Holographic Entanglement Entropy in Boundary QFT

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Outline

- Boundary Quantum Field Theory (BQFT)
- Holographic Entanglement Entropy in BQFT
  - Pure AdS spacetime
  - Schwarzschile-AdS black hole
  - Araki-Lieb Inequality

E-J Chang, C-J Chou and YY, arXiv:1805.06117, *Phys.Rev.* D98 (2018)

Holographic BQFT

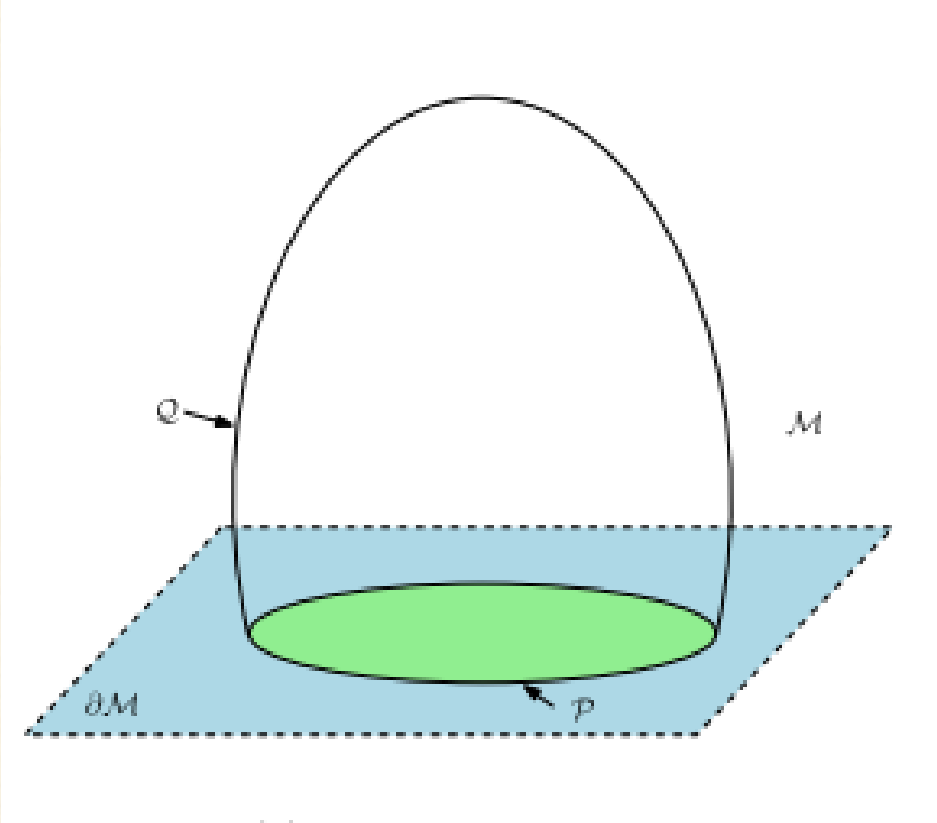
$$S = S_{\mathcal{M}} + S_{Q} + S_{\partial\mathcal{M}} + S_{\mathcal{P}}$$

$$S_{\mathcal{M}} = \int_{\mathcal{M}} \sqrt{-g}(R - 2\Lambda_{\mathcal{M}})$$

$$S_{Q} = \int_{Q} \sqrt{-h}\left(R_{Q} - 2\Lambda_{Q} + 2K\right)$$

$$S_{\partial\mathcal{M}} = \int_{\partial\mathcal{M}} \sqrt{-\gamma}(2K')$$

$$S_{\mathcal{P}} = \int_{\mathcal{P}} \sqrt{-\sigma}(2\theta)$$



[Takayanagi, Chu, Miao, Guo]

Boundary Condition

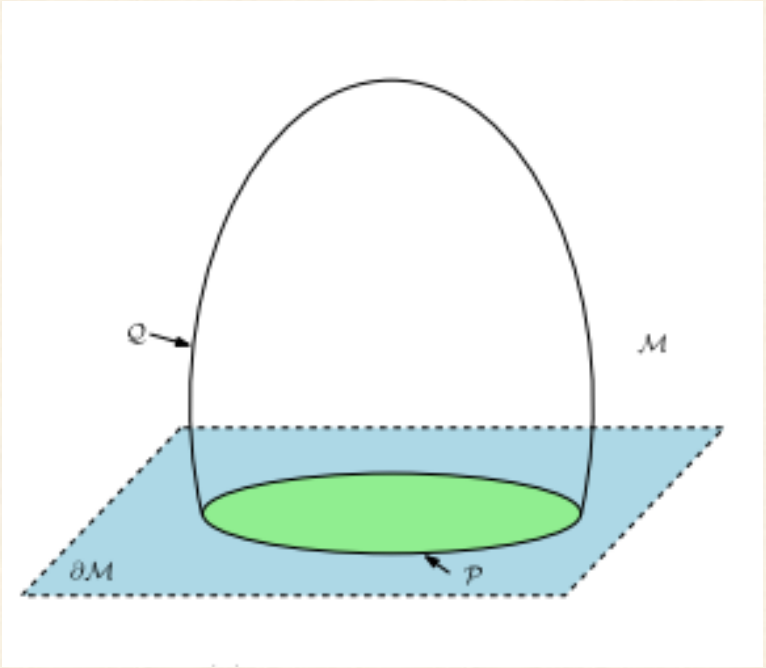
- Equation of motion

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda_{\mathcal{M}} g_{ab} = 0$$

- Boundary condition

$$R_{Q\,ab} + 2K_{ab} - \left(\frac{1}{2} R_Q + K - \Lambda_Q\right) h_{ab} = 0$$

$$\rightarrow (d-1)\left(R_Q + 2K\right) - 2(d+1)\Lambda_Q = 0$$



[Takayanagi, Chu, Miao, Guo]

Holographic Entanglement Entropy

$$S_A = \min_X \frac{Area(\mathcal{E}_{\mathcal{A}})}{4G_N^{(d+2)}}$$

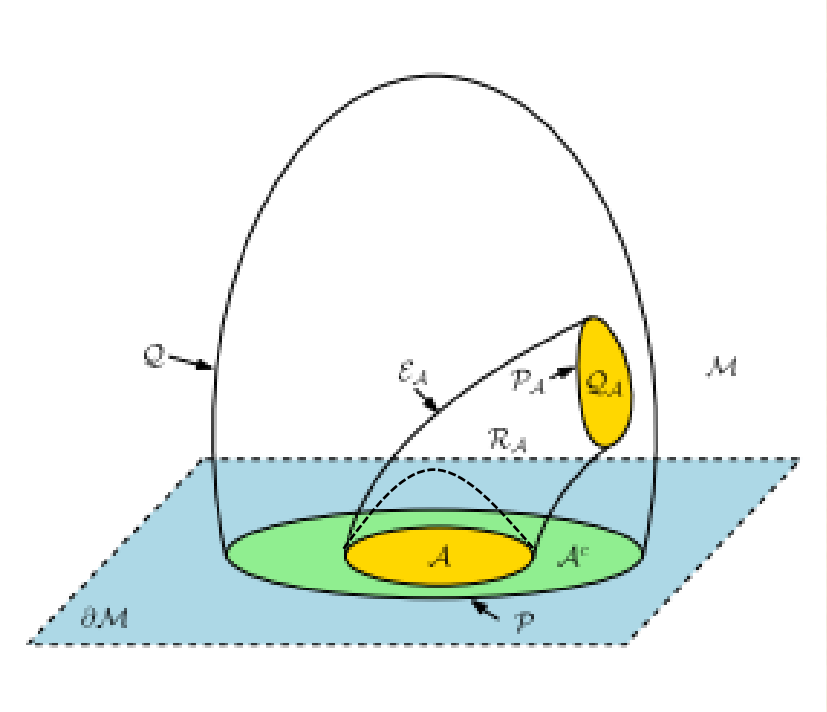
$$X = \left\{ \mathcal{E}_{\mathcal{A}} \middle| \begin{matrix} \mathcal{E}_{\mathcal{A}}|_{\partial\mathcal{M}} = \partial\mathcal{A} \\ \partial\mathcal{R}_{\mathcal{A}} = \mathcal{E}_{\mathcal{A}} \cup \mathcal{A} \end{matrix} \right\}$$

[Ryu, Takayanagi, Hubeny, Rangamani]

$$X = \left\{ \mathcal{E}_{\mathcal{A}} \middle| \begin{matrix} \mathcal{E}_{\mathcal{A}}|_{\partial\mathcal{M}} = \partial\mathcal{A}, \mathcal{E}_{\mathcal{A}}|_{Q} = \mathcal{P}_{\mathcal{A}} \\ \partial\mathcal{R}_{\mathcal{A}} = \mathcal{E}_{\mathcal{A}} \cup \mathcal{A} \cup Q_{\mathcal{A}} \end{matrix} \right\}$$

[Chu, Miao, Guo, Astaneh, Berthiere, Solodukhin]

Homology Constraint



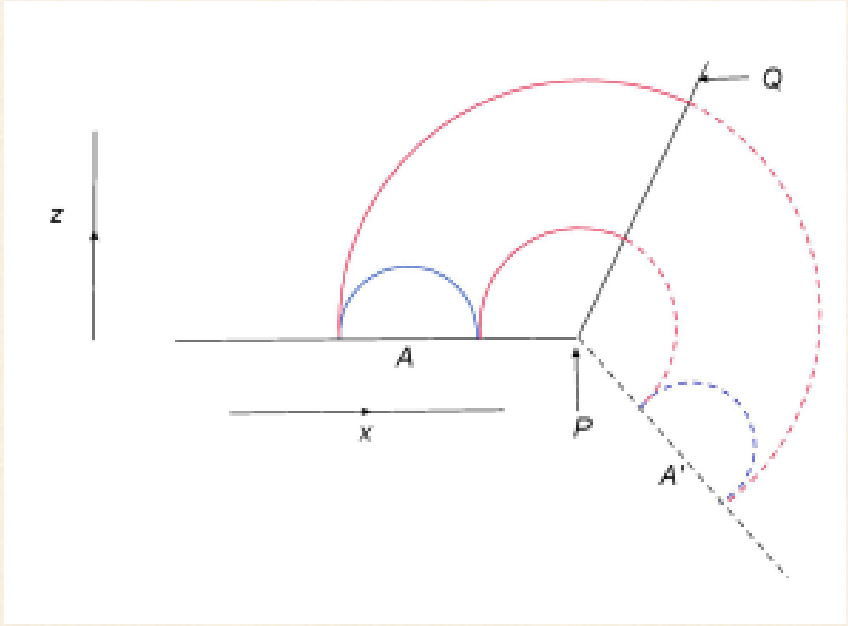
A Solution in AdS_3

- Solution of Q

  $x = P + \sinh(\rho)z$

- Entanglement entropy

$$S_{\mathcal{A}} = \begin{cases} \log\left(\frac{2l}{\epsilon}\right), & d \geq d_c \\ \rho + \frac{1}{2}\log\left(\frac{4d(d+2l)}{\epsilon^2}\right), & d \leq d_c \end{cases}$$

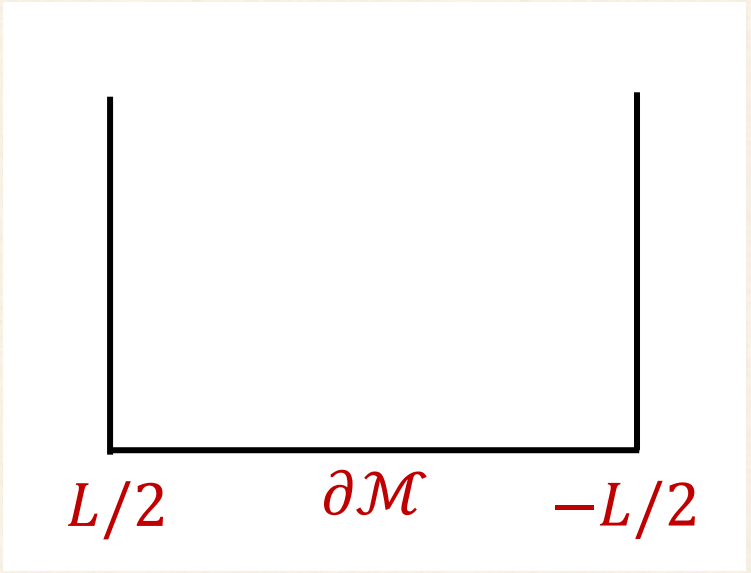


[Chu, Miao, Guo]

A Simple Solution in AdS_{d+2}

$$AdS_{d+2}:\ ds^2_{\mathcal{M}} = \frac{l^2_{AdS}}{z^2}\left(-dt^2 + dz^2 + \sum_{i=1}^{d} dx_i^2\right)$$

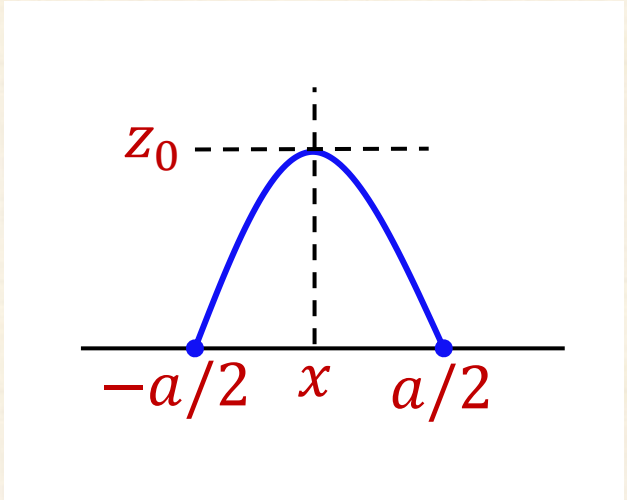
$$AdS_{d+1}:\ ds^2_{Q} = \frac{l^2_{AdS}}{z^2}\left(-dt^2 + dz^2 + \sum_{i=2}^{d} dx_i^2\right)$$



$$R_Q = -\frac{d(d+1)}{l^2_{AdS}}, \qquad K_{ab} = 0, \qquad \Lambda_Q = -\frac{d(d-1)}{2l^2_{AdS}}$$

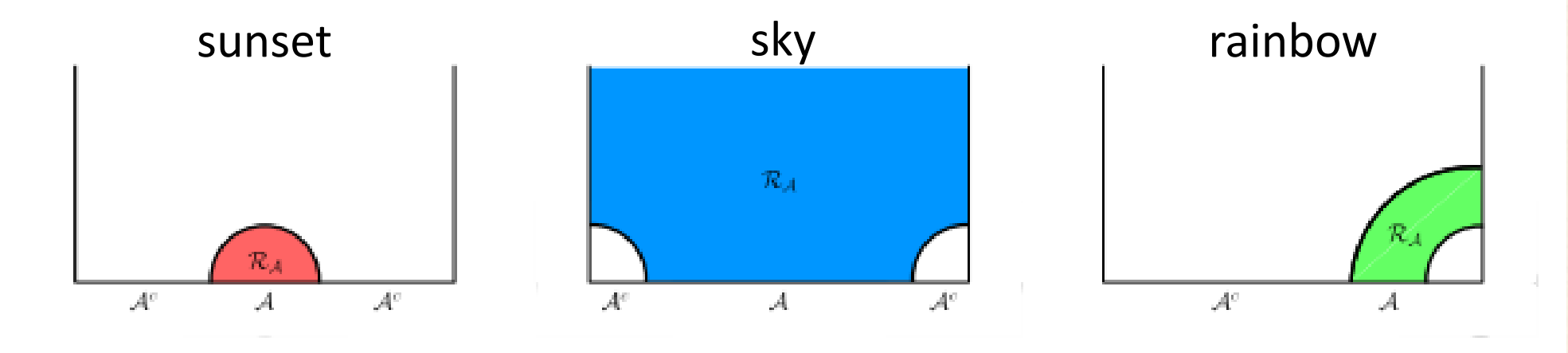
Minimal Surface in AdS_{d+2}

$$a = 2z_0\sqrt{\pi}\,\frac{\Gamma\left(\frac{d+1}{2d}\right)}{\Gamma\left(\frac{1}{2d}\right)}$$



$$S_{EE}^{\mathcal{A}}(a) = \frac{l_{AdS}^{d}}{2(d-1)G_N^{(d+2)}}\left[\left(\frac{L}{\epsilon}\right)^{d-1} - \left(\frac{L}{z_0}\right)^{d}\frac{a}{2L}\right]$$

Entanglement Wedge in AdS_{d+2}

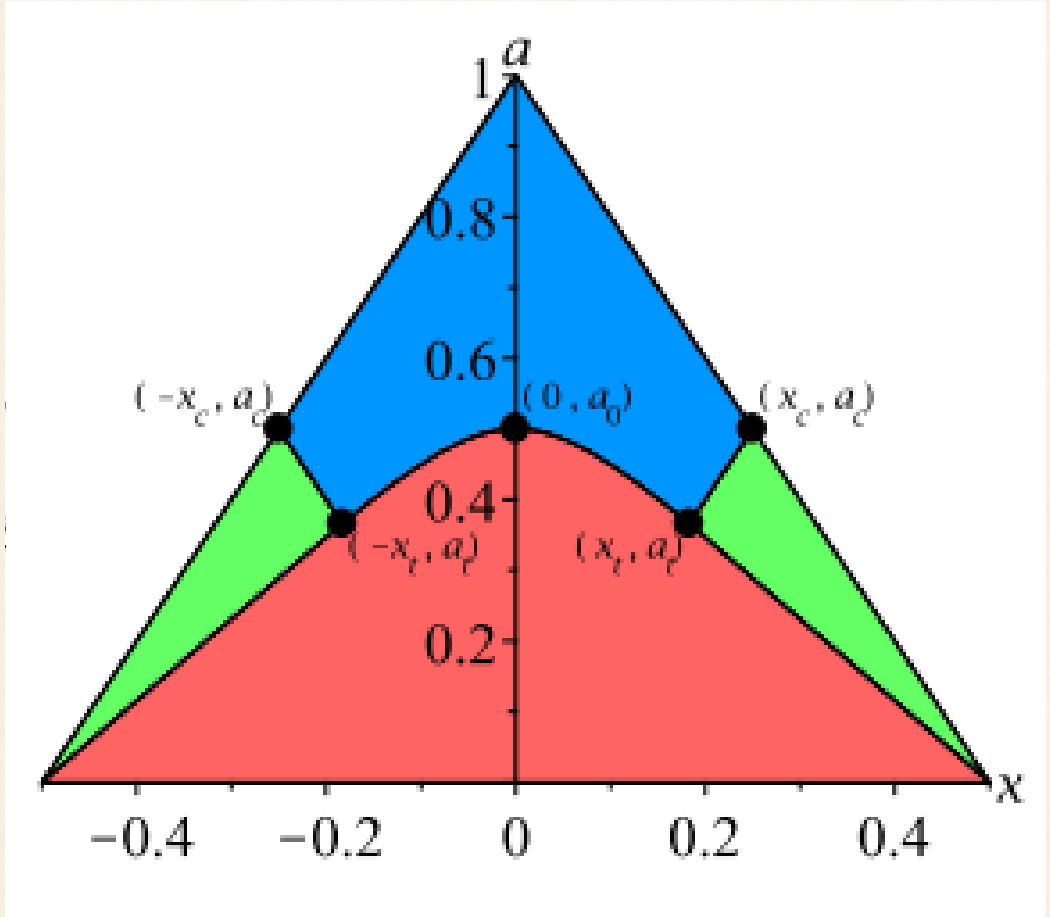


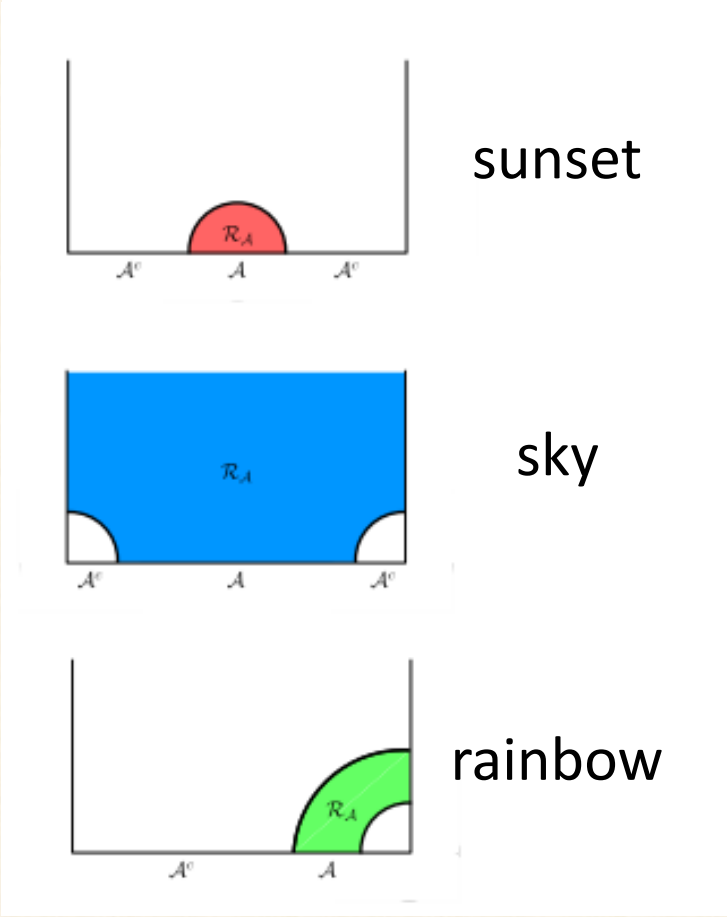
$$S^{\mathcal{A}}_{sunset} = S^{\mathcal{A}}_{EE}(a)$$

$$S^{\mathcal{A}}_{sky} = \frac{1}{2}S^{\mathcal{A}}_{EE}(l-a+2|x|) + \frac{1}{2}S^{\mathcal{A}}_{EE}(l-a-2|x|)$$

$$S^{\mathcal{A}}_{rainbow} = \frac{1}{2}S^{\mathcal{A}}_{EE}(l+a-2|x|) + \frac{1}{2}S^{\mathcal{A}}_{EE}(l-a-2|x|)$$

Phase Diagram in AdS_{d+2}





Holographic BQFT at Finite T

$$S = S_{\mathcal{M}} + S_{\mathcal{Q}} + S_{\partial\mathcal{M}} + S_{\mathcal{P}}$$

$$S_{\mathcal{M}} = \int_{\mathcal{M}} \sqrt{-g}(R - 2\Lambda_{\mathcal{M}})$$

$$S_{\mathcal{Q}} = \int_{\mathcal{Q}} \sqrt{-h}\left(R_{\mathcal{Q}} - 2\Lambda_{\mathcal{Q}} + 2K\right)$$

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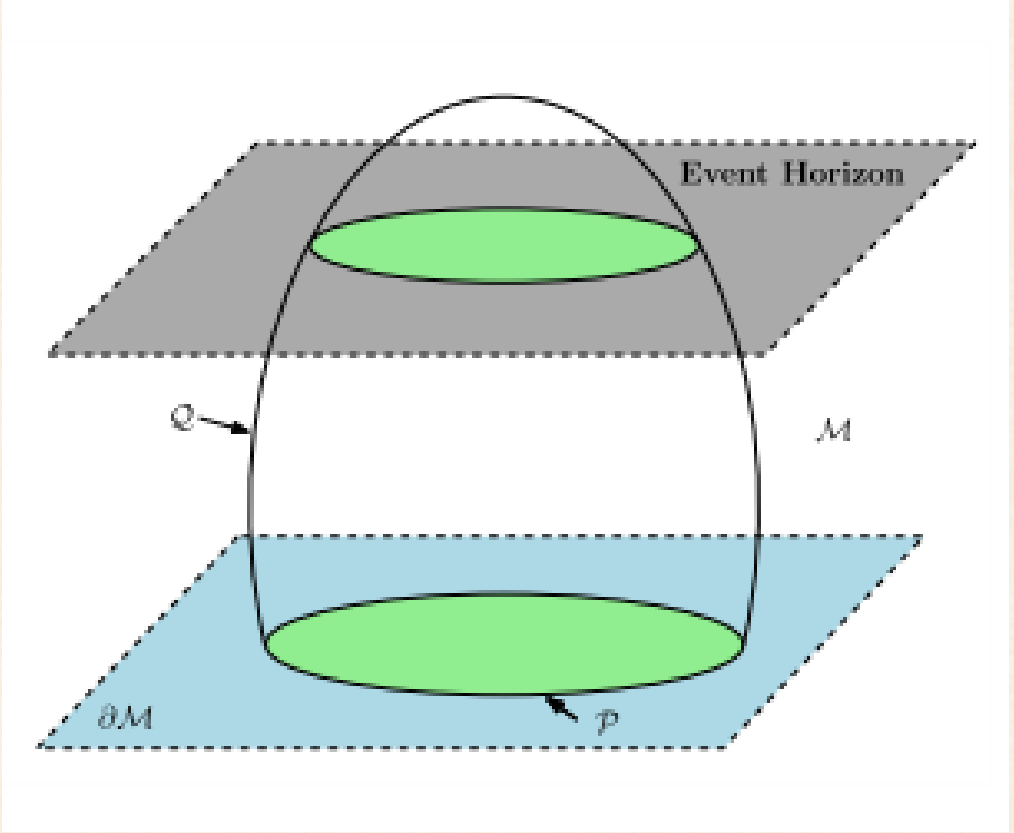


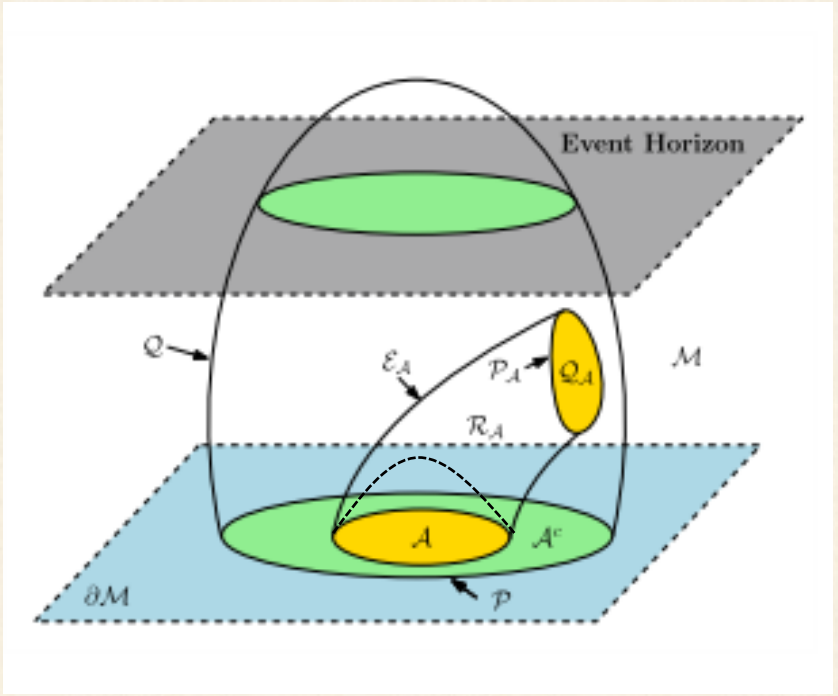
Holographic Entanglement Entropy

$$S_A = \min_X \frac{Area(\mathcal{E}_\mathcal{A})}{4G_N^{(d+2)}}$$

$$X = \left\{ \mathcal{E}_\mathcal{A} \middle| \begin{matrix} \mathcal{E}_\mathcal{A}|_{\partial\mathcal{M}} = \partial\mathcal{A} \\ \partial\mathcal{R}_\mathcal{A} = \mathcal{E}_\mathcal{A} \cup \mathcal{A} \end{matrix} \right\}$$

$$X = \left\{ \mathcal{E}_\mathcal{A} \middle| \begin{matrix} \mathcal{E}_\mathcal{A}|_{\partial\mathcal{M}} = \partial\mathcal{A}, \mathcal{E}_\mathcal{A}|_Q = \mathcal{P}_\mathcal{A} \\ \partial\mathcal{R}_\mathcal{A} = \mathcal{E}_\mathcal{A} \cup \mathcal{A} \cup \mathcal{Q}_\mathcal{A} \end{matrix} \right\}$$

Homology Constraint



Schwarzschild-AdS Black Hole

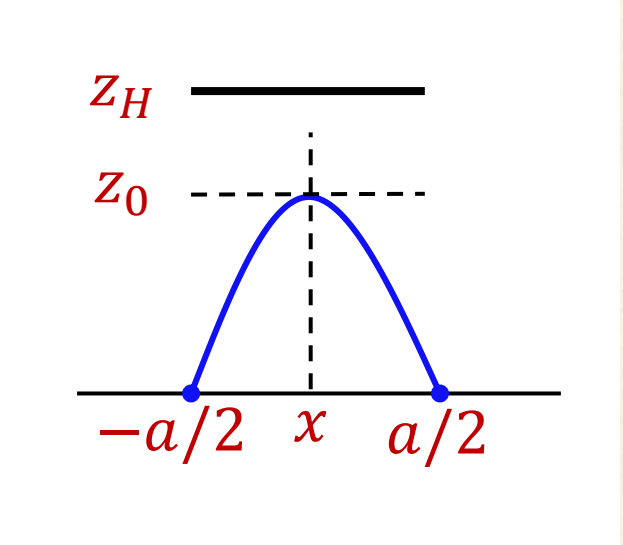
$$ds^2_{\mathcal{M}} = \frac{l^2_{AdS}}{z^2}\left(-g(z)dt^2 + \frac{dz^2}{g(z)} + \sum_{i=1}^{d} dx_i^2\right), \qquad g(z) = 1 - \frac{z^{d+1}}{z_H^{d+1}}$$

$$ds^2_{Q} = \frac{l^2_{AdS}}{z^2}\left(-g(z)dt^2 + \frac{dz^2}{g(z)} + \sum_{i=2}^{d} dx_i^2\right)$$

$$R_Q = -\frac{d(d+1)}{l^2_{AdS}}, \qquad K_{ab} = 0, \qquad \Lambda_Q = -\frac{d(d-1)}{2l^2_{AdS}}$$

Minimal Surface in Black Hole

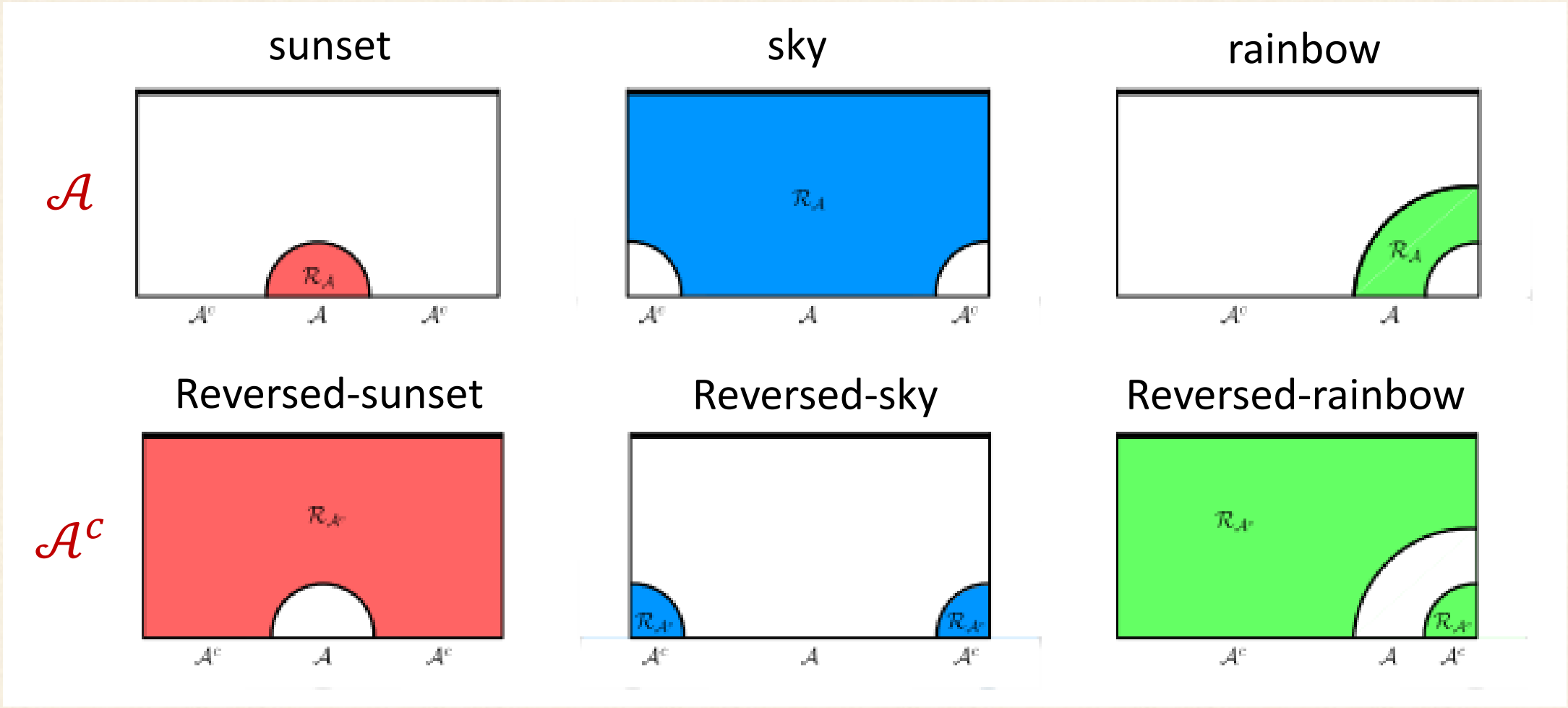
$$a = 2z_0 \int_0^1 \frac{v^d dv}{\sqrt{(1-(bv)^{d+1})(1-v^{2d})}} \,, b = \frac{z_0}{z_H}$$



$$S_{EE}^{\mathcal{A}}(a) = \frac{l_{AdS}^d}{2(d-1)G_N^{(d+2)}} \left[\left(\frac{L}{\epsilon}\right)^{d-1} - \left(\frac{L}{z_0}\right)^d \frac{a}{2L} \right]$$

$$- \frac{bz_0}{d-1} \int_0^1 \frac{(d-3)v - (d+3)v^{2d+1}}{\sqrt{(1-(bv)^{d+1})(1-v^{2d})}}$$

Entanglement Wedge in BH



Entanglement Entropy in BH

$$S^{\mathcal{A}}_{sunset} = S^{\mathcal{A}}_{EE}(a)$$

$$S^{\mathcal{A}}_{sky} = \frac{1}{2} S^{\mathcal{A}}_{EE}(l - a + 2|x|) + \frac{1}{2} S^{\mathcal{A}}_{EE}(l - a - 2|x|) + S_{BH}$$

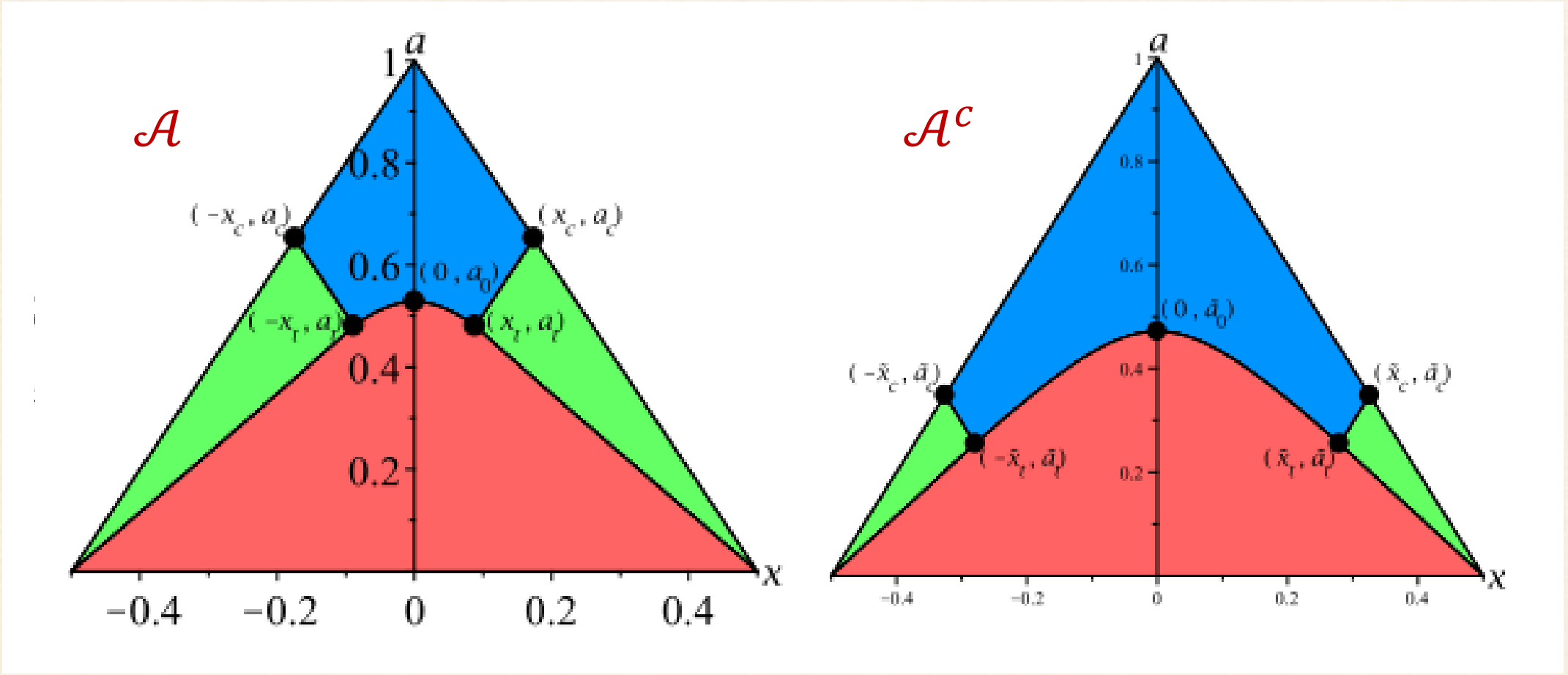
$$S^{\mathcal{A}}_{rainbow} = \frac{1}{2} S^{\mathcal{A}}_{EE}(l + a - 2|x|) + \frac{1}{2} S^{\mathcal{A}}_{EE}(l - a - 2|x|)$$

$$S^{\mathcal{A}^c}_{R-sunset} = S^{\mathcal{A}}_{EE}(a) + S_{BH}$$

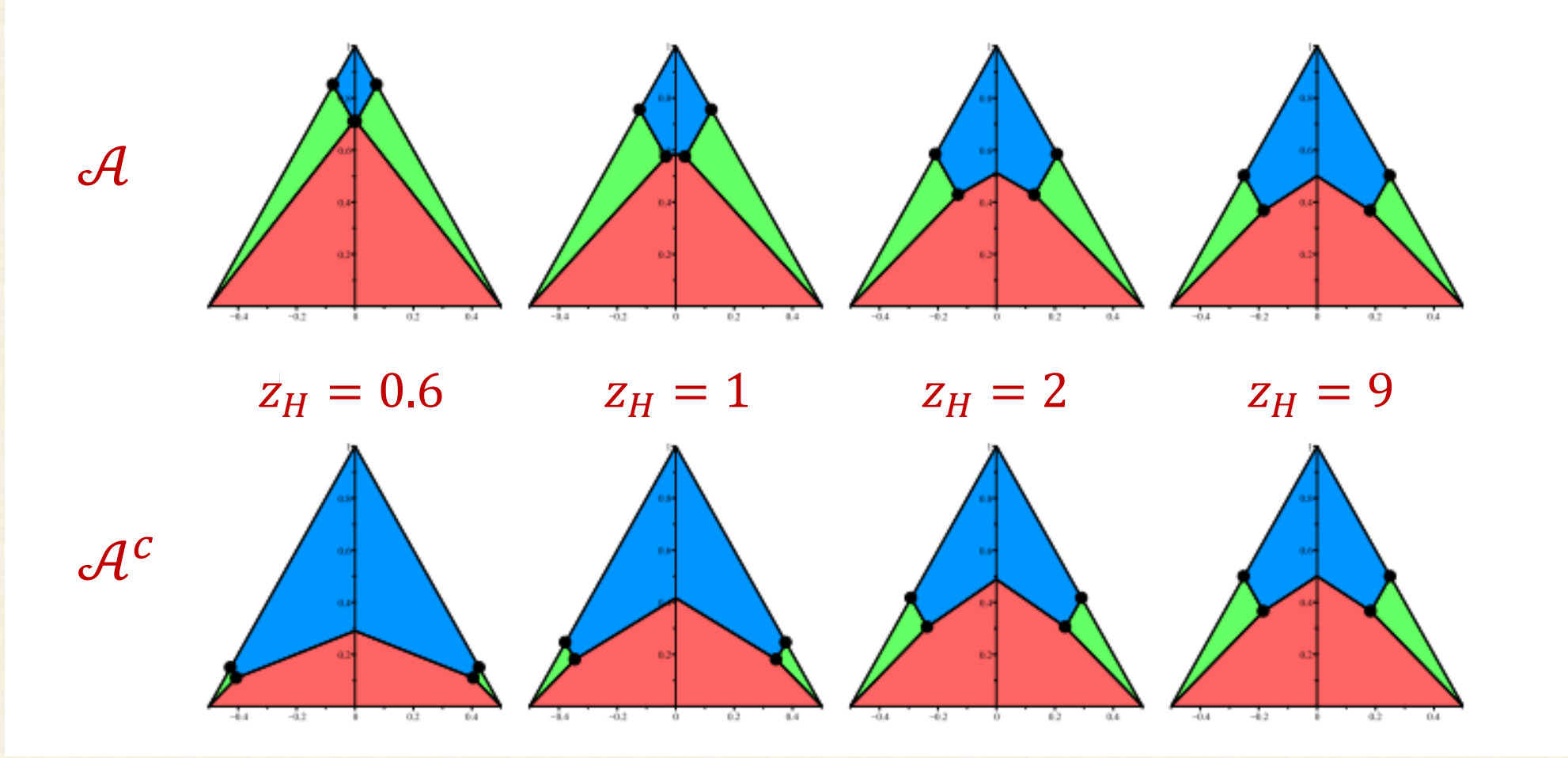
$$S^{\mathcal{A}^c}_{R-sky} = \frac{1}{2} S^{\mathcal{A}}_{EE}(l - a + 2|x|) + \frac{1}{2} S^{\mathcal{A}}_{EE}(l - a - 2|x|)$$

$$S^{\mathcal{A}^c}_{R-rainbow} = \frac{1}{2} S^{\mathcal{A}}_{EE}(l + a - 2|x|) + \frac{1}{2} S^{\mathcal{A}}_{EE}(l - a - 2|x|) + S_{BH}$$

Phase Diagram in BH at $z_H = 1.5$

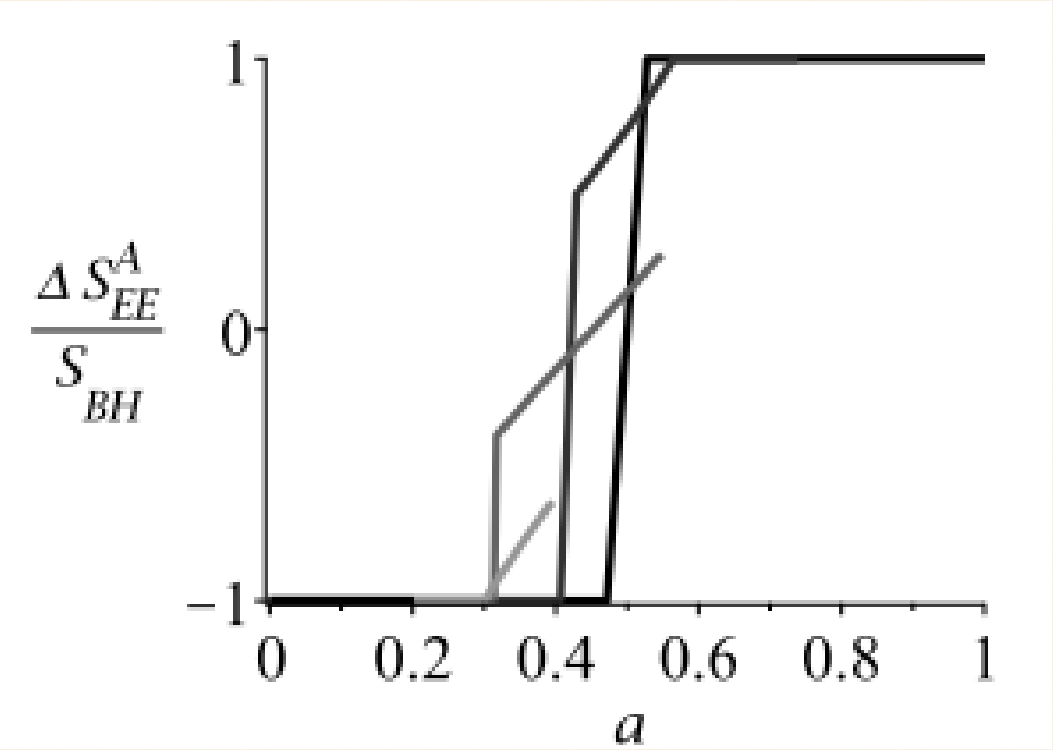


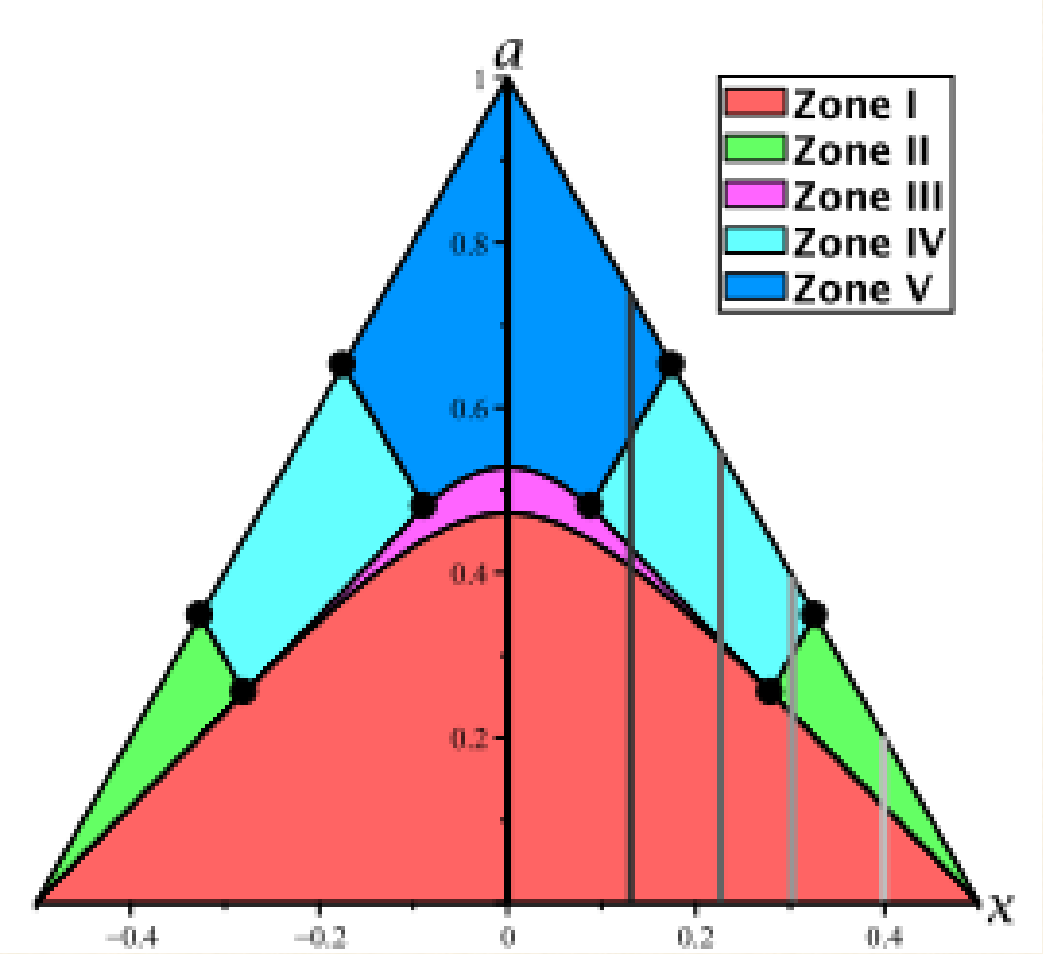
Phase Diagram at Different T



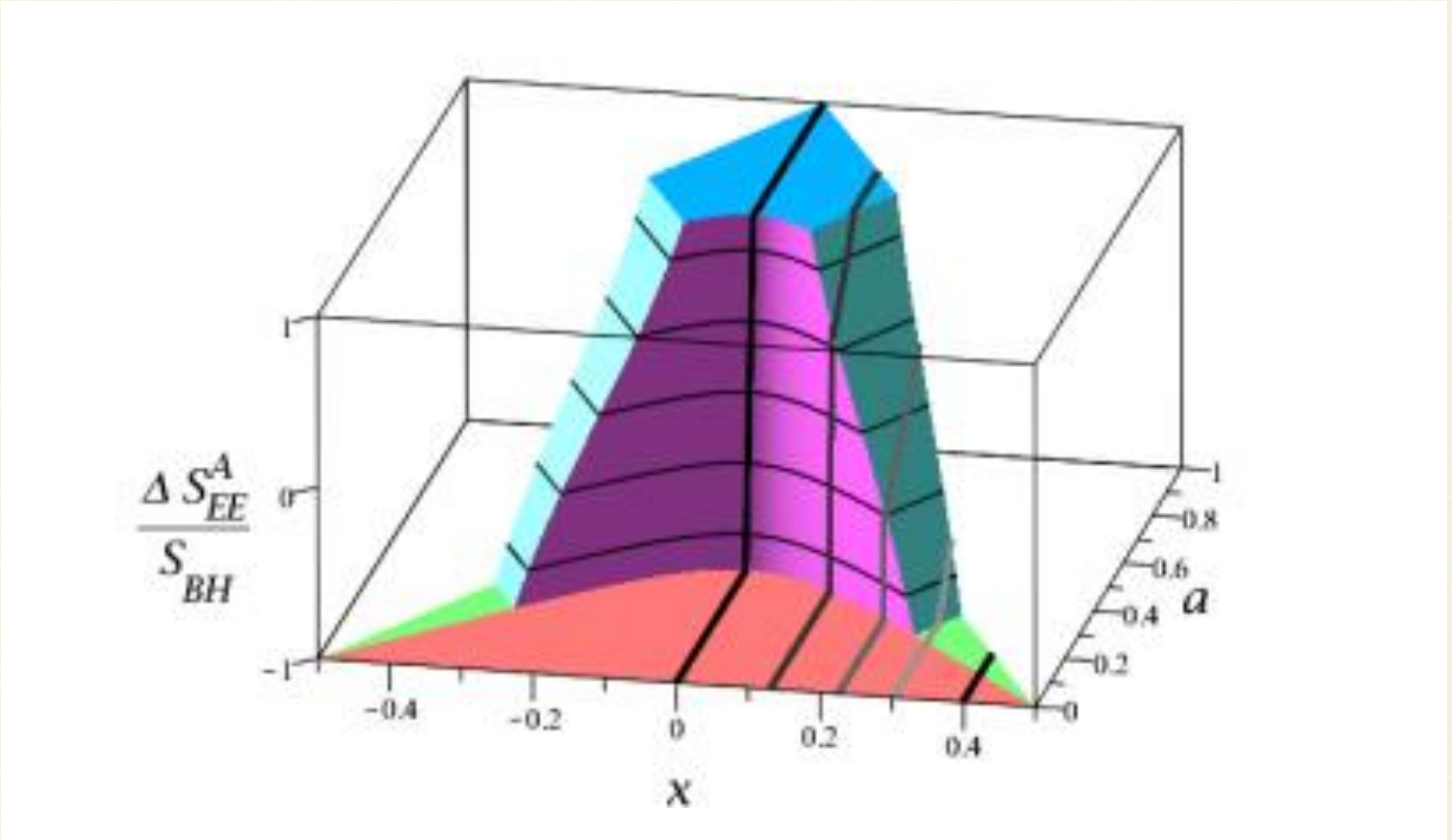
Araki-Lieb Inequality

$$\left|\Delta S_{EE}^{\mathcal{A}}\right| = \left|S_{EE}^{\mathcal{A}} - S_{EE}^{\mathcal{A}^c}\right| \leq S_{BH}$$





Entanglement Plateau



Summary

- Holographic BQFT: pure AdS and black hole
- Holographic entanglement entropy
  - Phase diagram of HEE
  - Araki-Lieb Inequality and entanglement plateau