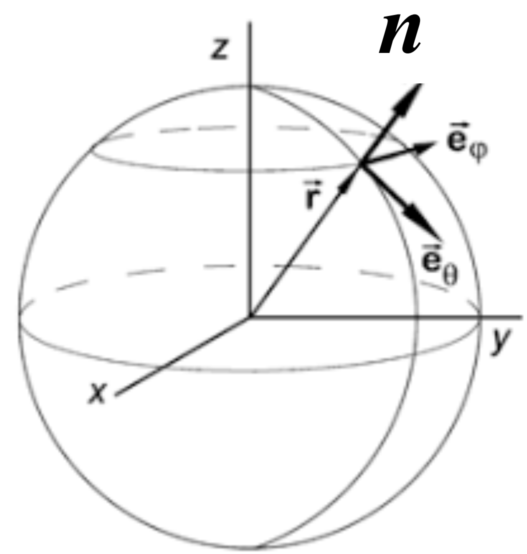
Gravitational Waves

$$h_{ij}(t,\boldsymbol{x}) = \sum_{P=+,\times} \int_{-\infty}^{\infty} df \int_{S^2} d\boldsymbol{n} h_P(f,\boldsymbol{n}) e^{2\pi i f(-t+\boldsymbol{n}\cdot\boldsymbol{x})} e_{ij}^P(\boldsymbol{n}). \tag{1}$$

Here, the bases for transverse-traceless tensor $\boldsymbol{e}^P$ ($P = +, \times$) are given as

$$\boldsymbol{e}^+ = \hat{\boldsymbol{e}}_\theta \otimes \hat{\boldsymbol{e}}_\theta - \hat{\boldsymbol{e}}_\phi \otimes \hat{\boldsymbol{e}}_\phi, \qquad \boldsymbol{e}^\times = \hat{\boldsymbol{e}}_\theta \otimes \hat{\boldsymbol{e}}_\phi + \hat{\boldsymbol{e}}_\phi \otimes \hat{\boldsymbol{e}}_\theta,$$

$$\begin{pmatrix} \langle h_+(f,\boldsymbol{n})h_+^*(f',\boldsymbol{n}')\rangle & \langle h_+(f,\boldsymbol{n})h_\times^*(f',\boldsymbol{n}')\rangle \\ \langle h_\times(f,\boldsymbol{n})h_+^*(f',\boldsymbol{n}')\rangle & \langle h_\times(f,\boldsymbol{n})h_\times^*(f',\boldsymbol{n}')\rangle \end{pmatrix}$$
$$= \frac{1}{2}\delta_D^2(\boldsymbol{n}-\boldsymbol{n}')\delta_D(f-f')$$
$$\times \begin{pmatrix} I(f,\boldsymbol{n}) + Q(f,\boldsymbol{n}) & U(f,\boldsymbol{n}) - iV(f,\boldsymbol{n}) \\ U(f,\boldsymbol{n}) + iV(f,\boldsymbol{n}) & I(f,\boldsymbol{n}) - Q(f,\boldsymbol{n}) \end{pmatrix},$$



Q= <++> - <xx>
U is the same as in
a frame rotated by **π**/8

$$\boldsymbol{e}^R = \frac{(\boldsymbol{e}^+ + i\boldsymbol{e}^\times)}{\sqrt{2}}, \qquad \boldsymbol{e}^L = \frac{(\boldsymbol{e}^+ - i\boldsymbol{e}^\times)}{\sqrt{2}}$$

$$h_R = \frac{(h_+ - ih_\times)}{\sqrt{2}}, \qquad h_L = \frac{(h_+ + ih_\times)}{\sqrt{2}}$$

$$\begin{pmatrix} \langle h_R(f,\boldsymbol{n})h_R(f',\boldsymbol{n}')^*\rangle & \langle h_L(f,\boldsymbol{n})h_R(f',\boldsymbol{n}')^*\rangle \\ \langle h_R(f,\boldsymbol{n})h_L(f',\boldsymbol{n}')^*\rangle & \langle h_L(f,\boldsymbol{n})h_L(f',\boldsymbol{n}')^*\rangle \end{pmatrix}$$
$$= \frac{1}{2}\delta_D(\boldsymbol{n}-\boldsymbol{n}')^2\delta_D(f-f')$$
$$\times \begin{pmatrix} I(f,\boldsymbol{n}) + V(f,\boldsymbol{n}) & Q(f,\boldsymbol{n}) - iU(f,\boldsymbol{n}) \\ Q(f,\boldsymbol{n}) + iU(f,\boldsymbol{n}) & I(f,\boldsymbol{n}) - V(f,\boldsymbol{n}) \end{pmatrix}.$$

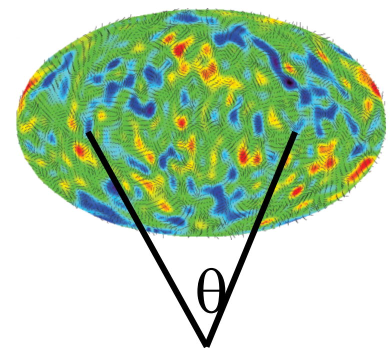
GWB Anisotropy and Polarization Angular Power Spectra

Decompose the GWB sky into a sum of spherical harmonics:

$$T(\theta,\varphi) = \Sigma_{lm}\ a_{lm}\ Y_{lm}\ (\theta,\varphi), \quad V(\theta,\varphi) = \Sigma_{lm}\ b_{lm}\ Y_{lm}\ (\theta,\varphi)$$

$$(Q - iU)\ (\theta,\varphi) = \Sigma_{lm}\ a_{4,lm}\ {}_{4}Y_{lm}\ (\theta,\varphi)$$

$$(Q + iU)\ (\theta,\varphi) = \Sigma_{lm}\ a_{-4,lm}\ {}_{-4}Y_{lm}\ (\theta,\varphi)$$



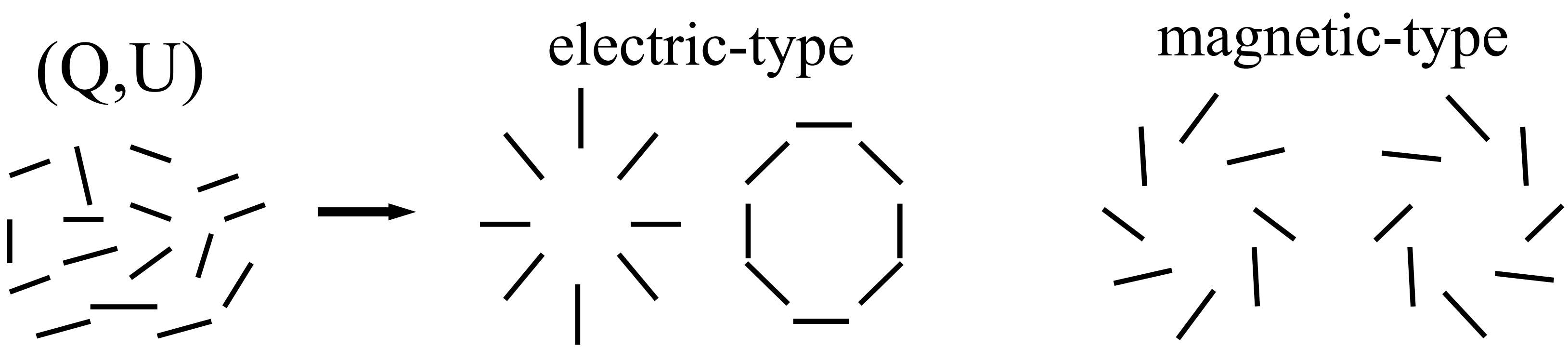
l = 180 degrees/ θ

$C^{T}_{l} = \Sigma_m\ (a^*_{lm}\ a_{lm})$ anisotropy power spectrum

$C^{V}_{l} = \Sigma_m\ (b^*_{lm}\ b_{lm})$ circular polarization power spectrum

$C^{E}_{l} = \Sigma_m\ (a^*_{4,lm}\ a_{4,lm} + a^*_{4,lm}\ a_{-4,lm})$ E-polarization power spectrum

$C^{B}_{l} = \Sigma_m\ (a^*_{4,lm}\ a_{4,lm} - a^*_{4,lm}\ a_{-4,lm})$ B-polarization power spectrum



Collisionless Boltzman Equation for Gravitons e.g. Bartolo et al. 19

$$ds^2 = a^2(\eta)\left[-e^{2\Phi}d\eta^2 + (e^{-2\Psi}\delta_{ij} + h_{ij})dx^i dx^j\right]$$

$$\frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial x^i}\frac{dx^i}{d\eta} + \frac{\partial f}{\partial q}\frac{dq}{d\eta} + \frac{\partial f}{\partial n^i}\frac{dn^i}{d\eta} = 0$$

Graviton phase space distribution function

$$f = f(\eta, x^i, q, \hat{n}^i)$$

$$\frac{\partial f}{\partial \eta} + n^i\frac{\partial f}{\partial x^i} + \left[\frac{\partial \Psi}{\partial \eta} - n^i\frac{\partial \Phi}{\partial x^i} + \frac{1}{2}n^i n^j\frac{\partial h_{ij}}{\partial \eta}\right] q\frac{\partial f}{\partial q} = 0$$

$$\delta f \equiv -q\frac{\partial \bar{f}}{\partial q}\Gamma(\eta, \vec{x}, q, \hat{n})$$

Newtonian potential k-mode

$$\Psi = \Phi \equiv T_\Phi(\eta, k)\,\hat{\zeta}(\vec{k})$$

Transfer function

Initial scalar power spectrum

GW background k-mode

$$h_{ij} \equiv \sum_{\lambda=\pm 2} e_{ij,\lambda}(\hat{k})h(\eta, k)\hat{\xi}_\lambda(k^i)$$

Transfer function

Initial tensor power spectrum

$$\Gamma(\hat{n}) = \sum_\ell \sum_{m=-\ell}^{\ell} \Gamma_{\ell m}\, Y_{\ell m}(\hat{n}) \qquad e^{i\vec{k}\cdot\vec{x}} = 4\pi \sum_{lm} i^l j_l(kx) Y^*_{lm}(\hat{k}) Y_{lm}(\hat{x})$$

Scalar contribution

$$\frac{\Gamma_{\ell m,S}}{4\pi\,(-i)^\ell} = \int \frac{d^3k}{(2\pi)^3}\,\zeta(\vec{k}) Y^*_{\ell m}(\hat{k})\,\mathcal{T}_\ell^{(0)}(k,\,\eta_0,\,\eta_{\rm in})$$

$\vec{x}_0 = 0$

η_0 today

$\eta_{\rm in}$ initial

$\boxed{T_\Phi(\eta_{\rm in}, k) = 3/5}$

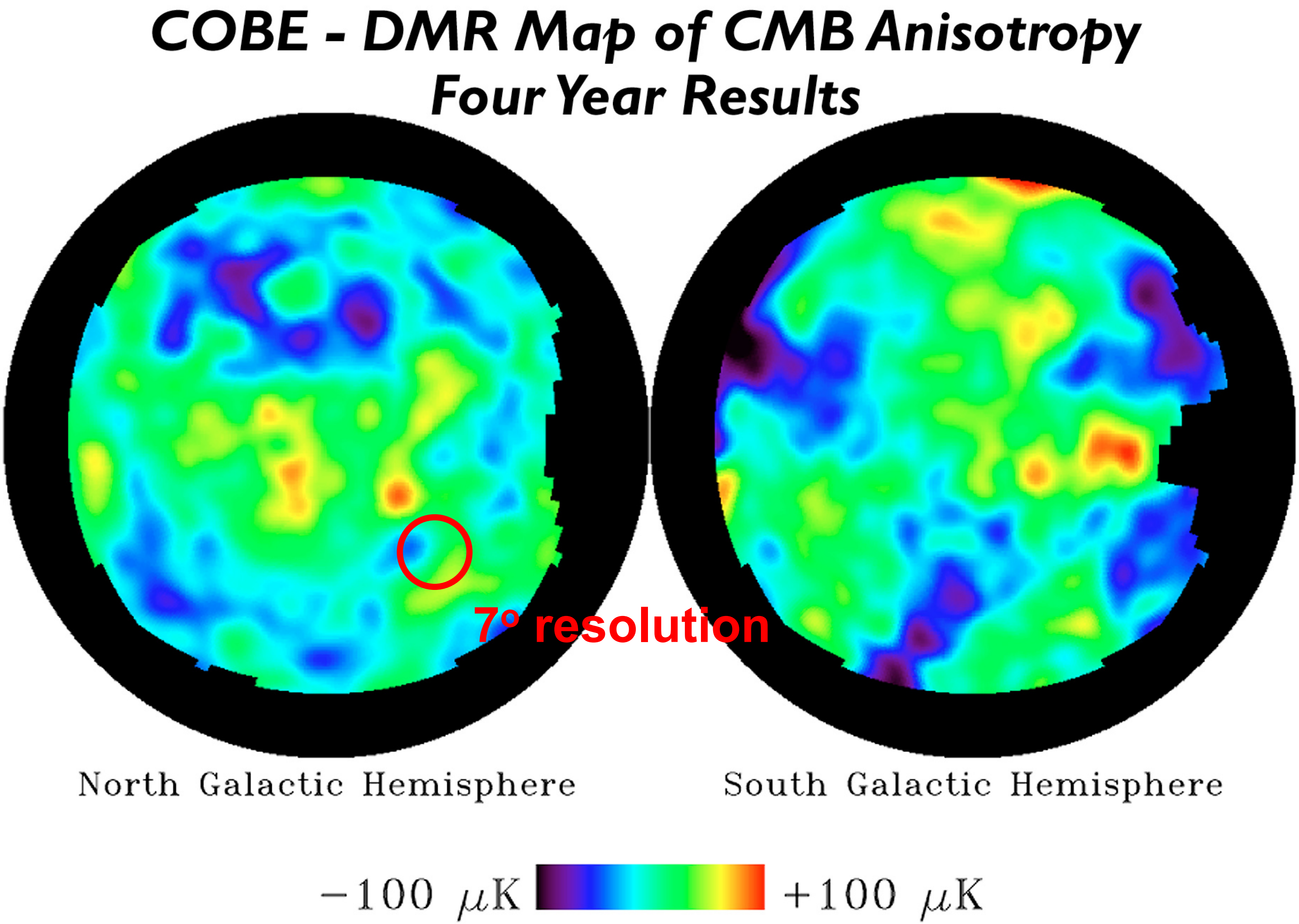
where the scalar transfer function $\mathcal{T}_\ell^{(0)}$ is the sum of a term analogous to the SW effect for CMB photons, $T_\Phi(\eta_{\rm in}, k)\, j_\ell[k(\eta_0 - \eta_{\rm in})]$, plus the analog of the ISW term, $\int_{\eta_{\rm in}}^{\eta_0} d\eta'\, [T'_\Psi(\eta, k) + T'_\Phi(\eta, k)]\, j_\ell[k(\eta - \eta_{\rm in})]$. Finally,

Tensor contribution

$$\mathcal{T}_\ell^{(\pm 2)} = \frac{1}{4}\sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \int_{\eta_{\rm in}}^{\eta_0} d\eta\, h'(\eta,\, k)\, \frac{j_\ell\left[k\left(\eta_0 - \eta\right)\right]}{k^2\left(\eta_0 - \eta\right)^2}$$

Sachs-Wolfe or Integrated Sachs-Wolfe effects – gravitational redshift of gravitons

GWB Anisotropy Map due to SW and ISW Effects



Gravitational Wave Anisotropies from Primordial Black Holes

N. Bartolo,[a,b,c] **D. Bertacca,**[a] **V. De Luca,**[d] **G. Franciolini,**[d] **S. Matarrese,**[a,b,c,e] **M. Peloso,**[a,b] **A. Ricciardone,**[b] **A. Riotto,**[d,f] **G. Tasinato**[g]

[a]Dipartimento di Fisica e Astronomia Galileo Galilei Universita‘ di Padova, 35131 Padova, Italy

[b]INFN, Sezione di Padova, 35131 Padova, Italy

[c]INAF - Osservatorio Astronomico di Padova, I-35122 Padova, Italy

[d]Department of Theoretical Physics and Center for Astroparticle Physics (CAP), CH-1211 Geneva 4, Switzerland

[e]Gran Sasso Science Institute, I-67100 L'Aquila, Italy

[f]CERN,Theoretical Physics Department, Geneva, Switzerland

[g]Department of Physics, Swansea University, Swansea, SA2 8PP, UK

Abstract. An observable stochastic background of gravitational waves is generated whenever primordial black holes are created in the early universe thanks to a small-scale enhancement of the curvature perturbation. We calculate the anisotropies and non-Gaussianity of such stochastic gravitational waves background which receive two contributions, the first at formation time and the second due to propagation effects. We conclude that a sizeable magnitude of anisotropy and non-Gaussianity in the gravitational waves would suggest that primordial black holes may not comply the totality of the dark matter.

Primordial black hole seeds or density (scalar) perturbation associated GWs

$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta} - \vec{\nabla}^2\right] h_{ij} = 0$$

Free gravitational wave equation

De Sitter vacuum fluctuations during inflation lead to almost scale-invariant primordial gravitational waves

$P_h = 8\pi GH^2$ and $\Delta_\zeta^2 = \langle\zeta\zeta\rangle = (\delta\rho/\rho)^2 \sim 2\text{x}10^{-9}$ on CMB scales

$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta} - \vec{\nabla}^2\right] h_{ij} = 16\pi G T_{ij}$$

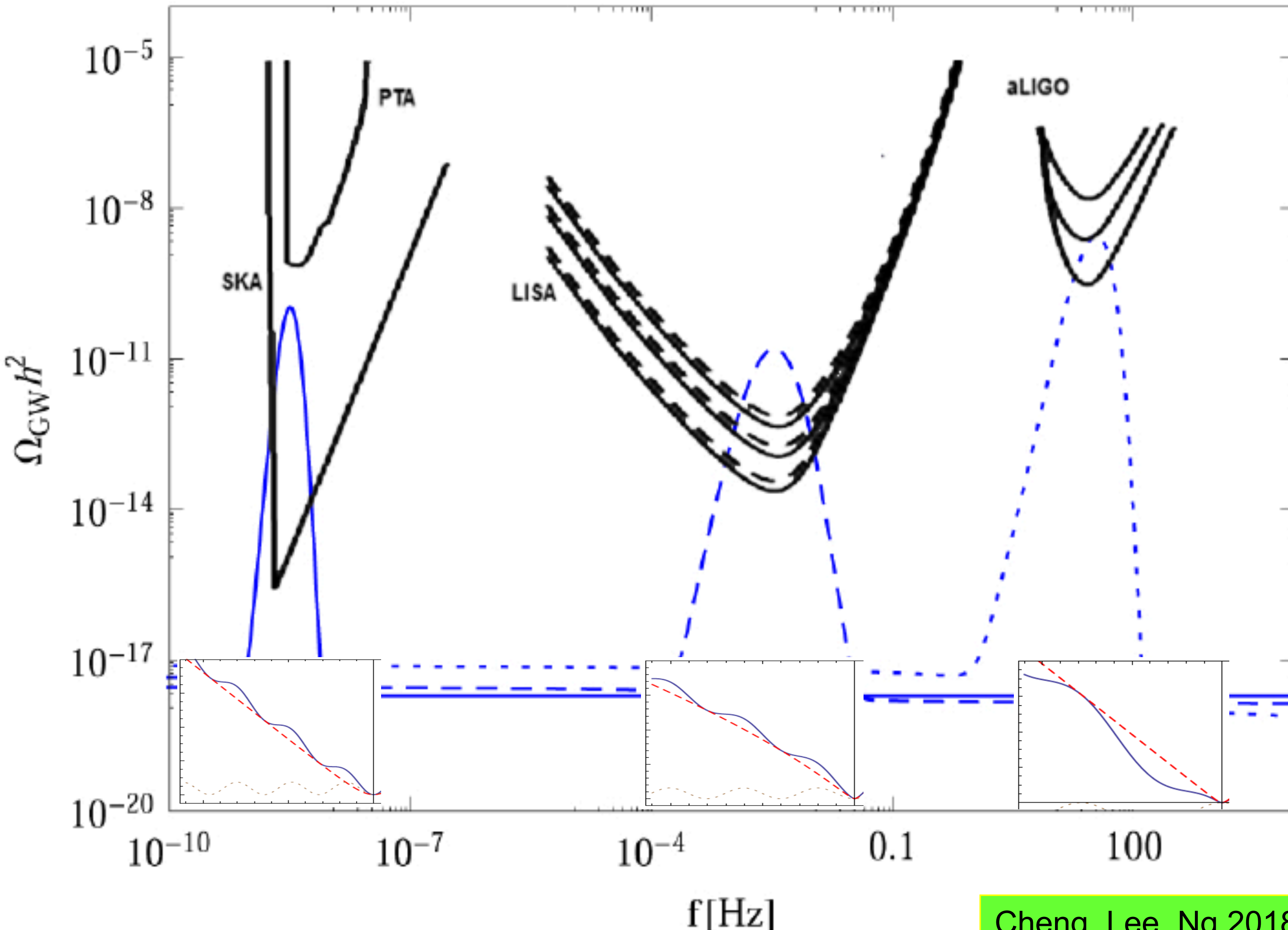
Stress due to transverse traceless part of 2nd order curvature perturbation $T_{ij}(\zeta^2)$

$\Delta_\zeta^2 = \langle\zeta\zeta\rangle = (\delta\rho/\rho)^2 \sim 10^{-3}$

When large curvature perturbation re-enter the horizon during the radiation-dominated era and collapse to form PBHs, they induce gravitational waves at short wavelengths

Ananda, Clarkson,Wands 2007, Baumann, Steinhardt,Takahashi,Ichiki 2007

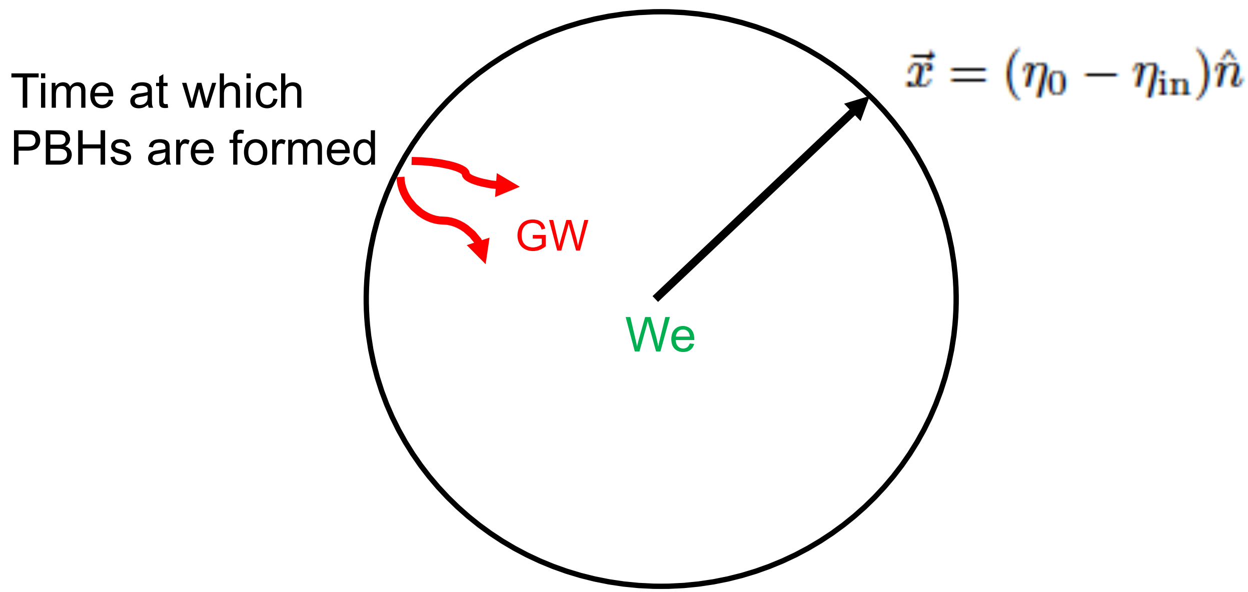
GWs associated with PBHs in modulated axion inflation



Cheng, Lee, Ng 2018

$$\Gamma(\eta,\vec{x},q,\hat{n}) = \int \frac{d^3k}{(2\pi)^3}\Gamma(\eta,\vec{k},q,\hat{n})e^{i\vec{k}\cdot\vec{x}}$$

k-mode



Initial value

$$\frac{\Gamma_{\ell m,I}(q)}{4\pi(-i)^\ell} = \int \frac{d^3k}{(2\pi)^3}\Gamma\left(\eta_{\rm in},\,\vec{k},\,q\right)\times Y^*_{\ell m}(\hat{k})\,j_\ell\left[k\left(\eta_0-\eta_{\rm in}\right)\right]$$

Anisotropic GWB from non-Gaussian long wavelength modes

The non-Gaussianity of the primordial scalar perturbations is parametrized by

$$\zeta(\vec{k}) = \zeta_g(\vec{k}) + \frac{3}{5} f_{\rm NL} \int \frac{d^3p}{(2\pi)^3} \zeta_g(\vec{p})\, \zeta_g(\vec{k}-\vec{p}),$$

Planck $-11.1 \leq f_{\rm NL} \leq 9.3$, at 95% C.L.

$$\text{Non Gaussian } \zeta \to \langle \zeta^4 \rangle = \left(1 + \frac{24}{5} f_{\rm NL} \zeta_L\right) \left(\langle \zeta_s^2 \rangle \langle \zeta_s^2 \rangle + \langle \zeta_s^2 \rangle \langle \zeta_s^2 \rangle + \langle \zeta_s^2 \rangle \langle \zeta_s^2 \rangle\right)$$

long wavelength mode short wavelength mode

$$\Gamma(\eta_{\rm in}, \vec{x}, q, \hat{n}) = \frac{3}{5} \tilde{f}_{NL}(q) \int \frac{d^3k}{(2\pi)^3} \zeta_L(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$$

$$\boxed{\tilde{f}_{\rm NL}(k) \equiv \frac{8\, f_{\rm NL}}{4 - \frac{\partial \ln \bar{\Omega}_{\rm GW}(\eta, k)}{\partial \ln k}}}$$

which is similar to the SW effect due to the scalar mode but modulated by a factor of f_{NL}

Cosmological GW spectral energy density

tion h_{ij} is gauged to be transverse-traceless. The latter can be decomposed into two polarization unit tensors as

$$h_{ij}(\eta,\vec{x}) = \sum_{\lambda=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^{\frac{3}{2}}} h_\lambda(\eta,\vec{k})\epsilon_{ij}^{\lambda}(\hat{k})e^{i\vec{k}\cdot\vec{x}}, \qquad (2)$$

where $h_\lambda(\eta,\vec{k})$ is a Gaussian random field that defines the power spectrum of tensor perturbation,

$$\langle h_\lambda(\eta,\vec{k})h^*_{\lambda'}(\eta,\vec{k}')\rangle = \delta(\vec{k}-\vec{k}')\frac{2\pi^2}{k^3}\mathcal{P}_h^{\lambda\lambda'}(\eta,k). \qquad (3)$$

In the following, we will assume that $\mathcal{P}_h^{\lambda\lambda'}(\eta,k) = \delta_{\lambda\lambda'}\mathcal{P}_h(\eta,k)$. Then, the spectral energy density of the GWs relative to the critical density is given by

$$\Omega_{\rm GW}(\eta,k,\hat{k}) \equiv \frac{k}{\rho_c}\frac{d\rho_{\rm GW}}{dk d^2\hat{k}} = \frac{1}{96\pi}\left(\frac{k}{aH}\right)^2 \bar{\mathcal{P}}_h(\eta,k), \quad (4)$$

where $\rho_c = 3M_p^2H^2$ and the overbar denotes taking a time-average. For k-modes that re-enter the horizon

GW observables

The method adopted in current GW experiments for detecting SGWB is to correlate responses of different detectors to the GW strain amplitude. This allows us to filter out detector noises and obtain a large signal-to-noise ratio [4]. Let $h(\vec{x})$ be the response of a detector located at $\vec{x}$ with a pair of arms d_{ij}; then, we have

$$h(\vec{x}) \equiv d^{ij} h_{ij}(\vec{x}) = \sum_{\lambda=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^{\frac{3}{2}}} h_\lambda(\vec{k}) F^\lambda(\hat{k}) e^{i\vec{k}\cdot\vec{x}}, \tag{6}$$

where $F^\lambda = d^{ij}\epsilon^\lambda_{ij}(\hat{k})$ is the beam-pattern function. Hence, using Eq. (3) the response correlation between two detectors a and b, located at $\vec{x}_a = \vec{x} + \vec{r}/2$ and $\vec{x}_b = \vec{x} - \vec{r}/2$ respectively, is given by

$$\langle h_a(\vec{x}_a) h_b(\vec{x}_b)\rangle = \sum_{\lambda=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^3} P_h(k) F^\lambda_a(\hat{k}) F^\lambda_b(\hat{k}) e^{i\vec{k}\cdot\vec{r}}, \tag{7}$$

GW power spectrum

$$P_h(k) \equiv (2\pi^2/k^3)\mathcal{P}_h(k)$$

Non-gaussian density perturbations

Newtonian potential

$$\Phi(\vec{x}) = \phi(\vec{x}) + f_{NL}\left(\phi(\vec{x})^2 - \langle\phi(\vec{x})^2\rangle\right)$$

$$\phi(\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^{\frac{3}{2}}}\phi(\vec{k})e^{i\vec{k}\cdot\vec{x}},$$

$$\langle\phi(\vec{k})\phi^*(\vec{k'})\rangle = \delta(\vec{k}-\vec{k'})P_\phi(k).$$

$$\langle\Phi\left(\vec{x}+\vec{r}/2\right)\Phi\left(\vec{x}-\vec{r}/2\right)\rangle_{\text{subvolume}} = \int_{|\vec{k}|>k_{\min}} \frac{d^3\vec{k}}{(2\pi)^3}e^{i\vec{k}\cdot\vec{r}}P_\phi(k)\left[1+4f_{NL}\int_{|\vec{p}|<k_{\min}} \frac{d^3\vec{p}}{(2\pi)^{\frac{3}{2}}}\phi(\vec{p})\cos\left(\frac{\vec{p}\cdot\vec{r}}{2}\right)e^{i\vec{p}\cdot\vec{x}}\right].$$

subvolume =
observable universe

For $\vec{p}\cdot\vec{r} \ll 1$, we have the position dependent power spectrum,

$$P_\Phi(k,\vec{x}) \simeq P_\phi(k)\left[1+4f_{NL}\int_{|\vec{p}|<k_{\min}} \frac{d^3\vec{p}}{(2\pi)^{\frac{3}{2}}}\phi(\vec{p})e^{i\vec{p}\cdot\vec{x}}\right].$$

Induced GW power spectrum

$$P_h(k,\vec{x}) \simeq P_h(k)\left[1 + 8f_{NL}\int_{|\vec{p}|<k_{\min}} \frac{d^3\vec{p}}{(2\pi)^{\frac{3}{2}}}\phi(\vec{p})e^{i\vec{p}\cdot\vec{x}}\right], \tag{15}$$

so the observed spectrum is given by $P_h(k,\hat{k}) = P_h(k, x_h\hat{k})$, where $x = x_h$ is the comoving distance to the horizon re-entry and $k_{\min} = \pi/x_h$. Let us expand

$$P_h(k,\hat{k}) \simeq P_h(k)\left[1 + f_{NL}\sum_{lm} a_{lm}Y_{lm}(\hat{k})\right],$$

$$a_{lm} = 32\pi i^l \int_{|\vec{p}|<k_{\min}} \frac{d^3\vec{p}}{(2\pi)^{\frac{3}{2}}} j_l(px_h)\phi(\vec{p})Y^*_{lm}(\hat{p}). \tag{16}$$

$$x_h = \eta_0 - \eta_c \simeq \eta_0$$

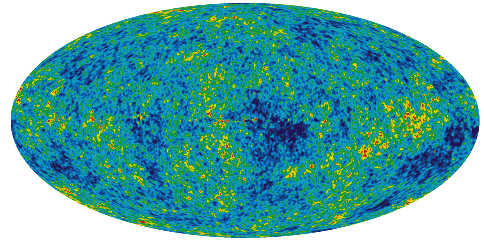
which is the superhorizon-mode contribution ($p\eta_0 < \pi$) of the SW effect

$$\boxed{\text{WMAP 5-year data: } f_{NL} = -100 \pm 100, \quad l < 100}$$

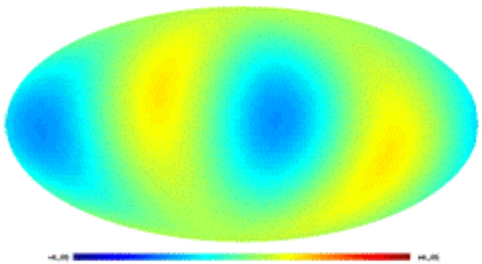
Implication

- These non-Gaussian superhorizon-mode density perturbations have been used to explain the large-scale CMB anomalies
- Anisotropy in GWB may give an independent evidence of the non-Gaussianity.

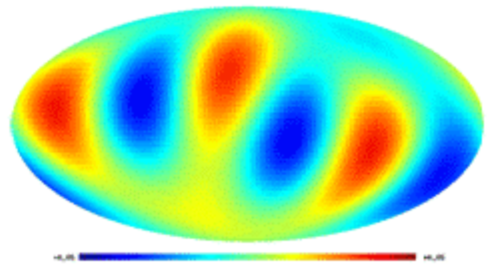
WMAP3
CMB sky map



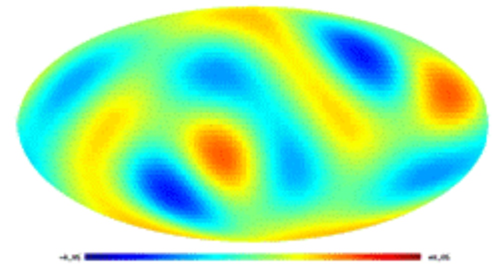
$$\delta T(\theta,\phi)=\sum_{l,m} a_{lm} Y_{lm}(\theta,\phi)$$



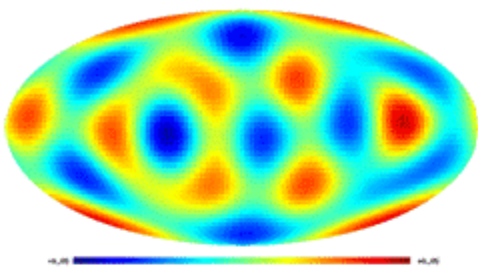
$\ell = 2$



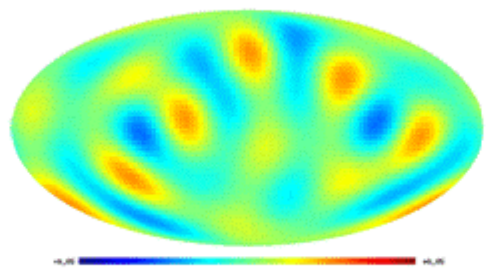
$\ell = 3$



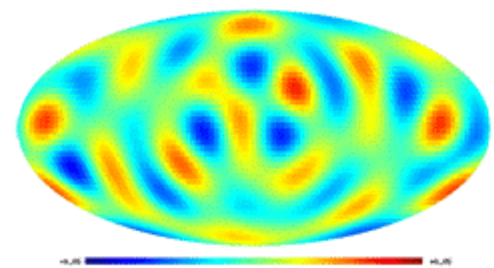
$\ell = 4$



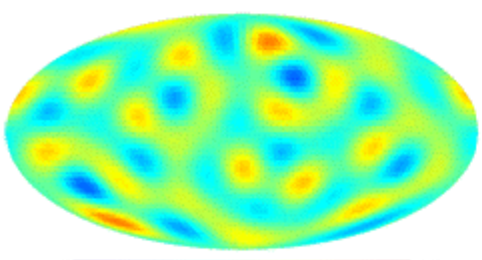
$\ell = 5$



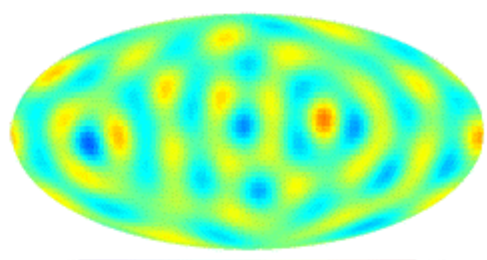
$\ell = 6$



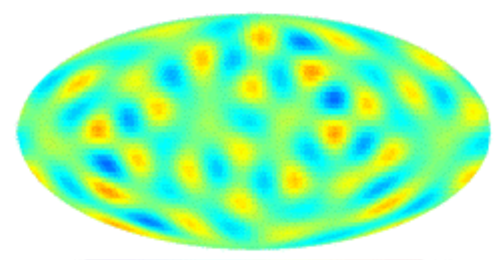
$\ell = 7$



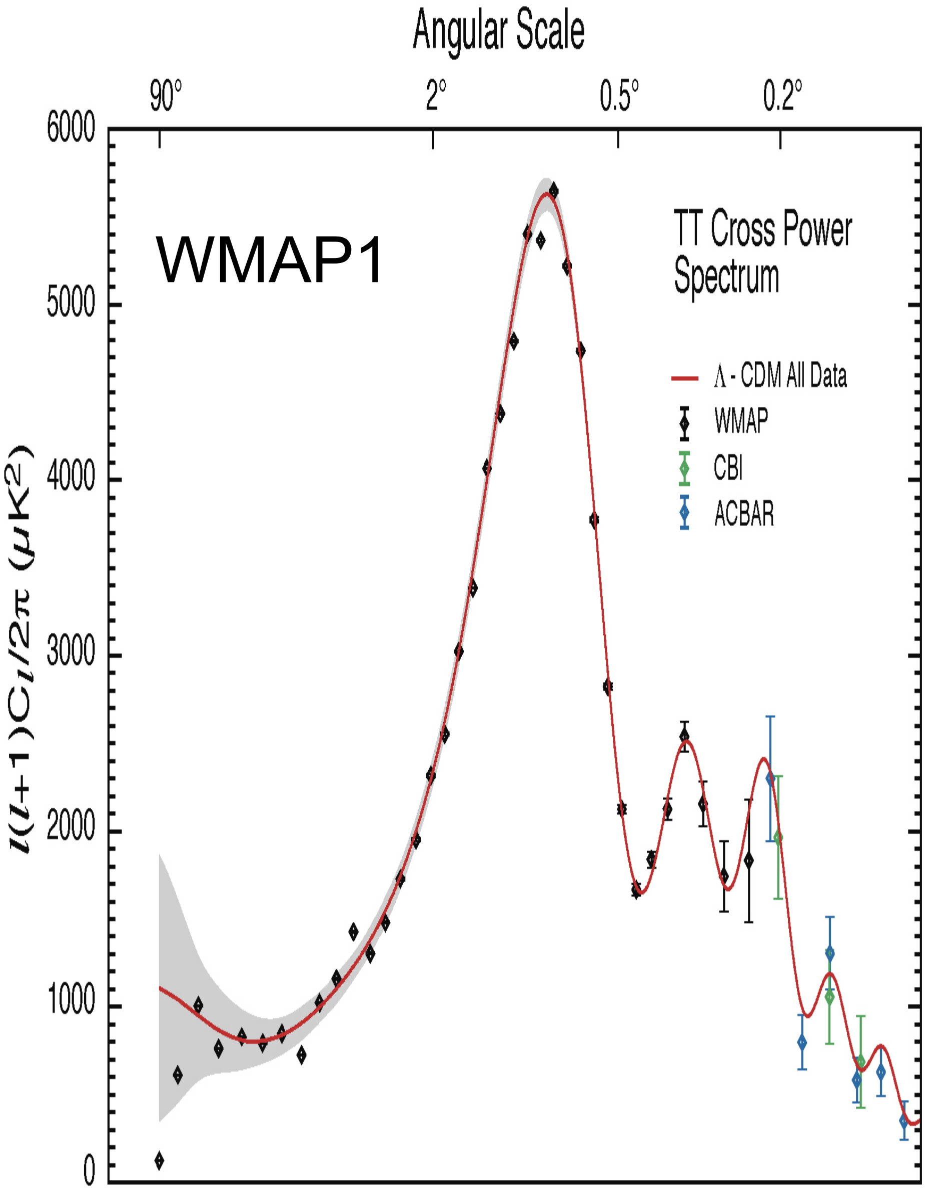
$\ell = 8$

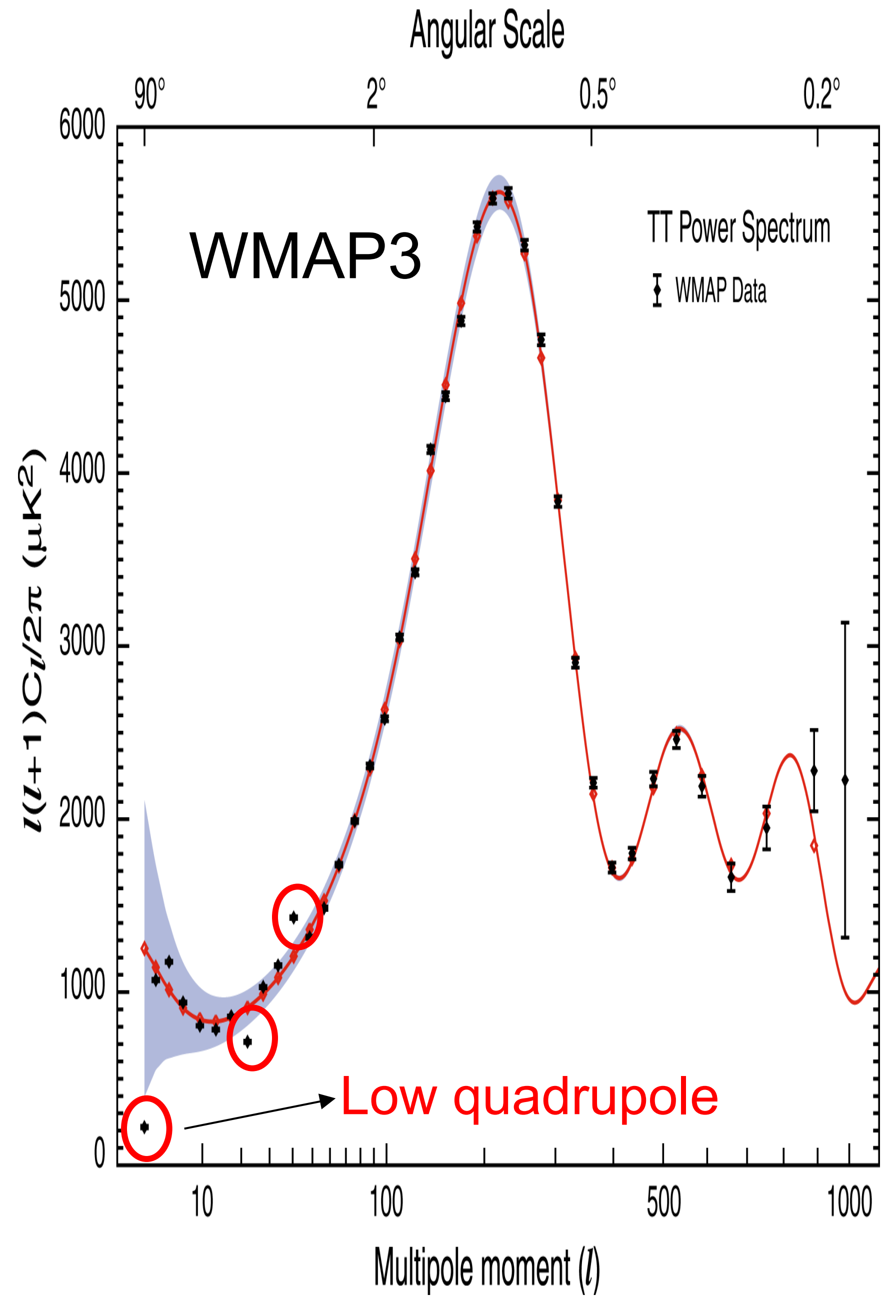


$\ell = 9$



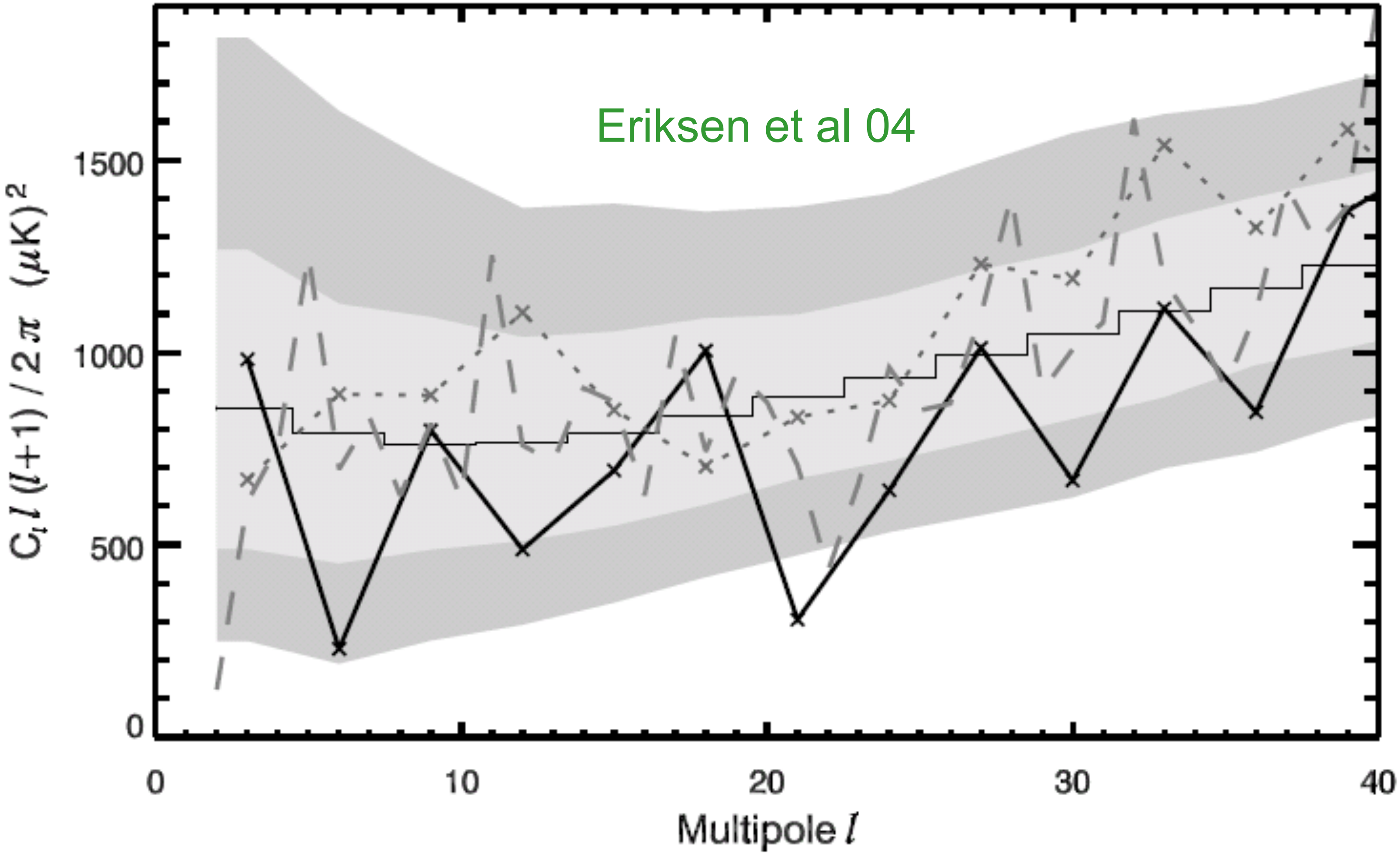
$\ell = 10$


Angular Scale
90°
2°
0.5°
0.2°
WMAP1
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
$l(l+1)C_l/2\pi$ (μK^2)
6000
5000
4000
3000
2000
1000
0


Angular Scale
90°
2°
0.5°
0.2°
WMAP3
TT Power Spectrum
WMAP Data
$l(l+1)C_l/2\pi$ (μK^2)
6000
5000
4000
3000
2000
1000
0
Low quadrupole
10
100
500
1000
Multipole moment (l)

South-North Power Asymmetry

Eriksen et al 04
Park 04

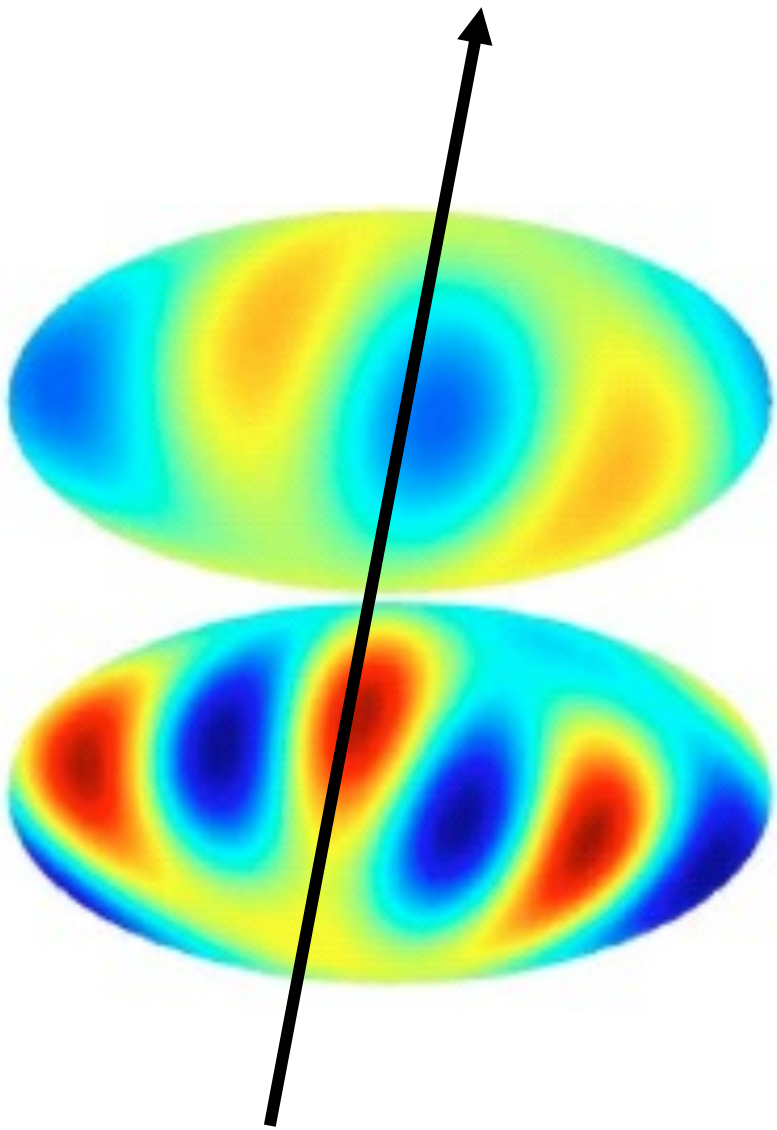


northern hemisphere
southern hemisphere
full sky

North pole $(80^o, 57^o)$

Land & Magueijo 05

“Axis of Evil” $(l,b)\sim(-110°,60°)$

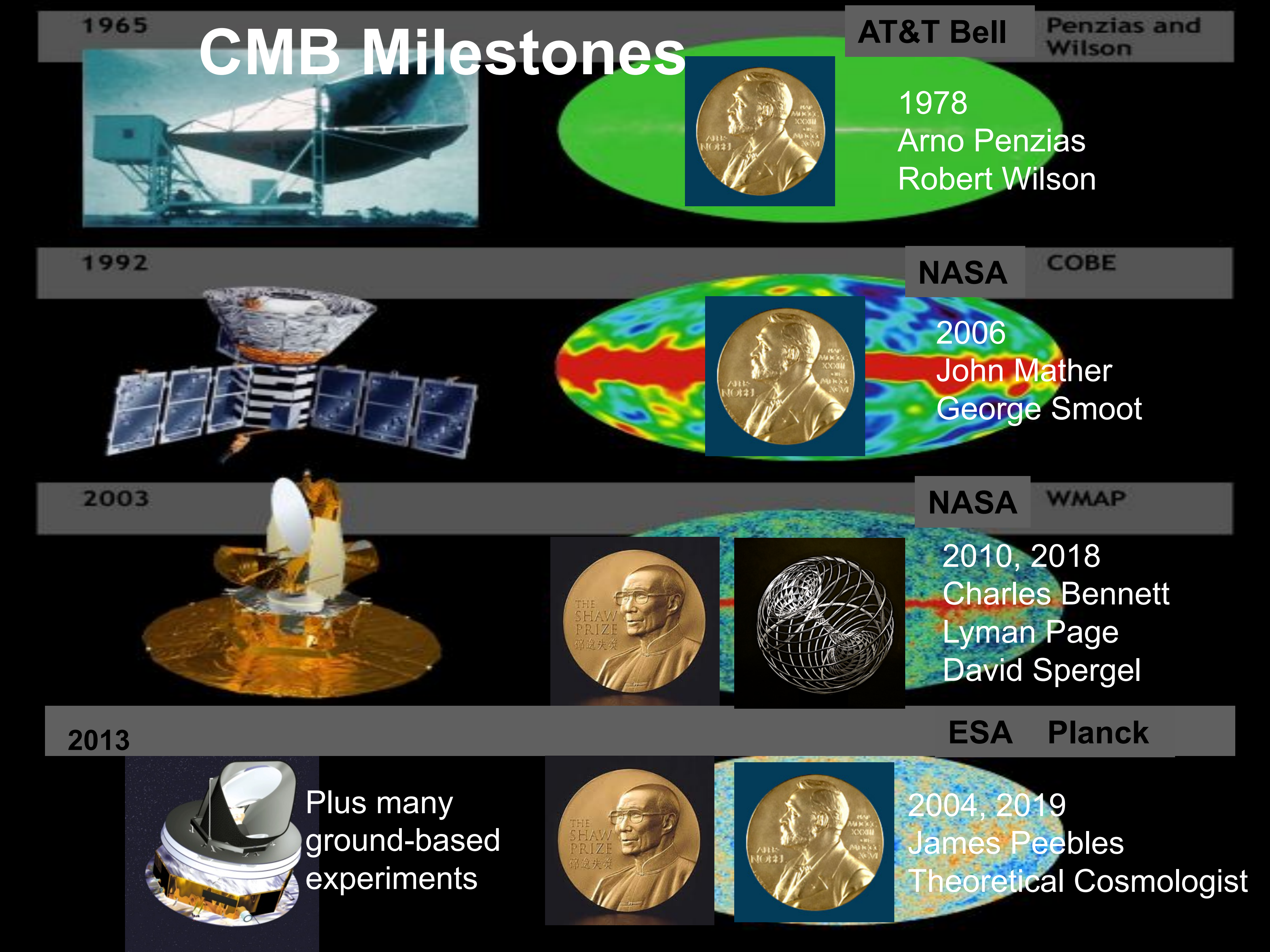


l=2, quadrupole

l=3, octopole

Conclusion

- GWB is a main goal in GW experiments
- GWB monopole and Doppler dipole
- GWB anisotropy and polarization
- Correlate with other cosmological data such as LSS, CMB
- In analogous to CMB, GWB is a deep probe of the early Universe
- Perspectives


CMB Milestones
1965
AT&T Bell
Penzias and Wilson
1978
Arno Penzias
Robert Wilson
1992
NASA
COBE
2006
John Mather
George Smoot
2003
NASA
WMAP
2010, 2018
Charles Bennett
Lyman Page
David Spergel
2013
ESA Planck
Plus many ground-based experiments
2004, 2019
James Peebles
Theoretical Cosmologist