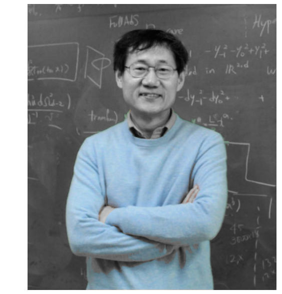


Based on [JY, 1906.08815]
[Y. Qi, S. Sin, JY, 1906.00996]

Fields and Strings 2019

Schwarzian Theory and Chaos

Junggi Yoon

October 28, 2019
NTHU, Taiwan

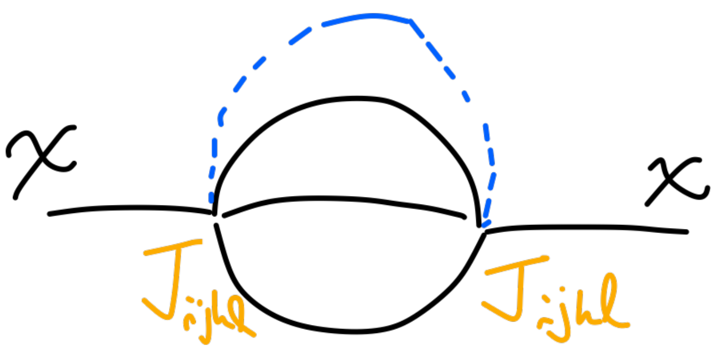
imagine **the impossible**

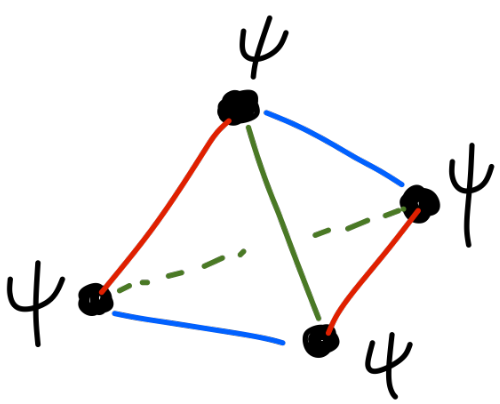
KOREA INSTITUTE FOR ADVANCED STUDY

“Universality” of Schwarzian Theory

$$S = \int d\tau \left(-\frac{c}{12}\mathrm{Sch}[\phi(\tau),\tau] - \frac{c}{24}[\phi'(\tau)]^2 \right)$$

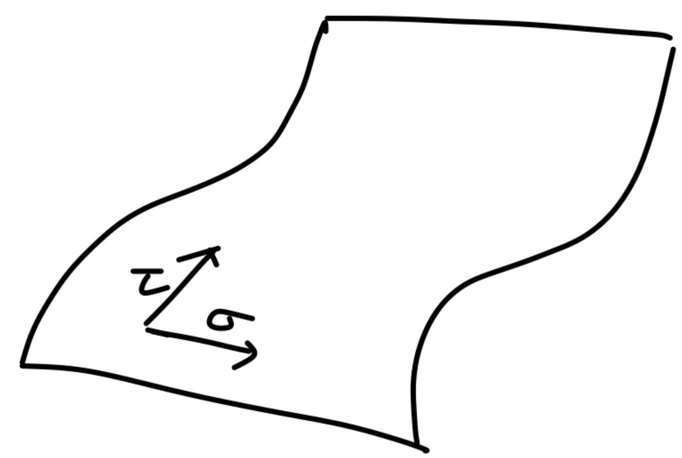
$$\mathrm{Sch}[\phi(\tau),\tau] \equiv \frac{\phi'''}{\phi'} - \frac{3}{2}\left(\frac{\phi''}{\phi'}\right)^2$$





✓ SYK(-like) models
[Maldacena, Stanford], …

✓ 2D JT gravity on the nearly-AdS2
[Maldacena, Stanford, Yang], …



✓ String worldsheet models
[de Boer, Llabres, Pedraza, Vegh]
[Murata],[Banerjee, Kundu, Poojary]

Large c Expansion

✳ Including measure in path integral [Witten, Stanford] $\left(\frac{1}{g^2} \sim c \right)$

$$\int \mu[f] \mathcal{D}f \, \mathcal{O}[f] \, e^{-S[f]}$$

measure

$$S = \frac{1}{2} \int_0^{2\pi} d\tau \left[\frac{1}{g^2} \left(\frac{\phi''}{\phi'} \right)^2 - \frac{1}{g^2} (\phi')^2 + \frac{\psi'' \psi'}{(\phi')^2} - \psi' \psi \right]$$

✳ Large c (=1/g²) expansion:

$$\phi(\tau) = \tau + g \, \epsilon(\tau) \longrightarrow S = -\frac{\pi}{g^2} + S^{(2)} + g S^{(3)} + g^2 S^{(4)} + \mathcal{O}(g^3)$$

$S^{cl} = -\frac{\pi}{g^2}$

saddle

fluctuation

Feynman Diagrams

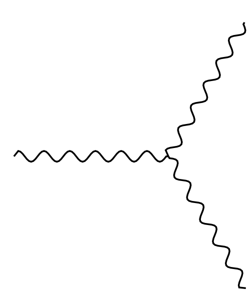
$$S = -\frac{\pi}{g^2} + S^{(2)} + gS^{(3)} + g^2 S^{(4)} + \mathcal{O}(g^3)$$

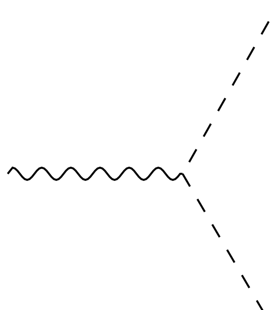
$\langle \epsilon_{-n} \epsilon_n \rangle_{\text{free}} =$ ϵ  ϵ

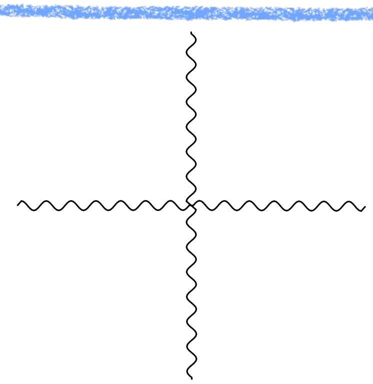
$$\sim \frac{1}{n^2(n^2-1)} \sim \mathcal{O}(g^0)$$

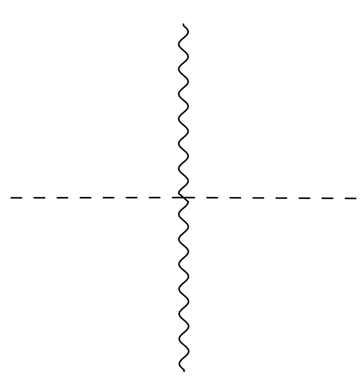
$\langle \psi_{-n} \psi_n \rangle_{\text{free}} =$ ψ ψ

$$\sim \frac{1}{n(n^2-1)} \sim \mathcal{O}(g^0)$$

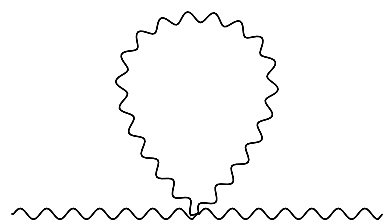
 $\sim \mathcal{O}(g^1)$

 $\sim \mathcal{O}(g^1)$

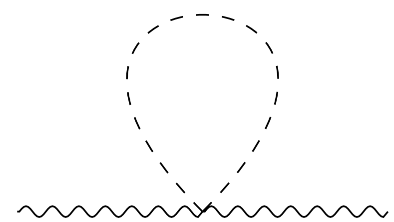
 $\sim \mathcal{O}(g^2)$

 $\sim \mathcal{O}(g^2)$

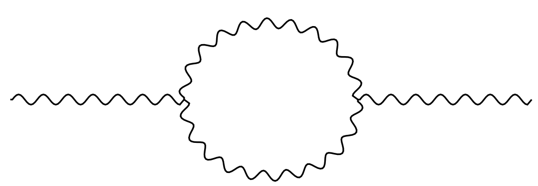
Cancellation of Divergences


diverge

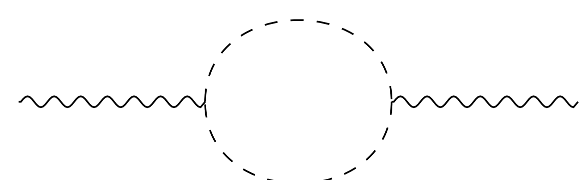
$+$


diverge

$=$ **finite**


diverge

$+$

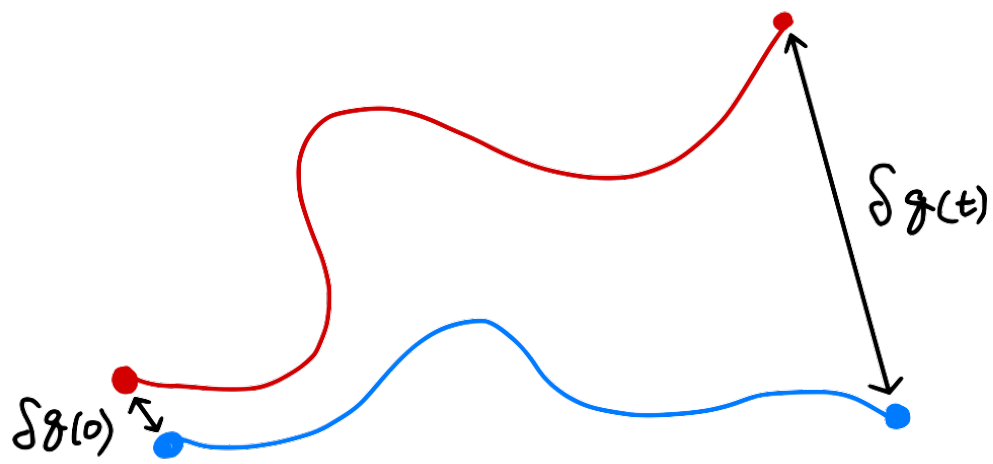

diverge

$=$ **finite**

Quantum Chaos

- ✴ (Classical) Butterfly Effect: The sensitivity of the system to the initial condition

$$\frac{\delta q(t)}{\delta q(0)} = \{q(t), p(0)\} \sim e^{\lambda t}$$



- ✴ Quantum version

$$\langle [V(t), W(0)]^2 \rangle_\beta \;\sim \langle V(t)W(0)V(t)W(0)\rangle_\beta \;-\; \langle V(t)V(t)W(0)W(0)\rangle_\beta$$

KIAS

Quantum Chaos

$$\langle [V(t), W(0)]^2 \rangle_\beta \sim \langle V(t)W(0)V(t)W(0)\rangle_\beta - \langle V(t)V(t)W(0)W(0)\rangle_\beta$$

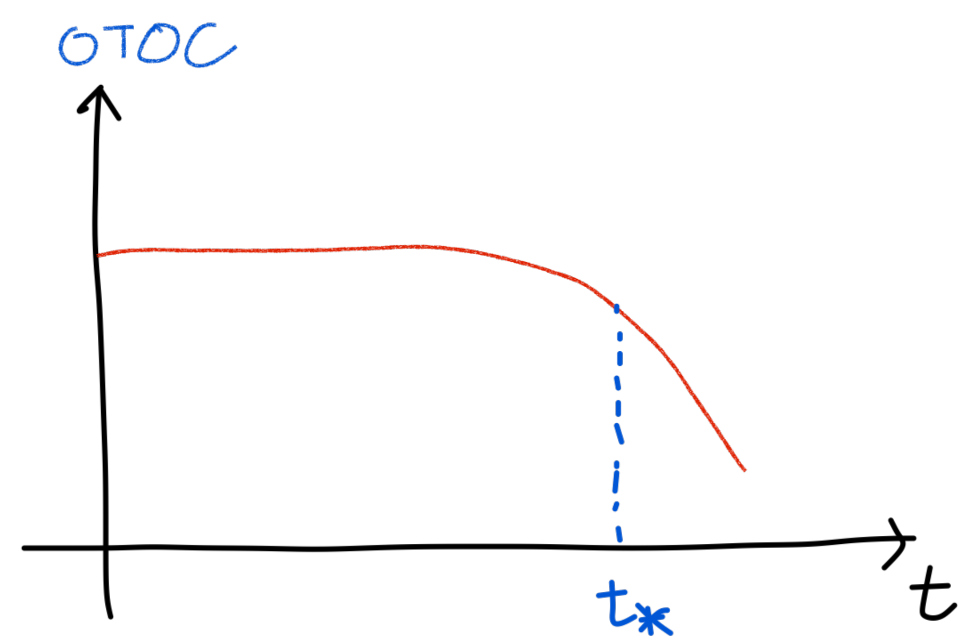
✳ The out-of-time-ordered correlator (OTOC)

$$\langle V(t)W(0)V(t)W(0)\rangle_\beta \sim 1 - \epsilon\, e^{\lambda t}$$

✓ Lyapunov exponent: λ

✓ Scrambling time $t_* \sim \frac{1}{\lambda} \log \frac{1}{\epsilon}$

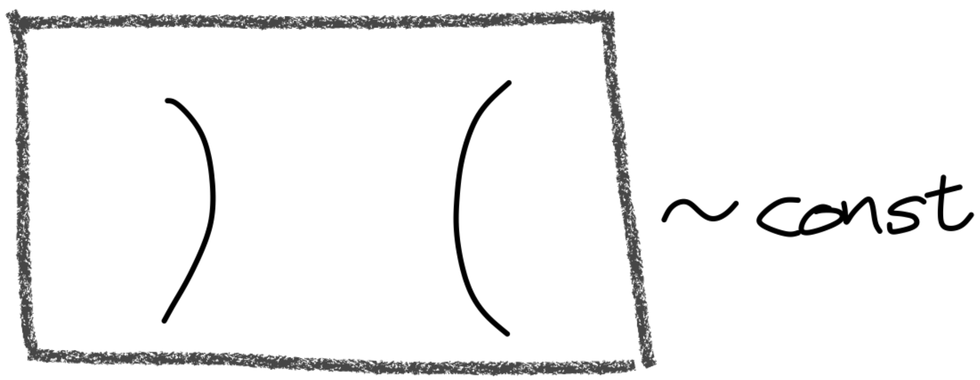
$\epsilon \sim 1/N$ (vector)
or $1/N^2$ (matrix)

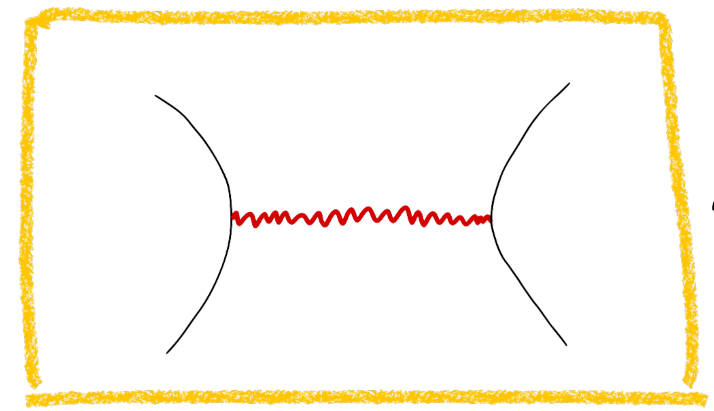


✳ Bound on Chaos [Maldacena, Shenker, Stanford, 1503.01409]

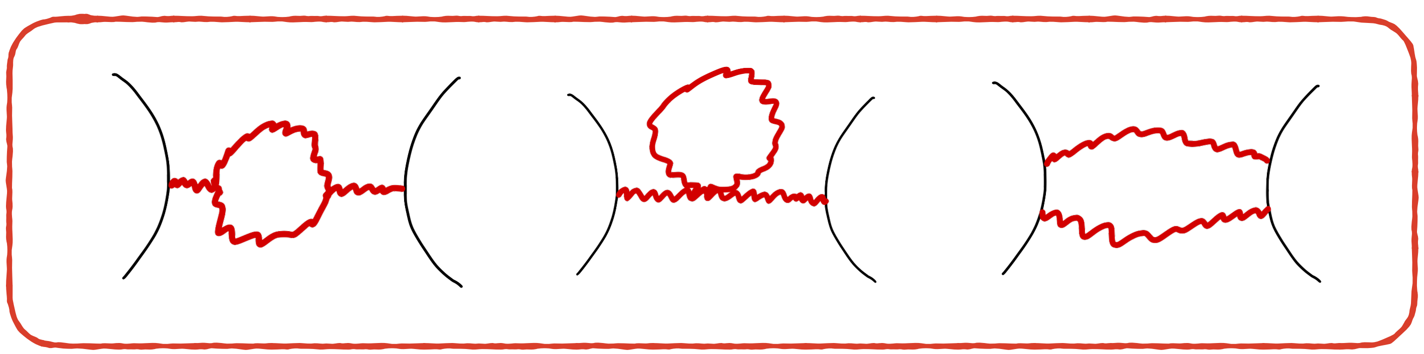
$$\lambda \leq \frac{2\pi}{\beta}$$

Out-of-time-ordered Correlator



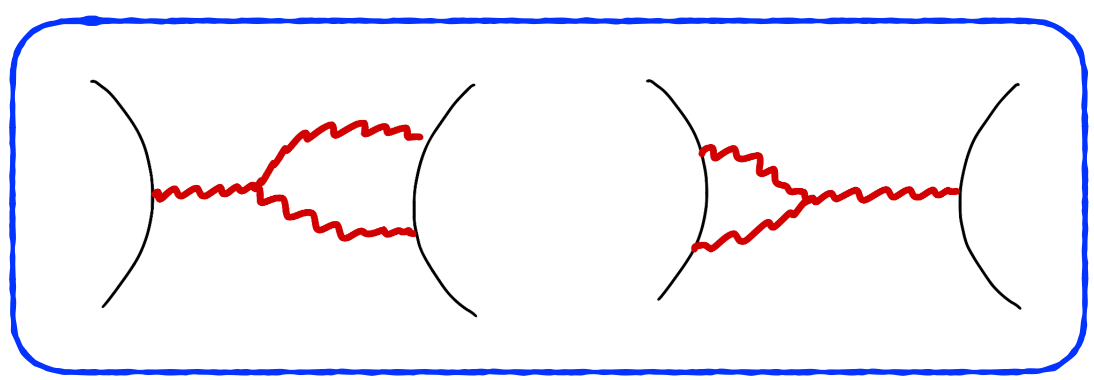


$$\sim \frac{1}{c} e^{\frac{2\pi t}{\beta}}$$



$$\sim \mathcal{O}(\frac{1}{c^2})$$

Speed up the exponential growth



$$\sim \mathcal{O}(\frac{1}{c^2})$$

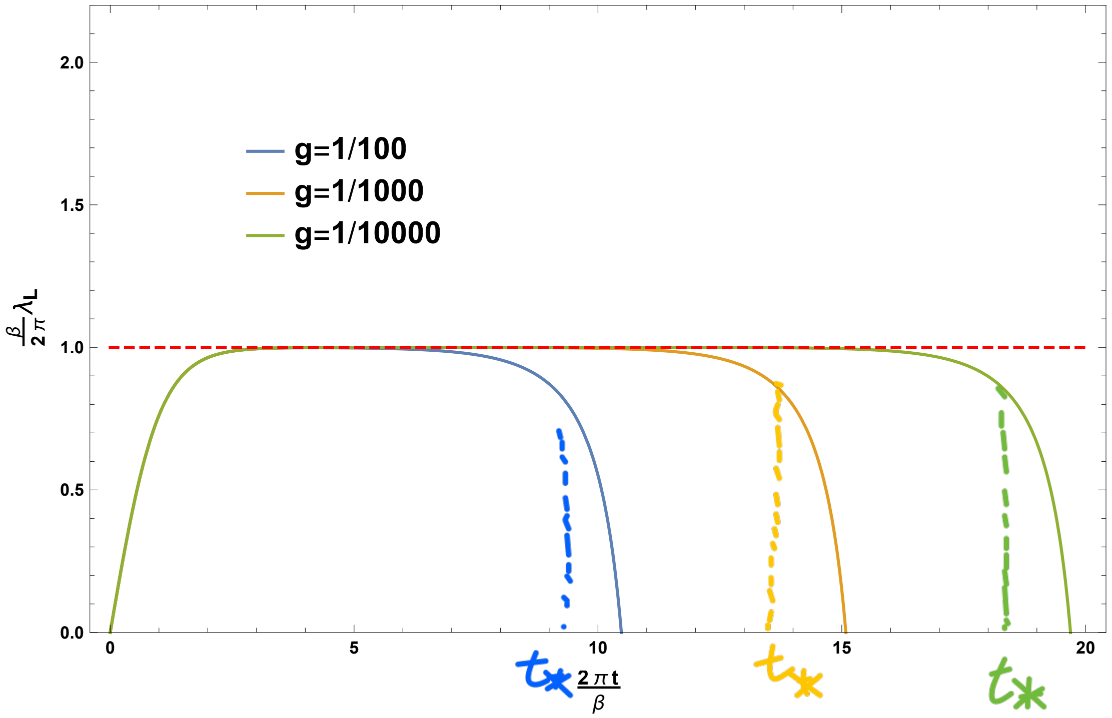
Slow down the exponential growth

Lyapunov Exponent

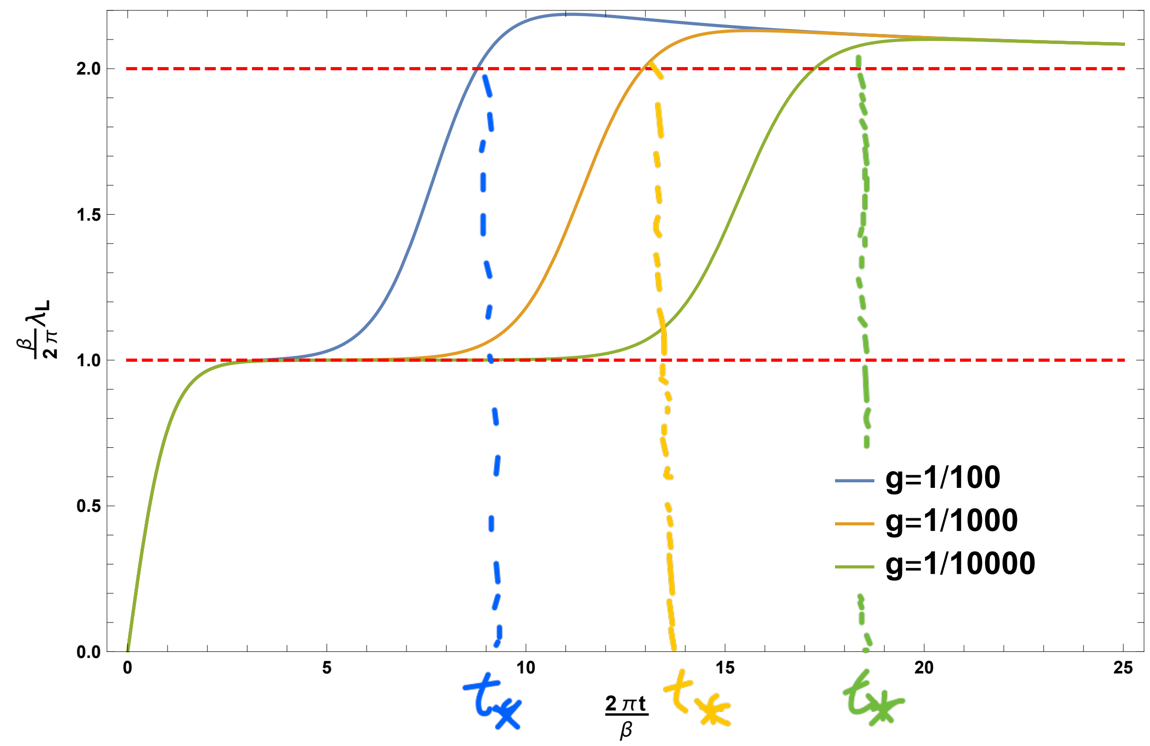
$$\lambda(t) = \frac{d}{dt}\log[\mathcal{F}_d - \mathcal{F}(t)]$$

const

OTOC



Total contribution



Loop contribution only

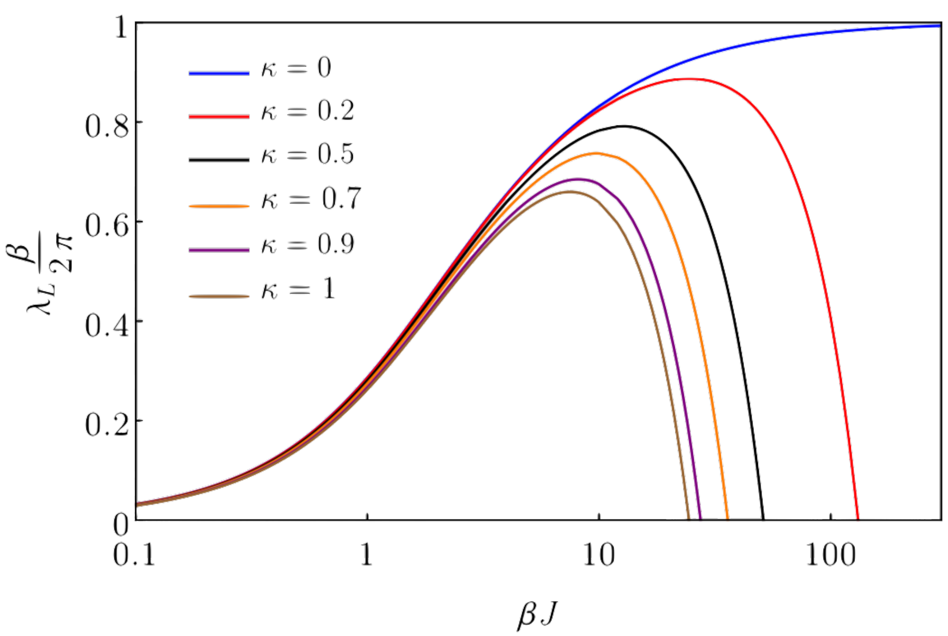
KIAS

Chaotic/Integrable Transition

[Banerjee, Altman], [Garcia-Garcia, Loureiro, Romero-Bermudez, Tezuka], [Nosaka, Rosa, JY], [Maldacena, Qi]

$$H = J_{ijkl}\chi_i\chi_j\chi_k\chi_l + i\kappa K_{kl}\chi_k\chi_l$$

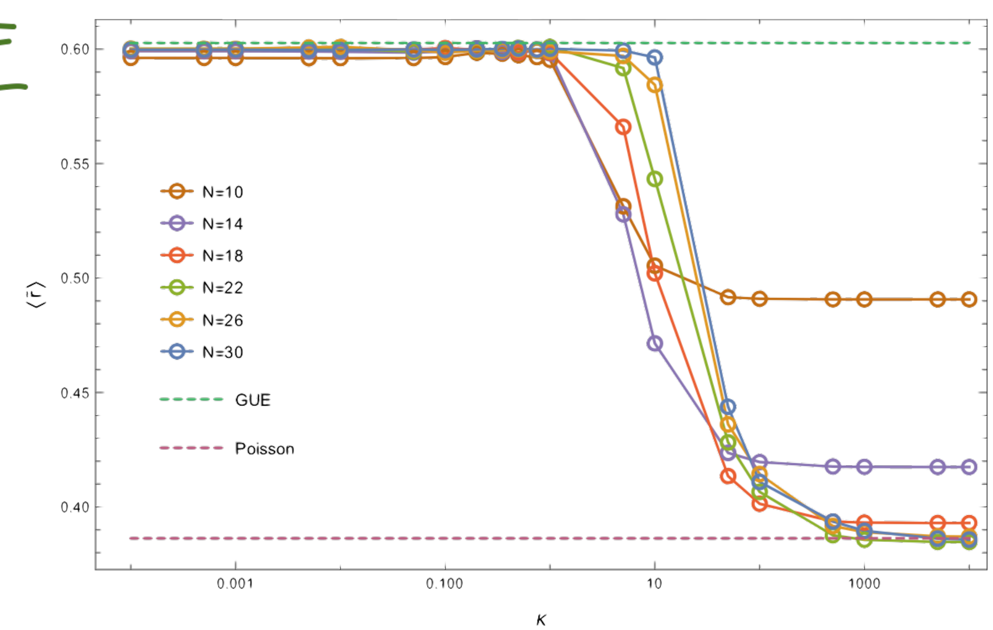
chaotic non-chaotic



Lyapunov Exponent

from [Garcia-Garcia, Loureiro, Romero-Bermudez, Tezuka]

GUE



Poisson

(averaged) r-parameter

[Nosaka, Rosa, JY]

$$\left\langle \frac{\min(E_{i+1}-E_i, E_{i+2}-E_{i+1})}{\max(E_{i+1}-E_i, E_{i+2}-E_{i+1})} \right\rangle$$

General Effective Action

- Reparametrization symmetry
  - Broken to SL(2)

$$S_{\text{eff}} = -\frac{c}{12}\int d\tau\, \text{Sch}\left[\tan\frac{\pi\phi(\tau)}{\beta}, \tau\right]$$

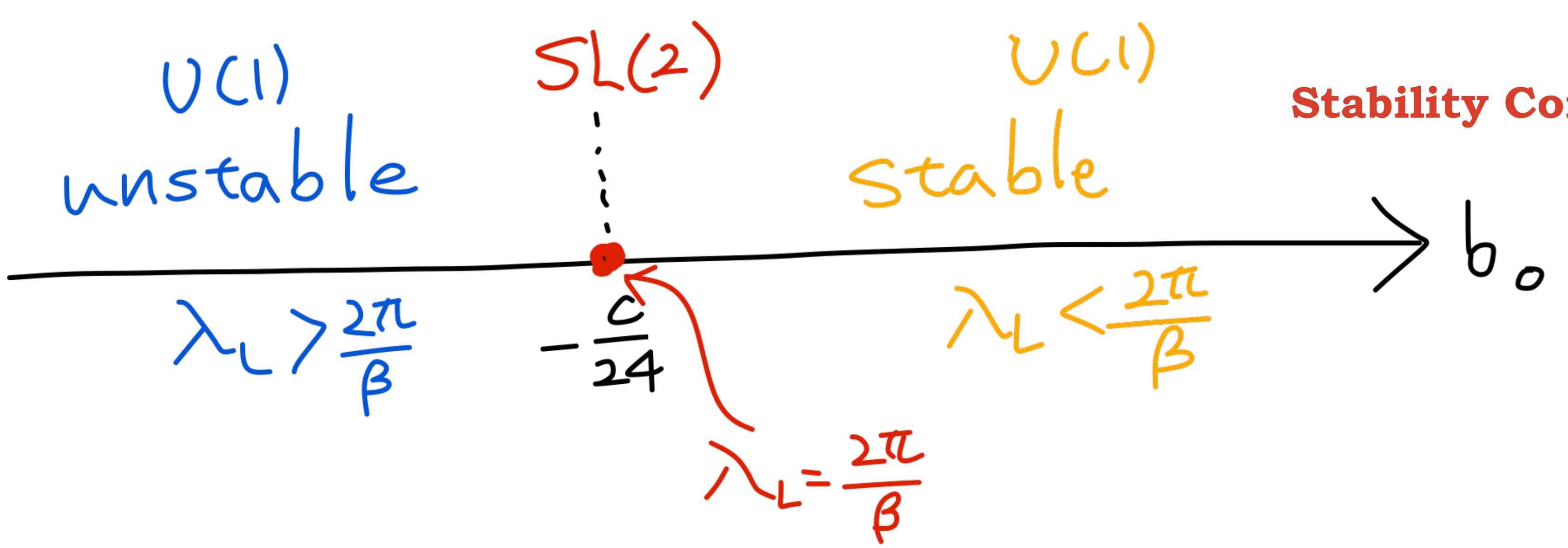
SL(2) invariant

  - Broken to U(1)

$$S_{\text{eff}} = \int d\tau \left(-\frac{c}{12}\text{Sch}\left[\tan\frac{\pi\phi(\tau)}{\beta}, \tau\right] + b_0'[\phi'(\tau)]^2\right)$$

SL(2) inv. U(1) inv.

- Lyapunov exponent: $\lambda_L = \frac{2\pi}{\beta}\sqrt{\frac{24}{c}|b_0|}$ $\left(b_0 = b_0' - \frac{c}{24}\right)$



Stability Condition [E. Witten, 1988]

Future Works

- Relation to 6j symbol of SL(2)
- Generalization to W_3 algebra [P. Narayan, JY, 1903.08761]
- Higher dimensional generalization
- Generalization to de Sitter space