Entropy generation and decay of the cosmological constant in de Sitter space

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Introduction

- The universe with a cosmological constant $\Lambda = 3H^2$ is given by dS space:

$$dS_4: \ ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \ a(t) = e^{Ht}$$

- In Gibbons-Hawking formula, the dS entropy is proportional to the area of the horizon:

$$S = \frac{1}{4G_N} \cdot 4\pi(1/H)^2 = \frac{\pi}{G_N H^2} \qquad G_N: \text{ Newton's const.}$$

- If $\Lambda \propto H^2$ and $S \propto H^{-2}$ anomalously scale with time, there may exist an unified understanding of these time evolutions

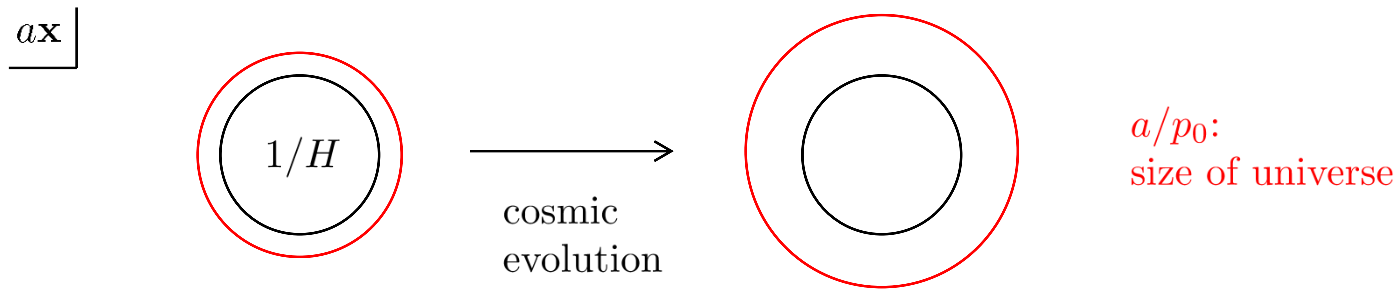
- What is the mechanism to give rise to such time evolutions?

Quantum IR effects

'82 A. Vilenkin, L. H. Ford,
A. D. Linde,
A. A. Starobinsky

- The propagator of a massless, minimally coupled scalar field has a secular growing term:

$$\langle \varphi^2(x) \rangle \simeq \int_{p=p_0}^{p=Ha} \frac{d^3p}{(2\pi)^3} \frac{H^2}{2p^3} = \frac{H^2}{4\pi^2} \underline{\log(a/a_0)} \qquad a_0 \equiv p_0/H$$



- In the presence of the light scalar field, physical quantities may acquire time dependent quantum effects through internal propagators

If $m^2 \sim H^2$, such IR effects do not appear as $\int d^3p/p^{2\nu},\ \nu = \sqrt{9/4 - m^2/H^2}$

What we did

- Gravitational fluctuations on the dS background also induce quantum IR effects. In contrast to scalar fields, the fine-tuning of the quadratic term is not necessary

→ dS space is unstable due to self-fluctuations?

'96 N. C. Tsamis, R. P. Woodard

- We derived the effective action up to the one-loop level and found that the dimensionless coupling $g = G_N H^2/\pi$ is dynamically screened by soft gravitons

- Furthermore, we found that the time evolution of the corresponding dS entropy can be identified with the von Neumann entropy of the conformal zero mode

Gravitational propagator

$$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}[R - 6H^2]$$

$$\kappa^2 = 16\pi G_N$$

$$g_{\mu\nu} = a^2 e^{2\omega}\eta_{\mu\rho}(e^h)^\rho{}_\nu, \;\; h^\mu{}_\mu = 0$$

$$S_{\rm GF} = \frac{1}{\kappa^2}\int d^4x\big[-\frac{1}{2}a^2\eta^{\mu\nu}F_\mu F_\nu\big]$$

$$F_\mu = \partial_\rho h^\rho{}_\mu - 2\partial_\mu\omega + 2Hah^0{}_\mu + 4Ha\delta^0{}_\mu\omega$$

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Quantum IR effects come from

$$h^{00} \simeq 2\omega, \quad \langle\omega(x)\omega(x')\rangle \simeq -\frac{3}{16}\kappa^2\langle\varphi(x)\varphi(x')\rangle$$

$$\langle\tilde{h}^{ij}(x)\tilde{h}^{kl}(x')\rangle = (\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk} - \frac{2}{3}\delta^{ij}\delta^{kl})\kappa^2\langle\varphi(x)\varphi(x')\rangle$$

$\tilde{h}^{ij}$: traceless

$$\langle b^i(x)\bar{b}^j(x')\rangle = \delta^{ij}\kappa^2\langle\varphi(x)\varphi(x')\rangle$$

b^μ: FP ghost

φ: massless, minimally coupled scalar field

$h^{00} - 2\omega$, h^{0i}, b^0 with $m^2 \sim H^2$ are neglected

One-loop effective action

On a homogeneous, isotropic background $\hat{g}_{\mu\nu} = a^2 \eta_{\mu\nu}$,

$$\frac{1}{\kappa^2} \int d^4x \sqrt{-\hat{g}} \big[\hat{R}(1 + \langle 4\omega^2 \rangle) - 6H^2(1 + \langle 8\omega^2 \rangle) - \hat{R}^{\mu}{}_{\nu} \langle 2h^{\nu}{}_{\mu}\omega \rangle \big]$$

$$\simeq \frac{1}{\kappa^2} \int d^4x \sqrt{-\hat{g}} \big[\hat{R} a_c^{-2\gamma} - 6H^2 a_c^{-4\gamma} - (\hat{R}^0{}_0 - \hat{R}^1{}_1) 2\gamma \log a_c \big]$$

$$\because \langle \omega^2 \rangle = -\frac{\gamma}{2} \log a_c, \gamma = \frac{3}{8} \frac{\kappa^2 H^2}{4\pi^2}$$

The $\sqrt{-\hat{g}}\hat{R}$, $\sqrt{-\hat{g}}$ terms imply an anomalous scaling

$$\frac{1}{\kappa^2(t)} = \frac{1}{\kappa^2} a_c^{-2\gamma}, \ \frac{H^2(t)}{\kappa^2(t)} = \frac{H^2}{\kappa^2} a_c^{-4\gamma} \ \Rightarrow \ H^2(t) = H^2 a_c^{-2\gamma}$$

$$\Rightarrow \ a = a_c^{1+\gamma} \qquad\qquad a_c = \frac{1}{-H\tau}$$

However, if $a \neq a_c$, the covariance is not manifest: $\hat{R}^0{}_0 - \hat{R}^1{}_1 = -2\gamma H^2 a_c^{-2\gamma}$

Inflaton as a counter term

Introducing an inflaton f with $\frac{1}{\kappa^2}\left(\frac{V'}{V}\right)^2 = \gamma,$

$$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\left[R - 6H^2V(f) - \frac{\kappa^2}{2}g^{\mu\nu}\partial_\mu f\partial_\nu f\right]$$

$$V = 1 - \sqrt{\gamma}\kappa f = a_c^{-2\gamma}$$

we can eliminate the non-covariant term

$$\hat{R}^0{}_0 - \hat{R}^1{}_1 + \frac{\kappa^2}{2}a^{-2}\partial_0 f\partial_0 f = 0$$

After the conformal transformation,

$$\frac{1}{\kappa^2}\int d^4x\sqrt{-\hat{g}}\left[\hat{R} - 6H^2a_c^{-2\gamma}\right]$$

$$\frac{1}{\kappa^2(t)} = \frac{1}{\kappa^2}, \quad H^2(t) = H^2a_c^{-2\gamma}$$

$$\kappa^2 = 16\pi G_N$$

Soft gravitons screen the dimensionless coupling: $g(t) = G_N H^2 a_c^{-2\gamma}/\pi$

Time evolution of dS entropy

The dS entropy increases with the decay of the coupling:

$$S = 1/g(t) = \underline{\frac{\pi}{G_N H^2} a_c^{2\gamma}}$$

Considering the conformal zero mode, the same result is reproduced

$$\begin{aligned} S &= \log Z \\ &= \frac{1}{16\pi G_N} \int \sqrt{\hat{g}} d^4x [\hat{R}\langle e^{2\omega}\rangle - 6H^2\langle e^{4\omega}\rangle] \\ &= \frac{1}{16\pi G_N} \cdot \frac{8\pi^2}{3H^4} \cdot [12H^2\langle e^{2\omega}\rangle - 6H^2\langle e^{4\omega}\rangle] \\ &= \frac{\pi}{G_N H^2}(1 - \langle 4\omega^2\rangle) = \underline{\frac{\pi}{G_N H^2} a_c^{2\gamma}} \end{aligned}$$

$$\because \; F = \cancel{U} - TS = -T \log Z$$

by rotating $dS_4 \to S_4$ (adiabatically)

The time evolution is local: $S = \frac{\pi}{G_N H^2}(1 + 2\gamma \log a_c) = \frac{\pi}{G_N H^2} + 3Ht$ $\qquad \gamma = 3g/2$

von Neumann entropy

Up to the one-loop level, the distribution function of the conformal zero mode is described by the following Fokker-Planck equation:

$$\frac{\partial}{\partial t}\rho(t,\omega) = \frac{\gamma}{2}\cdot\frac{H}{2}\frac{\partial^2}{\partial\omega^2}\rho(t,\omega)$$

No drift term

$$\rho(t,\omega) = \sqrt{\frac{4\xi(t)}{\pi g}}\exp\Big\{-\frac{4\xi(t)}{g}\omega^2\Big\}, \quad \xi(t) = \frac{1}{1+6Ht}$$

From the solution, the von Neumann entropy can be constructed

$$\begin{aligned} S &= -\mathrm{tr}(\rho\log\rho) \\ &= \frac{1}{2}\Big\{1+\log\pi g - \log 4\xi(t)\Big\} \\ &\to \frac{1}{2}\log(1+6Ht) \ \sim \underline{3Ht} \text{ at } Ht \ll 1 \end{aligned}$$

The local time evolution can be reproduced in von Neumann formula

β function

The dS entropy is given by the action: $S(t) = \log Z(t)$

$$S(t) = \frac{1}{g} - \frac{1}{2}\log \xi(t)$$

Since the bare action $S_B = \frac{1}{g(t)} + \frac{1}{2}\log \xi(t)$ is time independent,

$$\beta(g(t)) = -\frac{1}{2}g^2(t), \quad \beta(g(t)) \equiv \frac{\partial}{\partial \log(1+6Ht)} g(t)$$

$g(t) = G_N H^2(t)/\pi$ is asymptotically free toward future

Solving it exactly, we obtain the global time evolution:

$$g(t) = \frac{2}{\log(1+6Ht) + \frac{2}{g}}$$

Summary

- In order to obtain the covariance of the effective Einstein action, it is necessary to introduce an inflaton as a counter term

- The effective action shows that the dimensionless coupling $g = G_N H^2/\pi$ is screened by soft gravitons

- The dS entropy increases with the decay of the coupling, and it can be identified with the von Neumann entropy of the conformal zero mode

- Deriving the one-loop β function and solving it exactly, we found that the coupling logarithmically decays with time: $g \propto 1/\log Ht$

Physical implications

Applying $g \propto 1/\log Ht$ into inflation and dark energy, we obtained the following implications:

- For inflation,

$$\text{Scalar spectral index: } 1 - n_s = \frac{3}{2N_*} + \frac{1}{2N_* \log N_*} \sim 0.033$$

$$\text{Tensor to scalar ratio: } r = \frac{4}{N_* \log N_*} \sim 0.016 \qquad \text{for } N_* = 60$$

- For dark energy,

$$\text{Equantion of state: } w = w_0 + w_a(1-a)$$

$$w_0 = -1 + \frac{1}{3e} = -0.877\cdots\,, \quad w_a = -\frac{2}{3e^2} = -0.090\cdots$$

These predictions are still consistent with the observations