Symmetry Breaking and Phase Transition in $Z_2 \times Z_2$ Invariant Two Scalar Model

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Based on JH, H Kawai arXiv:1905.05656 [hep-th],
and works in progress with H Kawai& K Sakai

Introduction

- Phase structure of phi fourth theory is well-known.

$$V = m^2\phi^2 + \lambda\phi^4$$

… broken/unbroken phases depend on the sign of m^2.

- Phase structure of two scalar theory is unknown.

$$V = m_1^2\phi^2 + m_2^2\psi^2 + \rho\phi^4 + \rho'\psi^4 + \kappa\phi^2\psi^2$$

- In 4 dim, we found that in the massless case one scalar field have a vacuum expectiation value:

$$V = \rho\phi^4 + \rho'\psi^4 + \kappa\phi^2\psi^2 \Rightarrow \langle\phi\rangle \neq 0, \langle\psi\rangle = 0$$

- This implies two scalar theory has non-trivial phase structure.

Introduction

- We study $O(N) \times O(N)$ scalar model in 3 or 4 dimensions:

$$\mathcal{L} = N\left[\frac{1}{2}(\partial_\mu \phi_i)^2 + \frac{1}{2}(\partial_\mu \psi_i)^2 + \frac{m_0^2}{2}\phi_i^2 + \frac{m'^2_0}{2}\psi_i^2 + \frac{\rho_0}{8}(\phi_i^2)^2 + \frac{\kappa_0}{4}\phi_i^2\psi_i^2 + \frac{\rho'_0}{8}(\psi_i^2)^2\right]$$

$(i = 1, \ldots, \underline{\underline{N}})$

- For $N = 1$, This model corresponds to $Z_2 \times Z_2$ invariant two scalar model.
- In the large N limit $(N \to \infty)$, we can calculate the effective potential exactly, including all-order loop effects.

Summary

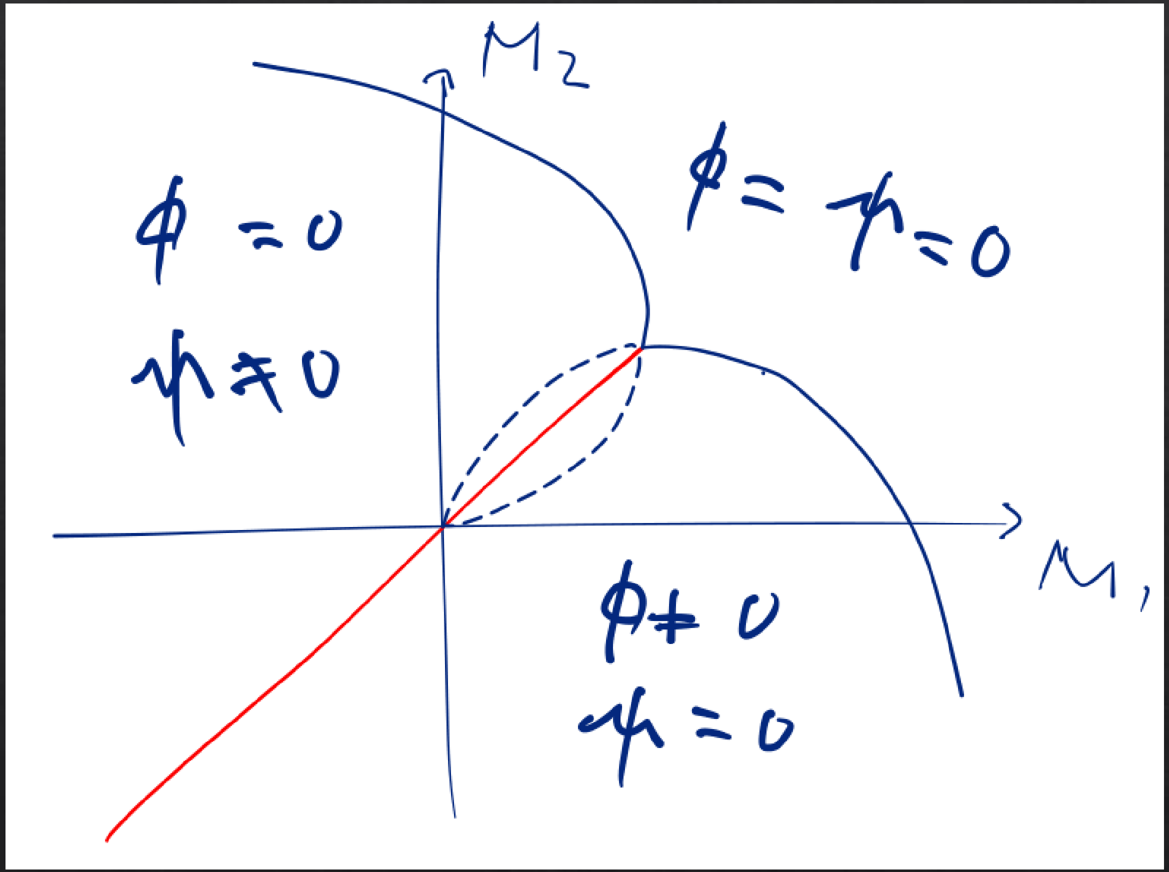
◇ Today, I talk about ⋯

1. Phase diagram in 3 dim.

- $0 < \rho_0 < \kappa_0$

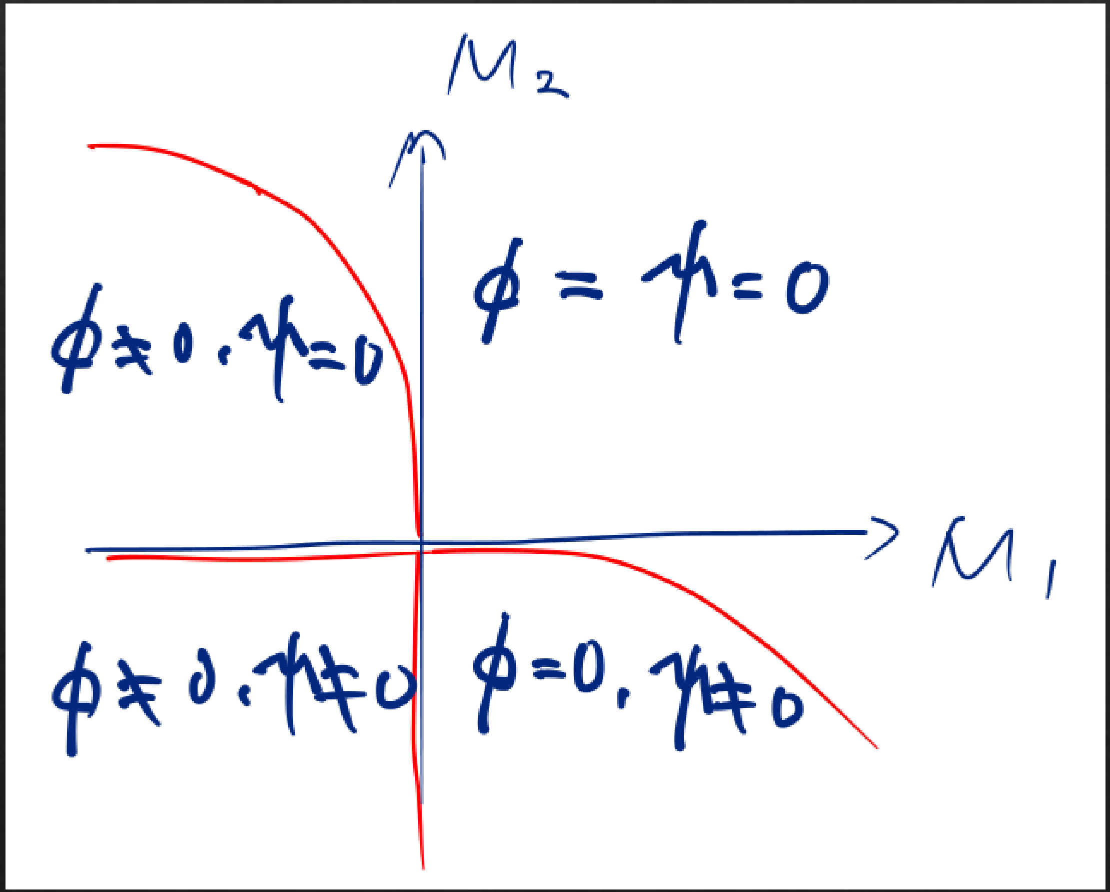
$$\left(M_i := (\lambda^{-1} m^2)_i, \lambda^{-1} := \begin{pmatrix} \rho_0 & \kappa_0 \\ \kappa_0 & \rho_0 \end{pmatrix} \right)$$

(We take $\rho_0 = \rho_0'$ for simplicity)

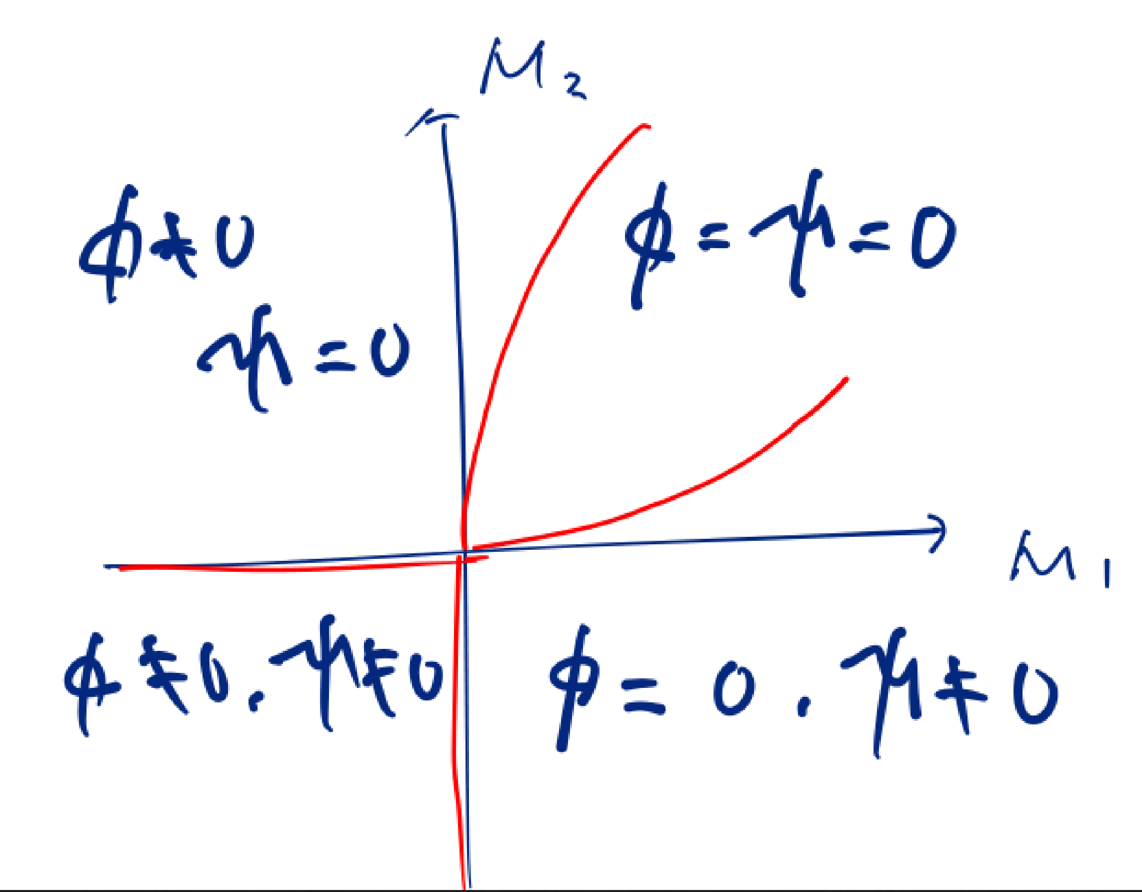


Summary

- $0 < \kappa_0 < \rho_0$



- $\kappa_0 < 0 < \rho_0$



Summary

2. The massless ($m_1 = m_2 = 0$) case in 4 dim:

$$\langle\psi\rangle^2 = 0, \langle\phi\rangle^2 = \mu_*^2 \exp\left(-32\pi^2 \lambda_{22}^{-1}(\mu_*)\right)$$

$(\mu_* \cdots \rho(\mu_*) = 0)$

◈ This can be applied for dynamical generation of Weak scale.

(SM + two SM singlet scalars (Dark Matter?))

$$\mathcal{L} = \left|\partial_\mu H\right|^2 - \frac{\eta}{2}\phi^2|H|^2 + \frac{\eta'}{2}\psi^2|H|^2 + \lambda|H|^4$$

$$+ \frac{1}{2}\left(\partial_\mu\phi\right)^2 + \frac{1}{2}\left(\partial_\mu\psi\right)^2 + \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2\psi^2 + \frac{\rho'}{4!}\psi^4$$

Contents

Model

- We study $O(N) \times O(N)$ invariant two scalar model:

$$\mathcal{L} = N\left[\frac{1}{2}(\partial_\mu \phi_i)^2 + \frac{1}{2}(\partial_\mu \psi_i)^2 + \frac{m_0^2}{2}\phi_i^2 + \frac{{m'}_0^2}{2}\psi_i^2 + \frac{\rho_0}{8}(\phi_i^2)^2 + \frac{\kappa_0}{4}\phi_i^2\psi_i^2 + \frac{\rho_0'}{8}(\psi_i^2)^2\right]$$

$$(\, i = 1, \dots, \underline{\underline{N}}\,)$$

- We calculate the effective potential in $N \to \infty$ limit.
- Point: rewriting the tree potential in quadratic form of fields, introducing auxiliary fields.

Calculation of Effective potential

◈ Rewriting the tree-level potential:

$$\frac{\mathcal{L}}{N} = (kin.) + \underbrace{\frac{1}{2}\begin{pmatrix} m_0^2 & m'^2_0 \end{pmatrix}}_{= M_0^{2\,t}} \underbrace{\begin{pmatrix} \phi_i^2 \\ \psi_i^2 \end{pmatrix}}_{= \Phi^2} + \frac{1}{8}\begin{pmatrix} \phi_i^2 \\ \psi_i^2 \end{pmatrix}^t \underbrace{\begin{pmatrix} \rho_0 & \kappa_0 \\ \kappa_0 & \rho'_0 \end{pmatrix}}_{= \lambda} \begin{pmatrix} \phi_i^2 \\ \psi_i^2 \end{pmatrix}$$

$$= (kin.) + \frac{1}{2} M_0^{2\,t} \Phi^2 + \left(\frac{\lambda}{2}\Phi^2\right)^t \frac{\lambda^{-1}}{2} \left(\frac{\lambda}{2}\Phi^2\right)$$

$$= (kin.) + \left(\frac{\lambda}{2}\Phi^2 + M_0^2\right)^t \frac{\lambda^{-1}}{2} \left(\frac{\lambda}{2}\Phi^2 + M_0^2\right)$$

$$= (kin.) - C^t \frac{\lambda^{-1}}{2} C + C^t \lambda^{-1} \left(\frac{\lambda}{2}\Phi^2 + M_0^2\right)$$

$C \coloneqq \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$:auxiliary fields

Calculation of Effective potential

$$\frac{\mathcal{L}}{N} = \frac{1}{2}\phi_i(-\Delta^2 + c_1)\phi_i + \frac{1}{2}\psi_i(-\Delta^2 + c_2)\psi_i - C^t\frac{\lambda^{-1}}{2}C + C^t\lambda^{-1}M^2$$

◈ We set $\phi_i(x) = \phi\delta_{i,1} + \widehat{\phi_i}(x), \psi_i(x) = \psi\delta_{i,1} + \widehat{\psi_i}(x)$

and drop the linear terms of $\widehat{\phi_i}(x)$, $\widehat{\psi_i}(x)$

$$= \frac{1}{2}\widehat{\phi_i}(-\Delta^2 + c_1)\widehat{\phi_i} + \frac{1}{2}\widehat{\psi_i}(-\Delta^2 + c_2)\widehat{\psi_i} - C^t\frac{\lambda^{-1}}{2}C + C^t\lambda^{-1}M^2 + \frac{1}{2}C^t\begin{pmatrix}\phi^2\\ \psi^2\end{pmatrix}$$

$$\therefore \int DcD\phi_i D\psi_i \exp(-S)$$

$$\propto \int Dc \exp\left(-N\int d^4x \sum_i \frac{\mathrm{tr}}{2}\log(-\Delta^2 + c_i) - C^t\frac{\lambda^{-1}}{2}C + C^t\lambda^{-1}M^2 + \frac{1}{2}C^t\begin{pmatrix}\phi^2\\ \psi^2\end{pmatrix}\right)$$

Calculation of Effective potential

$$\int DcD\phi_i D\psi_i \exp(-S) \propto \int Dc \exp\left(-N\int d^4x \sum_i \frac{\mathrm{tr}}{2}\log(-\Delta^2+c_i) - C^t\frac{\lambda^{-1}}{2}C + C^t\lambda^{-1}M_0^2 + \frac{1}{2}C^t\begin{pmatrix}\phi^2\\ \psi^2\end{pmatrix}\right)$$

◈ C integral is equivalent to substituting the value of the stationary point for C in the large N limit:

$$\exp\left(-\int d^4x V_{\mathrm{eff}}\right) \propto \int DcD\phi_i D\psi_i \exp(-S)$$

$$\propto \exp\left(-N\int d^4x \sum_i \frac{\mathrm{tr}}{2}\log(-\Delta^2+c_i) - C^t\frac{\lambda^{-1}}{2}C + C^t\lambda^{-1}M_0^2 + \frac{1}{2}C^t\begin{pmatrix}\phi^2\\ \psi^2\end{pmatrix}\right)$$

Calculation of Effective potential

$$\therefore \ \frac{V_{\text{eff}}}{N} = \sum_i \frac{\text{tr}}{2} \log(-\Delta^2 + c_i) - C^t \frac{\lambda^{-1}}{2} C + C^t \lambda^{-1} M_0^2 + \frac{1}{2} C^t \begin{pmatrix} \phi^2 \\ \psi^2 \end{pmatrix}$$

- The values of c_i are determined by $\partial_{c_i} V_{\text{eff}} = 0$
- Using $\frac{\text{tr}}{2} \log(-\Delta^2 + c_i) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \log(k^2 + c_i) = \frac{\Lambda}{4\pi} c_i - \frac{\sqrt{c_i}^3}{12\pi} + O\left(\frac{1}{\Lambda}\right)$

$$\frac{V_{\text{eff}}}{N} = \frac{\Lambda}{4\pi} c_i - \frac{\sqrt{c_i}^3}{12\pi} - C^t \frac{\lambda^{-1}}{2} C + C^t \lambda^{-1} M_0^2 + \frac{1}{2} C^t \begin{pmatrix} \phi^2 \\ \psi^2 \end{pmatrix}$$

- We need to renormalize the linear divergence of Λ:

$$\frac{\Lambda}{4\pi} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda^{-1} M_0^2 =: \lambda^{-1} M^2$$

Vacuum

$$\therefore\ \frac{V_{\text{eff}}}{N} = -\,C^t\frac{\lambda^{-1}}{2}C + C^t\lambda^{-1}M^2 + \frac{1}{2}C^t\begin{pmatrix}\phi^2\\ \psi^2\end{pmatrix} - \sum_i \frac{\sqrt{c_i}^{\,3}}{12\pi}$$

◈ The extreme condition is

$$0 = \begin{pmatrix}\partial_\phi\\ \partial_\psi\end{pmatrix}V_{\text{eff}} = \begin{pmatrix}\widetilde{\partial_\phi}\\ \widetilde{\partial_\psi}\end{pmatrix}V_{\text{eff}} = \begin{pmatrix}c_1\phi\\ c_2\psi\end{pmatrix}$$

the derivative of explicit ϕ,ψ dependence remains from $\partial_{c_i}V_{\text{eff}} = 0$.

◈ In order to find minimum, we solve $\partial_{c_i}V_{\text{eff}} = 0$ in the cases

$(1)\, c_1 = c_2 = 0$ $(2)\, c_1 = \psi = 0, c_2 \neq 0$

$(2')\, c_2 = \phi = 0, c_1 \neq 0$ $(3)\, \phi = \psi = 0, c_1 \neq 0, c_2 \neq 0$

and compare the value of V_{eff}.

Phase structure in 3 dim

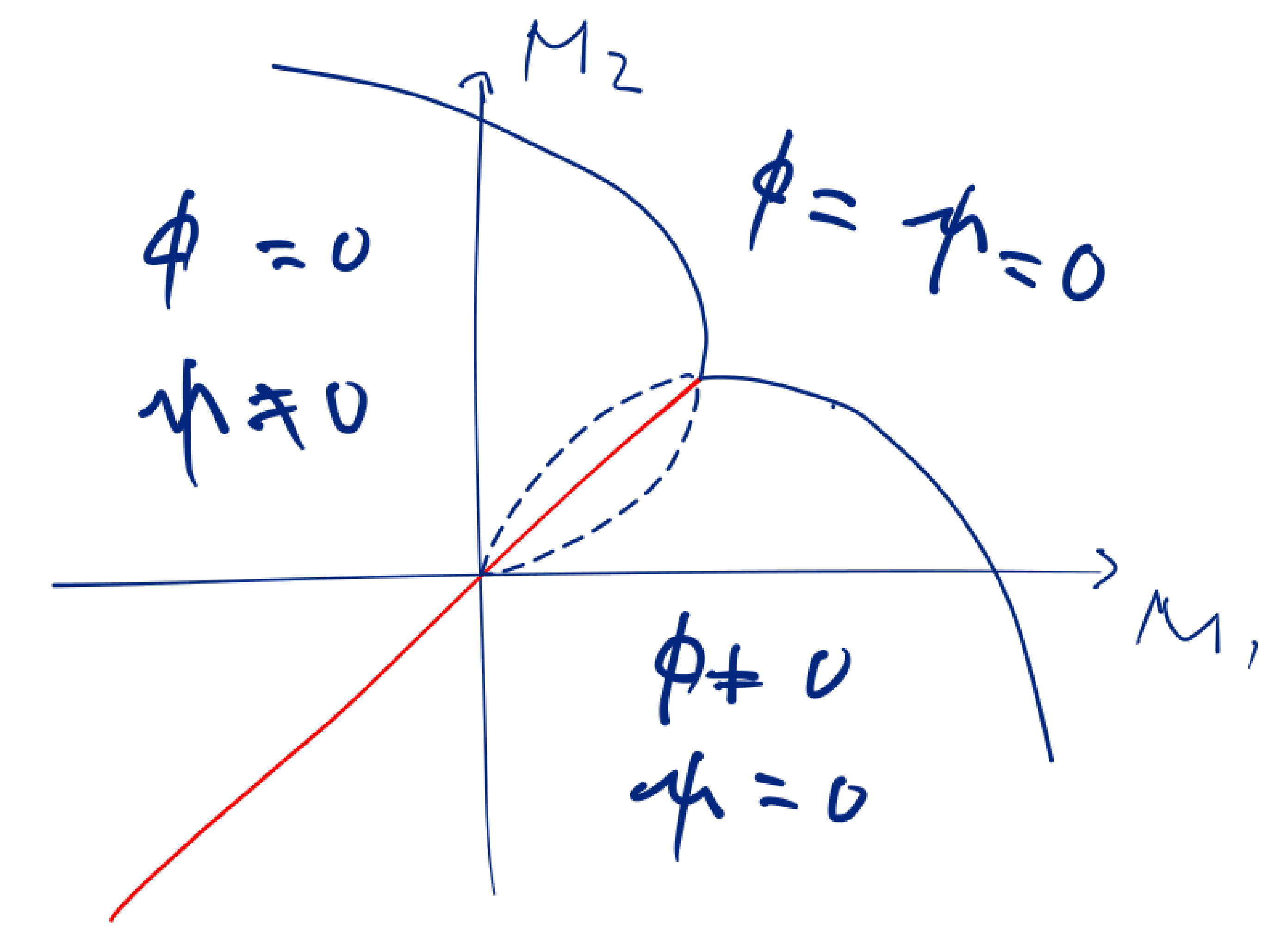
◈ Results

- $0 < \rho_0 < \kappa_0$

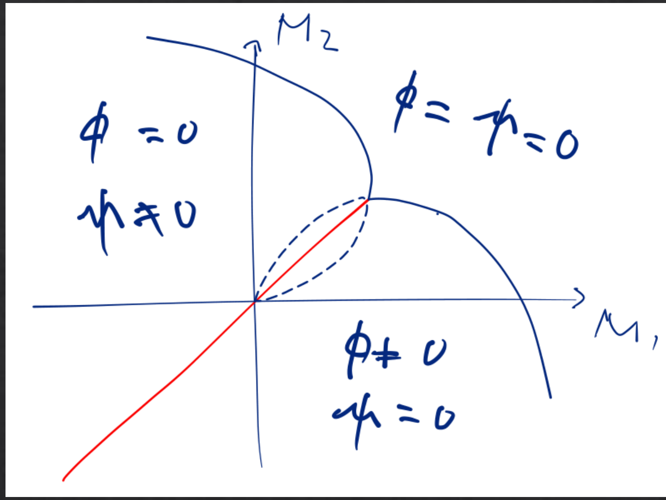
$(M_i := (\lambda^{-1} m^2)_i$

$\lambda^{-1} := \begin{pmatrix} \rho_0 & \kappa_0 \\ \kappa_0 & \rho_0 \end{pmatrix})$

(we take $\rho_0 = \rho_0'$
for simplicity)



Phase structure in 3 dim



◈ Comments ($0 < \rho_0 < \kappa_0$):

1. There is **no phase** where vacuum expectation values of **both fields are nonzero**.

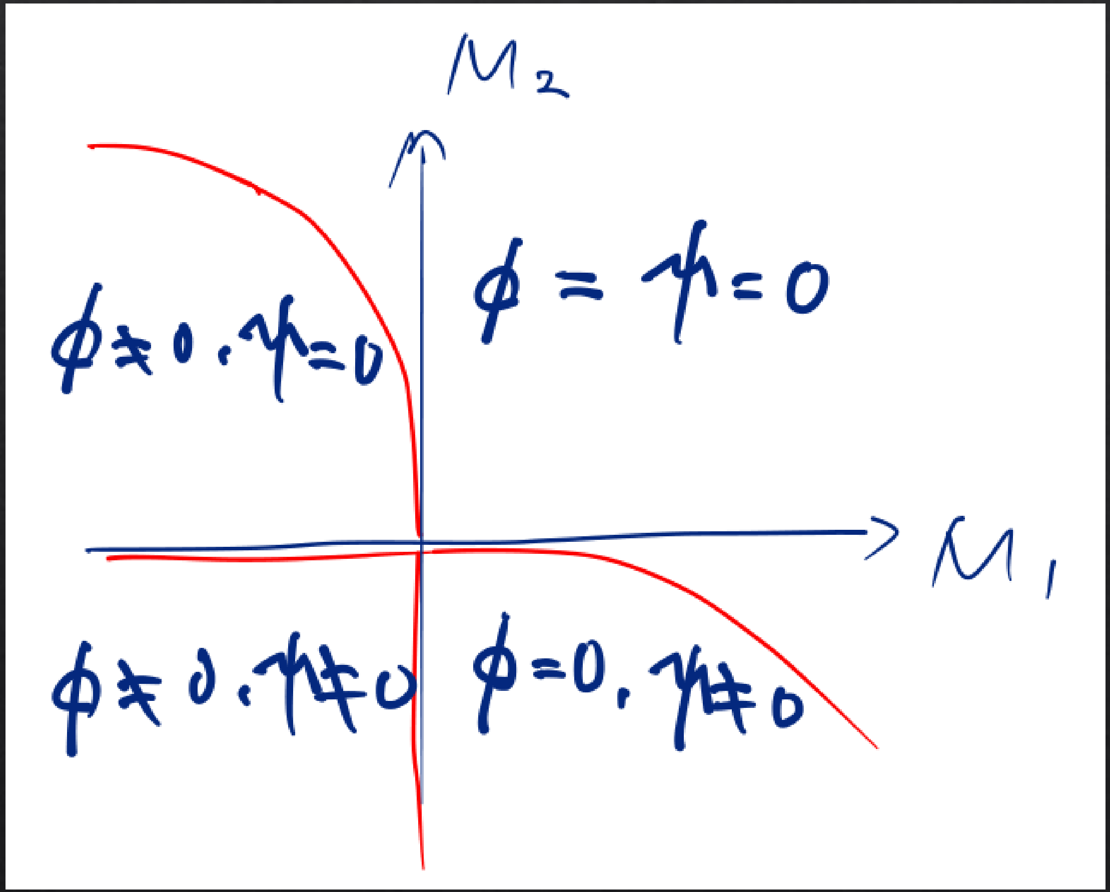
   ← $\kappa\phi^2\psi^2$ term in the potential prevents it.

2. **One broken phase** exist in a region where **mass squares are positive**.

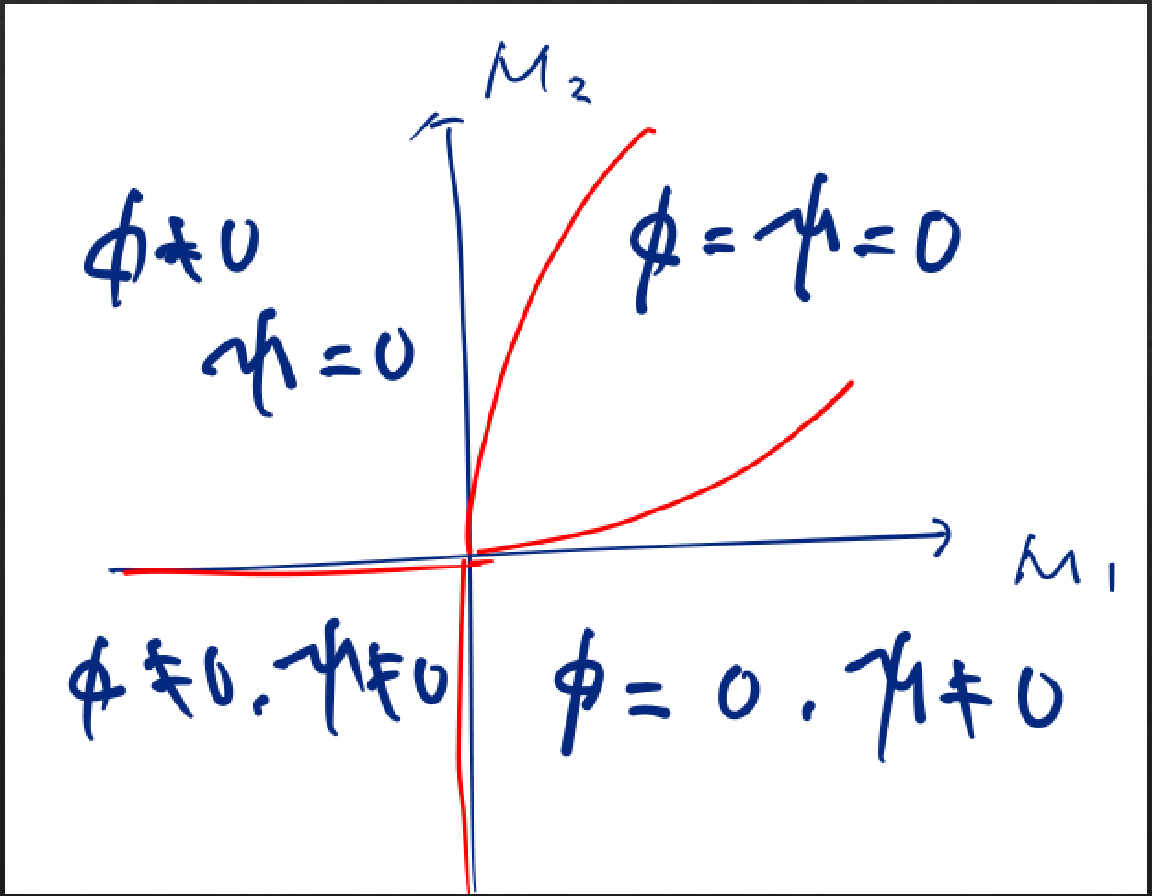
← because of the loop contribution to the effective potential?

Phase structure in 3 dim

- $0 < \kappa_0 < \rho_0$



- $\kappa_0 < 0 < \rho_0$



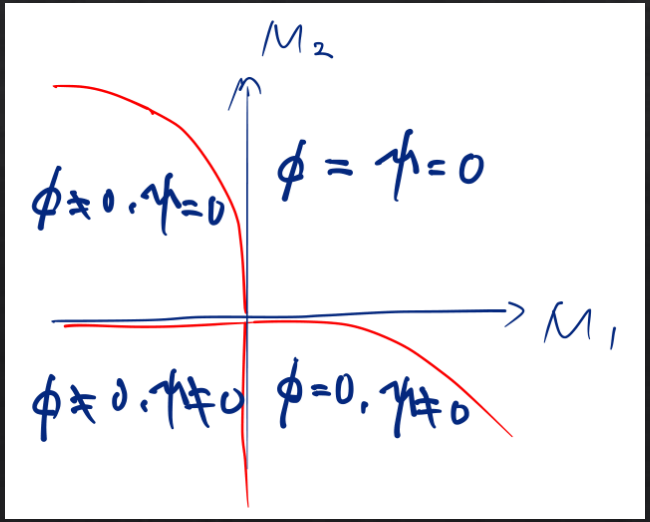
Phase diagram in 3 dim

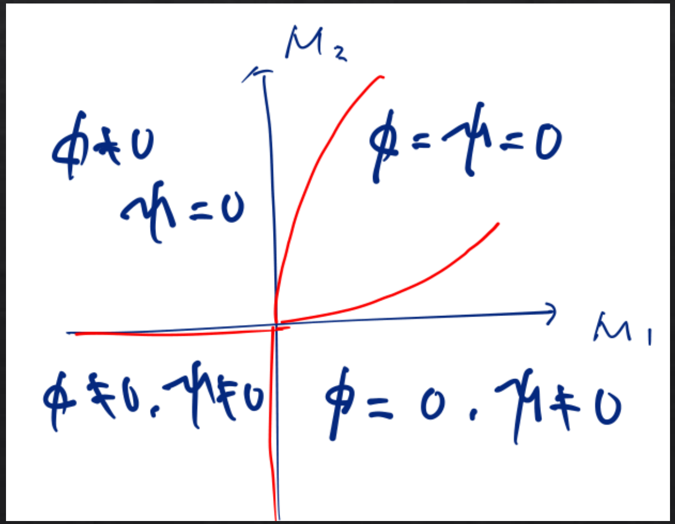
◈ Comments ($\kappa_0 < \rho_0$):

1. The phase diagram are essentially same as phi fourth theory.
2. The boundaries between both broken and one broken phases ($M_1 = 0, M_2 < 0$ and (1↔2)) are the boundary where the potential can be negative:

$$V = \frac{\lambda^{-1}}{2}\left(\frac{\lambda}{2}\Phi^2 + m^2\right)^2$$

$$= \frac{\lambda}{2}\left(\frac{1}{2}\Phi^2 + M\right)^2$$





Massless $O(N)\times O(N)$ invariant model in 4 dim

- In 4 dim, we have not analyzed the entire phase sturucture.
- We studied massless $O(N)\times O(N)$ invariant model in 4 dim.

(the renormalized masses are zero: $m_1^2 = m_2^2 = 0$)

$$\mathcal{L} = N\left[\frac{1}{2}(\partial_\mu\phi_i)^2 + \frac{1}{2}(\partial_\mu\psi_i)^2 + \frac{\rho}{8}(\phi_i^2)^2 + \frac{\kappa}{4}\phi_i^2\psi_i^2 + \frac{\rho'}{8}(\psi_i^2)^2 + (\text{counter terms})\right]$$

- As in 3 dim, the effective potential can be calculated exactly in large N limit.
- (Here, we take $\rho < \rho'$ without loss of generality.)

Effective potential in 4 dim

- Calculation is the same as 3 dim,

but the momentum integral differs:

$$\mathrm{tr}\log(-\Delta^2 + c_i) = \int \frac{d^4k}{(2\pi)^4}\log(k^2 + c_i) = \frac{c_i^2}{32\pi^2}\left(\log\frac{c_i}{\Lambda^2} - \frac{1}{2}\right) + \frac{c_i}{32\pi^2}\Lambda^2$$

- The effective potential is

$$\frac{V_{\mathrm{eff}}}{N} = \sum_i \frac{c_i^2}{64\pi^2}\left(\log\frac{c_i}{\Lambda^2} - \frac{1}{2}\right) + \frac{c_i}{32\pi^2}\Lambda^2 - C^t\frac{\lambda^{-1}}{2}C + C^t\lambda^{-1}M_0^2 + \frac{1}{2}C^t\begin{pmatrix}\phi^2\\ \psi^2\end{pmatrix}$$

Effective potential in 4 dim

$$\frac{V_{\rm eff}}{N} = \sum_i \frac{c_i^2}{64\pi^2}\left(\log\frac{c_i}{\Lambda^2} - \frac{1}{2}\right) + \frac{c_i}{32\pi^2}\Lambda^2 - C^t \frac{\lambda^{-1}}{2} C + C^t \lambda^{-1} M_0^2 + \frac{1}{2} C^t \begin{pmatrix} \phi^2 \\ \psi^2 \end{pmatrix}$$

- We need to renormalize the quadratic and log divergence of Λ

$$\begin{cases} \dfrac{\Lambda^2}{32\pi^2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda^{-1} M_0^2 = 0 \\ \dfrac{1}{64\pi^2} \log\dfrac{\mu^2}{\Lambda^2} I_2 - \dfrac{\lambda^{-1}}{2} = -\dfrac{\lambda^{-1}(\mu)}{2} \end{cases}$$

Here, we consider the case where renormalized masses are 0

$\therefore$

$$\frac{V_{\rm eff}}{N} = \sum_i \frac{c_i^2}{64\pi^2}\left(\log\left(\frac{c_i}{\mu^2}\right) - \frac{1}{2}\right) - C^t \frac{\lambda^{-1}}{2} C + \frac{1}{2} C^t \begin{pmatrix} \phi^2 \\ \psi^2 \end{pmatrix}$$

Vacuum

$$\frac{V_{\text{eff}}}{N} = \sum_i \frac{c_i^2}{64\pi^2}\left(\log\left(\frac{c_i}{\mu^2}\right) - \frac{1}{2}\right) - C^t \frac{\lambda^{-1}}{2} C + \frac{1}{2} C^t \begin{pmatrix} \phi^2 \\ \psi^2 \end{pmatrix}$$

- After comparing the values of V_{eff}, we find the minimum point is

$$\psi^2 = 0, \phi^2 = \mu^2 \exp\left(-32\pi^2 \lambda_{22}^{-1}\right)$$

$$\left(c_1 = 0,\ c_2 = 2\lambda_{12}^{-1} \mu^2 \exp\left(-32\pi^2 \lambda_{22}^{-1}\right)\right)$$

- Can we use this to explain the origin of Weak scale,

  by coupling the model to the standard model?

Weak Scale

- Let us change the Higgs Sector of SM:

$$\mathcal{L} = |\partial_\mu H|^2 - \frac{\eta}{2}\phi^2|H|^2 + \frac{\eta'}{2}\psi^2|H|^2 + \lambda|H|^4 + \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\psi)^2 + \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2\psi^2 + \frac{\rho'}{4!}\psi^4$$

- ϕ and ψ are SM singlet scalars.
- The vacuum expectation value of ϕ gives the negative mass term of Higgs fields:

$$-\frac{\eta}{2}\langle\phi\rangle^2|H|^2 + \lambda|H|^4 \simeq -\frac{m_H^2}{2}|H|^2 + \lambda|H|^4$$

= Weak scale ($\langle H\rangle = 246$ GeV) is reproduced dynamically!

Summary

- We studied $O(N) \times O(N)$ scalar model in 3 or 4 dimensions in the large N limit.
- In 3 dim, it has non-trivial phase structure.
- In 4 dim, we have not studied completely yet,

  but one scalar field has a vacuum expectation value even in

  the massless case.
- This can be used for dynamical generation of Weak scale.

Future Works

- We continue to study the phase structure in 4 dim.
  and order of phase transition.
- Models with more than 2 scalar fields are interesting.
- If the $\frac{1}{N}$ correction is considered, what does it become?
  These results are universal or peculiar to the large N limit?

Weak Scale

- ϕ, ψの質量

$$m_\phi^2 = \frac{d^2}{d\phi^2} V_{\text{eff}} \Big|_{\phi=\langle\phi\rangle} = \frac{\kappa^2(\mu_*)}{32\pi^2} \langle\phi\rangle^2$$

$$m_\psi^2 = \frac{\kappa(\mu_*)}{2} \langle\phi\rangle^2$$

- ϕ, ψは何者?

Higgsの真空期待値と質量を再現するための条件から

$$m_\psi^2 \geq (0.6\ \text{TeV})^2$$

ψはHiggs portal scalar dark matter?

ϕは未知のスカラー場