Higher spin symmetry in the IIB matrix model with the operator interpretation

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Plan of Talk

- Introduction
- A review of the operator interpretation
  - Definition
  - Advantages
- Higher spin symmetries
  - Set up
  - Analysis on gauge symmetries
  - Discussion on equations of motions
- Summary

Introduction

Introduction

The IIB Matrix Model

$\rightarrow$ A candidate for the nonperturbative formulation of string thoery [N. Ishibashi, H. Kawai, Y. Kitazawa, A. Tsuchiya(1996)]

$$S_{\mathrm{IIB}} = -\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[A_a, A_b]^2 + \frac{1}{2}\bar{\Psi}\Gamma^a[A_a, \Psi]\right)$$

A_a, Ψ : $N \times N$ Hermitian matrices

- The existence of the spacetime is not assumed; it emerges from the matrices
- Rich dynamics $\rightarrow$
  - Reproduction of the light-cone Hamiltonian of SFT [M. Fukuma, H. Kawai, Y. Kitazawa, A. Tsuchiya(1997)]
  - Emergence of (3+1) D expanding spacetime [S. Kim, J Nishimura, A Tsuchiya(2011)], …
  - Emergence of chiral zero modes in the fermionic sector [J Nishimura, A Tsuchiya(2013)], …

Introduction

On the other hand, the physical interpretation of the matrices is still unclear. There are several interpretation.

- In many works … Coordinate interpretation

  - Matrices represent the coordinate of some object embedded in the flat spacetime (string, brane, universe *etc.*).
  - Their fluctuations are field variables on it.

$$A_a \sim X_a + a_a$$

(Embedding coor.) + (Field)

- An interesting variation … Noncommutative interpretation

  - Matrices form some algebra and noncommutative spacetimes emerge from it.
  - Their fluctuations are described with noncommutative field theory.

ex) $[A_a, A_b] = i\epsilon_{abc}A_c \quad \Rightarrow$ Fuzzy S^2

  - An sophisticated realization of gravity and higher spin symmetries

  → Prof. H. Steinacker's talk

Introduction

- In this talk … Operator interpretation

[M.Hanada, H.Kawai, Y.Kimura (2005)]

  - Matrices represent operators which act on some functional space.

$$(A_a \cdot f)(x) = \int dy K_a(x,y) f(y)$$
$$\sim \left(a_a(x) + a_a{}^{\mu}\partial_\mu + a_a{}^{\mu\nu}\partial_\mu\partial_\nu + \cdots\right) f(x)$$

  - The generalization of momentum
    (If one consider $A_a \sim i\partial_a$ acting on functions defined on a flat spacetime.
    → It is momentum.)

  - It is natural that matrices include the notion of the momentum. In the IIB matrix model as large-*N* reduction of Yang-Mills theory, momenta are absorbed in the matrices.

This interpretation has many interesting aspects (symmetries, dynamics, *etc.*).
I'll explain them soon, but firstly give more concrete definition

- A review of the operator interpretation

Definition

On what functional space are matrices as operators defined?

- Naively $A_a \in End(C^\infty(\mathcal{M}))$, $\mathcal{M}$: spacetime

$$A_a \stackrel{?}{=} a_a(x) + \frac{1}{2}[a_a{}^\mu(x), i\nabla_\mu]_+ + \frac{1}{2}[a_a{}^{\mu\nu}(x), i\nabla_\mu i\nabla_\nu]_+ + \cdots$$

Difficulty :

The above operator is an vector operator.
(Each component is not defined globally in an independent way.)

On the other hand, each of $A_1, A_2, \cdots$ is independently well-defined.

$\Downarrow$

$\cancel{(A_a = \nabla_a)}$ We cannot interpret matrices as derivative operator straightforwardly.

ex) $A_1 A_2$ vs. $\nabla_1 \nabla_2 = \partial_1 \nabla_2 - \Gamma^c_{12} \nabla_c$

Definition

In order to resolve such difficulty…

- We introduce a principal bundle

$$E_{\rm prin} \sim \mathcal{M} \times G, \quad G:$$ Lorentz group

$$C^\infty(E_{\rm prin}) \ni f(x,g)$$: functions in the regular representation of Lorentz group

- Then we define derivative operators on $C^\infty(E_{\rm prin})$.

ex) $$\nabla_{(a)} := R_{(a)}{}^b(g^{-1})\nabla_b$$

$R_{(a)}{}^b(g^{-1})$: Matrix element for the vector rep. of G

Each of $\nabla_{(1)}, \nabla_{(2)}, \cdots$
- is defined globally.
- belongs to $End(C^\infty(E_{\rm prin}))$.

$(A_a = i\nabla_{(a)})$ OK!

Definition

More details…

Consider $f(x,g) \in C^\infty(E_{\text{prin}})$. $\quad \rightarrow \quad (h \cdot f)(x,g) = f(x, h^{-1}g)$

An important feature of the regular representation

$$V_r \otimes V_{\text{reg}} \simeq V_{\text{reg}} \oplus \cdots \oplus V_{\text{reg}}$$

$$f_a(x,g) \simeq \underline{R_{(a)}{}^{\langle r \rangle b}(g^{-1})} f_b(x,g)$$

matrix elements of g^{-1}
in the r-rep.

h

h

$$R_a{}^{\langle r \rangle b}(h) f_b(x, h^{-1}g)$$

Transforming as $(r \otimes \text{reg})$ - rep..
The components are mixed.

$$R_{(a)}{}^{\langle r \rangle b}(h^{-1}g) f_b(x, h^{-1}g)$$

Transforming as reg. rep..
Each component transforms independently.

Definition

Matrices as general derivative operators acting on $C^{\infty}(E_{\mathrm{prin}})$

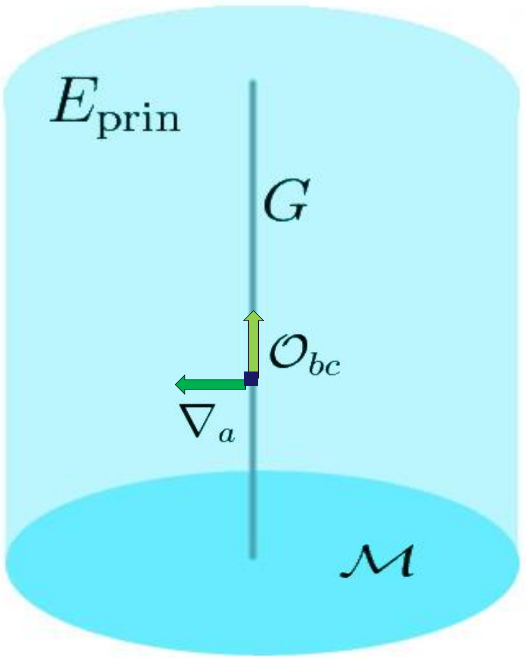
- We expand a “matrix” to derivatives of infinite degree, both in spacetime-manifold and in fiber-bundle directions.
- We further expand each coefficient field of regular representation to those of various representations.

$$A_a = \hat{a}_{(a)}(x,g) + \frac{1}{2}[\hat{a}_{(a)}{}^{\mu}(x,g), i\nabla_{\mu}]_{+} + \cdots ,$$

$$\hat{a}_{(a)}(x,g) = a_{(a)}(x,g) + \frac{1}{2}[a_{(a)}{}^{bc}, \underline{\mathcal{O}_{bc}}]_{+} + \cdots ,$$

Lorentz generator

$$a_{(a)}(x,g) = \sum_{r:\mathrm{irr.rep.}} R^{\langle r\rangle}{}_{i}{}^{j}(g)\; a^{\langle r\rangle}{}_{(a)}{}^{i}{}_{j}(x)$$ Peter-Weyl thm



Advantages

Equations of Motion

When we pose an ansatz of the form $A_a = i\nabla_{(a)}$,

$$[A^{(a)}, [A_{(a)}, A_{(b)}]] = 0$$

$$\Updownarrow \qquad \left(A_{(a)} A_{(b)} \cdots = R_{(a)}{}^{c}(g^{-1})\, R_{(b)}{}^{d}(g^{-1}) \cdots A_c\, A_d \cdots\right)$$

$$\begin{aligned} 0 &= [\nabla^a, [\nabla_a, \nabla_b]] \\ &= [\nabla^a, R_{ab}{}^{cd}\mathcal{O}_{cd}] \\ &= \left(\nabla^a R_{ab}{}^{cd}\right)\mathcal{O}_{cd} + R_{ab}{}^{ac}\nabla_c \end{aligned}$$

$$\Updownarrow$$

$$R_{ab} = 0$$

- A curved spacetime $\mathcal{M}$ emerges as a classical sol.
- We can also show that the fluctuations on this sol. is fields living on $\mathcal{M}$.

Advantages

As a part of symmetries…

U(N) symm. as the matrix model

$\supset$ *U*(1) , diffeomorphism, local Lorentz symmetries

$$\delta A_a = i[\Lambda, A_a]$$

ex) around flat background, modes with lower degrees of derivative

$$A_a = R_{(a)}^{\ \ b}(g^{-1})[\partial_b + \underbrace{f_b(x)}_{\text{gauge field}} + \underbrace{e_b^{\ \mu}(x)}_{\text{vielbein fluctuation}} i\partial_\mu + \underbrace{\omega_b^{\ cd}(x)}_{\text{spin connection}} \mathcal{O}_{cd} + \cdots]$$

$$\Lambda = \lambda(x)$$

$$\delta f_a = -e_a^{\ \mu}\partial_\mu \lambda, \quad \delta e_a^{\ \mu} = 0, \quad \delta \omega_a^{\ bc} = 0.$$

$$\Lambda = \frac{1}{2}[\lambda^\mu(x), i\partial_\mu]_+$$

$$\delta f_a = \lambda^\mu \partial_\mu f_a, \quad \delta e_a^{\ \mu} = -e_a^{\ \nu}\partial_\nu \lambda^\mu + \lambda^\nu \partial_\nu e_a^{\ \mu}, \quad \delta \omega_a^{\ bc} = \lambda^\mu \partial_\mu \omega_a^{\ bc}.$$

$$\Lambda = \frac{1}{2}[\lambda^{bc}, \mathcal{O}_{bc}]_+$$

$$\delta f_a = i\lambda_a^{\ b} f_b, \quad \delta e_a^{\ \mu} = i\lambda_a^{\ b} e_b^{\ \mu}, \quad \delta \omega_a^{\ bc} = -e_a^{\ \mu}\partial_\mu \lambda^{bc} + 2i\lambda_d^{\ [b}\omega_a^{\ c]d} + i\lambda_a^{\ d}\omega_d^{\ bc}.$$

Advantages

Other features and advantages of the op. interpretation

- *U*(N) symmetry appears to allow some kind of mass terms.
  …However, such terms is not induced by loop correction in the presence of SUSY.
  [KS (2017)]

- The effective action is written in the form of "products of the integration of local operators." [Y.Asano, H.Kawai, A.Tsuchiya (2012)]

$$S = \sum_i S_i + \sum_{i,j} c_{ij} S_i S_j + \sum_{i,j,k} c_{ijk} S_i S_j S_k + \cdots , \quad S_i = \int d^d x \sqrt{-g} O_i(x)$$

Actions of this form can lead to resolution of fine-tuning problem.
[Y.Hamada, H.Kawai, K.Kawana (2015)]

- Infinitely many massless fields of various representations.

  $\rightarrow$ Physically meaningful DoF？ The structure of gauge symmetries？

We analyze this aspect by studying the bosonic part in the flat background.

Higher spin symmetry

Settings

We focus on the zero-modes with respect to the bandle-dependence.

$$A_a = R_{(a)}{}^{b}(g^{-1})\left[a_b(x,\cancel{g}) + a_b{}^{\mu}(x,\cancel{g})\partial_\mu + \cdots\right]$$

In the following we will write this as A_a.

An useful technique : "Semiclassical limit"

Replacing derivatives with c-numbers: $i\partial_\mu \longrightarrow p_\mu, \;\; \mathcal{O}_{bc} \longrightarrow t_{bc}$

ex) $$A_a = \frac{1}{2}[a_a{}^{\mu\nu}(x), i\partial_\mu i\partial_\nu]_+ \;\longrightarrow\; a_a{}^{\mu\nu}(x)p_\mu p_\nu$$

Commutators → Poisson brackets

$$-i[f,h] \to \{f,h\} = \frac{\partial f}{\partial p_\mu}\frac{\partial h}{\partial x^\mu} - \frac{\partial f}{\partial x^\mu}\frac{\partial h}{\partial p_\mu} + i(\mathcal{M}_{ab}g)_{ij}\left(\frac{\partial f}{\partial t_{ab}}\frac{\partial h}{\partial g_{ij}} - \frac{\partial f}{\partial g_{ij}}\frac{\partial h}{\partial t_{ab}}\right) + i f^{ab}{}_{cd,ef}\; t_{ab}\frac{\partial f}{\partial t_{cd}}\frac{\partial h}{\partial t_{ef}}$$

Derivatives in the spacetime directions

Derivatives in the bundle directions

(~ We ignore the complication of anti-commutators, but taking into account commutators.)

Settings

Notations

$$X^{(\mu_1\cdots\mu_n)} \longrightarrow X^{\mu(n)},$$

$$Y^{(\mu_1\cdots\mu_n}X^{\mu_{n+1}\cdots\mu_{n+m})} \longrightarrow Y^{\mu(n)}X^{\mu(m)},$$

$$\partial_{\mu_1}\cdots\partial_{\mu_n} \longrightarrow \partial_{\mu(n)}, \quad p_{\mu_1}\cdots p_{\mu_n} \longrightarrow p_{\mu(n)},$$

$$Y^{(c_1\cdots c_n)}X^{(d_1\cdots d_n)}t_{c_1d_1}\cdots t_{c_nd_n} \longrightarrow Y^{c(n)}X^{d(n)}t_{c(n)d(n)}$$

Analysis on gauge symmetries

Higher spin gauge symm. in field theories [C. Fronsdal(1978)]

$$a_{\mu_1\mu_2\cdots\mu_s}(x) \quad : \text{symmetric}, \qquad a^{\nu\lambda}{}_{\nu\lambda\mu_5\cdots\mu_s}(x) = 0$$

$$\delta a_{\mu_1\mu_2\cdots\mu_s}(x) = \partial_{(\mu_s}\lambda_{\mu_1\cdots\mu_{s-1})}(x), \qquad \lambda^{\nu}{}_{\nu\mu_3\cdots\mu_{s-1}} = 0$$

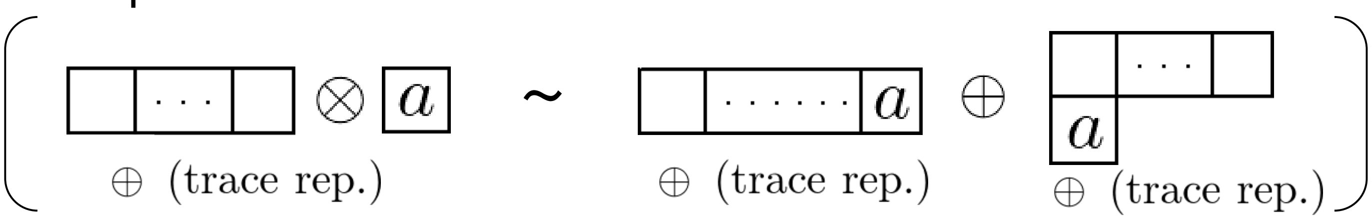
We would like to find out the same symm. In the matrix model.

With a naïve parametrization,

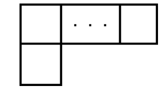
$$A_a = p_a + a_a{}^{\mu_1\cdots\mu_{s-1}}(x)p_{\mu_1}\cdots p_{\mu_{s-1}}, \qquad \Lambda = \lambda^{\mu_1\cdots\mu_{s-1}}(x)p_{\mu_1}\cdots p_{\mu_{s-1}}$$

$$\delta A_a = \{A_a, \Lambda\} \;\Leftrightarrow\; \delta a_a{}^{\mu_1\cdots\mu_{s-1}} = \partial_a\lambda^{\mu_1\cdots\mu_{s-1}} + O(a\times\lambda)$$

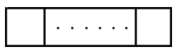
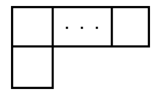
・This trsf. fails to eliminate the longitudinal components, because $a_a{}^{\mu_1\cdots\mu_{s-1}}$ has non-totally symmetric part.



Analysis on gauge symmetries

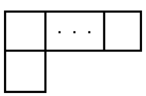
Introduction of an additional gauge parameter removing  part.

$$\Lambda = \underline{\lambda^{\mu(s-1)}} p_{\mu(s-1)} + \underline{\lambda^{c,d\mu(s-2)}} t_{cd} p_{\mu(s-2)}$$

Symmetrized indices

$$b^{a,\mu(s-1)} := \frac{s-1}{s}\left(a^{a|\mu(s-1)} - a^{\mu|a\mu(s-2)}\right)$$

:  part of $a^{a|\mu(s-1)}$

Its trsf. law is

$$\delta b^{a,\mu(s-1)} = \frac{s-1}{s}\left(\partial^a \lambda^{\mu(s-1)} - \partial^\mu \lambda^{a\mu(s-2)}\right) - \lambda^{a,\mu(s-1)} + O(b \times \lambda)$$

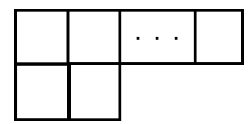
However, $\delta A_a = \{A_a, \Lambda\} \ni \partial_a \lambda^{c,d\mu(s-2)} t_{cd}$

Some additional field required in order to absorb this variation.

Analysis on gauge symmetries

$$A^a = p^a + \ a^{a|\mu(s-1)} p_{\mu(s-1)} + \omega^{a|c,d\mu(s-2)} t_{cd} p_{\mu(s-2)}$$

$$\Lambda = \lambda^{\mu(s-1)} p_{\mu(s-1)} + \lambda^{c,d\mu(s-2)}\, t_{cd}\, p_{\mu(s-2)} + \frac{1}{2} \underline{\lambda^{c_1 c_2, d_1 d_2 \mu(s-3)}}\, t_{c_1 d_1} t_{c_2 d_2}\, p_{\mu(s-3)}$$



Decomposing the new field as $\omega^{a|c,d\mu(s-2)} = \omega^{ac,d\mu(s-2)} + \omega^{[a|c],d\mu(s-2)}$,
their trsf. laws are

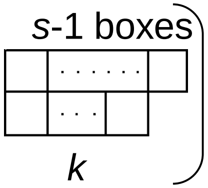
$$\begin{cases} \delta\omega^{ac,d\mu(s-2)} = \partial^{(a}\lambda^{c),d\mu(s-2)} - \lambda^{ac,d\mu(s-2)} \\ \delta\omega^{[a|c],d\mu(s-2)} = -\dfrac{2(s-1)}{s}\delta(\partial^{[a}a^{c]|d\mu(s-2)}) \end{cases}$$

- $\omega^{ac,d\mu(s-2)}$ can be removed, but $\delta A^a = \{A^a, \Lambda\} \ni \partial^a \lambda^{c_1 c_2, d_1 d_2 \mu(s-3)}\, t_{c_1 d_1} t_{c_2 d_2} p_{\mu(s-3)}$ is…
- $\omega^{[a|c],d\mu(s-2)}$ can be written in terms of $a^{a|\mu(s-1)}$ by imposing "the generalized torsion-free condition":

$$\omega^{[a|c],d\mu(s-2)} = -\frac{2(s-1)}{s}\partial^{[a}a^{c]|d\mu(s-2)}$$

Analysis on gauge symmetries

We repeat the same procedure. Introducing $\omega^{a|b(k),\mu(s-1)}$ $(1 \leq k \leq s-1)$, and analyzing the gauge trsf. laws, we find that



part can be removed by gauge trsf., while the other part can be written with the lower-rank field.

$$\delta\omega^{ab(k),\mu(s-1)} = \partial^{(a}\lambda^{b(k)),\mu(s-1)} - \lambda^{ab(k),\mu(s-1)} \qquad (k \leqq s\text{-}2)$$

$$\omega^{[a|b]b(k-1),\mu(s-1)} = -\frac{2(s-1-k)}{s}\partial^{[a}\omega^{b]|b(k-1),\mu(s-1)} \qquad (k \leqq s\text{-}1) \qquad \cdots(\bigstar)$$

(★) is consistent with the Bianch id. for the lowest GTCs;

$$\partial^{[a}\omega^{b|c],\mu(s-1)} \propto \partial^{[a}\partial^{b}a^{c]|\mu(s-1)} = 0$$

As for the highest-rank field $\omega^{a|b(s-1),\mu(s-1)}$, we have no gauge parameter to remove it. However, the GTC for it can be solved and the gauge variation for it is consistent.

just like that the spin connection can be written with vielbein through torsion-free cond..

$$\omega^{[\mu|a],b} = -\partial^{[\mu}e^{a]|,b} \quad \Rightarrow \quad \omega^{\mu|a,b} = \omega^{\mu|a,b}(e)$$

Analysis on gauge symmetries

As a result,

The suitable parametrization is

$$A^a = a^{a|\mu(s-1)} p_{\mu(s-1)} + \sum_{n=1}^{s-1} \frac{1}{n!} A^a_{(n)},$$

$$A^a_{(n)} = \omega^{a|c(n),d(n)\mu(s-1-n)}\, t_{c(n)d(n)}\, p_{\mu(s-1-n)}$$

Using the gauge trsf.s and GTC,s it can be written with totally-symmetric part of $a^{a|\mu(s-1)}$.

$$A^a = \sum_{n=0}^{s-1} \frac{1}{n!} \partial^{c(n)} a^{ad(n)\mu(s-1-n)} t_{c(n)d(n)} p_{\mu(s-1-n)},$$

Furthermore, there is a recidual gauge symmetries holding the above gauge-fixing;

$$\Lambda = \sum_{n=0}^{s-1} \frac{s-n}{n!} \partial^{c(n)} \lambda^{d(n)\mu(s-1-n)} t_{c(n)d(c)} p_{\mu(s-1-n)}$$

$$\delta A^a = \{A^a, \Lambda\} \Leftrightarrow \delta a^{a\mu(s-1)} = \partial^a \lambda^{\mu(s-1)} + O(a \times \lambda)$$

$$\sim \partial^{(a} \lambda^{\mu(s-1))}$$

Discussion on equations of motions

The structures of EoM

Apparently, the single equation of matrix leads to *s* equations of the field:

$$[A^b,[A^a,A_b]]=0 \;\Leftrightarrow\; \underbrace{(\mathrm{EOM}_1)^{a\mu(s-1)}}_{=0} p_{\mu(s-1)} + \underbrace{(\mathrm{EOM}_2)^{c,ad\mu(s-2)}}_{=0} t_{cd}p_{\mu(s-2)} + \cdots = 0$$

We find that among the equations, only the (*s*-1)-th and *s*-th ones is nontrivial; the others are identically zero.

$$\begin{aligned}[A^b,[A_a,A_b]]=0 \;\Leftrightarrow\; & 0\cdot p_{\mu(s-1)} + 0\cdot t_{cd}p_{\mu(s-2)} + \cdots \\ & + \underbrace{\left(\Box\partial^{c(s-2)}a^{a\mu d(s-2)} - \partial^{c(s-2)}\partial^{(a}\partial_b a^{\mu)bd(s-1)} + \partial^{a\mu}\partial^{c(s-2)}a_b^{\;bd(s-2)}\right)}_{1} t_{c(s-2)d(s-2)}p_{\mu} \\ & + \underbrace{\left(\partial^a\partial^{c(s-1)}\partial^b a_b{}^{d(s-1)} - \Box\partial^{c(s-1)}a^{ad(s-1)}\right)}_{2} t_{c(s-1)d(s-1)} = 0\end{aligned}$$

(EoM (2) can be derived from EoM (1) by taking a derivative and anti-symmetrizing the indices.)

$$\Rightarrow\quad \boxed{\partial^{c(s-2)}\left[\Box a^{a\mu d(s-2)} - \partial^{(a}\partial_b a^{\mu)bd(s-1)} + \partial^{a\mu}a_b^{\;bd(s-2)}\right] = 0.}$$

Discussion on equations of motions

Rewriting the EoM…

$$\eta_{cc} R^{c(s),d(s)} = 0,$$

$$R^{c(s),d(s)} = \partial^{c(s)} a^{d(s)}$$ with [c- and d-] indices unti-symmetrized.

: the generalized curvature

[B. de Witt & D. Z. Freedman (1980)]

$$\begin{array}{|c|c|c|}\hline c & \cdots\cdots & c \\ \hline d & \cdots\cdots & d \\ \hline \end{array}$$

It is the only gauge-invariant quantity without a traceless condition
(and we indeed have no such condition in the matrix model)

In conclusion,

・higher spin fields in the matrix model with the operator interpretation represent the generalized curvature, rather than Fronsdal fields.

・The EoM is the condition that the trace of the generalized curvature vanishes.

・It includes s derivatives, making it difficult to understand the dynamics.

Summary

Summary

・We have studied the gauge symmetries of higher spin fields that emerge in the IIB matrix model with the operator interpretation.

・In order to equip the appropriate gauge symmetries, we have to introduce auxiliary fields of non-totally symmetric representation. However, a part of them can be written with totally symmetric fields, while the rest part can be removed by gauge transformations.

・We have still residual gauge symmetry that remove the longitudinal components of the totally-symmetric field.

・As far as we fous on the kinetic terms of EoM, the field is generalized curvatures, not Fronsdal fields.

Open questions

- Does the homogeneous terms in gauge trsf.s cause any problem? $(\delta a \sim \lambda + O(\{a\} \times \{\lambda\})$
- What is the physical meaning of the generalized curvatures in the matrix model? (EoM for the spin-*s* curvature contains *s* derivatives… how do we discuss the stability?)
- What about fields in the general class, which depends on the bundle coor. *g*?

Thank you!

Back up

Definition

More details…

$U_i,\ U_j$: local patches of $\mathcal{M}$

For $x \in U_i \cap U_j,\ t_{ij}(x)$: transition function

Then
$$\begin{aligned}\nabla^{[i]}_{(a)} &= R_{(a)}{}^{b}(g^{-1})\nabla^{[i]}_{b} \\ &\to R_{(a)}{}^{b}(g^{-1})R_{b}{}^{c}(t_{ij}(x))\nabla^{[j]}_{c} \\ &= R_{(a)}{}^{c}(((t_{ij}(x)^{-1}g)^{-1})\nabla^{[j]}_{c} \\ &= \nabla^{[j]}_{(a)}\end{aligned}$$

This equation means that each of $\nabla_{(1)}, \nabla_{(2)}, \cdots$ is a scalar operator, hence defined globally.