Schwinger Effect in Inflaton-Driven Electric Field

Hiroyuki Kitamoto（NCTS）

Based on Phys. Rev. D98 (2018) 103512

[arXiv:1807.03753]

Introduction
（No-go of anisotropic inflation）

- Concerning the primordial universe, we find no significant evidence for violation of rotational symmetry from the current status of cosmic microwave background observations ’13 J. Kim, E. Komatsu, ’18 Planck Collaboration

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}+\mathbf{k}') P(\mathbf{k})$$

$$P(\mathbf{k}) = P_0(|\mathbf{k}|) \left\{ 1 + g_* (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}})^2 \right\}, \quad |g_*| \lesssim 10^{-2}$$

$\hat{\mathbf{n}}$: preferred direction

- From a theoretical viewpoint, an anisotropic inflation can be obtained if an U(1) gauge field has a classical value like an inflaton

$$A_i \neq 0$$

- In fact, if the gauge field respects the conformal symmetry as its kinetic term is canonical, the electromagnetic field decays with the cosmic expansion and then there is no anisotropic hair

Introduction
（Model with a canonical kinetic term）

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2) \qquad H \equiv \frac{1}{a}\frac{da}{dt} \simeq \text{const.}$$

$$S_{\text{gauge}} = \int \sqrt{-g} d^4x \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}\right] = \int d^4x \left[-\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma} F_{\mu\nu}F_{\rho\sigma}\right]$$

conformal symmetry

$$\Rightarrow \quad \frac{d^2}{d\tau^2} A = 0 \Leftrightarrow \frac{d}{d\tau} A = \text{const.}$$

temporal gauge: $A_0 = 0$

homogeneity: $A_i = A(\tau)\delta_i{}^1$

$$\Rightarrow \quad E_{\text{phys}} = -a^{-2}\frac{d}{d\tau}A \propto a^{-2}$$

The electric field decays with the cosmic expasion $\Rightarrow$ Isotropic inflation

If the conformal symmetry is broken, this discussion does not hold true

Introduction
（Model with a dilatonic coupling）

'09, '10 M. Watanabe,
S. Kanno, J. Soda

$$S_{\rm bg} = \int \sqrt{-g} d^4x \left[\frac{M_{\rm pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) - \frac{1}{4} \underline{f^2(\varphi)} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right]$$

Solving the classical field eqs. by use of the ansatz: $f(\varphi) = \exp\left\{ \frac{2c}{M_{\rm pl}^2} \int d\varphi \; \frac{V}{\partial_\varphi V} \right\}$,

$$f = \left(a^{-4} + q a^{-4c}\right)^{\frac{1}{2}} \;\rightarrow a^{-2} \quad \text{for } c > 1 \qquad q\text{: integration const.}$$

$$E_{\rm phys} = -f a^{-1} \frac{d}{dt} A = \underline{f^{-1}} a^{-2} E \;\rightarrow E \qquad E = \frac{\sqrt{3\epsilon_V (c-1)}}{c} M_{\rm pl} H$$

$$\epsilon_V \equiv \frac{1}{2} \left(\frac{M_{\rm pl} \partial_\varphi V}{V} \right)^2$$

we obtain a persistent electric field (inflaton-driven electric field)

$c - 1 \lesssim 10^{-7}$ to satisfy the observational bound $g_* = 24 \frac{c-1}{c} N^2 \lesssim 10^{-2}$

Motivation

- We consider the case that a charged test scalar field exists

$$S_{\rm test} = \int \sqrt{-g} d^4x \ \left[-g^{\mu\nu}(\partial_\mu + ieA_\mu)\phi^*(\partial_\nu - ieA_\nu)\phi - m^2\phi^*\phi \right]$$

- A strong electric field leads to the pair production of charged particles (Schwinger effect), and the pair production induces the U(1) current

$$\tilde{j} = 2e \int \frac{d^3k}{(2\pi)^3} \ v_{\mathbf{k}} n_{\mathbf{k}}$$

$n_{\mathbf{k}}$: particle number

$v_{\mathbf{k}}$: velocity of particle

- It is reasonable to conjecture that if we take into account the Schwinger effect, the induced current screens the inflaton-driven electric field

- Evaluating the induced current, and solving the field eqs. with it, we verify the no-anisotropic hair conjecture for inflation

Validity of WKB approximation

Klein–Gordon eq.: $\left\{\frac{d^2}{d\tau^2}+\omega_{\mathbf{k}}^2(\tau)\right\}\tilde{\phi}_{\mathbf{k}}(x)=0$ $\quad\quad\tilde{\phi}=a\phi$

$$\omega_{\mathbf{k}}^2=(k_1-eA)^2+k_2^2+k_3^2+(m^2-2H^2)a^2$$

$$A=-\frac{E}{3H}a^{1+2}$$

+2 comes from f

At $a\to 0$, the WKB approximation is trivially valid

$$\omega_{\mathbf{k}}\simeq|\mathbf{k}|\quad\Rightarrow\quad\omega_{\mathbf{k}}^{-4}\left(\frac{d\omega_{\mathbf{k}}}{d\tau}\right)^2\simeq 0,\quad\omega_{\mathbf{k}}^{-3}\frac{d^2\omega_{\mathbf{k}}}{d\tau^2}\simeq 0$$

At $a\to\infty$, the validity is ensured due to the presence of f

$$\omega_{\mathbf{k}}\simeq\frac{eE}{3H}a^{1+2}\quad\Rightarrow\quad\omega_{\mathbf{k}}^{-4}\left(\frac{d\omega_{\mathbf{k}}}{d\tau}\right)^2\simeq 9\left(\frac{eE}{3H^2}a^2\right)^{-2},\quad\omega_{\mathbf{k}}^{-3}\frac{d^2\omega_{\mathbf{k}}}{d\tau^2}\simeq 12\left(\frac{eE}{3H^2}a^2\right)^{-2}$$

Particle number and Induced current

In the semiclassical picture,

$$n_{\mathbf{k}} = \exp\left\{4\ \mathrm{Im} \int^{\tau_*} d\tau'\ \omega_{\mathbf{k}}(\tau')\right\}, \quad \omega_{\mathbf{k}}(\tau_*) \equiv 0$$

'61 V. L. Pokrovskii,
I. M. Khalatnikov

$$\tilde{j} = 2e \int \frac{d^3k}{(2\pi)^3}\ v_{\mathbf{k}} n_{\mathbf{k}}, \quad v_{\mathbf{k}} = (k_1 - eA)/\omega_{\mathbf{k}}$$

$\tilde{j}_0 = 0,$
$\tilde{j}_i = \tilde{j}(t)\delta_i{}^1$

We evaluate the late time behavior at $\frac{eE}{H^2}a^2 \gg 1$:

$$\tilde{j} \simeq \frac{e^3E^2}{4\pi^3}\frac{a^7}{7H}\exp\left\{-\pi\frac{m^2 - 2H^2}{eEa^2}\right\}$$

At $\frac{|m^2-2H^2|}{eEa^2} \ll 1$, the contribution from the mass term becomes irrelevant

$$\tilde{j} \simeq \frac{e^3E^2}{4\pi^3}\frac{a^7}{7H}$$

Field eqs. with Induced current

$$\begin{cases} V = 3M_{\rm pl}^2 H^2 \\ 3H\dfrac{d}{dt}\varphi + \partial_\varphi V - f^{-1}\partial_\varphi f \cdot E_{\rm phys}^2 = 0 \\ \dfrac{d}{dt}\left(f a^2 E_{\rm phys}\right) + \underline{a^{-1}\tilde{j}} = 0 \end{cases}$$

Solving them by use of the ansatz: $f(\varphi) = \exp\left\{\frac{2c}{M_{\rm pl}^2}\int d\varphi\ \frac{V}{\partial_\varphi V}\right\}$,

$$f = a^{-2}\left\{1 - \frac{1}{1+\frac{3}{2}\frac{1}{c-1}} \cdot \frac{e^3 E}{4\pi^3}\frac{a^6}{42H^2}\right\}$$

for $c > 1$

$$E_{\rm phys} = E\left\{1 - \frac{\frac{3}{2}\frac{1}{c-1}}{1+\frac{3}{2}\frac{1}{c-1}} \cdot \frac{e^3 E}{4\pi^3}\frac{a^6}{42H^2}\right\}$$

Considering the first-order backreaction, the electric field decays with the cosmic expansion $\Rightarrow$ No anisotropic hair may exist also in this model

Summary

- In the inflation theory with a dilatonic coupling between the inflaton and the U (1) gauge field, a persistent electric field (and then an anisotropic inflation) is obtained as a solution of the classical field eqs.

- We investigated the pair production of scalar particles in the inflaton-driven electric field. In particular, we evaluated the induced current due to the pair production

- Solving the field eqs. with the induced current, we found that the first-order backreaction screens the electric field with the cosmic expansion

- The result indicates that the no-go of anisotropic inflation holds true regardless of whether the dilatonic coupling is present or not

Open problems

- In order to prove the no-anisotropic hair conjecture completely, the whole time evolution of the electric field should be investigated

- For the investigation, we need to evaluate the induced current on general backgrounds $E_{\rm phys},\ f$

e.g. if the WKB approximation is valid,

$$\tilde{j} \simeq \frac{e^3}{4\pi^3} \int_{t_0}^{t} dt'\ a^3(t') f^{-2}(t') E_{\rm phys}^2(t') \exp\left\{ -\pi \frac{m^2 - 2H^2}{e f^{-1}(t') E_{\rm phys}(t')} \right\}$$

- The investigation of the pair production of charged fermions in the inflaton-driven electric field is another future subject