UV Behavior of $\mathcal{N} = 8$ Supergravity at Five Loops

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based on

Phys.Rev. D98 no.8, 086021 (1804.09311)
Phys.Rev. D96 no.12, 126012 (1708.06807)
Phys.Rev.Lett. 118 no.18, 181602 (1701.02519)

With Bern,, Carrasco,,Edison, Johansson, Parra-Martine, Roiban, Zeng

Motivation

- Gravity theories are with dimensionful couplings: non-renormalizable, UV divergent at some loop order

- Can supersymmetry help make gravity UV finite?

- More symmetries, more restriction on counter terms, better UV behavior
  ➡ maximal SUSY+ gravity:

  $4D \quad \mathcal{N} = 8 \quad \text{SUGRA}$ a perturbatively UV-finite theory?

- Some signs: UV-finiteness at L-loop 4-point, $L < 5$.

Bern, Carrasco, Dixon, Dunbar, Johansson, Kosower, Perelstein, Roiban, Rozowsky

Motivation

- critical dimension: characterizes the UV behavior
  the dimension where the UV div. first appears

$$\mathscr{A}\,\Big|_{\substack{\ell \gg k_i \\ \text{leading}}} \sim \int (d\ell^{D_c})^L \ell^{-x}, \; D_c = \frac{x}{L} \quad .$$

- Interestingly, $D_c^{L,\,\mathcal{N}=4\ \text{SYM}} = D_c^{L,\,\mathcal{N}=8\ \text{SUGRA}}, \quad L < 5\,.$

Bern, Carrasco, Dixon, Dunbar, Johansson, Kosower, Perelstein, Roiban, Rozowsky

- KLT relation: tree level scattering amplitudes

$$(\mathcal{N} = 4\ \text{SYM})^2 \sim (\mathcal{N} = 8\ \text{SUGRA})$$

all loops UV finite $\xrightarrow{??}$ all loops UV finite

Motivation

- the next target:
  5-loop 4-point scattering amplitude of $\mathcal{N} = 8$ SUGRA

- check the UV behavior: explicit computation

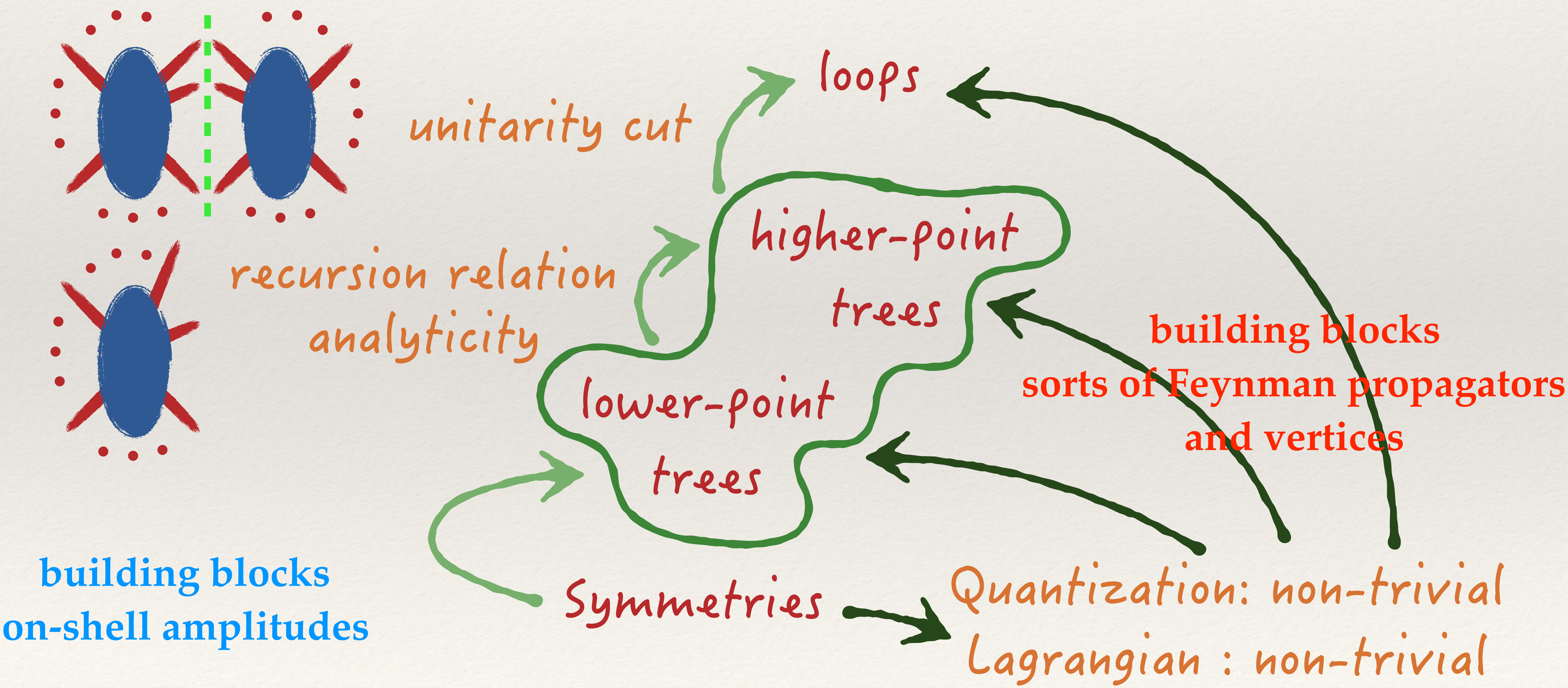
  Lagrangian formalism (almost undoable, hard)

  On-shell methods (more efficient, no so hard)

On shell methods

Modern on-shell approach:
recycle known info.
no gauge redundancy
efficiency

Lagrangian formalism:
compute case by case
gauge redundancy
complication



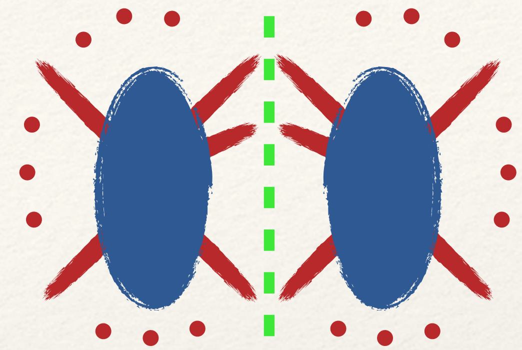
On shell methods

tree amplitudes: symmetries + analyticity

loop amplitudes: tree amp. + unitarity

On shell methods

Unitarity Cut



$$S = 1 + iT\,, \qquad S\,S^\dagger = 1$$

$$\Rightarrow \quad 2\,\mathrm{Im}\,T = T\,T^\dagger$$

results from $i\epsilon$ part of propagators:
taking imaginary part is taking the cuts,
(putting the relevant propagators on-shell)

the product
of sub-amplitudes.

Generalized Unitarity Cut

Bern, Dixon, Dunbar, Kosower

recursively apply cuts to a loop amplitude,
(cuts on a loop amplitude)=(a product of tree amplitudes)

On shell methods

guiding principle:

a correct loop amp. needs
to satisfy all unitarity cuts

On shell methods

How to reconstruct a loop integral:

Design an ansatz,
the ansatz is required to satisfy all unitarity cuts.

The size of ansatz for SUGRA is usually
too large to be controlled.

Color-kinematic duality is helpful.

Tree level: KLT $\Longleftrightarrow$ color-kinematic duality

On shell methods

Idea: loop version of KLT
two copies of $\mathcal{N} = 4$ SYM give $\mathcal{N} = 8$ SUGRA

spectrum: $(\mathcal{N} = 4 \text{ SYM}) \otimes (\mathcal{N} = 4 \text{ SYM}) = (\mathcal{N} = 8 \text{ SUGRA})$

- the size of ansatz for $\mathcal{N} = 4$ is more accessible, less combinatorial possibilities.

$$[\text{cubic vertices}]_{\mathcal{N}=4} = \mathrm{M}, \quad [\text{cubic vertices}]_{\mathcal{N}=8} = \mathrm{M}^2$$

$$[\text{propagators}]_{\mathcal{N}=4} = [\text{propagators}]_{\mathcal{N}=8}$$

- represent the ansatz by cubic diagrams, each cubic vertex is a color factor f_{abc} .

On shell methods

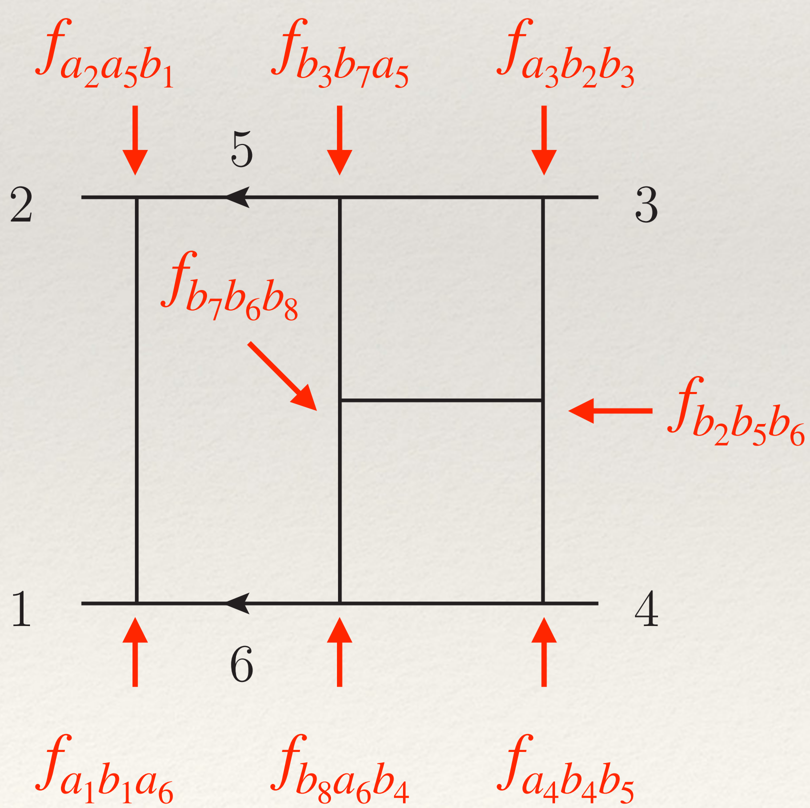
use cubic diagrams to represent the basis of our loop integrand

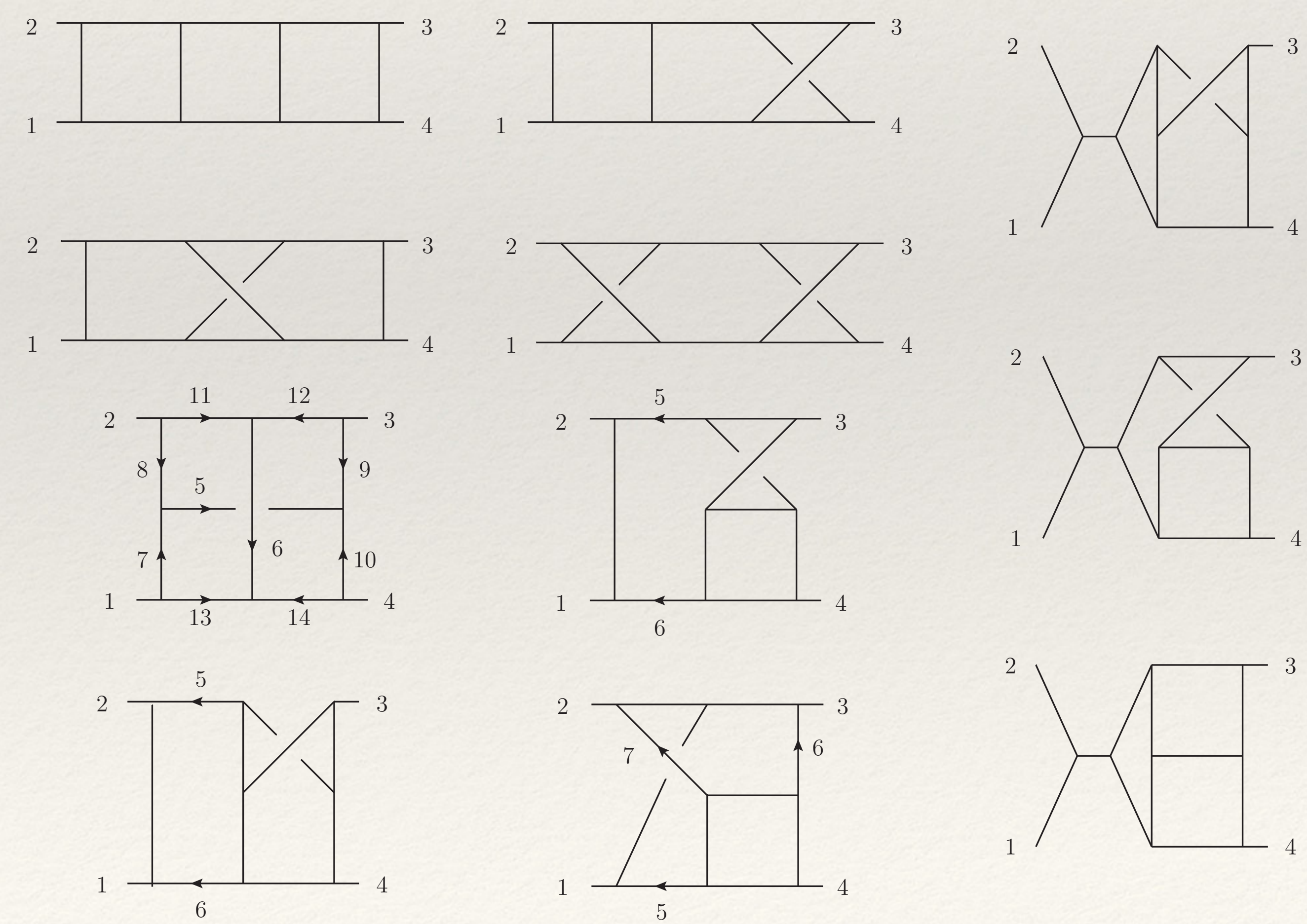
parameters determined by unitarity cuts

$$\frac{f_{a_1b_1a_6}\cdots f_{a_2a_5b_1}}{\ell_5^2\ell_6^2\cdots}\left[Y(1)k_1\cdot k_2+Y(2)\ell_5^2+Y(3)\ell_6^2+\ldots\right] = \frac{c_k n_k}{\ell_5^2\ell_6^2\cdots}$$

c_k : color factor
fixed by graph

n_k : kinematic factor
the part needs an ansatz





Color-Kinematic Duality

Bern, Carrasco, Johansson

$$A^{\mathcal{N}=4} = \sum_i \int d\ell_1^D \ldots d\ell_L^D \frac{c_i n_i}{P_{\alpha_i}}$$

- For a given diagram with color factor $c_i = \prod_{q_i} f_{q_i}$, there must exist c_j and c_k such that $c_i + c_j + c_k = 0$ by Jacobi identity
- If we can find a representation, where n_i's satisfy the same algebraic eqs. as c_i's, we can get

$$M^{\mathcal{N}=8} = \sum_i \int d\ell_1^D \ldots d\ell_L^D \frac{\tilde{n}_i n_i^{ck}}{P_{\alpha_i}}$$

Color-Kinematic Duality

color-kinematic duality

$$A^{\mathcal{N}=4} \xrightarrow{c_i \to n_i} M^{\mathcal{N}=8}$$

works in L-loop, $L < 5$

ck rep. is still unavailable for $L = 5$

Generalized Double Copy

No more ansatz…

For an arbitrary known $\mathcal{N}=4$ SYM rep.:

$$A^{\mathcal{N}=4} = \sum_i \int d\ell_1^D \ldots d\ell_L^D \frac{c_i n_i}{P_{\alpha_i}}$$

- a given diagram with color factor $c_i = \prod f_{q_i}$, there must exist c_j and c_k such that $c_i + c_j + c_k = 0$ by Jacobi identity, but, in general, $n_i + n_j + n_k \neq 0$.
- $n_i + n_j + n_k \equiv J_{\ldots}$, on a cut

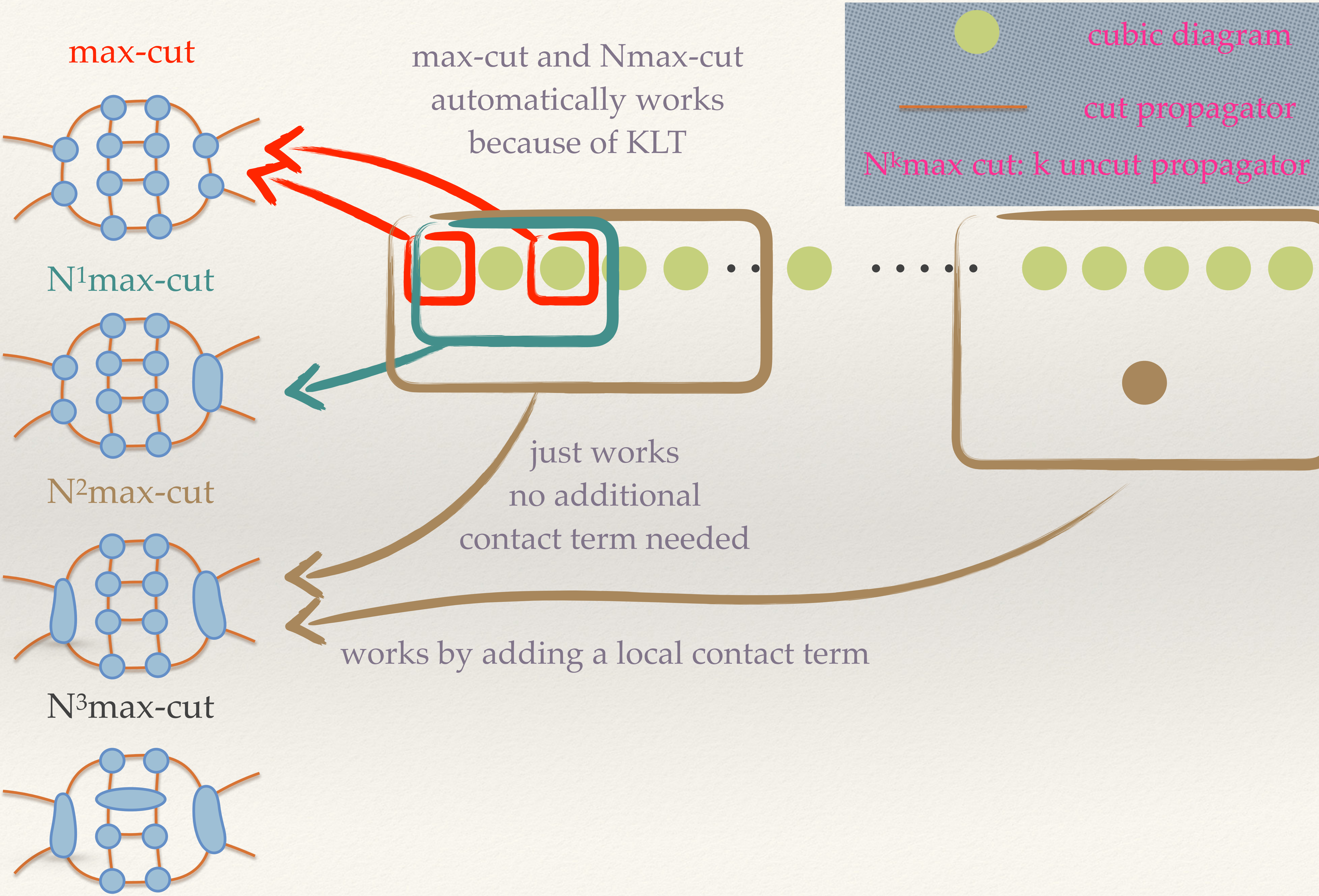
Bern, WMC, Carrasco, Johansson, Roiban, PRL

$$\left[(\text{the result of a } \mathcal{N}=8 \text{ cut}) - \sum_i \frac{n_i^2}{P_{\alpha_i}}\right]\Bigg|_{cut} = g(J_{\ldots}, \text{inverse pp.})$$

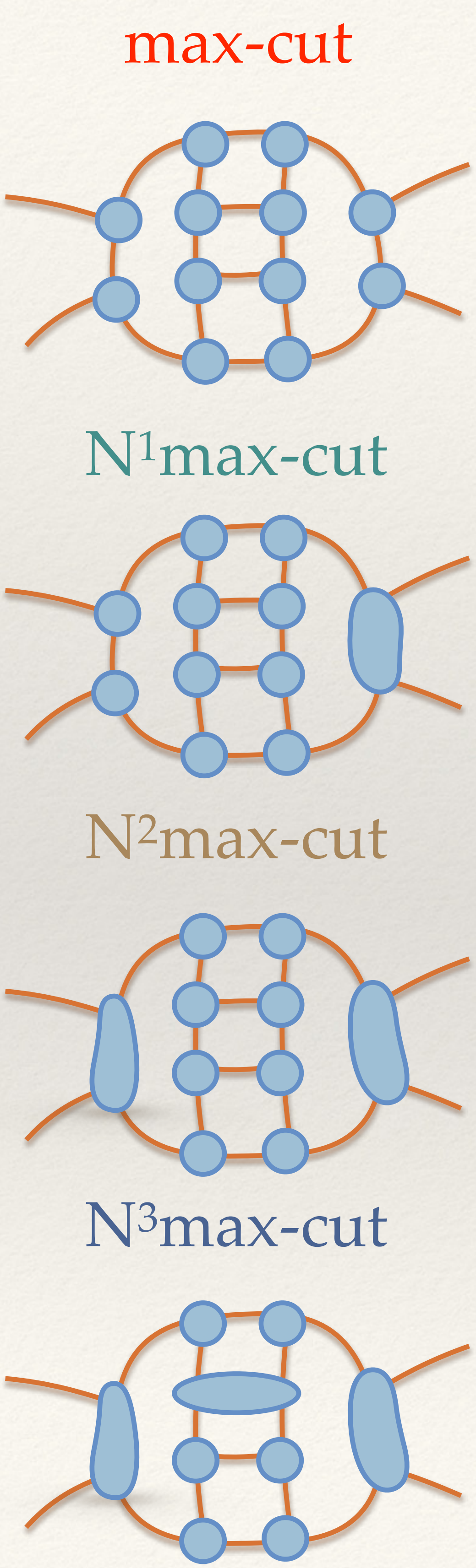
naive double copy

cut discrepancy

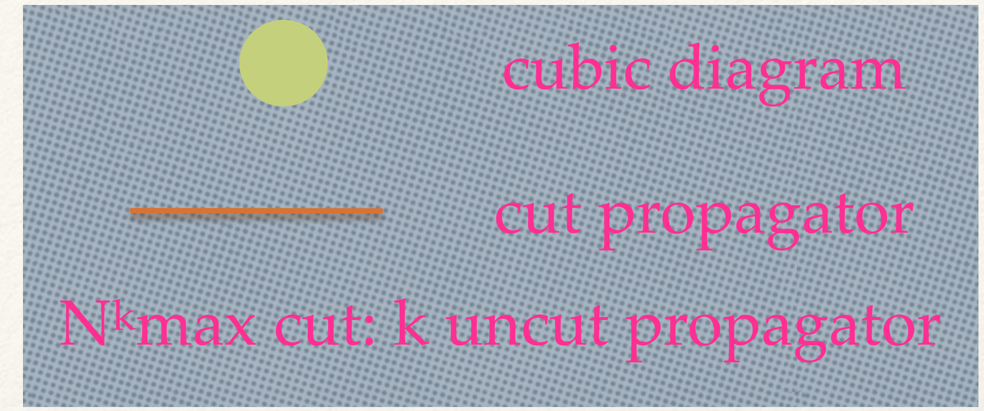
Systematic organization of cuts

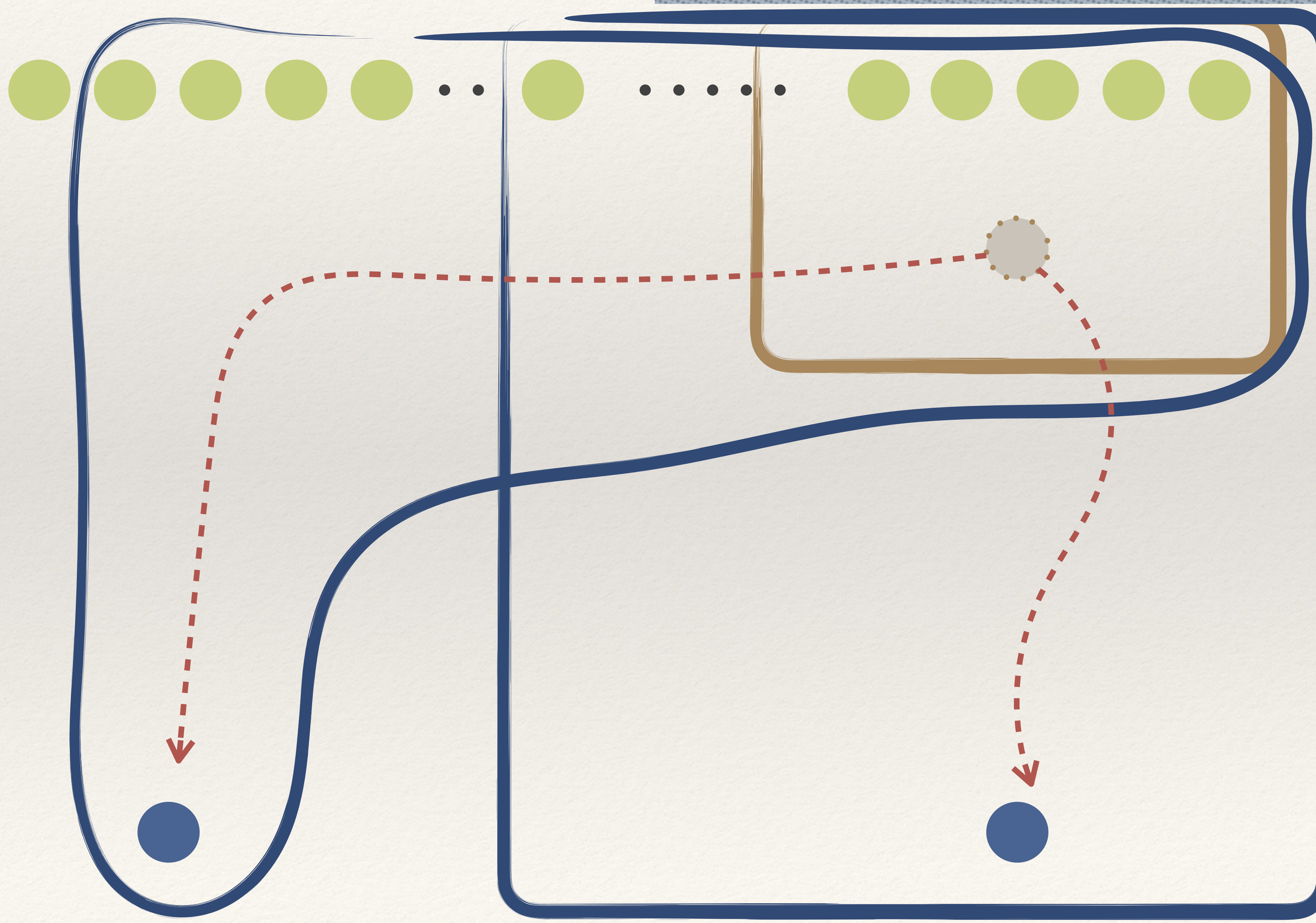


Systematic organization of cuts



the two non-local terms include
some duplicated information





Contact Term Approach

- We need to work level by level in order to make every contact term local
- N^2 contact terms: $I_2^i|_{cut}$ = (N^2max-cut-i) - ($I_0\,|_{cut}$) $\quad I_2^i|_{cut} \overset{\text{off-shell}}{\Longrightarrow} I_2^i$
- complete result satisfies all max-cut, N^1max-cut, N^2max-cut: $P_2 = I_0 + \sum_i I_2^i$
- N^3 contact terms: $I_3^i|_{cut}$ = (N^3max-cut-i) - ($P_2\,|_{cut}$) $\quad I_3^i|_{cut} \overset{\text{off-shell}}{\Longrightarrow} I_3^i$
- complete result satisfies all max-cut,…, N^3max-cut: $P_3 = I_0 + \sum_i I_2^i + \sum_i I_3^i$
- For $\mathcal{N}$=8 5-loop, the result is

Bern, WMC, Carrasco,,Edison, Johansson, Parra-Martine, Roiban, Zeng

Level	No. Diagrams	No. Nonvanishing Diagrams
0	752	649
1	2,781	0
2	9,007	1,306
3	17,479	2,457
4	22,931	2,470
5	20,657	1,335
6	13,071	256
total	86,678	8,473

Five-loop results

- check N[7]max-cuts and N[8]max-cuts to make sure everything is correct
- small external momenta expansion to extract UV divergence

see Beneke, Vladimirov, Marcus, Sagnotti, Smirnov

The results

Bern, WMC, Carrasco,,Edison, Johansson, Parra-Martine, Roiban, Zeng

first order

leading log divergence

$$\mathcal{M}_4^{(5)}\Big|_{k\to 0} \sim \int (d\ell^{D_c-2\epsilon})^5 \frac{\ell^{10}}{(\ell^2)^{16}} \Rightarrow 5D_c - 22 = 0 \Rightarrow D_c = \frac{22}{5}$$

$$\mathcal{M}_4^{(5)}\big|_{\text{leading}}^{D=22/5} = 0$$

second order

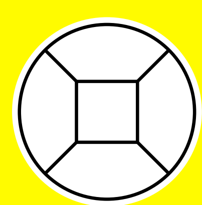
$$\frac{k\cdot\ell}{(\ell^2 + k\cdot\ell + \dots)}$$

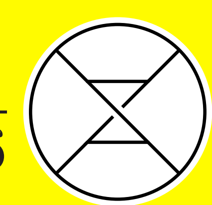
no contribution in the first order

$$\frac{(k\cdot\ell)(k\cdot\ell)}{(\ell^2 + k\cdot\ell + \dots)^2} \sim \frac{1}{\ell^2} \Rightarrow D_c = \frac{24}{5}$$

critical dim. is raised

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16\times 629}{25}\left(\frac{\kappa}{2}\right)^{12}(s^2+t^2+u^2)^2 stu M_4^{\text{tree}}\left(\frac{1}{48}\,\text{[diagram 1]} + \frac{1}{16}\,\text{[diagram 2]}\right)$$





Main Results

- the critical dimensions start to be different between $\mathcal{N}$=8 SUGRA and $\mathcal{N}$=4 SYM at five-loop, but the SUGRA is still finite at $D = 4$.
- The result suggests the existence of D^8R^4 operator at $D = 24/5$.
- D^8R^4 operator is responsible for 7-loop div. at $D = 4$.
- UV-divergence at 7-loop? need to compute 6-loop first.
- Some consistent patterns for vacuum diagrams from higher loop to lower loop, useful for obtaining higher loop result?

Thank you!