Widening the axion scales and coupling hierarchies

Kiwoon Choi

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Institute for Basic Science

Outline

- Some generic features of axions or axion-like particles (ALPs)
- Axions or axion-like particles (ALPs) in particle physics and cosmology
- Widening the axion field range & coupling hierarchy

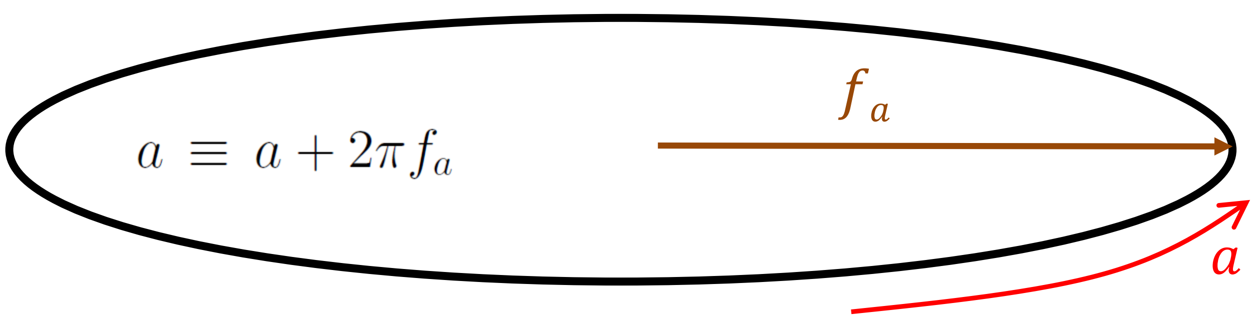
Some generic features of axions or ALP

- Axions or axion-like particles (ALP) are some of the best motivated candidates for physics beyond the Standard Model of particle physics.

- Typically axions correspond to the pseudo-Nambu-Goldstone boson of a non-linearly realized (= spontaneously broken) global U(1) symmetry:

$$U(1)_{\rm PQ}: \quad a \quad \to \quad a + \text{constant} \quad \text{(Shift symmetry)}$$

- Axions in theories with sensible UV completion are **periodic fields**, so characterized by a mass scale **f_a** (= **axion decay constant or axion scale**) which corresponds to the periodicity of the canonically normalized axion field:



- Again in theories with sensible UV completion, axion has its **scalar partner ρ** whose vacuum expectation value determines f_a :

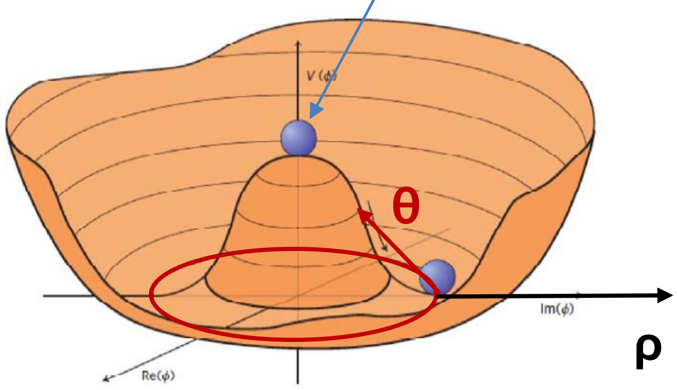
Two type of axions depending on the nature of scalar partner:

__**Field-theoretic axion**__ (open string axion)

$$\phi = \frac{1}{\sqrt{2}}\rho \exp(i\theta) \quad \left(\theta = \frac{a}{f_a}\right)$$

$$\Rightarrow \quad f_a = \langle \rho \rangle$$

$f_a=0$ at the point which allows 4D EFT description



$$U(1)_{\rm PQ}: \ \phi \to e^{i\alpha}\phi \quad (a \to a + \alpha f_a)$$

<u>String-theoretic axion</u> (closed string axion) whose scalar partner **ρ** describes some coupling or the volume of (some part of) internal space.

For each string-theoretic axion, there exists corresponding instanton which couples to the axion and thereby induces an axion-dependent amplitude

Euclidean action of instanton

$$\propto \exp\left(-S_{\rm ins}(\rho) + i\frac{a}{f_a(\rho)}\right)$$

$$S_{\rm ins}(\rho) \propto \frac{1}{(\text{coupling})^n} \quad \text{or volume of (some part of) internal scape}$$

$$\Rightarrow \quad f_a = c(\rho,\Phi)\frac{M_{\rm Planck}}{S_{\rm ins}(\rho)} \qquad \left(\Phi = \text{generic moduli other than } \rho\right)$$

$$c = \mathcal{O}(1) \;\text{ if the compactification does not involve a suppression factor} \ll 1/S_{\rm ins}$$

$$c \ll 1 \;\text{ in case with a suppression by } 1/(\text{large volume}) \text{ or small warp factor} \ll 1/S_{\rm ins}$$

$$* \quad \frac{f_a}{M_{\rm Planck}} \to 0 \;\text{ at either } \rho \to \infty \;(S_{\rm ins}(\rho) \to \infty) \;\text{ or } \Phi \to \infty \;(c(\rho,\Phi) \to 0)$$

(Swampland which does not allow 4D EFT description)

Axions associated with the string dilaton and overall volume modulus:

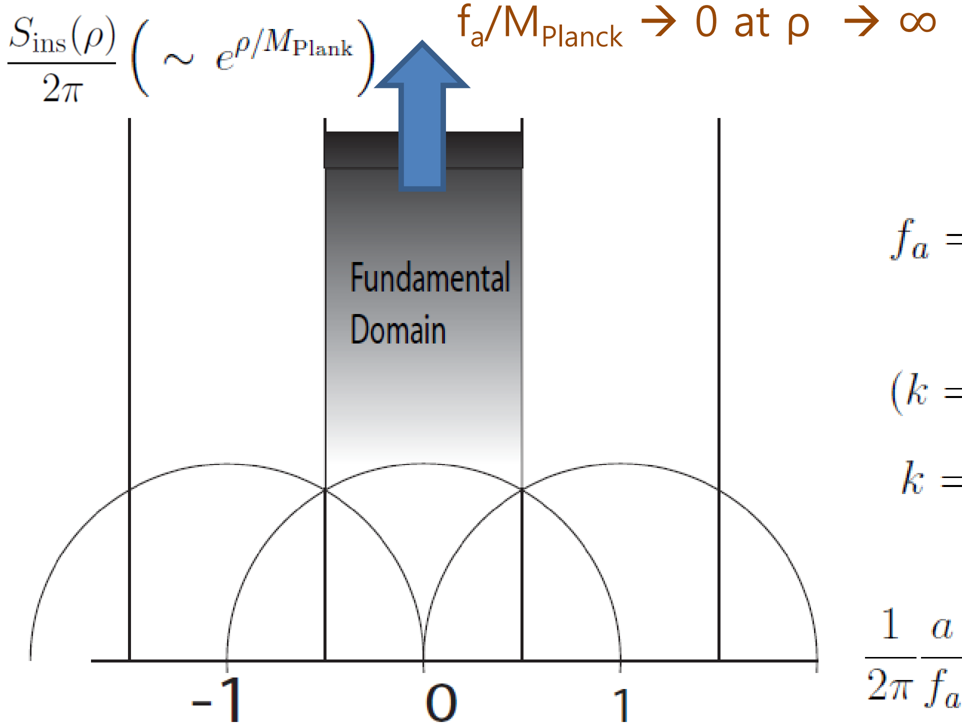
$$Z = \frac{1}{2\pi}\left[S_{\rm ins}(\rho) + i\frac{a}{f_a(\rho)}\right]$$

Duality invariance under

$Z \to \frac{1}{Z}$ (or its variant) : strong-weak coupling duality or large-small radius duality

$Z \to Z + i$: axion periodicity

f_a/M_{Planck} → 0 at ρ → ∞ (swampland)



$$f_a = c\frac{M_{\rm Planck}}{S_{\rm ins}} \quad \left(c = \sqrt{\frac{k}{2}}\right)$$

KC, Kim '85
Svrcek, Witten '06

($k = 1$ for $\rho =$ string dilaton and

$k = 3$ for $\rho =$ overal volume modulus)

Many of the physical properties of axion are determined by f_a .

Axion appears through the angular field variable $\theta = \frac{a}{f_a}$.

➔ All axion couplings and the axion mass $\propto \frac{1}{f_a}$

Low energy effective lagrangian at scales below f_a , including the axion couplings to the Standard Model:

$$\mathcal{L}_{\text{axion}} = \frac{1}{2}(\partial_\mu a)^2 + \frac{\partial_\mu a}{f_a} J^\mu + \Delta\mathcal{L} + \text{higher derivative couplings}$$

PQ-invariant derivative couplings with

$$J^\mu = c_\psi \bar{\psi}\bar{\sigma}^\mu \psi + i c_H \left(H^\dagger D^\mu H - D^\mu H^\dagger H\right) + \ldots$$

$$U(1)_{\text{PQ}} : \; a \to a + \text{constant}$$

PQ-breaking non-derivative couplings:

$$-V\left(\frac{a}{f_a}\right) + \frac{c_A}{32\pi^2}\frac{a}{f_a} F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$$

$$+ \epsilon_H m_H^2 \frac{a}{f_a}|H|^2 + \ldots$$

Quite often, axion masses are generated by non-perturbative effects, e.g. some instantons, and so exponentially suppressed:

$$m_a^2 \propto \exp\left(-S_{\rm ins}\right)$$

Weak gravity conjecture on axion field range: Arkani-Hamed et al '07

For an axion $\boldsymbol{a}$ with field range $2\pi f_a$ defined at the scale of quantum gravity, there exists an instanton which couples to $\boldsymbol{a}$ and therefore generates m_a, satisfying

$$1/f_a \geq O(S_{\rm ins}/M_{\rm Planck})$$

(axion-instanton coupling is stronger than the gravity)

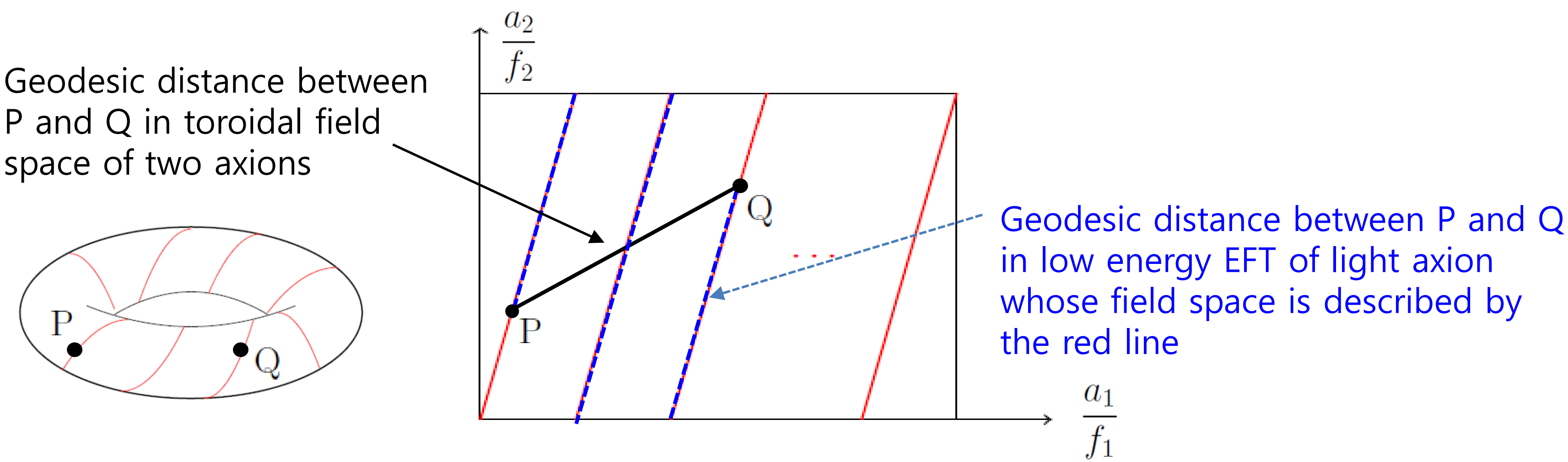
➔ Field range of light axions at the scale of quantum gravity is bounded as

$$f_a \lesssim \mathcal{O}\left(\frac{M_{\rm Planck}}{S_{\rm ins}}\right) = \mathcal{O}\left(\frac{M_{\rm Planck}}{\ln(M_{\rm Planck}/m_a)}\right)$$

String theoretic axions at the compactification scale satisfy the WGC bound.

However, implication of the WGC on low energy axion physics can be quite model-dependent:

* In models with multiple axions, the field distance in low energy effective theory can differ from the distance in the underlying UV theory if some of the axions get heavy masses and are integrated out in low energy effective theory.



* Dynamical role of the instantons suggested by the WGC is model-dependent also, e.g those instantons can be the dominant source of the axion mass in some cases or just gives a small correction in some other cases.

Axions (or ALP) in particle physics & cosmology

- <u>Axions to solve the naturalness problems (through cosmological excursion)</u>

1) **QCD axion** to solve the strong CP problem

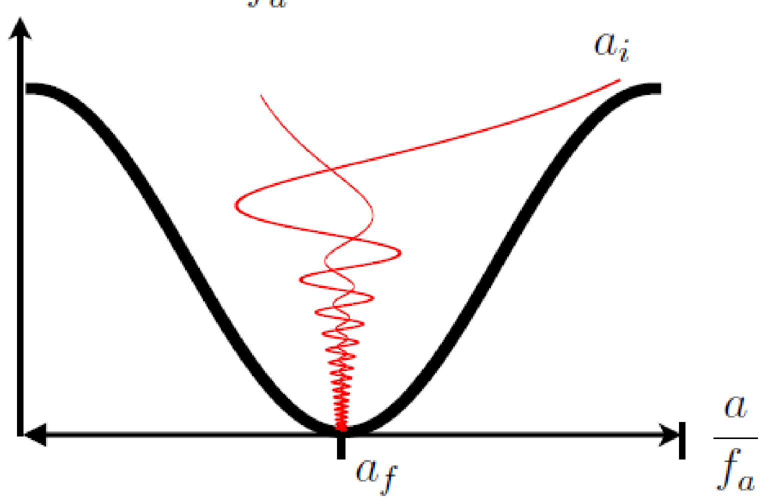
Peccei, Quinn
Weinberg; Wilczek

U(1)$_{PQ}$ dominantly broken by the QCD anomaly:

$$\Delta\mathcal{L} = \frac{1}{32\pi^2}\frac{a}{f_a}G^{a\mu\nu}\tilde{G}^a_{\mu\nu} + \ldots \quad \Rightarrow \quad V(a) = -m_\pi^2 f_\pi^2\sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}$$

$V(a)$ $\qquad \dfrac{\langle a\rangle}{f_a} \equiv \theta_{\rm QCD}$

$$m_a \sim 5\times 10^{-6}\left(\frac{10^{12}\,{\rm GeV}}{f_a}\right)\,{\rm eV}$$



Dynamical relaxation of $\theta_{\rm QCD}$ through the cosmological evolution of axion:

Generic $\quad (\theta_{\rm QCD})_i = \dfrac{a_i}{f_a}$

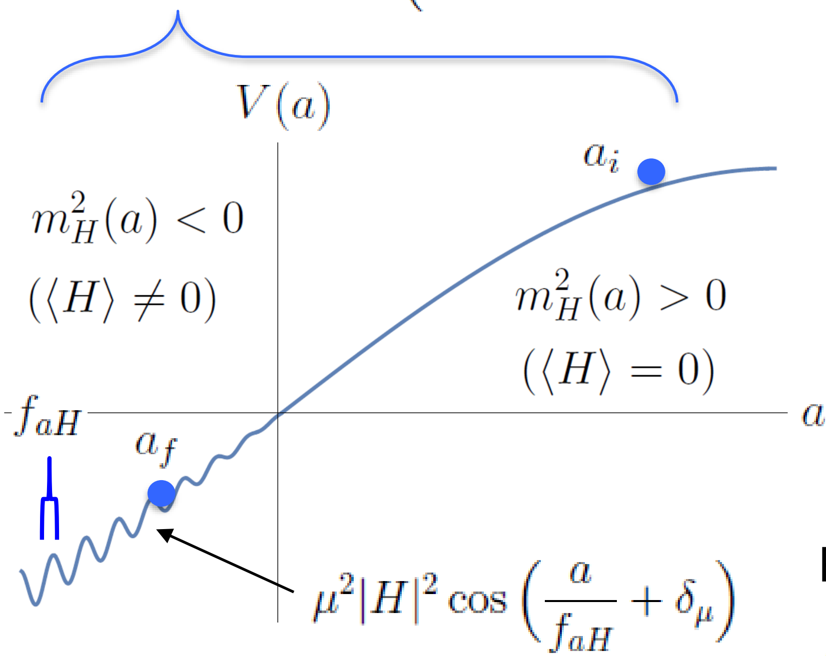
$\Rightarrow$ Nearly vanishing $\quad (\theta_{\rm QCD})_f = \dfrac{a_f}{f_a} \simeq 0$

2) **Relaxion** to solve the weak scale hierarchy problem

Graham, Kaplan, Rajendran '15

$$V(a,H) = \Lambda_1^4 \cos\left(\frac{a}{f_a}+\delta_1\right) + |H|^2\left(\Lambda_2^2\cos\left(\frac{a}{f_a}+\delta_2\right) + \Lambda_3^2 + \mu^2\cos\left(\frac{a}{f_{aH}}+\delta_\mu\right)\right) + \ldots$$

f_a $\quad \left(\Lambda_{1,2,3} \sim \text{Higgs mass cutoff} \equiv \Lambda \gg v = \langle H\rangle = 246\text{GeV} \gtrsim \mu\right)$



Cosmological excursion of relaxion to relax the Higgs mass:

$$a = a_i \;\rightarrow\; a = a_f \quad \left(|a_i - a_f| \sim f_a\right)$$

$$m_H^2(a_i) \sim \Lambda^2 \quad \Rightarrow \quad m_H^2(a_f) \simeq v^2 \ll \Lambda^2$$

Relaxion stabilization condition:

$$\frac{\Lambda^4}{f_a} \sim \frac{v^2\mu^2}{f_{aH}} \lesssim \frac{v^4}{f_{aH}} \quad \Rightarrow \quad \frac{f_a}{f_{aH}} \gtrsim \left(\frac{\Lambda}{v}\right)^4 \gg 1$$

➔ Weak scale hierarchy from the relaxion scale (or coupling) hierarchy

3) **Natural inflation** to solve the horizon and flatness problems with an axion-like inflaton whose potential is well controlled over large field range:

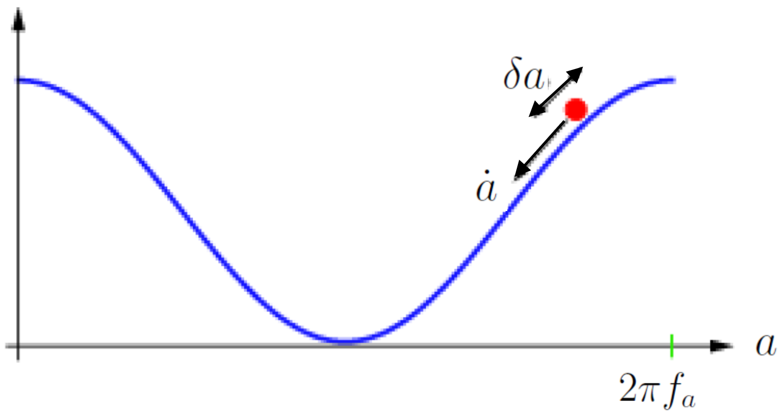
Freese, Frieman, Olinto, '90

$$V(a) = \Lambda_{\rm inf}^4 \left(1 - \cos\left(\frac{a}{f_a}\right)\right)$$

$$\Rightarrow \quad f_a \sim \sqrt{N_e} M_{\rm Planck}$$

$$\left(N_e = \text{number of e-foldings} \gtrsim 60\right)$$

$$m_a \sim H/\sqrt{N_e} \sim 10^{12}\,{\rm GeV}$$

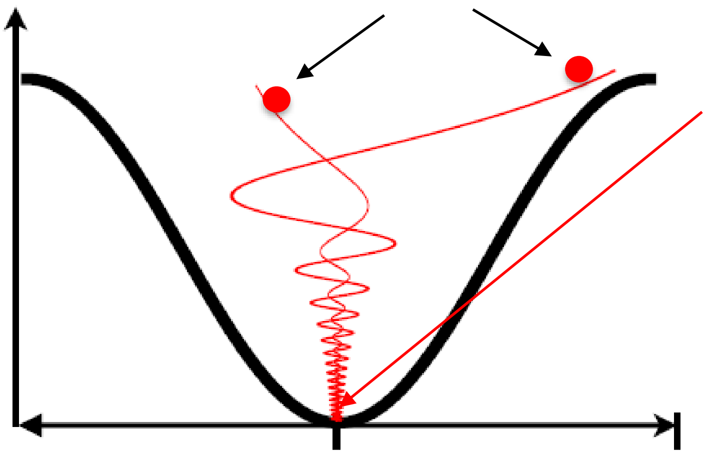


➢ Axions constituting the dark sector

1) Axion (or ALP) dark matter

Preskill, Wise, Wilczek; Abott, Sikivie; Dine, Fischler '83

initial axion field misalignment $a_i \sim f_a$



Coherently oscillating axion field today, which constitutes DM with relic mass density

$$\Omega_a h^2 \sim 0.2 \left(\frac{m_a}{10^{-22}\,\mathrm{eV}}\right)^{1/2} \left(\frac{f_a}{10^{17}\,\mathrm{GeV}}\right)^{\frac{3n+8}{2n+4}}$$

$$m_a(T) \propto T^{-n}$$

around the time when $m_a(T) \sim H(T)$

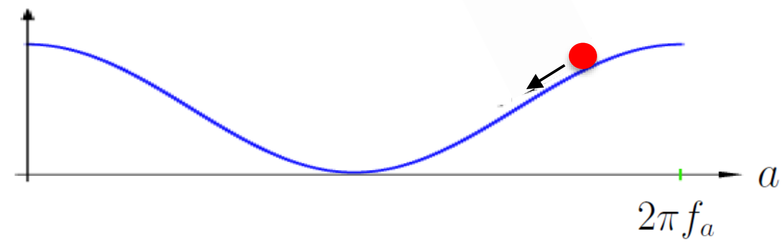
* QCD axion dark matter: $f_a \sim 10^{12}\,\mathrm{GeV}, \quad m_a \sim 5\times 10^{-6}\,\mathrm{eV} \quad (n \simeq 4)$

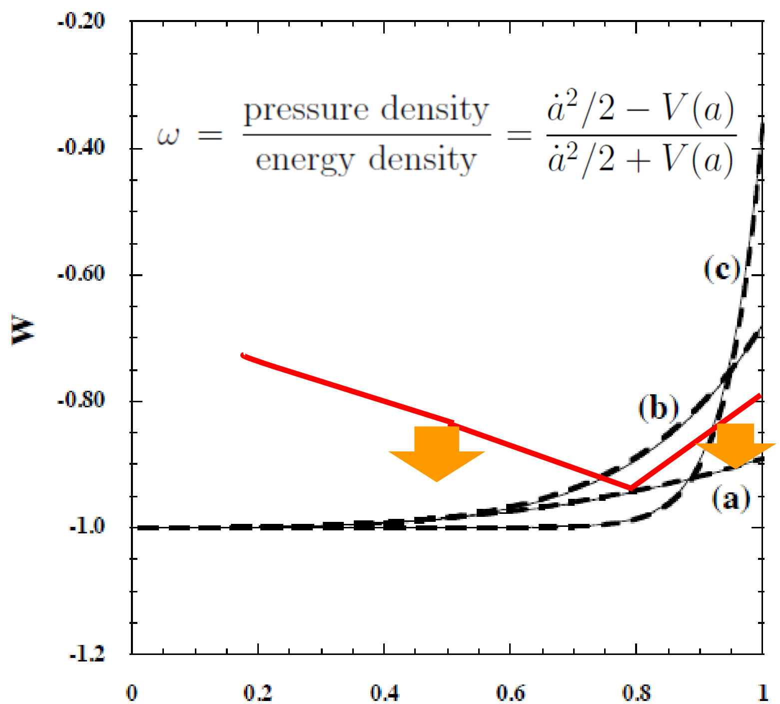
* Fuzzy dark matter: $f_a \sim 10^{17}\,\mathrm{GeV}, \quad m_a \sim 10^{-22}\,\mathrm{eV} \quad (n = 0)$

2) **Quintessence axion** for dark energy

KC '99; Kim, Nilles '02

$$V_{\rm DE} = \mu^4\left(1+\cos\left(\frac{a}{f_a}\right)\right)\quad (\mu\sim 10^{-3}\,{\rm eV})$$





$(a)\ \frac{f_a}{M_{\rm Pl}} = \frac{a_i}{f_a} = 0.5$

$(b)\ \frac{f_a}{M_{\rm Pl}} = 0.3,\quad \frac{a_i}{f_a} = 0.25$

$(c)\ \frac{f_a}{M_{\rm Pl}} = 0.1,\quad \frac{a_i}{f_a} = 10^{-3}$

$$\Rightarrow\quad f_a \gtrsim 0.5 M_{\rm Planck}$$

$$m_a \lesssim 10^{-32}\,{\rm eV}$$

Tsujikawa '13

$R(t)/R(t_0)\quad (R = \text{scale factor of the expanding universe})$

$$= \frac{1}{1+z}$$

Widening the axion scale and coupling hierarchies

Some axions introduced in particle physics and cosmology require intriguing hierarchical pattern of couplings and scales.

- Axions or ALP which require $f_a \gg f_a(\mathrm{WGC})$

$$\left(f_a(\mathrm{WGC}) \equiv \frac{M_{\mathrm{Planck}}}{S_{\mathrm{ins}}} \sim \frac{M_{\mathrm{Planck}}}{\ln(M_{\mathrm{Planck}}/m_a)} \right)$$

* Natural inflation:

$$f_a \sim \sqrt{N_e} M_{\mathrm{Planck}} \quad \Rightarrow \quad \frac{f_a}{f_a(\mathrm{WGC})} \sim \sqrt{N_e} \ln(M_{\mathrm{Planck}}/H_{\mathrm{inf}}) \sim 10^2$$

* Quintessence axion:

$$f_a \sim M_{\mathrm{Planck}} \quad \Rightarrow \quad \frac{f_a}{f_a(\mathrm{WGC})} \sim \ln\left(M_{\mathrm{Planck}}/10^{-33}\mathrm{eV}\right) \sim 10^2$$

* Original version of relaxion model:

$$f_a \gtrsim 10^{12}\Lambda \left(\frac{\Lambda}{246\,\mathrm{GeV}}\right)^4 \gtrsim 10^{23}\,\mathrm{GeV} \quad \left(\Lambda = \text{Higgs mass cutoff scale} \gtrsim 10\,\mathrm{TeV}\right)$$

- Axions with hierarchical couplings and scales:

$$\frac{1}{4}\left(g_{a\gamma} a F^{\mu\nu}\tilde{F}_{\mu\nu} + g_{ag} G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}\right) + \mu^2 \cos\left(\frac{a}{f_{aH}}\right)|H|^2 + \ldots \quad \left(a \equiv a + 2\pi f_a\right)$$

Typical axion models predict

$$g_{a\gamma} \sim \frac{e^2}{8\pi^2}\frac{1}{f_a} \sim \frac{10^{-3}}{f_a}, \quad g_{ag} \sim \frac{g_3^2}{8\pi^2}\frac{1}{f_a} \sim \frac{g_3^2}{e^2} g_{a\gamma} \sim 10^2 g_{a\gamma}, \quad f_{aH} \sim f_a$$

$$\text{(around the QCD scale)}$$

However some axions require

* $g_{a\gamma} f_a \gg 1$: Gauge field production by rolling axion in dissipative axion inflation model and axion-driven magnetogenesis scenario

Turner, Widrow '88; Grrestson et al '92
Anber, Sorbo '09

* $\frac{f_a}{f_{aH}} \gtrsim \left(\frac{\Lambda}{246\,\mathrm{GeV}}\right)^4 \gg 1$: Relaxion

* $g_{a\gamma} \gg g_{ag}$: Photophilic QCD axion

Higaki et al '15; Farina et al '16;
Agrawa et al '17

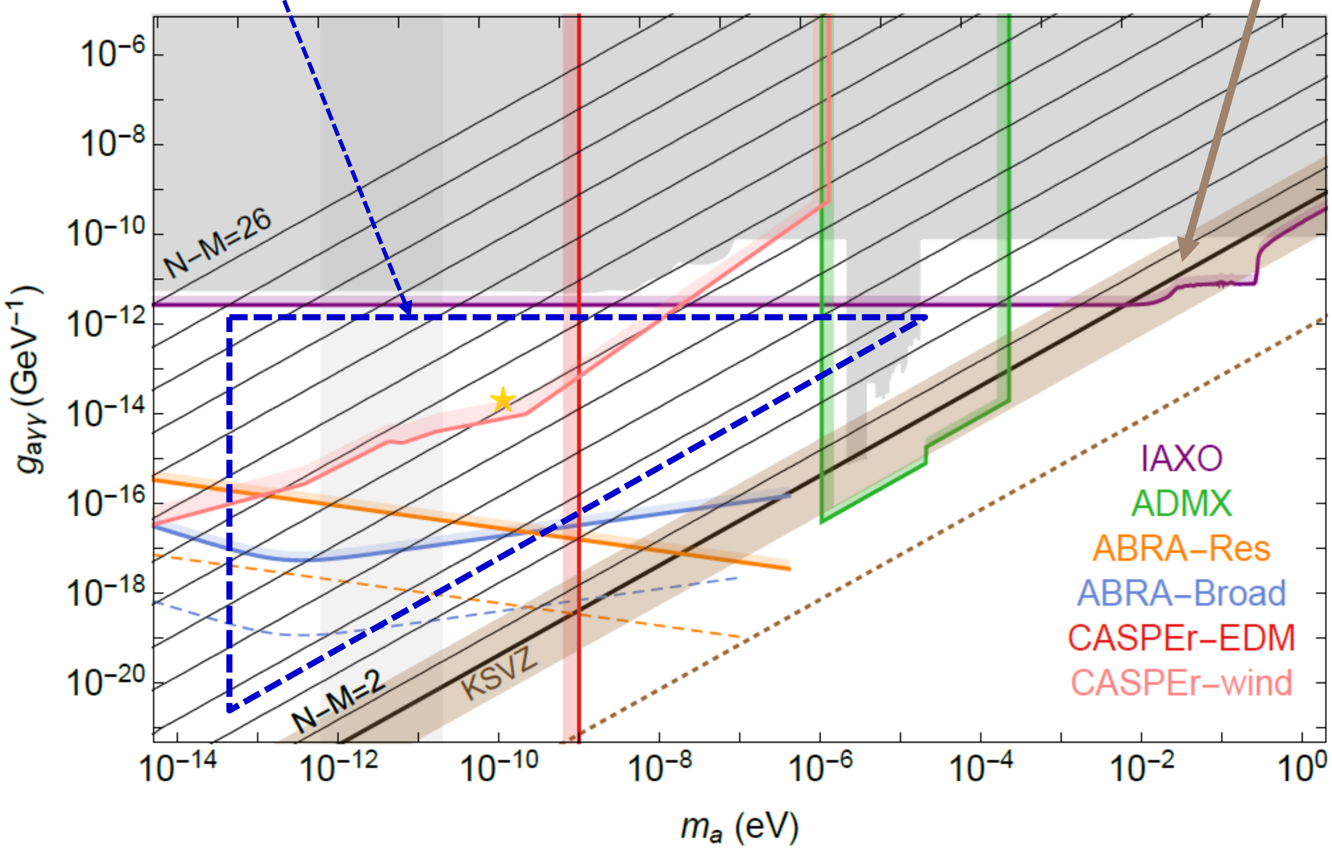
These hierarchical pattern of parameters are stable against quantum corrections, so technically natural.

* Photophilic QCD axion above the brown band

$$g_{a\gamma} \gg 10^{-2} g_{ag}$$

Brown band for ordinary QCD axion

$$g_{a\gamma} \sim (e^2/g_3^2)\, g_{ag} \sim 10^{-2} g_{ag}$$



Farina et al '16

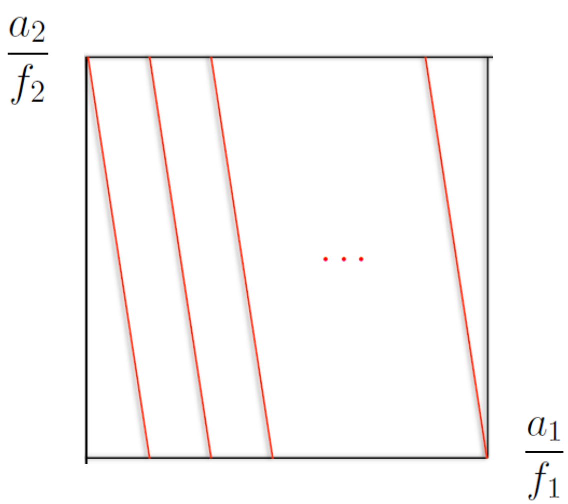
QCD axion to solve the strong CP problem

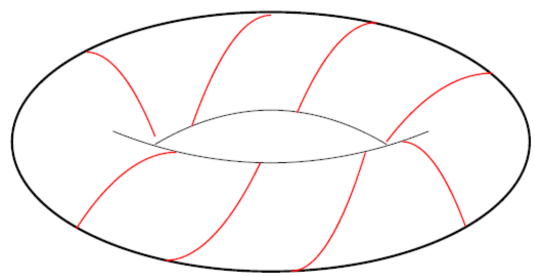
$$\longrightarrow \quad m_a \sim 10^{-3} \left(\frac{g_{ag}}{10^{-10}\,\mathrm{GeV}^{-1}} \right) \mathrm{eV}$$

Dynamical mechanism to enlarge the axion field range and/or generate hierarchical pattern of axion couplings

Alignment to enlarge the axion field range: Kim, Nilles, Peloso '05

$$V\left(\frac{a_1}{f_1},\frac{a_2}{f_2}\right) = -\Lambda^4\cos\left(\frac{a_2}{f_2}+n\frac{a_1}{f_1}\right)$$





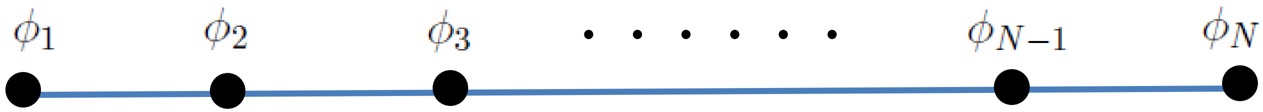
a = canonically normalized light axion describing the flat direction (red line)

In the limit $n \gg 1$, a is aligned with a_2 (KNP alignment) and its field range is enhanced by $n \gg 1$.

$$2\pi f_a = 2\pi\sqrt{n^2 f_2^2 + f_1^2} \simeq n \times 2\pi f_2 \quad \left(f_1 \sim f_2\right)$$

Clockwork mechanism to exponentially enlarge the axion field range and also generate exponential hierarchy among the axion couplings:

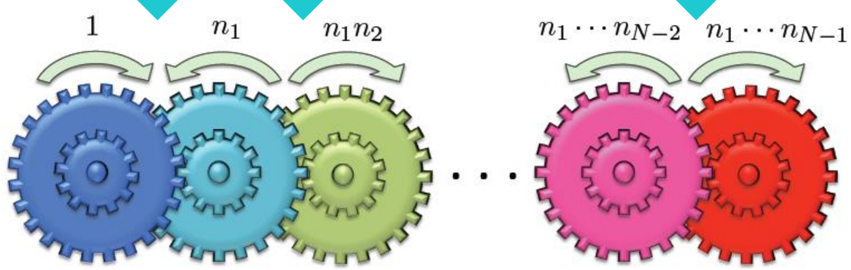
KC, Kim, Yun '14; KC, Im '15; Kaplan, Rattazzi '15



$$V_{\rm CW} = \sum_{i=1}^{N} \lambda_i \left(|\phi_i|^2 - \frac{f^2}{2} \right) + \left(\kappa_1 \phi_1^{n_1} \phi_2 + \kappa_2 \phi_2^{n_2} \phi_3 + \ldots \kappa_{N-1} \phi_{N-1}^{n_{N-1}} \phi_N + {\rm h.c} \right)$$

$$\phi_k = \frac{1}{\sqrt{2}} \rho_k \exp(i\theta_k)$$

$$\left(\langle \rho_k \rangle = f \right)$$



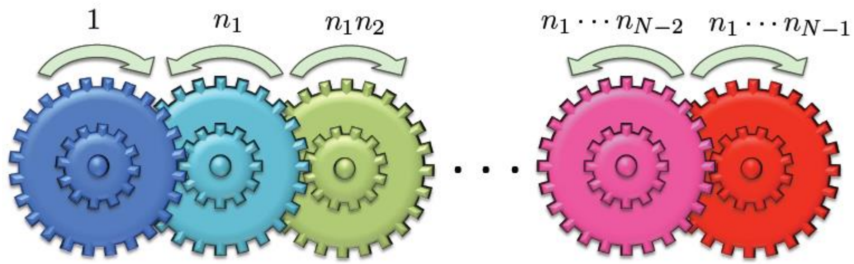
$\theta_k =$ rotation angle of the k-th gear

$$\Rightarrow \quad \theta_2 = -n_1 \theta_1, \quad \theta_3 = -n_2 \theta_2, \quad \ldots \ , \quad \theta_N = -n_{N-1} \theta_{N-1}$$

Single full rotation of the first gear causes an exponentially many rotations of the last gear, which means that the field range of the light axion $\boldsymbol{a}$ describing the collective rotation of the clockwork gears is exponentially enhanced:

$$f_a = f \sqrt{1 + n_1^2 + \ldots + (n_1 n_2 \ldots n_{N-1})^2} \sim n_1 n_2 \ldots n_{N-1} f$$

Light axion quasi-localized in discrete theory space in clockwork axion model:

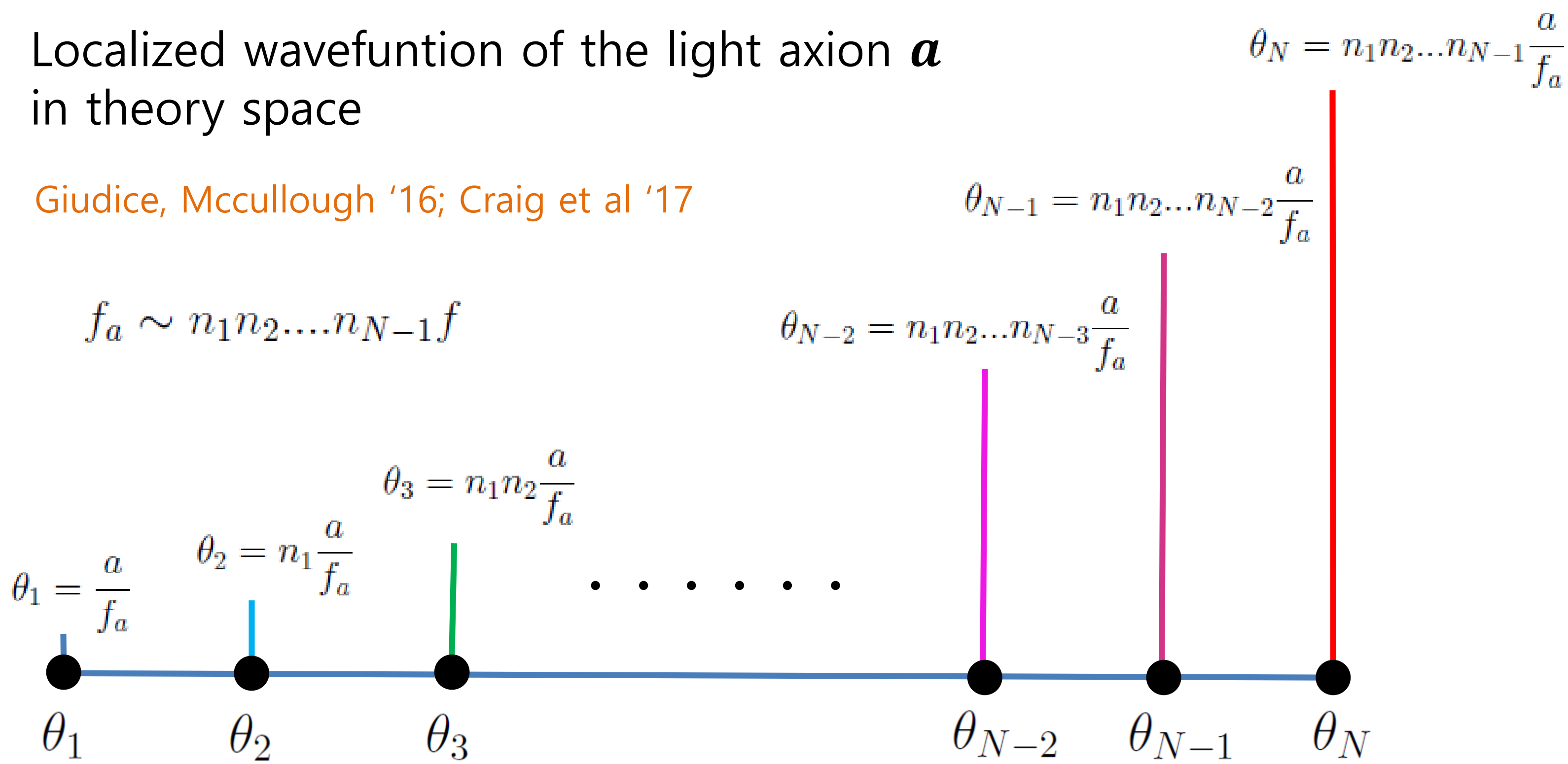


$$\theta_2 = -n_1\theta_1, \quad \theta_3 = -n_2\theta_2, \quad \dots \ , \quad \theta_N = -n_{N-1}\theta_{N-1}$$

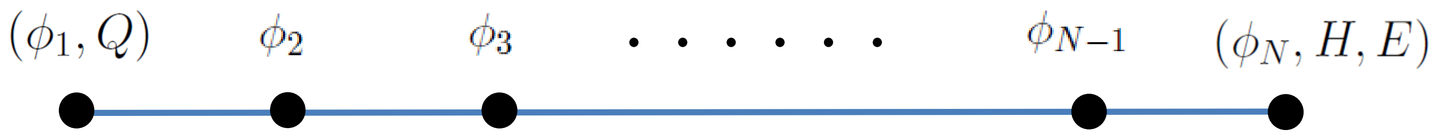
$\Rightarrow$ Localized wavefuntion of the light axion $\boldsymbol{a}$ in theory space

Giudice, Mccullough '16; Craig et al '17

$$f_a \sim n_1 n_2 \dots n_{N-1} f$$



Hierarchical axion couplings to the SM:



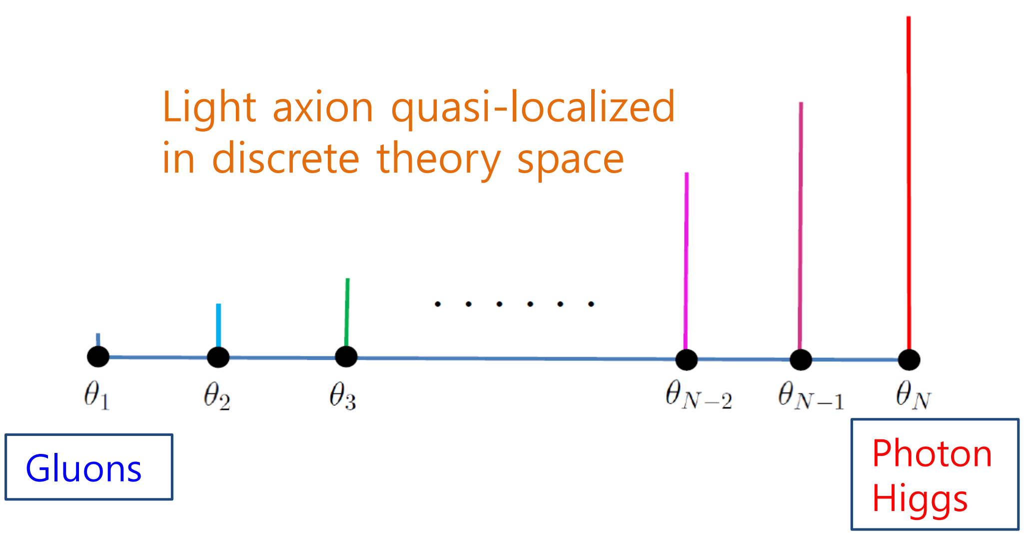
$Q = SU(2)_W \times U(1)_Y$-singlet vector-like quark

$E = U(1)_Y$-charged vector-like lepton

$$\mathcal{L}_{\rm int} = y_1\phi_1\bar{Q}_L Q_R + y_N\phi_N\bar{E}_L E_R + \mu\phi_N|H|^2 + {\rm h.c} \quad \left(\phi_k = \frac{1}{\sqrt{2}}\rho_k \exp(i\theta_k)\right)$$

$$\Rightarrow \quad \theta_1\frac{g_3^2}{32\pi^2}G^{a\mu\nu}\tilde{G}^a_{\mu\nu} + \theta_N\frac{g_1^2}{32\pi^2}B^{\mu\nu}\tilde{B}_{\mu\nu} + \sqrt{2}\mu f\cos(\theta_N)|H|^2$$

$$= \frac{1}{32\pi^2}\frac{a}{f_a}\left(g_3^2 G^{a\mu\nu}\tilde{G}^a_{\mu\nu} + (n_1 n_2 \ldots n_{N-1})e^2 F^{\mu\nu}\tilde{F}_{\mu\nu}\right) + \sqrt{2}\mu f\cos\left(\underbrace{n_1 n_2\ldots n_{N-1}\frac{a}{f_a}}_{\frac{a}{f_{aH}}}\right)|H|^2$$



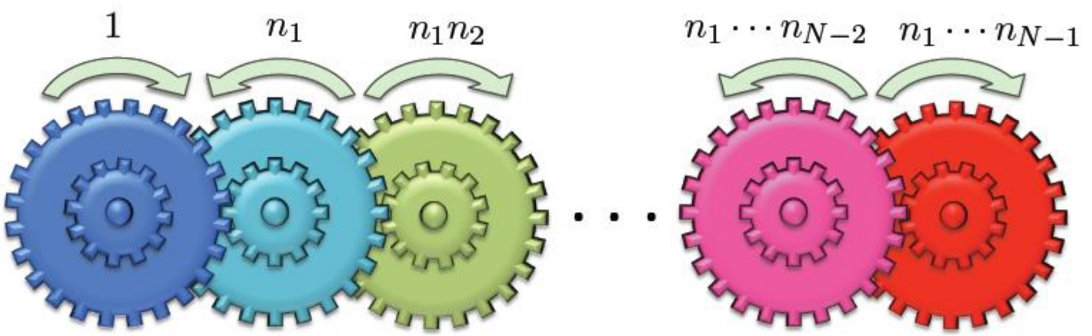
$$\theta_1 = \frac{a}{f_a}, \quad \theta_N = n_1 n_2 \ldots n_{N-1}\frac{a}{f_a}$$

$$\Rightarrow \quad \frac{g_{a\gamma}}{g_{ag}} = (n_1 n_2 \ldots n_{N-1})\frac{e^2}{g_3^2}$$

$$\frac{f_a}{f_{aH}} = n_1 n_2 \ldots n_{N-1}$$

Axions are periodic fields, so their non-derivative couplings are appropriately quantized.

➔ To enlarge the field range and/or to generate hierarchical non-derivative couplings of light axion, we need to generate a **large integer coefficient**, not a large (or small) continuous coefficient. This can not be achieved by a localization in continuous extra dimension.



$$\Rightarrow \quad n_1 n_2 n_3 \ldots n_{N-1} \gg 1$$

Conclusion

- Studies of various issues in particle physics and cosmology suggest that there may exist multiple light axions having very different field ranges and masses.

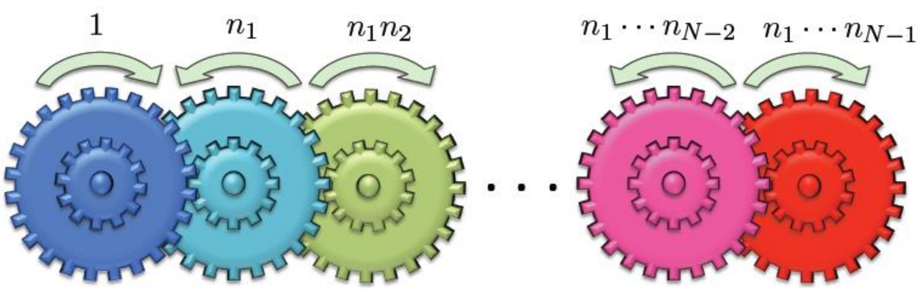
- Some axions require hierarchical pattern of couplings and scales, which is difficult to be achieved in conventional axion models. This motivates a mechanism to widen the axion field range and the coupling hierarchies:

$$\frac{1}{4}\left(g_{a\gamma} a F^{\mu\nu}\tilde{F}_{\mu\nu} + g_{ag} G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}\right) + \mu^2 \cos\left(\frac{a}{f_{aH}}\right)|H|^2 + \ldots \quad \left(a \equiv a + 2\pi f_a\right)$$

$$f_a \gg f_a(\mathrm{WGC}) \quad \left(f_a(\mathrm{WGC}) \equiv \frac{M_{\mathrm{Planck}}}{S_{\mathrm{ins}}} \sim \frac{M_{\mathrm{Planck}}}{\ln(M_{\mathrm{Planck}}/m_a)}\right)$$

$$g_{a\gamma} f_a \gg 1 \qquad \frac{f_a}{f_{aH}} \gtrsim \left(\frac{\Lambda}{246\,\mathrm{GeV}}\right)^4 \gg 1 \qquad g_{a\gamma} \gg g_{ag}$$

- Clockwork mechanism can generate an exponentially large integer coefficient, which would result in an exponentially enlarged field range and exponential hierarchy among the quantized couplings of light axion.



$$\Rightarrow \quad n_1 n_2 n_3 \ldots n_{N-1} \gg 1$$