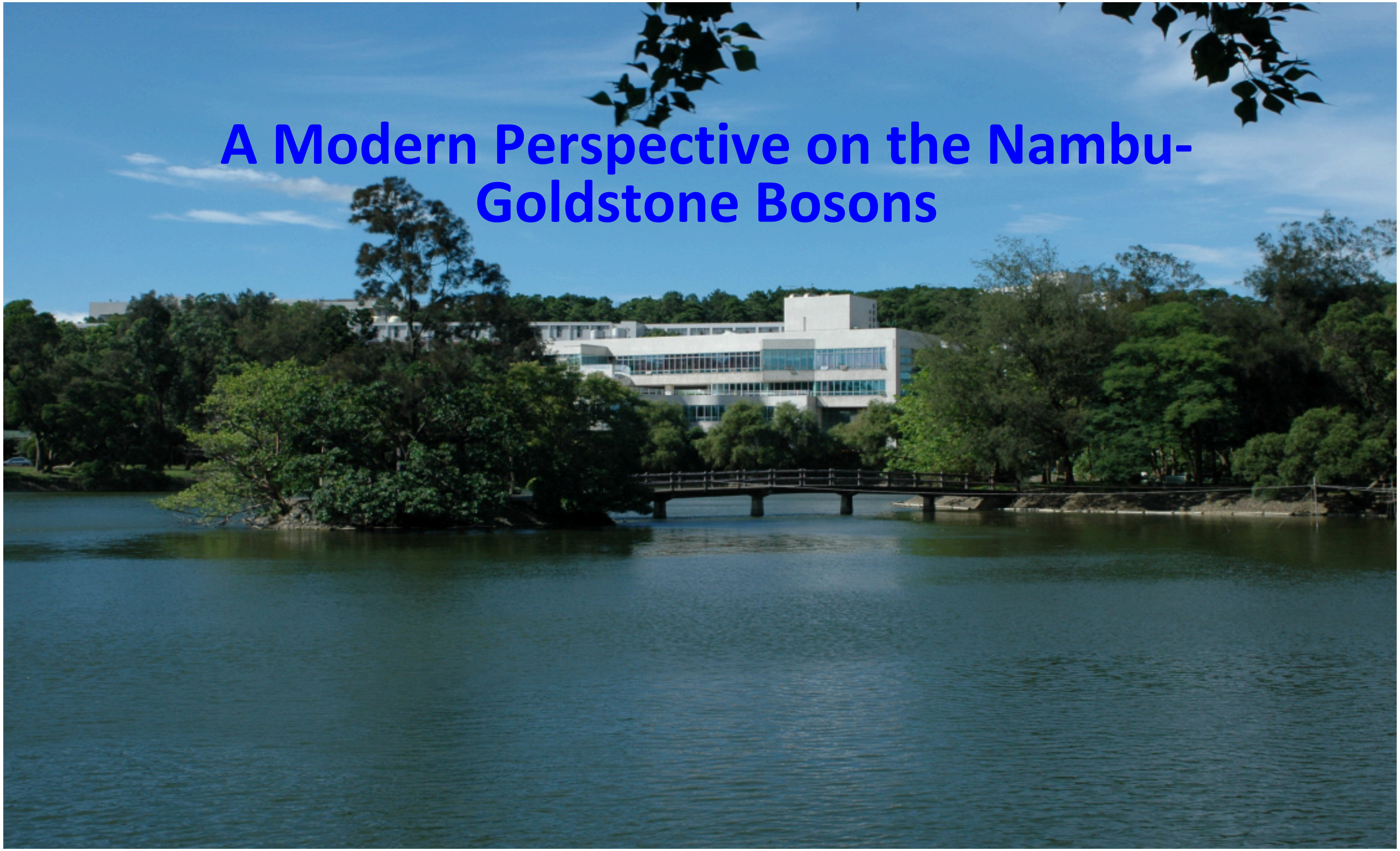
A Modern Perspective on the Nambu-Goldstone Bosons



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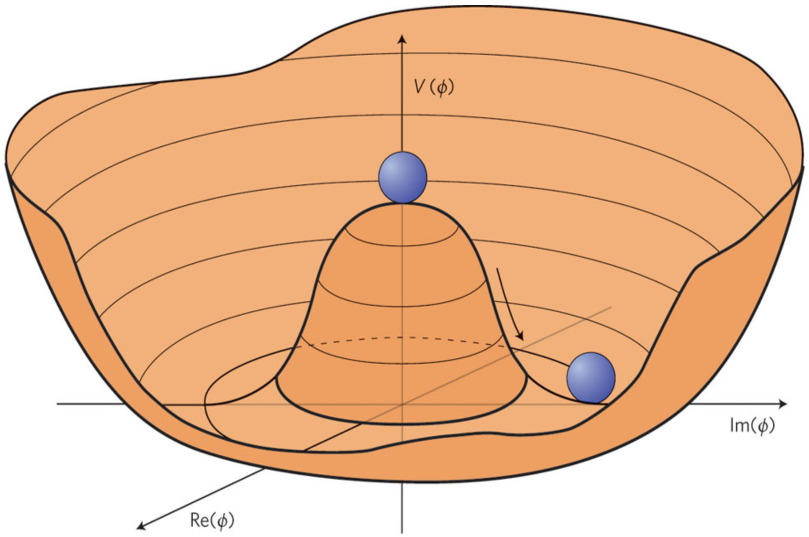
The typical "textbook" example of SSB starts with the following potential energy:

$$\phi = \phi_1 + i\phi_2$$
$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$



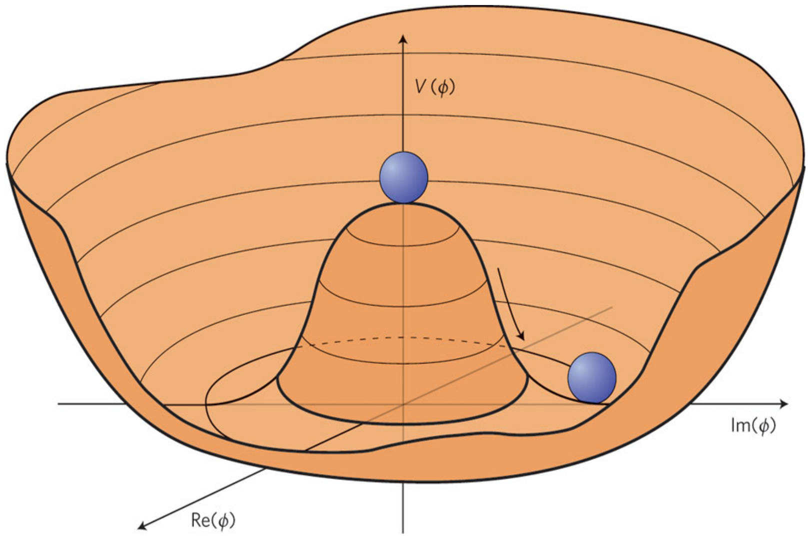
Particle physicists call it the "Mexican-hat potential."

For condensed matter physicists this is known as the "Ginzburg-Landau" potential.

The potential has a 2-dimensional rotation symmetry.

But the ground state is not unique, and can be obtained by minimizing the potential energy:

$$\langle\phi\rangle_\alpha \equiv {}_\alpha\langle 0|\phi|0\rangle_\alpha = \frac{v}{\sqrt{2}}\, e^{i\frac{1}{\sqrt{2}v}\alpha}\,, \qquad v \equiv \sqrt{\mu^2/\lambda}$$



- There is an infinite number of ground state, parameterized by the angle parameter “alpha”:

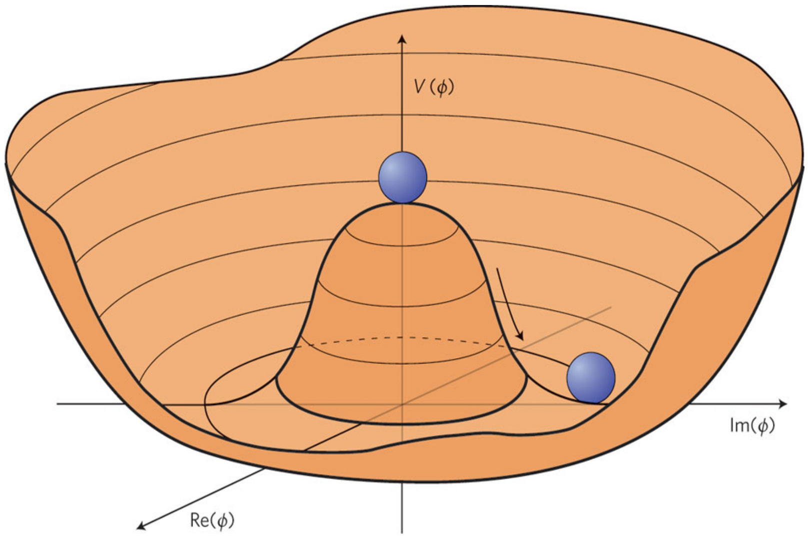
$$|\mathrm{VAC}\rangle = \{|0\rangle_\alpha;\ \alpha \in [0, 2\pi)\}$$

- But once an “alpha” is chosen, the rotational invariance is hidden.

To see the NGB explicitly, it's best to go to "polar coordinate:"

$$\phi(x) = \frac{\rho(x)}{\sqrt{2}} \, e^{i \frac{1}{\sqrt{2}v} \pi(x)}$$

In this parameterization, the ground state is $\langle \rho \rangle_\alpha = v \ , \quad \langle \pi \rangle_\alpha = \alpha$



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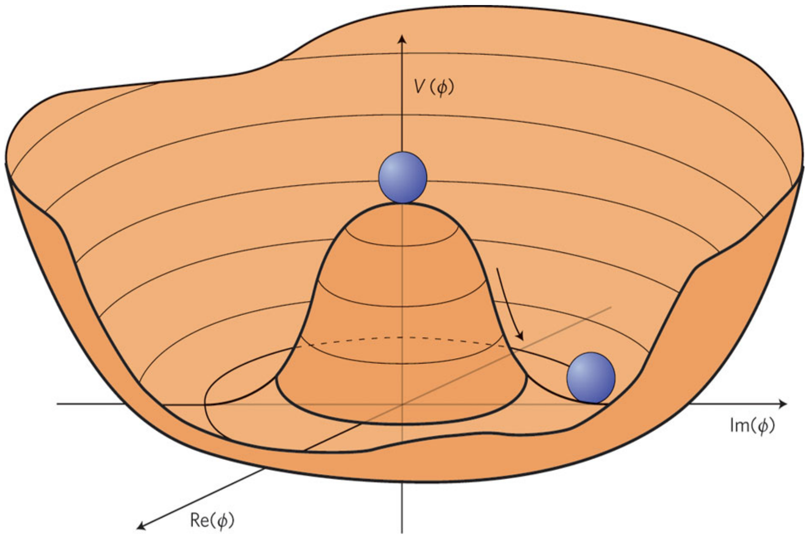


Expanding with respect to the ground state:

$$\rho \to \rho + \langle \rho \rangle_\alpha$$
$$\pi \to \pi + \langle \pi \rangle_\alpha$$

The angular mode disappears from the potential:

$$V(\phi) = -\frac{\mu^2}{2}(\rho + v)^2 + \frac{\lambda}{4}(\rho + v)^4$$

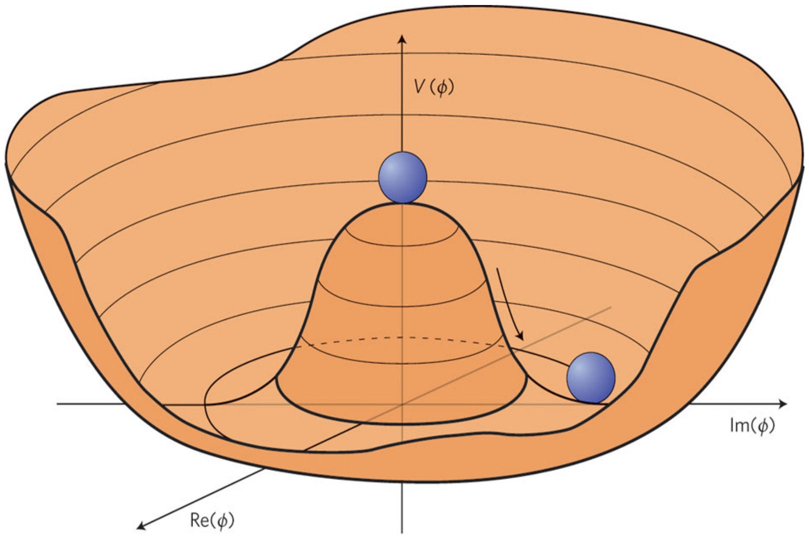
→ **Pi-mode is massless!**

After all, it's the excitation along degenerate ground states.

The “kinetic energy” gives

$$\partial_\mu \phi^* \partial^\mu \phi = \frac{1}{2}\partial_\mu \rho \partial^\mu \rho + \frac{1}{2}\partial_\mu \pi \partial^\mu \pi + \frac{1}{v}\rho\, \partial_\mu \pi \partial^\mu \pi + \frac{\rho^2}{2v}\partial_\mu \pi \partial^\mu \pi$$

Notice the alpha-dependence drops out!



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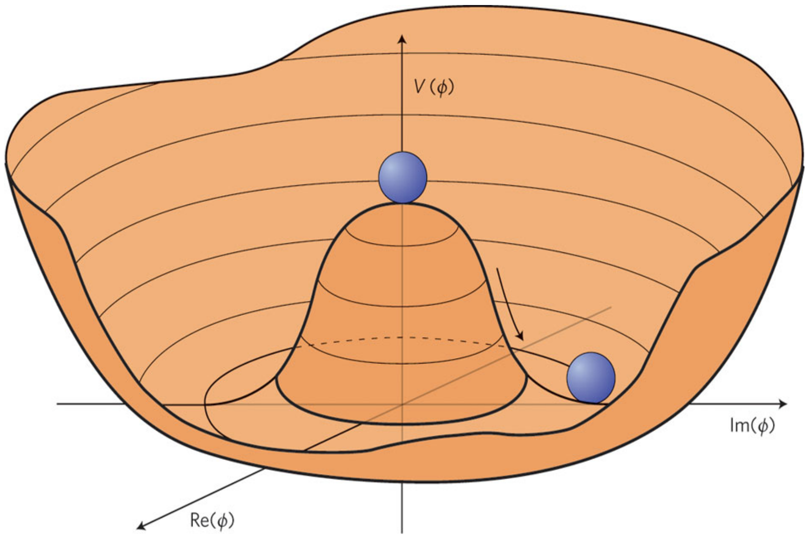

Recall that, under rotation by theta-angle,

$$\langle \pi \rangle_\alpha \rightarrow \langle \pi \rangle_{\alpha+\theta} = \alpha + \theta$$

This is how rotational invariance manifest
Itself in the "polar coordinate!"

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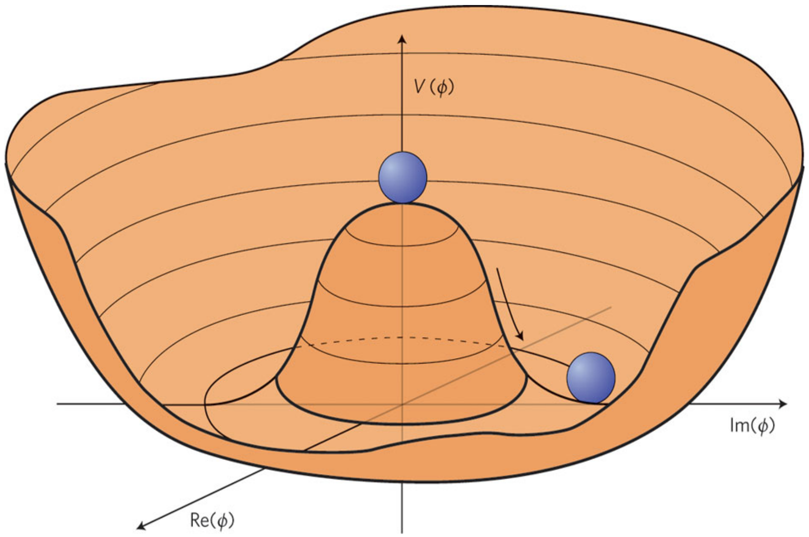


Equivalently, This is the same as shifting the Pi-mode by a constant:

$$\pi \to \pi + \theta$$

Rotational symmetry implies the dynamics must be independent of the constant shift!

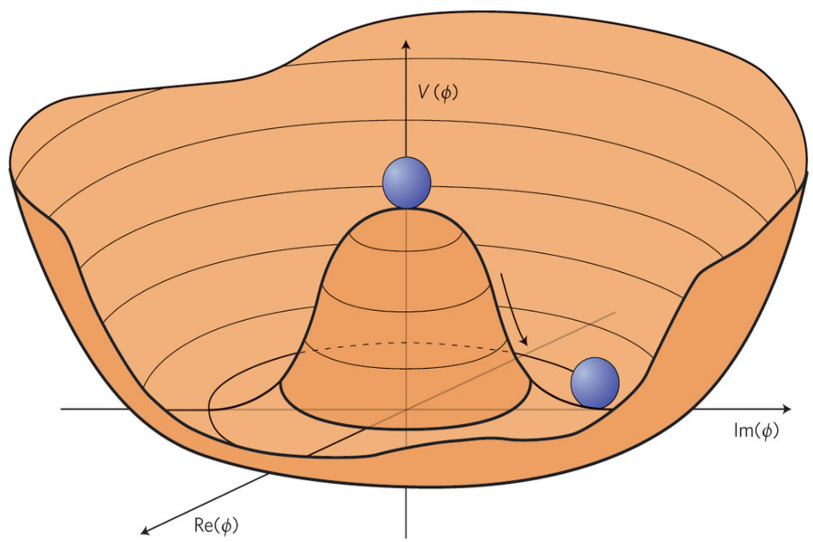
We call this a "shift symmetry."

In this example, the full symmetry group “G” of the system is the 2-d rotation, the U(1) symmetry:

$$\text{Broken symmetry } G = U(1)$$

The ground state $\langle \pi \rangle_\alpha = \alpha$ breaks the U(1) symmetry completely. There is no residual symmetry that leaves the ground invariant.

$$\text{Unbroken symmetry } H = \emptyset$$



There could be more complicated cases.

For example, let's consider n real scalars:

$$\vec{\phi} = (\phi_1, \cdots, \phi_n)$$

$$V(\vec{\phi}) = -\mu^2 \vec{\phi} \cdot \vec{\phi} + \lambda \left(\vec{\phi} \cdot \vec{\phi} \right)^2$$

$$\langle \vec{\phi} \rangle = (v, 0, \cdots, 0)$$

Then

$$\text{Broken symmetry } G = O(n)$$

$$\text{Unbroken symmetry } H = O(n-1)$$

The NGB mode can be parameterized by

$$\vec{\phi} = v(\sigma, \pi^1, \cdots, \pi^{n-1}) \ , \quad \sigma = \sqrt{1 - \pi^2}$$

$$\frac{1}{2}\left(\partial_\mu \vec{\phi}\right)^2 \to \frac{1}{2}\left[(\partial_\mu \vec{\pi})^2 + \frac{(\vec{\pi} \cdot \partial_\mu \vec{\pi})^2}{1 - \pi^2}\right]$$

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Key observations:

- Interactions of NGB, in general, are horribly nonlinear.
- NGBs are always "derivatively coupled," due to the shift symmetry:

$$\vec{\pi} \to \vec{\pi} + \vec{\theta} + \cdots$$

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Absent in the simple U(1) case.
No one ever worked it out, to our knowledge.

More generally, there is a well-defined procedure to write down NGB effective actions for arbitrary G and H.

(Coleman, Callan, Wess and Zumino, Phys. Rev. 1969.)

The general procedure requires prior knowledge of "G" and "H".

That is, one need to know the full symmetry group of the Mexican-hat potential first!

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The conventional wisdom:

Nonlinearity in NGB interactions is due to the “nonlinearly realized” group G in the UV.

let's review CCWZ briefly:

standard forms, which are described in detail. The mathematical problem is equivalent to that of finding all (nonlinear) realizations of a (compact, connected, semisimple) Lie group which become linear when restricted to a given subgroup. The relation between linear representations and nonlinear realizations is

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$$\xi = e^{i\Pi/f}, \quad \Pi = \pi^a X^a,$$

$$g\,\xi = \xi'\,U(g,\xi) \qquad \Pi' = \Pi'(\Pi, g)$$

This is a complicated mess. So complicated that CCWZ didn't want to deal with it!

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$$\xi = e^{i\Pi/f}, \quad \Pi =$$

This is the shift symmetry mentioned in the beginning.

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However, we know that for

$$g = e^{i\varepsilon^a X^a}, \quad \pi^{a\,\prime} = \pi^a + \varepsilon^a + \cdots$$

Instead, CCWZ looked for objects that have “simple” transformation properties under the action of G.

These are contained in the Cartan-Maurer one-form:

$$\xi^\dagger \partial_\mu \xi \;=\; i\mathcal{D}^a_\mu X^a + i\mathcal{E}^i_\mu T^i \equiv i\mathcal{D}_\mu + i\mathcal{E}_\mu$$

They are the “Goldstone covariant derivative” and the “associated gauge field”,

$$\mathcal{D}_\mu \to U\mathcal{D}_\mu U^{-1} \;, \quad \mathcal{E}_\mu \to U\mathcal{E}_\mu U^{-1} - (\partial_\mu U)U^{-1}$$

upon which the complete effective lagrangian can be built (apart from the topological terms)

$$\mathcal{L}_{eff} = \frac{f^2}{2}\mathrm{Tr}\mathcal{D}_\mu \mathcal{D}^\mu + \cdots$$

In this fashion, CCWZ circumvents the problem of working out how the pions transform under the broken G:

$$\xi = e^{i\Pi/f}, \qquad g\,\xi = \xi'\,U(g,\xi)$$

$$\Pi' = \Pi'(\Pi, g)$$

Not needed for writing down the EFT.

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Not needed for writing down the EFT.

CCWZ is extremely powerful, but it adopts a “top-down” perspective, which requires knowing ahead of time what the broken group “G” is in the UV.

It also obscures the fact that Goldstone bosons are infrared degrees of freedoms that connect different vacua.

Consider two different G's and H's, which both contain a NGB charged under (a U(1) subgroup of) H

$G_1 = SU(2);\ H_1 = U(1)$

$$|\partial_\mu \phi|^2 - \frac{1}{3f^2}\left|\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*\right|^2 + \frac{8}{45 f^4}\left|\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*\right|^2 |\phi|^2 - \frac{16}{315 f^6}\left|\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*\right|^2 |\phi|^4 + \cdots\ ,$$

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$$-\frac{16}{315f^6}\,|\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*|^2\,|\phi|^4 + \cdots\;\;,$$

$$G_2 = SU(5);\;\; H_2 = SO(5)$$

$$|\partial_\mu \Phi|^2 - \frac{1}{48f^2}\,|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*|^2 + \frac{1}{1440f^4}\,|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*|^2\,|\Phi|^2$$
$$-\frac{1}{80640f^6}\,|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*|^2\,|\Phi|^4 + \cdots\;\;,$$

Indeed, the NGB effective interactions look different.

Let’s recap the conventional wisdom from the past four decades:

- SSB occurs when the ground state is not invariant under the full symmetry of the system.

- Nambu-Goldstone modes are long wavelength, “gapless” excitations over the degenerate ground states.

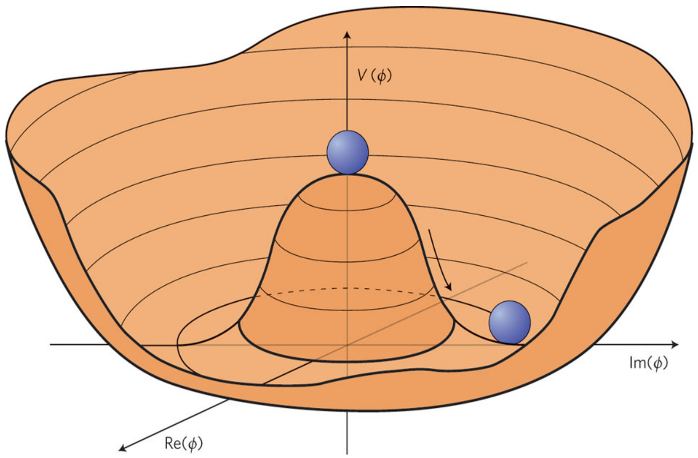
- NGBs are “derivatively coupled,” due to a shift symmetry.

- Effective interactions of NGB are dependent on both the full symmetry group G in the UV and the unbroken group H in the IR.

Let's talk about something that is not usually emphasized in the textbook. Recall the ground state is characterized by

$$|\mathrm{VAC}\rangle = \{|0\rangle_\alpha;\ \alpha \in [0, 2\pi)\}$$

Now let's bring in Quantum Mechanics...



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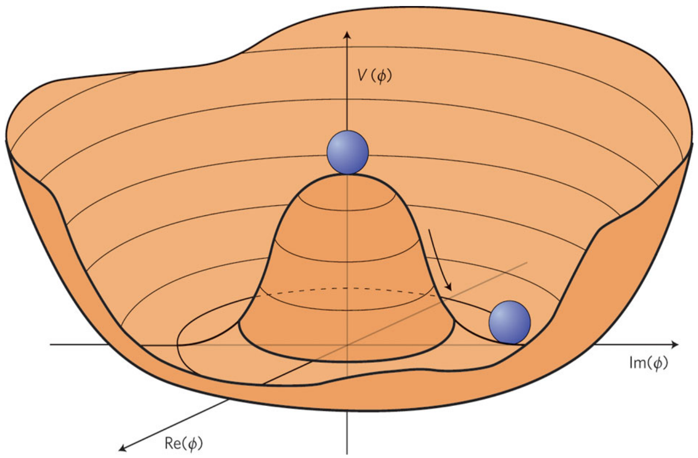
$$|\tilde{0}\rangle = \int \frac{d\alpha}{2\pi} |0\rangle_\alpha \ , \qquad R(\theta)|\tilde{0}\rangle = |\tilde{0}\rangle$$

This superposition of alpha-state is invariant under rotation. Why couldn't it be the "ground state"?



This shows an important ingredient for SSB to occur is the "vacuum superselection rule",

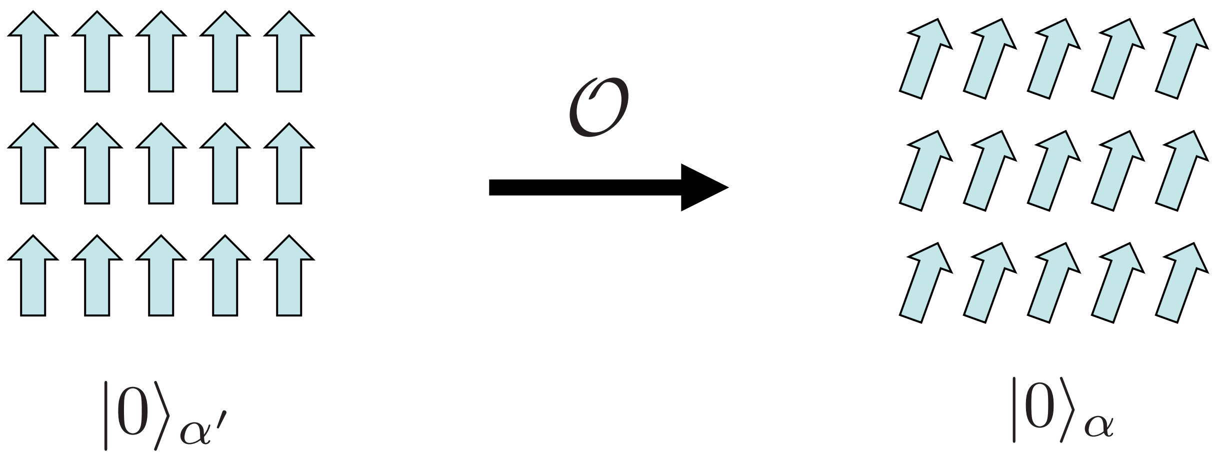
$$ {}_{\alpha}\langle 0|\mathcal{O}|0\rangle_{\alpha'} = 0 $$

for any Hermitian local operator "O".

Then any true ground state must carry a definite "alpha" and cannot be a superposition of alpha-states.

An intuitive way to understand the superselection rule is this:

$$ {}_{\alpha}\langle 0|\mathcal{O}|0\rangle_{\alpha'} = $$

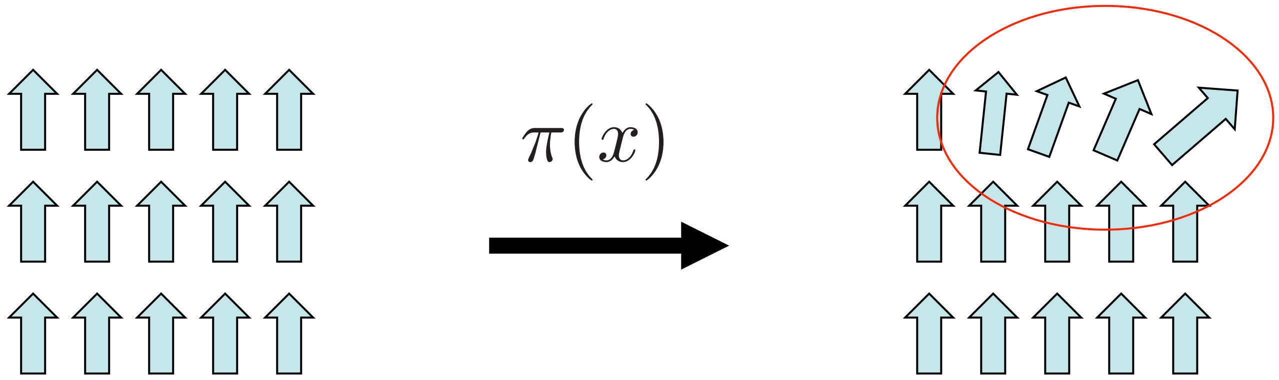


The energy cost to flip the alpha-direction everywhere is proportional to the size of the system.

If the system has an infinite volume, the energy cost is infinite.

→ **SSB only occurs for systems with infinite volume!**

On the other hand, a "local excitation" only costs finite energy and ought to exist:



Such "local excitations" are precisely the NGB modes!

The superselection rule has an important implication for the scattering amplitudes of NGBs:

$$\lim_{p^\mu \to 0} \ {}_\alpha\langle f|i + \pi(p)\rangle_\alpha = 0$$

ie, the scattering matrix elements involving a zero-momentum NGB must vanish!

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Recall in Quantum Mechanics that a momentum eigenstate

$$|\vec{k}\rangle = e^{ik\cdot x}$$

Then a zero-momentum eigenstate has a constant wave function, ie it flips the direction of the ground state everywhere in the system:

$$\lim_{p^\mu \to 0} \pi(p)|0\rangle_\alpha \sim |0\rangle_{\alpha'}$$

This implies

$$\lim_{p^\mu \to 0} |i + \pi(p)\rangle_\alpha \ \sim \ |i\rangle_{\alpha'}$$

Therefore the superselection rule tells us

$$\lim_{p^\mu \to 0} {}_\alpha\langle f|i + \pi(p)\rangle_\alpha \ \sim \ {}_\alpha\langle f|i\rangle_{\alpha'} = 0$$

This property of the NGB scattering amplitudes is derived, in a different fashion, in the context of pions in low-energy QCD by Adler in 1960’s.

It is now known as the Adler’s zero condition.

In a quantum field theory, one can show that the Adler's zero condition,

$$\lim_{p^\mu \to 0} {}_\alpha\langle f | i + \pi(p) \rangle_\alpha = 0$$

follows directly from the shift symmetry acting on the NGB:

$$\pi \to \pi + \epsilon + \cdots$$

This is hardly surprising, as the shift symmetry is an indication of the existence of other degenerate ground states!

The study on the "soft limit" of scattering amplitudes in QFT's is a very old subject, dated back to the early days of 20th century.

- In both Quantum Electrodynamics and Gravity, scattering amplitudes with one soft gauge boson factorize universally:

$$\lim_{\tau\to 0} M_{n+1}(p_1, \cdots, p_n; \tau p_{n+1}) = \left(\frac{1}{\tau} + \tau^0 + \cdots\right) M_n(p_1, \cdots, p_n)$$

- For NGBs, the Adler's zero condition states

$$\lim_{\tau\to 0} M_{n+1}(p_1, \cdots, p_n; \tau p_{n+1}) = \mathcal{O}(\tau)$$

In recent years, there is a growing community of theorists working on "scattering amplitudes."

One of the guiding principles is to define a QFT not by a Hamiltonian or a Lagrangian, but instead by its scattering amplitudes.

This raises the question:

Can we (re)construct the NGB interactions by imposing the Adler's zero condition on the scattering amplitudes?

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One of the guiding principles is to define a QFT not by a Hamiltonian or a Lagrangian, but instead by its scattering amplitudes.

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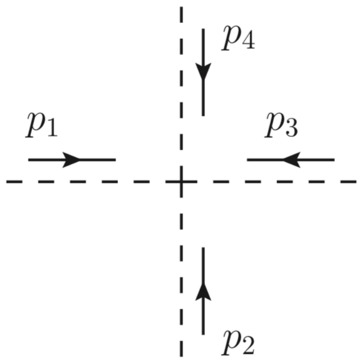
Can we (re)construct the NGB interactions by imposing the Adler's zero condition on the scattering amplitudes?

This turns out to be a very powerful constraint and allows us to construct the complete NGB interactions without ever referring to the full symmetry group "G" in the UV.

To see how this works, let's start with the simplest 4-pt amplitude.

Let's assume there is a notion of "ordering" among the NGBs, then the Adler's zero condition uniquely determines



$$p_i^2 = 0 \; ; \sum_{i=1}^{4} p_i = 0$$

$$M_4(p_1, p_2, p_3, p_4) = c\frac{p_2 \cdot p_4}{f^2} = c\frac{p_1 \cdot p_3}{f^2}$$

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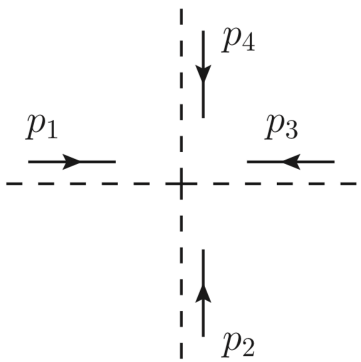

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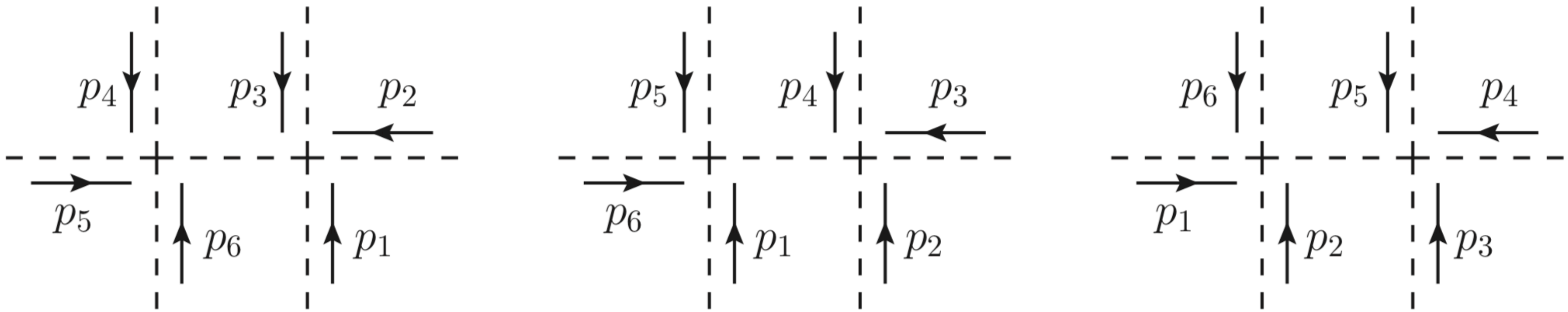
Some observations:

- There is no constant term in the amplitude.
- The notion of "ordering" can be achieved by assigning a discrete quantum number on the NGB.
- "f" is a dimensionful parameter, while "c" is an arbitrary number, which could be absorbed into the normalization of "f".

Once we have the 4-pt amplitude, Quantum Mechanics implies the following contributions to the 6-pt amplitude must exist:



$$\frac{1}{f^2}\left(\frac{s_{13}s_{46}}{P^2_{123}}+\frac{s_{24}s_{15}}{P^2_{234}}+\frac{s_{35}s_{26}}{P^2_{345}}\right)$$

$$s_{ij} = (p_i + p_j)^2$$
$$P^2_{ijk} = (p_i + p_j + p_k)^2$$

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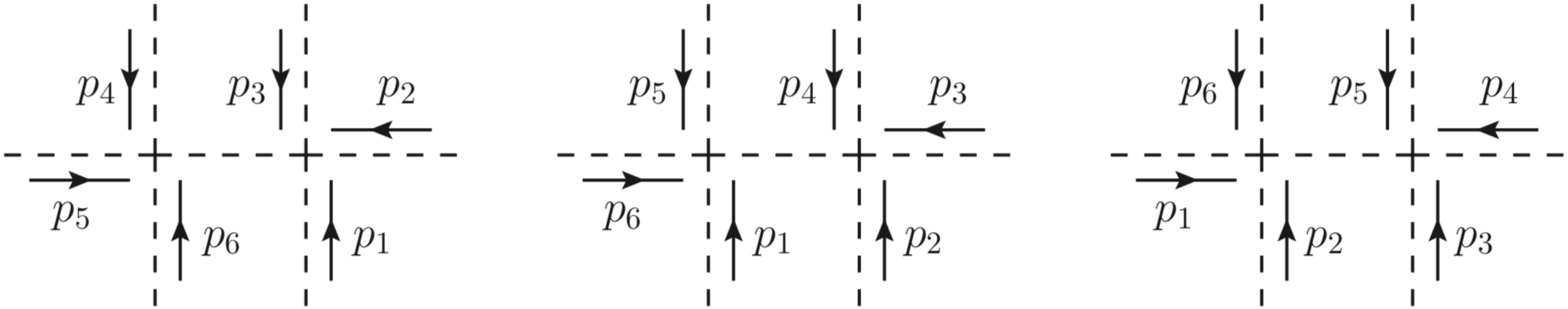
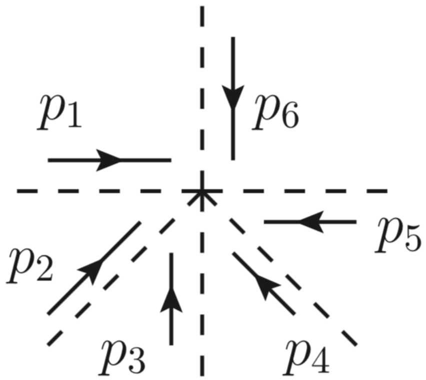


$$\frac{1}{f^2}\left(\frac{s_{13}s_{46}}{P_{123}^2}+\frac{s_{24}s_{15}}{P_{234}^2}+\frac{s_{35}s_{26}}{P_{345}^2}\right)$$

$$s_{ij}=(p_i+p_j)^2$$
$$P_{ijk}^2=(p_i+p_j+p_k)^2$$

This expression doesn't satisfy the Adler's zero condition!

The resolution is to introduce an additional contribution, the “contact interaction,”



It turns out imposing the Adler’s condition also uniquely fixes this 6-pt contact interaction:

$$M_6 = \frac{1}{f^2}\left(\frac{s_{13}s_{46}}{P_{123}^2} + \frac{s_{24}s_{15}}{P_{234}^2} + \frac{s_{35}s_{26}}{P_{345}^2}\right) - \frac{1}{f^2}P_{135}^2$$

This process is then continued to 8-pt amplitudes and so on.

In the end all "tree-level" amplitudes of NGBs can be reconstructed simply by assuming:

- There is a notion of "ordering," which arises due to some discrete quantum numbers, given by the "unbroken group" H.
- The vanishing "soft limit" in the scattering amplitudes.

The important observation here is that only IR data are needed.

It is not necessary to know the full symmetry group "G", unless one is interested in the absolute normalization of "f".

This program was initiated by Susskind and Frye in 1969 up to 8-pt amplitude and completed, Feynman-diagrammatically, to all tree-amplitudes by Cheung et. al. in 1611.03137.

It turned out that it is possible to construct the full quantum effective action using only the IR data, by "completing" the shift symmetry to higher orders:

$$\pi^{a\,\prime} = \pi^a + [F_1(\mathcal{T})]_{ab}\,\varepsilon^b\;, \quad F_1(\mathcal{T}) = \sqrt{\mathcal{T}}\cot\sqrt{\mathcal{T}} \qquad \mathcal{T}_{ab} = \frac{2}{f^2}(T^i)_{ac}(T^i)_{db}\,\pi^c\pi^d$$

The Lagrangian invariant under the full shift symmetry is

$$\mathcal{L}^{(2)} = \frac{1}{2}[F_2(\mathcal{T})^2]_{ab}\;\partial_\mu\pi^a\partial^\mu\pi^b\;, \quad F_2(\mathcal{T}) = \frac{\sin\sqrt{\mathcal{T}}}{\sqrt{\mathcal{T}}}$$

Again the only free parameter is the normalization of "f".

Low: 1412.2145; 1412.2146
Low and Yin: 1709.08639

We have discovered the following universality:

Given different broken groups “G” in the UV, NGBs carrying the same IR data will have identical interactions, up to the normalization of “f”.

Going back to the earlier example,

$$G_1 = SU(2);\ \ H_1 = U(1)$$

$$|\partial_\mu\phi|^2 - \frac{1}{3f^2}\left|\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*\right|^2 + \frac{8}{45f^4}\left|\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*\right|^2|\phi|^2$$
$$-\frac{16}{315f^6}\left|\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*\right|^2|\phi|^4 + \cdots\ ,$$

$$G_2 = SU(5);\ \ H_2 = SO(5)$$

$$|\partial_\mu\Phi|^2 - \frac{1}{48f^2}\left|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*\right|^2 + \frac{1}{1440f^4}\left|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*\right|^2|\Phi|^2$$
$$-\frac{1}{80640f^6}\left|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*\right|^2|\Phi|^4 + \cdots\ ,$$

Both ϕ and Φ carry identical IR data:

They are both charged under a U(1) (sub)group of H.

The universality then implies their interactions should be identical, up to the normalization of “f”.

Going back to the earlier example,

$$G_1 = SU(2);\ \ H_1 = U(1)$$

$$|\partial_\mu \phi|^2 - \frac{1}{3f^2}\left|\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*\right|^2 + \frac{8}{45 f^4}\left|\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*\right|^2 |\phi|^2$$
$$-\frac{16}{315 f^6}\left|\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*\right|^2 |\phi|^4 + \cdots \ ,$$

$$G_2 = SU(5);\ \ H_2 = SO(5)$$

$$|\partial_\mu \Phi|^2 - \frac{1}{48 f^2}\left|\Phi^* \partial_\mu \Phi - \Phi \partial_\mu \Phi^*\right|^2 + \frac{1}{1440 f^4}\left|\Phi^* \partial_\mu \Phi - \Phi \partial_\mu \Phi^*\right|^2 |\Phi|^2$$
$$-\frac{1}{80640 f^6}\left|\Phi^* \partial_\mu \Phi - \Phi \partial_\mu \Phi^*\right|^2 |\Phi|^4 + \cdots \ ,$$

If we make f → 4f in the first case, the two Lagrangians are identical!

The modern perspective on NBG:

- The Adler’s zero can be taken as the defining property of NGB. Moreover, it is a consequence of the vacuum superselection rule.
- There is a universality class for each NGB carrying the same charge under the unbroken group.
- The nonlinearity in the NGB interactions arises entirely from IR physics.
- What’s being “broken” in the UV is irrelevant, other than determining the normalization of “f”.

What is this universality good for?

After the discovery of the Higgs boson in 2012, many important questions remain unanswered. One of the most intriguing questions is

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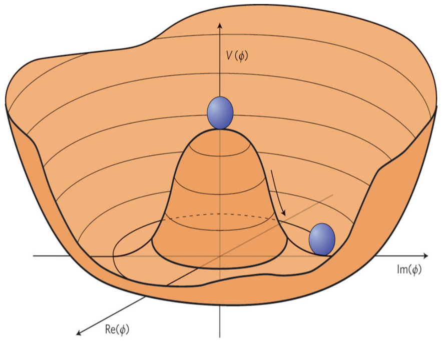
What is the Higgs made of?

The question can be rephrased slightly:

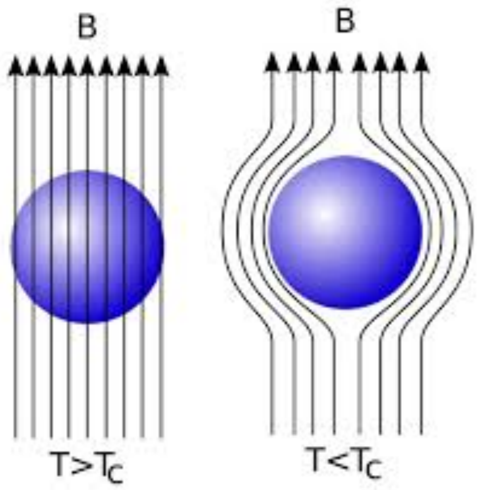
What is the microscopic theory giving rise to the Higgs boson and its potential?

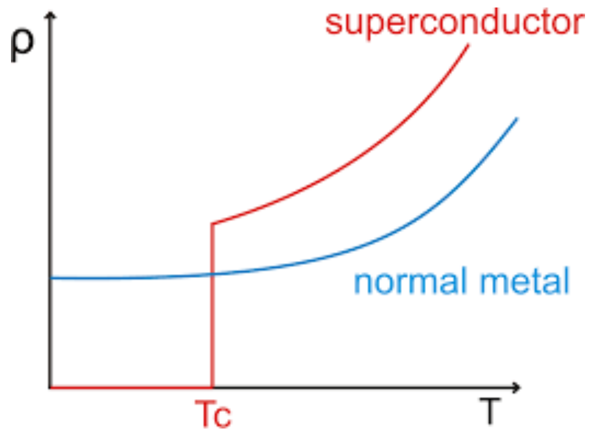
$$V(H) = -\mu^2|H|^2 + \lambda|H|^4$$



Our colleagues in condensed matter physics are very used to asking, and studying, this kind of questions.

One of the most beautiful examples is the superconductivity discovered in 1911:





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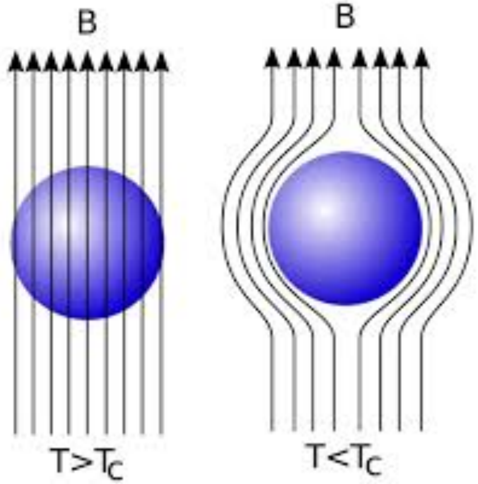

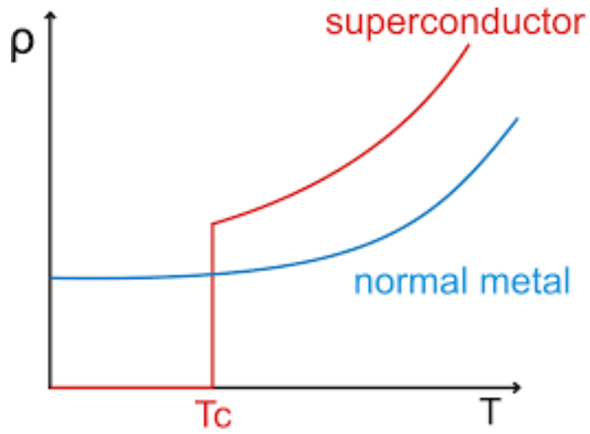

Ginzburg-Landau theory from 1950 offered a **macroscopic** (ie effective) theory for conventional superconductivity,

$$V(\Psi) = \alpha(T)|\Psi|^2 + \beta(T)|\Psi|^4 \qquad \alpha(T) \approx a^2(T - T_c) \qquad \text{and} \qquad \beta(T) \approx b^2$$

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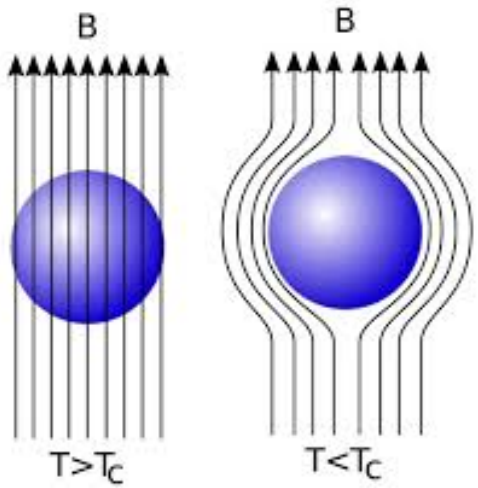

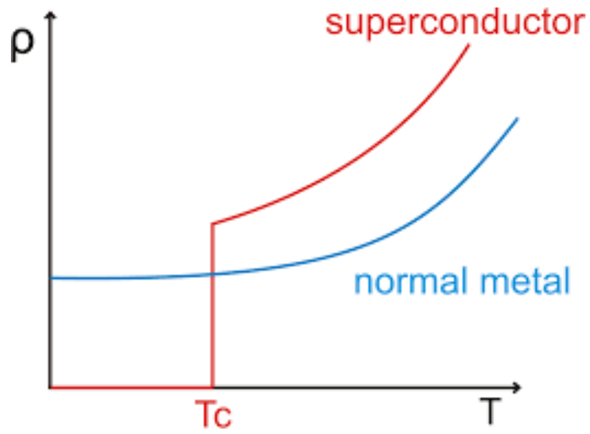

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What is the **microscopic** origin of the Ginzburg-Landau potential for superconductivity?

In 1957 Bardeen, Cooper and Schrieffer provided the **microscopic** (fundamental) theory that allows one to

1) interpret $|\Psi|^2$ as the number density of Cooper pairs

2) calculate coefficients of $|\Psi|^2$ and $|\Psi|^4$ in the potential.

In 1957 Bardeen, Cooper and Schrieffer provided the **microscopic** (fundamental) theory that allows one to

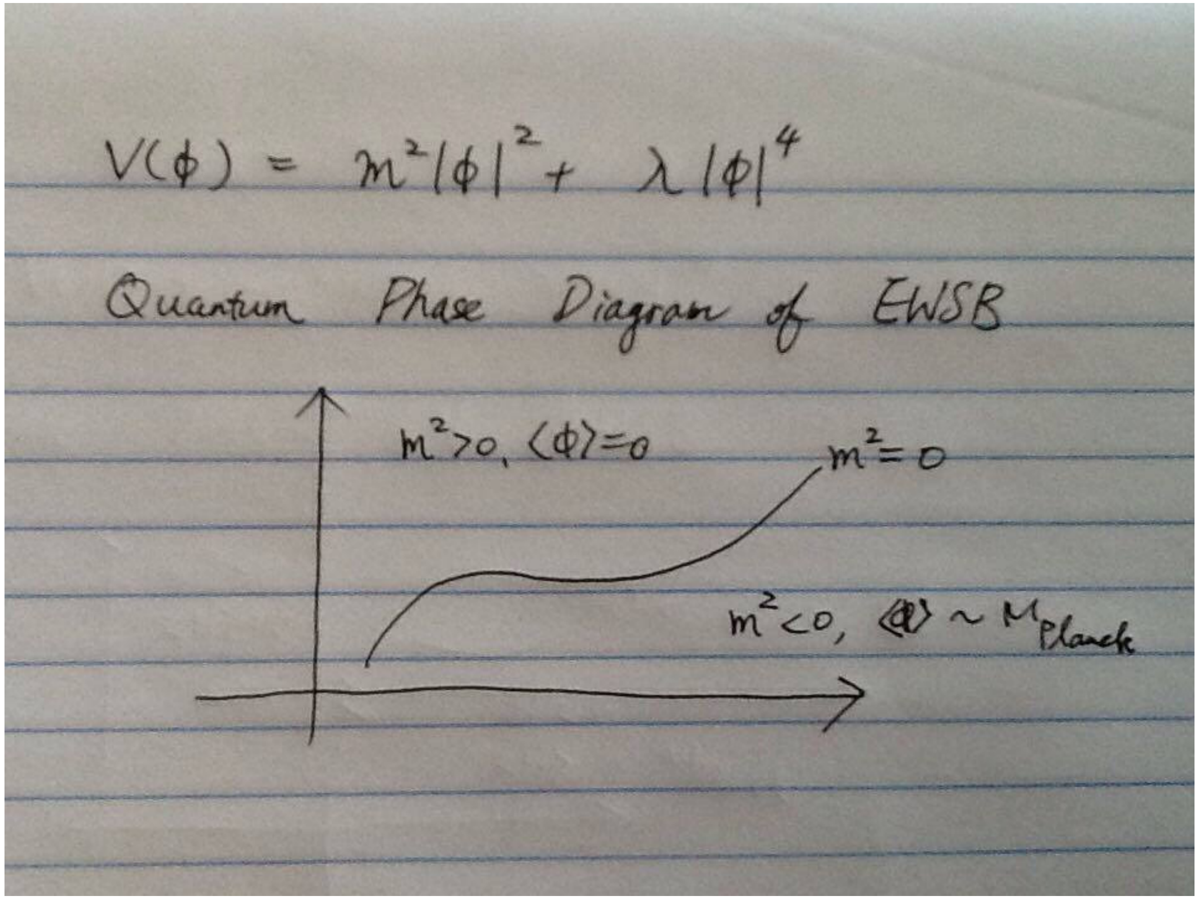
1) interpret $|\Psi|^2$ as the number density of Cooper pairs

2) calculate coefficients of $|\Psi|^2$ and $|\Psi|^4$ in the potential.

We do not know the corresponding **microscopic** theory for the Higgs boson.

In fact, we have NOT even measured the Ginzburg-Landau potential of the Higgs!

The question can be reformulated in terms of **Quantum Criticality**:



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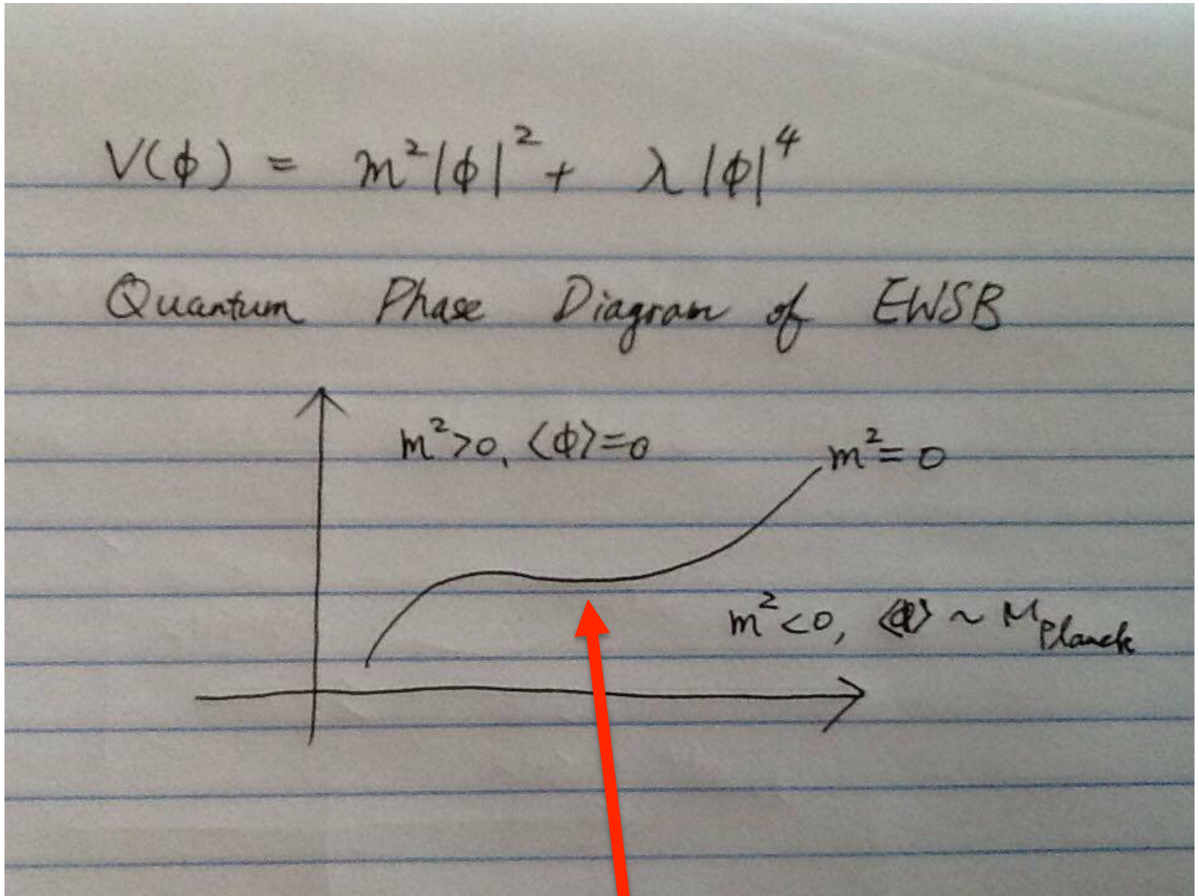

Mh=125 GeV. We are sitting extremely close to the criticality. **WHY??**

One appealing possibility – the critical line is selected dynamically, because one cannot write down any relevant operators giving rise to the Higgs mass.

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This is the analogy of BCS theory for electroweak symmetry breaking. It goes by the name of "technicolor," which is strongly disfavored experimentally.

The compatibility with the data can be improved by postulating new global symmetries above the weak scale, such that the Higgs boson arises as a (pseudo) Nambu-Goldstone boson.

➔ This class goes by the name of "composite Higgs models."

The other popular possibility is that the critical line is a locus of enhanced symmetry.

➔ This is the (broken) supersymmetry.

A survey of composite Higgs models from several years ago:

$\mathcal{G}$	$\mathcal{H}$	C	N_G	$\mathbf{r}_{\mathcal{H}} = \mathbf{r}_{SU(2)\times SU(2)}\ (\mathbf{r}_{SU(2)\times U(1)})$	Ref.
SO(5)	SO(4)	✓	4	$\mathbf{4} = (\mathbf{2},\mathbf{2})$	[11]
SU(3) × U(1)	SU(2) × U(1)		5	$\mathbf{2}_{\pm 1/2} + \mathbf{1}_0$	[10, 35]
SU(4)	Sp(4)	✓	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	[29, 47, 64]
SU(4)	$[SU(2)]^2 \times U(1)$	✓*	8	$(\mathbf{2},\mathbf{2})_{\pm 2} = 2 \cdot (\mathbf{2},\mathbf{2})$	[65]
SO(7)	SO(6)	✓	6	$\mathbf{6} = 2 \cdot (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	–
SO(7)	G_2	✓*	7	$\mathbf{7} = (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$	[66]
SO(7)	SO(5) × U(1)	✓*	10	$\mathbf{10}_0 = (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$	–
SO(7)	$[SU(2)]^3$	✓*	12	$(\mathbf{2},\mathbf{2},\mathbf{3}) = 3 \cdot (\mathbf{2},\mathbf{2})$	–
Sp(6)	Sp(4) × SU(2)	✓	8	$(\mathbf{4},\mathbf{2}) = 2 \cdot (\mathbf{2},\mathbf{2})$	[65]
SU(5)	SU(4) × U(1)	✓*	8	$\mathbf{4}_{-5} + \bar{\mathbf{4}}_{+5} = 2 \cdot (\mathbf{2},\mathbf{2})$	[67]
SU(5)	SO(5)	✓*	14	$\mathbf{14} = (\mathbf{3},\mathbf{3}) + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$	[9, 47, 49]
SO(8)	SO(7)	✓	7	$\mathbf{7} = 3 \cdot (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	–
SO(9)	SO(8)	✓	8	$\mathbf{8} = 2 \cdot (\mathbf{2},\mathbf{2})$	[67]
SO(9)	SO(5) × SO(4)	✓*	20	$(\mathbf{5},\mathbf{4}) = (\mathbf{2},\mathbf{2}) + (\mathbf{1}+\mathbf{3},\mathbf{1}+\mathbf{3})$	[34]
$[SU(3)]^2$	SU(3)		8	$\mathbf{8} = \mathbf{1}_0 + \mathbf{2}_{\pm 1/2} + \mathbf{3}_0$	[8]
$[SO(5)]^2$	SO(5)	✓*	10	$\mathbf{10} = (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	[32]
SU(4) × U(1)	SU(3) × U(1)		7	$\mathbf{3}_{-1/3} + \bar{\mathbf{3}}_{+1/3} + \mathbf{1}_0 = 3 \cdot \mathbf{1}_0 + \mathbf{2}_{\pm 1/2}$	[35, 41]
SU(6)	Sp(6)	✓*	14	$\mathbf{14} = 2 \cdot (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + 3 \cdot (\mathbf{1},\mathbf{1})$	[30, 47]
$[SO(6)]^2$	SO(6)	✓*	15	$\mathbf{15} = (\mathbf{1},\mathbf{1}) + 2 \cdot (\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$	[36]

Table 1: Symmetry breaking patterns $\mathcal{G} \to \mathcal{H}$ for Lie groups. The third column denotes whether the breaking pattern incorporates custodial symmetry. The fourth column gives the dimension N_G of the coset, while the fifth contains the representations of the GB's under $\mathcal{H}$ and $SO(4) \cong SU(2)_L \times SU(2)_R$ (or simply $SU(2)_L \times U(1)_Y$ if there is no custodial symmetry). In case of more than two SU(2)'s in $\mathcal{H}$ and several different possible decompositions we quote the one with largest number of bi-doublets.

Bellazzni, Csaki and Serra:1401.2457

In fact, the theory space of composite Higgs models is infinite:

Different composite Higgs models choose different symmetry-breaking patterns G/H.

Conventional wisdom:

Effective actions based on different G/H are different.

Each time a young hot shot comes up with a new model, we need to work out the experimental consequences all over again.

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This begs the question:

Are there universal predictions of a composite Higgs boson that are independent of the symmetry-breaking pattern?

It turned out that a common ingredient for pretty much all viable compoiste Higgs models, is H always contains a SO(4) subgroup, under which the 125 GeV Higgs is the fundamental representation.

Universality implies effective interactions of the 125 GeV Higgs in a composite Higgs model are identical, up to the normalization of "f".

In particular, we can show that the (multi)Higgs couplings to two electroweak gauge bosons are universal.

Da Liu, IL and Zhewei Yin: 1805.00489; 1809.09126

Schematically, the universality predicts that

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu h \partial^\mu h + b_{nh}\left(\frac{h}{v}\right)^n \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right)$$

$$\begin{aligned}
b_h &= 1 - 2\xi & b_{2h} &= 2\sqrt{1-\xi}\ , \\
b_{3h} &= -\frac{4}{3}\xi\sqrt{1-\xi}\ , & b_{4h} &= \frac{1}{3}\xi(2\xi - 1)\ , \\
b_{5h} &= \frac{4}{15}\xi^2\sqrt{1-\xi}\ , & b_{6h} &= \frac{2}{45}\xi^2(1-2\xi)\ , \\
&\ldots & &\ldots
\end{aligned}$$

The shift symmetry relates all these different couplings.

Experimental confirmation of the shift symmetry would be a striking indication on the NGB nature of the 125 GeV Higgs boson.

→ Opens up a new experimental frontier.

One way to “detect” the presence of the shift symmetry is to measure HVV and HHVV couplings to see if they are controlled by the same parameter.

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$$\mathcal{L}_{\rm NL} = \sum_i \frac{m_W^2}{m_\rho^2}\left(C_i^h \mathcal{I}_i^h + C_i^{2h}\mathcal{I}_i^{2h} + C_i^{3V}\mathcal{I}_i^{3V}\right)$$

$\mathcal{I}_i^h$
(1) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$
(2) $\frac{h}{v} Z_{\mu\nu} Z^{\mu\nu}$
(3) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$
(4) $\frac{h}{v} Z_{\mu\nu} A^{\mu\nu}$

$\mathcal{I}_i^{2h}$
(1) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$
(2) $\frac{h^2}{v^2} Z_{\mu\nu} Z^{\mu\nu}$
(3) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$
(4) $\frac{h^2}{v^2} Z_{\mu\nu} A^{\mu\nu}$

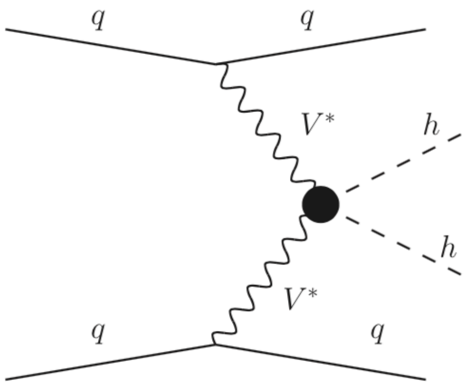
Some examples of "Universal Relations" are

$$\frac{C_3^{2h}}{C_3^h} = \frac{C_4^{2h}}{C_4^h} = \frac{1}{2}\cos\theta = \frac{1}{2}\sqrt{1-\xi}$$

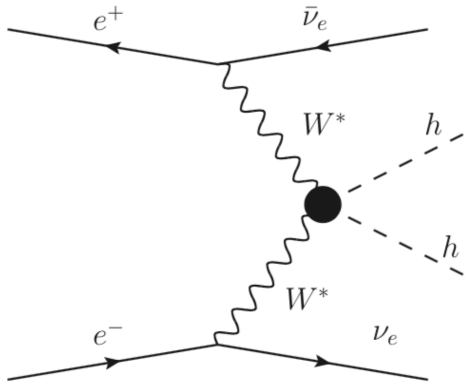
$$\frac{s_{2w}\,C_1^{2h} - c_{2w}\,C_3^{2h}}{s_{2w}\,C_1^h - c_{2w}\,C_3^h} = \frac{s_{2w}\,C_2^{2h} - c_{2w}\,C_4^{2h}}{s_{2w}\,C_2^h - c_{2w}\,C_4^h} = \frac{\cos 2\theta}{2\cos\theta} \approx \frac{1}{2}\left(1 - \frac{3}{2}\xi\right)$$

Z. Yin, D. Liu and IL: 1805.00489; 1809.09126

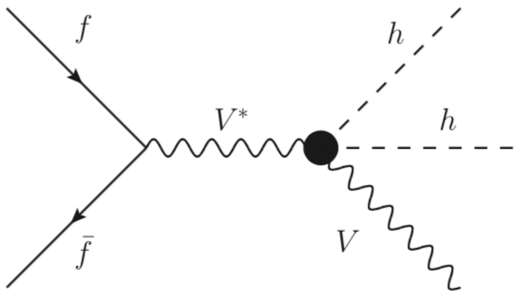
More generally, the HHVV coupling can be probed in the following channels:



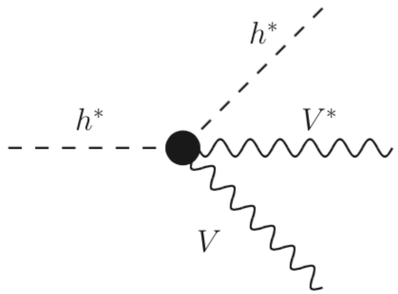
(a) Double Higgs production through vector boson fusion at a hadron collider.



(b) Double Higgs production through vector boson fusion at a lepton collider.

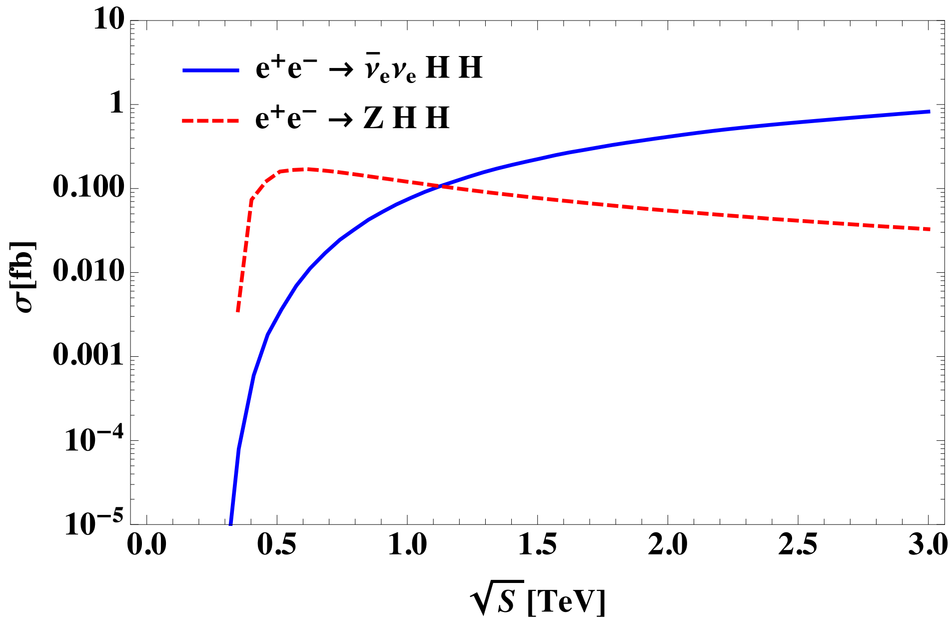


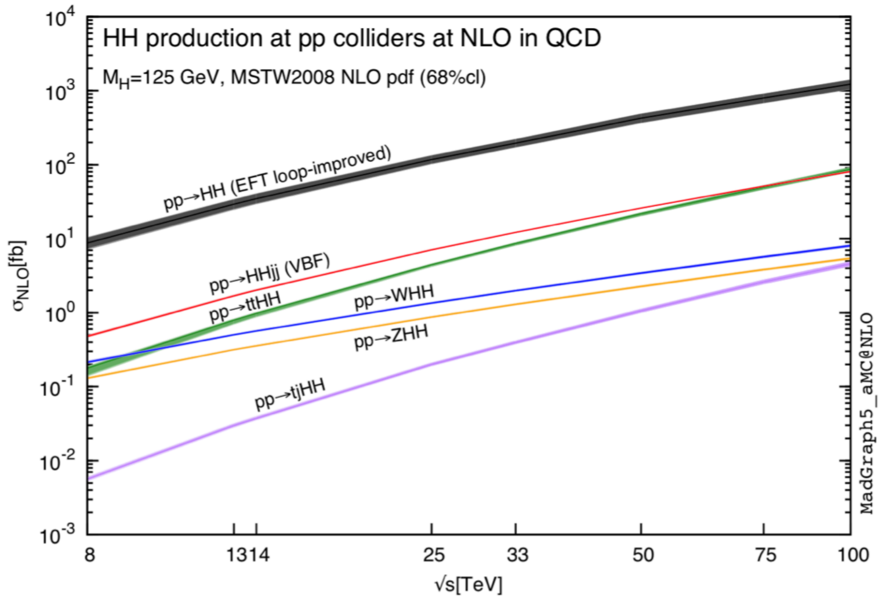
(c) Double Higgs production in association with a vector boson.



(d) Off-shell Single Higgs decay.

The rate at future colliders:





1401.7340

This is the future frontier of precision Higgs physics!

Concluding Remarks:

- Effective actions of NGBs are defined entirely by the infrared properties, independent of the broken group G.

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- Interactions of a Nambu-Goldstone Higgs boson with two gauge bosons are dictated by shift symmetry and universal.

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- Interactions of a Nambu-Goldstone Higgs boson with two gauge bosons are dictated by shift symmetry and universal.

- Testing the shift symmetry in Higgs couplings could drive future experimental programs in the study of Higgs boson.