Optical tweezers
and the
ponderomotive force

Colloquium at Tsinghua University

2019-5-01

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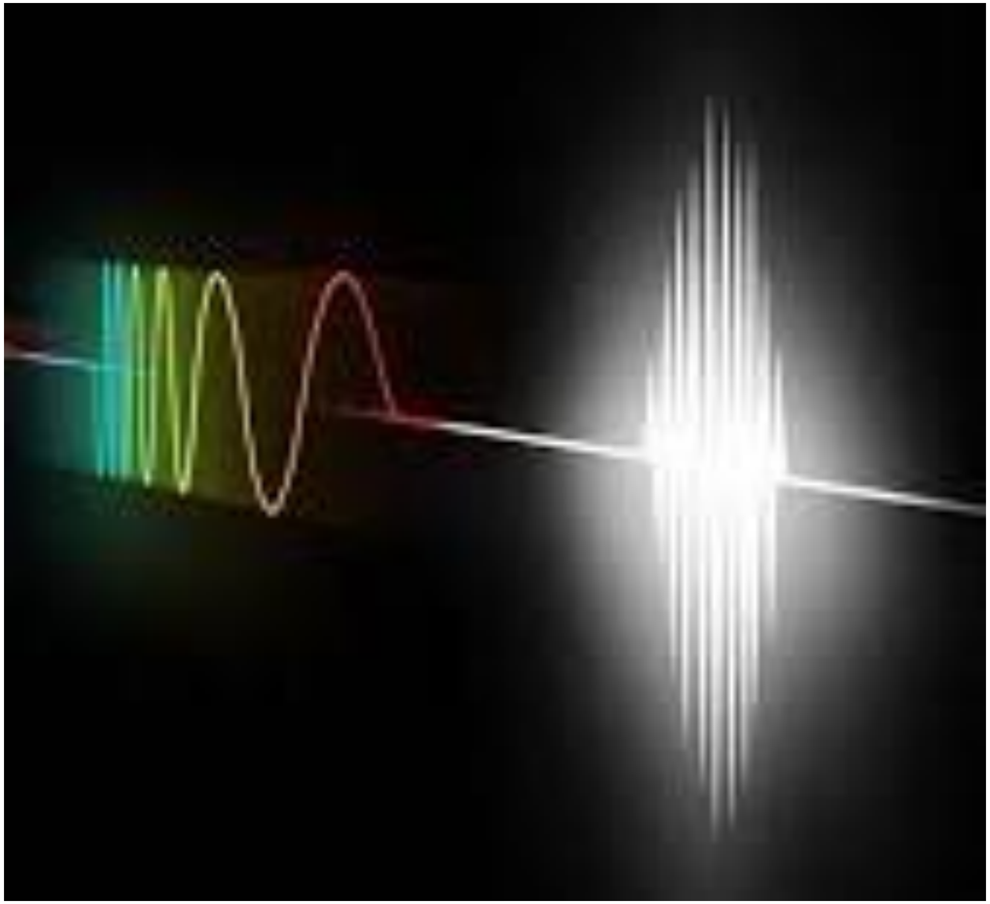
2018 Nobel Prize in Physics

- two parts
- both for laser physics



Arthur Ashkin

optical tweezers and applications to biological systems

Gerard Mourou &
Donna Strickland

generating high-intensity, ultra-short optical pulses

Arthur Ashkin

Born 1922

Age 96

https://www.youtube.com/watch?v=ZGZrjOnwXyk

How to win a Nobel Prize

- Stay healthy
- Live to 96 or more

“I didn’t realise I’m the oldest ever! So I just about made it, huh? Because you can’t be dead and win.”

Ashkin

… invented optical tweezers that grab particles, atoms, viruses and other living cells with their laser beam fingers. This new tool allowed Ashkin to realise an old dream of science fiction – using the radiation pressure of light to move physical objects.

He succeeded in getting laser light to push small particles towards the centre of the beam and to hold them there. Optical tweezers had been invented.

A major breakthrough came in 1987, when Ashkin used the tweezers to capture living bacteria without harming them.

He immediately began studying biological systems and optical tweezers are now widely used to investigate the machinery of life.

Our intersection with Ashkin

Response of a liquid surface to the passage of an intense laser pulse

HM Lai and K Young, PRA 14, 2329 (1976)

ABSTRACT

The temporal hydrodynamic response of the free surface of a transparent liquid dielectric due to the passage of an intense, focused laser pulse is considered theoretically, and the calculated focal length of the surface lens is in satisfactory agreement with the recent experimental result of Ashkin and Dziedzic. An understanding of this effect is important for answering the question of the momentum of light in a material medium.

The story

(A) Road to optical tweezers (our view)

1. Momentum of light and radiation pressure
2. Ashkin experiment & correct explanation
3. Key idea of ponderomotive force
4. Optical tweezers

The story

(B) Theoretical subtleties

5. Peierls' insight
6. The extra EM effect
7. Equilibrium under long-range forces

(A) Road to optical tweezers

1. Momentum of light & radiation pressure

In vacuum

Poynting vector $\vec{S} \sim \vec{E} \times \vec{B}$

Momentum density $\vec{g} \sim \vec{E} \times \vec{B}$

up to factors of c

In material medium

- Minkowski ?? $\vec{g} \sim \vec{D} \times \vec{B}$
- Abraham ?? $\vec{g} \sim \vec{E} \times \vec{H}$

Controversy and mistakes by Minkowski, Abraham, Einstein, Taub, …

Symmetry $$T_{\mu\nu} = T_{\nu\mu} \quad ??$$

Formal manipulations $$\frac{\partial}{\partial t} g = \partial \Pi + f \quad ??$$

$$P \Rightarrow D \Rightarrow \nabla \bullet D = \rho_{\mathrm{f}}$$

Averaging

But for stress tensor

$$P(\partial E) \Rightarrow \partial(ED)$$

Average a quadratic expression

Does P see average E?

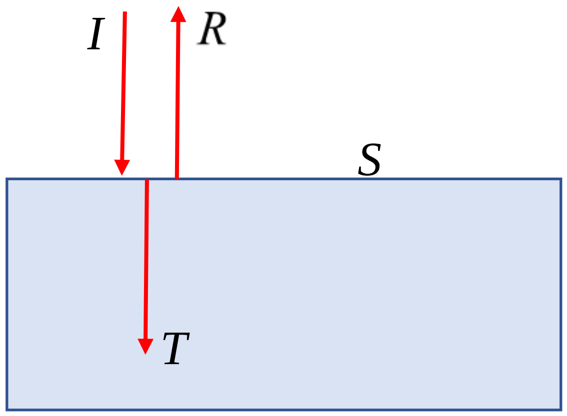
Rudolf Peierls

- Nuclear bomb
- Umklapp

File:Sir Rudolf Ernst Peierls.jpg

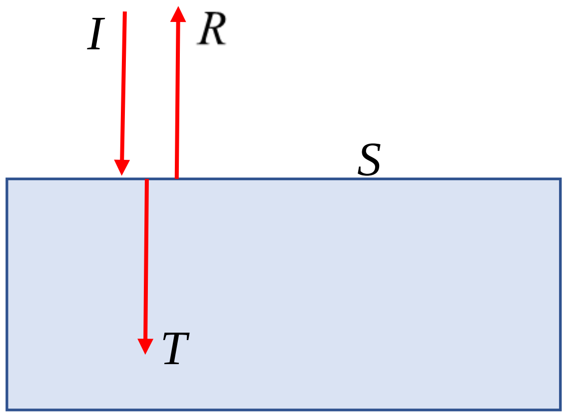
- Early 1970s: revived issue
- Experiment !

2. Ashkin experiment & correct explanation



- Measure movement of S
- Determine momentum in T

In effect measuring radiation pressure



- Observe *S*
- Determine *T*

- Surface bulged up!

Ashkin and Dziedzic, PRL 30, 139 (1973)

Radiation Pressure on a Free Liquid Surface

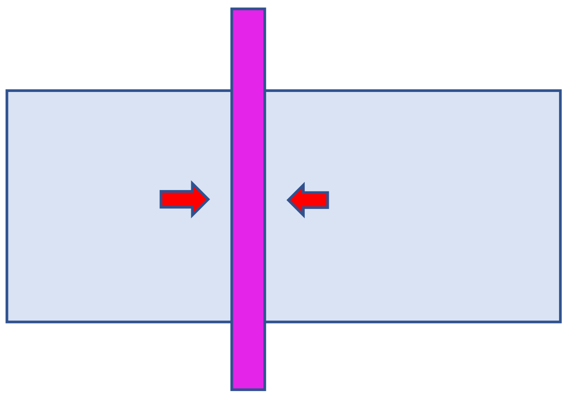
ABSTRACT

The force of radiation pressure on

the free surface of a transparent liquid dielectric has been observed … It is shown that light on either entering or leaving the liquid exerts a net outward force at the liquid surface. … The data relate to the understanding of the momentum of light in dielectrics.

Wrong!

Correct explanation



- Molecules like to go to region of high intensity
- It is a transverse effect!

- Ashkin started with radiation pressure: a “longitudinal” effect
- Our analysis shows it is a “transverse” effect
- Transverse attraction towards the axis of the beam is the essence of optical tweezers

- Also experiment by Jones et al
- Submerged mirrors
- Similar acoustic effects
- Later analyzed in detail

Jones and Leslie, Proc Roy Soc, A360, 347 (1978)

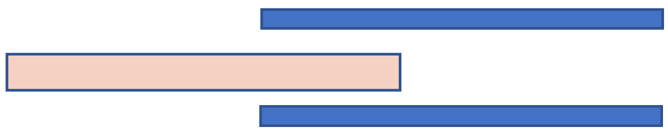
Lai, Ng and Young, PRA 20, 1060 (1984)

3. Key idea of ponderomotive force

A dielectric is attracted to a region of high $\vec{E}^2$

Why?

Dielectric likes to go into plate



$$\text{Energy} \propto ED = \frac{D^2}{\kappa} \propto \frac{\sigma^2}{\kappa}$$

$$\varepsilon = \kappa\, \varepsilon_0$$

Dipoles like high electric field (squared)

$$\vec{f} = (\vec{p}\cdot\vec{\nabla})\vec{E}$$

$$f_i = p_j\,\partial_j\,E_i = \alpha E_j\,(\partial_j\,E_i)$$

$$= \alpha E_j\,(\partial_i\,E_j) \qquad \text{assuming statics}$$

$$= (\alpha/2)\,\partial_i(E_j E_j)$$

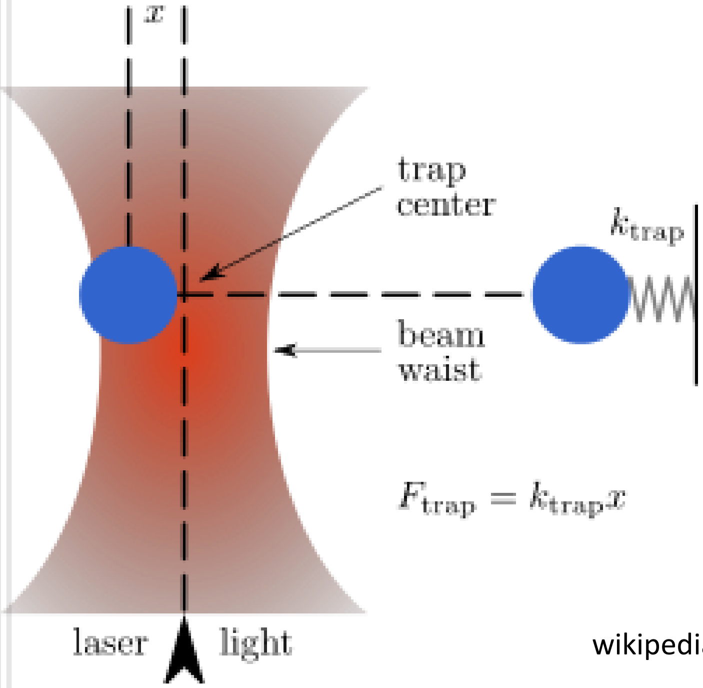
$$\vec{f} = -\vec{\nabla}\Phi$$

$$\boxed{\Phi = -(\alpha/2)\,\vec{E}^2}$$

4. Optical tweezers

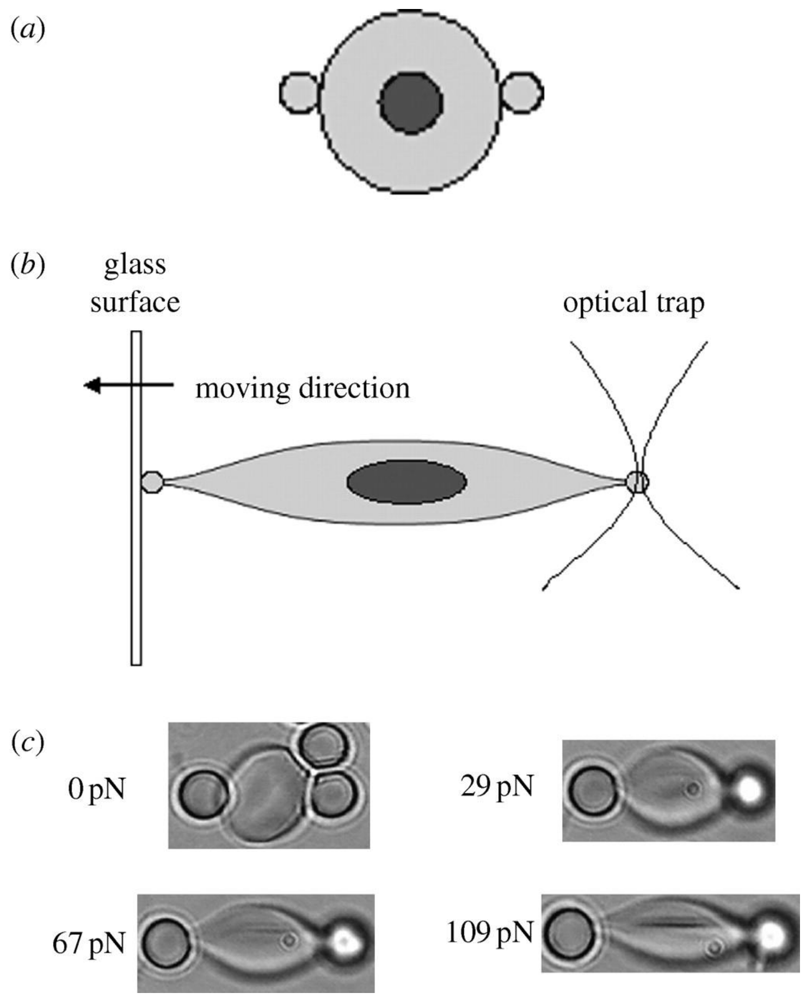
Tweezer

A dielectric material is attracted to regions of high $\vec{E}^2$



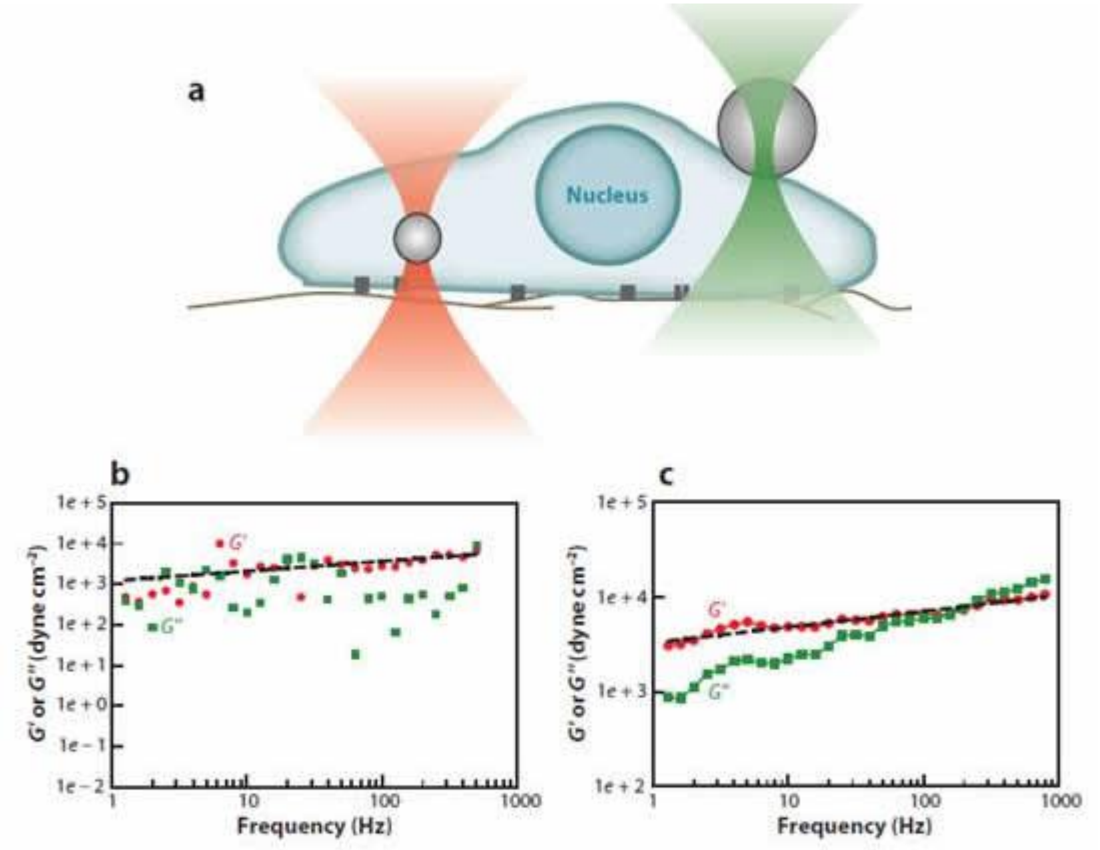
wikipedia

Cell



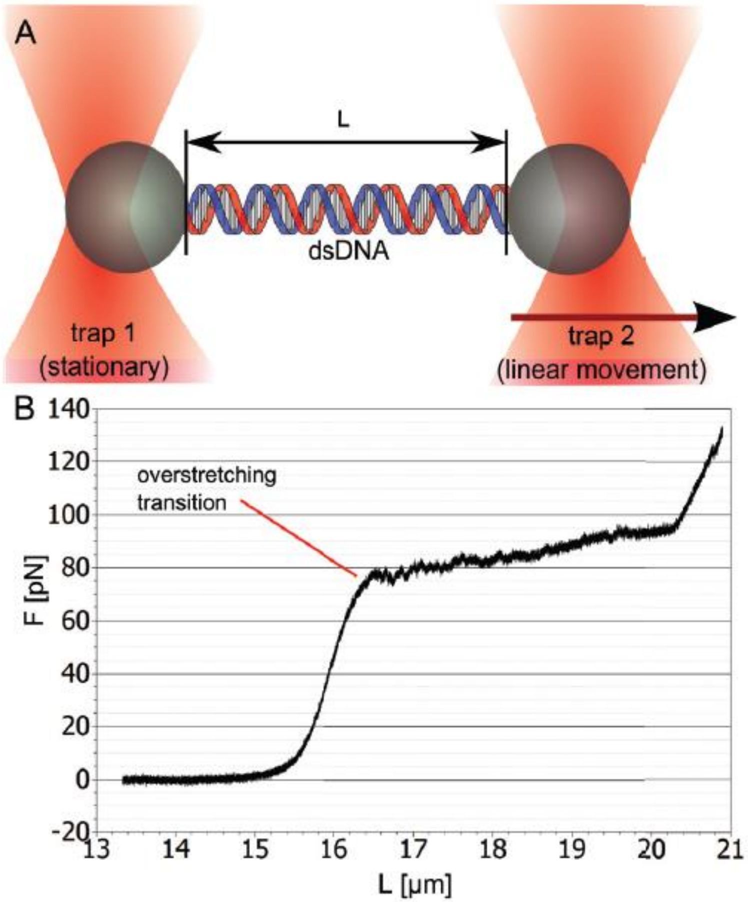
Journal of The Royal Society

Cell



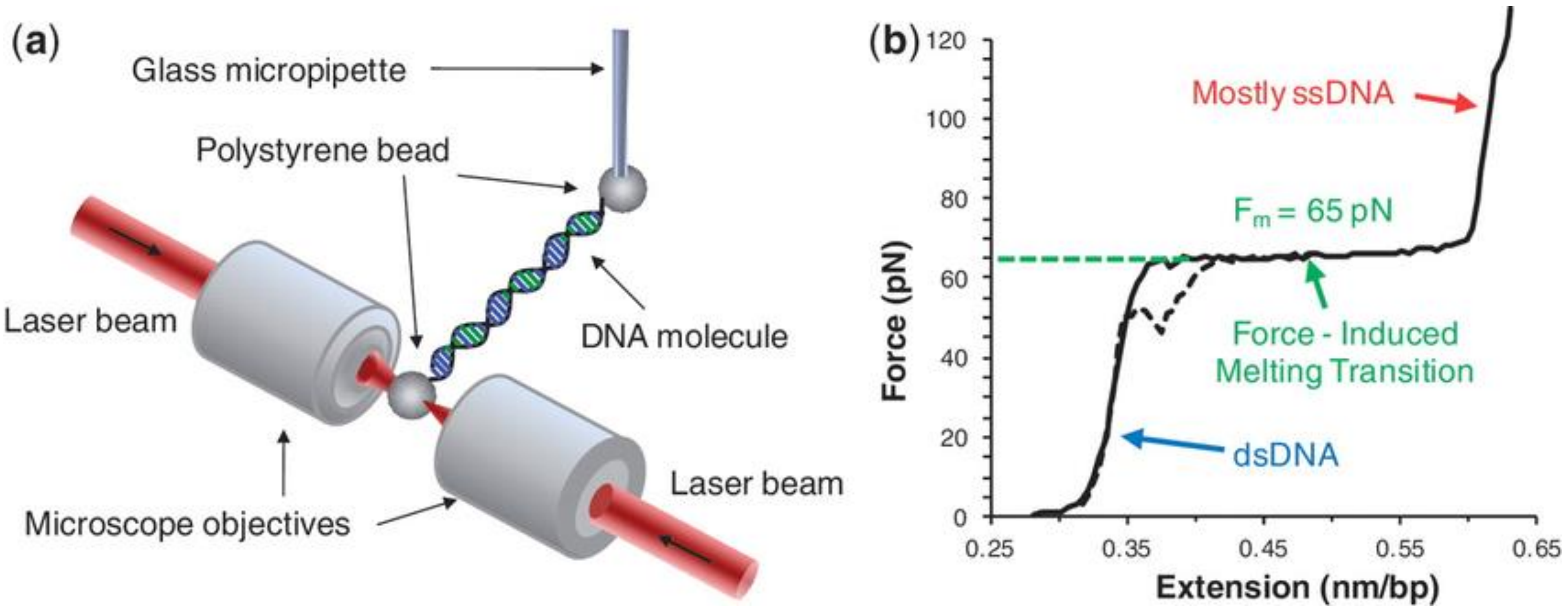
Lehigh University

DNA



www.americanlaboratory.com

DNA



www.researchgate.net

(B) Theoretical subtleties

- All the above is correct for single dipole, or dilute fluid of dipoles
- What is exact answer?
- Controversies
- High orders in $(\kappa - 1)$

5. Peierls' insight

- Dielectric made up of (point) dipoles
- Know what happens microscopically
- There can be no ambiguity

$$f \sim p\,\partial E$$

Peierls, Proc Roy Soc A347, 475 (1976)

Reminder: Clausius-Mossotti

$p \propto E'$ Local field

$P \propto \rho p \propto \rho E$, $\rho =$ number density

$\kappa - 1 = C\rho$

Extra factor of $E'/E = (\kappa + 2)/3$

$$\frac{\kappa - 1}{\kappa + 2} = \frac{C}{3}\rho$$

$$f \sim p \partial E'$$

- Now need the ratio of field gradients
- Local vs macroscopic

Peierls

- Calculated ratio of field gradients for plane wave
- Obtain force on dipole
- Hence both force on dielectric, and momentum in EM pulse

But …

But …

1. Minor arithmetic mistake
2. Did not do more general case, e.g., general static fields
3. Missed one subtle point

6. The extra EM effect

- Calculate force on dipole
- In particular the part proportional to intensity

- Model of molecule =

  hard sphere + dipole moment

$$f = f_M + f_E$$

- Extended Peierl’s calculation to general static fields
- But the force due to the electric term was not curl-free!
- No way it can be balanced by pressure!

Subtle point

$$f_M \sim \rho_2 \sim \exp(-\beta U) \sim 1 - \beta U$$

$$U \sim pp \sim E^2$$

- Mechanical (hard sphere) force contains a term proportional to E^2
- When this term is added
  - Total force is a gradient
  - Static case agrees with known results

$$f = f_M + f_E$$

$$= f_{M0} + f_{M2} + f_E$$

$$= -\nabla p_0 + f_P$$

Same function of density and temp as in zero field

Ponderompotive force

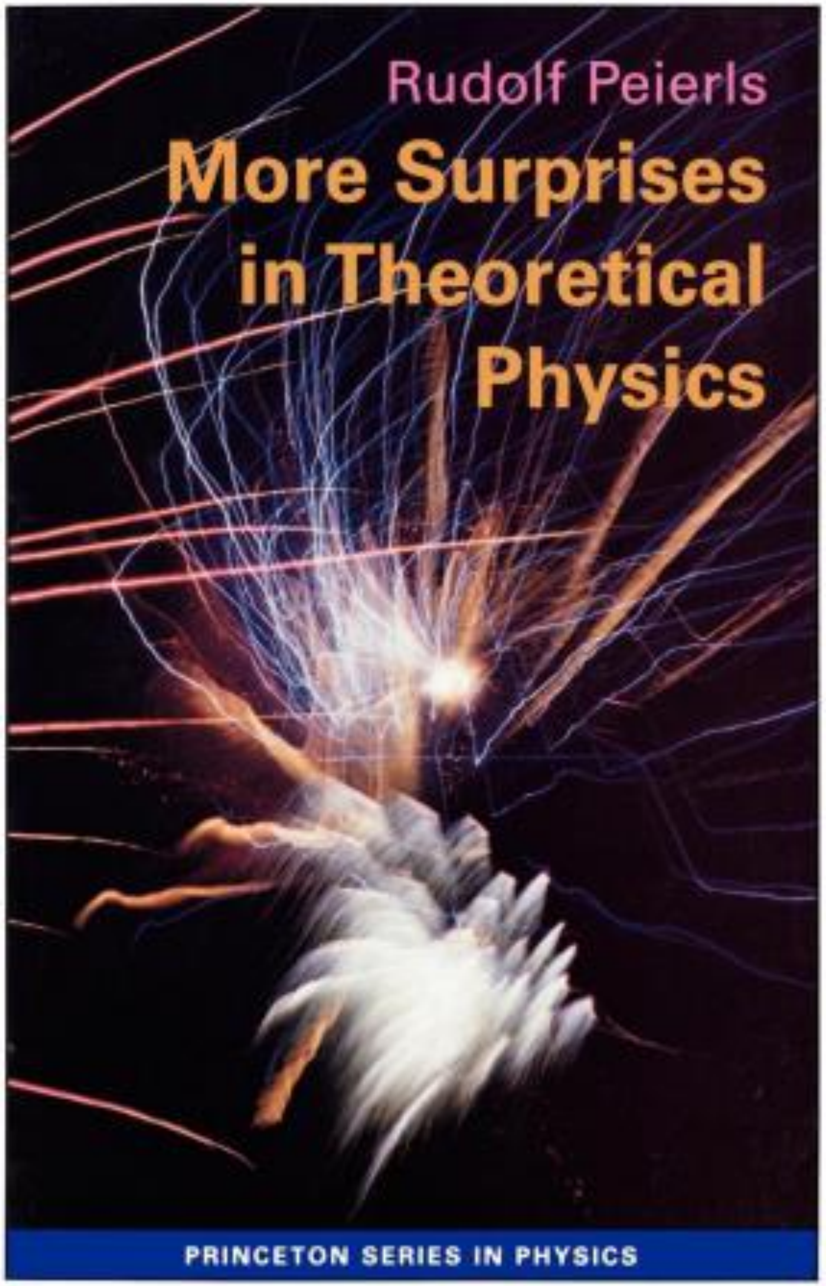
In electrostatic case

$$\vec{f}_P = -(\vec{\nabla}\kappa)U + \vec{\nabla}[\rho(\partial\kappa/\partial\rho)U]$$

$$U = \frac{1}{2}\varepsilon_0 \vec{E}^2$$

- Helmholtz
- First term: force on interface
- Otherwise pure gradient

Lai, Suen and Young, PRL 47, 177 (1981)
Lai, Suen and Young, PRA 25, 1755 (1982)



General QM

Condensed Matter

- Structure of Bismuth and the Jones
- Peierls Transition
- Momentum & Pseudomomentum
- Momentum of a Sound Wave
- Momentum & Pseudomomentum of Light
- Superconducting Sphere

Statistical Mechanics

Transport Problems

Nuclear Physics

Field Theory

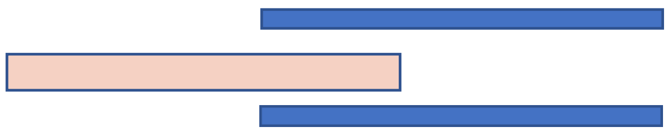
Hydrodynamics

7. Equilibrium under long range forces

- But how was the static case obtained?
- Why do we believe it?
- Most textbooks are (slightly) wrong
- Most reliable: Landau & Lifschitz
- Imagine virtual displacement …

Just like this …

What to keep fixed?



$$\text{Energy} \sim ED = \frac{D^2}{\kappa} \square \frac{\sigma^2}{\kappa}$$

$$\varepsilon = \kappa\, \varepsilon_0$$

Usual derivation of equilibrium

$$\delta(S_1+S_2)-\beta\,\delta(U_1+U_2)-\mu\,\delta(N_1+N_2)=0$$

$$\frac{\partial S_1}{\partial T_1}-\beta\frac{\partial U_1}{\partial T_1}=0 \quad\Rightarrow\quad \frac{1}{T_1}=\beta \quad\Rightarrow\quad T \text{ uniform } =\beta^{-1}$$

Likewise chemical potential = uniform = μ

- Second condition gives expression of force
- But all depends on additivity

But …

- Do we keep $\vec{E}$ fixed or $\vec{D}$ fixed in variation?
- Neither !
- Rather, constrain

Depends on local temp and density

$$\delta \int a(\vec{r}) \vec{\nabla} \bullet [\kappa(\vec{r}) \vec{E}(\vec{r})] d^3 r +$$

$$\delta \int \vec{b}(\vec{r}) \bullet [\vec{\nabla} \times \vec{E}(\vec{r})] d^3 r$$

- Infinite number of local Lagrange multipliers

- No longer obvious that temperature and chemical potential will be uniform
- But miraculous cancellation of terms in dielectric case
- Not unexpected: there is underlying microscopic theory without a dielectric constant

Lai, Suen and Young, PR A34, 1458 (1986)

Generalize to gravity

- Temperature affects energy, source of gravity
- There could be “problems”
- Temperature not uniform in equilibrium !
- Arises from these local Lagrange multipliers
- Tolman

Suen and Young, PR A35, 406 (1987) ; PR A35, 411 (1987) ; Class & Quan Grav 5, 1447 (1988)

Heuristically the same thing ?

- Energy (something that responds to gravitational field) likes region with strong gravitational field [Energy ~ kT]
  - Tiny effect but conceptually curious
- Dielectric (something that responds to electric field) likes region of high electric field
  - Optical tweezers