Quantum Gravity and Naturalness

NCTS-KU workshop

September 5, 2019

Hikaru KAWAI

(Kyoto Univ.)

Plan of the talk

1. Higgs at Planck scale

2. Fine tunings by nature itself

3. Emergence of SM scale from Planck scale

Appendix A
   Low energy effective theory of QG
Appendix B
   Multiverse and naturalness

1. Higgs at Planck scale

Desert

SM is good to high energy scales.

Experimentally LHC
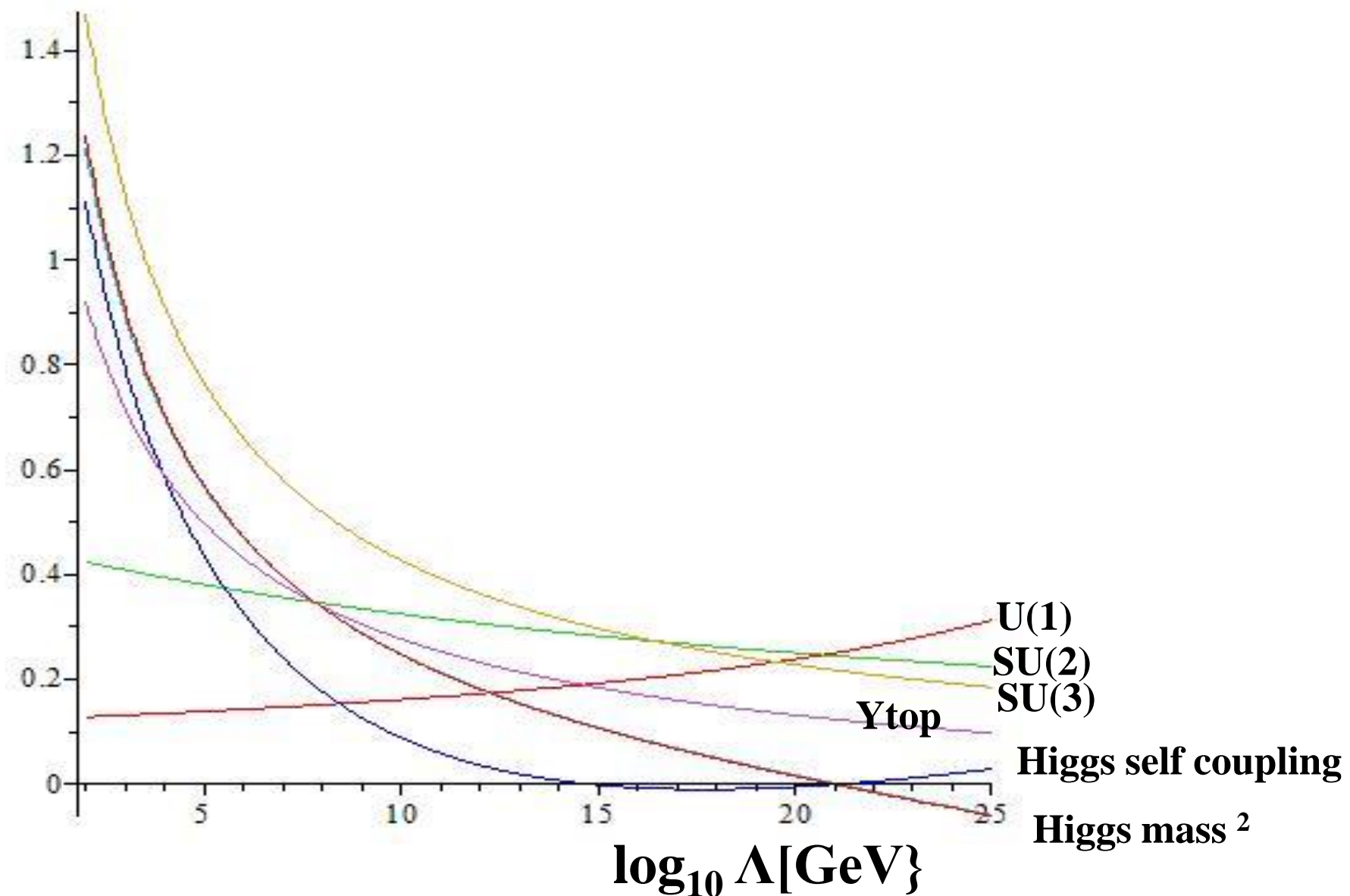 SM is good at least **below a few TeV.**
  No signal for new particles or physics.
 Especially no indication of low energy SUSY.

Theoretically   UV region of SM by RG
  No contradiction **below Planck (string) scale.**
  All the couplings are small and the perturbative
  picture is very good up to Planck scale.

U(1)
SU(2)
SU(3)
Ytop
Higgs self coupling
Higgs mass ^{2}

$\log_{10} \Lambda [\text{GeV}\}$

It is natural to imagine that SM is directly connected to the string scale dynamics such as quantum gravity without large modification.
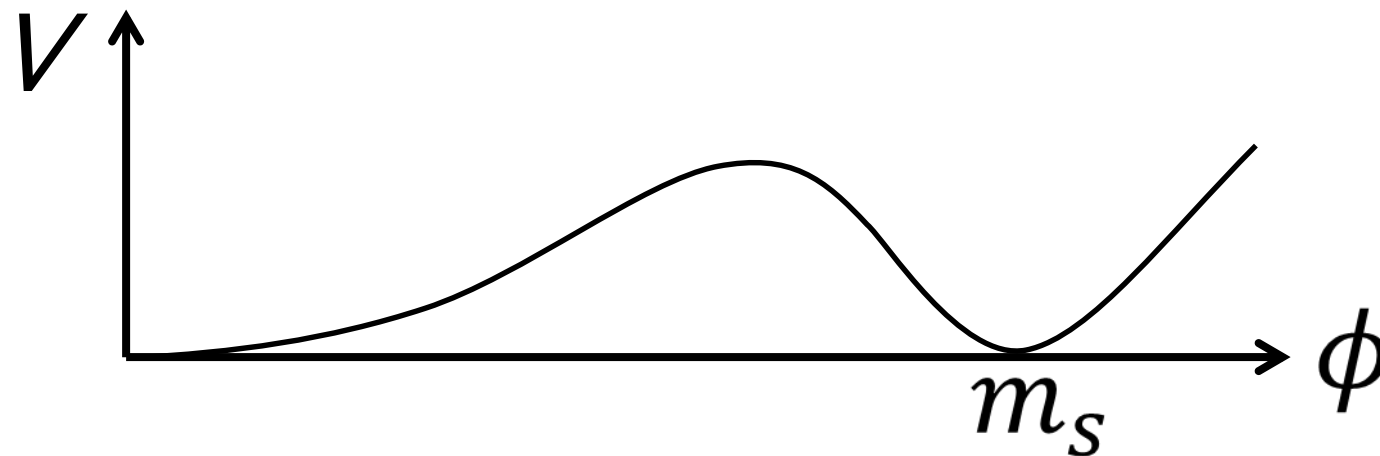
SM m_s

↑
desert

string scale
dynamics
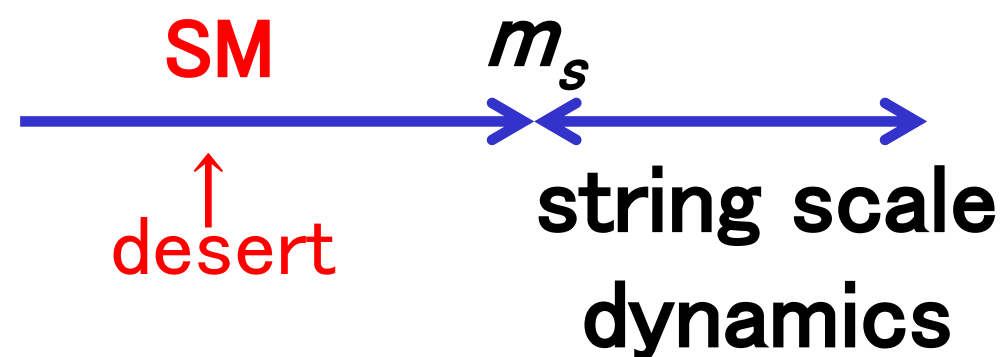
Triple coincidence

RG analyses indicate that three quantities,

$$\lambda_B, \beta_\lambda(\lambda_B), m_B$$
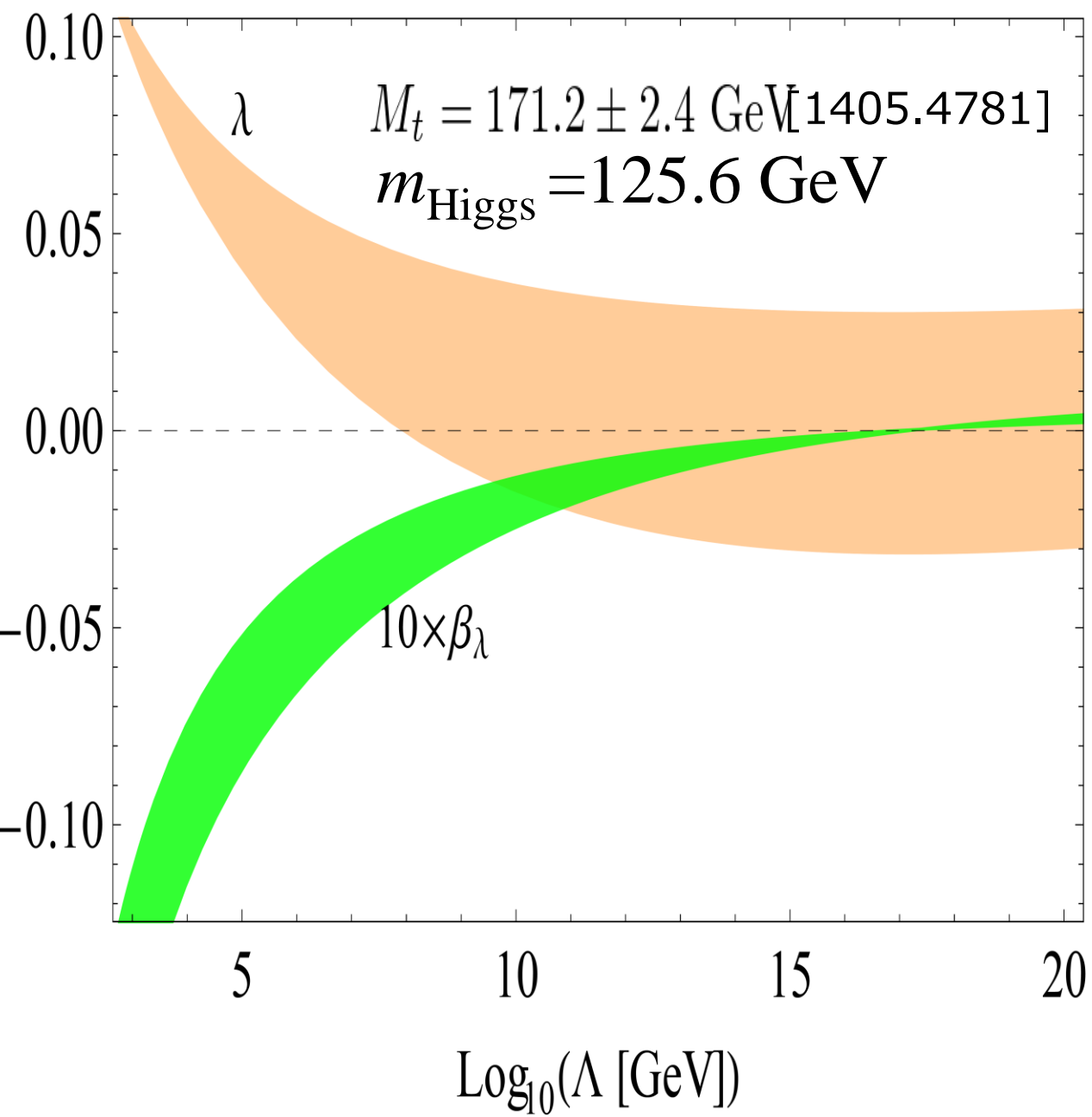
become zero around the string scale.

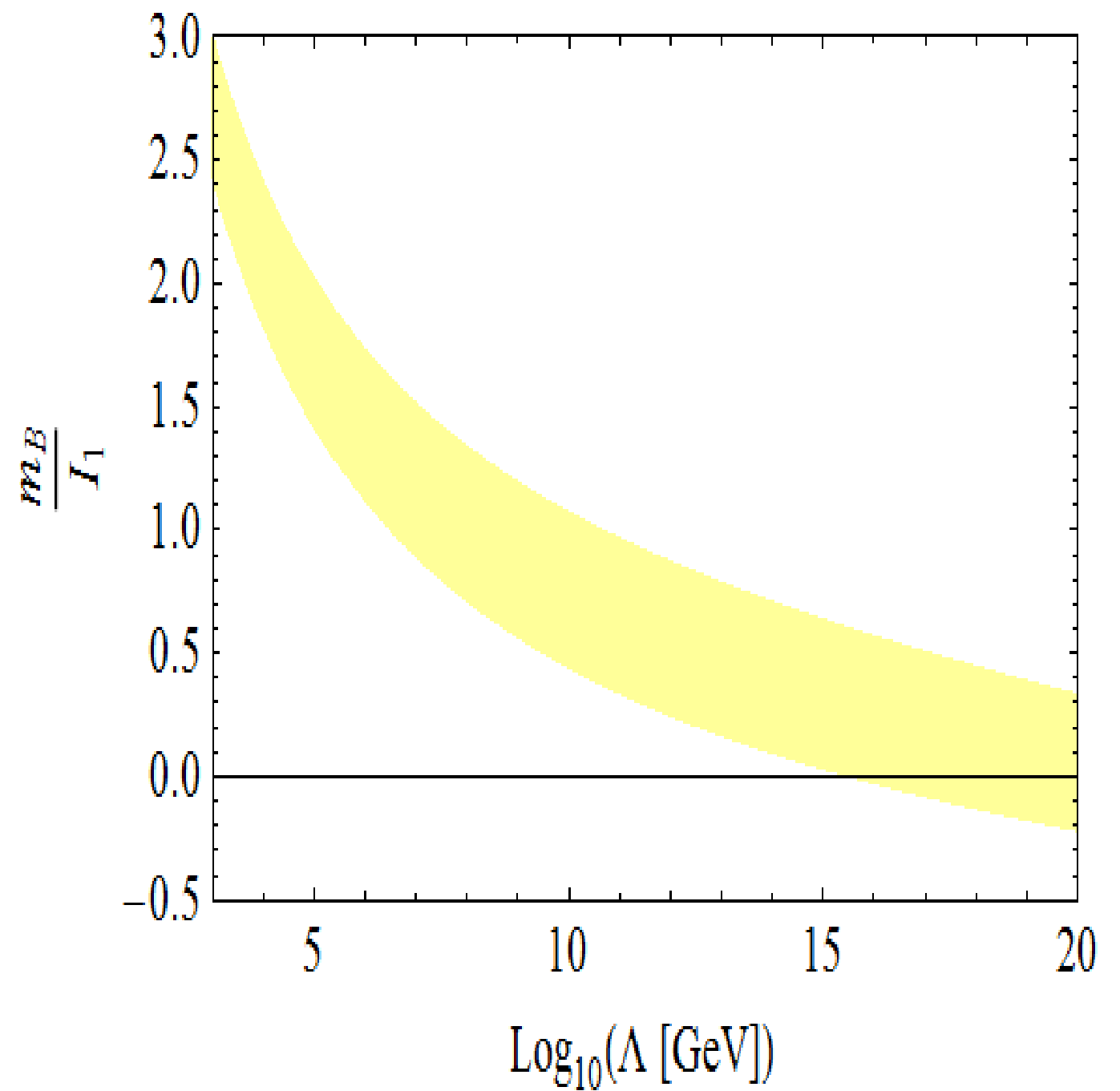**The Higgs potential becomes flat (or zero) around the string scale.**

Froggatt and Nielsen '95.
Multiple Point Criticality Principle (MPP)

Higgs self coupling

Bare mass

$M_t = 171.2 \pm 2.4 \text{ GeV}$ [1405.4781]

$m_{\text{Higgs}} = 125.6 \text{ GeV}$

λ

$10 \times \beta_\lambda$

$\text{Log}_{10}(\Lambda \text{ [GeV]})$

$\frac{m_B}{F_1}$

$\text{Log}_{10}(\Lambda \text{ [GeV]})$

[Hamada, Oda, HK, 1210.2538, 1308.6651]

Higgs potential

$$V(\varphi) = \frac{\lambda(\varphi)}{4}\varphi^4$$

Legend:
- $M_t = 171.0796\,\text{GeV}$ (red)
- $M_t = 171.0798\,\text{GeV}$ (blue)
- $M_t = 171.0800\,\text{GeV}$ (green)

$m_{\text{Higgs}} = 125.6\ \text{GeV}$

Axes: vertical $\frac{V}{\text{GeV}^4}$, horizontal $\frac{\mu}{\text{GeV}}$
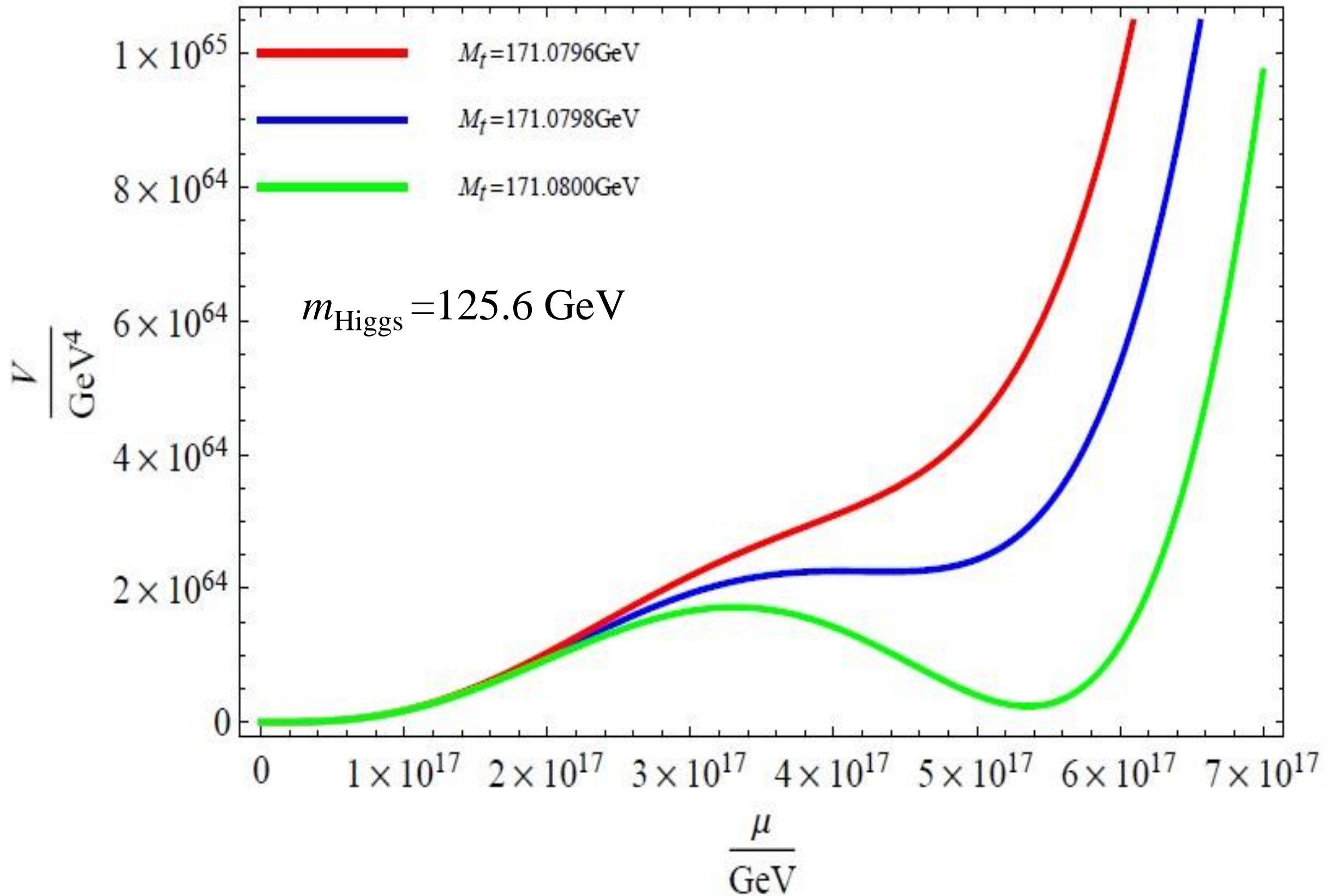
Higgs inflation

Higgs potential may be flat around the string scale.

It suggests the possibility of Higgs inflation.

(1) Critical Higgs inflation

We trust the flat potential including the inflection point.
We assume that nature does fine tunings on the SM parameters if necessary.
We introduce a non-minimal coupling of order 10.
Hamada, Oda, Park and HK, PhysRevLett.112.241301
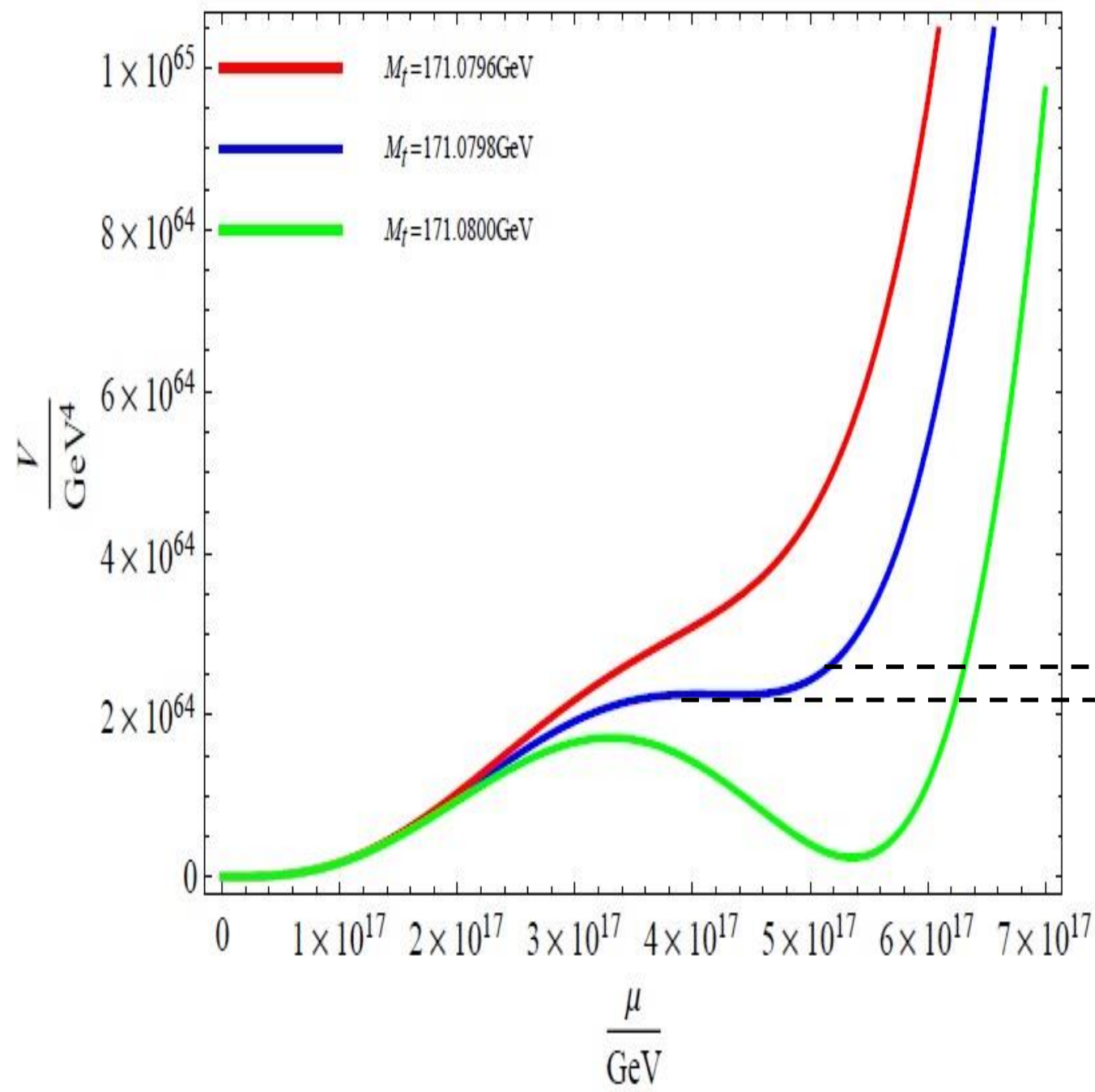
(2) General bounds

We trust the effective potential only below the string scale, and try to make bounds on the physical parameters.
Hamada, Nakanish, Oda and HK: arXiv: 1709.9035
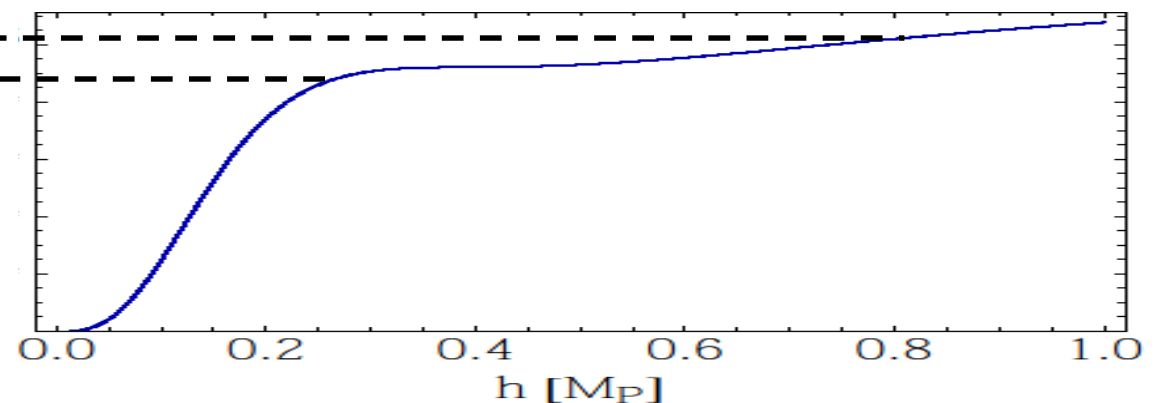
(1) Critical Higgs inflation

If we introduce a non-minimal coupling $\xi R \varphi^2$
a realistic Higgs inflation is possible.

ξ can be as small as 10.

In the Einstein frame the
effective potential becomes

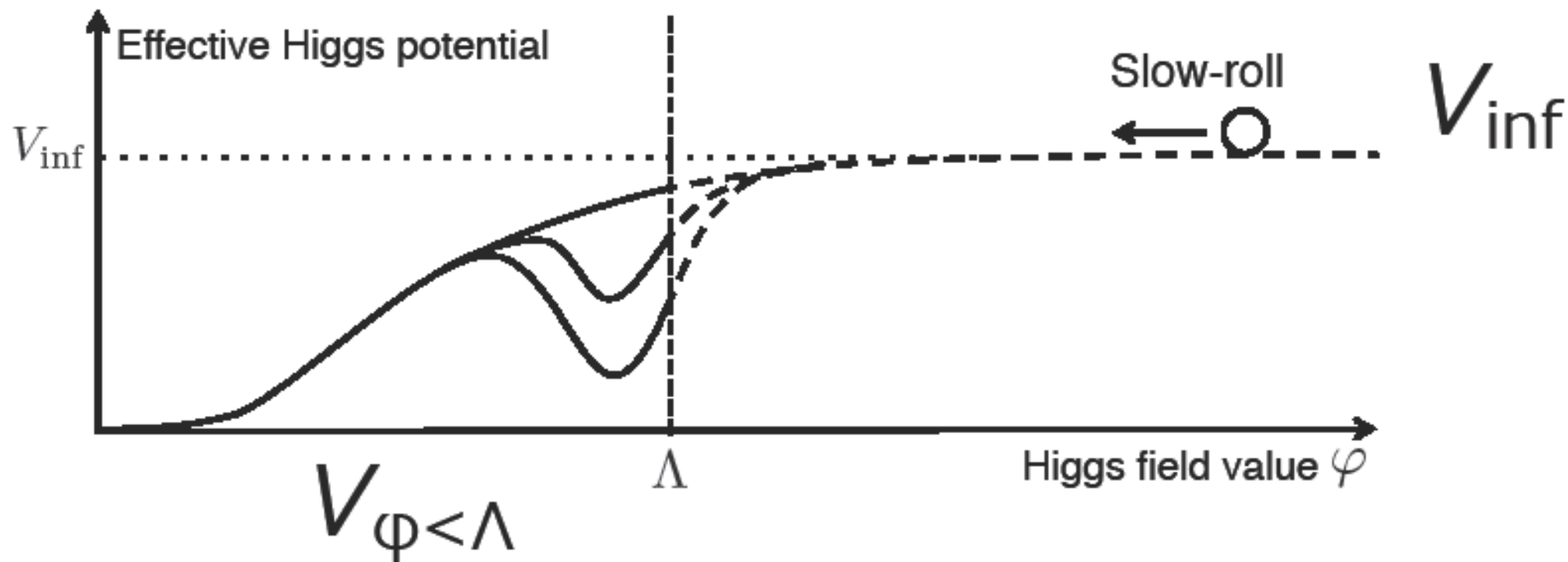$$V(\varphi_h), \quad \varphi_h = \frac{h}{\sqrt{1 + \xi h^2 / M_P^2}}.$$

(2) General bounds

We simply assume that the non-minimal couplings ξ's are not so large: $\xi < 100$.
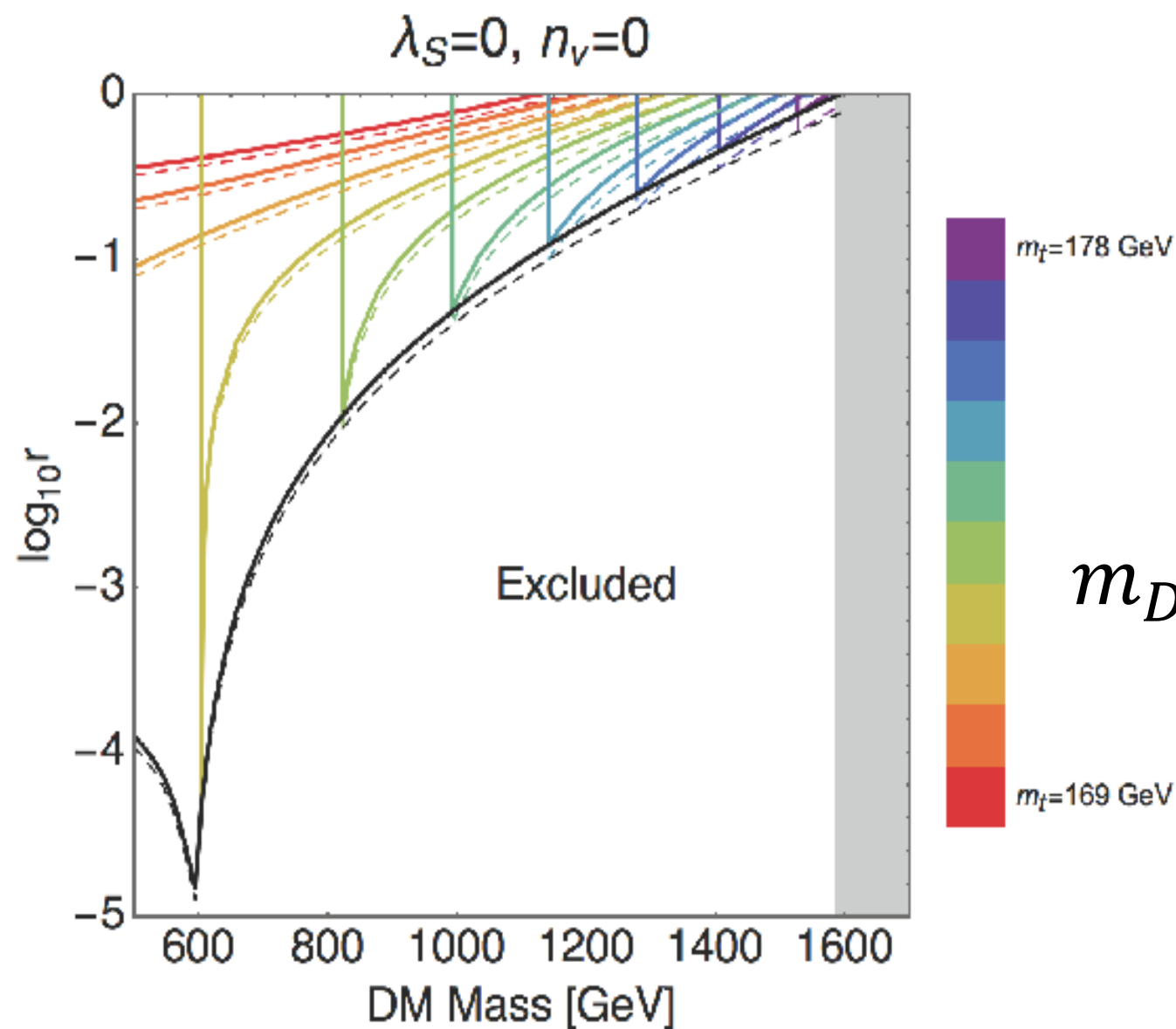
Then we can trust field theory up to

$$\frac{M_P}{\sqrt{\xi}} \sim 10^{17} \, \text{GeV}.$$

We then have a lower bound on the vacuum energy at the inflation.

Effective Higgs potential

V_{inf} .... Slow-roll ←O ... V_{inf}

$V_{\varphi < \Lambda}$

Λ Higgs field value φ

- ✦ From low energy potential $V_{\varphi < \Lambda}$, we get lower bound:
$V_{inf} > V_{\varphi < \Lambda}^{max}$.

- ✦ In slow-roll, $A_s = 0.068\ V_{inf}/r$ (=2.2×10^{-9}, fixed)

- ✦ So we have lower bound: $r > V_{\varphi < \Lambda}^{max}/(3.2 \times 10^{16} \text{GeV})^4$.

SM + Higgs portal scalar dark matter $(+ \nu_R)$

$$\mathcal{L} \sim -\frac{\kappa}{2} S^2 H^2$$

$$m_{DM} = \kappa \times 3.2 \text{ TeV}$$

✦ Vertical line from potential positivity.

SM around Planck scale

- **Desert**

  SM is valid to the string scale at least theoretically.

  SM might be directly connected to string theory without large modification.

- **Marginal stability**

  Higgs field is near the stability bound.

- **Zero bare mass**

  The bare Higgs mass is close to zero at the string scale.

  It implies that Higgs is a massless state of string theory.

- **Flat potential and Higgs inflation**

  Higgs self coupling and its beta function become zero at the string scale.

  Higgs potential might be flat around the string scale.

  It suggests the possibility of Higgs inflation.

2. Fine tunings by nature itself

There are several attempts to extend the conventional framework of the local field theory in order to solve the fine tuning problem.

- asymptotic safety
    - Shaposhnikov, Wetterich
- multiple point criticality principle
    - Froggatt, Nielsen.
- classical conformality
    - Meissner, Nicolai,
    - Foot, Kobakhidze, McDonald, Volkas
    - Iso, Okada, Orikasa.
- baby universe and multi-local action
    - Coleman
    - Okada, Hamada, Kawana, Sakai, HK

They are related.

MPP of Froggatt and Nielsen

Imagine a system that is described by the path integral of not the canonical ensemble

$$\int [d\varphi] \exp(-S[\varphi]),$$

but the micro canonical ensemble

$$\int [d\varphi] \delta(S[\varphi] - C),$$

or an even more general ensemble

$$\int [d\varphi] f(S_1[\varphi], S_2[\varphi], \cdots).$$

Still the system is equivalent to the ordinary field theory in the large space-time volume limit.

But the parameters of the corresponding field theory are automatically fixed such that the vacuum is at a multiple criticality point.

Integrating coupling constants

In fact we can show that the low energy effective theory of QG / string theory is given by the multi-local action:

$$S_{\text{eff}} = f\left(S_1, S_2, \cdots\right)$$

$$= \sum_i c_i\, S_i + \sum_{ij} c_{ij}\, S_i S_j + \sum_{ijk} c_{ijk}\, S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)}\, O_i(x).$$

Here O_i are local scalar operators such as

$$1\,,\, R\,,\, R_{\mu\nu} R^{\mu\nu}\,,\, F_{\mu\nu} F^{\mu\nu},\, \overline{\psi}\gamma^\mu D_\mu \psi\,,\, \cdots\,.$$

Because S_eff is a function of S_i's , we can express $\exp(iS_\text{eff})$ by a Fourier transform as

$$\exp\left(iS_\text{eff}(S_1,S_2,\cdots)\right)=\int d\lambda\, w(\lambda_1,\lambda_2,\cdots)\exp\left(i\sum_i \lambda_i S_i\right),$$

where λ_i's are Fourier conjugate variables to S_i's, and w is a function of λ_i's .

Then the path integral for S_eff becomes

$$Z=\int[d\phi]\exp(iS_\text{eff})=\int d\lambda\, w(\lambda)\int[d\phi]\exp\left(i\sum_i \lambda_i S_i\right).$$

Because O_i are local operators, $\sum_i \lambda_i S_i$ is an ordinary local action where λ_i are regarded as the coupling constants.

Therefore the system is the ordinary field theory, but we have to integrate over the coupling constants with some weight $w(\lambda)$.

Nature does fine tunings

$$Z = \int [d\phi] \exp(iS_{\text{eff}}) = \int d\lambda\, w(\lambda) \underbrace{\int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right)}_{= Z(\lambda)}$$

$$= \int d\lambda\, w(\lambda) Z(\lambda).$$

Ordinary field theory

If a small region $\lambda \sim \lambda^{(0)}$ dominates the λ integral, it means that the coupling constants are fixed to $\lambda^{(0)}$.

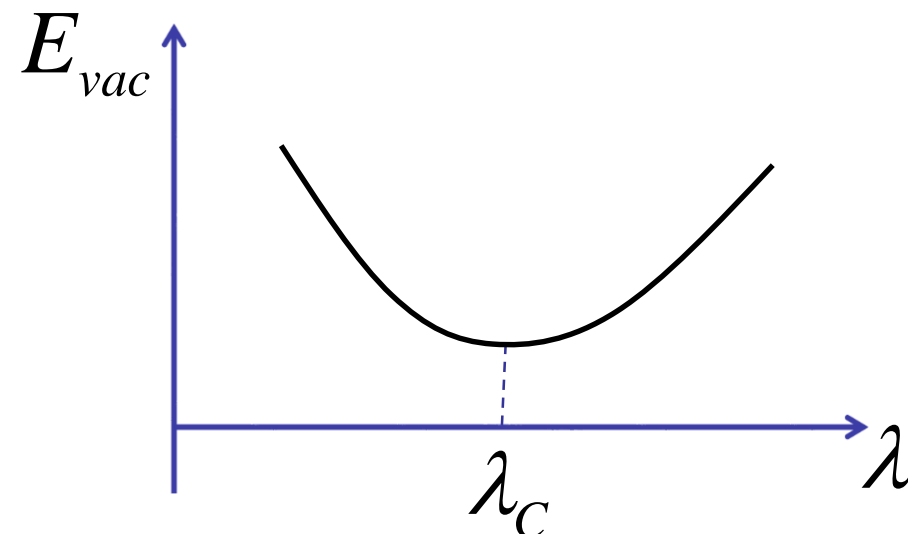# MPP can be explained. '14 '15 Hamada, Kawana, HK

Since our universe has been cooled down for long time, we can approximate $Z(\lambda) = \exp\left(-iVE_{vac}(\lambda)\right).$

1) extremum

If $E_{vac}(\lambda)$ is smooth and has an extremum,

$$\sim \frac{\sqrt{2\pi}}{\sqrt{i\,V|E''(\lambda_c)|}}\,\delta(\lambda - \lambda_c) + O\left(\frac{1}{V}\right)$$

Thus λ **is fixed to** λ_C **in the limit** $V \to \infty$.

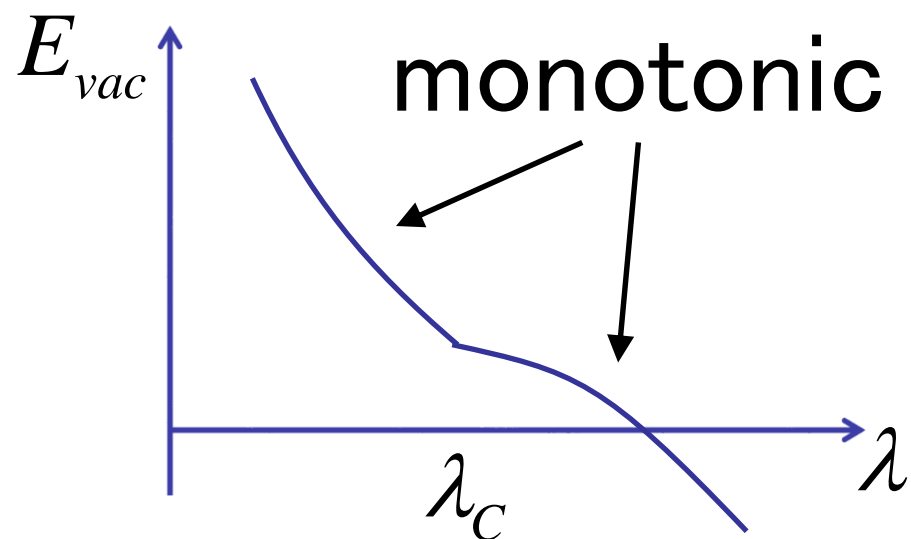## 2) Kink (need not be an extremum)

If $E_{vac}(\lambda)$ has a kink (first order phase transition),

$$Z(\lambda) = \exp\big(-iVE_{vac}(\lambda)\big)$$

$$\sim \frac{i}{V}\left(\frac{1}{E_{vac}{}'(\lambda_c + 0)} - \frac{1}{E_{vac}{}'(\lambda_c - 0)}\right)\delta(\lambda - \lambda_c) + O\left(\frac{1}{V^2}\right)$$

Thus λ is fixed to λ_C in the limit $V \to \infty$. $\Rightarrow$ MPP

$$\int_a^b dx\, \exp(iVx)\,\varphi(x)$$

$$= \left[\frac{1}{iV}\exp(iVx)\,\varphi(x)\right]_a^b + O\left(\frac{1}{V^2}\right)$$

$$\int_a^b dx\, \exp(iVf(x))\,\varphi(x)$$

$$= \left[\frac{1}{iV}\exp(iVf(x))\,\frac{1}{f'(x)}\varphi(x)\right]_a^b + O\left(\frac{1}{V^2}\right)$$

$(f$ is monotonic$)$

E_{vac}

monotonic

λ_C

λ

Generalization

If we consider the time evolution of the universe, the definition of $Z(\lambda)$ is not a priori clear.
For example we need to specify the initial and final sates.

Even if we do not know the precise form of $Z(\lambda)$, we expect that a similar mechanism to the previous case, $Z(\lambda) = \exp\big(-iVE_{vac}(\lambda)\big)$, works.

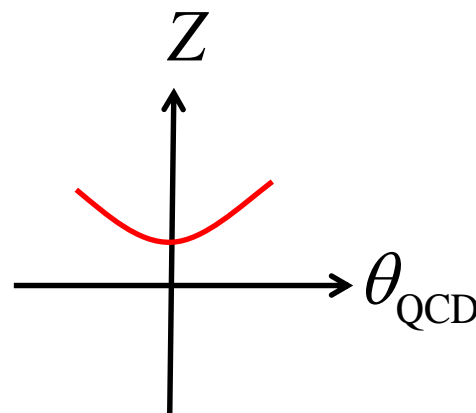**Some of the couplings can be determined without knowing the details of $Z(\lambda)$.**

(1) Symmetry example $\boldsymbol{\theta_{QCD}}$ Nielsen, Ninomiya

1. It becomes important only after the QCD phase transition.

2. The mass spectrum of hadrons is invariant under
$$\boldsymbol{\theta_{QCD} \to -\theta_{QCD}}\,.$$

$\Rightarrow$ It is expected that Z is even in $\boldsymbol{\theta_{QCD}}$.

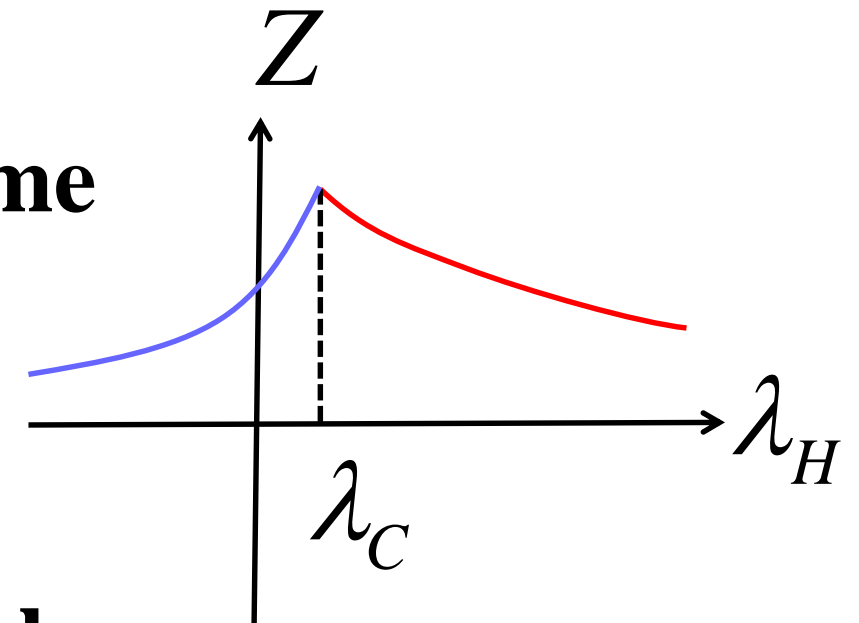$\Rightarrow$ $\boldsymbol{\theta_{QCD}}$ is tuned to 0.

(2) Edge or drastic change

Conditions:
1. **Physics changes drastically at some value of the couplings.**
2. **Z is monotonic elsewhere.**

Z

λ_C

λ_H

$\Rightarrow$ **The couplings are tuned to the value.**

Examples:
Higgs self coupling,
Cosmological constant,
Classical conformality,
...

finite

∞

Open questions

- Higgs mass?
- Degenerate vacuum or flat potential?
- Small cosmological constant?
- Too much big fix?

$\Rightarrow$ We need the precise form of $Z(\lambda)$.

$\Rightarrow$ wave function of multiverse

3. Emergence of SM scale from Planck scale

Weak scale as a non-perturbative effect

Basic assumption:

SM is directly connected to string theory without large modification.
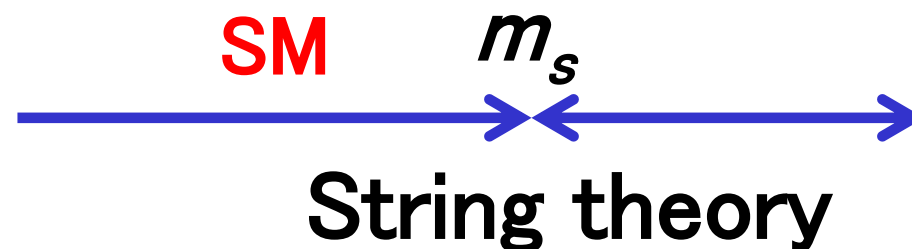
The fundamental scale of string theory is only the Planck scale.

Question:

How does the weak scale appear?

Everybody's guess:

Weak scale must appear as a **non-perturbative effect**.

Then it is related to the Planck scale as

$$m_H = M_P \, e^{-\text{const.}/g_s}.$$

And the **large hierarchy** is naturally understood.

Problem:

Show a phenomenologically acceptable mechanism.

Various possibilities:

1. QCD like dimensional transmutation.

$$\Lambda_{\text{QCD}} = \Lambda \, e^{-\text{const.}/g_0^2}$$

   Not compatible with weakly coupled Higgs.

2. Coleman-Weinberg mechanism.
 a) Original idea is to explain SSB of SM from the massless Higgs.
   Not acceptable. $m_H \ll v_H$
 b) Additional gauge + complex scalar
   Make a mass scale independently to SM sector. Then transfer it to SM through VEV.
   Possible to make an acceptable model.

3. Even simpler (simplest) model.

Two real scalars.

$$\mathcal{L}_{\phi\varphi} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\varphi)^2 - V$$

$$V = \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2\varphi^2 + \frac{\rho'}{4!}\varphi^4$$

For a while we assume the $Z_2 \times Z_2$ invariance.

$$Z_2: \quad \phi \rightarrow -\phi$$

$$Z_2: \quad \varphi \rightarrow -\varphi$$

More important assumption is the classical conformality.

What is classical conformality?

Classical conformality

= "renormalized masses are 0"

This sounds nonsense for normal fields theorists: There is **no quantum mechanical symmetry** that guarantees masslessness of scalars.

On the other hand, once we accept the existence of **self-tuning mechanism**, classical conformality is one of the natural choices.

For example, we can take the MPP:
"Coupling constants are fixed to critical points."

There are many kinds of critical points.

(1) Critical points for cosmological evolution

Critical values that the time evolution of universe changes drastically when they are changed.

We assume that the universe starts from $\langle \phi \rangle = 0$.

$m^2 > 0 \Rightarrow \langle \phi \rangle = 0$ is metastable.

$\Rightarrow$ Universe remains in that state for a while.

$m^2 < 0 \Rightarrow \langle \phi \rangle = 0$ is unstable.

$\Rightarrow$ Universe transitions quickly to another state.

So the time evolution of the universe changes drastically at the point $m^2 = 0$.

This is nothing but the classical conformality.

(2) Critical points for the vacuum energy

Critical values that the phase of the vacuum changes when they are changed.

We will see that this actually happens when $m^2 = m_c^2$.

Maybe there are other types of critical point. In any case, renormalized masses are fixed to some values.

For a while, we concentrate on the case (1).

3-1 SSB of the 2 scalar model

Two real scalar model

$$\mathcal{L}_{\phi\varphi} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\varphi)^2 - V$$

$$V = \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2\varphi^2 + \frac{\rho'}{4!}\varphi^4$$

Classical conformality:

$$\frac{\partial}{\partial\phi^2}V_{\text{eff}}\Big|_{\phi=\varphi=0} = 0, \quad \frac{\partial}{\partial\varphi^2}V_{\text{eff}}\Big|_{\phi=\varphi=0} = 0.$$

Result

For a large region of the parameter space, one of the fields has non-zero vacuum expectation value.

This is contrary to the expectation from the tree potential.

RG analysis

Beta functions:

$$\beta_\rho = \frac{3}{16\pi^2}\left(\rho^2 + \kappa^2\right)$$

$$\beta_{\rho'} = \frac{3}{16\pi^2}\left(\rho'^2 + \kappa^2\right)$$

$$\beta_\kappa = \frac{1}{16\pi^2}\left(\rho\kappa + \rho'\kappa + 4\kappa^2\right)$$

Assumption: cut off scale = Planck scale,

$$\rho_0, \rho_0', \kappa_0 > 0.$$

When we decrease the renormalization point, one of the couplings becomes zero.
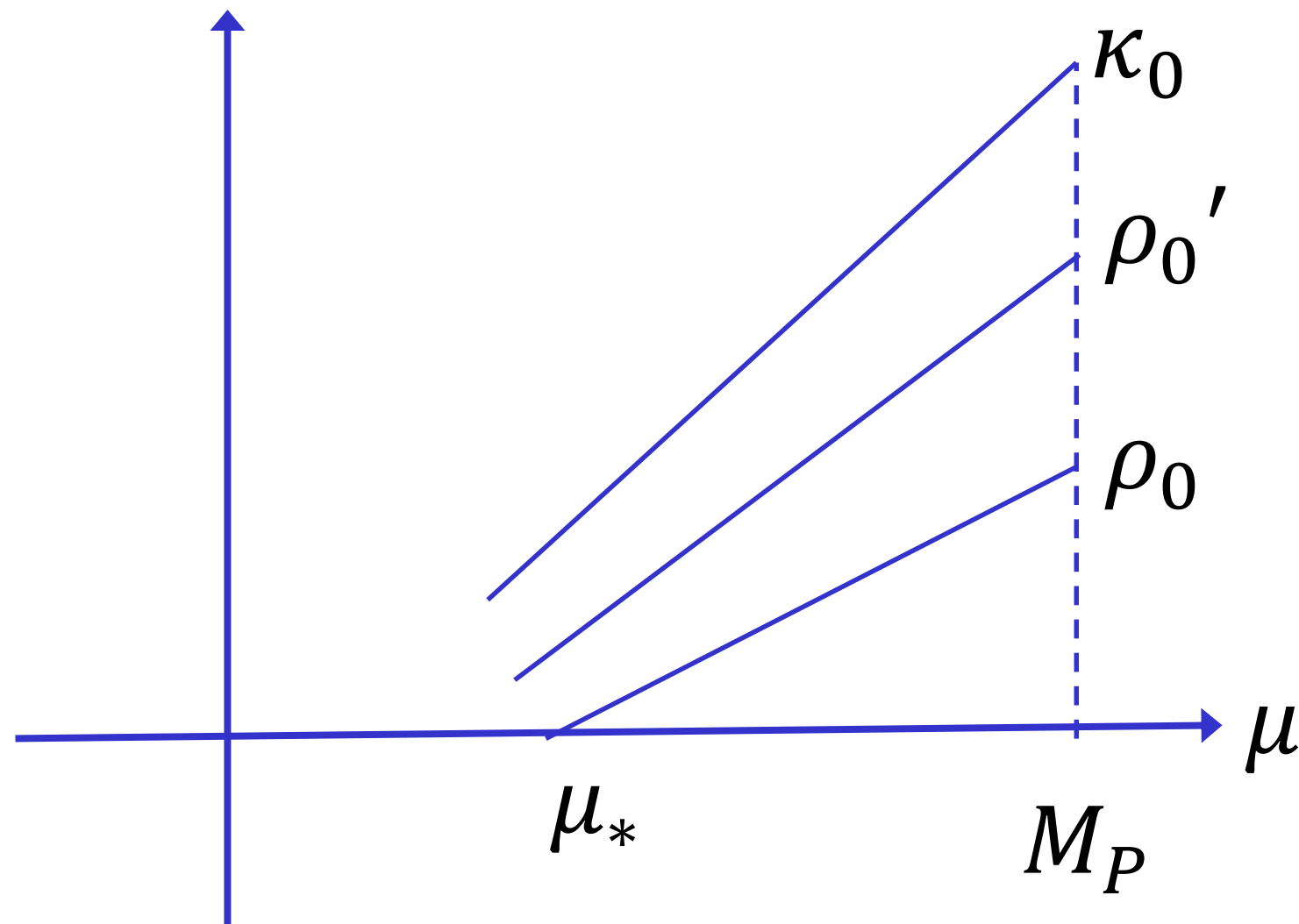
We assume ρ becomes zero first at $\mu = \mu_*$.

This is possible if $\kappa_0 \gg \rho_0' > \rho_0$.

Then it is expected

$\langle \phi \rangle \neq 0 \Rightarrow \varphi$ becomes massive through $\frac{\kappa}{4}\phi^2\varphi^2$

$\Rightarrow \langle \varphi \rangle = 0$

One-loop effective potential
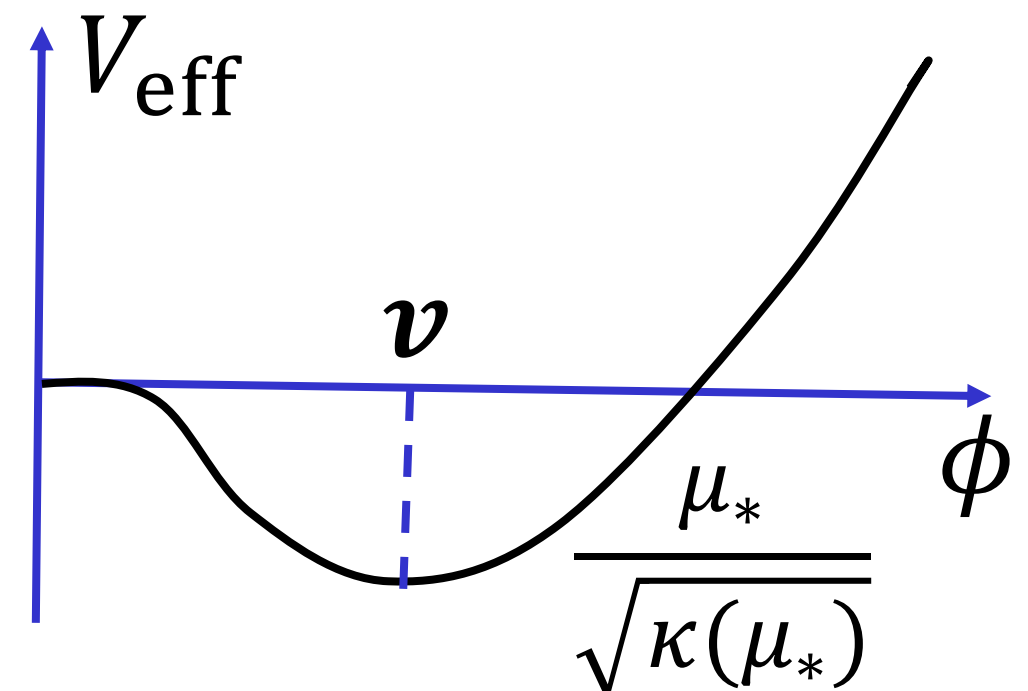
Effective potential for $\langle \varphi \rangle = 0$:

$V_{\text{eff}}(\phi, \varphi = 0)$ $\qquad$ ϕ loop $\qquad\qquad\qquad$ φ loop

$$= \frac{\rho(\mu)}{4!} \phi^4 + \frac{\rho(\mu)^2}{256\pi^2} \phi^4 \log\left(\frac{\rho(\mu)\phi^2}{\mu^2}\right) + \frac{\kappa(\mu)^2}{256\pi^2} \phi^4 \log\left(\frac{\kappa(\mu)\phi^2}{\mu^2}\right)$$

If we take $\mu = \mu_*$, this becomes

$$= \frac{\kappa(\mu_*)^2}{256\pi^2} \phi^4 \log\left(\frac{\kappa(\mu_*)\phi^2}{\mu_*^2}\right)$$

$$\Rightarrow \langle \phi \rangle = v = \text{const.} \frac{\mu_*}{\sqrt{\kappa(\mu_*)}}$$

A mass scale v emerges.

Relation between v and M_P

For simplicity we consider the case
$$\rho_0 < \rho_0' \ll \kappa_0 \ll 1.$$

Then for $\mu_* \leq \mu \leq M_P$

$$\kappa(\mu) \sim \kappa_0, \ \beta_\rho \sim \frac{3\kappa_0^2}{16\pi^2} .$$

$$\Rightarrow \rho(\mu) \sim \rho_0 + \frac{3\kappa_0^2}{16\pi^2} \log\left(\frac{\mu}{M_P}\right)$$

Thus we have

$$\rho(\mu_*) = 0 \Rightarrow \mu_* \sim M_P \exp\left(-\frac{16\pi^2}{3}\frac{\rho_0}{\kappa_0^2}\right)$$

$$\Rightarrow v \sim M_P \frac{1}{\sqrt{\kappa_0}} \exp\left(-\frac{16\pi^2}{3}\frac{\rho_0}{\kappa_0^2}\right)$$

Non−perturbative

Masses of the particles

mass of ϕ : $M_\phi^2 = \dfrac{d^2}{d\phi^2} V_{\text{eff}}\Big|_{\phi=v} = \dfrac{\kappa(\mu_*)^2}{32\pi^2} v^2$

mass of φ: $M_\varphi^2 = \dfrac{\kappa(\mu_*)}{2} v^2$

$\Rightarrow$ $\dfrac{M_\phi^2}{M_\varphi^2} = \dfrac{\kappa(\mu_*)}{16\pi^2}$

The general pattern is

$$v \gg M_\varphi \gg M_\phi\,.$$

For example,

$\kappa(\mu_*) = 1$ $\Rightarrow v : M_\varphi : M_\phi = 1 : 0.7 : 0.06\,,$

$\kappa(\mu_*) = 0.1 \Rightarrow v : M_\varphi : M_\phi = 1 : 0.2 : 0.006\,.$

3-2 Coupling to SM

Incorporating two real scalar model into SM

Total action:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\phi\varphi} - V_{H\phi\varphi}$$

$$V_{H\phi\varphi} = -\frac{\eta}{2}\phi^2|H|^2 + \frac{\eta'}{2}\varphi^2|H|^2$$

We assume that $\mathcal{L}_{SM}$ does not contain the Higgs mass term because of the classical conformality.

Rigorously speaking, we should consider the mixing between Higgs and ϕ:

$$\tan(\theta) \sim \frac{v_H}{v}, \qquad v_H \sim 250\text{GeV}, \quad v = \langle\phi\rangle.$$

If $v \gg v_H$, the mixing is small.

In that case, the Higgs potential is given by

$$\lambda|H|^4 - \frac{\eta}{2}\langle\phi\rangle^2|\mathrm{H}|^2,$$

which means

$$m_H^2 = \eta\, v^2,$$

and we can reproduce the spontaneous symmetry breaking in the weak scale

$$\langle H\rangle = \frac{2m_H}{\sqrt{\lambda}}.$$

Weak scale is generated indirectly but non-perturbatively from the Planck scale, as

$$M_P \to \langle\phi\rangle \to \langle H\rangle.$$

φ as dark matter

Property of φ

no vev: $\langle \varphi \rangle = 0$

heavy but not too heavy: $M_\varphi^2 = \dfrac{\kappa(\mu_*)}{2} v^2$

couples to Higgs: $\dfrac{\eta'}{2} \varphi^2 |H|^2$

It is natural to regard φ as the Higgs portal scalar dark matter.

Parameters of the model

Two scalar model has 3 parameters ρ, ρ', κ in addition to M_P.

ρ is replaced by $\langle \phi \rangle = v$.

κ gives the ratios of M_ϕ, M_φ, v .

ρ' is the self coupling of φ.

Coupling to the Higgs has 2 parameters.

η is determined by $m_H^2 = \eta\, v^2$.

η' gives coupling between Higgs and φ .

$\Rightarrow$ **Only κ is new compared with the Higgs portal scalar dark matter scenario.**

Examples

(1) $\kappa = 0.1, M_\varphi = 1\text{TeV}$

$\quad \Rightarrow M_\phi = 25\text{GeV}, v = 4.5\text{TeV}, \theta = 0.057$

(2) $\kappa = 0.1, M_\varphi = 6\text{TeV}$

$\quad \Rightarrow M_\phi = 151\text{GeV}, v = 27\text{TeV}, \theta = 0.020$

If we assume the critical Higgs inflation, (1) seems typical.

It predicts a light scalar ~ **25GeV**, which may be tested in near future experiments.

3-3 Other criticalities than classical conformality

Two real scalar model revisited

$$\mathcal{L}_{\phi\varphi} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\varphi)^2 - V$$

$$V = \frac{m_0^2}{2}\phi^2 + \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2\varphi^2 + \frac{\rho'}{4!}\varphi^4$$

As we have discussed,

classical conformality is the assumption that the bare mass should be tuned so that the renormalized mass becomes zero:

$$m^2 = 0.$$

This is a critical point in that the time evolution of universe drastically changes at $m^2 = 0$.

But there is another kind of critical point.

$$V_{\text{eff}}(\phi, \varphi = 0) =$$

Classical conformality

$$\frac{\kappa(\mu_*)^2}{256\pi^2}\phi^4 \log\left(\frac{\kappa(\mu_*)\phi^2}{\mu_*^2}\right)$$
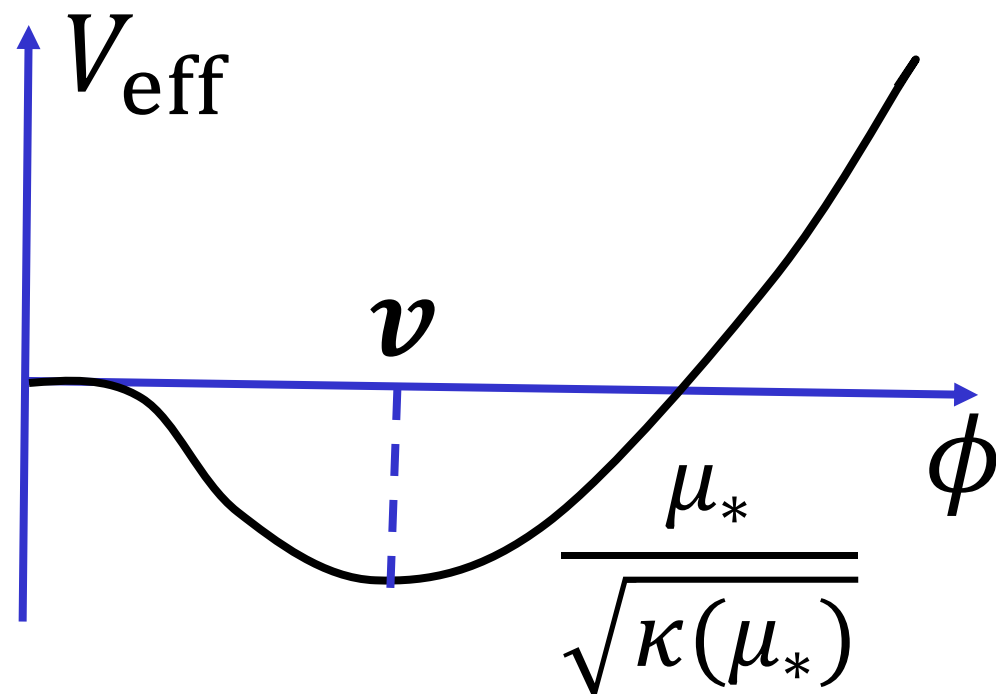
1-st order phase transition

$$\frac{m_C^2}{2}\phi^2 + \frac{\kappa(\mu_*)^2}{256\pi^2}\phi^4 \log\left(\frac{\kappa(\mu_*)\phi^2}{\mu_*^2}\right)$$
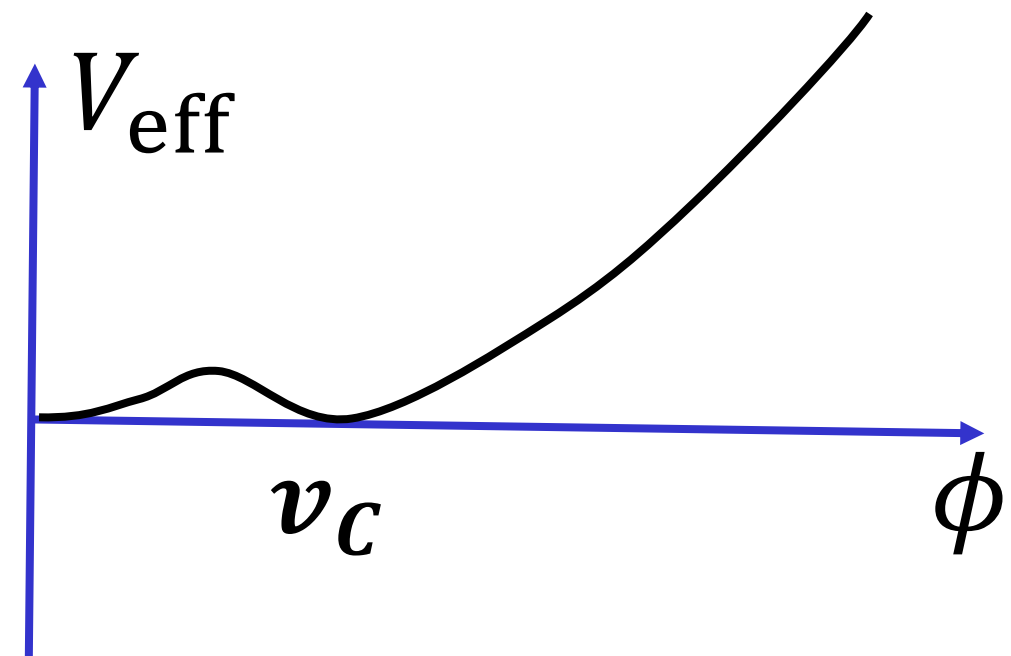
$$v_C = v/\sqrt{e}$$

$$M'_\phi = M_\phi/\sqrt{2}$$

1-st order phase transition point is as plausible as classically conformality.

We can not tell which one is favored by nature unless we know the precise mechanism of MPP.

At any rate,

the generated mass scales v, M_ϕ, M_φ differ only by numerical factors.

We can say that the weak scale emerges from the Planck scale non-perturbatively.

Further generalization

So far we have assumed $Z_2 \times Z_2$ symmetry.

Here we assume Z_2 only for φ .

Then the action has 5 parameters with positive mass dimensions (relevant parameters):

$$\mathcal{L}_{\phi\varphi} = \frac{1}{2}\,(\partial\phi)^2 + \frac{1}{2}\,(\partial\varphi)^2 - V$$

$$V = g\phi + \frac{m^2}{2}\phi^2 + \frac{h}{3!}\phi^3 + \frac{\sigma}{2}\phi\varphi^2 + \frac{m'^2}{2}\varphi^2$$

$$+ \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2\varphi^2 + \frac{\rho'}{4!}\varphi^4$$

We can set $g = 0$ by shifting ϕ.

$\Rightarrow$ 4 relevant parameters.

The problem is to find tetra critical points in the space of 4 parameters m^2, h, σ, m'^2 .

Instead of seeking the general solutions, we construct a special solution.

First we take the conditions $m^2 = m'^2 = 0$.
In fact these are criticality conditions because the behavior of $V_{\text{eff}} \sim \frac{m^2}{2} \phi^2 + \frac{m'^2}{2} \varphi^2$

around $\phi = \varphi = 0$ changes drastically depending on the signs of m^2 and m'^2 .

Then we take the condition $\sigma = 0$.

Again this is a criticality condition because the behavior of $V_{\text{eff}} \sim \frac{h}{3!}\phi^3 + \frac{\sigma}{2}\phi\varphi^2$ changes drastically depending on the signs of σ .

Then the only remaining parameter is h. We determine it by the criticality condition of the effective potential as in the case of 1-st order phase transition:

$$V_{\text{eff}}(\phi, \varphi = 0) = \frac{h}{6}\phi^3 + \frac{\kappa(\mu_*)^2}{256\pi^2}\phi^4 \log\left(\frac{\kappa(\mu_*)\phi^2}{\mu_*^2}\right)$$
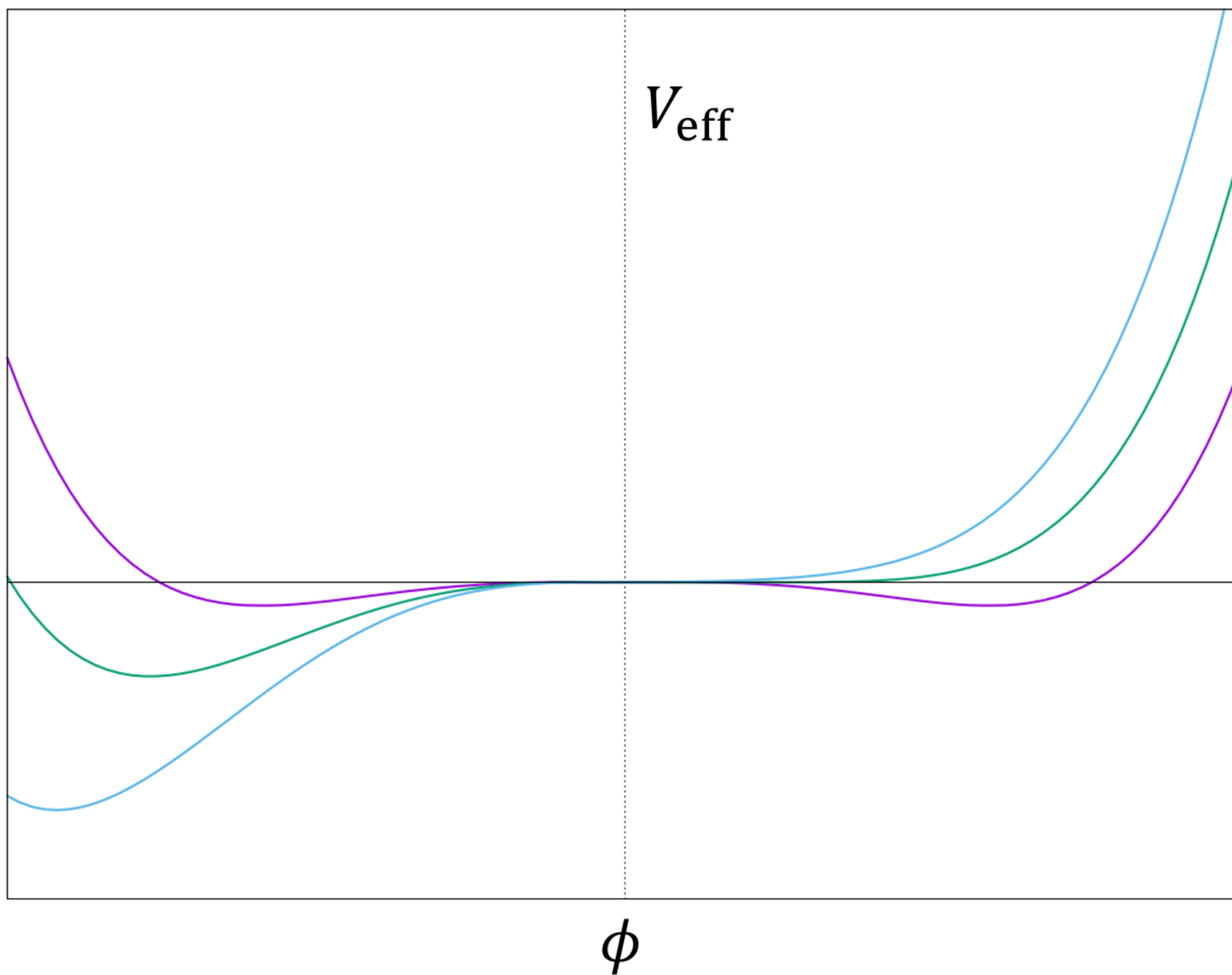
$$V_{\text{eff}}(\phi, \varphi = 0) = \frac{h}{6}\phi^3 + \frac{\kappa(\mu_*)^2}{256\pi^2}\phi^4 \log\left(\frac{\kappa(\mu_*)\phi^2}{\mu_*^2}\right)$$
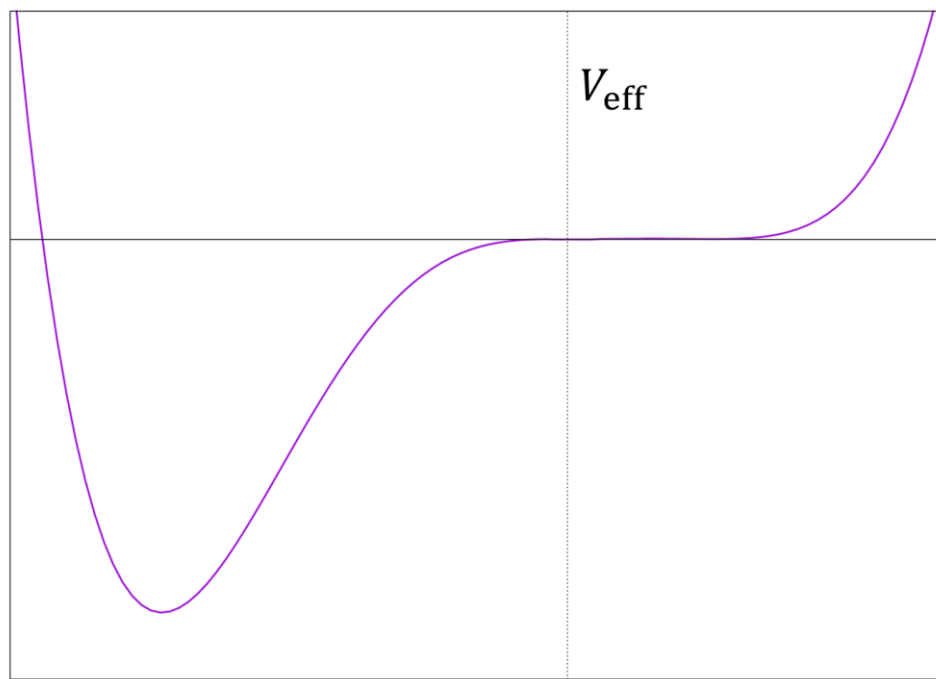
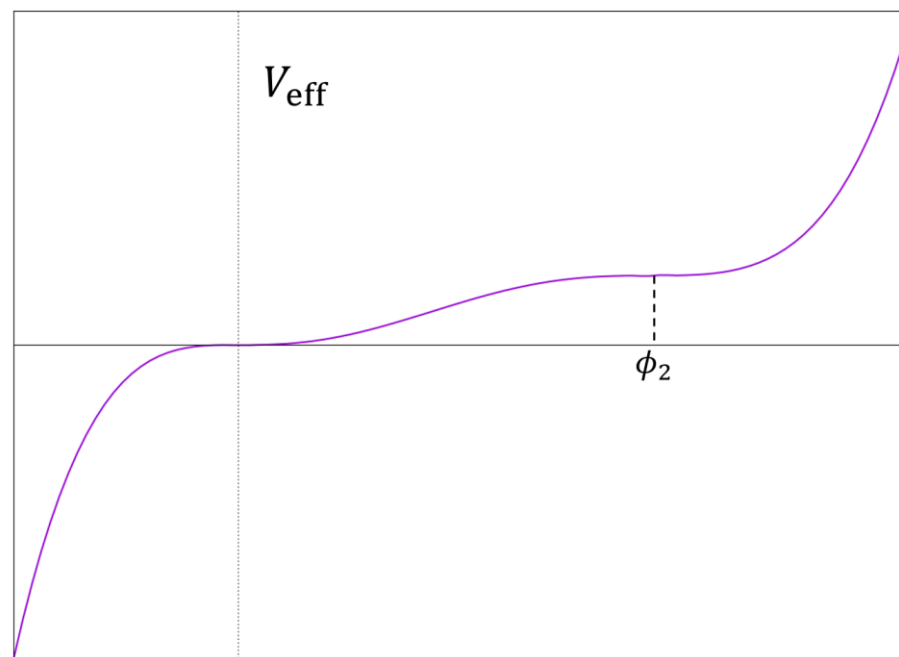$$h_1 = 0.71 \frac{\kappa^2 v}{32\pi^2}, \quad \phi_1 = 0.47v$$
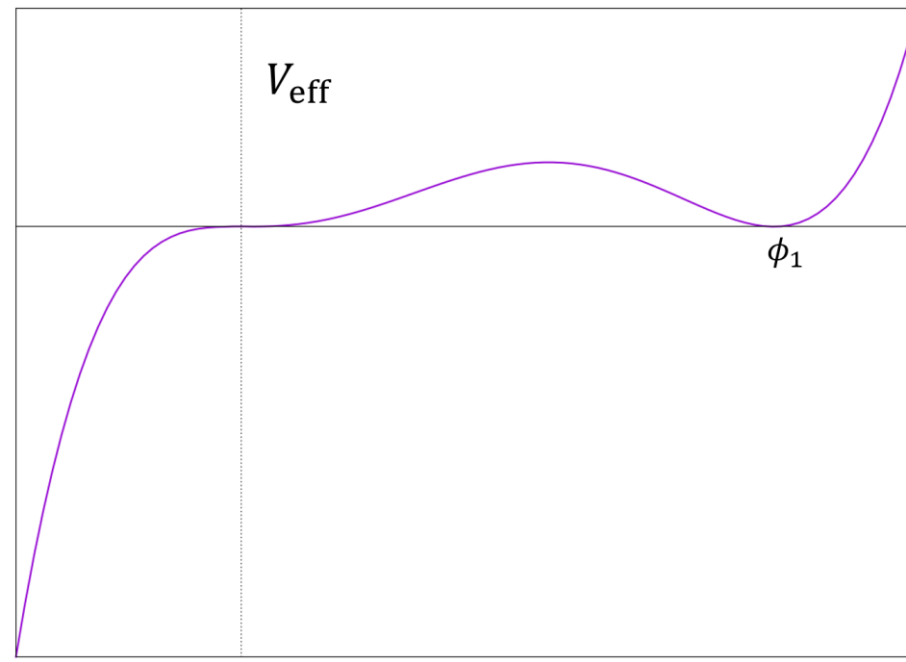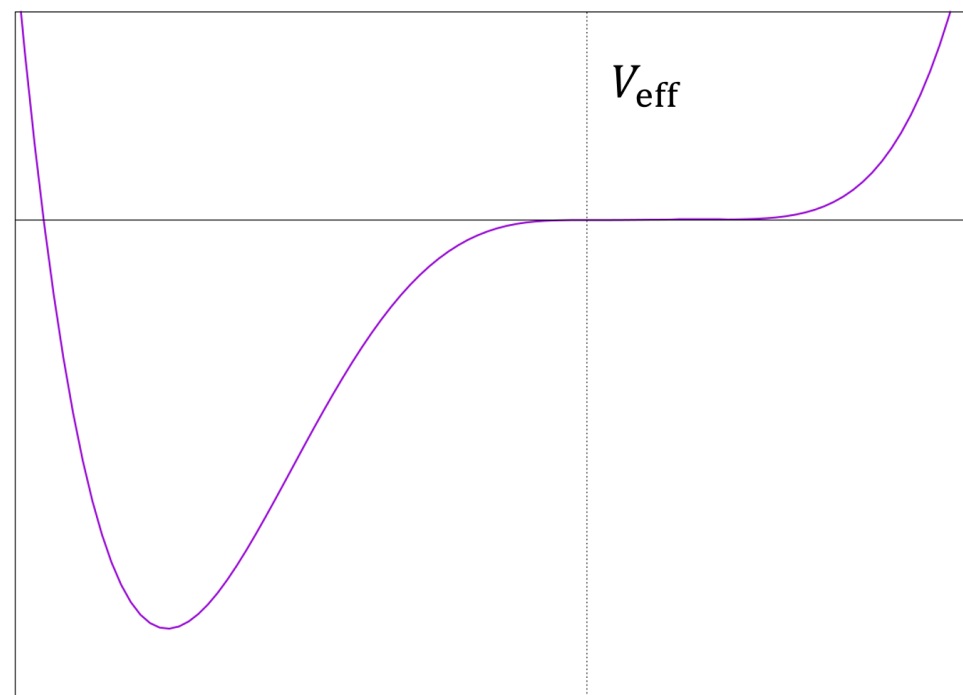
$$h_2 = 0.74 \frac{\kappa^2 v}{32\pi^2}, \quad \phi_2 = 0.37v$$

These do not have Z_2 symmetry for ϕ.

$\Rightarrow$ No cosmological domain wall problem.

Mass scales v, M_ϕ, M_φ are similar to the previous ones.

Again the weak scale emerges from the Planck scale non-perturbatively.

Summary

In wide classes of quantum gravity or string theory, the low energy effective action has **the multi local form**:

$$S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots.$$

The fine tuning problem might be solved by the dynamics of such action.

We need a good definition of the path integral for such action, but as an ad hoc attempt we can use the **generalized MPP** and make non-trivial predictions.
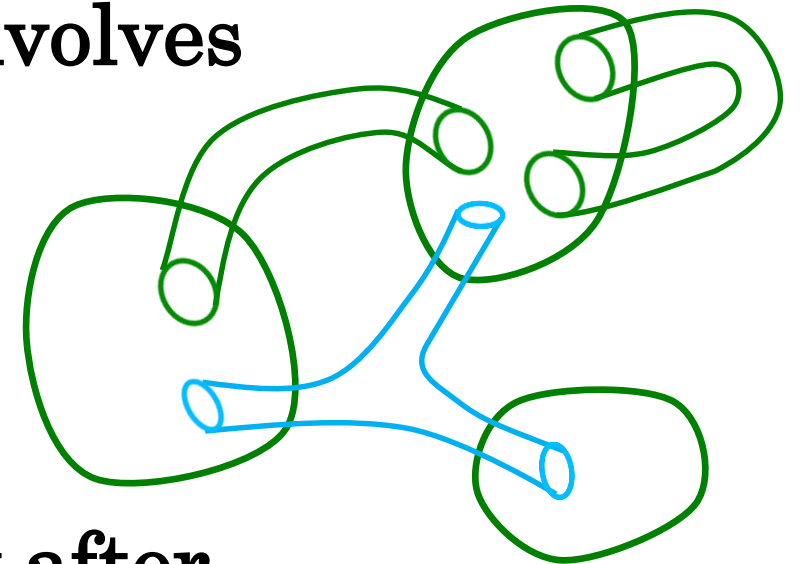
Appendix A
Low energy effective theory of quantum gravity/string theory

Consider Euclidean path integral which involves
the summation over topologies,

$$\sum_{\text{topology}} \int [dg] \exp(-S) .$$

We consider the low energy effective theory after
integrating out the short-distance configurations.

Among such configurations there should be a wormhole-
like configuration in which a thin tube connects two points
on the universe. Here, the two points may belong to either
the same universe or different universes.

If we see such configuration from the side of the large
universe(s), it looks like two small punctures.

But the effect of a small puncture is equivalent to an
insertion of a local operator.

Therefore, a wormhole contributes to the path integral as

$$\int [dg] \sum_{i,j} c_{ij} \int d^4x \, d^4y \sqrt{g(x)} \sqrt{g(y)} \, O^i(x) \, O^j(y) \, \exp(-S) \, .$$
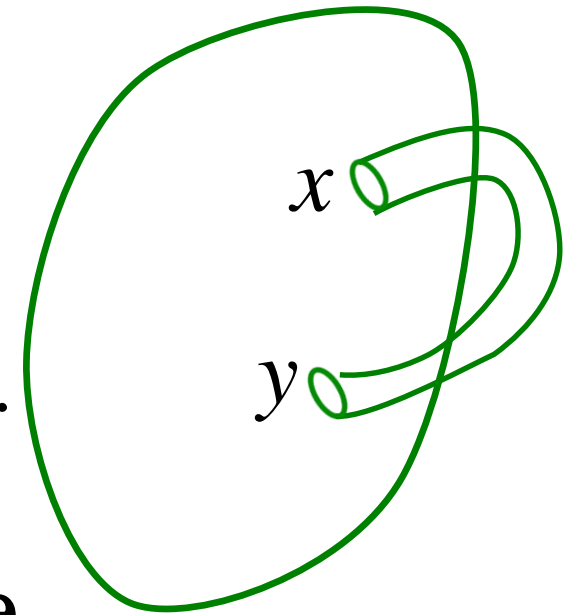
Summing over the number of wormholes, we have

$$\sum_{N=0}^{\infty} \frac{1}{n!} \left(\sum_{i,j} c_{ij} \int d^4x \, d^4y \sqrt{g(x)} \sqrt{g(y)} \, O^i(x) \, O^j(y) \right)^n$$

$$= \exp \left(\sum_{i,j} c_{ij} \int d^4x \, d^4y \sqrt{g(x)} \sqrt{g(y)} \, O^i(x) \, O^j(y) \right) .$$

Thus wormholes contribute to the path integral as

$$\int [dg] \exp \left(-S + \sum_{i,j} c_{ij} \int d^4x \, d^4y \sqrt{g(x)} \sqrt{g(y)} \, O^i(x) \, O^j(y) \right) .$$

bifurcated wormholes
$\Rightarrow$ cubic terms, quartic terms, ...

The effective action becomes a multi-local form

$$S_{\text{eff}} = \sum_i c_i \, S_i + \sum_{ij} c_{ij} \, S_i S_j + \sum_{ijk} c_{ijk} \, S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

By introducing the Laplace transform

$$\exp\left(-S_{\text{eff}}\left(S_1, S_2, \cdots\right)\right) = \int d\lambda \, w\left(\lambda_1, \lambda_2, \cdots\right) \exp\left(-\sum_i \lambda_i S_i\right),$$

we can express the path integral as

$$Z = \int [d\phi] \exp(-S_{\text{eff}}) = \int d\lambda \, w(\lambda) \int [d\phi] \exp\left(-\sum_i \lambda_i S_i\right).$$

Coupling constants are not merely constant, but they should be integrated.
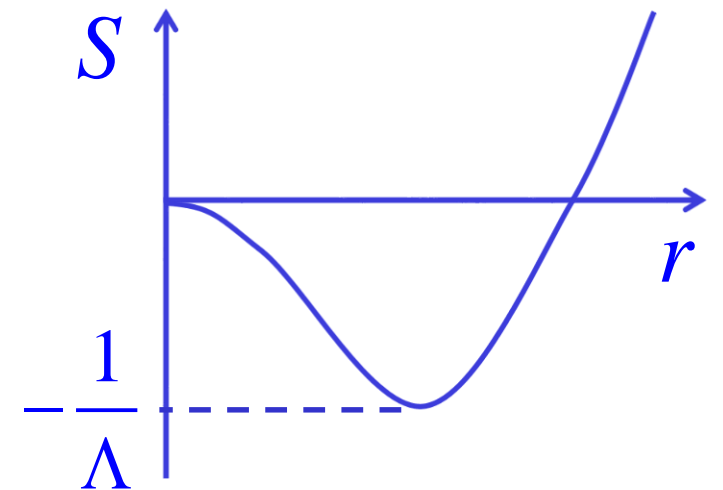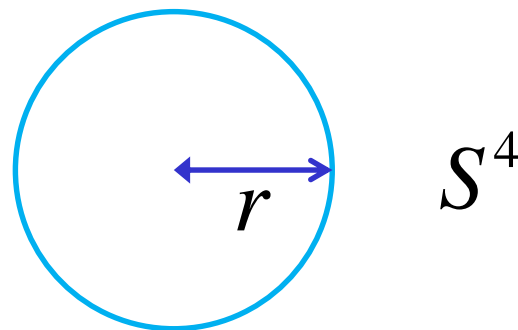
including multiverse

$$Z = \int d\lambda \, w(\lambda) \int [d\phi] \exp(-S(\lambda))$$

$$= \int d\lambda \, w(\lambda) \sum_{n=0}^{\infty} \frac{1}{n!} Z_{\text{single}}{}^{n}$$

$$= \int d\lambda \, w(\lambda) \exp(Z_{\text{single}}).$$

n
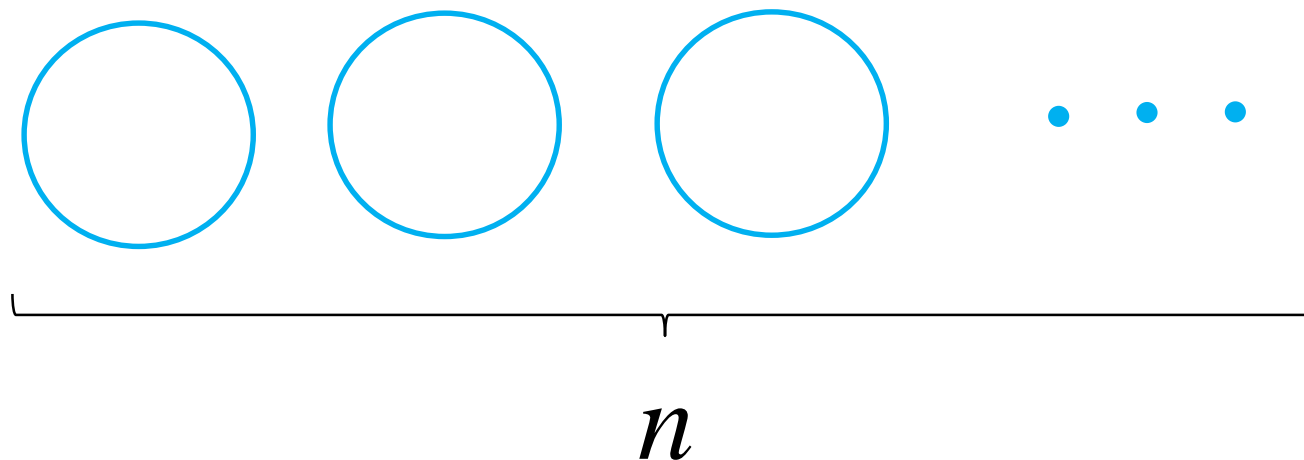
Coleman's "solution" to the cosmological constant problem

$$Z = \int d\Lambda\, w(\Lambda) \int [dg]\, \exp\left(-\int \sqrt{g}\, R - \Lambda \int \sqrt{g}\,\right).$$

$$\sim \int d\Lambda\, w(\Lambda) \int dr\, \exp\left(-\left(-r^2 + \Lambda\, r^4\right)\right)$$

$$\sim \int d\lambda\, w(\Lambda) \begin{cases} \exp(1/\Lambda), & \Lambda > 0 \\ \text{no solution,} & \Lambda < 0 \end{cases}$$

$\Lambda \sim 0$ dominates irrespectively of $w(\Lambda)$.

Difficulty
Problem of the Wick rotation

WDW eq.
$$H_{\text{total}}\big|\Psi\big\rangle = 0$$

$$H_{\text{total}} = H_{\text{universe}} + H_{\text{matter}} + H_{\text{graviton}} + \cdots$$

$$H_{\text{universe}} = -\left(\frac{1}{2}p_a{}^2 + \cdots\right) \quad \leftarrow \text{wrong sign}$$
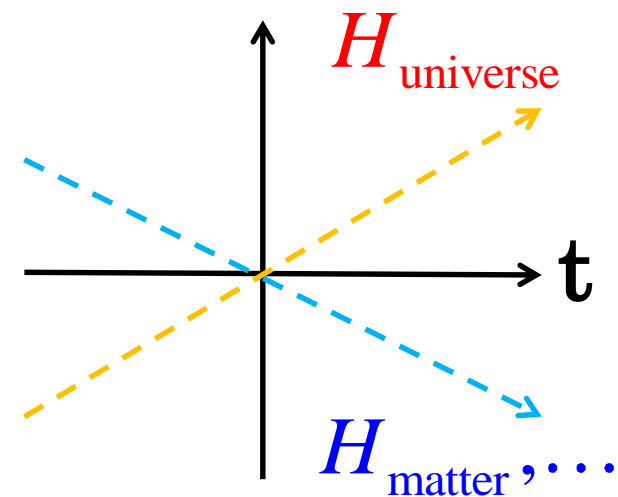
⇓

a : radius of the universe

"Ground state" does not exist.

Wick rotation is not well defined.

H_{matter} is bounded from below.

H_{universe} is bounded from above.

We expect that the physics with gravity should be expressed in Lorentzian signature, but the low energy effective theory is still given by the multi local action.

In fact we obtain the same effective Lagrangian in the IIB matrix model with Lorentzian signature.

Covariant derivatives as matrices

Y. Kimura, M. Hanada and HK

The basic question :

In the large-N reduced model, a background of simultaneously diagonalizable matrices $A_\mu^{(0)} = P_\mu$ corresponds to the flat space, if the eigenvalues are uniformly distributed.

In other words, the background $A_\mu^{(0)} = i\partial_\mu$ represents the flat space.

How about curved space?

Is it possible to consider some background like

$$A_\mu^{(0)} = i\nabla_\mu \ ?$$

Actually, there is a way to express the covariant derivatives on any D-dim manifold by D matrices.

More precisely, we consider

M: any D-dimensional manifold,

φ_α: a regular representation field on M.

Here the index α stands for the components of the regular representation of the Lorentz group $SO(D-1, 1)$.

The crucial point is that for any representation r, its tensor product with the regular representation is decomposed into the direct sum of the regular representations:

$$V_r \otimes V_{reg} \cong V_{reg} \oplus \cdots \oplus V_{reg}.$$

In particular the Clebsh-Gordan coefficients for the decomposition of the tensor product of the **vector** and the **regular** representaions

$$V_{vector} \otimes V_{reg} \cong V_{reg} \oplus \cdots \oplus V_{reg}$$

are written as $C_{(a)\alpha}{}^{b,\beta}$, $(a = 1,..,D)$.

Here b and β are the dual of the vector and the regular representation indices on the LHS. (a) indicates the a-th space of the regular reprezentation on the RHS, and α is its index.

Then for each a ($a = 1..D$)

$$\psi_\alpha = C_{(a)\alpha}{}^{b,\beta} \nabla_b \varphi_\beta$$

is a regular representation field on M.

In other words, if we define $\nabla_{(a)}$ by

$$\left(\nabla_{(a)}\varphi\right)_\alpha = C_{(a)\alpha}{}^{b,\beta} \nabla_b \varphi_\beta \, ,$$

each $\nabla_{(a)}$ is an endomorphism on the space of the regular representation field on M.

Thus we have seen that the covariant derivatives on any D dimensional manifold can be expressed by D matrices.

Therefore any *D*-dimensional manifold *M* with $D \leq 10$ can be realized in the space of the IIB matrix model as

$$A_a{}^0 = \begin{cases} \nabla_{(a)}, & a = 1, \cdots, D \\ 0, & a = D+1, \ldots, 10 \end{cases},$$

where $\nabla_{(a)}$ is the covariant derivative on *M* multiplied by the C-G coefficients.
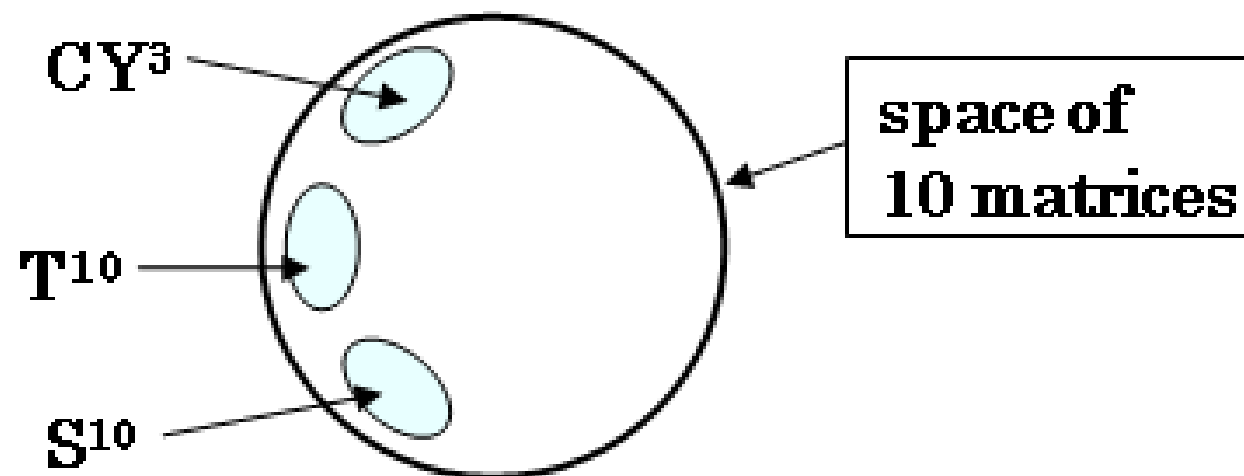
Low energy effective action of IIB matrix model

A. Tsuchiya, Y. Asano and HK

We have seen that any D-dim manifold is contained in the space of D matrices. Therefore IIB matrix model should contain the effects of the topology change of space-time.

As was pointed out by Coleman some years ago, such effects give significant corrections to the low energy effective action.

It is interesting to consider the low energy effective action of the IIB matrix model.

Actually we can show that **if we integrate out the heavy states** in the IIB matrix model, the remaining low energy effective action is not a local action but has a special form, which we call *the multi-local action*:

$$S_{\mathrm{eff}} = \sum_i c_i\, S_i + \sum_{ij} c_{ij}\, S_i S_j + \sum_{ijk} c_{ijk}\, S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)}\, O_i(x).$$

Here O_i are local scalar operators such as

$$1\,,\, R\,,\, R_{\mu\nu} R^{\mu\nu}\,,\, F_{\mu\nu}\, F^{\mu\nu},\, \overline{\psi}\gamma^\mu D_\mu \psi\,,\, \cdots.$$

S_i are parts of the conventional local actions.
The point is that S_{eff} is a function of S_i's.

This is essentially the consequence of the well-known fact that the effective action of a matrix model contains multi trace operators.

More precisely, we first decompose the matrices A_a into the background $A^0{}_a$ and the fluctuation ϕ :

$$A_a = A^0{}_a + \phi_a.$$

Here we assume that the background $A^0{}_a$ contains only the low energy modes, and ϕ contains the rest. We also assume that this decomposition can be done in a $SU(N)$ invariant manner.

Then we integrate over ϕ to obtain the low energy effective action.

Substituting the decomposition into the action of the IIB matrix model, and dropping the linear terms in ϕ, we obtain

$$S = \frac{1}{4} Tr \Big(\left[A_a^0, A_b^0 \right]^2$$

$$+ 2 \left[A_a^0, \phi_b \right]^2 + \left[A_a^0, A_b^0 \right] \left[\phi_a, \phi_b \right] - 2 \left[A_a^0, \phi_b \right] \left[A_b^0, \phi_a \right]$$

$$+ 4 \left[A_a^0, \phi_b \right] \left[\phi_a, \phi_b \right] + \left[\phi_a, \phi_b \right]^2 + \text{fermion} \Big).$$

In principle, the 0-th order term $S_0 = \frac{1}{4} Tr \left(\left[A_{(a)}^0, A_{(b)}^0 \right]^2 \right)$ **can be evaluated with some UV regularization, which should give a local action.**

The one-loop contribution is obtained by the Gaussian integral of the quadratic part.

Then the result is given by a double trace operator as usual:

$$W = \sum K^{abc\cdots,\,pqr\cdots}\, Tr\big(A_a^0 A_b^0 A_c^0 \cdots\big)\, Tr\big(A_p^0 A_q^0 A_r^0 \cdots\big)$$

The crucial assumption here is that both of the diffeomorphism and the local Lorentz invariance are realized as a part of the SU(N) symmetry.

Then each trace should give a local action that is invariant under the diffeomorphisms and the local Lorentz transformations:

$$S_{\text{eff}}{}^{\text{1-loop}} = \sum_{ij} \frac{1}{2} c_{ij} S_i S_j, \quad S_i = \int d^D x \sqrt{g(x)}\, O_i(x).$$

Similar analyses can be applied for higher loops.

In the two loop order, from the planar diagrams we have a cubic form of local actions

$$S_{\text{eff}}{}^{\text{2-loop Planar}} = \sum_{i,j,k} \frac{1}{6} c_{ijk} S_i S_j S_k,$$

while non-planar diagrams give a local action

$$S_{\text{eff}}{}^{\text{2-loop NP}} = \sum_i c_i' S_i.$$
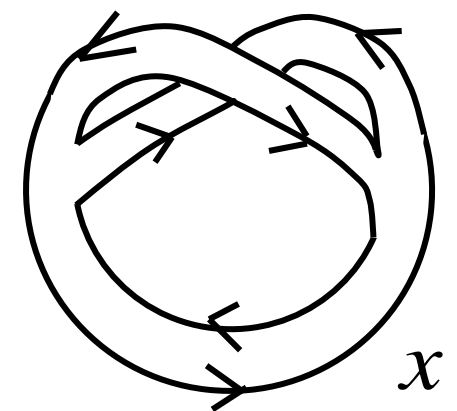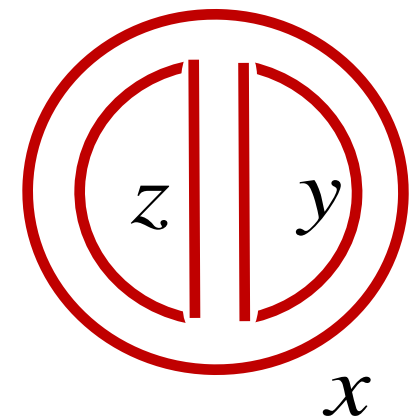
We have seen that

the low energy effective theory of the IIB matrix model is given by *the multi-local action*:

$$S_{\text{eff}} = \sum_i c_i \, S_i + \sum_{ij} c_{ij} \, S_i S_j + \sum_{ijk} c_{ijk} \, S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

This reminds us of the theory of baby universes by Coleman.

Appendix B
Multiverse and naturalness

3-1. Partition function of the multiverse

It is natural to apply this action to the multiverse.

$$Z = \int [d\phi] \exp(i S_{\text{eff}}) = \int d\lambda\, w(\lambda) \int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right).$$

$$\int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right) = \sum_{n=0}^{\infty} \frac{1}{n!} Z_1^{\,n}$$

$$Z_1 = \int [d\phi]_{\text{single universe}} \exp\left(i \sum_i \lambda_i S_i \right)$$

$$n$$
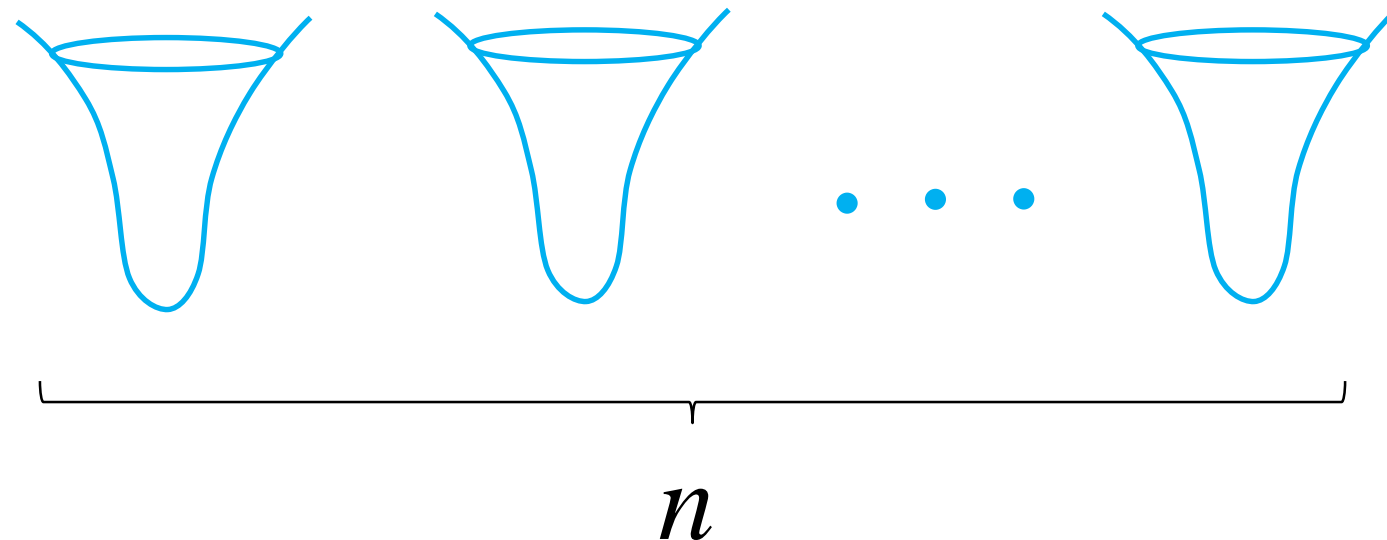
$$= \int d\lambda\, w(\lambda) \exp\left(Z_1(\lambda) \right).$$

Path integral for a universe

If the initial and final states are given, the path integral is evaluated as usual: (mini superspace)

$|f\rangle$

$$Z_1(\lambda) = \int [d\phi] \exp(iS)$$

$$= \langle f | \int [da\, dp\, dN] \exp\left(i\int_0^1 dt\, (p\dot{a} - NH_\lambda)\right)|i\rangle \quad T$$

$$= \langle f | \int_{-\infty}^{\infty} dT \exp\left(-iT\hat{H}_\lambda\right)|i\rangle$$

$|i\rangle$

$$= \langle f | \delta\left(\hat{H}_\lambda\right)|i\rangle$$

$$\hat{H}_\lambda = -\frac{1}{2}\frac{1}{\sqrt{a}} p^2 \frac{1}{\sqrt{a}} - a^3 U(a)$$

$$= \langle f | \phi_{E=0} \rangle \langle \phi_{E=0} | i \rangle$$

$$U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{matt}}{a^3} - \frac{C_{rad}}{a^4}$$

$$\langle \phi_E | \phi_{E'} \rangle = \delta(E - E')$$

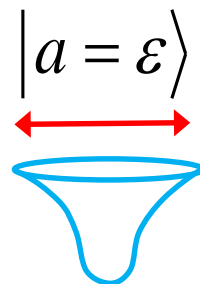a : radius of the universe

Question:
Is there a natural choice for them?

Initial state

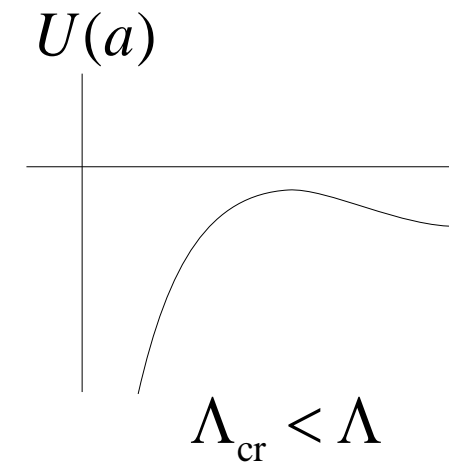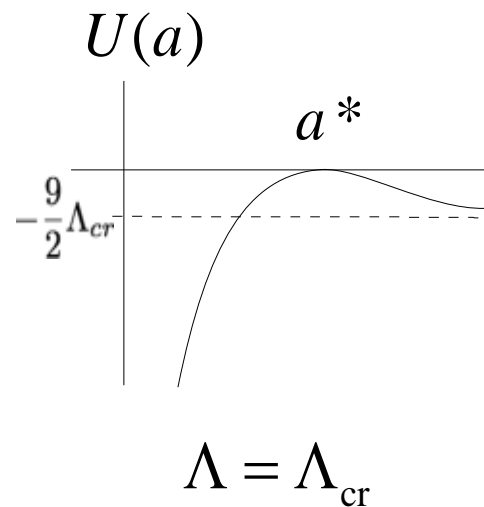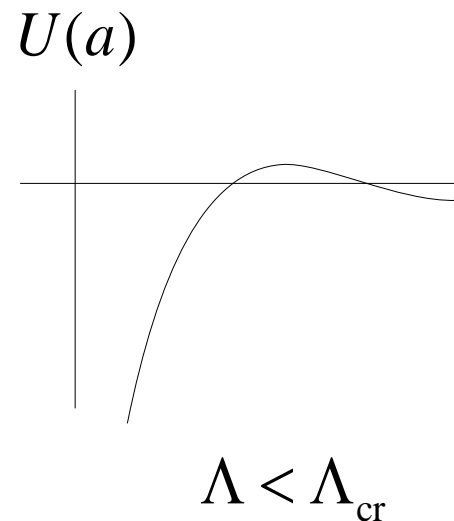For the initial state, we assume that the universe emerges with a small size ε.

$$|i\rangle = \mu |a = \varepsilon\rangle \otimes |matter \cdots\rangle,$$

μ : probability amplitude of a universe emerging.

$$|a = \varepsilon\rangle$$

Evolution of the universe S^3 topology

$U(a)$

$U(a)$

$a*$

$-\frac{9}{2}\Lambda_{cr}$

$U(a)$

$\Lambda < \Lambda_{cr}$

$\Lambda = \Lambda_{cr}$

$\Lambda_{cr} < \Lambda$

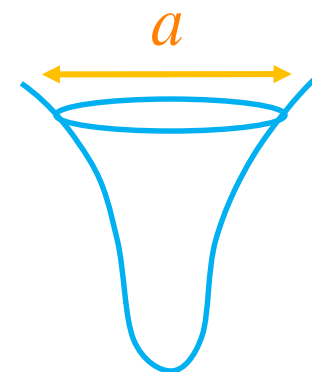$\Lambda \sim$ curvature

$\sim$ energy density

WKB solution

$$\phi_{E=0}(a,\lambda) \sim \frac{1}{\sqrt{a^{-1}\, p(a,\lambda)}} \sin\left(\int_0^z da' p(a',\lambda) + \alpha \right)$$

$$S = \int d^4x \sqrt{-g}\left(R - \Lambda + matter \right)$$

a

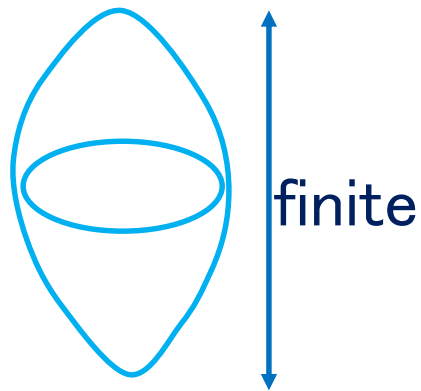with $p(a,\lambda) = \sqrt{-2a^4 U(a)}.$

$$U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{matt}}{a^3} - \frac{C_{rad}}{a^4}$$

For the final state, we have two possibilities.

Final state: case 1 $\Lambda < \Lambda_{cr}$

finite

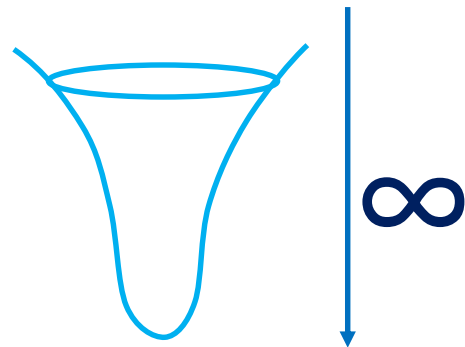The universe is closed.
We assume the final state is

$$\left|f\right\rangle = \mu'\left|a = \varepsilon\right\rangle \otimes \left|matter\cdots\right\rangle.$$

The path integral

$$Z_1(\lambda) = \left\langle f\left|\delta\left(\hat{H}_\lambda\right)\right|i\right\rangle$$

$$\sim const\left|\phi_{E=0}(\varepsilon)\right|^2.$$

Final state: case 2
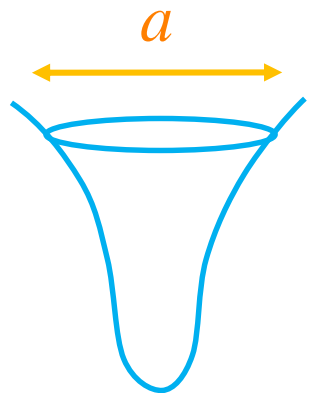
$$\Lambda > \Lambda_{\text{cr}}$$

∞

The universe is open.

It is not clear how to define the path integral for the universe:

$$Z_1(\lambda) = \int [d\phi] \exp(i S).$$

As an ad hoc assumption we consider

$$|f\rangle = \lim_{a_{IR} \to \infty} c \sqrt{a_{IR}} \, |a_{IR}\rangle \otimes |matter \cdots\rangle.$$

a

Then the partition function becomes

$$Z_1(\lambda) = \mu\, c\, \sqrt{a_{IR}}\, \phi_{E=0}(a_{IR})\, \phi^*_{E=0}(\varepsilon)$$

$$\sim \mu\, c\, \sqrt{a_{IR}}\, \frac{1}{\sqrt{a_{IR}}\, \sqrt[4]{\Lambda}}\, \sin\left(a_{IR}{}^3 \sqrt{\Lambda} + \alpha'\right)\phi^*_{E=0}(\varepsilon)$$

$$\sim \mu\, c\, \frac{1}{\sqrt[4]{\Lambda}}\, \sin\left(a_{IR}^3 \sqrt{\Lambda} + \alpha'\right)\phi^*_{E=0}(\varepsilon).$$

$$\phi_{E=0}(a) \sim \frac{1}{\sqrt{a^{-1}\, p(a)}}\, \sin\left(\int_0^z da'\, p(a') + \alpha\right)$$

$$p(a,\lambda) = \sqrt{-2a^4 U(a)} \qquad U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{mat}}{a^3} - \frac{C_{rad}}{a^4}$$
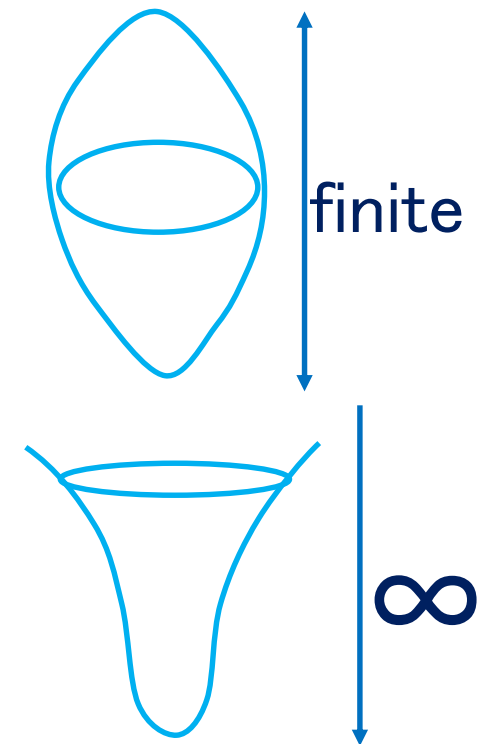
The result does not depend on a_{IR} except for the phase which come from the classical action.

Thus we have

the path integral for a universe $Z_1(\lambda)$

for $\Lambda < \Lambda_{cr}$ *const* of order 1 ,

for $\Lambda > \Lambda_{cr}$ $const\, \dfrac{1}{\sqrt[4]{\Lambda}} \sin\left(a_{IR}^{\,3} \cdot \sqrt{\Lambda} + \alpha' \right).$

finite

∞

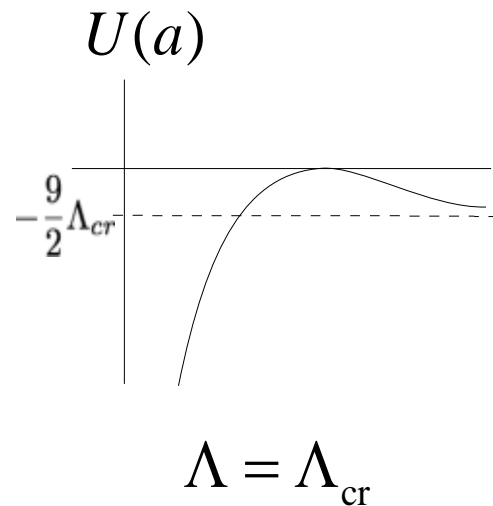**Then the λ integration for the multiverse**

$$Z = \int d\lambda\, w(\lambda) \exp\big(Z_1(\lambda) \big).$$

has a large peak at $\Lambda(\lambda) \sim \Lambda_{cr}$, which means that the cosmological constant at the late stages of the universe almost vanishes.

3-2. Maximum Entropy Principle

Maximum entropy principle

For simplicity we assume the S^3 topology of the space and that all matters decay to radiation at the late stages.

$U(a)$

$$U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{\text{rad}}}{a^4}$$

$-\frac{9}{2}\Lambda_{cr}$

$\Lambda_{\text{cr}} \sim 1/C_{\text{rad}}$

$\Lambda = \Lambda_{\text{cr}}$

Then the multiverse partition function is given by

$$Z = \int d\lambda\, w(\lambda) \exp\big(Z_1(\lambda)\big)$$

$$\sim \exp\left(\text{const}\, \frac{1}{\sqrt[4]{\Lambda_{cr}}} \right) \sim \exp\big(\text{const}\, \sqrt[4]{C_{\text{rad}}(\lambda)} \big).$$

Maximum entropy principle (MEP)

The low energy couplings are determined in such a way that the entropy at the late stages of the universe is maximized.

There are many ways to obtain MPP:

Suppose that we pic up a universe randomly from the multiverse. Then the most probable universe is expected to be the one that has the maximum entropy.

Okada and HK '11

Flatness of the Higgs potential

We may understand the flatness of the Higgs potential as a consequence of MEP.

If we accept the inflation scenario in which universe pops out from nothing and then inflates, most of the entropy of the universe is generated at the stage of reheating just after the inflation stops. Therefore the potential of the inflaton should be tuned in such a way that inflation occurs.

Furthermore, if the Higgs field plays the role of inflaton, the above analysis asserts that the SM parameters are tuned such that the Higgs potential becomes flat at high energy scale.

3-3. Probabilistic interpretation of multiverse wave function
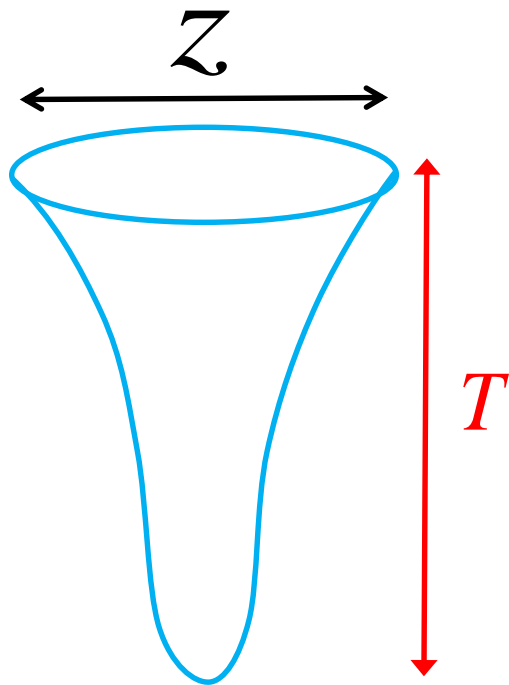
Probabilistic interpretation (1)

postulate
$$\psi(z) = \mu\, \phi_{E=0}(z)$$

$$|\psi(z)|^2\, dz \propto \text{probability of finding a universe of size } z$$

meaning of this measure

$$\int dz\, |\phi_{E=0}(z)|^2 \sim \int dz\, \frac{1}{z\, p(z)} \qquad \phi_{E=0}(z) \sim \frac{1}{\sqrt{z\, p(z)}} \exp\!\left(i\int^z dz'\, p(z')\right)$$

$$= \int dz\, \frac{1}{\dot{z}} \ = \int dT \qquad H = z\left(-\frac{1}{2}p^2 + \cdots\right) \rightarrow \dot{z} = \frac{\partial H}{\partial p} = -zp$$
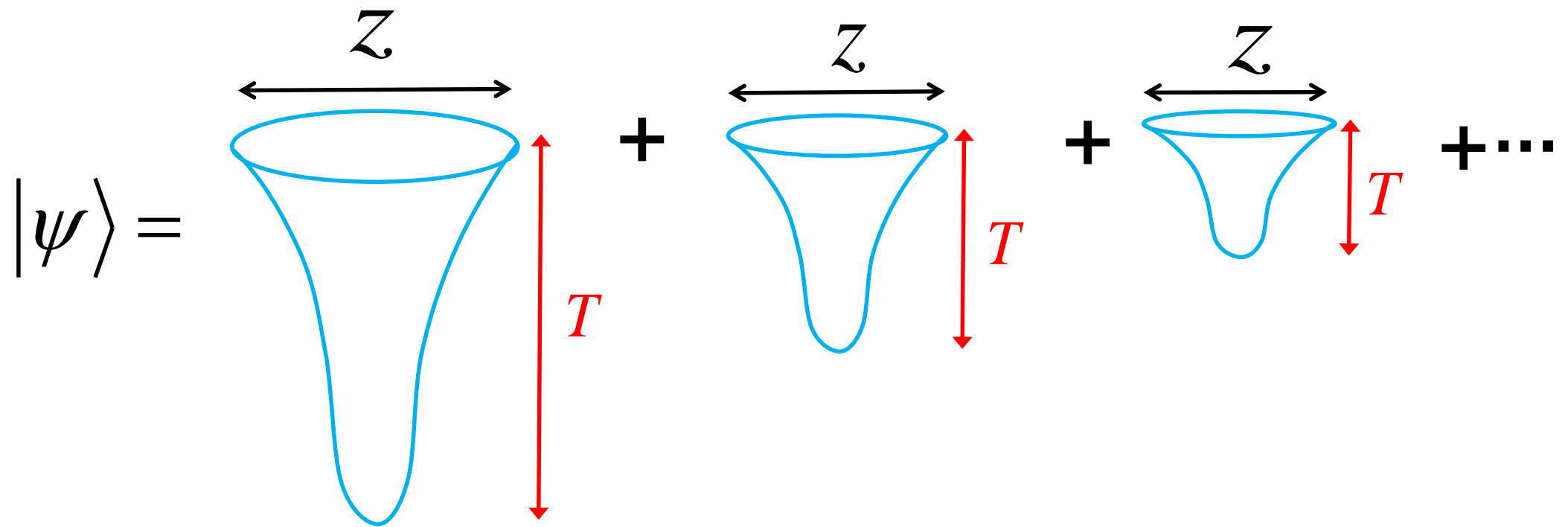
z

T

T : **age of the universe**

the time that has passed after the universe is created

$$\Rightarrow \ |\psi(z)|^2\, dz \sim |\mu|^2\, dT$$

$$|\mu|^2 = \text{probability of a universe emerging in unit time}$$

Probabilistic interpretation (2)

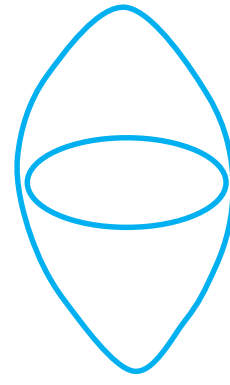$|\psi\rangle$ is a superposition of the universe with various age,

$$|\psi(z)|^2 \, dz \sim |\mu|^2 \, dT$$ gives the probability of finding a universe of age $T \sim T + dT$.
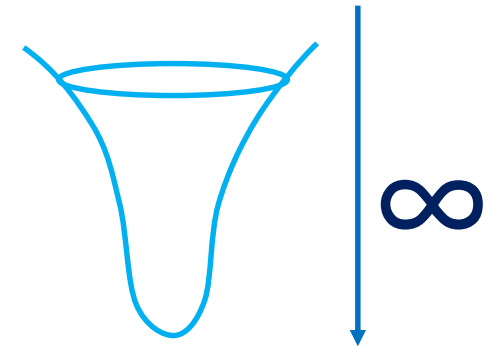
Lifetime of the universe

dimensionless

$$\int |\psi(z)|^2\, dz \sim |\mu|^2 \int dT = |\mu|^2 \times (\text{life time of the universe})$$
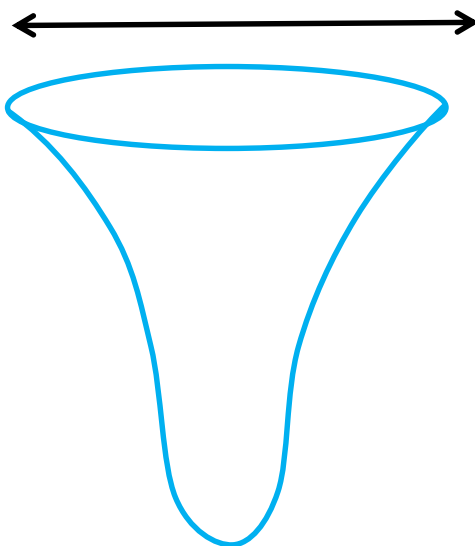
finite

∞

infrared cutoff

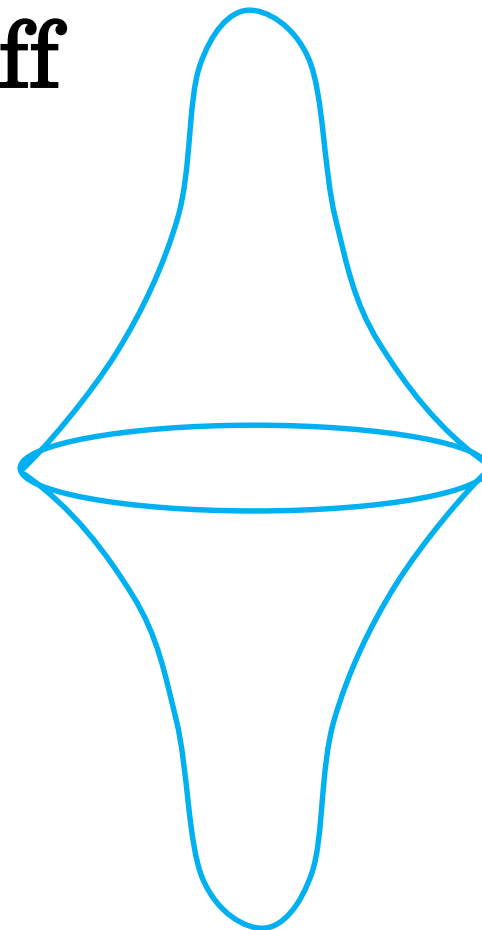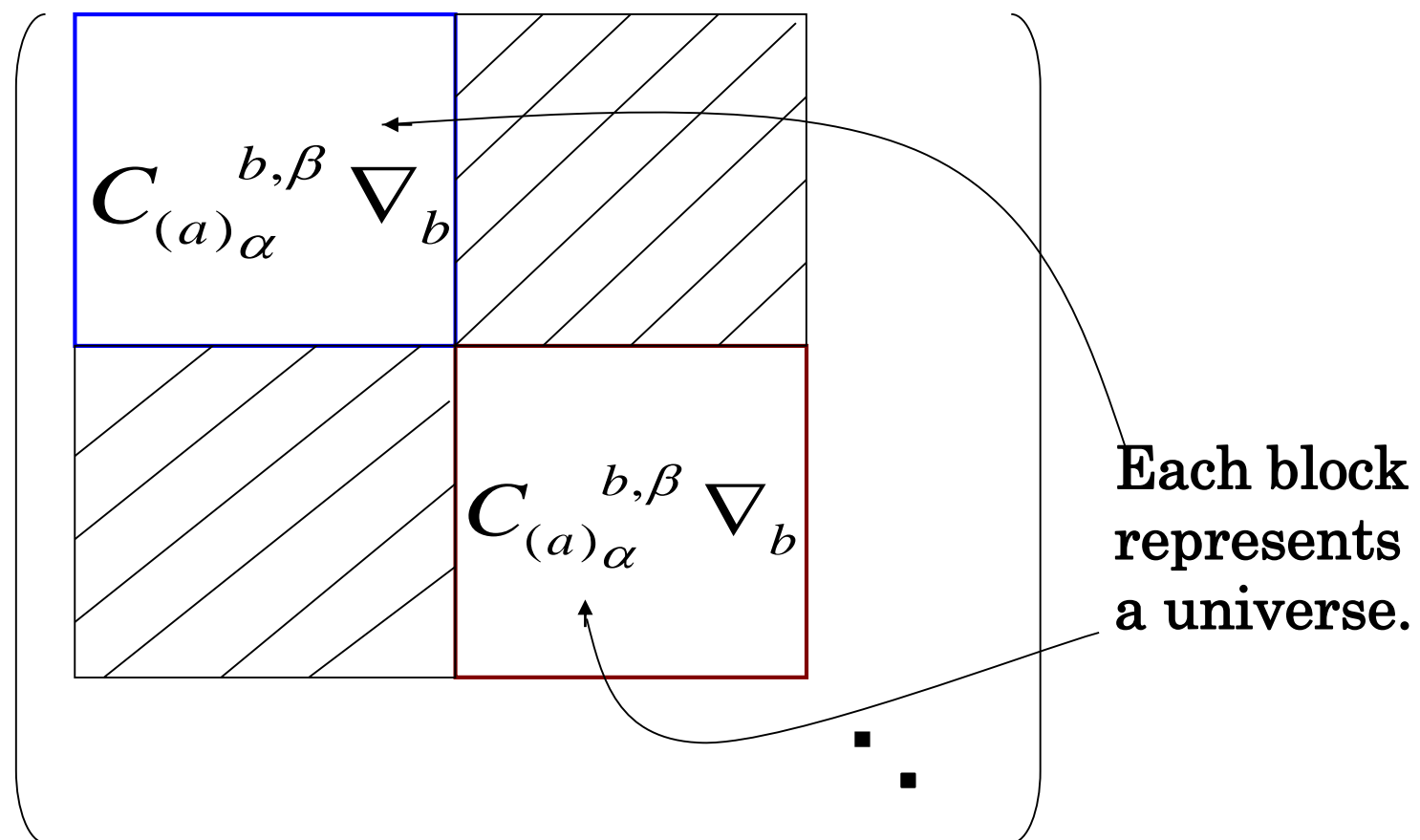We introduce an infrared cutoff for the size of universes.

z_{IR}

ceases to exist   or   bounces back

Wave Function of the multiverse (1)

Okada, HK

Multiverse appears naturally in quantum gravity / string theory.

$$C_{(a)\,\alpha}^{\ \ b,\beta}\nabla_b$$

$$C_{(a)\,\alpha}^{\ \ b,\beta}\nabla_b$$

Each block represents a universe.

matrix model
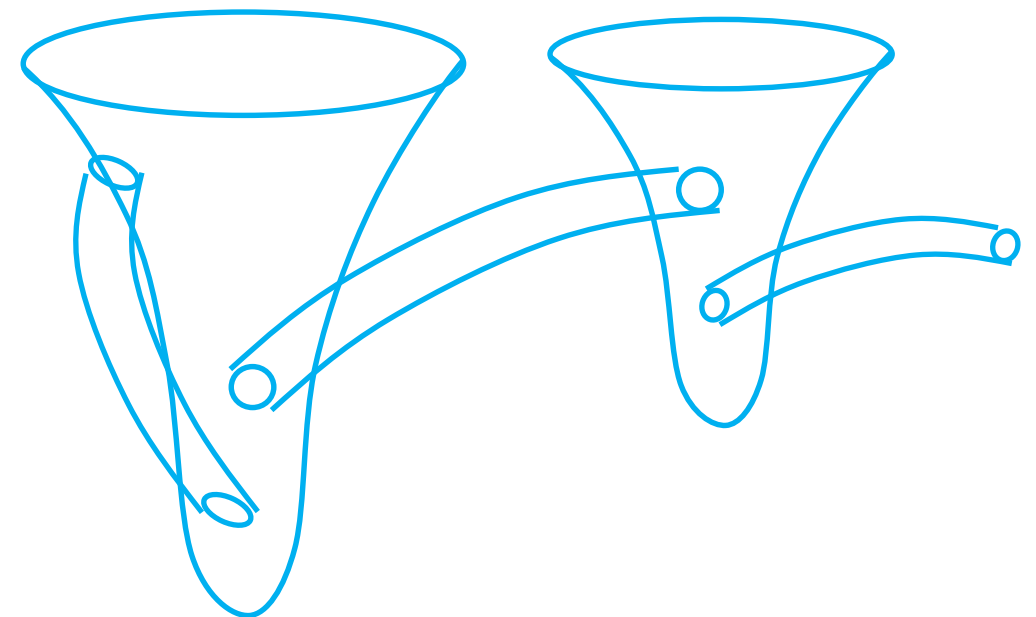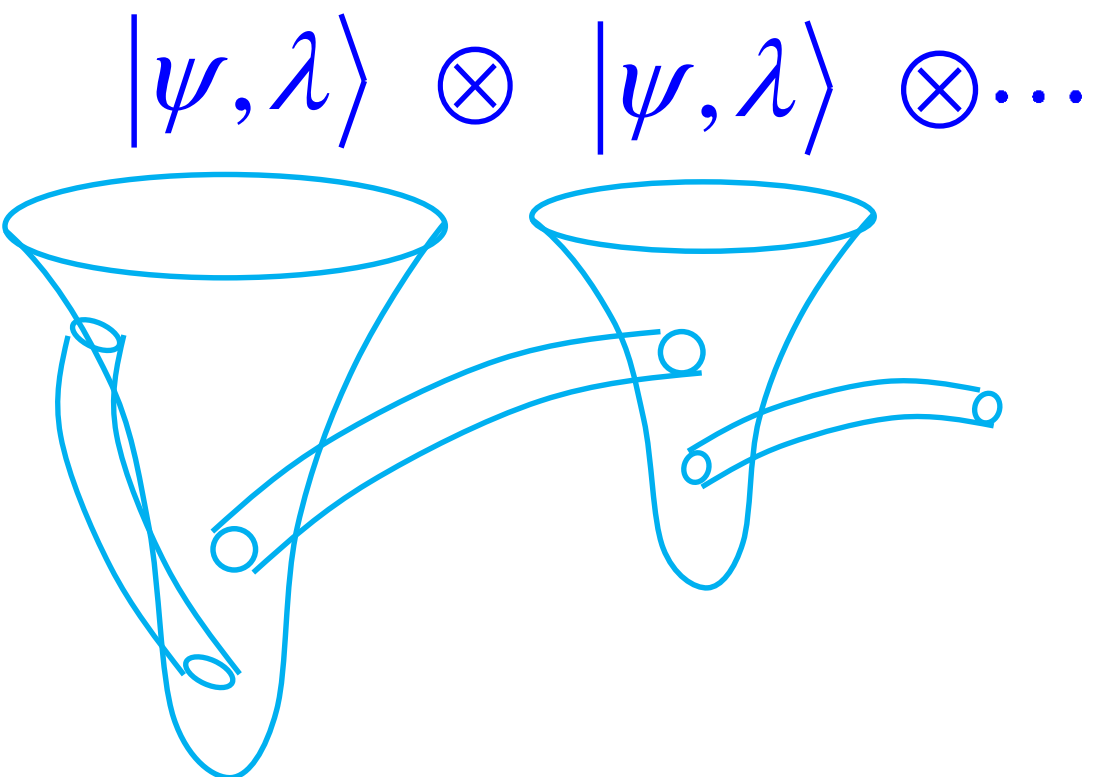
quantum gravity

Wave Function of the multiverse (2)

The multiverse sate is a superposition of N-verses.

$$\left| \Psi_{\text{multi}} \right\rangle = \int d\lambda \, w(\lambda) \sum_{N=0}^{\infty} \left| \Psi_N, \lambda \right\rangle \otimes \left| \lambda \right\rangle \quad \leftarrow Z = \int d\lambda \, w(\lambda) \int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right)$$

$$\Psi_N\left(z_1, \cdots, z_N, \lambda \right) = \psi\left(z_1, \lambda \right) \cdots \psi\left(z_N, \lambda \right)$$

$$\left| \psi, \lambda \right\rangle \otimes \left| \psi, \lambda \right\rangle \otimes \cdots$$

$$\left| \Psi_{\text{multi}} \right\rangle = \int d\lambda \, w(\lambda) \sum_N$$

Wave Function of the multiverse (3)

Probabilistic interpretation

$$\left| \Psi_{\text{multi}} \right\rangle = \int d\lambda \, w(\lambda) \sum_{N=0}^{\infty} \left| \Psi_N, \lambda \right\rangle \otimes \left| \lambda \right\rangle$$

$$\Psi_N \left(z_1, \cdots, z_N, \lambda \right) = \psi \left(z_1, \lambda \right) \cdots \psi \left(z_N, \lambda \right)$$

$\left| \Psi_N \left(z_1, \cdots, z_N, \lambda \right) \right|^2 dz_1 \cdots dz_N \left| w(\lambda) \right|^2 d\lambda$ represents

the probability of finding N universes with size

$$z_1 \sim z_1 + dz_1, \ \cdots, \ z_N \sim z_N + dz_N$$

and finding the coupling constants in

$$\lambda \sim \lambda + d\lambda.$$

Probability distribution of λ

$$P(\lambda) = \sum_{N=0}^{\infty} \int \frac{dz_1 \cdots dz_N}{N!} \left| \Psi_N(z_1, \cdots, z_N, \lambda) \right|^2 \left| w(\lambda) \right|^2$$

$$= \exp\left(\int dz \left| \psi(z, \lambda) \right|^2 \right) \left| w(\lambda) \right|^2 \qquad \leftarrow \Psi_N(z_1, \cdots, z_N, \lambda) = \psi(z_1, \lambda) \cdots \psi(z_N, \lambda)$$

$$\leftarrow |\psi\rangle = \mu |\phi_{E=0}\rangle$$

$$= \exp\left(\left| \mu \tau(\lambda) \right|^2 \right) \left| w(\lambda) \right|^2$$

$$\tau(\lambda) = \int dz \left| \phi_{E=0}(z, \lambda) \right|^2 \sim \text{(life time of the universe)}$$

$\tau(\lambda)$ can be very large.

$\Downarrow$

λ is chosen in such a way that $\tau(\lambda)$ is maximized, irrespectively of $w(\lambda)$.

If we accept the probabilistic interpretation of the multiverse wave function, the coupling constants are chosen in such a way that the lifetime of the universe becomes maximum.
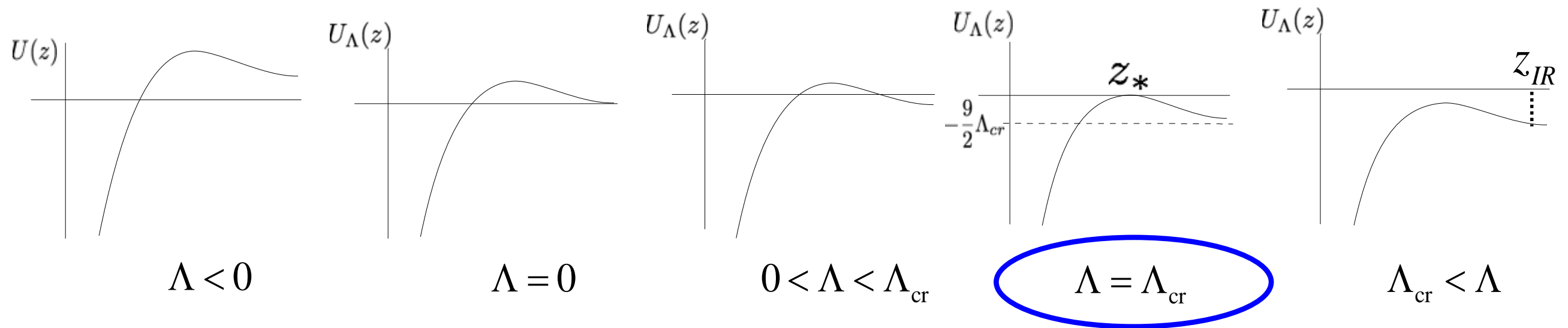
Cosmological constant

What value of Λ maximizes $\int dz \left| \mu \phi_{E=0}(z, \lambda) \right|^2$?

WKB sol $\phi_{E=0}(z, \lambda) \sim \dfrac{1}{\sqrt{z\, p(z, \lambda)}}$ with $p(z, \lambda) = \sqrt{-2U(z)}$. $\quad$ S^3 topology

$$U(z) = \frac{1}{z^{2/3}} - \Lambda - \frac{C_{matt}}{z} - \frac{C_{rad}}{z^{4/3}}$$

assuming all matters decay to radiation

$\Lambda < 0 \qquad\qquad \Lambda = 0 \qquad\qquad 0 < \Lambda < \Lambda_{cr} \qquad\qquad \Lambda = \Lambda_{cr} \qquad\qquad \Lambda_{cr} < \Lambda$

The cosmological constant in the far future is predicted to be very small.

$\Lambda \sim$ curvature $\sim$ energy density

$\Lambda_{cr} \sim 1/C_{rad}$ (extremely small)

The other couplings（Big Fix）

$$P(\lambda) = \exp\left(\left|\mu\tau(\lambda)\right|^2\right)\left|w(\lambda)\right|^2 \qquad \leftarrow \tau(\lambda) = \int dz \left|\phi_{E=0}(z,\lambda)\right|^2$$

The exponent is divergent, and regulated by the IR cutoff：

$$\int dz \left|\phi_{E=0}(z,\lambda)\right|^2 \sim \int_0^{z_{IR}} \frac{1}{z\sqrt{\Lambda_{\rm cr}}} \sim \sqrt{C_{\rm rad}}\,\log z_{IR}. \qquad \leftarrow \Lambda_{\rm cr} \sim 1/C_{\rm rad}$$

assuming all matters decay to radiation

MEP

λ are determined in such a way that $C_{\rm rad}(\lambda)$
is maximized.
Again we have MEP.