THE POSITIVE LANDSCAPE

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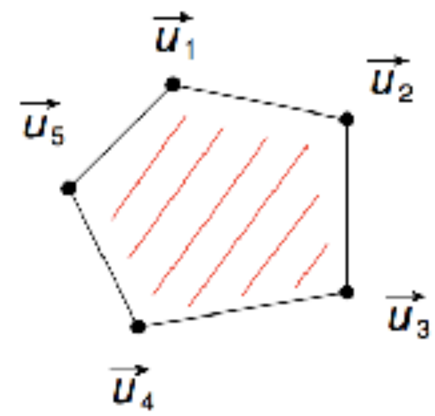
GEOMETRY FROM POSITIVITY

Let's start with a primitive notion of positivity in physics

- The position of center of mass



$$\vec{A} = \frac{\sum_{i=1}^{5} m_i \vec{u}_i}{\sum_{i=1}^{5} m_i}$$

The center of mass is always "inside" the polygon because m>0

Writing things projectively,

$$A^I = \begin{pmatrix} 1 \\ \vec{A} \end{pmatrix}, \quad U_i^I = \begin{pmatrix} 1 \\ \vec{u}_i \end{pmatrix}$$

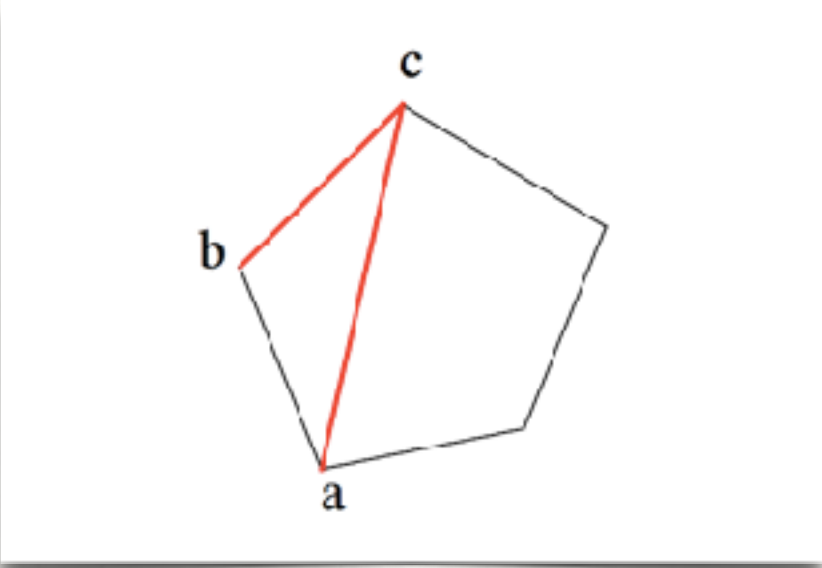
The inside of the polygon is inside the CONVEX HULL

$$A^I = w_1 U_1^I + \cdots + w_n U_n^I, \quad w_i > 0, \sum_{i=1}^{n} w_i = 1$$

GEOMETRY FROM POSITIVITY

We can define the polygon through positivity

The line (bc) is a boundary but (ac) is not. This is because while the hull (A) is ONLY on one side of (bc), it can be on either side of (ac), including on (ac)



$$det[A, U_a, U_c] = 0$$

Thus for boundaries if must satisfy

$$det[A, U_i, U_j] > 0, \quad \forall A$$

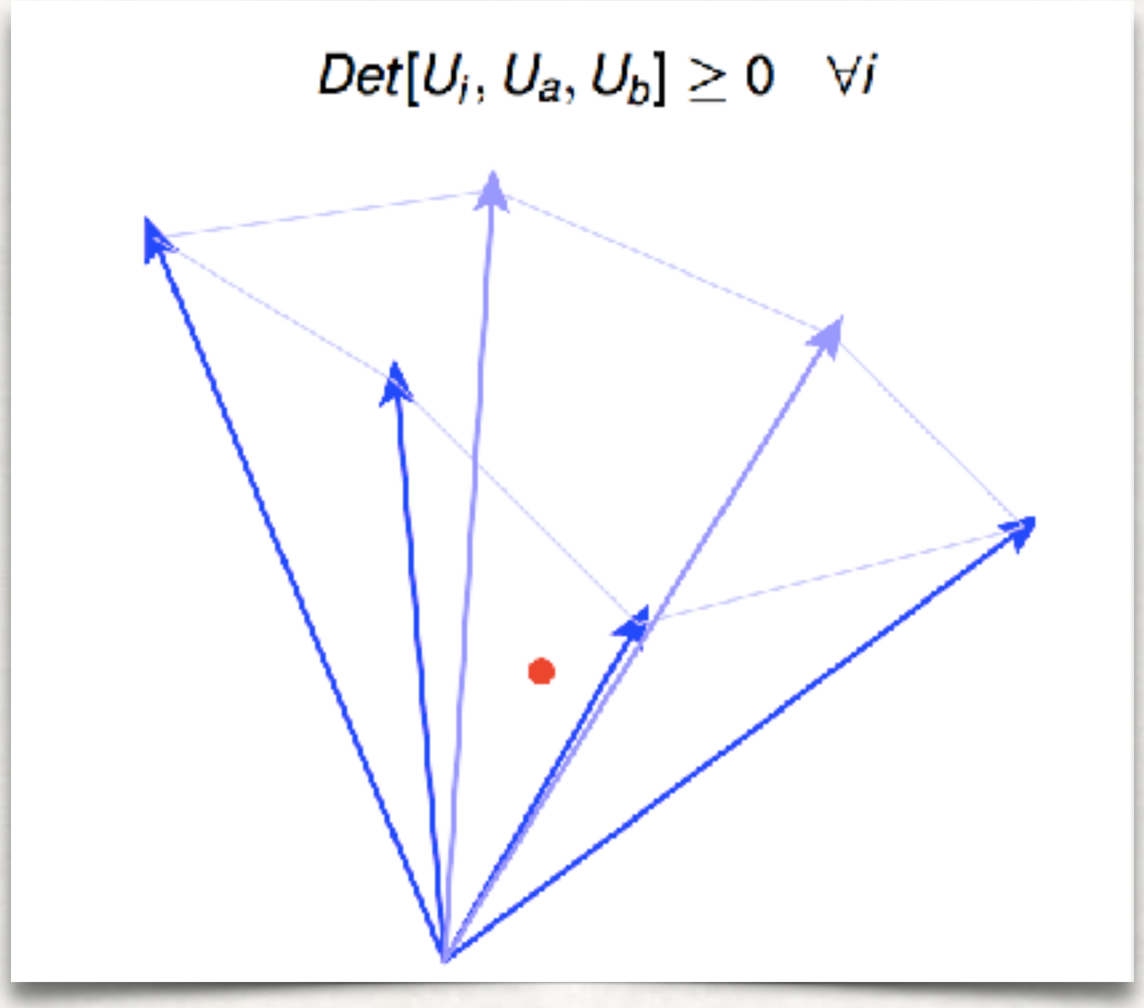
With

$$A^I = w_1 U_1^I + \cdots + w_n U_n^I, \quad w_i > 0, \sum_{i=1}^{n} w_i = 1$$

POLYTOPE POSITIVITY

We can define the polytope through its facets. This entails:

- Determine which vectors are the vertices
- Determine which set of the vertices form facets:



We need to compute all possible determinants to fully determine the geometry!

THE CYCLIC POLYTOPE

A new form of positivity trivializes the problem!

Let's say we have a set of vectors, with a well defined ordering. If its ordered determinant is positive:

$$det[U_{i_1}, U_{i_2}, \cdots, U_{i_k}] > 0, \quad \forall\, i_1 < i_2 < i_3 \cdots < i_k$$

The convex hull of U_i is a cyclic polytope

- It's boundaries are known

$$d = 2: \ (i, i+1), \quad d = 3: \ (0, i, i+1), (i, i+1, \infty), \quad d = 4: (i, i+1, j, j+1) \cdots$$

Exp: Let $A = aU_6 + bU_9$ with $a, b > 0$

$$\begin{aligned} Det[A, U_4, U_5, U_7, U_8] &= a\, Det[U_6, U_4, U_5, U_7, U_8] + b\, Det[U_9, U_4, U_5, U_7, U_8] \\ &= a\, Det[U_4, U_5, U_6, U_7, U_8] + b\, Det[U_4, U_5, U_7, U_8, U_9] \\ &= a\,(positive) + b\,(positive) \end{aligned}$$

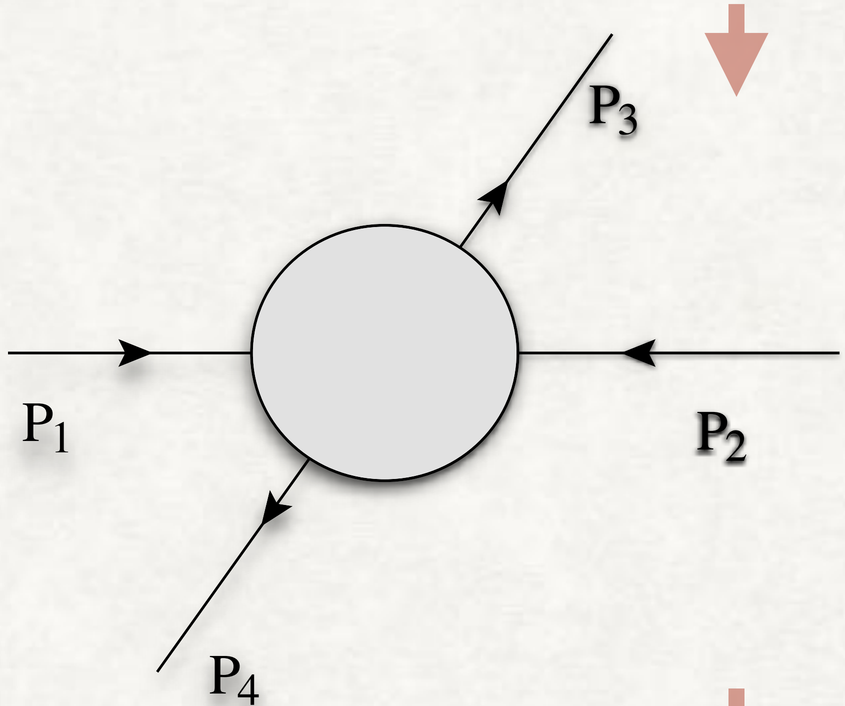
- Do we encounter convex hull problems physics?
- Do we find cyclic polytopes in such scenario?

CONVEX HULL

- Do we encounter convex hull problems physics?

Any effective field theory at low energies has a description as

$$\int dx^4 \frac{1}{2}\phi\Box\phi + a_0\phi^4 + a_1(\partial\phi)^2\phi^2 + a_2(\partial\phi)^4 + \cdots$$



$s = (P_1 + P_2)^2$

$t = (P_1 + P_4)^2$

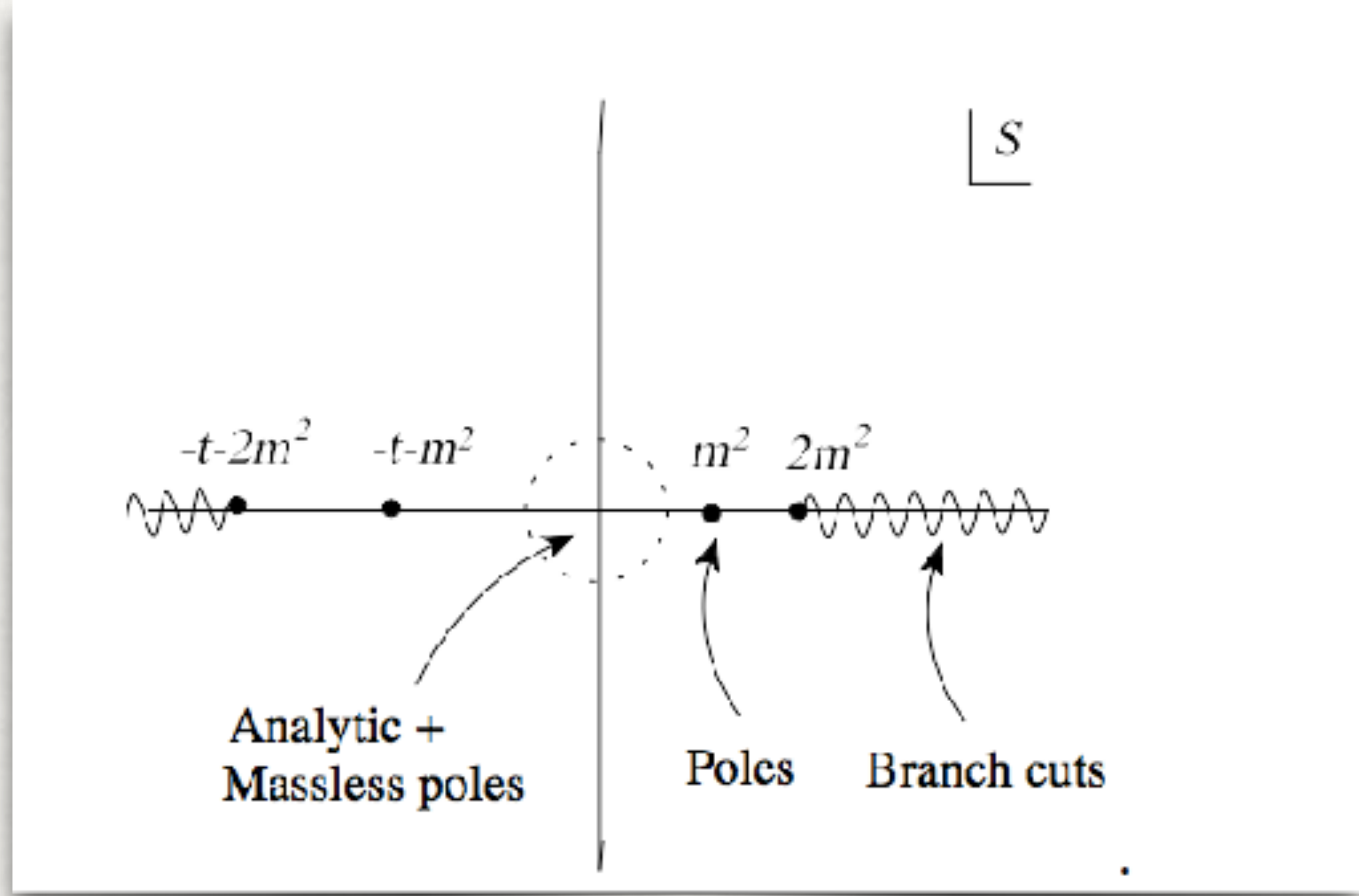
$$M^{IR}(s,t) = \{massless\ poles\} + \sum_{k,q} g_{k,q} s^{k-q} t^q,$$

CONVEX HULL

Any effective field theory at low energies has a description as

$$M^{IR}(s,t) = \{massless\ poles\} + \sum_{k,q} g_{k,q} s^{k-q} t^q,$$

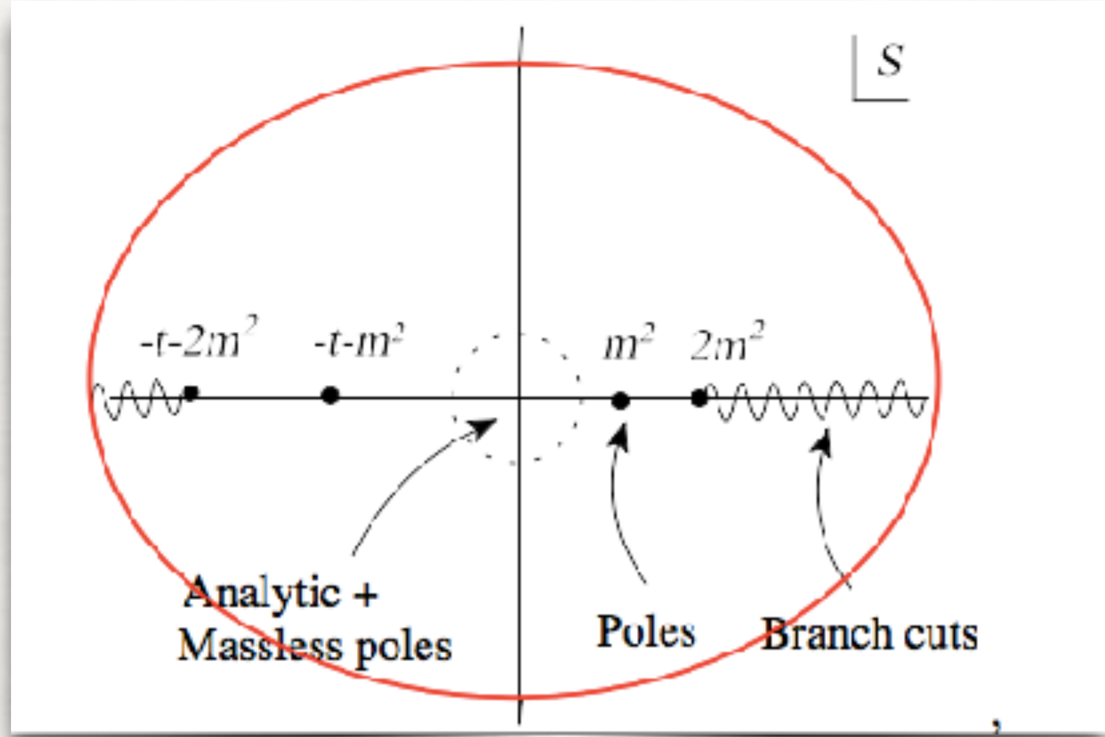
In the complex plane, the couplings for the higher dimension operators are related to the singularities and the discontinuity of its UV completion



no thresholds for t<<1

CONVEX HULL

In the complex plane, the couplings for the higher dimension operators are related to the singularities and the discontinuity of its UV completion



We can perform a contour integral

$$I = \frac{i}{2\pi} \int_{\infty} \frac{ds}{s^{n+1}} M(s,t)$$

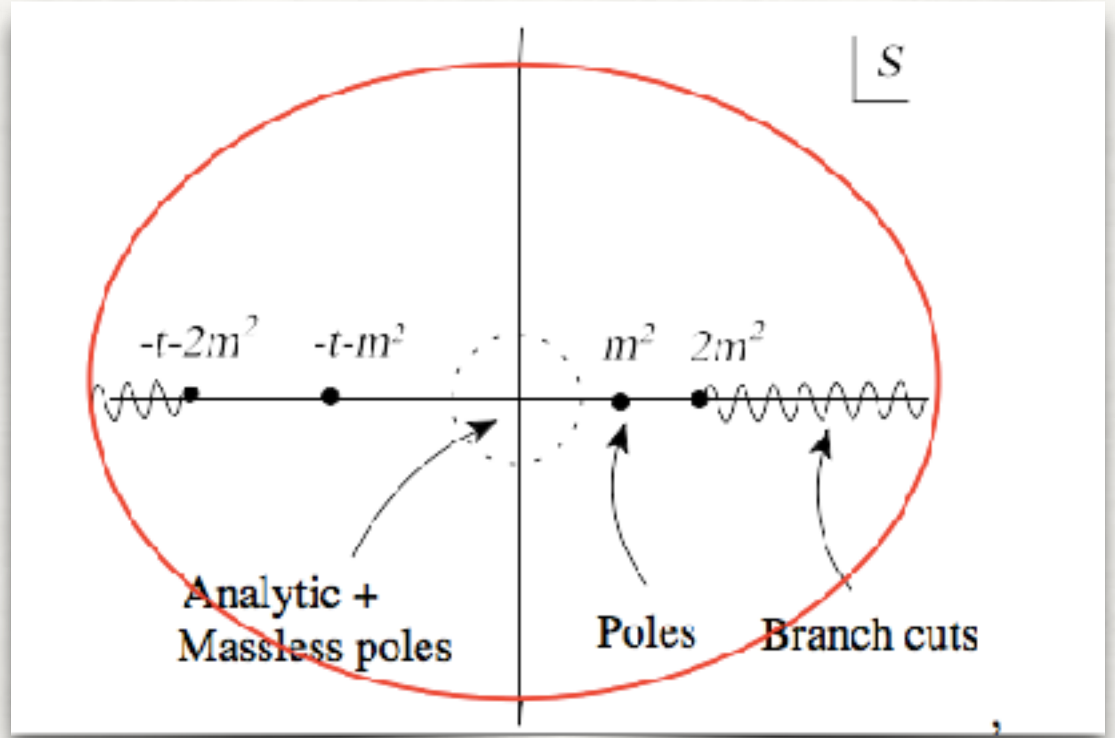
From the origin

$$M^{IR}(s,t) = \{massless\ poles\} + \sum_{k,q} g_{k,q} s^{k-q} t^q,$$

$$\rightarrow \sum_q g_{n+q,q} t^q$$

CONVEX HULL

In the complex plane, the couplings for the higher dimension operators are related to the singularities and the discontinuity of its UV completion



We can perform a contour integral

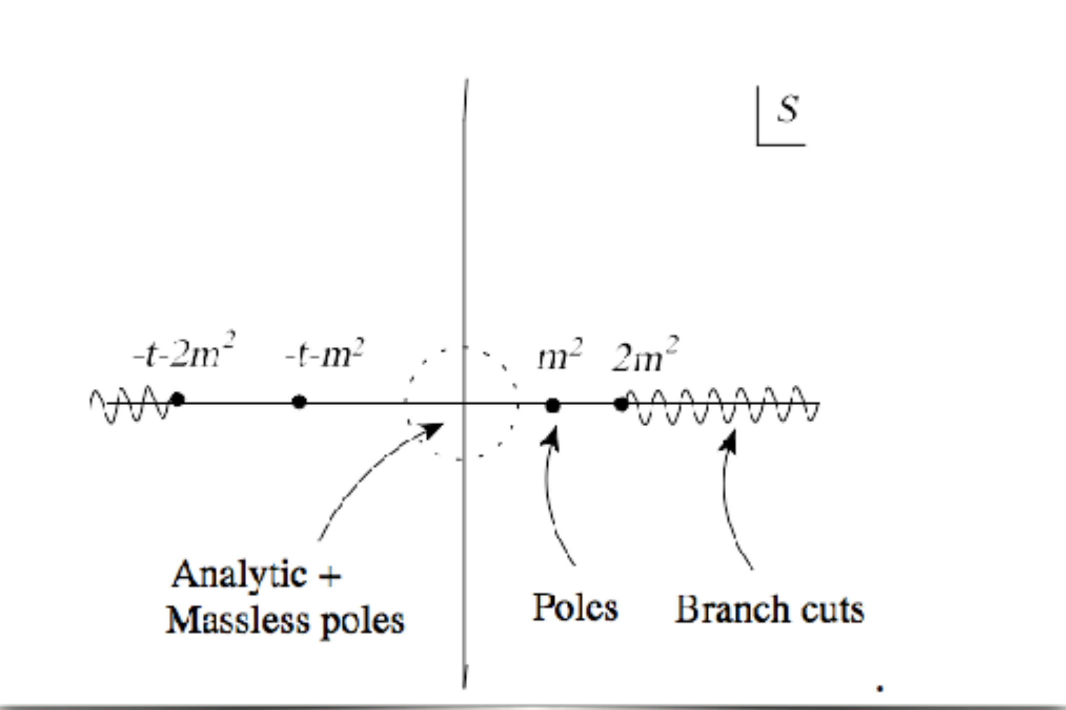
$$I = \frac{i}{2\pi} \int_{\infty} \frac{ds}{s^{n+1}} M(s,t)$$

From the discont

$$\left(\sum_a \frac{\mathsf{p}_a G^{\alpha}_{\ell_a}(1+2\frac{t}{m_a^2})}{(m_a^2)^{n+1}} + \sum_b \int ds' \mathsf{p}_{b,\ell}(s') \frac{G^{\alpha}_{\ell}(1+2\frac{t}{s'})}{(s')^{n+1}} + \{u\} \right)$$

CONVEX HULL

In the complex plane, the couplings for the higher dimension operators are related to the singularities and the discontinuity of its UV completion



$$I = \frac{i}{2\pi}\int_{\infty} \frac{ds}{s^{n+1}} M(s,t)$$

$$\sum_q g_{n+q,q} t^q = \left(\sum_a \frac{\mathsf{p}_a G^{\alpha}_{\ell_a}(1+2\frac{t}{m_a^2})}{(m_a^2)^{n+1}} + \sum_b \int ds' \mathsf{p}_{b,\ell}(s') \frac{G^{\alpha}_{\ell}(1+2\frac{t}{s'})}{(s')^{n+1}} + \{u\} \right).$$

CONVEX HULL

$$\sum_q g_{n+q,q} t^q = \left(\sum_a \frac{\mathsf{p}_a G^{\alpha}_{\ell_a}(1 + 2\frac{t}{m_a^2})}{(m_a^2)^{n+1}} + \sum_b \int ds' \mathsf{p}_{b,\ell}(s') \frac{G^{\alpha}_{\ell}(1 + 2\frac{t}{s'})}{(s')^{n+1}} + \{u\} \right).$$

Since we have no idea what the nature of the spectrum is, we do not assume anything, a continuous spectrum can be approximated by an infinite spectrum.

In other words, we are simply identifying

$$g_{k,q} = \sum_a \mathsf{p}_a \left[x_a^{k+1} v_{\ell_a,q} 2^q \right] \qquad x_a = \frac{1}{m_a^2} \qquad \mathfrak{p}_a > 0$$

where the vectors v come from the expansion of Gegenbauer (Legendre) polynomials.

CONVEX HULL

In other words, we are identifying

$$g_{k,q} = \sum_a \mathfrak{p}_a \left[x_a^{k+1} v_{\ell_a,q} 2^q \right]$$

$$x_a = \frac{1}{m_a^2}$$

with

$$\mathfrak{p}_a > 0$$

we have a convex hull!

	m^0	$\frac{1}{m^2}$	$\frac{1}{m^4}$	$\frac{1}{m^6}$	$\cdots$
t^0	$g_{0,0}$	$g_{1,0}$	$g_{2,0}$	$g_{3,0}$	$\cdots$
t^1		$g_{0,1}$	$g_{2,1}$	$g_{3,1}$	$\cdots$
t^2			$g_{2,2}$	$g_{3,2}$	$\cdots$
t^3				$g_{3,3}$	$\cdots$

$$\vec{g}_2 = \begin{pmatrix} g_{2,0} \\ g_{2,1} \\ g_{2,2} \end{pmatrix} \in \sum_a \mathfrak{p}'_a v_{\ell_a} \quad \mathfrak{p}'_a > 0$$

CONVEX HULL

$$\sum_q g_{n+q,q} t^q = \left(\sum_a \frac{\mathsf{p}_a G^{\alpha}_{\ell_a}(1+2\frac{t}{m_a^2})}{(m_a^2)^{n+1}} + \sum_b \int ds' \mathsf{p}_{b,\ell}(s') \frac{G^{\alpha}_{\ell}(1+2\frac{t}{s'})}{(s')^{n+1}} + \{u\} \right).$$

Put in another way, we are relating the low energy couplings to the expansion of

$$\begin{aligned} M(s,t) &= -\sum_a \mathsf{p}_a \frac{P_{\ell_a}\left(1+\frac{2t}{m_a^2}\right)}{s-m_a^2} \\ &= \sum_a \mathsf{p}_a \frac{1}{m_a^2} \left(1+\frac{s}{m_a^2}+\frac{s^2}{m_a^4}+\cdots\right)_{locality} \left(v_{\ell_a,0}+v_{\ell_a,1}\frac{t}{m_a^2}+v_{\ell_a,2}\frac{t^2}{m_a^4}\cdots\right)_{unitarity} \end{aligned}$$

We have that the coupling constants of higher dimensional operators must line in the convex hull of TWO geometries stemming from Locality and Unitarity

THE CYCLIC POLYTOPE

- Do we encounter convex hull problems physics?

We do! All the time! But it's useless since we always have infinite number of vectors

- Do we find a cyclic polytope?

YES!

THE CYCLIC POLYTOPE

Recall that EFT are bounded by two geometries

$$M(s,t) = -\sum_a p_a \frac{P_{\ell_a}\left(1+\frac{2t}{m_a^2}\right)}{s-m_a^2}$$

$$= \sum_a p_a \frac{1}{m_a^2}\left(1+\frac{s}{m_a^2}+\frac{s^2}{m_a^4}+\cdots\right)_{locality}\left(v_{\ell_a,0}+v_{\ell_a,1}\frac{t}{m_a^2}+v_{\ell_a,2}\frac{t^2}{m_a^4}\cdots\right)_{unitarity}$$

Let's consider the matrix consists of these Taylor vectors ordered with spin

$$(v_0^I, v_1^I, \cdots, v_8^I) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 6 & 10 & 15 & 21 & 28 \\ 0 & 0 & \frac{3}{2} & \frac{15}{2} & \frac{45}{2} & \frac{105}{2} & 105 & 189 \\ 0 & 0 & 0 & \frac{5}{2} & \frac{35}{2} & 70 & 210 & 525 \\ 0 & 0 & 0 & 0 & \frac{35}{8} & \frac{315}{8} & \frac{1575}{8} & \frac{5775}{8} \\ 0 & 0 & 0 & 0 & 0 & \frac{63}{8} & \frac{693}{8} & \frac{2079}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{231}{16} & \frac{3003}{16} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{429}{16} \end{pmatrix}.$$

All ordered determinants are positive!

$$det[v_{\ell_1} v_{\ell_2} \cdots] > 0, \quad \forall\, \ell_1 > \ell_2 > \cdots$$

THE CYCLIC POLYTOPE

Changing to external spinning states, correspond to operators of the form

R^2, R^2F, F^4, ..

one finds the same!

$$h{=}1: \begin{pmatrix} \frac{1}{4} & \frac{5}{4} & \frac{15}{4} & \frac{35}{4} & \frac{35}{2} & \frac{63}{2} & \frac{105}{2} & \frac{165}{2} \\ 0 & \frac{3}{4} & \frac{21}{4} & 21 & 63 & \frac{315}{2} & \frac{693}{2} & 693 \\ 0 & 0 & \frac{7}{4} & \frac{63}{4} & \frac{315}{4} & \frac{1155}{4} & \frac{3465}{4} & \frac{9009}{4} \\ 0 & 0 & 0 & 15 & \frac{165}{4} & \frac{495}{2} & \frac{2145}{2} & \frac{15015}{4} \\ 0 & 0 & 0 & 0 & \frac{495}{64} & \frac{6435}{64} & \frac{45045}{64} & \frac{225225}{64} \\ 0 & 0 & 0 & 0 & 0 & \frac{1001}{64} & \frac{15015}{64} & \frac{15015}{8} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1001}{32} & \frac{17017}{32} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1989}{32} \end{pmatrix} \qquad h{=}2: \begin{pmatrix} \frac{1}{16} & \frac{9}{16} & \frac{45}{16} & \frac{165}{16} & \frac{495}{16} & \frac{1287}{16} & \frac{3003}{16} & \frac{6435}{16} \\ 0 & \frac{5}{16} & \frac{55}{16} & \frac{165}{8} & \frac{715}{8} & \frac{5005}{16} & \frac{15015}{16} & \frac{5005}{2} \\ 0 & 0 & \frac{33}{32} & \frac{429}{32} & \frac{3003}{32} & \frac{15015}{32} & \frac{15015}{8} & \frac{51051}{8} \\ 0 & 0 & 0 & \frac{91}{32} & \frac{1365}{32} & \frac{1365}{4} & \frac{7735}{4} & \frac{69615}{8} \\ 0 & 0 & 0 & 0 & \frac{455}{64} & \frac{7735}{64} & \frac{69615}{64} & \frac{440895}{64} \\ 0 & 0 & 0 & 0 & 0 & \frac{1071}{64} & \frac{20349}{64} & \frac{101745}{32} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{4845}{128} & \frac{101745}{128} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{10659}{128} \end{pmatrix}$$

All ordered determinants are positive!

$$det[v_{\ell_1} v_{\ell_2} \cdots] > 0, \quad \forall\, \ell_1 > \ell_2 > \cdots$$

THE CYCLIC POLYTOPE

Recall that EFT are bounded by two geometries

$$M(s,t) = -\sum_a p_a \frac{P_{\ell_a}\left(1+\frac{2t}{m_a^2}\right)}{s-m_a^2}$$

$$= \sum_a p_a \frac{1}{m_a^2}\left(1+\frac{s}{m_a^2}+\frac{s^2}{m_a^4}+\cdots\right)_{locality}\left(v_{\ell_a,0}+v_{\ell_a,1}\frac{t}{m_a^2}+v_{\ell_a,2}\frac{t^2}{m_a^4}\cdots\right)_{unitarity}$$

Consider fixed mass dimension

	m^0	$\frac{1}{m^2}$	$\frac{1}{m^4}$	$\frac{1}{m^6}$	$\cdots$
t^0	$g_{0,0}$	$g_{1,0}$	$g_{2,0}$	$g_{3,0}$	$\cdots$
t^1		$g_{0,1}$	$g_{2,1}$	$g_{3,1}$	$\cdots$
t^2			$g_{2,2}$	$g_{3,2}$	$\cdots$
t^3				$g_{3,3}$	$\cdots$

$$\vec{g}_2 = \begin{pmatrix} g_{2,0} \\ g_{2,1} \\ g_{2,2} \end{pmatrix} \in \sum_a p'_a v_{\ell_a} \quad p'_a > 0$$

THE CYCLIC POLYTOPE

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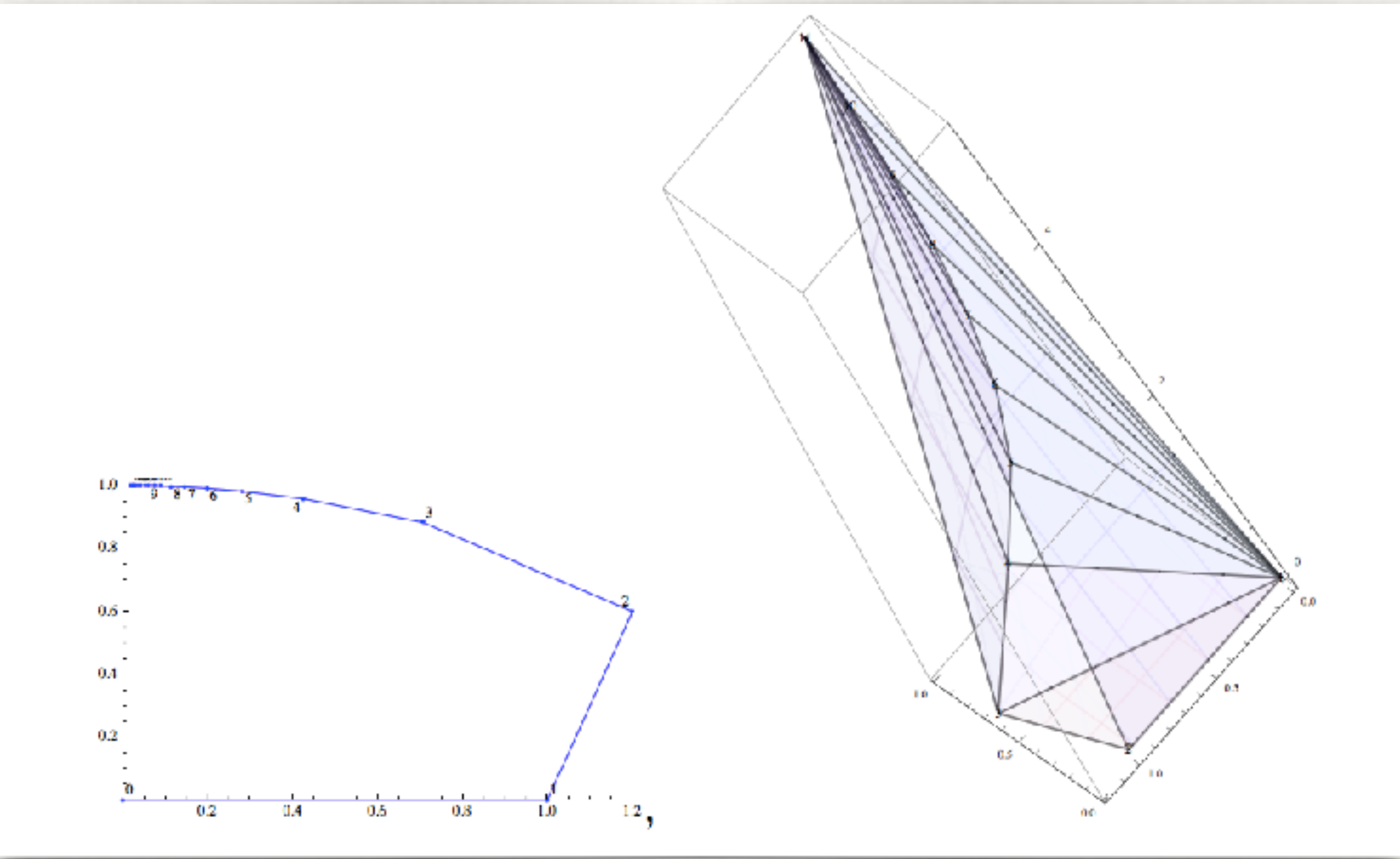
	m^0	$\frac{1}{m^2}$	$\frac{1}{m^4}$	$\frac{1}{m^6}$	$\cdots$
t^0	$g_{0,0}$	$g_{1,0}$	$g_{2,0}$	$g_{3,0}$	$\cdots$
t^1		$g_{1,1}$	$g_{2,1}$	$g_{3,1}$	$\cdots$
t^2			$g_{2,2}$	$g_{3,2}$	$\cdots$
t^3				$g_{3,3}$	$\cdots$

$$\vec{g}_2 = \begin{pmatrix} g_{2,0} \\ g_{2,1} \\ g_{2,2} \end{pmatrix} \rightarrow Det[\vec{g}_2, v_\ell, v_{\ell+1}] > 0$$

THE MAGIC OF CYCLIC POLYTOPE

Since we have cyclic polytopes, we can now fully explore the geometry since we TRIVIALLY KNOW where all the boundaries are!



Gives bounds for the coupling of 4 and 6 derivative higher dimension operators

THE MOMENT CURVE

Recall that EFT are bounded by two geometries

$$M(s,t) = -\sum_a p_a \frac{P_{\ell_a}\left(1+\frac{2t}{m_a^2}\right)}{s-m_a^2}$$

$$= \sum_a p_a \frac{1}{m_a^2}\left(1+\frac{s}{m_a^2}+\frac{s^2}{m_a^4}+\cdots\right)_{locality}\left(v_{\ell_a,0}+v_{\ell_a,1}\frac{t}{m_a^2}+v_{\ell_a,2}\frac{t^2}{m_a^4}\cdots\right)_{unitarity}$$

Consider fixed degree in angles

	m^0	$\frac{1}{m^2}$	$\frac{1}{m^4}$	$\frac{1}{m^6}$	$\cdots$
t^0	$g_{0,0}$	$g_{1,0}$	$g_{2,0}$	$g_{3,0}$	$\cdots$
t^1		$g_{1,1}$	$g_{2,1}$	$g_{3,1}$	$\cdots$
t^2			$g_{2,2}$	$g_{3,2}$	$\cdots$
t^3				$g_{3,3}$	$\cdots$

$$\begin{pmatrix} g_{1,1} \\ g_{2,1} \\ g_{3,1} \end{pmatrix} \in \sum_a p'_a \begin{pmatrix} \frac{1}{m_a^2} \\ \frac{1}{m_a^4} \\ \frac{1}{m_a^6} \end{pmatrix} \qquad p'_a > 0$$

$$(1, x, x^2, \cdots, x^a), \quad x \in R^+$$

The convex hull of points on a moment curve

THE MOMENT CURVE

$$(1, x, x^2, \cdots, x^a), \quad x \in R^+$$

Organizing the couplings for fixed t power into the Hankel matrix ($g'_k \equiv g_{k,i}$)

$$K(g') = \begin{pmatrix} 1 & g'_1 & \cdots & g'_{p-1} \\ g'_1 & g'_2 & \cdots & g'_p \\ \vdots & \vdots & \vdots & \vdots \\ g'_{p-1} & g'_p & \cdots & g'_{2p-2} \end{pmatrix},$$

If $\{g'_i\}$ lies in the convex hull of half-moment curves, then all minors of $K[g']$ is positive!

$$i \in even: \quad Det \begin{pmatrix} 1 & g'_1 & \cdots & g'_{\frac{i}{2}} \\ g'_1 & g'_2 & \cdots & g'_{\frac{i}{2}+1} \\ \vdots & \vdots & \vdots & \vdots \\ g'_{\frac{i}{2}} & g'_{\frac{i}{2}+1} & \cdots & g'_i \end{pmatrix} \geq 0, \qquad i \in odd: \quad Det \begin{pmatrix} g'_1 & g'_2 & \cdots & g'_{\frac{i+1}{2}} \\ g'_2 & g'_3 & \cdots & g'_{\frac{i+3}{2}} \\ \vdots & \vdots & \vdots & \vdots \\ g'_{\frac{i+1}{2}} & g'_{\frac{i+3}{2}} & \cdots & g'_i \end{pmatrix} \geq 0$$

IMPLICATIONS I

Lets consider the standard model. In the IR we only have U(1) + gravity. Lets compactly to 3 dimensions. The EFT for integrating out the massive states are

$$\Gamma = \int \mathrm{d}^3x\sqrt{-g}\left[\frac{M_3}{2}R - \frac{1}{4}\sum_i F_i^2\right] + \text{C.S.} + \text{H.O.}$$

where the H.O. are in terms of field strengths, starting with

$$\text{H.O.} = \sum_{i,j,k,l} c_{ijkl}(F_i \cdot F_j)(F_k \cdot F_l)$$

with the coefficients parametrized as:

$$C_{ijkl} \sim \mathcal{O}(z^4) + \mathcal{O}(z^2) + \mathcal{O}(z^0)$$ with $$z_{ai} \equiv \frac{q_{ai}g_i\sqrt{M_3}}{|m_a|}$$

The all order coefficients can be computed from the one-loop four point massive amplitude

IMPLICATIONS I

Lets consider in 3 dimensions where in the IR we only have U(1) + gravity (compactify standard model!) The EFT for integrating out the massive states are

$$\Gamma = \int \mathrm{d}^3x\sqrt{-g}\left[\frac{M_3}{2}R - \frac{1}{4}\sum_i F_i^2\right] + \text{C.S.} + \text{H.O.}$$

where the H.O. are in terms of field strengths, starting with

$$\text{H.O.} = \sum_{i,j,k,l} c_{ijkl}(F_i \cdot F_j)(F_k \cdot F_l)$$

with the coefficients parametrized as: (Andriolo, Junghans, Noumi, Shiu)

$$\sum_a \frac{1}{1920\pi|m_a|M_3^2}\cdot\begin{cases}\left[\frac{7}{8}z_{ai}z_{aj}z_{ak}z_{al} + \frac{3}{2}z_{ai}z_{aj}\delta_{kl} - z_{ai}z_{ak}\delta_{jl}\right.\\ \qquad\left. + \frac{1}{2}\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl}\right] & \text{(scalars)}\\ \left[z_{ai}z_{aj}z_{ak}z_{al} + z_{ai}z_{aj}\delta_{kl} - \frac{3}{2}z_{ai}z_{ak}\delta_{jl}\right.\\ \qquad\left. - \frac{1}{2}\delta_{ij}\delta_{kl} + \frac{3}{2}\delta_{ik}\delta_{jl}\right] & \text{(fermions)}\,.\end{cases}$$

with

$$z_{ai} \equiv \frac{q_{ai}g_i\sqrt{M_3}}{|m_a|}$$

IMPLICATIONS I

1. The dominant contribution is for the largest z (for charged states) and the lightest m (for neutral states).

$$\sum_a \frac{1}{1920\pi|m_a|M_3^2}\cdot\begin{cases}\left[\frac{7}{8}z_{ai}z_{aj}z_{ak}z_{al}+\frac{3}{2}z_{ai}z_{aj}\delta_{kl}-z_{ai}z_{ak}\delta_{jl}\right.\\ \qquad\left.+\frac{1}{2}\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}\right] & \text{(scalars)}\\ \left[z_{ai}z_{aj}z_{ak}z_{al}+z_{ai}z_{aj}\delta_{kl}-\frac{3}{2}z_{ai}z_{ak}\delta_{jl}\right.\\ \qquad\left.-\frac{1}{2}\delta_{ij}\delta_{kl}+\frac{3}{2}\delta_{ik}\delta_{jl}\right] & \text{(fermions)}\,.\end{cases}$$

In the standard model electron has the largest z~10^22

2. We can get compute the coefficient to all order in derivatives from the 1-loop amp

$$\begin{aligned}&\frac{3840}{\pi}A_4^{one-loop}\Big|_{m\to\infty}\\ =\;& e^4\left[\frac{7(s^2+st+t^2)}{8m^5}+\frac{75stu}{448m^7}+\frac{7(s^2+st+t^2)^2}{192m^9}+\dots\right]+\\ &+e^2\left[\frac{10(s^2+st+t^2)^2}{mstu}+\frac{s^2+st+t^2}{2m^3}+\frac{51stu}{112m^5}\right.\\ &\left.+\frac{5(s^2+st+t^2)^2}{112m^7}+\frac{53stu(s^2+st+t^2)}{2112m^9}+\dots\right]\\ &+\left[\frac{3(s^2+st+t^2)}{2m}+\frac{3stu}{28m^3}+\frac{(s^2+st+t^2)^2}{56m^5}+\frac{(5stu(s^2+st+t^2)}{616m^7}\right.\\ &\left.+\frac{5(5s^6+15s^5t+37s^4t^2+49s^3t^3+37s^2t^4+15st^5+5t^6)}{36608m^9}+\dots\right],\end{aligned}$$

IMPLICATIONS I

Recall that being inside the convex hull of moment curves imply:

$$i \in even: \quad \text{Det}\begin{bmatrix} g_0 & g_1 & \cdots & g_{\frac{i}{2}} \\ g_1 & g_2 & \cdots & g_{\frac{i}{2}+1} \\ \vdots & \vdots & \vdots & \vdots \\ g_{\frac{i}{2}} & g_{\frac{i}{2}+1} & \cdots & g_i \end{bmatrix} \geq 0, \qquad i \in odd: \quad \text{Det}\begin{bmatrix} g_1 & g_2 & \cdots & g_{\frac{i+1}{2}} \\ g_2 & g_3 & \cdots & g_{\frac{i+3}{2}} \\ \vdots & \vdots & \vdots & \vdots \\ g_{\frac{i+1}{2}} & g_{\frac{i+3}{2}} & \cdots & g_i \end{bmatrix} \geq 0$$

This puts constraint on Z

```
In[100]:= For[l = 1, l ≤ 2, l++, M = Table[Table[List4[[i]], {i, 1 + j, 2 + l + j}], {j, 0, l + 1}];
   Print[M // MatrixForm]; Print[{Reduce[(Det[M] /. {z → 1}) > 0] // N, l}]]
```

$$\begin{pmatrix} \frac{7z^4}{30720} + \frac{3z^2}{7680} & \frac{1}{215040} + \frac{z^2}{86016} + \frac{7z^4}{737280} & \frac{5}{28114944} - \frac{5z^2}{9371648} + \frac{43z^4}{95420416} \\ \frac{1}{215040} - \frac{z^2}{86016} + \frac{7z^4}{737280} & \frac{5}{28114944} + \frac{5z^2}{9371648} + \frac{43z^4}{95420416} & \frac{7}{869007360} + \frac{7z^2}{267386880} + \frac{73z^4}{3208642560} \\ \frac{5}{28114944} + \frac{5z^2}{9371648} + \frac{43z^4}{95420416} & \frac{7}{869007360} + \frac{7z^2}{267386880} - \frac{73z^4}{3208642560} & \frac{15}{37933285376} + \frac{3z^2}{2231369728} + \frac{37z^4}{31004295168} \end{pmatrix}$$

```
{Z < -3.68657 || -1.36709 < Z < 1.36709 || Z > 3.68657, 1}
```

$$\begin{pmatrix} \frac{7z^4}{30720} + \frac{3z^2}{7680} & \frac{1}{215040} + \frac{z^2}{86016} + \frac{7z^4}{737280} & \frac{5}{28114944} + \frac{5z^2}{9371648} + \frac{43z^4}{95420416} & \frac{7}{869007360} + \frac{7z^2}{267386880} - \frac{73z^4}{3208642560} \\ \frac{1}{215040} + \frac{z^2}{86016} + \frac{7z^4}{737280} & \frac{5}{28114944} + \frac{5z^2}{9371648} + \frac{43z^4}{95420416} & \frac{7}{869007360} + \frac{7z^2}{267386880} + \frac{73z^4}{3208642560} & \frac{15}{37933285376} + \frac{3z^2}{2231369728} + \frac{37z^4}{31004295168} \\ \frac{5}{28114944} + \frac{5z^2}{9371648} + \frac{43z^4}{95420416} & \frac{7}{869007360} + \frac{7z^2}{267386880} + \frac{73z^4}{3208642560} & \frac{15}{37933285376} + \frac{3z^2}{2231369728} + \frac{37z^4}{31004295168} & \frac{11}{540226355200} + \frac{11z^2}{154350387200} + \frac{157z^4}{2442762649600} \\ \frac{7}{869007360} + \frac{7z^2}{267386880} + \frac{73z^4}{3208642560} & \frac{15}{37933285376} + \frac{3z^2}{2231369728} + \frac{37z^4}{31004295168} & \frac{11}{540226355200} + \frac{11z^2}{154350387200} + \frac{157z^4}{2442762649600} & \frac{91}{84073984819200} + \frac{13z^2}{3362959392768} + \frac{211z^4}{59785944760320} \end{pmatrix}$$

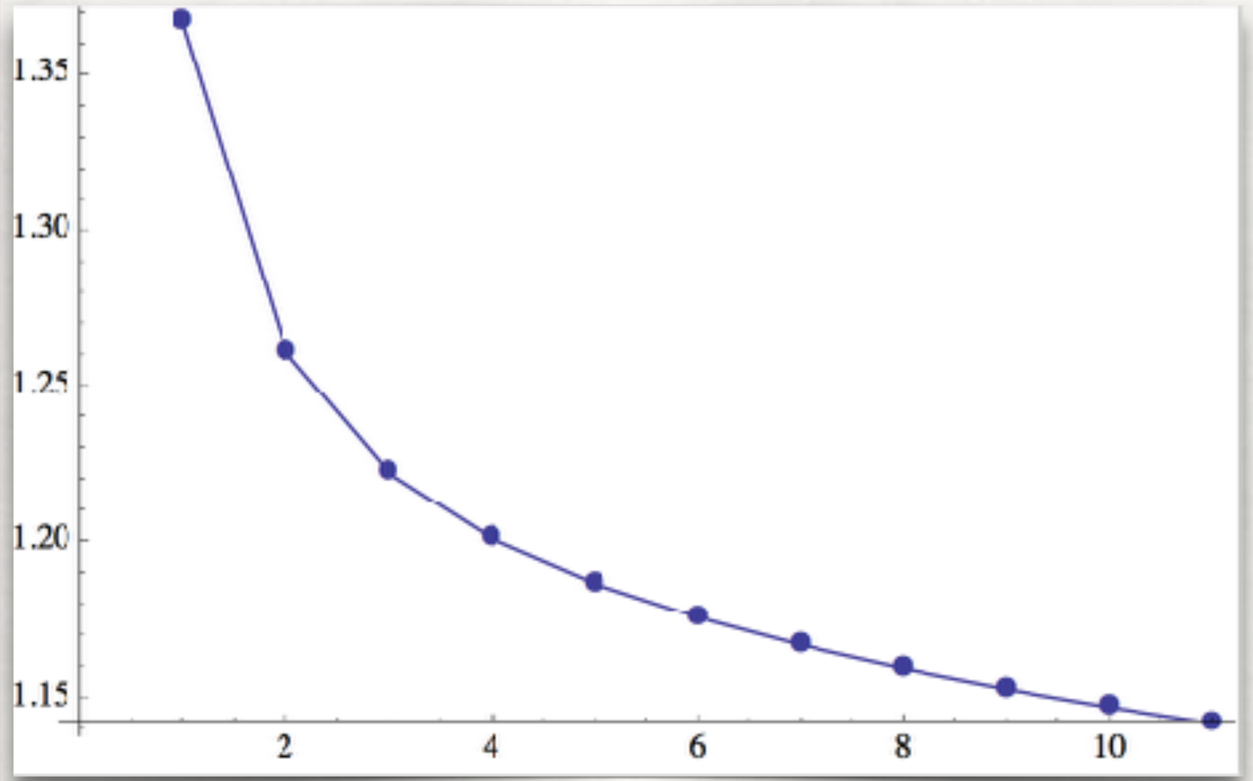
```
{Z < -5.36882 || -1.26089 < Z < 1.26089 || Z > 5.36882, 2}
```

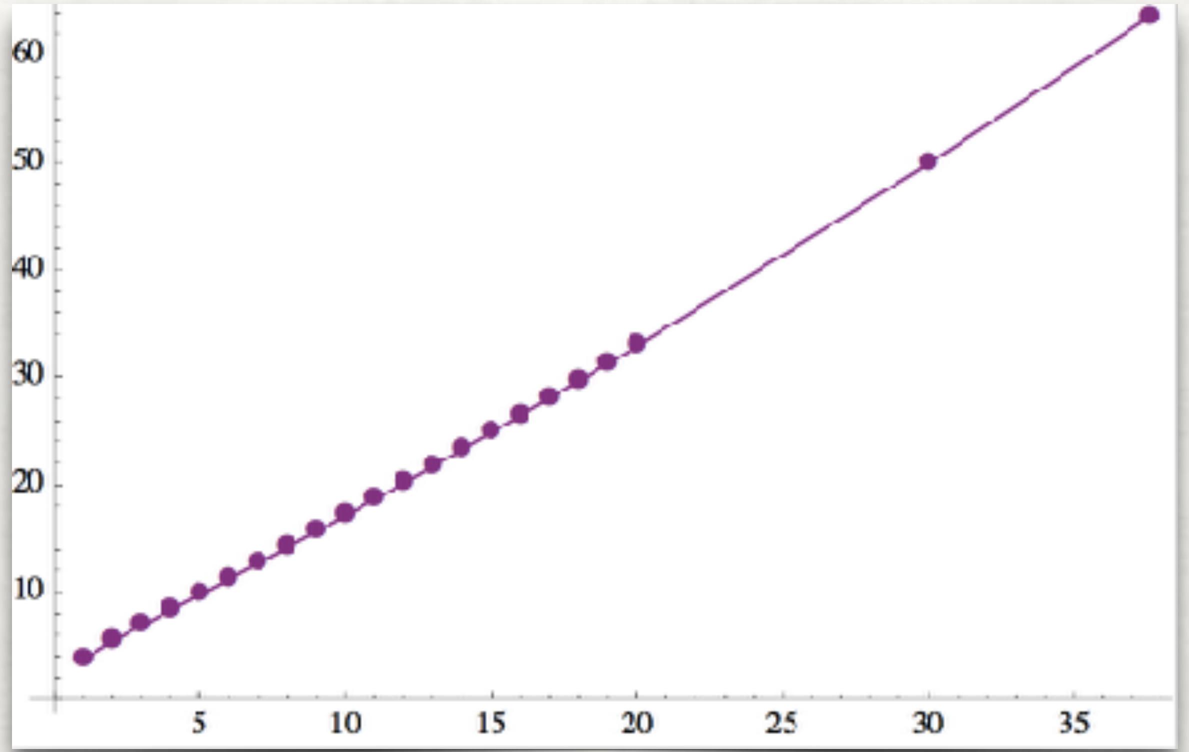
IMPLICATIONS I

We find (at 60 derivative order)

0<z<1.14 (a) and 124.28 <z (b)

However





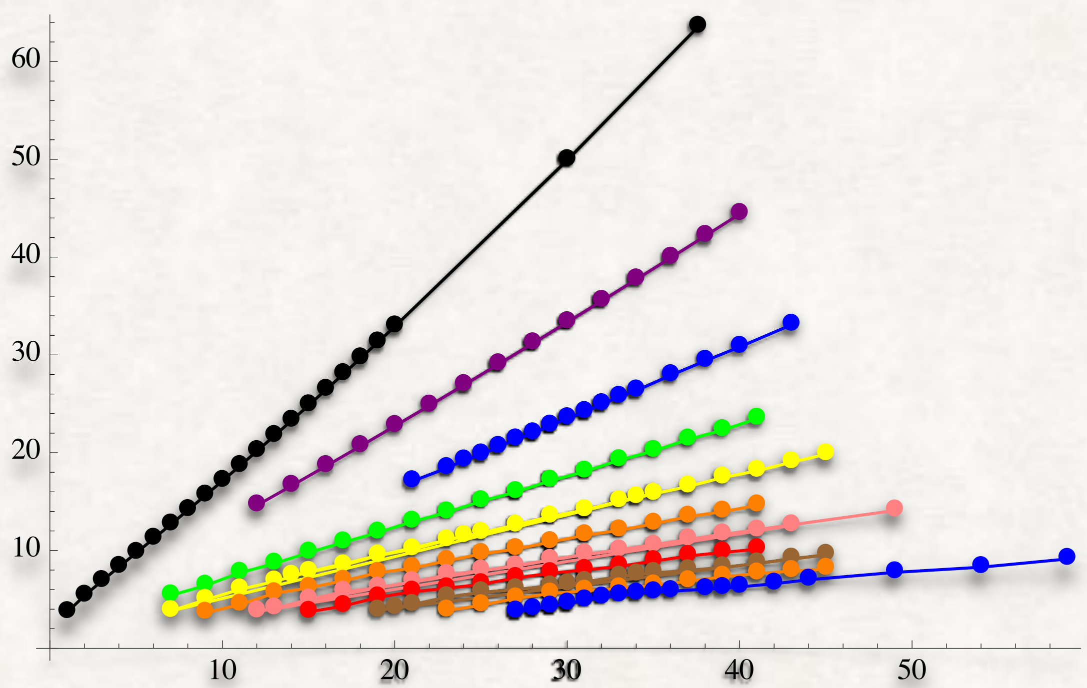
While a asymptotes, b is linearly rising. If the coefficients are dominated by the state with z>1.14 it is inconsistent! With electron giving z~10^22 we must have extremely light neutral states!

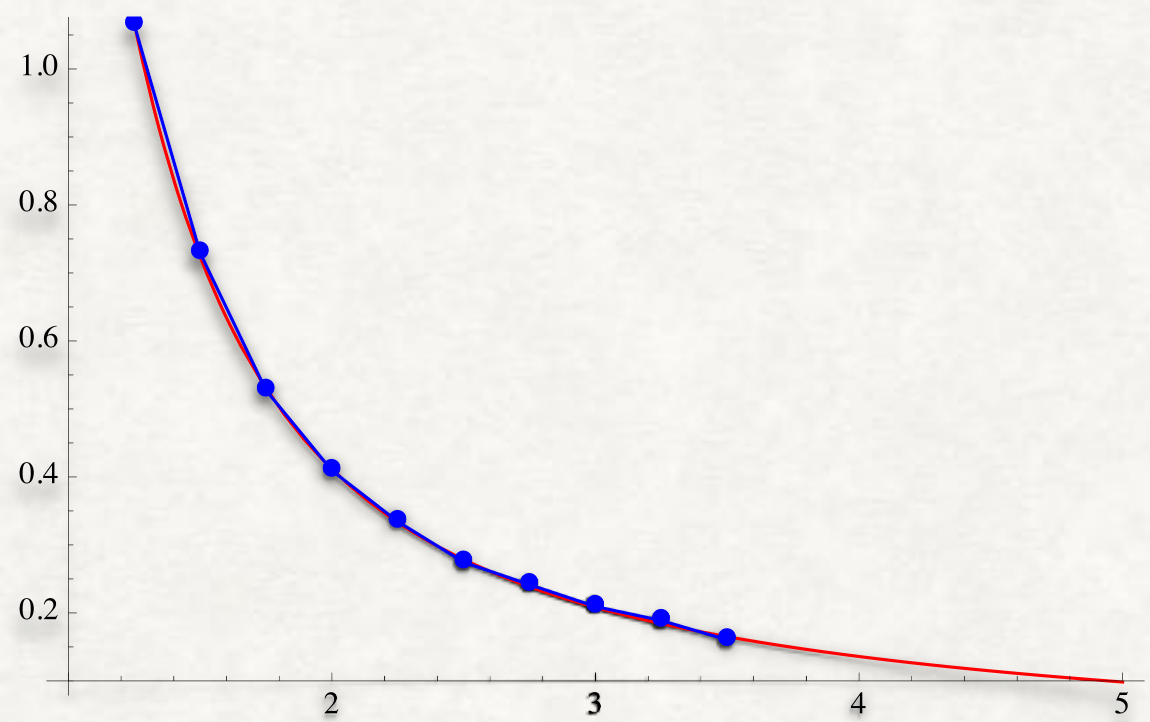
IMPLICATIONS I

The inclusion of light states imply that the z independent part of the coefficients are replaced by

$$\frac{1}{m^n M_{pl}^3}\left(\alpha_1 z^4+\alpha_2 z^2+\alpha_3\right) \to \frac{1}{m_e^n M_{pl}^3}\left(\alpha_1 z^4+\alpha_2 z^2+(1+\beta^n)\alpha_3\right) \quad \beta=\frac{m_e}{m_0}$$

We can plot the slope of b with respect to different **β**

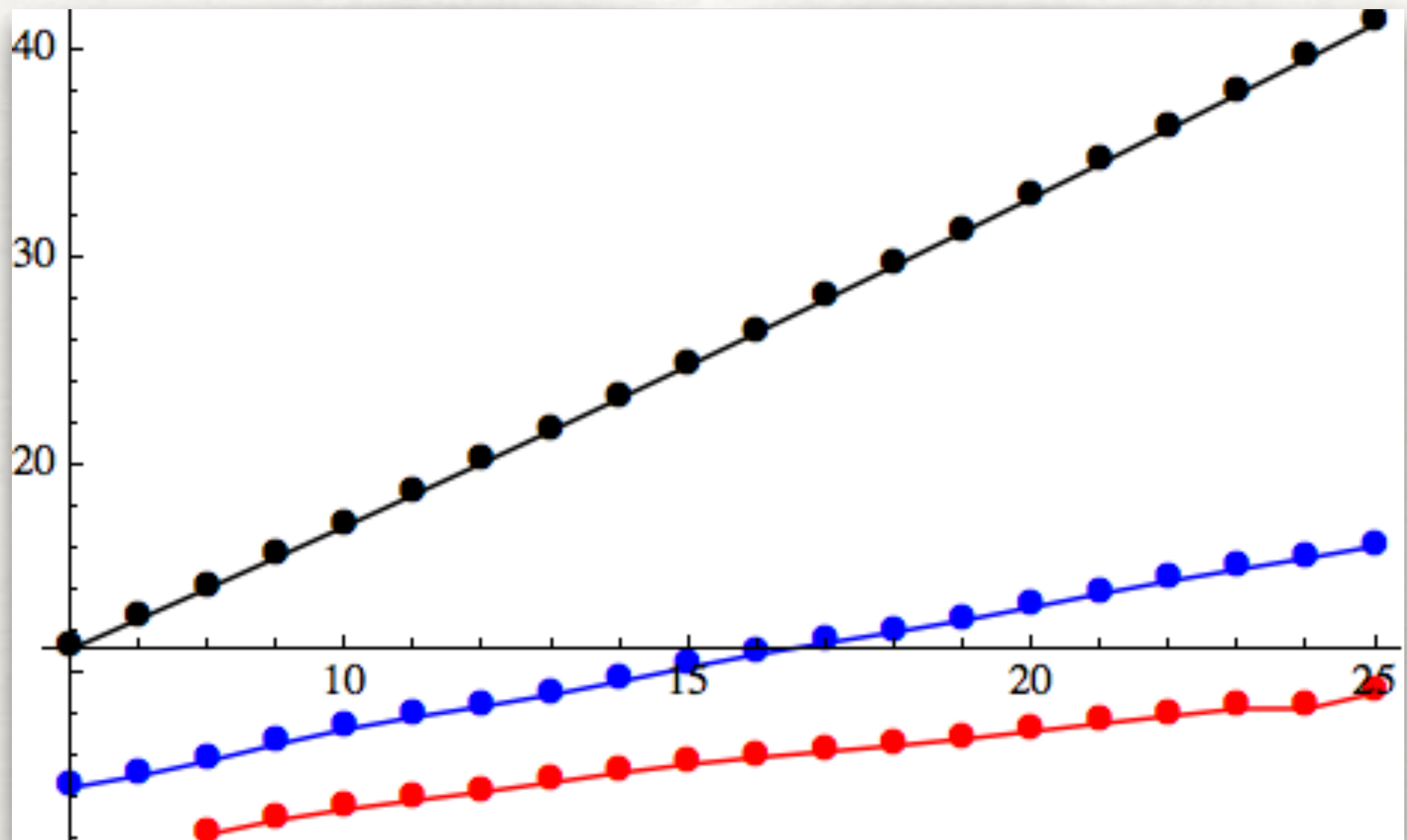




Find the critical **β** such that the slope vanishes gives us the upper bound of the massless states

IMPLICATIONS I

Adding nearly lying charged states with comparable z also reduces the slope



IMPLICATIONS I

In three dimensions if we have a charged state with z>>1 then

- We must have neutral states with **β**>1

or

- We must have closely lying charged states

IMPLICATIONS II

We can also begin to carve out the string landscape in an on-shell fashion!

Consider a string compactification where the space is

$$R^4 \otimes M_6$$

The world-sheet CFT is given by a product of free boson and fermions and a compact CFT. This implies the following monodromy relations

$$A\,(2134) + e^{i\pi(\alpha' k_1 \cdot k_2 + a_{12})} A\,(1234) + e^{i\pi\left(\alpha' k_1 \cdot k_2 + \alpha' k_2 \cdot k_3 + a_{12} + a_{23}\right)} A\,(1324) = 0$$

Low energy consistency sets all constants to zero

$$A\,(2134) + e^{i\pi\alpha' k_1 \cdot k_2} A\,(1234) + e^{i\pi\alpha'(k_1 \cdot k_2 + k_2 \cdot k_3)} A\,(1234) = 0$$

IMPLICATIONS II

We can also begin to carve out the string landscape in an on-shell fashion!

$$A\,(2134) + e^{i\pi\alpha' k_1\cdot k_2} A\,(1234) + e^{i\pi\alpha' (k_1\cdot k_2 + k_2\cdot k_3)} A\,(1234) = 0$$

This implies further constraint in the space of coupling constants! For exp:

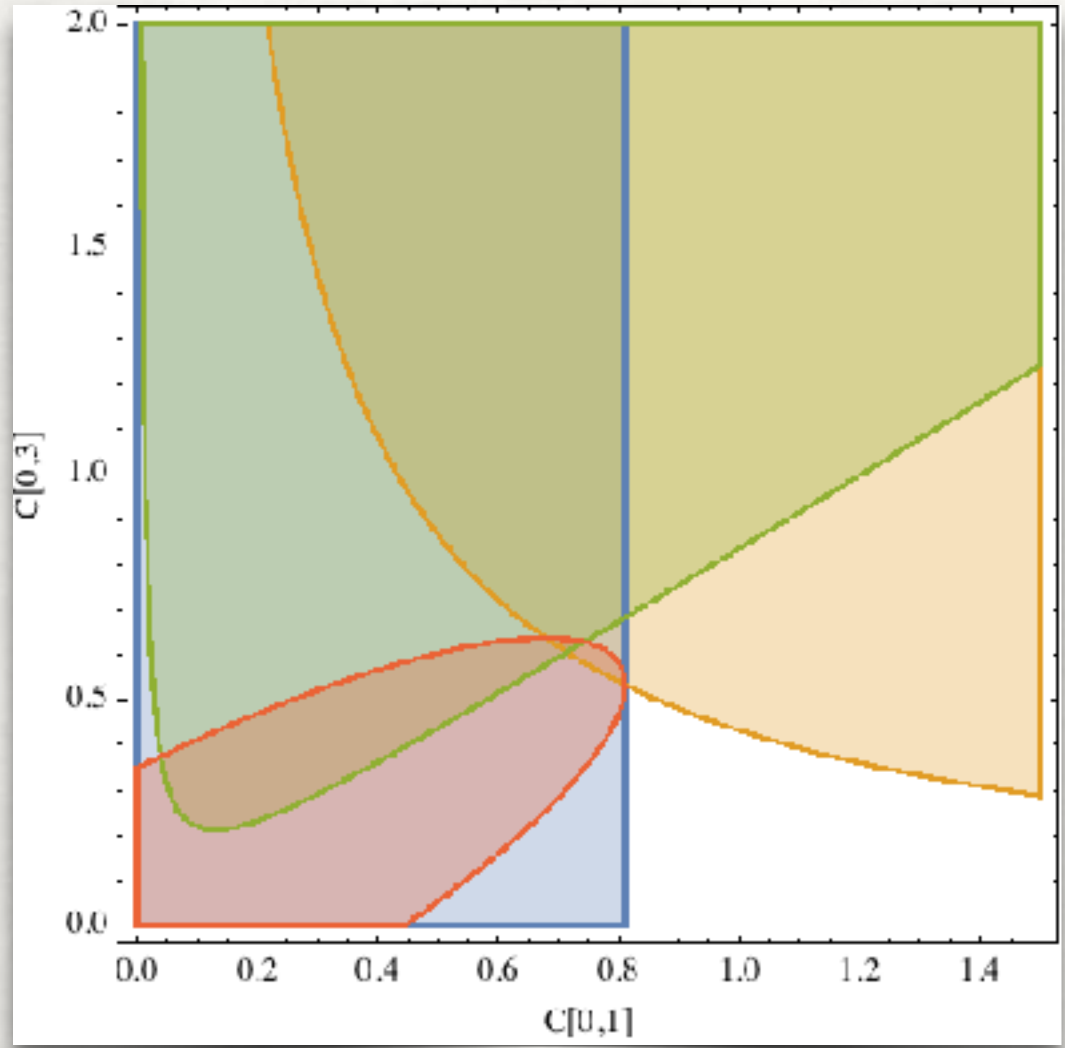
$$\begin{pmatrix} g_{-1,0} & g_{-1,-1} & & & \\ g_{00} & & & & \\ g_{1,0} & g_{1,1} & & & \\ g_{2,0} & g_{2,1} & g_{2,2} & & \\ g_{3,0} & g_{3,1} & g_{3,2} & g_{3,3} & \\ g_{4,0} & g_{4,1} & g_{4,2} & g_{4.3} & g_{4,4} \end{pmatrix} = \begin{pmatrix} 0 & 0 & & & \\ \zeta\,(2) & & & & \\ g_{1,0} & g_{1,0} & & & \\ \zeta\,(4) & \frac{\pi^4}{360} & \zeta\,(4) & & \\ g_{3,0} & 2g_{3,0} - \zeta\,(2)\, g_{1,0} & 2g_{3,0} - \zeta\,(2)\, g_{1,0} & g_{3,0} & \\ \zeta\,(6) & g_{4,1} & -\frac{\pi^6}{15120} + 2g_{4,1} & g_{4,1} & \zeta\,(6) \end{pmatrix}$$

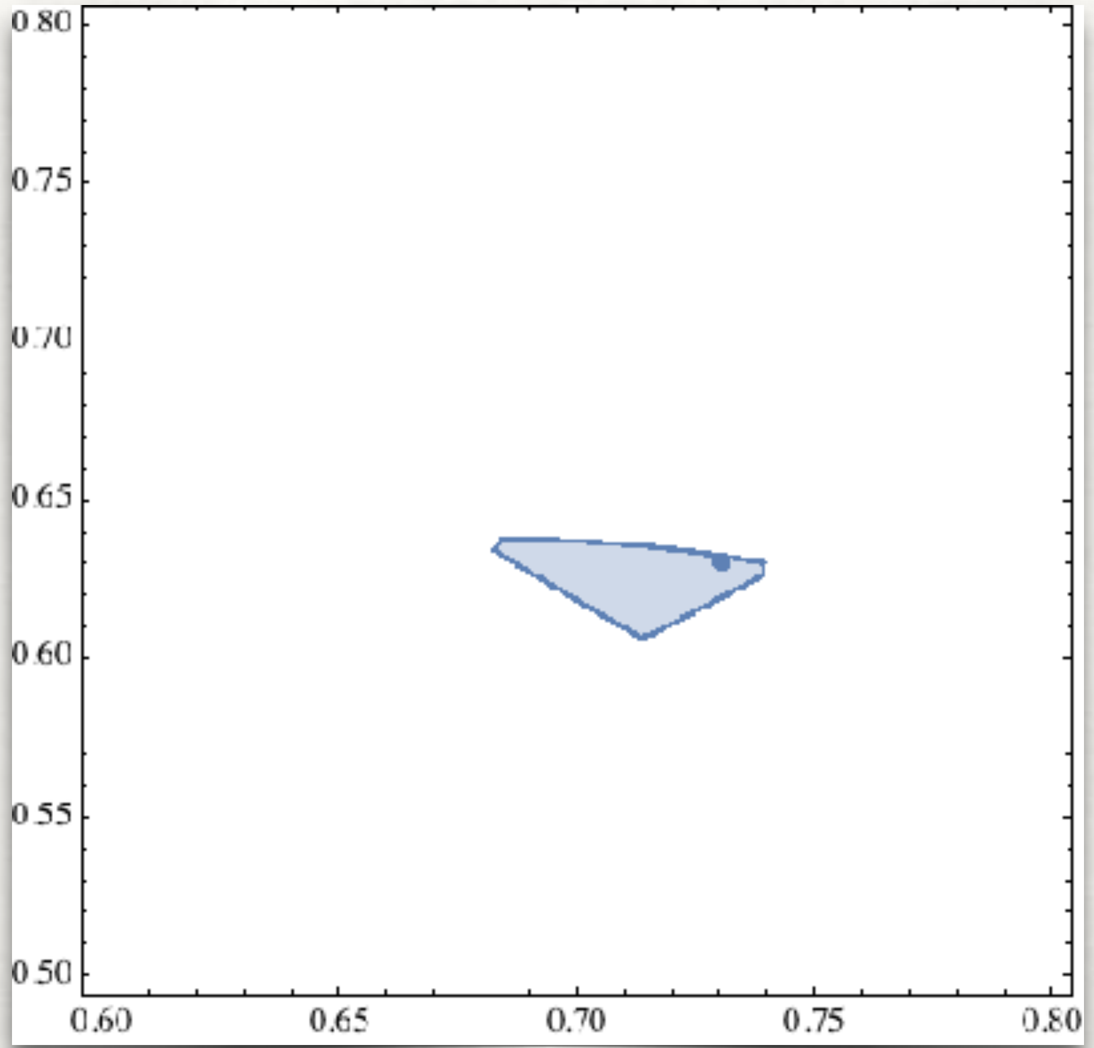
IMPLICATIONS II

We can also begin to carve out the string landscape in an on-shell fashion!

$$A\left(2134\right)+e^{i\pi\alpha' k_1\cdot k_2}A\left(1234\right)+e^{i\pi\alpha'(k_1\cdot k_2+k_2\cdot k_3)}A\left(1234\right)=0$$

This implies further constraint in the space of coupling constants! For exp:





SUMMARY

- **The union of physical principles (unitarity, locality, symmetries) in the UV, defines an IR avatar in the form of Positive geometries.**
- **Such geometries imposes constraint on the space of coupling constants in the EFT**
- **Examples: charge to mass ratios in 3D, tentative carving out the string landscape**
- **Extend the analysis for high order in derivatives, multi U(1)s, grave-photon mixed couplings**
- **IS there much more positivity out there?**
- **(YES!) Present for any planar Ising networks (see Pavel Galashin, Pavlo Pylyavskyy 1807.03282 Y-t H, Chia-Kai Kuo, Congkao Wen 1809.01231)**
- **(YES!) The geometry of CFT bootstrap (see tomorrow on arxiv :Nima Arkani-Hamed, Y-t H, Shu-Heng-Shao)**
- **(YES?)**