2d Anomaly, Orbifolding, and Boundary States

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Based on 1908.02918(v2) w/ Yang Zhou

and "Orbifolding D_{2l} type WZW model," (KK) to appear

Motivation: 3d story

$$I_{CS}[a] = \frac{k}{4\pi}\int_{3d} ada$$

- Canonical quantization $\quad [a_1(\mathbf{x}), a_2(\mathbf{y})] = \frac{2\pi i}{k}\delta^{(2)}(\mathbf{x}-\mathbf{y})$

$$e^{i\oint_{cycle1} a} e^{i\oint_{cycle2} a} = e^{\frac{2\pi i}{k}} e^{i\oint_{cycle2} a} e^{i\oint_{cycle1} a} \quad \text{(on } \mathbb{R}\times\mathbb{T}^2\text{)}$$

- new interpretation⋯anomaly of a generalized global sym.

Motivation: generalized global symmetry

[Gaiotto-Kapustin-Seiberg-Willett]

- Charges of the ordinary syms. are defined on a <u>time-slice</u>.

  ||

  codimension-one defect

- Charges are <u>conserved</u>.

  ||

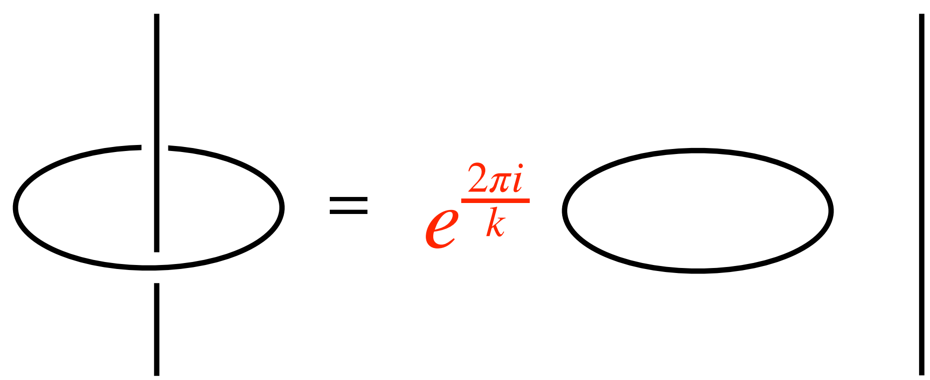
  topological

- They defined p-form sym. via invertible operators on topological defects with codimension-$(p+1)$.

Motivation: 1-form anomaly

- Classically, codmension ≥ 2 objects commute.
- But Wilson loops $e^{i\oint a}$ do not:

$$e^{i\oint_{cycle1} a} e^{i\oint_{cycle2} a} = e^{\frac{2\pi i}{k}} e^{i\oint_{cycle2} a} e^{i\oint_{cycle1} a} \quad (\textbf{on } \mathbb{R} \times \mathbb{T}^2)\,.$$



- The noncommutativity was interpreted as a 1-form $\mathbb{Z}_k$ anomaly.

Motivation: 3d↔2d

- 3d CS/2d RCFT (e.g. WZW) correspondence is known. e.g. [Moore-Seiberg '89]
- In 3d, commutativity (of lines) is well-defined, but in 2d, it is not.

Q: What is the 2d counterpart of the 3d 1-form anomaly?

Review of WZW

- These are class of 2d (diagonal) RCFTs labeled by $\hat{G}$ and k, $\hat{G}_k$ WZW model.

- Its primaries are parametrized by affine weights

$$\hat{\mu} = [\mu_0; \mu_1, \cdots, \mu_r]$$

where r is the rank of $\hat{G}$.

Review of WZW

- The theory has two 0-form syms. G_{center} and G_{auto} related by

$$S^\dagger G_{auto} S = G_{center} \quad (\text{"}S\textbf{-duality}\text{"})\,.$$

- G_{center} acts on a primary $\phi_{\hat{\mu}}$ by a phase while G_{auto} mixes primaries by acting on affine weights.

Example: $\widehat{SU(3)}_1$ WZW

- There are 3 primaries labeled by

$$\{[1;0,0],[0;1,0],[0;0,1]\}\ .$$

- $G_{center} \simeq \mathbf{Z}_3$ acts on them by

$$\phi_{\widehat{[1;0,0]}} \mapsto \phi_{\widehat{[1;0,0]}}, \quad \phi_{\widehat{[0;1,0]}} \mapsto \omega\phi_{\widehat{[0;1,0]}}, \quad \phi_{\widehat{[0;0,1]}} \mapsto \omega^2\phi_{\widehat{[0;0,1]}}\ .$$

- $G_{auto} \simeq \mathbf{Z}_3$ acts on affine weights by

$$[\mu_0;\mu_1,\mu_2] \mapsto [\mu_2;\mu_0,\mu_1]\ .$$

Verlinde lines

(=charges in this talk)

- These syms. are generated by topological defect lines, which have codim. 1, called Verlinde lines. **[Verlinde '88]**

- More generally, in diagonal RCFTs, primaries $\overset{\exists 1 \text{ to } 1}{\longleftrightarrow}$ Verlinde lines (also for non-sym. lines).

[Moore-Seiberg '89]

e.g. in $\widehat{SU(3)}_1$ WZW, $\phi_{\widehat{[1;0,0]}} \leftrightarrow \mathscr{L}_{id}$, $\quad \phi_{\widehat{[0;1,0]}} \leftrightarrow \mathscr{L}_g$, $\quad \phi_{\widehat{[0;0,1]}} \leftrightarrow \mathscr{L}_{g^2}$.

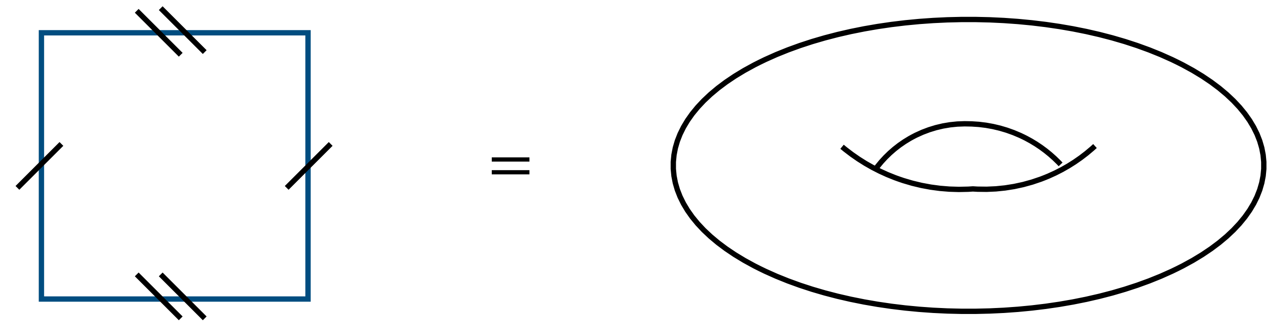
2d Anomaly, Orbifolding,

Result 1

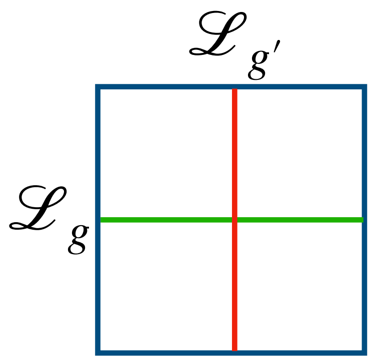
and Boundary States

Result 1: detecting anomaly

- We proposed to detect the anomaly in question by putting the 2d diagonal RCFTs on a two-torus;



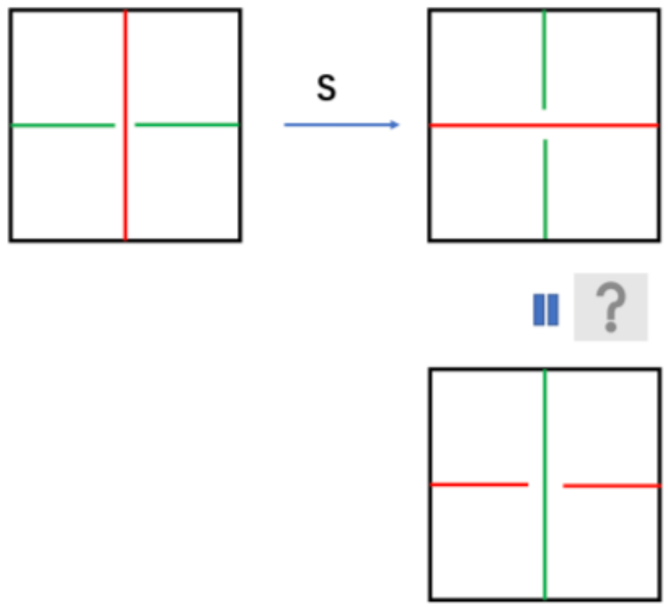
- More concretely, we twist the torus partition functions by two Verlinde lines



$$= Z_{(\mathscr{L}_g, \mathscr{L}_{g'})},$$

perform modular S-trans., and check $SZ_{(\mathscr{L}_g, \mathscr{L}_{g'})} \stackrel{?}{=} Z_{(\mathscr{L}_{g'}, \mathscr{L}_g)}$.

Result 1: detecting anomaly



$$SZ_{(\mathscr{L}_g,\mathscr{L}_{g'})} \stackrel{?}{=} Z_{(\mathscr{L}_{g'},\mathscr{L}_g)}$$

e.g. $\widehat{SU(N)}_k$ WZW is anomaly free iff $k \in N\mathbb{Z}$.

What is the anomaly?

- Our proposal: mixed anomaly between G_{center} and G_{auto}.
- Some tests;

test1) $\widehat{SU(3)}_3$ $\quad Z_{orb}^{\mathbb{Z}_3} = |\chi_{[3;0,0]} + \chi_{[0;3,0]} + \chi_{[0;0,3]}|^2 + 3|\chi_{[1;1,1]}|^2$

test2) $\widehat{SU(3)}_1$ $\quad Z_{orb}^{\mathbb{Z}_3} = |\chi_{[1;0,0]}|^2 + \chi_{[0;1,0]}\bar{\chi}_{[0;0,1]} + \chi_{[0;0,1]}\bar{\chi}_{[0;1,0]}$

test3,4,...) minimal models

Comment: what about T?

[Numasawa-Yamaguchi]
[KK, to appear]

- Since $g \in \mathbb{Z}_N$ obey $g^N = id$, we must have

$$T^N Z_{(\mathscr{L}_{id}, \mathscr{L}_g)} = Z_{(\mathscr{L}_{g^N}, \mathscr{L}_g)} = Z_{(\mathscr{L}_{id}, \mathscr{L}_g)}\,.$$

- However, sometimes nontrivial phases appear on RHS

$$Z_{(\mathscr{L}_{g^N}, \mathscr{L}_g)} = e^{i\alpha(k,N)} Z_{(\mathscr{L}_{id}, \mathscr{L}_g)}\,.$$

- The nontrivial phase was identified as a mixed anomaly between $\mathbb{Z}_N$ and $SL(2,\mathbb{Z})$, which obstructs orbifolding.

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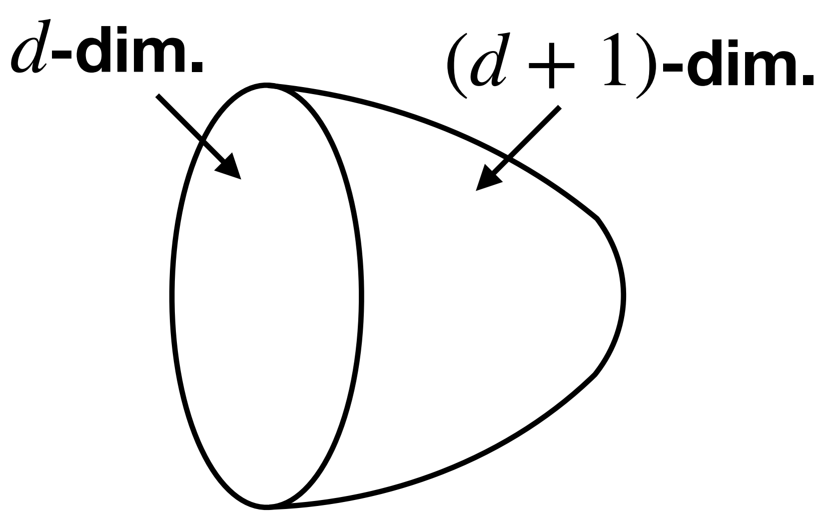
Result 2

Result 2: boundary

- Anomaly inflow:

$$Z[A + d\lambda] = e^{i\alpha(A,\lambda)} Z[A]$$

One usually can save the gauge inv. by extending background fields to $(d+1)$-dim. bulk;



$$\widetilde{Z}[A] := Z[A] \exp\left(\int_{(d+1)} f(A) \right)$$

with

$$\widetilde{Z}[A + d\lambda] = \widetilde{Z}[A] \, .$$

Result 2: boundary

- Simple fact: $\partial^2 = 0$.

- So if anomaly inflow works, anomalous theories cannot have boundaries.

anomalous$\Rightarrow$no boundary

or

$\exists$boundary$\Rightarrow$non-anomalous

[Han-Tiwari-Hsieh-Ryu][Jensen-Shaverin-Yarom]

Result 2: boundary

- Boundary states (given by Cardy states) are also parameterized by affine weights $|\hat{\mu}\rangle_c$. **[Cardy '89]**

- The center $g_c \in G_{center}$ acts on it by its "S-dual" $g_a = S g_c S^\dagger \in G_{auto}$

$$g_c : |\hat{\mu}\rangle_c \mapsto |g_a \hat{\mu}\rangle_c \,.$$

- So if $\exists \hat{\mu}$ s.t. $g_a \hat{\mu} = \hat{\mu}$, the corresponding boundary state is invariant.

Example: $\widehat{SU(3)}_k$ WZW

- Recall $G_{auto} : \hat{\mu} = [\mu_0; \mu_1, \mu_2] \mapsto [\mu_2; \mu_0, \mu_1]$.

- So if $\mu_0 = \mu_1 = \mu_2$, i.e., $k = \mu_0 + \mu_1 + \mu_2 \in 3\mathbb{Z}$, ∃inv. boundary state, and the theories are anomaly-free.

"Anomaly-decoupling"

[Yang Zhou '19]

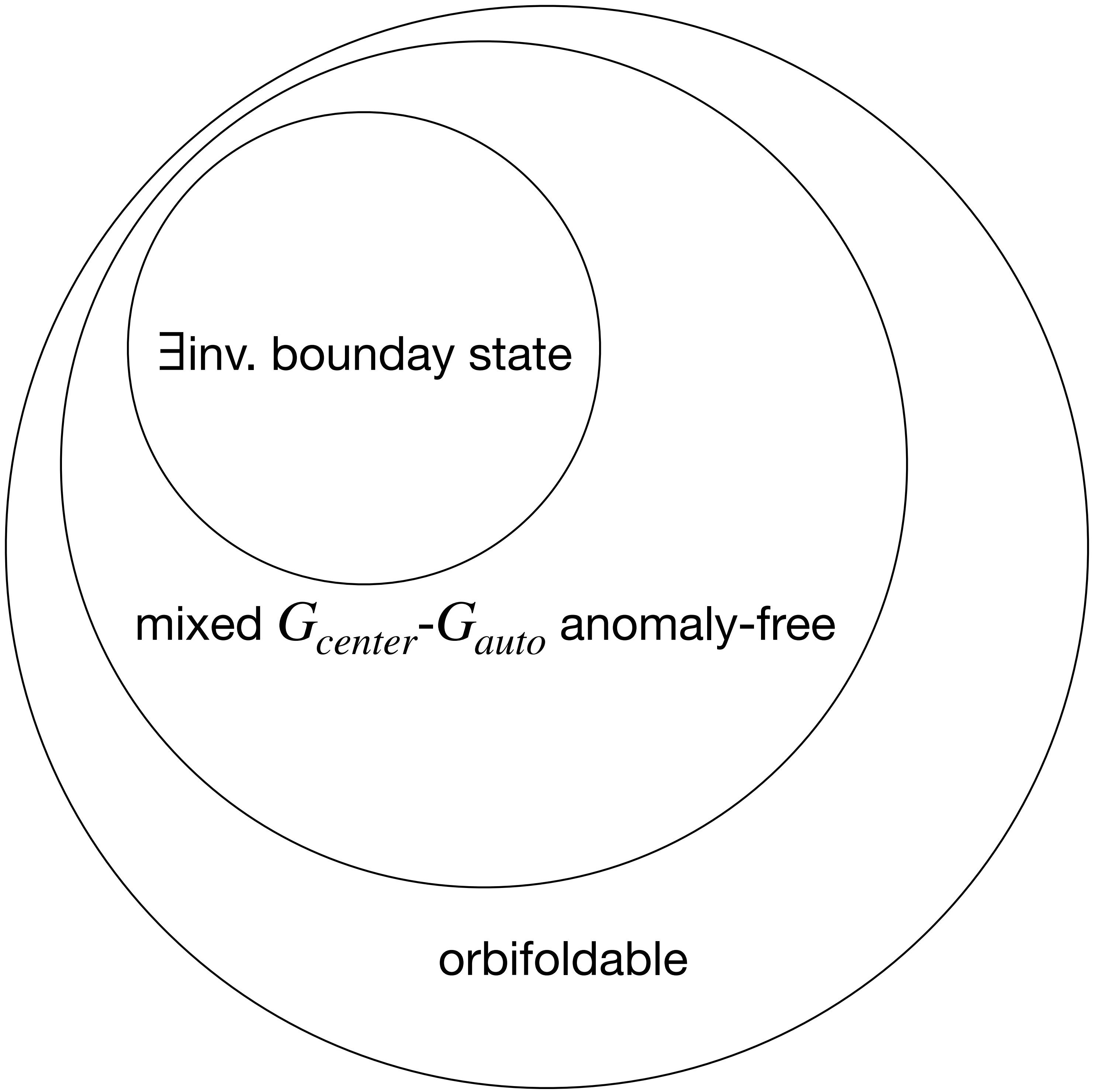
- Conjecture: If a theory with global internal sym. $G = G_1 \times G_2 \times \cdots$ admit a G_1-inv. boundary state, G_1 is free of 't Hooft anomaly and of mixed anomaly between the other $G_{j \neq 1}$s.

- We checked this conjecture holds in all WZW.

Summary of results

$\widehat{G}$	G_{center}	CS_3	$SZ_{(\mathcal{L}_g,\mathcal{L}_g)}\Big\vert = Z_{(\mathcal{L}_g,\mathcal{L}_g)}\Big\vert$	$\vert g_a\hat{\mu}\rangle_c = \vert\hat{\mu}\rangle_c$
$\widehat{SU(r+1)}$	$\mathbb{Z}_{r+1}$	$k \in (r+1)\mathbb{Z}$	$k \in (r+1)\mathbb{Z}$	$k \in (r+1)\mathbb{Z}$
$\widehat{SO(2r+1)}$	$\mathbb{Z}_2$	$k \in \mathbb{Z}$	$k \in \mathbb{Z}$	$k \in \mathbb{Z}$
$\widehat{Sp(2r)}$	$\mathbb{Z}_2$	$rk \in 2\mathbb{Z}$	$rk \in 2\mathbb{Z}$	$rk \in 2\mathbb{Z}$
$\widehat{SO(4l)}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$k \in 2\mathbb{Z}$	$lk \in 2\mathbb{Z}$	$k \in 2\mathbb{Z}$
$\widehat{SO(4l+2)}$	$\mathbb{Z}_4$	$k \in 4\mathbb{Z}$	$k \in 4\mathbb{Z}$	$k \in 4\mathbb{Z}$
$\widehat{E_6}$	$\mathbb{Z}_3$		$k \in 3\mathbb{Z}$	$k \in 3\mathbb{Z}$
$\widehat{E_7}$	$\mathbb{Z}_2$		$k \in 2\mathbb{Z}$	$k \in 2\mathbb{Z}$

Summary of results



Discussion

- We considered discrete syms. in 2d diagonal RCFTs from the viewpoint of anomaly, orbifold, and boundary state.

3d 1-form anomaly↔2d mixed G-$SGS^\dagger$ 0-form anomaly

Future directions

- What about continuous syms. or more general CFTs?
- RG flows between WZWs?

Appendix

$SZ = Z$ anomaly in minimal models

- In WZW, G_{auto} can be read from fusion rules

$$\phi_{g_a} \times \phi_{\hat{\mu}} = \phi_{g_a \hat{\mu}} \,.$$

- Critical Ising: Its anomaly-free $\mathbb{Z}_2$ is generated by the Verlinde line $\mathscr{L}_\varepsilon$. $\varepsilon \times id = \varepsilon$, $\varepsilon \times \varepsilon = id$, $\varepsilon \times \sigma = \sigma$ suggests $id \leftrightarrow \varepsilon$, preserved in

$$Z_{orb}^{\mathbb{Z}_2} = |\chi_{id}|^2 + |\chi_\varepsilon|^2 + |\chi_\sigma|^2 \,.$$

- 3-state Potts: Its anomalous $\mathbb{Z}_3$ is generated by $\mathscr{L}_{C_{13}^{(1)}}$. Its fusion rule suggests cyclic permutations of $\{C_{11}, C_{13}^{(1)}, C_{13}^{(2)}\}$, $\{C_{21}, C_{23}^{(1)}, C_{23}^{(2)}\}$, broken in

$$Z_{orb}^{\mathbb{Z}_3} = |\chi_{C_{11}}|^2 + |\chi_{C_{21}}|^2 + \chi_{C_{13}^{(1)}} \bar{\chi}_{C_{13}^{(2)}} + \chi_{C_{13}^{(2)}} \bar{\chi}_{C_{13}^{(1)}} + \chi_{C_{23}^{(1)}} \bar{\chi}_{C_{23}^{(2)}} + \chi_{C_{23}^{(2)}} \bar{\chi}_{C_{23}^{(1)}} \,.$$