Reflected Entropy and Entanglement Wedge Cross Section with the First Order Correction

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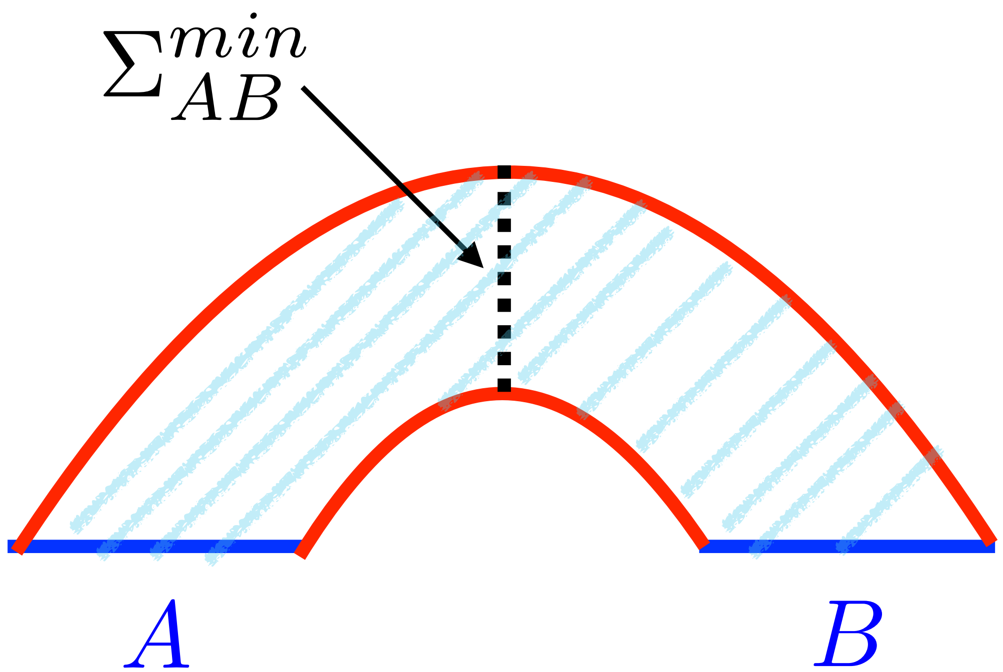
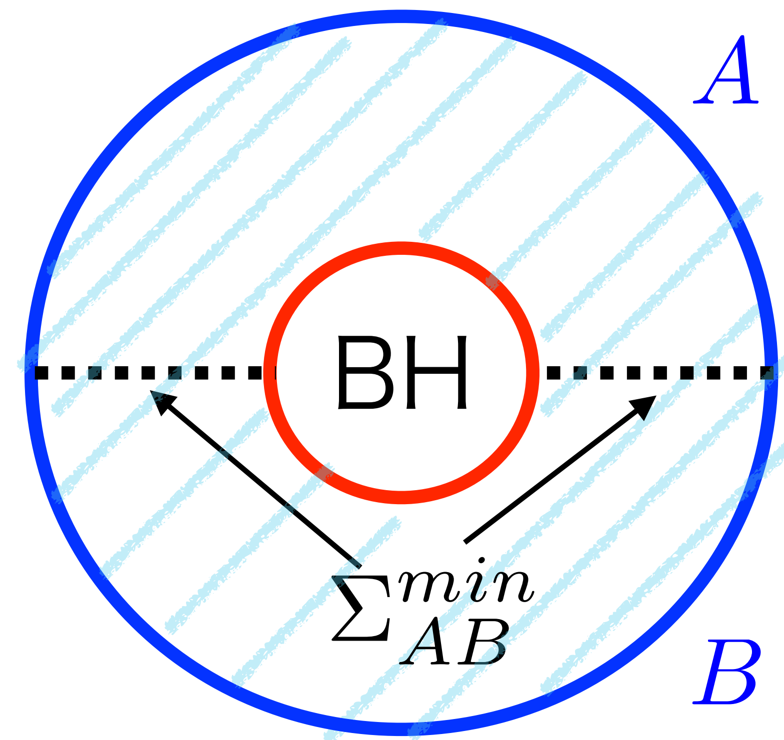
Entanglement wedge cross section

$$E_W(A:B) = \frac{\text{Area}[\Sigma_{AB}^{min}]}{4G_N}$$

minimal cross section
in entanglement wedge





Proposals of the dual quantities

- Entanglement of purification [T. Takayanagi, K. Umemoto, 2017] [P. Nguyen et al., 2017]
- Logarithmic negativity [J. Kudler-Flam, S. Ryu, 2018]
- Odd entropy [K. Tamaoka, 2018]
- Reflected entropy [S. Dutta, T. Faulkner, 2019]

Reflected entropy $S_R(A:B)$

entanglement entropy of a canonical purification of ρ_{AB}

$$\rho_{AB}$$
$$=\frac{1}{2}|+\rangle_A|+\rangle_B\langle+|_A\langle+|_B$$
$$+\frac{1}{2}|-\rangle_A|-\rangle_B\langle-|_A\langle-|_B$$

$$|\sqrt{\rho_{AB}}\rangle \in \mathcal{H}_A\otimes\mathcal{H}_B\otimes\mathcal{H}_A^\star\otimes\mathcal{H}_B^\star$$
$$:=\frac{1}{\sqrt{2}}|+\rangle_A|+\rangle_B|+\rangle_{A^\star}|+\rangle_{B^\star}$$
$$+\frac{1}{\sqrt{2}}|-\rangle_A|-\rangle_B|-\rangle_{A^\star}|-\rangle_{B^\star}$$

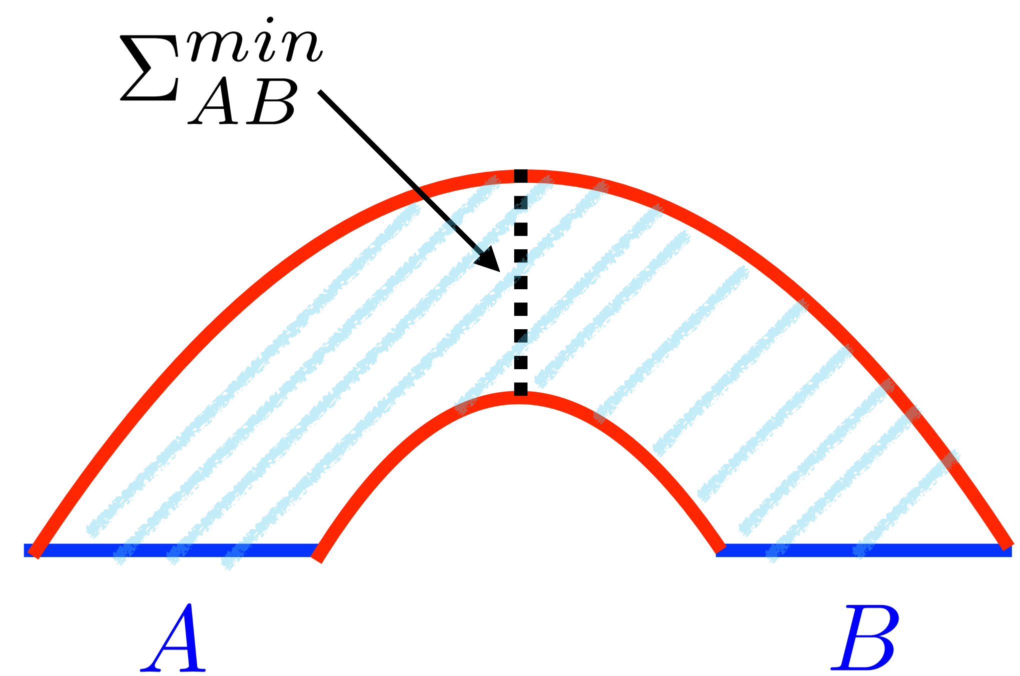
$$S_R(A:B) := -\mathrm{Tr}_{AA^\star}[\rho_{AA^\star}\log\rho_{AA^\star}]$$

$$\rho_{AA^\star} := \mathrm{Tr}_{BB^\star}|\sqrt{\rho_{AB}}\rangle\langle\sqrt{\rho_{AB}}|$$

Holographic duality

[S. Dutta, T. Faulkner, 2019]

$$S_R(A:B) = 2E_W(A:B)$$



Generalization by a replica index m

[S. Dutta, T. Faulkner, 2019]

$$S_{mR}(A:B) = 2E_{mW}(A:B)$$

$S_{mR}(A:B):$ reflected entropy with a canonical purification of ρ_{AB}^m

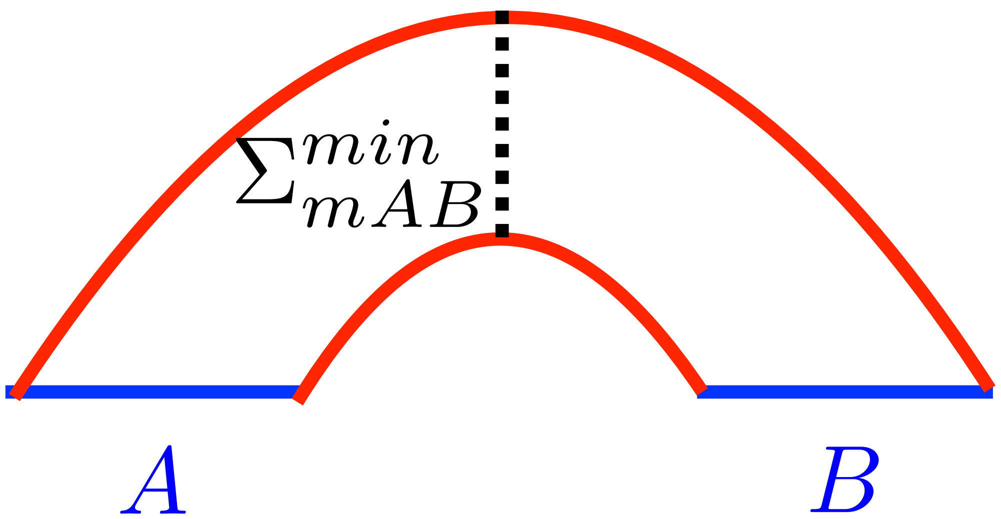
$E_{mW}(A:B):$ entanglement wedge cross section in the gravity dual of ρ_{AB}^m

Our motivation

We want to check and understand this relation by a simple example.

What we did

We explicitly check $S_{mR}(A:B) = 2E_{mW}(A:B)$ up to the first order in $m-1$.



$$S_{mR}(A:B) = \frac{c}{3}\log\left[\frac{1+\sqrt{z}}{1-\sqrt{z}}\right] - \frac{2c(m-1)}{3}\frac{\sqrt{z}\log z}{1-z} + \cdots$$

$\updownarrow$ exactly matched with $c = \frac{3}{2G_N}$

$$2E_{mW}(A:B) = \frac{1}{2G_N}\log\left[\frac{1+\sqrt{z}}{1-\sqrt{z}}\right] - \frac{m-1}{G_N}\frac{\sqrt{z}\log z}{1-z} + \cdots$$

Outline

1. Review of $S_R(A:B) = 2E_W(A:B)$ with thermal state and black hole

2. Our computation of $S_{mR}(A:B)$ and $E_{mW}(A:B)$ up to the first order in $m-1$

Thermofield double and two-sided AdS black hole

[J. M. Maldacena, 2001]

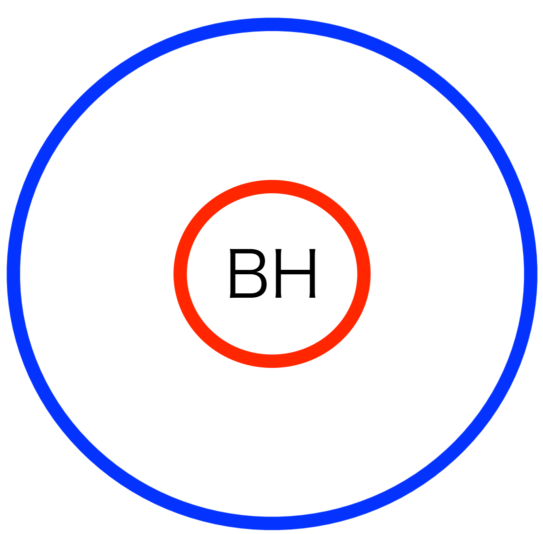
Thermal state on $\mathcal{H}_{AB}$

$$\rho_{AB} = \sum_n \frac{e^{-\beta E_n}}{Z} |n\rangle_{AB} \langle n|_{AB}$$

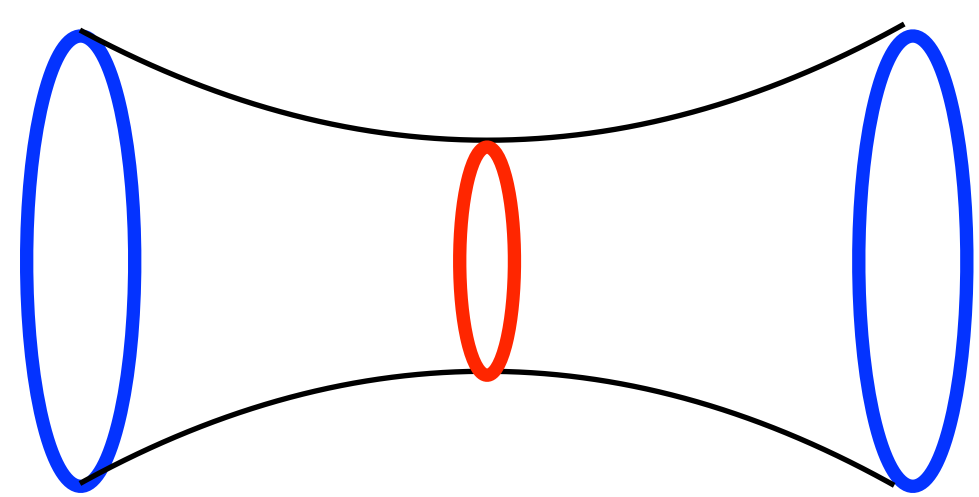
Thermofield double on $\mathcal{H}_{AB} \otimes \mathcal{H}^\star_{AB}$

$$|\sqrt{\rho_{AB}}\rangle = \sum_n \frac{e^{-\beta E_n/2}}{\sqrt{Z}} |n\rangle_{AB} |n\rangle_{A^\star B^\star}$$

one-sided AdS BH



two-sided AdS BH



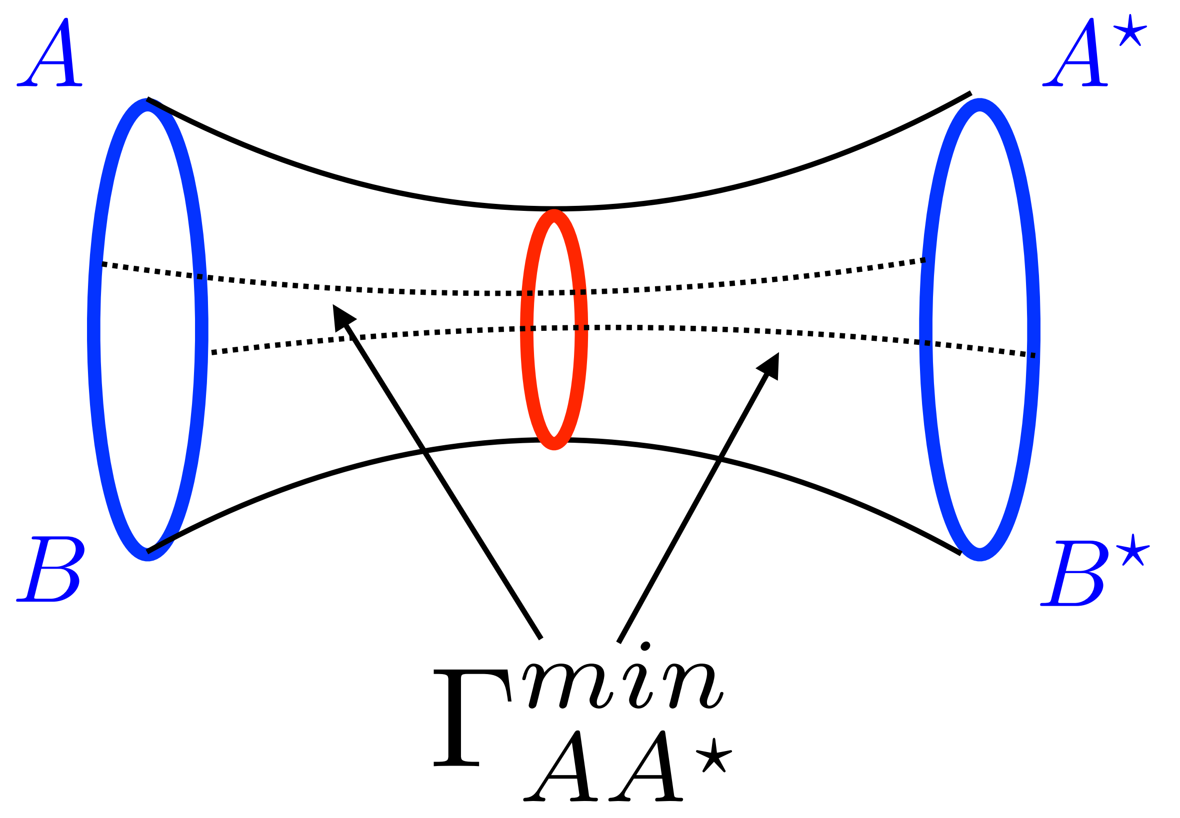
Reflected entropy of thermal state

= Entanglement entropy of thermofield double

= Holographic entanglement entropy in two-sided AdS black hole

$$S_R(A:B)$$
$$= S_{AA^\star}$$
$$= \frac{\mathrm{Area}[\Gamma^{min}_{AA^\star}]}{4G_N}$$

two-sided AdS BH

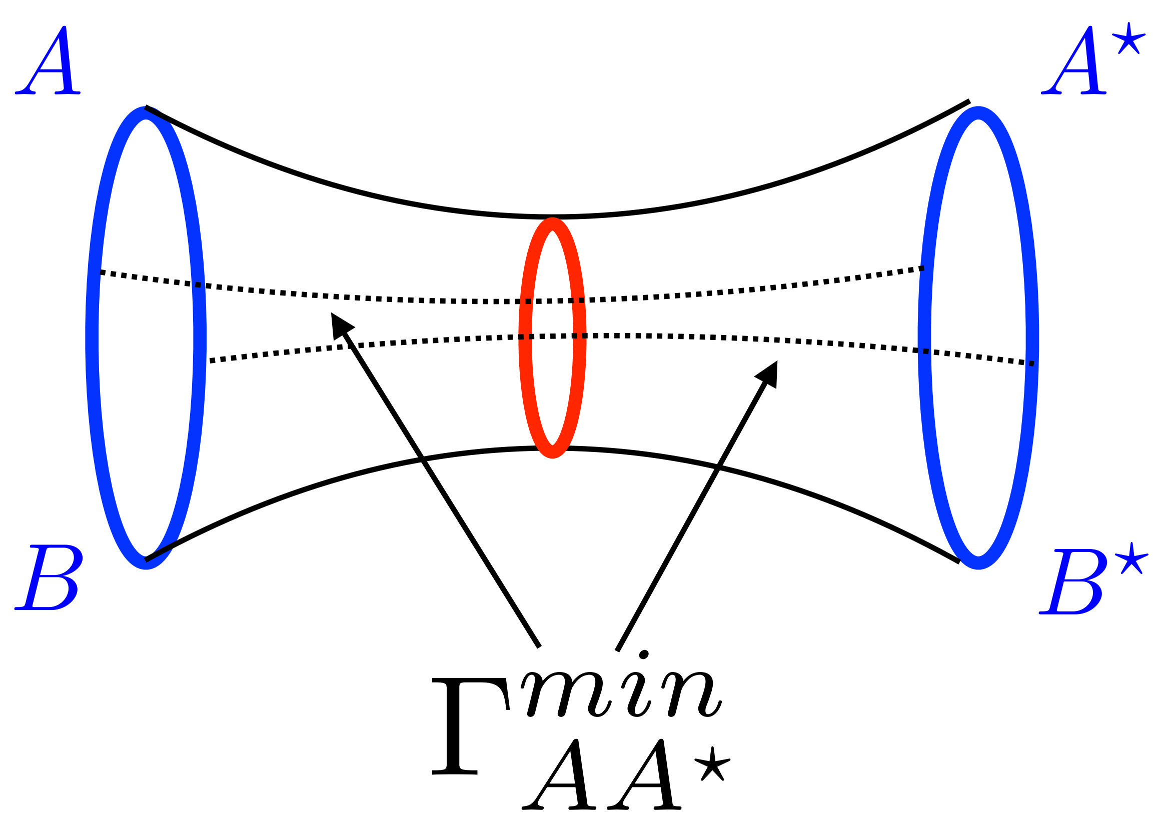


Holographic duality

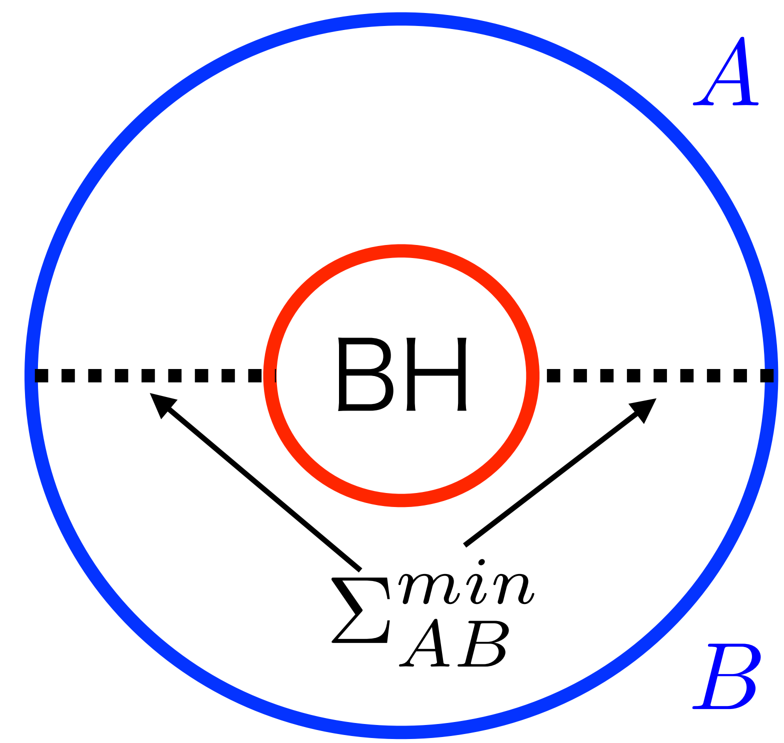
[S. Dutta, T. Faulkner, 2019]

$$S_R(A:B) = \frac{\text{Area}[\Gamma^{min}_{AA^\star}]}{4G_N} = 2\frac{\text{Area}[\Sigma^{min}_{AB}]}{4G_N} = 2E_W(A:B)$$

two-sided AdS BH



one-sided AdS BH



Outline

1. Review of $S_R(A:B) = 2E_W(A:B)$ with thermal state and black hole

2. Our computation of $S_{mR}(A:B)$ and $E_{mW}(A:B)$ up to the first order in $m-1$

Generalization by a replica index m

$$S_{mR}(A:B) = 2E_{mW}(A:B)$$

[S. Dutta, T. Faulkner, 2019]

$S_{mR}(A:B)$: reflected entropy with a canonical purification of ρ_{AB}^m

$E_{mW}(A:B)$: entanglement wedge cross section in the gravity dual of ρ_{AB}^m

Our motivation

We want to check and understand this relation
by a simple example
(the first order correction in $m-1$).

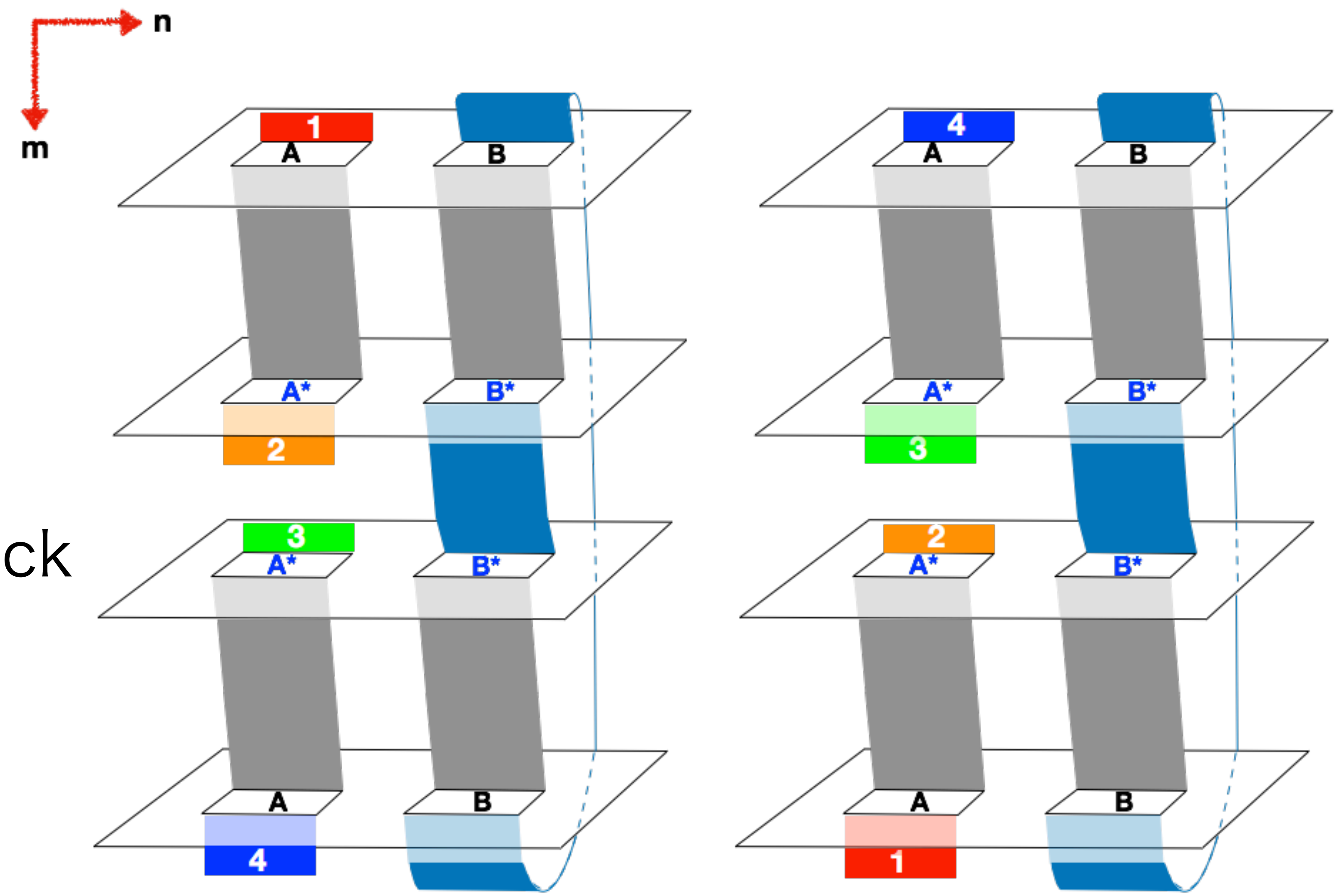
Computation of $S_{mR}(A:B)$

$$S_{mR}(A:B) = \lim_{n\to 1} \frac{1}{1-n} \log \frac{\left\langle \sigma_{g_A}(x_1)\sigma_{g_A^{-1}}(x_2)\sigma_{g_B}(x_3)\sigma_{g_B^{-1}}(x_4)\right\rangle_{CFT^{\otimes mn}}}{\left(\left\langle \sigma_{g_m}(x_1)\sigma_{g_m^{-1}}(x_2)\sigma_{g_m}(x_3)\sigma_{g_m^{-1}}(x_4)\right\rangle_{CFT^{\otimes m}}\right)^n}$$

- In the 2d holographic CFT, the four point function can be approximated by a single conformal block. [T. Hartman, 2013]

- We used a perturbative expansion of conformal block in $m-1$ and $n-1$. [A. L. Fitzpatrick, J. Kaplan, M. T. Walters, 2014]

Replica manifold $(m=4, n=2)$

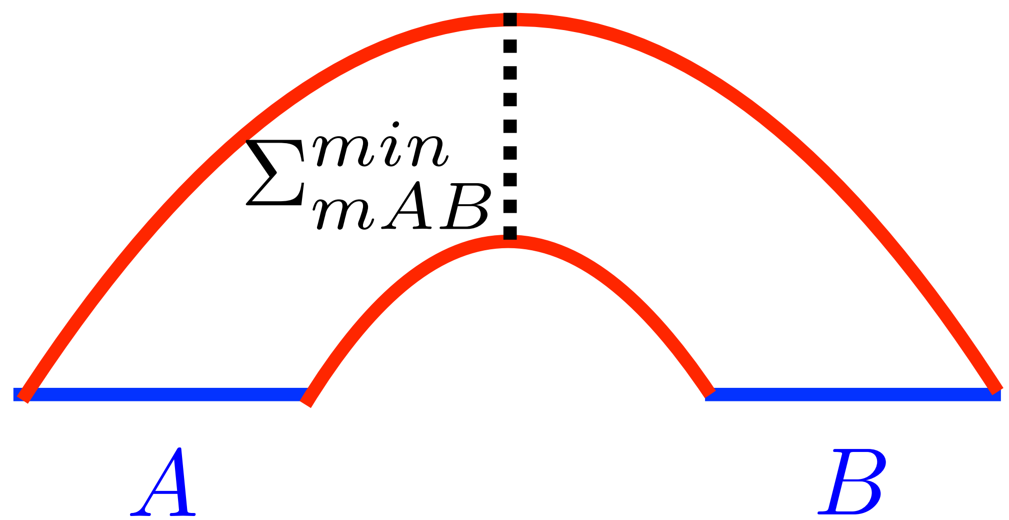


Computation of $E_{mW}(A:B)$

$$E_{mW}(A:B) = \frac{\text{Area}[\Sigma_{mAB}^{min}]}{4G_N}$$

entanglement wedge cross section in the gravity dual of ρ_{AB}^m

- The gravity dual of ρ_{AB}^m includes the backreaction from cosmic branes.
  [X. Dong, 2016]



Red surface are cosmic branes with tension T_m .

- At the first order in $m-1$, we can apply the computation method in [X. Dong, 2016] .

Check of $S_{mR}(A:B) = 2E_{mW}(A:B)$

reflected entropy with a canonical purification of ρ_{AB}^m

$$S_{mR}(A:B) = \frac{c}{3}\log\left[\frac{1+\sqrt{z}}{1-\sqrt{z}}\right] - \frac{2c(m-1)}{3}\frac{\sqrt{z}\log z}{1-z} + \cdots$$

$\updownarrow$ exactly matched with $c = \frac{3}{2G_N}$

$$2E_{mW}(A:B) = \frac{1}{2G_N}\log\left[\frac{1+\sqrt{z}}{1-\sqrt{z}}\right] - \frac{m-1}{G_N}\frac{\sqrt{z}\log z}{1-z} + \cdots$$

entanglement wedge cross section in the gravity dual of ρ_{AB}^m

Summary

$$S_{mR}(A:B) = 2E_{mW}(A:B)$$

- We study the holographic duality between reflected entropy and entanglement wedge cross section.

- We explicitly compute and check this duality up to the first order in $m-1$.