Black resonators and geons in AdS_5

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and work in progress

with Keiju Murata, Jorge Santos, Benson Way

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Introduction

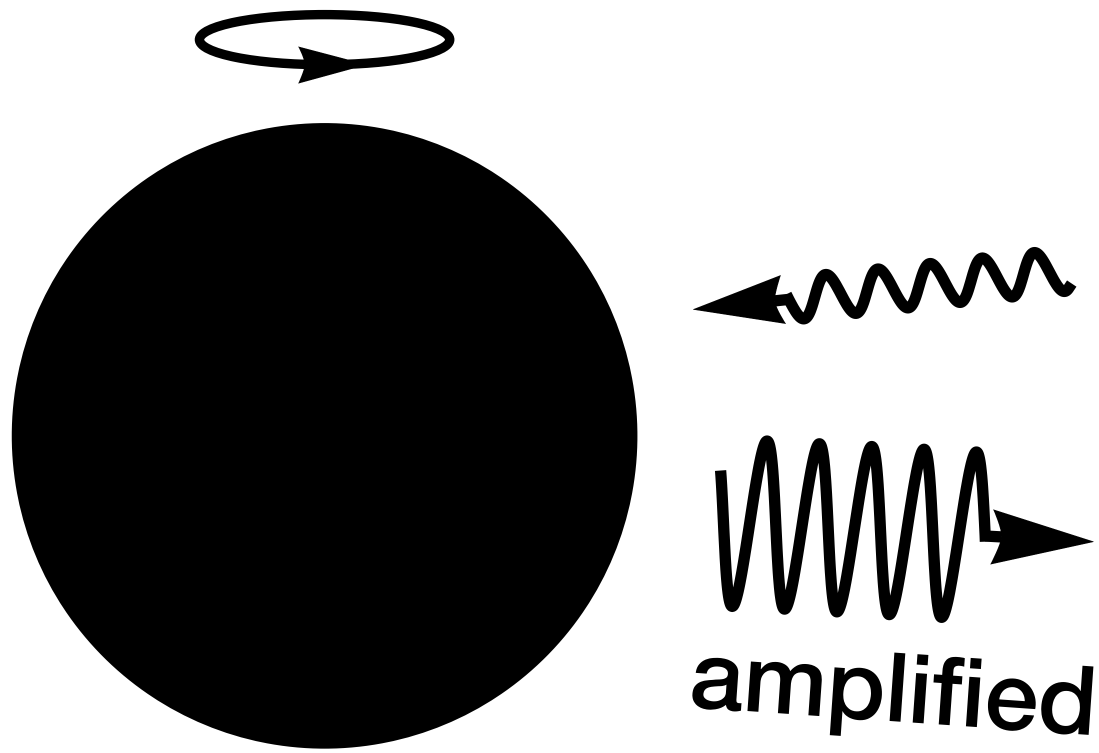
Motivations

Gravity in **higher dimensions** and **AdS spacetime**

Non-uniqueness and various black hole solutions

Instabilities and **dynamics** of such black holes

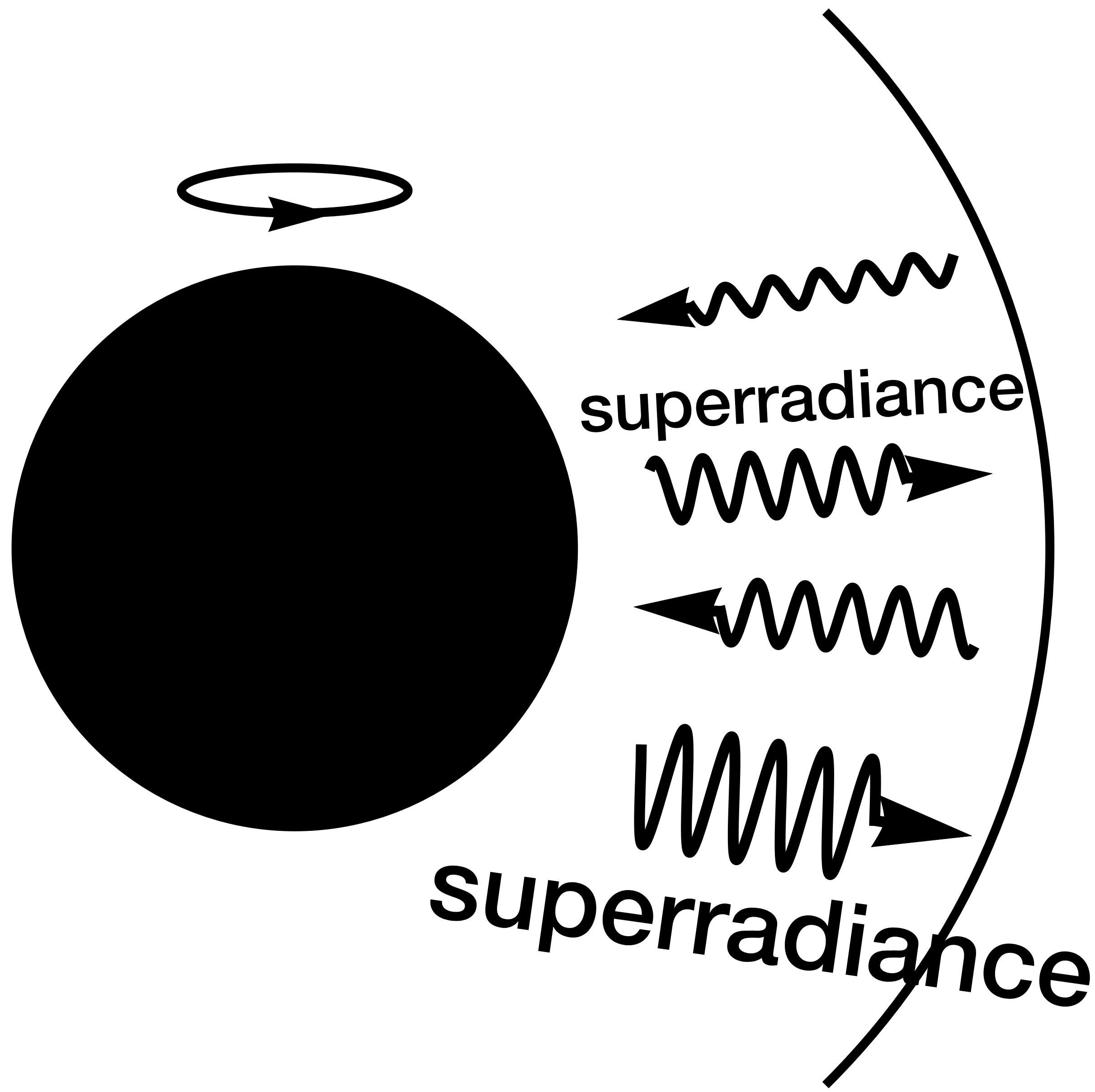
Superradiance



Rotational superradiance: Waves can be amplified by a rotating BH.

(cf. charged superradiance)

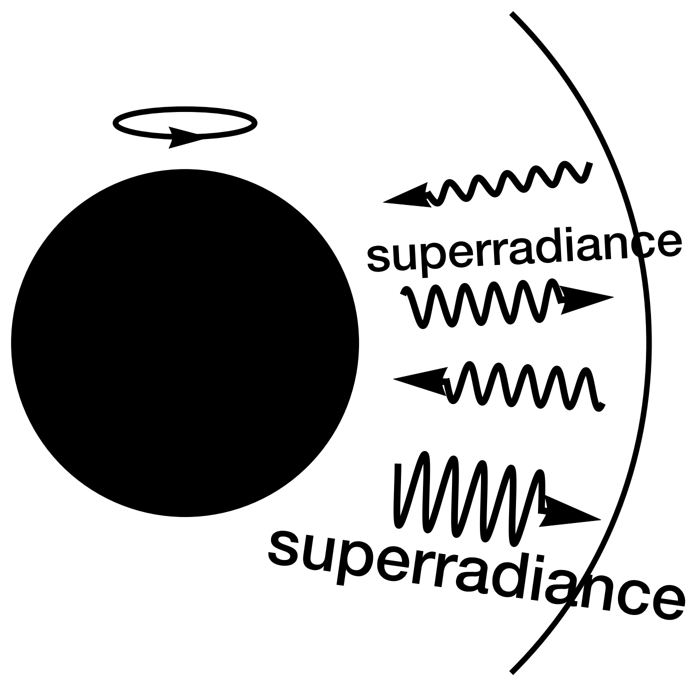
Superradiant instability

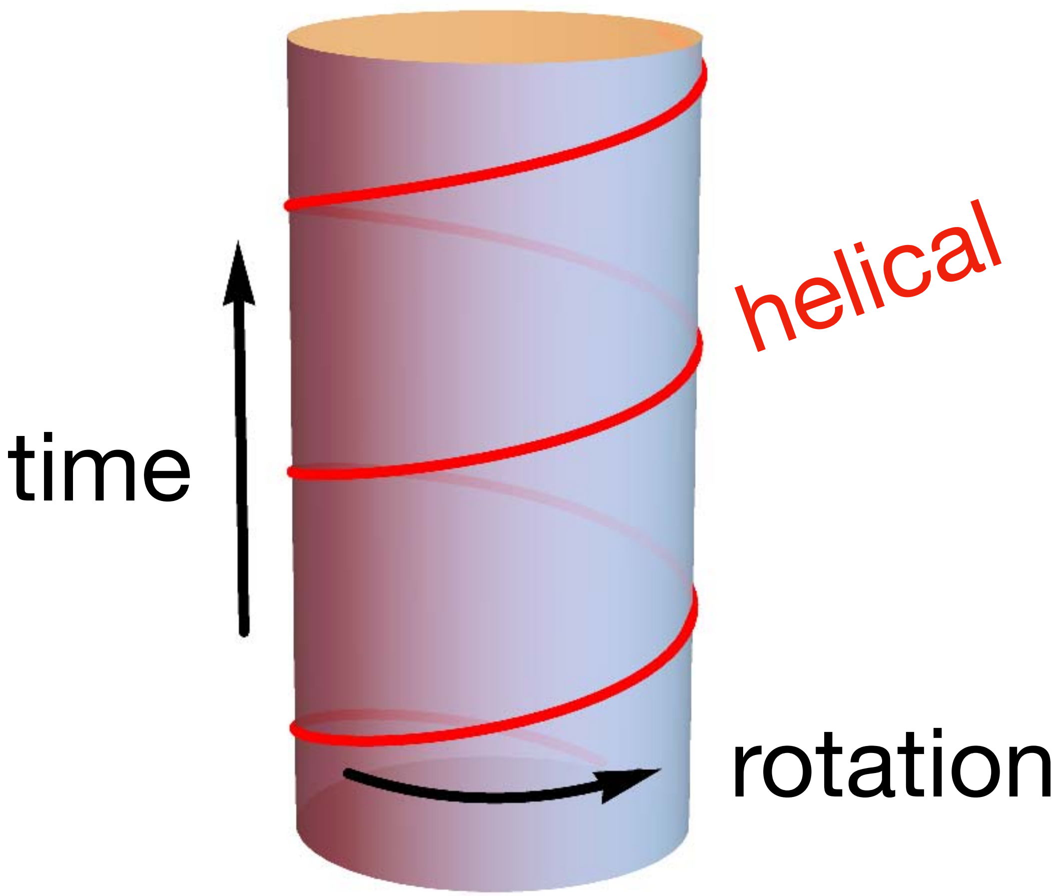


In AdS, the repetition of superradiance gives rise to an **instability.**

[Kunduri-Lucietti-Reall]

Backreaction





A new solution with less symmetries will bifurcate from the onset of the instability.

[Kunduri-Lucietti-Reall]

Black resonators

**Black holes with a single Killing vector field:
black resonators**

Óscar J. C. Dias,[1,*] Jorge E. Santos,[2,†] and Benson Way[2,‡]

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[2]*DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK*

We numerically construct asymptotically anti-de Sitter (AdS) black holes in four dimensions that contain only a single Killing vector field. These solutions, which we coin *black resonators*, link

arXiv:1505.04793 [hep-th]

Time-periodic black holes were constructed in 4D AdS and named **black resonators.**

This talk

Black resonators were first obtained by solving PDEs in 4D AdS.

[Dias-Santos-Way]

$$\mathrm{d}s^2 = \frac{L^2}{(1-y^2)^2}\Bigg[-y^2 q_1 \Delta(y)\left(\mathrm{d}\tau + y\, q_6 \mathrm{d}y\right)^2 + \frac{4y_+^2\, q_2 \mathrm{d}y^2}{\Delta(y)} + \frac{4y_+^2\, q_3}{2-x^2}\left(\mathrm{d}x + yx\sqrt{2-x^2}\, q_7 \mathrm{d}y + y^2 x\sqrt{2-x^2}\, q_8 \mathrm{d}\tau\right)^2$$
$$+ (1-x^2)^2 y_+^2 q_4 \left(\mathrm{d}\phi - y^2 q_5 \mathrm{d}\tau + \frac{x\sqrt{2-x^2} q_9 \mathrm{d}x}{1-x^2} + y\, q_{10} \mathrm{d}y\right)^2\Bigg], \quad (6)$$

In 5 dimensions, we can obtain a class of black resonators by solving ODEs.

[TI-Murata]

Geons

- coined by Wheeler as
  "gravitational and electromagnetic entities"

- horizonless

- appear from normal modes in global AdS

Black resonators smoothly reduce to geons in the zero-size limit of the horizon.

Contents

Myers-Perry AdS BH with equal angular momenta

Setup

5D pure Einstein gravity (AdS radius = 1)

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\,(R-2\Lambda)\,, \qquad \Lambda = -6$$

Asymptotically global AdS (RxS^3 boundary at r=∞)

$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_3^2$$

Isometries of 5D black holes

Schwarzschild: $R_t \times SO(4) = R_t \times SU(2) \times SU(2)$

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\Omega_3^2$$

General Myers-Perry: $R_t \times U(1) \times U(1)$

Myers-Perry with equal angular momenta:

$$R_t \times U(2) = R_t \times U(1) \times SU(2)$$

⇒ will be broken to a helical one

MPAdS$_5$ with equal angular momenta

$$ds^2 = -F(r)d\tau^2 + \frac{dr^2}{G(r)} + \frac{r^2}{4}\left[\underbrace{\sigma_1^2 + \sigma_2^2}_{S^2} + B(r)\underbrace{(\sigma_3 + 2H(r)d\tau)^2}_{S^1 \text{ fiber}}\right]$$

SU(2) invariant 1-forms (θ,ϕ,χ: Euler angles of S^3)

$$\begin{aligned}\sigma_1 &= -\sin\chi d\theta + \cos\chi \sin\theta d\phi \\ \sigma_2 &= \quad \cos\chi d\theta + \sin\chi \sin\theta d\phi \\ \sigma_3 &= d\chi + \cos\theta d\phi\end{aligned}$$

U(1) isometry: $\chi \to \chi + c$

(Note that $\sigma_1^2 + \sigma_2^2 = d\theta^2 + \sin^2\theta d\phi^2$)

Boundary condition of H(r) and frames

	Non-rotating frame	**Rotating frame**
Boundary	$\bar{H}(\infty) = 0$	$H(\infty) = \Omega$
Horizon	$\bar{H}(r_h) = -\Omega$	$H(r_h) = 0$

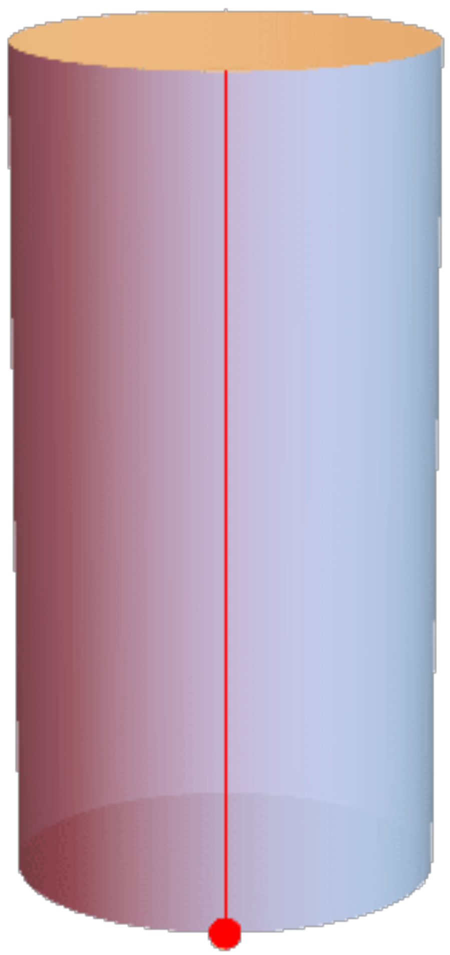
Rotating frame at infinity (τ,χ): The AdS boundary is rotating with Ω, and the horizon does not.

Non-rotating frame at infinity (t,ψ): vice versa.

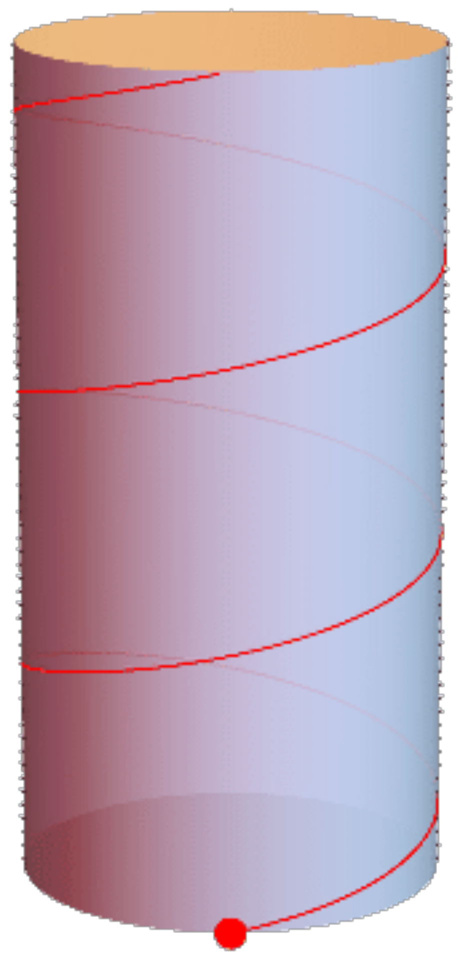
Transformation: $t = \tau, \quad \psi = \chi + 2\Omega\tau$

Null generator of horizon

$$K = \partial_\tau = \partial_t + \Omega \partial_{\psi/2}$$

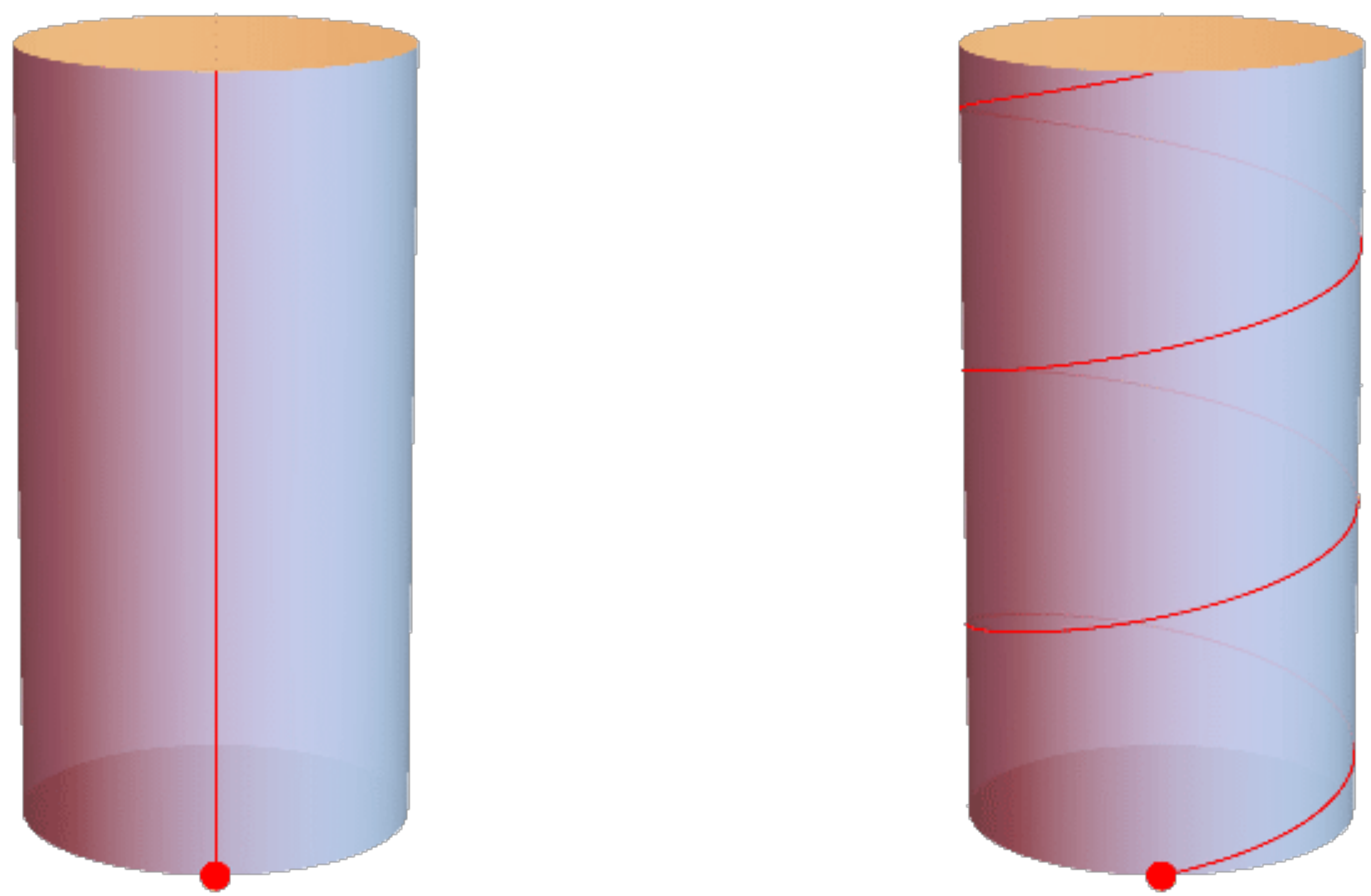


Rotating frame
(horizon is static)



Non-rotating frame
(horizon rotates)

Helical Killing vector



Rotating frame
(rotating boundary)

Non-rotating frame
(static boundary)

Helical Killing vector: $K = \partial_\tau = \partial_t + \Omega \partial_{\psi/2}$

Superradiant instability

U(1)-breaking perturbation

To break the U(1), we unbalance $\sigma_1^2 + \sigma_2^2$.

Perturbation in the rotating frame

$$\delta g_{\mu\nu} dx^\mu dx^\nu = \frac{r^2}{4}\delta\alpha(r)(\sigma_1^2 - \sigma_2^2)$$

In the non-rotating frame, this is **time periodic**.

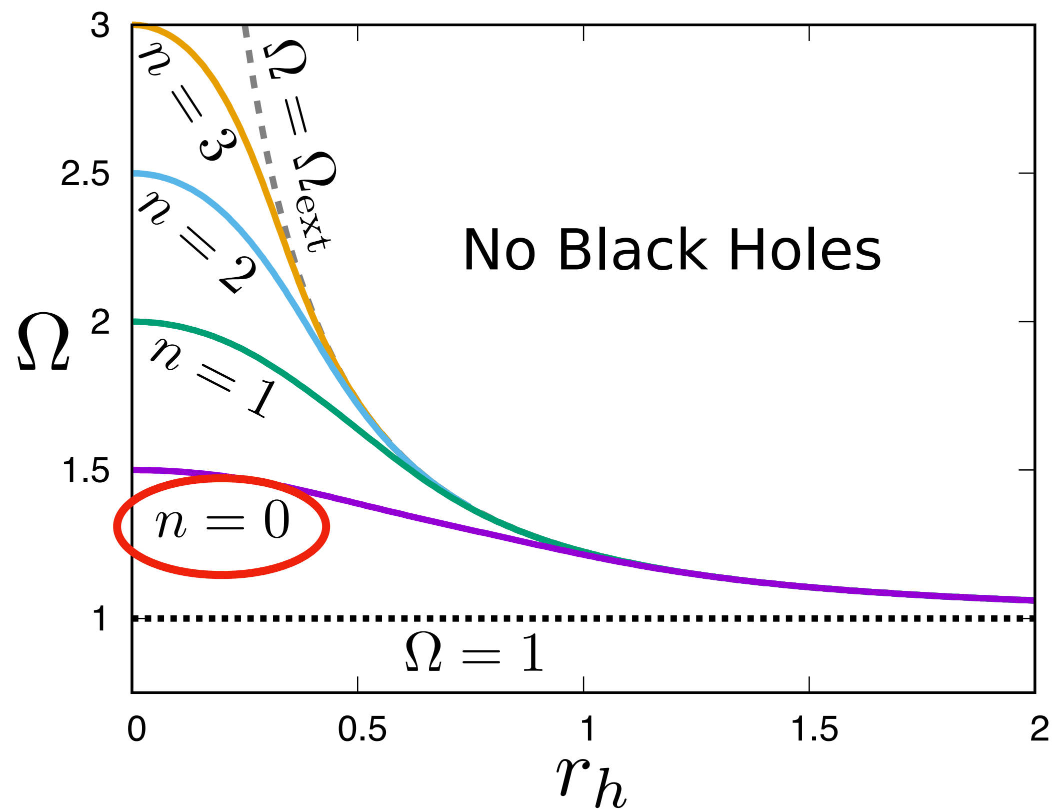
$$\delta g_{\mu\nu} dx^\mu dx^\nu = \frac{r^2}{2}\delta\alpha(r)(e^{4i\Omega t}\bar{\sigma}_+^2 + e^{-4i\Omega t}\bar{\sigma}_-^2)$$

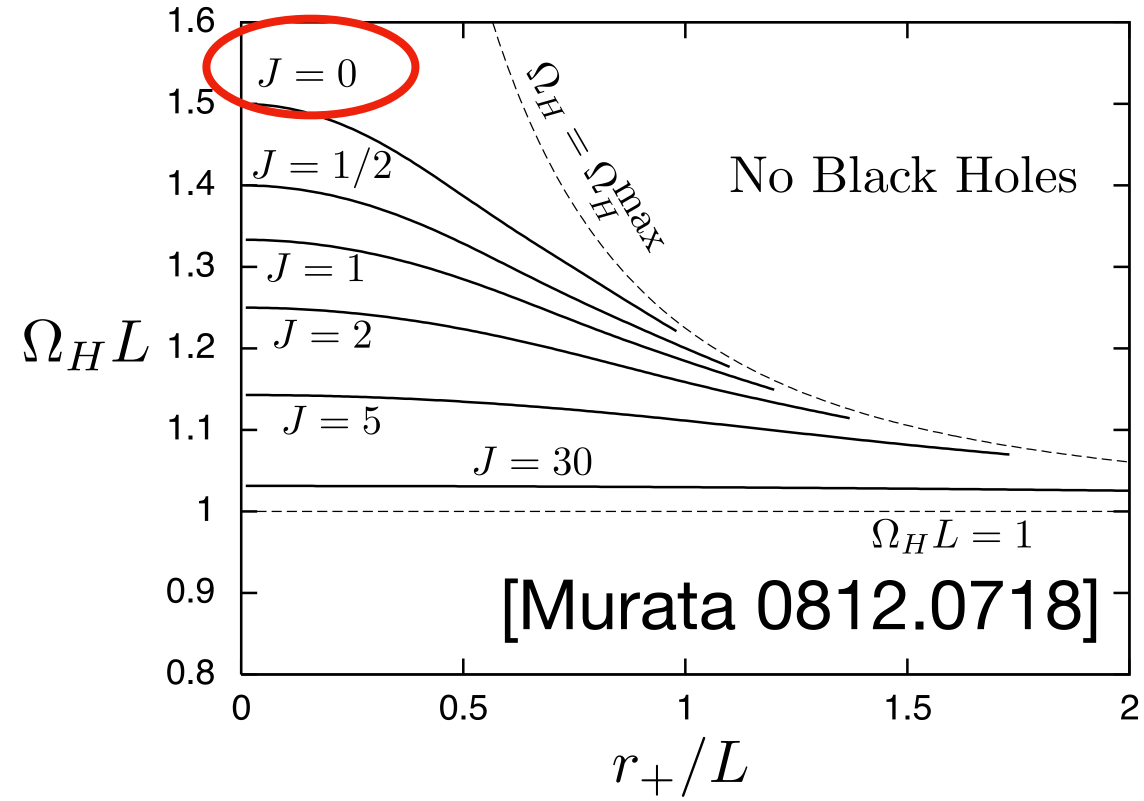
$$\sigma_\pm \equiv (\sigma_1 \mp i\sigma_2)/2 \qquad \sigma_\pm = e^{\pm 2i\Omega t}\bar{\sigma}_\pm$$

Onset of superradiant instability

As Ω is increased, the perturbation becomes unstable.

New solutions bifurcate from the onset of instabilities.





Black resonators and geons

Cohomogeneity-1 metric ansatz

In the rotating frame, we use

$$ds^2 = -(1+r^2)f(r)d\tau^2 + \frac{dr^2}{(1+r^2)g(r)}$$
$$+\frac{r^2}{4}\left[\alpha(r)\sigma_1^2 + \frac{1}{\alpha(r)}\sigma_2^2 + \beta(r)(\sigma_3 + 2h(r)d\tau)^2\right]$$

This is **time periodic** in the non-rotating frame.

$$\alpha\sigma_1^2 + \frac{1}{\alpha}\sigma_2^2 = 2\left(\alpha + \frac{1}{\alpha}\right)\bar{\sigma}_+\bar{\sigma}_- + \left(\alpha - \frac{1}{\alpha}\right)(e^{4i\Omega t}\bar{\sigma}_+^2 + e^{-4i\Omega t}\bar{\sigma}_-^2)$$

Isometries: $R_\tau \times SU(2)$

helical

Einstein equations

$$
\begin{aligned}
f' =& \frac{1}{r(1+r^2)^2 g\alpha^2(r\beta'+6\beta)}[4r^2h^2(\alpha^2-1)^2\beta \\
&+ r(r^2+1)g\{r(1+r^2)f\alpha'^2\beta - r^3h'^2\alpha^2\beta^2 - 2(2+3r^2)f\alpha^2\beta'\} \\
&- 4(1+r^2)f\{6r^2\alpha^2\beta(g-1)+3g\alpha^2\beta+(\alpha^2-\alpha\beta+1)^2-4\alpha^2\}] \\
g' =& \frac{1}{6r(1+r^2)^2 f\alpha^2\beta}[-4r^2h^2(\alpha^2-1)^2\beta \\
&+ r(1+r^2)g\{-r(1+r^2)f\alpha'^2\beta + r^3h'^2\alpha^2\beta^2 \\
&- (-r(1+r^2)f'+2f)\alpha^2\beta'\} + 4(1+r^2)f\{-6r^2\alpha^2\beta(g-1)-3g\alpha^2\beta \\
&\qquad + \alpha^4+4\alpha^3\beta-5\alpha^2\beta^2-2\alpha^2+4\alpha\beta+1\}] \\
h'' =& \frac{1}{2r^2(1+r^2)\alpha^2\beta fg}[8fh(\alpha^2-1)^2 \\
&- r(1+r^2)h'\alpha^2\{r(fg'\beta - f'g\beta+3fg\beta')+10fg\beta\}] \\
\alpha'' =& \frac{1}{2r^2(1+r^2)^2 f\alpha g\beta}[2r^2(r^2+1)^2 fg\alpha'^2\beta \\
&- r(r^2+1)\alpha\alpha'\{r(1+r^2)(fg\beta)'+2(3+5r^2)fg\beta\} \\
&- 8(\alpha^2-1)\{r^2h^2\beta(\alpha^2+1)-(1+r^2)f\alpha(\alpha-\beta)-(1+r^2)f\}] \\
\beta'' =& \frac{1}{(2r^2(1+r^2))fg\alpha^2\beta}[-2r^4gh'^2\alpha^2\beta^3 \\
&- r\alpha^2\beta'\{r(1+r^2)(f'g\beta+fg'\beta-fg\beta')+2(3+5r^2)fg\beta\} \\
&- 8f\beta(\alpha^4+\alpha^3\beta-2\alpha^2\beta^2-2\alpha^2+\alpha\beta+1)]
\end{aligned}
$$

Einstein equations

$$f' = \frac{1}{r(1+r^2)^2 g\alpha^2(r\beta' + 6\beta)}[4r^2h^2(\alpha^2-1)^2\beta$$

$$+ r(r^2+1)g\{r(1+r^2)f\alpha'^2\beta - r^3h'^2\alpha^2\beta^2 - 2(2+3r^2)f\alpha^2\beta'\}$$

Coupled **ODEs** for (f',g',h'',α'',β'').

$$g' = \frac{1}{6r(1+r^2)^2 f\alpha^2\beta}[-4r^2h^2(\alpha^2-1)^2\beta$$

$$+ r(1+r^2)g\{-r(1+r^2)f\alpha'^2\beta + r^3h'^2\alpha^2\beta^2$$

$$- (-r(1+r^2)f' + 2f)\alpha^2\beta'\} + 4(1+r^2)f\{-6r^2\alpha^2\beta(g-1) - 3g\alpha^2\beta$$

Boundary conditions:

1) **Asymptotically AdS** with h(r=∞)=Ω

2) Geon: **regular** at r=0

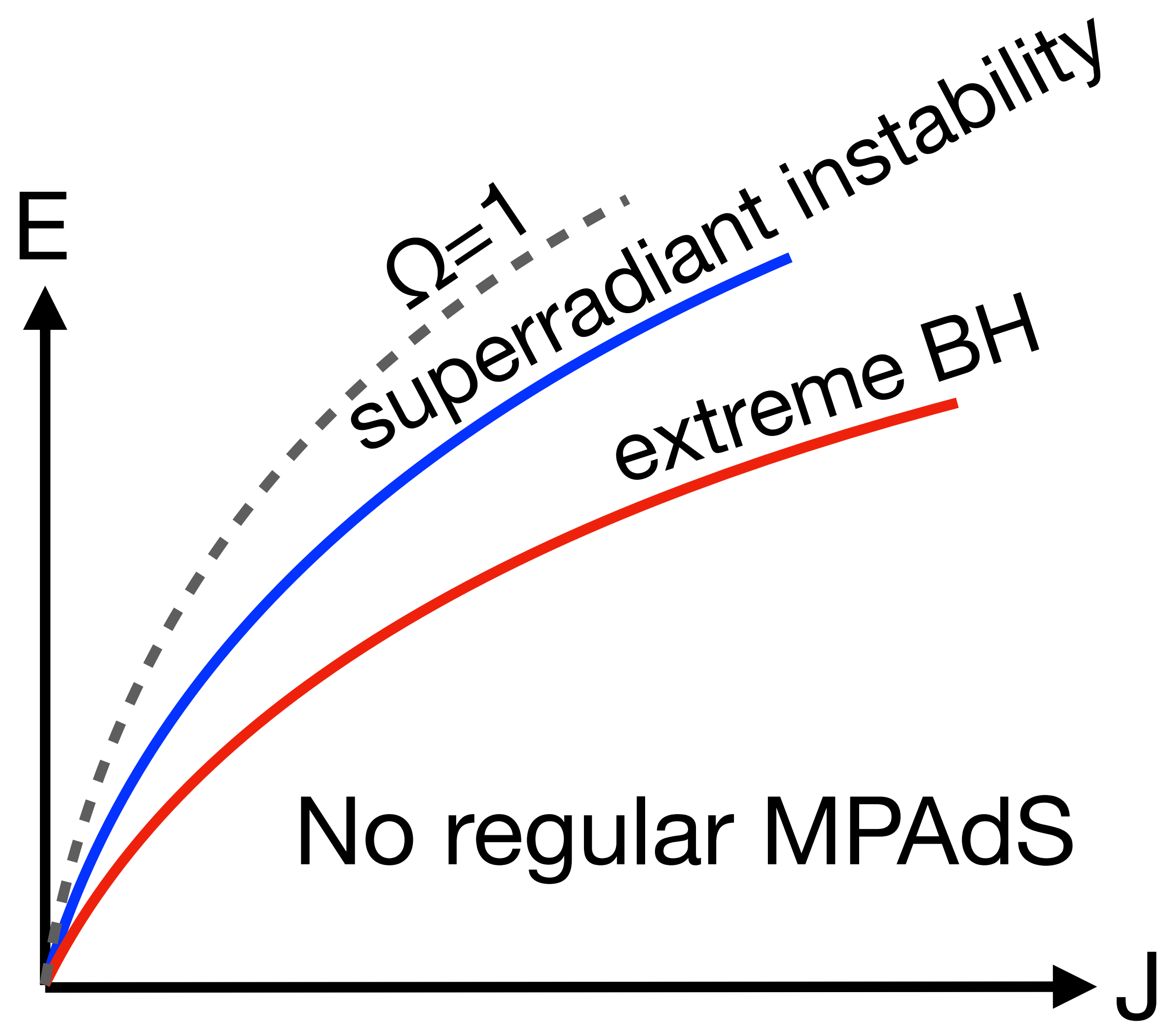
Black resonator: **horizon** at r=r_h

$$\beta'' = \frac{1}{(2r^2(1+r^2))fg\alpha^2\beta}[-2r^4gh'^2\alpha^2\beta^3$$

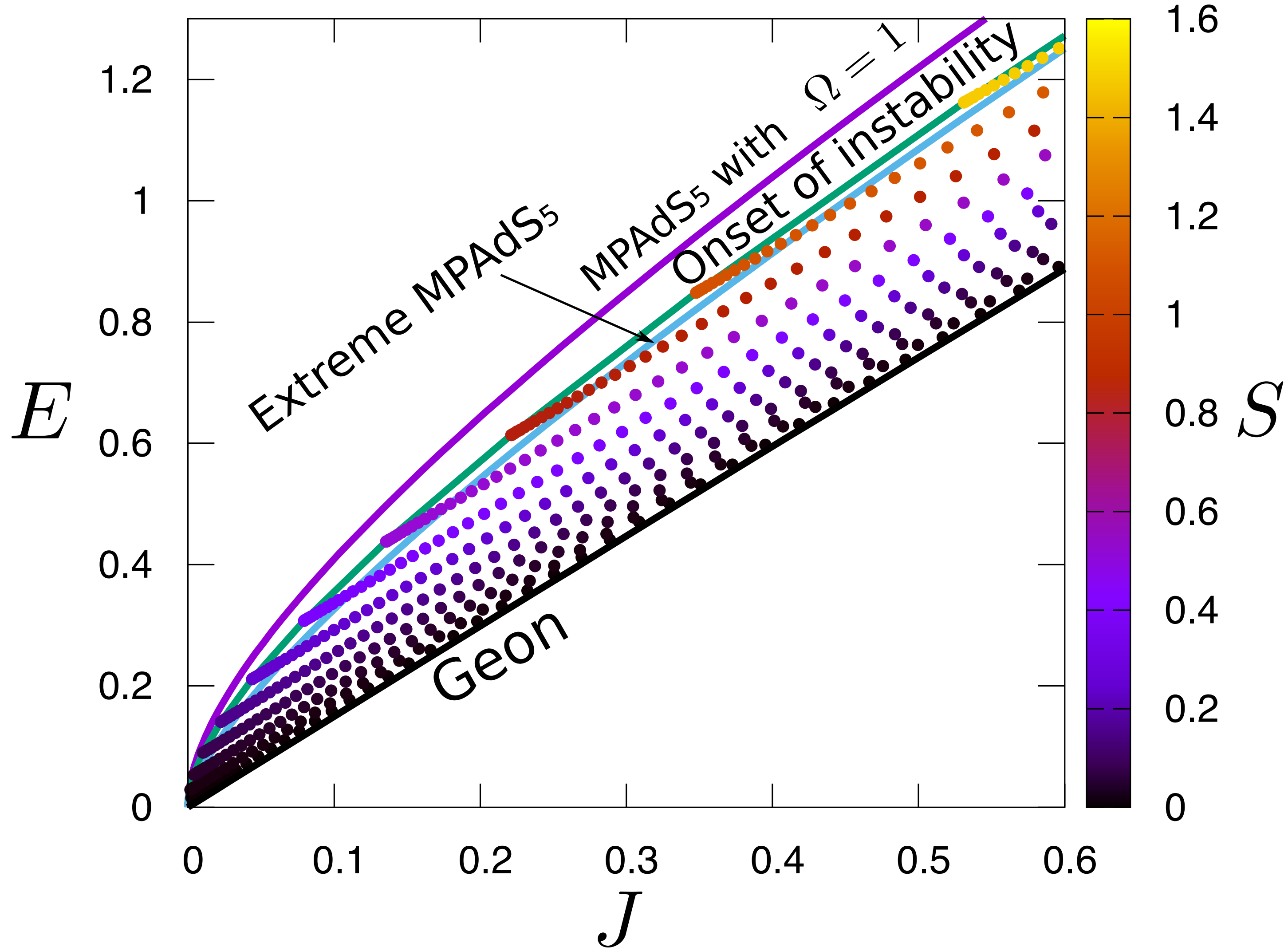
$$- r\alpha^2\beta'\{r(1+r^2)(f'g\beta + fg'\beta - fg\beta') + 2(3+5r^2)fg\beta\}$$

$$- 8f\beta(\alpha^4 + \alpha^3\beta - 2\alpha^2\beta^2 - 2\alpha^2 + \alpha\beta + 1)]$$

(E,J) diagram for MPAdS$_5$

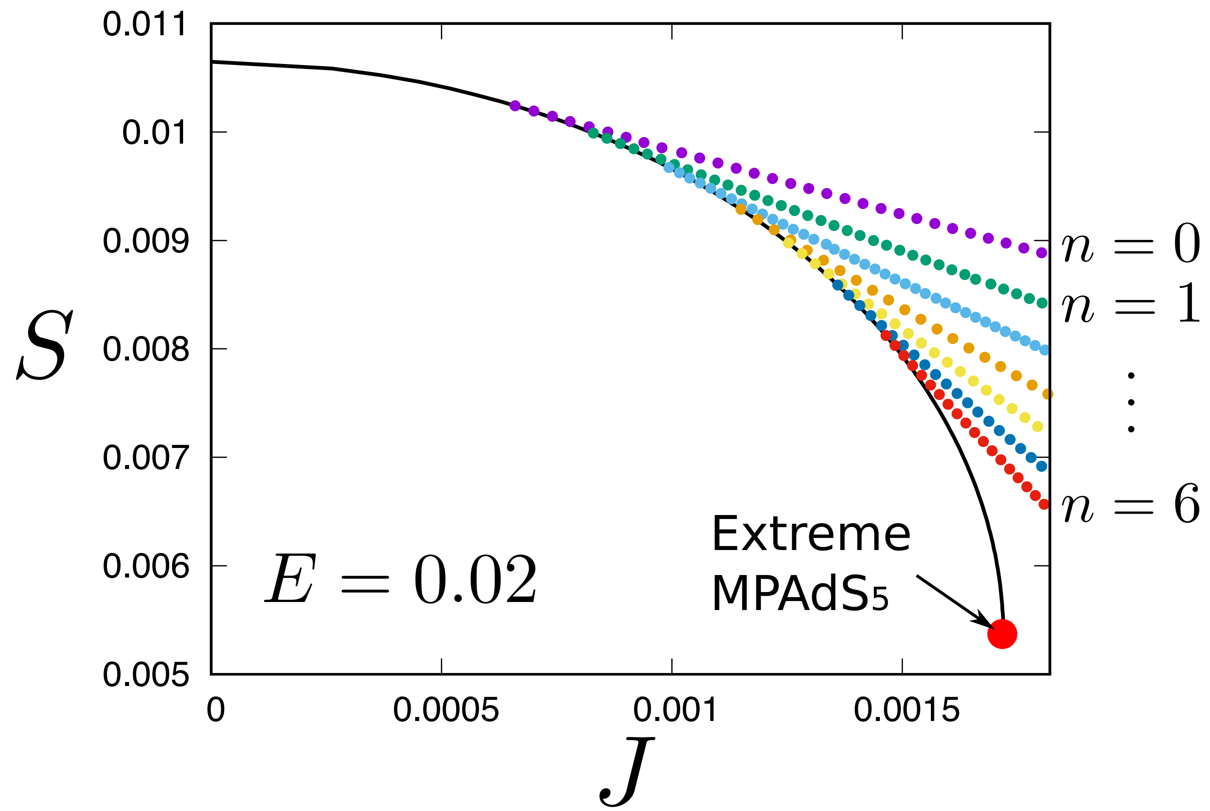


(E,J) diagram for black resonators



Black resonators extend to the (E,J) region beyond the limit of extreme MPAdS.

Entropy



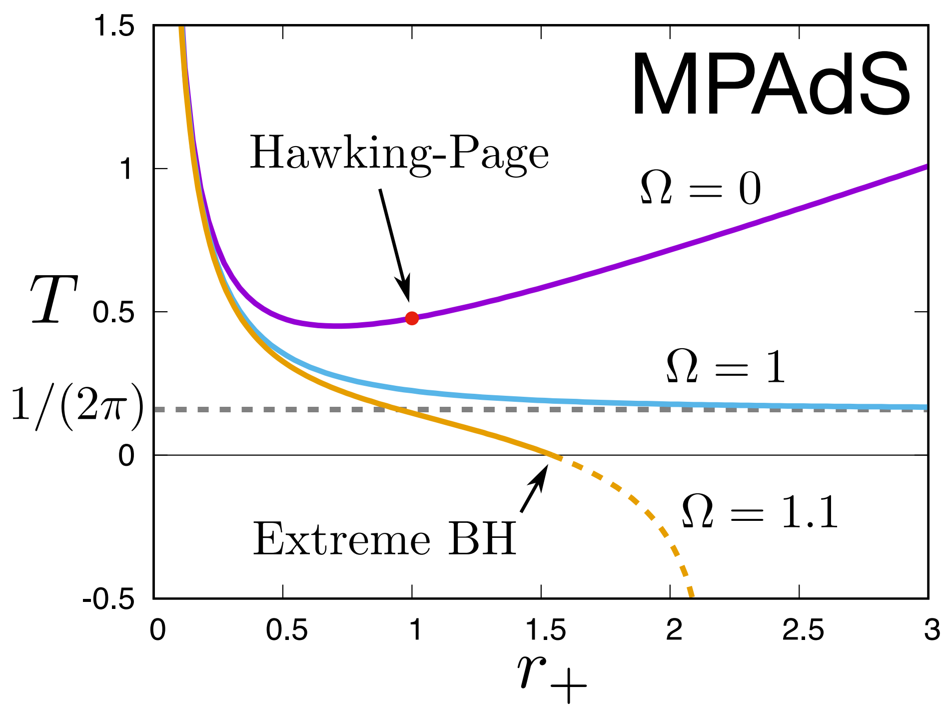
Black resonators have higher entropy than MPAdS.

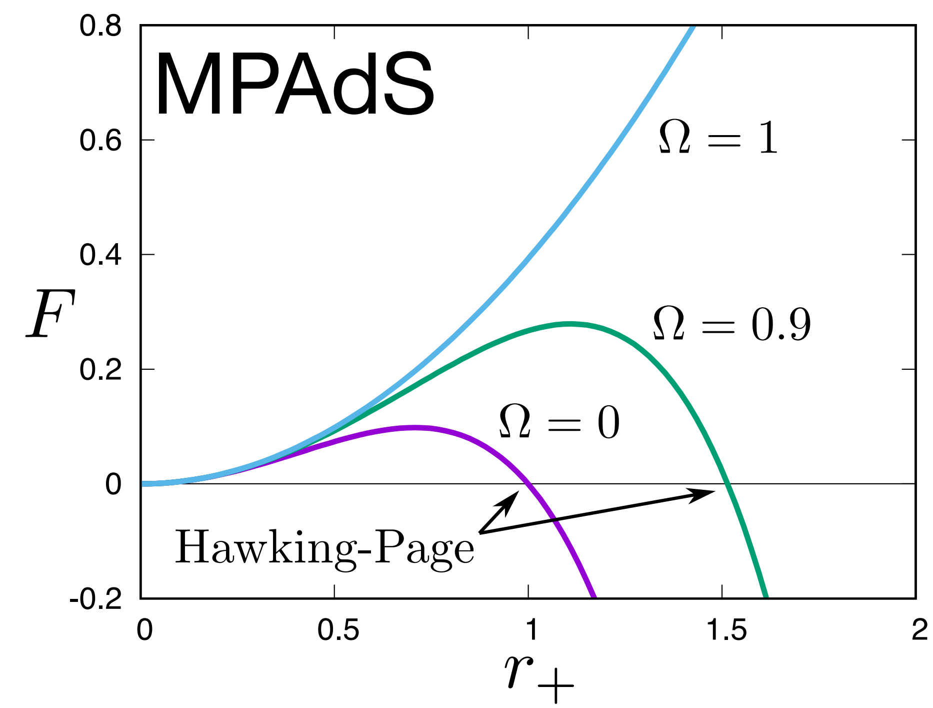
An unstable MPAdS can evolve into a black resonator.

Implications to AdS/CFT

The black resonators obtained so far have **Ω>1.**

But, in fact, BH with Ω>1 are **small BH.**





Are there (unstable) states dual to black resonators?

Ongoing results

Instability of black resonators

Theorem: a BH with $\Omega > 1$ is always unstable (against some perturbations).

[Green-Hollands-Ishibashi-Wald]

Black resonator solutions (obtained so far) have $\Omega > 1$. Hence, **they must be unstable.**

The cohomogeneity-1 metric is useful to study **perturbations of black resonators.**

Instability of black resonators

Scalar, Maxwell, and gravitational perturbations:

$$\nabla^2\phi = 0$$

$$\nabla^2 h_{\mu\nu} + 2R_{\mu\rho\nu\sigma}h^{\rho\sigma} = 0$$

$$\nabla^\mu F_{\mu\nu} = 0$$

We find instabilities in general perturbations which **break the SU(2) isometry.**

[TI-Murata-Santos-Way, to appear]

$$\Phi = e^{-i\omega\tau} \sum_{j,m,k} \Phi_{jmk}(r) Y^j_{mk}(\Omega_3)$$

Adding matter field

In the presence of a Maxwell field, we can obtain (uncharged) **black resonators dressed with photons.**

[TI-Murata, to appear]

$$
\begin{aligned}
ds^2 = &-(1+r^2)f(r)d\tau^2 + \frac{dr^2}{(1+r^2)g(r)} \\
&+ \frac{r^2}{4}\left[\alpha(r)\sigma_1^2 + \frac{1}{\alpha(r)}\sigma_2^2 + \beta(r)(\sigma_3 + 2h(r)d\tau)^2\right]
\end{aligned}
$$

and

$$A = \gamma(r)\sigma_1$$

Conclusion

We constructed **black resonators** and **geons** with a **cohomogeneity-1 metric in 5D AdS**.

They appear from the **superradiant instability** of **MPAdS BH with equal angular momenta.**

They have **helical** and **SU(2) isometries.**

This metric can be used to study properties of black resonators such as instability.