Gravity and linearized Schwarzschild Solution in the IKKT Matrix Model

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NCTS-Kyoto Joint Meeting, september 2019 how to formulate quantum theory of spacetime & gravity?

guidelines:

- simple, constructive
- gauge theory (Minkowski signature!)
- finite dof per volume (Planck scale!)
  $\rightarrow$ underlying d.o.f. non-geometric
- GR established only in IR regime
  space-time & gravity may emerge from other d.o.f.

  (cf. Navier-Stokes)

Matrix Models (of Yang-Mills type)

$S = Tr([X^\mu, X^\nu][X_\mu, X_\nu] + ...)$ provide such models!

- simple
- describe dynamical (noncomm.) spaces, gauge theory

  $$X^a \to U^{-1} X^a U$$

- well suited for quantization: $\int dX\, e^{-S[X]}$
  - generic models: serious UV/IR mixing problem
  - preferred model: maximal SUSY = IKKT model

    shares features of string theory, cut the “landscape”
- gravity?

summary of results to be discussed:

- (3+1)-dim. covariant quantum space-time solution
  (FRW cosmology, Big Bounce)
- tower of higher-spin modes, truncated at n.
  $\rightarrow$ higher-spin gauge theory, all d.o.f. required for gravity
  no ghosts!
- propagation governed by universal dynamical metric
  (Lorentz invar. only partially manifest)
- metric perturbations $\rightarrow$ massless graviton & scalar
- linearized Schwarzschild solution

HS, arXiv:1606.00769
HS, arXiv:1710.11495
M. Sperling, HS arXiv:1806.05907
M. Sperling, HS arXiv:1901.03522
HS arXiv:1905.07255, arXiv:1909.xxxxx

outline:

- the IKKT matrix model & NC gauge theory
- *4D covariant quantum spaces*:
  fuzzy H_n^4, cosmological space-time $\mathcal{M}_n^{3,1}$
- fluctuations $\rightarrow$ higher spin gauge theory
- metric perturbations, lin. Schwarzschild

The IKKT model

IKKT or IIB model Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S[Y,\Psi] = -Tr\Big([Y^a,Y^b][Y^{a'},Y^{b'}]\eta_{aa'}\eta_{bb'} \; + m^2 Y^a Y_a \; + \bar\Psi\gamma_a[Y^a,\Psi]\Big)$$

$$Y^a = Y^{a\dagger} \in Mat(N,\mathbb{C})\,, \qquad a = 0,...,9, \qquad N \to \infty$$

gauge invariance $Y^a \to UY^aU^{-1}$, $SO(9,1)$, ~~SUSY~~

- quantized Schild action for IIB superstring
- reduction of $10D$ SYM to point, N large
- equations of motion:

  $\Box Y^a + m^2 Y^a = 0, \qquad \Box \equiv \eta_{ab}[Y^a,[Y^b,.]]$
- quantization: $Z = \int dY d\Psi\, e^{iS[Y]}$, SUSY essential

strategy:

- look for solutions $\rightarrow$ space(time)

  generically non-commutative
- fluctuations $\rightarrow$ gauge theory, dynamical geometry, gravity ?!
- matrix integral = (Feynman) path integral, incl. geometry

(no holography)

relation with string theory:

- solutions = branes
- quantum effects (1-loop) $\rightarrow$ interactions consistent with IIB (etc.) (IKKT, cf. BFSS)

numerical studies possible & underway (Nishimura, Tsuchiya 1904.05919, Kim, Nishimura, Tsuchiya arXiv:1108.1540 ff) class of solutions: fuzzy spaces = quantized symplectic manifolds

$$X^a \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^{9,1}$$

$$[X^a, X^b] \sim \{x^a, x^b\} = i\theta^{ab}(x)$$ ... (quantized) Poisson bracket

algebra of functions on NC (=fuzzy) space: $\mathrm{End}(\mathcal{H})$

- Moyal-Weyl quantum plane $\mathbb{R}^4_\theta$:

$$[X^a, X^b] = i\theta^{ab}\,\mathbb{1}$$

quantized symplectic space $(\mathbb{R}^4, \omega)$

admits translations $X^a \to X^a + c^a \mathbb{1}$, no rotation invariance

- fuzzy 2-sphere S^2_N

$$X_1^2 + X_2^2 + X_3^2 = R_N^2, \qquad [X_i, X_j] = i\epsilon_{ijk}X_k$$

fully covariant under $SO(3)$ (Hoppe; Madore) emergent gravity on deformed $\mathcal{M}^4_\theta$?

H.S. 1003.4134 ff

cf. H.Yang, hep-th/0611174 ff

eff. metric encoded in $\Box = [X_a, [X^a, .]] \sim -e^\sigma(x)\Delta_G$

$$G^{\mu\nu} = e^{-\sigma}\theta^{\mu\mu'}\theta^{\mu\mu'}g_{\mu'\nu'}$$

fluctuations $X^a + \mathcal{A}^a(X)$ $\to$ dynamical metric $\to$ induced gravity?
(cf. Sakharov)

problems:

- $\theta^{\mu\nu}$ breaks Lorentz invariance $\to$ other terms possible
  (e.g. $R_{\mu\nu\alpha\beta}\theta^{\mu\nu}\theta^{\alpha\beta}$ D. Klammer, H.S. 0909.5298)
- full metric fluctuations require transversal brane excitations, non-linear treatment required
- huge cosm. constant

issues seem resolved for covariant quantum spaces:

4D covariant quantum spaces

- in 4D: symplectic form ω breaks local (Lorentz/Euclid.) invar.
- avoided on covariant quantum spaces

  example: fuzzy four-sphere S_N^4

  Grosse-Klimcik-Presnajder; Castelino-Lee-Taylor; Medina-o'Connor; Ramgoolam; Kimura; Abe; Karabail-Nair; Zhang-Hu 2001 (QHE); HS

- noncompact H_n^4 Hasebe 1207.1968 , M. Sperling, HS 1806.05907
- projection of H_n^4 $\to$ cosmological space-time $\mathcal{M}_n^{3,1}$

  HS, 1710.11495, 1709.10480

- $\mathcal{M}_n^{3,1}$ $\to$ gravity, lin. Schwarzschild

  M. Sperling, HS 1901.03522, HS 1905.07255

$\}$ $\to$ higher-spin gauge theory in IKKT model!

Euclidean fuzzy hyperboloid H_n^4 (=$EAdS_n^4$)

Hasebe arXiv:1207.1968, M. Sperling, HS 1806.05907

$\mathcal{M}^{ab}$ ... hermitian generators of $\mathfrak{so}(4,2)$,

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\eta_{ac}\mathcal{M}_{bd} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad} + \eta_{bd}\mathcal{M}_{ac}) \ .$$

$$\eta^{ab} = \mathrm{diag}(-1,1,1,1,1,-1)$$

choose "short" discrete unitary irreps $\mathcal{H}_n$ ("minireps", doubletons)

special properties:

- irreps under $\mathfrak{so}(4,1)$, multiplicities one, minimal oscillator rep.
- positive discrete spectrum

$$\mathrm{spec}(\mathcal{M}^{05}) = \{E_0, E_0+1, ...\}, \qquad E_0 = 1 + \frac{n}{2}$$

lowest eigenspace is $n+1$-dim. irrep of $SU(2)_L$: fuzzy S_n^2

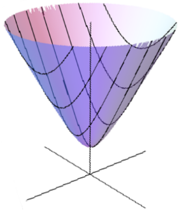
fuzzy hyperboloid H_n^4

def.

$$
\begin{aligned}
X^a &:= r\mathcal{M}^{a5}, \qquad a = 0, \ldots, 4 \\
[X^a, X^b] &= ir^2\mathcal{M}^{ab} =: i\Theta^{ab}
\end{aligned}
$$

5 hermitian generators X^a satisfy (cf. Snyder)

$$
\eta_{ab}X^aX^b = X^iX^i - X^0X^0 = -R^2\mathbb{1}, \qquad R^2 = r^2(n^2 - 4)
$$



one-sided hyperboloid in $\mathbb{R}^{1,4}$, covariant under $SO(4,1)$

note: induced metric = Euclidean AdS^4

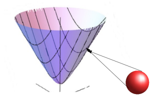
<u>oscillator construction</u>: 4 bosonic oscillators $[\psi_\alpha, \bar\psi^\beta] = \delta_\alpha^\beta$

$\mathcal{H}_n$ = suitable irrep in Fock space

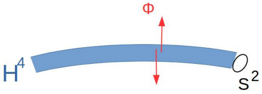
Then

$$\begin{aligned} \mathcal{M}_{ab} &= \bar\psi \Sigma_{ab} \psi, \qquad \gamma_0 = diag(1,1,-1,-1) \\ X^a &= r \bar\psi \gamma^a \psi \end{aligned}$$

fact: $H_n^4 \;=\;$ quantized $\mathbb{C}P^{1,2} = S^2$ bundle over H^4, selfdual $\theta^{\mu\nu}$



functions on $H_n^4 \overset{loc}{\cong} S^2 \times H^4$ = harmonics on $S^2 \times$ functions on H^4



local stabilizer acts on $S^2 \;\rightarrow\;$ harmonics = higher spin modes

fuzzy "functions" on H_n^4:

$$End(\mathcal{H}_n) \rightsquigarrow \quad HS(\mathcal{H}_n) = \int\limits_{\mathbb{C}P^{1,2}} f(m)\, |m\rangle\langle m| \;\cong\; \bigoplus_{s=0}^{n} \mathcal{C}^s$$

$\mathcal{C}^0$ = scalar functions on H^4: $\qquad \phi(X)$

$\mathcal{C}^1$ = selfdual 2-forms on H^4: $\qquad \phi_{ab}(X)\theta^{ab} \;=\; \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$

$\vdots$

$End(\mathcal{H}_n) \cong$ fields on H^4 taking values in $\mathfrak{hs} = \bigoplus \begin{array}{|c|c|c|}\hline & & \\ \hline & & \\ \hline \end{array} \ni \theta^{a_1 b_1} \ldots \theta^{a_s b_s}$

> higher spin modes = would-be KK modes on S^2

i.e. higher spin theory, truncated at n

M. Sperling, HS 1806.05907

H_n^4 is starting point for cosmological quantum space-times $\mathcal{M}_n^{3,1}$:

- exactly homogeneous & isotropic, Big Bounce
- on-shell higher-spin fluctuations obtained
- spin 2 metric fluctuations $\rightarrow$ gravity (linearized)

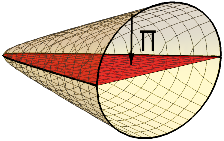
open FRW universe from H_n^4

HS 1710.11495

$\mathcal{M}_n^{3,1}$ = H_n^4 projected to $\mathbb{R}^{1,3}$ via

$$Y^\mu \sim y^\mu : \ \mathbb{C}P^{1,2} \to H^4 \subset \mathbb{R}^{1,4} \xrightarrow{\Pi} \mathbb{R}^{1,3} .$$

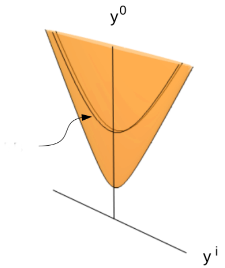
induced metric has Minkowski signature!



algebraically: $\mathcal{M}_n^{3,1}$ generated by

$$\boxed{Y^\mu := X^\mu, \quad \text{for} \ \mu = 0,1,2,3 \qquad \text{(drop } X^4 \text{)}}$$

geometric properties:



- $SO(3,1)$ manifest $\Rightarrow$ foliation into $SO(3,1)$-invariant space-like 3-hyperboloids H^3_τ
- double-covered FRW space-time with hyperbolic ($k = -1$) spatial geometries

$$ds^2 = d\tau^2 - a(\tau)^2 d\Sigma^2,$$

$d\Sigma^2$ ... $SO(3,1)$-invariant metric on space-like H^3

functions on $\mathcal{M}^{3,1}$:

generated by $X^\mu = r\mathcal{M}^{\mu 5} \sim x^\mu$ and $T^\mu = \frac{1}{R}\mathcal{M}^{\mu 4} \sim t^\mu$, with CR

$$\begin{aligned} \{t^\mu, x^\nu\} &= \sinh(\eta)\eta^{\mu\nu} \\ \{x^\mu, x^\nu\} &= \theta^{\mu\nu} \\ \{t^\mu, t^\nu\} &= -\frac{1}{r^2R^2}\theta^{\mu\nu} \end{aligned}$$

constraints

$$\begin{aligned} x_\mu x^\mu &= -R^2\cosh^2(\eta), \qquad x^4 = R\sinh(\eta) \\ t_\mu t^\mu &= r^{-2}\cosh^2(\eta)\,, \\ t_\mu x^\mu &= 0, \\ \theta^{\mu\nu} &= c(x^\mu t^\nu - x^\nu t^\mu) + b\epsilon^{\mu\nu\alpha\beta}x_\alpha t_\beta \end{aligned}$$

t^μ ... generates space-like S^2 fiber functions as higher-spin modes:

$$\phi \in \mathrm{End}(\mathcal{H}_\mathrm{n}) = \phi^{(0)} \oplus \phi^{(0)} \oplus ... \oplus \phi^{(\mathrm{n})}, \qquad \phi^{(\mathrm{s})} \in \mathcal{C}^\mathrm{s}$$

(selected by spin Casimir S^2)

2 points of view:

- functions on H^4_n: full $SO(4,1)$ covariance

  represent $\phi^{(s)}$ as

$$\begin{aligned} \phi^H_{a_1\ldots a_s} &\propto \{x^{a_1}, \ldots \{x^{a_s}, \phi^{(s)}\} \ldots\}_0 \\ \phi^{(s)} &= \{x^{a_1}, \ldots \{x^{a_s}, \phi^H_{a_1\ldots a_s}\} \ldots\} \end{aligned}$$

- functions on $\mathcal{M}^{3,1}_n$: reduced $SO(3,1)$ covariance

$$\phi^{(s)} = \phi^{(s)}_{\mu_1\ldots\mu_s}(x) t^{\mu_1} \ldots t^{\mu_s}$$

$t_\mu x^\mu = 0 \;\Rightarrow$ "space-like gauge" $\quad \boxed{x^{\mu_i}\phi^{(s)}_{\mu_1\ldots\mu_s} = 0}$

($\rightarrow$ no ghosts!)

$SO(3,1)$-invariant: substructure: derivation

$$
\begin{aligned}
D: \quad \mathcal{C}^s &\to \mathcal{C}^{s+1} \oplus \mathcal{C}^{s-1} \\
\phi &\mapsto D^+\phi + D^-\phi = \{\theta^{45}, \phi\} \\
\phi = \phi_{\mu_1\ldots\mu_s}(x) t^{\mu_1}\ldots t^{\mu_s} &\mapsto t^{\mu_1}\ldots t^{\mu_s} t^{\mu}\, \nabla^{(3)}_{\mu}\phi_{\mu_1\ldots\mu_s}(x)
\end{aligned}
$$

def.

$$
\begin{aligned}
\mathcal{C}^{(s,0)} &= \{\phi \in \mathcal{C}^s;\ D^-\phi = 0\} && \text{... primal fields} \\
\mathcal{C}^{(s+k,k)} &= (D^+)^k \mathcal{C}^{(s,0)} && \text{... descendants}
\end{aligned}
$$

(cf. CFT, but no highest weight modules!)

... spin s fields on H^3 in space-like gauge, $SO(4,1)$ spin $s+k$

$SO(4,2)$ - invariant integral = trace

$$\langle\phi,\phi'\rangle := \int\limits_{\mathbb{C}P^{1,2}} \omega^{\wedge 3}\, \phi\phi' = \int\limits_{H^4} dV[\phi\phi']_0$$

$[\phi\phi']_0$ ... average over S^2 fiber

$\mathcal{M}^{3,1}$ realization in IKKT model:

background solution:

$$T^\mu := \frac{1}{R}\mathcal{M}^{\mu 4}$$

satisfies

$$\Box T^\mu = 3R^{-2}T^\mu, \qquad \Box = [T^\mu,[T_\mu,.]]$$

- $[\Box,\mathcal{S}^2]=0, \quad \mathcal{S}^2 = [\mathcal{M}^{ab},[\mathcal{M}_{ab},\cdot]] + r^{-2}[X_a,[X^a,\cdot]]$

  ... spin Casimir, selects spin s sectors $\mathcal{C}^s$

  $\Rightarrow$ higher-spin expansion $\phi = \phi(X) + \phi_\mu(X)T^\mu + \ldots$ on $\mathcal{M}^{3,1}$
- $\Box \sim \alpha^{-1}\Box_G$ encodes eff. FRW metric $ds_G^2 = -dt^2 + a(t)^2 d\Sigma^2$, asymptotically coasting $a(t) \propto t$
- Big Bounce, initial $a(t) \sim t^{1/5}$ singularity

fluctuations & higher spin gauge theory

$$S[Y] = Tr(-[Y^\mu, Y^\nu][Y_\mu, Y_\nu] + m^2 Y^\mu Y_\mu) = S[U^{-1} YU]$$

background solution: $\overline{Y}^\mu = T^\mu$ ... $\mathcal{M}_n^{3,1}$

add fluctuations $Y^\mu = \overline{Y}^\mu + \mathcal{A}^\mu,$

gauge trafos $\mathcal{A}^\mu \to [\Lambda, \mathcal{A}^\mu] + [\Lambda, \overline{Y}^\mu], \qquad \Lambda \in End(\mathcal{H})$

expand action to second oder in $\mathcal{A}^\mu$

$$S[Y] = S[\overline{Y}] + \frac{2}{g^2}\mathrm{Tr}\mathcal{A}_\mu\Big(\underbrace{(\Box + \frac{1}{2}m^2)\delta^\mu_\nu + 2[[\overline{Y}^\mu, \overline{Y}^\nu], .]}_{\mathcal{D}^2} - \underbrace{[\overline{Y}^\mu, [\overline{Y}^\nu, .]]}_{g.f.}\Big)\mathcal{A}_\nu$$

$\mathcal{A}_\mu$ ... $\mathfrak{hs}$-valued field on $\mathcal{M}$, incl. spin 2 diagonalization & eigenmodes on $\mathcal{M}_n^{3,1}$: background $\overline{Y}^\mu = T^\mu$

M. Sperling, HS: 1901.03522, HS 1909.xxxxx

4 tangential $SO(3,1)$-covariant modes for each $\phi = \phi^{(s,k)}$:

$$\begin{aligned}
\mathcal{A}_\mu^{(+)}[D^-\phi] &:= \{x_\mu, D^-\phi\}_+ \\
\mathcal{A}_\mu^{(-)}[D^+\phi] &:= \{x_\mu, D^+\phi\}_- \\
\mathcal{A}_\mu^{(n)}[\phi] &:= D^+\{x_\mu, \phi\}_- \\
\mathcal{A}_\mu^{(g)}[\phi] &:= \{t_\mu, \phi\} \qquad \text{...pure gauge mode}
\end{aligned}$$

underlying $SO(4,2)$ extremely useful to show: are eigenmodes

$$\begin{aligned}
\left(\mathcal{D}^2 + \tfrac{1}{2}\mu^2\right) \mathcal{A}_\mu^{(+)}[D^-\phi^{(s)}] &= m^2\, \mathcal{A}_\mu^{(+)}[D^-\phi^{(s)}] \\
\left(\mathcal{D}^2 + \tfrac{1}{2}\mu^2\right) \mathcal{A}_\mu^{(-)}[D^+\phi^{(s)}] &= m^2\, \mathcal{A}_\mu^{(-)}[D^+\phi^{(s)}] \\
\left(\mathcal{D}^2 + \tfrac{1}{2}\mu^2\right) \mathcal{A}_\mu^{(g)}[\phi^{(s)}] &= m^2\, \mathcal{A}_\mu^{(n)}[\phi^{(s)}] \\
\left(\mathcal{D}^2 + \tfrac{1}{2}\mu^2\right) \mathcal{A}_\mu^{(n)}[\phi^{(s)}] &= m^2\, \mathcal{A}_\mu^{(n)}[\phi^{(s)}]
\end{aligned}$$

for $\Box\phi = m^2\phi$

4 regular on-shell modes $(\mathcal{D}^2 - \frac{3}{R^2})\mathcal{A} = 0$ for

$$
\begin{array}{rcl}
\mathcal{A}^{(+)}[D^-\phi^{(s)}] & \text{for} & \Box\phi^{(s)} = 0, \\
\mathcal{A}^{(-)}[D^+\phi^{(s)}] & \text{for} & \Box\phi^{(s)} = 0, \\
\mathcal{A}^{(g)}[\phi^{(s)}] & \text{for} & \Box\phi^{(s)} = 0, \\
\mathcal{A}^{(n)}[\phi^{(s)}] & \text{for} & \Box\phi^{(s)} = 0
\end{array}
$$

all 4 regular modes propagate in the same way!

$+$ one "special" mode

$$
\mathcal{A}^{(-)}[\phi^{(s,0)}] \qquad \text{for} \qquad \left(\Box + \frac{-2s}{R^2}\right)\phi^{(s,0)} = 0
$$

orthogonal to all regular modes, positive establish lin. independence by diagonalzing inner product matrix

$$\mathcal{G}^{(i,j)} = \left\langle \mathcal{A}^{(i)}[\phi'], \mathcal{A}^{(j)}[\phi] \right\rangle, \qquad i,j \in \{+,-,n,g\}$$

complete set of eigenmodes:

- for $\phi^{(s,k)}$, $k \neq s \neq 0$:
  4 independent modes, signature $(+++-)$
- for $\phi^{(s,0)}$, $s \neq 0$:
  3 independent modes, signature $(++-)$,
  plus special mode $\mathcal{A}^{(-)}[\phi^{(s,0)}]$, positive
- for $\phi^{(s,0)}$, $s \neq 0$:
  2 modes, signature $(+-)$
- for $\phi^{(s,s)}$:
  4 modes, signature $(++-0)$
  one null ”would-be pure gauge“ mode ... (?only off-shell?)

impose gauge-fixing $\{t^\mu, \mathcal{A}_\mu\} = 0$

$\rightarrow$ generically 2 physical modes for each $\Box\phi^{(s,k)}$ in

$$\mathcal{H}_{\rm phys} = \{\mathcal{D}^2\mathcal{A} = 0,\ \ \mathcal{A}\ \text{ gauge fixed}\}/\{\text{pure gauge}\}$$

pure gauge mode

$$\mathcal{A}^{(g)}_\mu[\phi] = \{t_\mu, \phi\}$$

results:

- no ghosts (cf. YM !), + scalar null modes for $\phi^{(s,s)}$
- all modes found, full control
- same propagation for all modes
  even though
  - $SO(3,1)$ only space-like
  - $\exists$ time-like VF $\tau = x_\mu\{t^\mu, .\} \sim \frac{\partial}{\partial t}$ (cosmic background!)

vielbein, metric & dynamical geometry

effective metric $G^{\mu\nu}$ extracted from kinetic term of (all) modes

$$-Tr[T^\alpha,\phi][T_\alpha,\phi] \sim \int e^\alpha\phi e_\alpha\phi = \int \gamma^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = \int \sqrt{|G|}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

vielbein

$$\begin{aligned} e^\alpha &:= \{T^\alpha, .\} = e^{\alpha\mu}\partial_\mu \\ e^{\alpha\mu} &= \sinh(\eta)\eta^{\alpha\mu} \end{aligned}$$

metric

$$\begin{aligned} G^{\mu\nu} &= \alpha\,\gamma^{\mu\nu}\,, \qquad \alpha = \sqrt{\frac{|\theta^{\mu\nu}|}{|\gamma^{\mu\nu}|}}\,, \\ \gamma^{\mu\nu} &= g_{\mu'\nu'}[\theta^{\mu'\mu}\theta^{\nu'\nu}]_{S^2} \end{aligned}$$

encoded in Laplacian $\Box_Y = [Y_\mu,[Y^\mu,.]] \sim \frac{1}{\sqrt{|G|}}\partial_\mu(\sqrt{|G|}G^{\mu\nu}\partial_\nu .)$:

$\to$ FLRW metric: $ds^2 = d\tau^2 - a(\tau)^2 d\Sigma^2$

- late times: $a(t) \approx \frac{3}{2}t, \qquad t \to \infty$ . ... coasting universe (no bad !)
- big bounce: $a(t) \propto (t-t_0)^{\frac{1}{5}}$

perturbed vielbein: $Y^\alpha = T^\alpha + \mathcal{A}^\alpha$

$$\begin{aligned} e^\alpha &= \{T^\alpha + \mathcal{A}^\alpha, .\} = e^{\alpha\mu}[\mathcal{A}]\partial_\mu \\ \delta_\mathcal{A}\gamma^{\mu\nu} &\sim \{\mathcal{A}^\mu, x^\nu\} + (\mu \leftrightarrow \nu) \end{aligned}$$

linearize & average over fiber $\rightarrow$ $h^{\mu\nu} = [\delta_\mathcal{A}\gamma^{\mu\nu}]_0$

coupling to matter:

$$S[\text{matter}] \sim \int_\mathcal{M} d^4x\, h^{\mu\nu}\, T_{\mu\nu}$$

gauge transformations

-of functions: $\phi \mapsto \{\Lambda, \phi\}$

spin 1 trafos: $\Lambda = v^\mu(x) t_\mu \ \in \mathcal{C}^1$:

$$\{v^\mu t_\mu, \phi\}_0 \quad = \frac{1}{3}\left(\sinh(\eta)\left(3v^\mu\partial_\mu + (\mathrm{div} v)\tau - \tau v^\mu \partial_\mu\right) + x_\gamma \varepsilon^{\gamma\mu\alpha\beta}\partial_\alpha v_\mu \partial_\beta\right)\phi$$

3 (rather than 4) diffeomorphisms !

due to invar. symplectic volume on $\mathbb{C}P^{1,2}$

-of gauge fields: $\mathcal{A}^\mu \mapsto \{\Lambda, T^\mu + \mathcal{A}^\mu\}$

-of gravitons:

$$\boxed{\delta G_{\mu\nu} = \nabla_\mu \mathcal{A}_\nu + \nabla_\mu \mathcal{A}_\nu,} \qquad \mathcal{A}_\mu = \{x_\mu, \Lambda\}_- \quad \text{... VF}$$

∇ ... covariant w.r.t. FLRW background

$$\nabla_\alpha \mathcal{A}^\alpha = \frac{1}{x_4^2}(x \cdot \mathcal{A}) \qquad \text{...(almost) volume preserving}$$

towards gravity on $\mathcal{M}^{3,1}$

linearized metric: $h^{\mu\nu} \propto \{\mathcal{A}^\mu, x^\nu\} + (\mu \leftrightarrow \nu)$

off-shell: contains all dof required for gravity

5+1+1 dof from $\mathcal{A}^{(-)}[\phi^{(2)}]$ + $\mathcal{A}^{(+)}[\phi^{(0)}]$ + $\mathcal{A}^{(n)}[D^+\phi^{(0)}]$ (null?),

+ 3 (!) pure gauge $\nabla_\mu \mathcal{A}_\nu + \nabla_\mu \mathcal{A}_\nu$

lin. Ricci:

$$\mathcal{R}^{\mu\nu}_{(\rm lin)}[h[\mathcal{A}]] \approx \frac{1}{2} \underbrace{\Box h_{\mu\nu}[\mathcal{A}]}_{h_{\mu\nu}[\mathcal{D}^2\mathcal{A}]\approx 0} - \frac{1}{4}(\{t_\mu, \{t_\nu, h\}\} + (\mu \leftrightarrow \nu))$$

(up to cosm. scales)

on-shell (vacuum) in M.M.:

$\mathcal{A}^-[\phi^{(2,0)}]$: $h \approx 0 \Rightarrow \mathcal{R}^{\mu\nu}_{(\rm lin)} \approx 0$ ... 2 graviton modes (massless !)

$\mathcal{A}^-[\phi^{(2,1)}]$: $h \approx 0 \Rightarrow \mathcal{R}^{\mu\nu}_{(\rm lin)} \approx 0$ ... trivial (on-shell)

$\mathcal{A}^-[\phi^{(2,2)}]$, $\mathcal{R}^{\mu\nu}_{(\rm lin)} \sim 0$ ... scalar mode (lin. Schwarzschild !)

(linearized) Schwarzschild solution

HS arXiv:1905.07255

focus on scalar metric perturbations $\mathcal{A}^-[D^+D^+\phi]$, gauge-fixed

can show: linearized Ricci-tensor vanishes, $\mathcal{R}^{\mu\nu}_{(\mathrm{lin})}[\delta G[\mathcal{A}]] \approx 0$

explicit metric fluctuation:

$$\delta G_{\mu\nu}\, dx^\mu dx^\nu \overset{\tau\to -2}{=} -4\phi'(dt^2 + a(t)^2 d\Sigma^2) \qquad \text{quasi-static}$$

$\to$ lin. Schwarzschild ($\approx$ Vittie)

$$ds^2 = (G_{\mu\nu} - \delta G_{\mu\nu})dx^\mu dx^\nu = -dt^2 + a(t)^2 d\Sigma^2 + \phi'(dt^2 + a(t)^2 d\Sigma^2)$$

$$\phi' \sim \frac{1}{\rho} e^{-\chi-3\eta} \sim \frac{e^{-\chi}}{\sinh(\chi)} \frac{1}{a(t)^2} \sim \frac{1}{\rho}\frac{1}{a(t)^2}$$

$\approx$ lin. Vittie solution on FRW with mass $m(t) \sim \frac{1}{a(t)}$

note: linearized approx. only valid in quasi-static case $\tau = -2$, otherwise large pure gauge contribution (cf. massive graviton)

discussion

- non- quasi-static case $\tau \sim x^\mu \partial_\mu \neq -2$:
  large pure gauge contribution (cf. massive graviton),
  $\rightarrow$ lin. approx. breaks down at late times
  for very long wavelengths: lin. approx. more reliable,

  $\rightarrow$ extra scalar metric mode, not Ricci-flat!

  would behave like $\approx$ dark matter !
- similar feature expected for would-be spin 1 graviton
- reliable treatment requires induced Einstein-Hilbert action S_{EH}
  - expect Ricci-flat solution (Schwarzschild, ...) to survive

    should recover inhomogeneous Einstein eq. $G_{\mu\nu} \propto T_{\mu\nu}$

    $\Rightarrow$ expect $\approx$ linearized GR at intermediate scale,
    good agreement with solar system tests
  - $\exists$ non-Ricci-flat mode(s), to be understood

- model is fully non-linear (to be understood)
- no cosm. const. $\int d^4x \sqrt{g}$ (?), replaced by YM-action, stabilizes $\mathcal{M}^{3,1}$

  $\to$ no cosm. const. problem ?!
- significant differences at cosmic scales,

  reasonable (coasting) cosmology without any fine-tuning !!

coupling to matter & eom:

for physical transverse traceless spin 2 modes $h_{\mu\nu}[\phi^{(2,0)}]$:

$$S_2[\mathcal{A}^{(-)}[\phi^{(2,0)}]] \propto -\int h^{\mu\nu}[\phi^{(2,0)}](\Box - R^{-2})(\Box_H - 2r^2)^{-1} h_{\mu\nu}[\phi^{(2,0)}]$$

leads to eom

$$(\Box - 2R^{-2})h_{\mu\nu} \sim -(\Box_H - 2r^2)T_{\mu\nu}$$

upon adding S_{EH} expect

$$\Big(\Box - \frac{\Box - R^{-2}}{\Box_H - 2r^2}\Big)h_{\mu\nu} \sim T_{\mu\nu}$$

... Einstein eq., with slight modifications, still no ghosts (?)

summary

- matrix models:
  natural framework for quantum theory of space-time & matter
- 3+1D covariant quantum cosmological FRW space-time solution
  $\rightarrow$ higher spin theory
  reg. BB, finite density of microstates
- fluctuations fully consistent (no ghosts or tachyons)
  all ingredients for (lin.) gravity
- quantized like Yang-Mills theory,
  good UV behavior (SUSY)
- $\rightarrow$ emergent gravity rather than GR
  extra scalar mode (only 3 diffeos), possibly dark matter/energy...

... seems to work!! needs to be elaborated

breaking $SO(4,1) \to SO(3,1)$ and sub-structure

consider

$$D\phi := -i[X^4, \phi], \qquad \text{respects } SO(3,1)$$

acts on spin s modes as follows

$$D = \underbrace{div^{(3)}\phi}_{D^-\phi} + \underbrace{t^\mu \nabla^{(3)}_\mu \phi}_{D^+\phi} : \quad \mathcal{C}^s \to \mathcal{C}^{s+1} \oplus \mathcal{C}^{s-1}$$

decomposition into $SO(3,1)$ irreps on $H^3 \subset H^4$

$$\mathcal{C}^{(s)} = \mathcal{C}^{(s,0)} \oplus \mathcal{C}^{(s,1)} \oplus \ldots \oplus \mathcal{C}^{(s,s)}$$

D^- resp. D^+ act as

$$D^- : \mathcal{C}^{(s,k)} \to \mathcal{C}^{(s-1,k-1)}, \qquad D^+ : \mathcal{C}^{(s,k)} \to \mathcal{C}^{(s+1,k+1)} .$$