Trapping Horizon and Negative Energy

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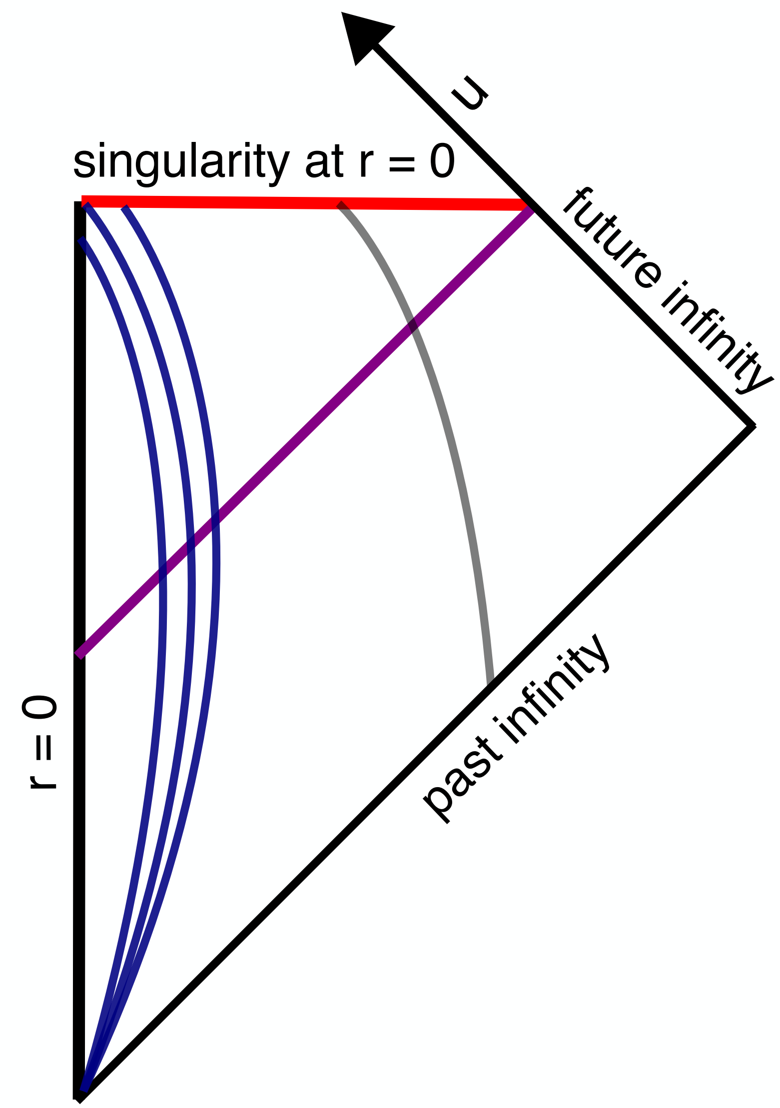
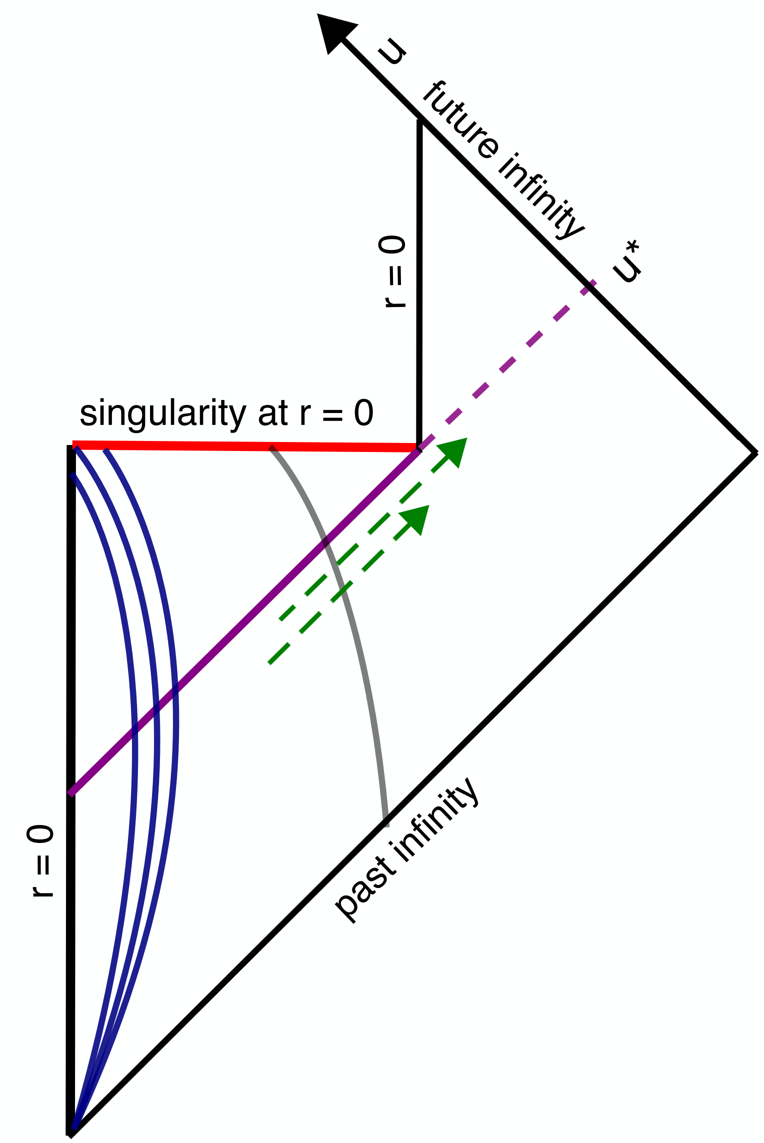
classical BH

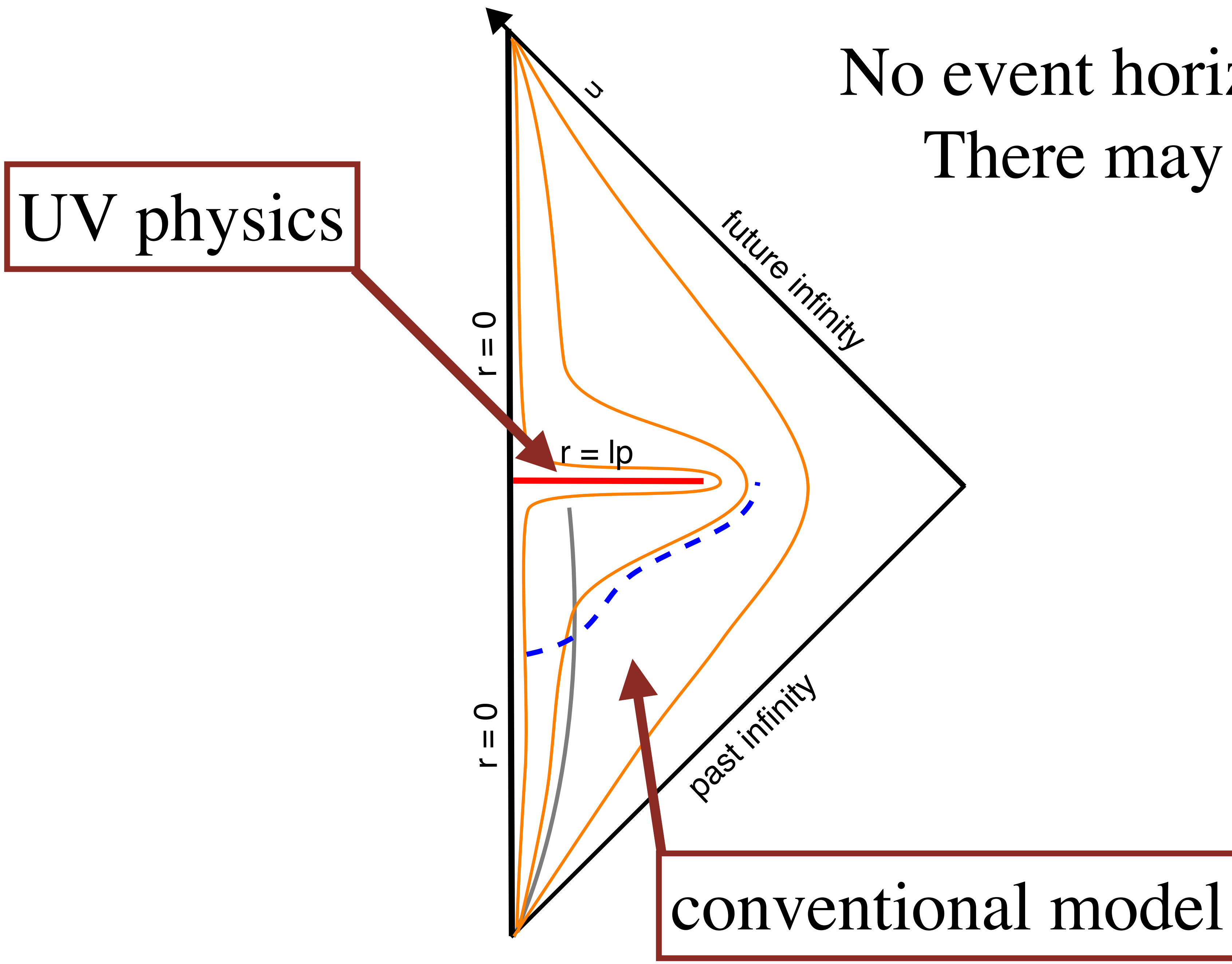


conventional model



assuming “no back-reaction” for Hawking radiation. [Hawking 1976]

Conventional model with singularity resolved



No event horizon if no singularity.
There may be trapping horizon.

Horizon means *trapping horizon*
or *apparent horizon* from now on.

Conventional Model

- A *static* black hole has balanced negative ingoing and outgoing vacuum energy fluxes at the horizon. → Null Energy Condition is violated.
- A *dynamical* black hole has Hawking radiation balanced with the negative outgoing vacuum energy flux at the horizon.
- Ingoing negative vacuum energy flux cancels mass.
- Claim: There is no high-energy event at the horizon. Low-energy effective theory holds. → paradox

Information Paradox

According to the conventional model:

- Matter collapses into horizon, black hole evaporates.
- Hawking radiation is created outside horizon.
- No chance to bring out the information of matter.
- Remnant?
- Information paradox is not just about information.
  (low-energy effective theories, black-hole thermodynamics, holography, …)

Information Paradox

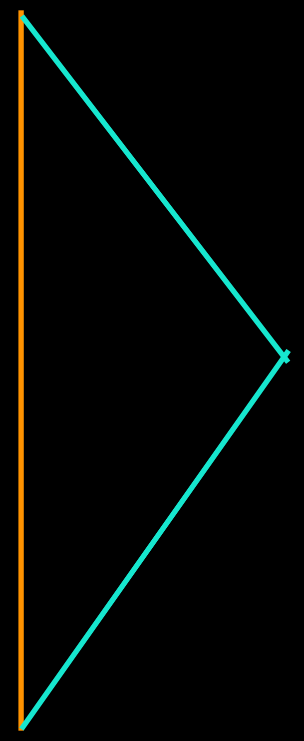
- Absence of high energy events => information loss
- Mathur:

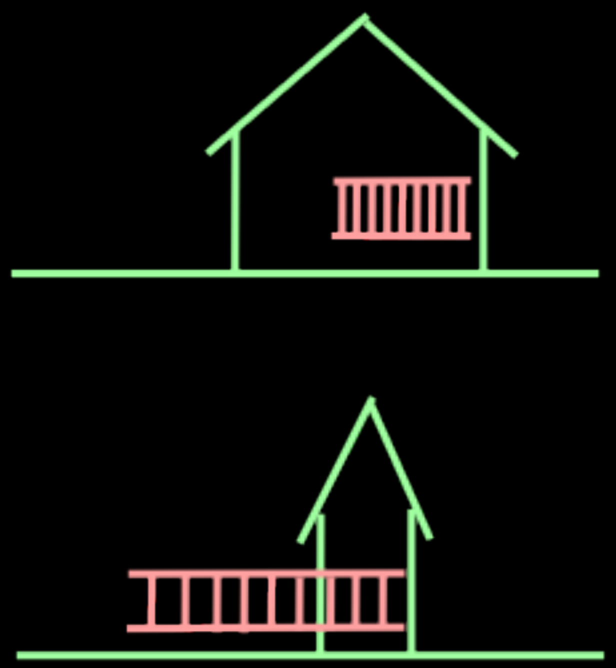
  "Niceness conditions" must be violated. (e.g. firewall)

  O(1) correction needed at horizon → fuzzball
- Where are the high energy events?

Two Types of Paradoxes

- Twin's paradox
- Barn and Ladder Paradox
- Ehrenfest Paradox





The Task

- What is really the geometry in time evolution according to semi-classical Einstein equations (with back reaction of vacuum energy)?

$$G_{\mu\nu} = \kappa T^{class}_{\mu\nu} + \kappa \langle \hat{T}_{\mu\nu} \rangle$$

- High energy events?

Basic Assumptions

- Semi-classical Einstein equation:

$$G_{\mu\nu} = \kappa T^{class}_{\mu\nu} + \kappa \langle \hat{T}_{\mu\nu} \rangle$$

- Spherical symmetry

$$ds^2 = -C(u,v)dudv + r^2(u,v)d\Omega^2$$

areal radius

- 1. Static black hole?

  2. Dynamic black-hole (formation/evaporation)?

What’s wrong with naive perturbation

- Perturbative expansion in κ is different when

$r \sim a + O(\kappa/a)$ $\qquad (\kappa \equiv 8\pi G_N, \quad \hbar = c = 1)$

$$G_{\mu\nu} = \kappa \langle \hat{T}_{\mu\nu} \rangle$$

$$ds^2 = -\left(1 - \frac{a}{r}\right) dt^2 + \frac{dr^2}{1 - a/r} + d\Omega^2$$

- Schwarzschild metric is only a good approximation at a few Planck lengths away from the Schwarzschild radius (in vacuum).

Static Black Holes

- The energy-momentum operator $\langle T_{\mu\nu} \rangle$ in curved spacetime is different for different QFTs.
- Conformal matters are convenient because of trace anomaly
- 2D massless field [Davies-Fulling-Unruh 1976][PMH-Matsuo 17 (1)][PMH-Matsuo 17 (2)]
- 4D conformal matter [Christensen-Fulling 1977][PMH-Kawai-Matsuo-Yokokura 18]
- Literature [Solodukhin 04, 06; Fabbri-Farese-Navarro-Salas-Olmo-Sanchis-Alepuz 05 (1), 05 (2)]

Static Black Holes

[Ho-Kawai-Matsuo-Yokokura, JHEP1811]

Schwarzschild background

$$ds^2 = -\left(1 - \frac{a}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{a}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\to -e^{\rho(r)}\left[dt^2 - \frac{dr^2}{F(r)}\right] + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,.$$

$$G_{\mu\nu} = \kappa\langle T_{\mu\nu}\rangle \qquad \kappa \equiv 8\pi G_{Newton}$$

$$\nabla^\mu\langle T_{\mu\nu}\rangle = 0 \qquad \begin{aligned} \rho(r) &= \rho_0(r) + \rho_1(r) + \cdots, \\ F(r) &= F_0(r) + F_1(r) + \cdots\,. \end{aligned}$$

The most general conserved energy-momentum tensor with spherical symmetry for static states:

$$\langle T^t{}_t \rangle = -\frac{1}{r(r-a)}[q + H(r) + G(r)] + \frac{1}{2}\langle T^\mu{}_\mu \rangle - 2\Theta(r),$$

$$\langle T^r{}_r \rangle = \frac{1}{r(r-a)}[q + H(r) + G(r)],$$

$$\langle T^\theta{}_\theta \rangle = \Theta(r) + \frac{1}{4}\langle T^\mu{}_\mu \rangle = \langle T^\phi{}_\phi \rangle \,.$$

$$H(r) \equiv \frac{1}{2}\int_a^r dr' \, \left(r' - \frac{a}{2}\right)\langle T^\mu{}_\mu(r') \rangle,$$

$$G(r) \equiv 2\int_a^r dr' \, \left(r' - \frac{3a}{2}\right)\Theta(r') \,.$$

$$\Theta(r) \equiv \langle T^\theta{}_\theta(r) \rangle - \frac{1}{4}\langle T^\mu{}_\mu \rangle \,.$$

3 Classes of Geometries

[PMH-Kawai-Matsuo-Yokokura 18]

- Wormhole-like neck $q < 0$

  Local minimum in *r* occurs at $r > a$ (*No event horizon.*)

- Event horizon $q = 0$

  Equivalent to a shift of Schwarzschild radius a. (*fine tuning*)

- No neck, no horizon $q > 0$

  Perturbation theory breaks down when $r - a \ll \kappa |q| a$

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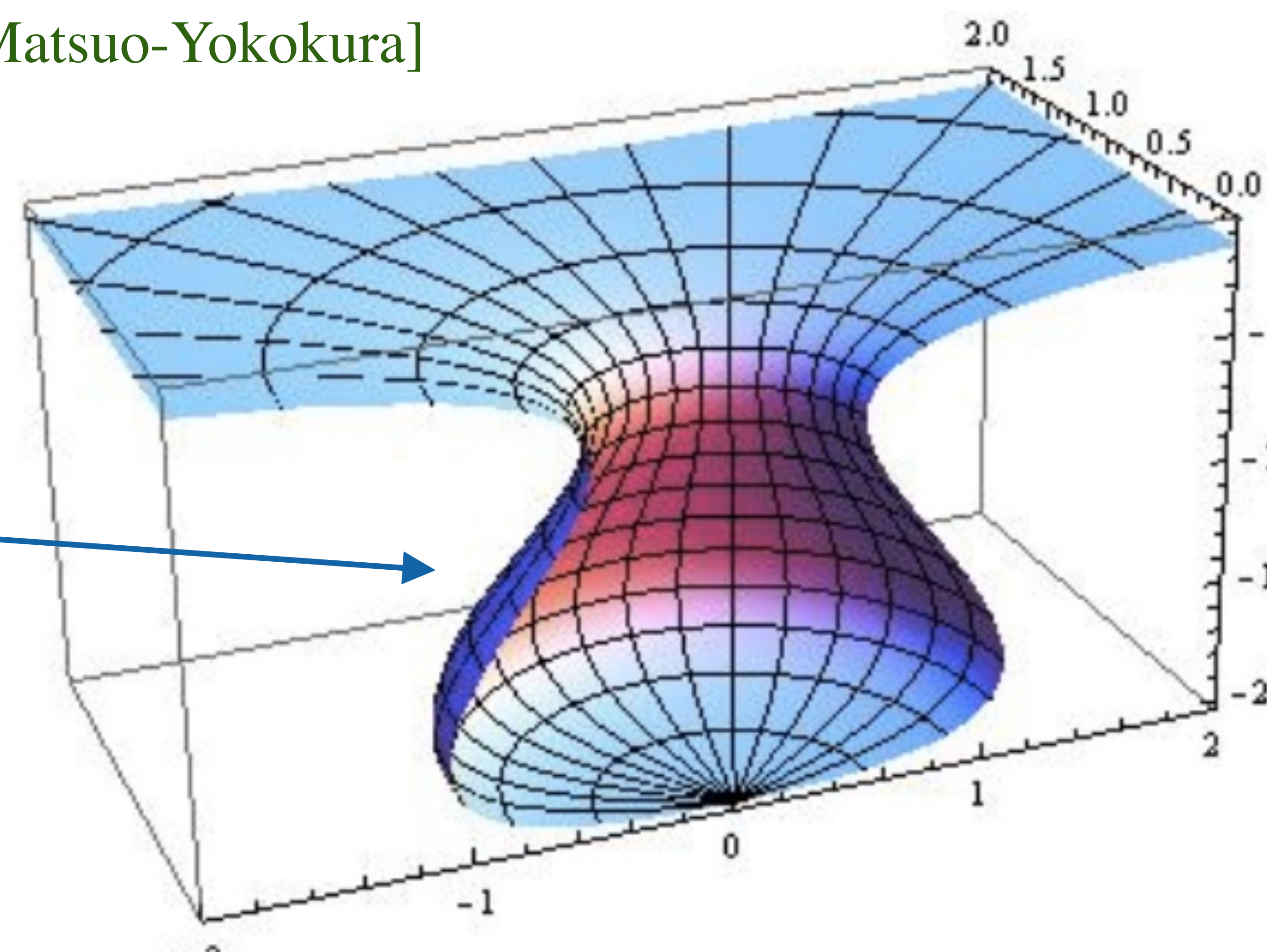
Wormhole-Like Solution: local minimum in areal radius

- vacuum energy + incompressible fluid
- No horizon
- *Large pressure* when matter resides inside the neck

[PMH-Matsuo 17 (1), (2), PMH-Kawai-Matsuo-Yokokura]

Radial distance is short



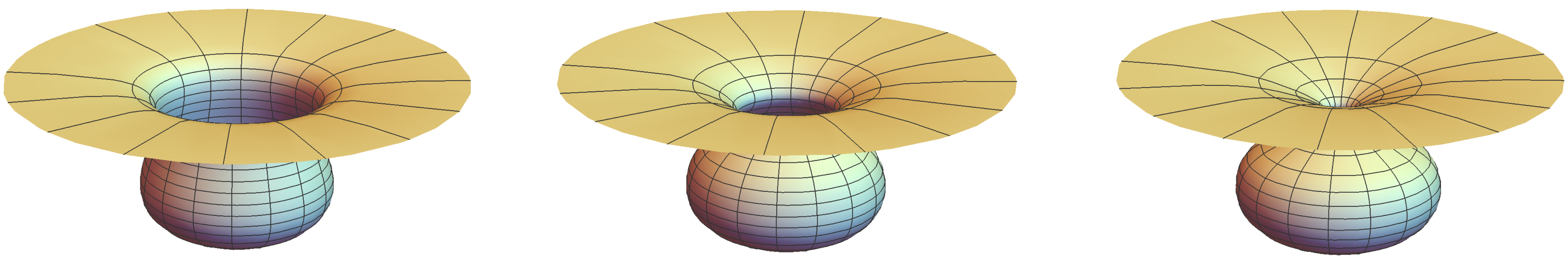
Dynamical Cases:
3 Scenarios

- apparent horizon: collapsing wormhole evaporated
  [PMH-Matsuo 18, PMH-Matsuo-Yang 19, PMH-Matsuo 19]
- apparent horizon: collapsing wormhole (nearly) decapitated [Parentani-Piran 1994]
- no apparent horizon: KMY Model
  [Kawai-Matsuo-Yokokura 13][Kawai-Yokokura 14, 15, 17]
- Different scenarios happen for different vacuum EMT.

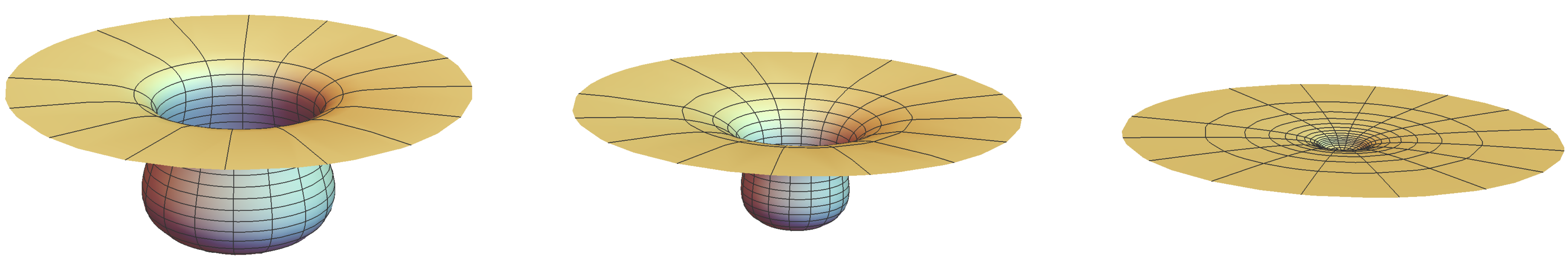
Black-Hole Geometry

[Ho-Matsuo, JHEP 1807]

Wheeler's Bag of Gold as Remnant



Everything Evaporated



Trapping Horizon

[Ho-Matsuo, JHEP 1807]

Spherical Symmetry

areal radius

$$ds^2 = -C(u,v)dudv + r^2(u,v)(d\theta^2 + \sin^2\theta d\phi^2)$$

Trapping Horizon

Minkowski space

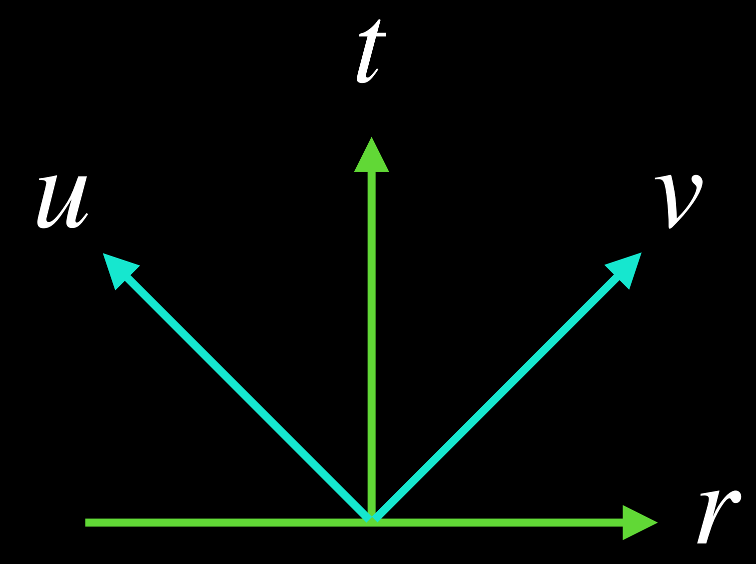
$$\partial_u r < 0, \quad \partial_v r > 0$$

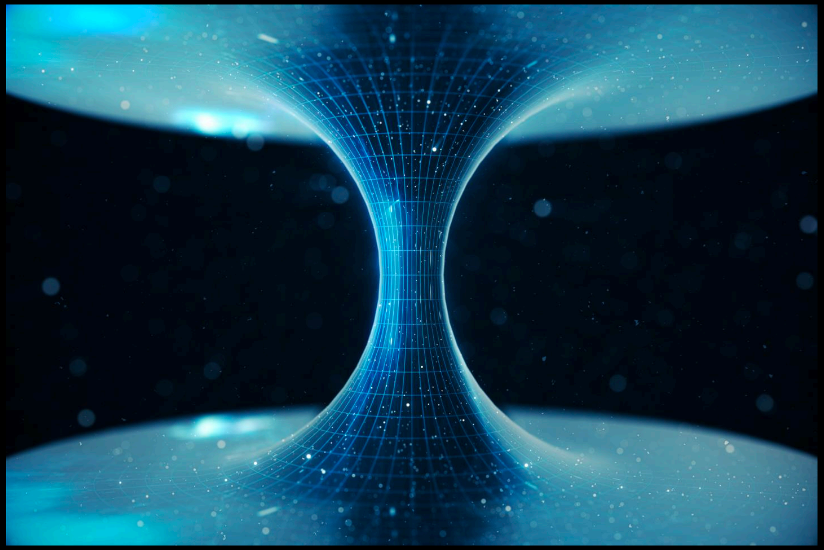
Trapped region

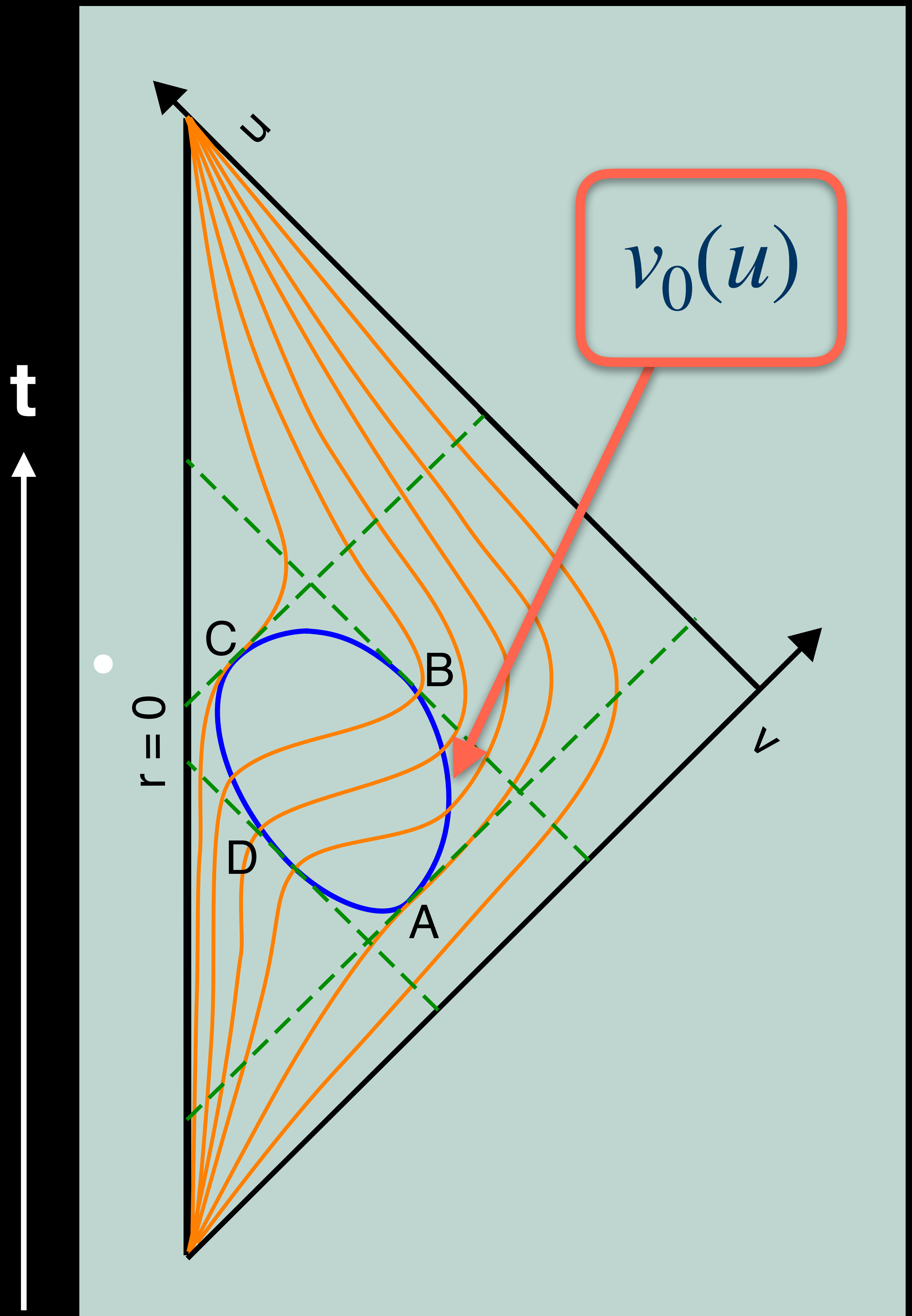
$$\partial_u r < 0, \quad \partial_v r < 0$$

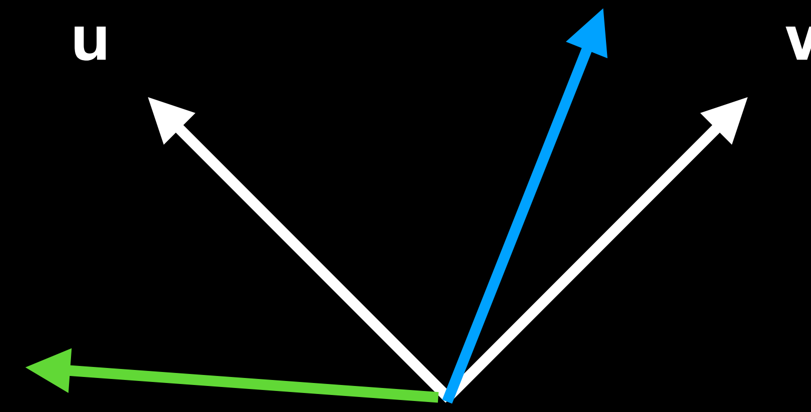
Trapping horizon

$$\partial_v r = 0, \quad (\partial_v^2 r > 0, \quad \partial_v^2 r < 0?)$$









- Time-like => vacuum

$$\frac{dv_0}{du} > 0 \quad \Rightarrow \quad T_{vv} < 0$$

- Space-like => matter

$$\frac{dv_0}{du} < 0 \quad \Rightarrow \quad T_{vv} > 0$$

Semi-Classical Einstein Equation

$$G_{uu} \equiv \frac{2\partial_u C \partial_u r}{Cr} - \frac{2\partial_u^2 r}{r} = \kappa T_{uu}$$

$$G_{vv} \equiv \frac{2\partial_v C \partial_v r}{Cr} - \frac{2\partial_v^2 r}{r} = \kappa T_{vv}$$

$$G_{uv} \equiv \frac{C}{2r^2} + \frac{2\partial_u r \partial_v r}{r^2} + \frac{2\partial_u \partial_v r}{r} = \kappa T_{uv}$$

$$G_{\theta\theta} \equiv \frac{2r^2}{C^3}\left(\partial_u C \partial_v C - C \partial_u \partial_v C\right) - \frac{4r}{C}\partial_u \partial_v r = \kappa T_{\theta\theta}$$

Semi-Classical Einstein Equation

$\partial_v r = 0$
@ trapping horizon

$$G_{uu} \equiv \frac{2\partial_u C \partial_u r}{Cr} - \frac{2\partial_u^2 r}{r} = \kappa T_{uu}$$

$$G_{vv} \equiv \frac{2\partial_v C \partial_v r}{Cr} - \frac{2\partial_v^2 r}{r} = \kappa T_{vv}$$

$$G_{uv} \equiv \frac{C}{2r^2} + \frac{2\partial_u r \partial_v r}{r^2} + \frac{2\partial_u \partial_v r}{r} = \kappa T_{uv}$$

$$G_{\theta\theta} \equiv \frac{2r^2}{C^3}\left(\partial_u C \partial_v C - C \partial_u \partial_v C\right) - \frac{4r}{C}\partial_u \partial_v r = \kappa T_{\theta\theta}$$

Energy-Momentum Tensor

around Trapping Horizon at $v = v_0(u)$

$$C(u,v) = C_0(u) + C_1(u)(v - v_0(u)) + \cdots$$

$$r(u,v) = a(u) + \frac{1}{2} r_2(u)(v - v_0(u))^2 + \cdots$$

Semi-classical Einstein equation ⇒

$$r_2(u) = -\frac{1}{2}\ell_p^2 a(0) T_{vv}^{(0)}(u)$$

$$\dot{v}_0(u) = \frac{C_0(u) - 2\ell_p^2 a^2(u) T_{uv}^{(0)}}{-2\ell_p^2 a^2(u) T_{vv}^{(0)}}$$

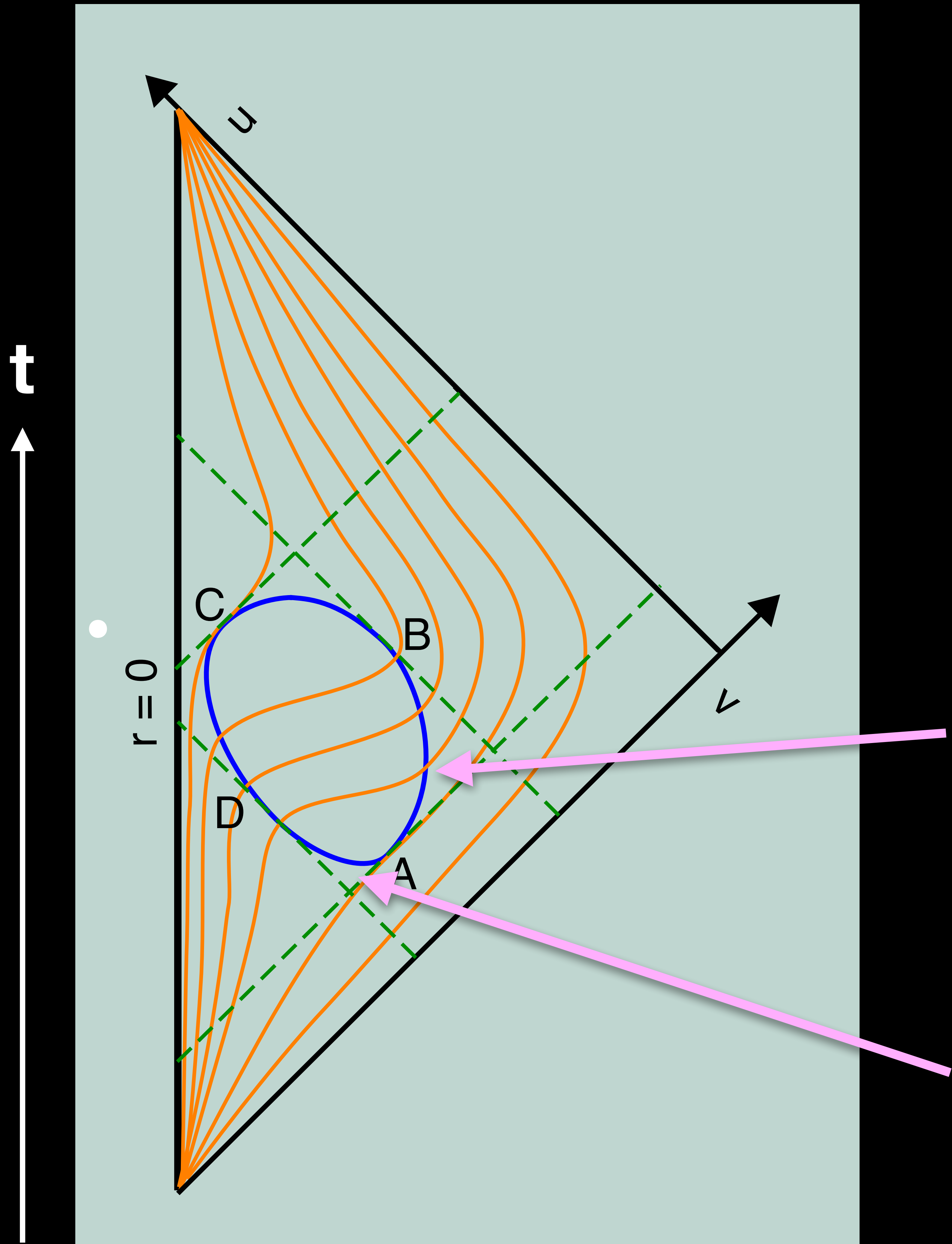
$$\ell_p^2 \equiv 8\pi G_N$$

$$\hbar = c = 1$$

Trapping Horizon in vacuum demands negative Tvv.

Trapping Horizon

Apparent Horizon



Outer Trapping Horizon in Vacuum

Outer Trapping Horizon in Matter

"No Drama" at Horizon

$$ds^2 = -C(u,v)dudv + r^2(u,v)(d\theta^2 + \sin^2\theta d\phi^2)$$

Residual gauge symmetry

$$u \to u'(u) \qquad v \to v'(v)$$

Gauge invariant quantities

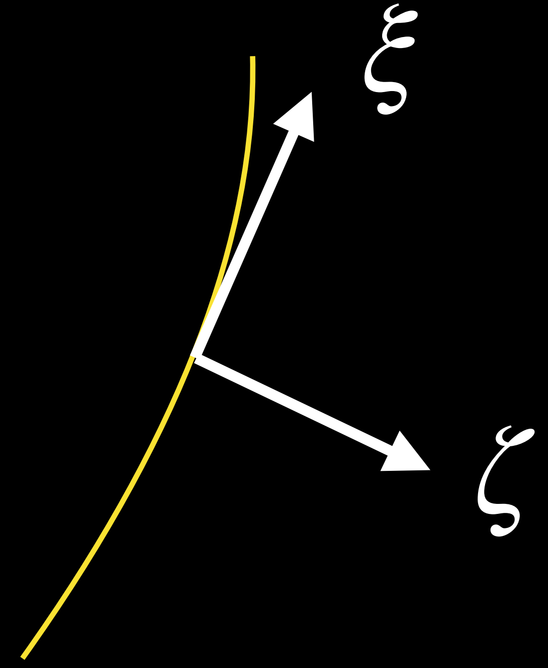
$$|R_{\theta\theta}| \sim \mathcal{O}\left(\frac{\ell_p^2}{a^2(u)}\right) \ll 1$$

$$det(R^{(2D)}) \sim \mathcal{O}\left(\frac{\ell_p^2}{a^6(u)}\right) \ll \mathcal{O}\left(\frac{1}{a^4(u)}\right)$$

EMT @ Outer Trapping Horizon in Vacuum

[Ho-Matsuo, JHEP 1807]

$$\mathscr{E} \equiv T_{\xi\xi} \simeq -T_{\xi\zeta} \simeq T_{\zeta\zeta}$$



$$\mathscr{P} \equiv 4\pi a^2(u)\mathscr{E}$$

Universal result:

$$\mathscr{E} \simeq -\frac{1}{2\ell_p^2 a^2}$$

$$\mathscr{P} \simeq -\frac{2\pi}{\ell_p^2}$$

$$\ell_p^2 \equiv 8\pi G_N$$

$$\hbar = c = 1$$

Black Hole's mass decreases because of this,
not Hawking Radiation.

Large Quantum Correction?

- Schwarzschild solution:

$$T_{\mu\nu} = 0 \quad \leftrightarrow \quad R_{\mu\nu} = 0$$

- Quantum correction:

$$\mathscr{E} \equiv T_{\mu\nu}\xi^{\mu}\xi^{\nu} \simeq -\frac{1}{2\ell_p^2 a^2(u)} \longrightarrow -\infty \qquad (\hbar \to 0)$$

- Non-perturbative gauge-invariant geometric quantity

- Gravitational effect of such a large vacuum energy?

Outlook

- O(1) correction at trapping horizon
- Not completely local, as Hawking radiation.
- Relevance in low-energy effective action?

Thank you!