Fermions in Geodesic Witten Diagrams

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Outline

1. Introduction
2. Detail of our result

Conformal field theory in theoretical physics

e.g. String theory, Critical phenomena, RG flow, ⋯

Recent progress of CFT

(They are not directly related to our paper.)

Numerical conformal bootstrap

[Rattazzi, Rychkov, Tonni, Vichi, 2008], ⋯

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, 2012],⋯

Out of time order correlation function in CFT

[Fitzpatrick, Kaplan, Walters, 2014], [Roberts, Stanford, 2014],⋯

Conformal block $W_{\Delta,l}(x_i)$

Basis of CFT's correlation function

$$
\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle
= \sum_{\mathcal{O}} \underline{C_{12\mathcal{O}}C_{34\mathcal{O}}}\,\underline{W_{\Delta,l}(x_i)}
$$

It depends
on the theory.

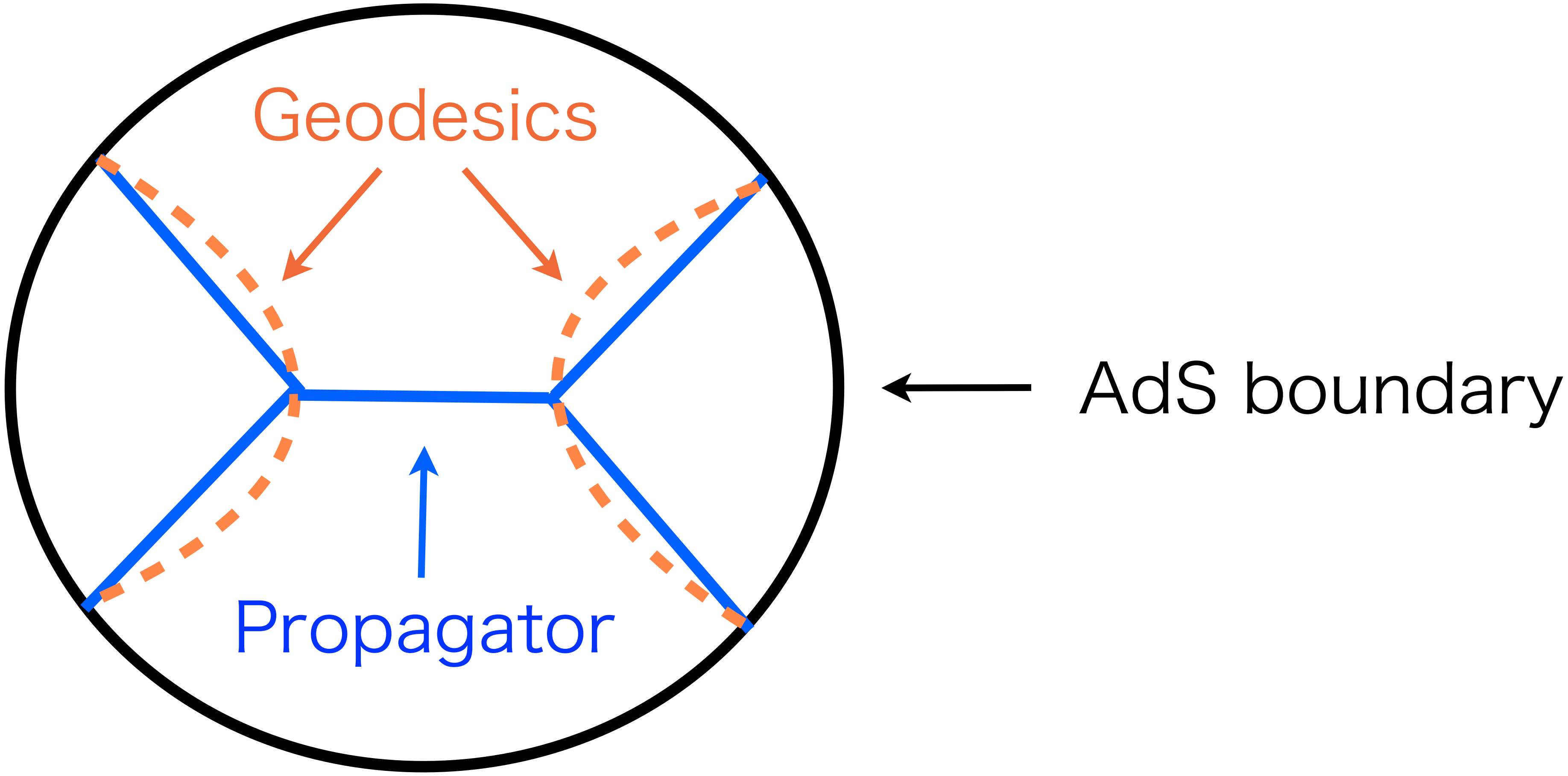
It doesn't depend
on the theory.

Conformal block is important for the recent progress of CFT.

Q What is the gravity dual of 4-pt conformal block?

A **4-point geodesic Witten diagram (GWD)**

Diagram for amplitude in AdS spacetime that interaction points are integrated over geodesics



[E. Hijano, P. Kraus, E. Perlmutter, R. Sniverly, 2015]

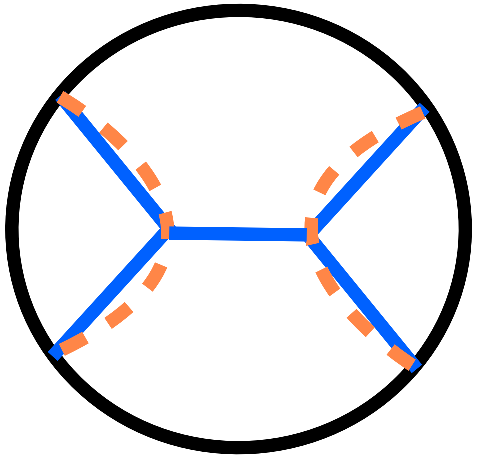
Our purpose

Generalization to fermion fields

Our motivation

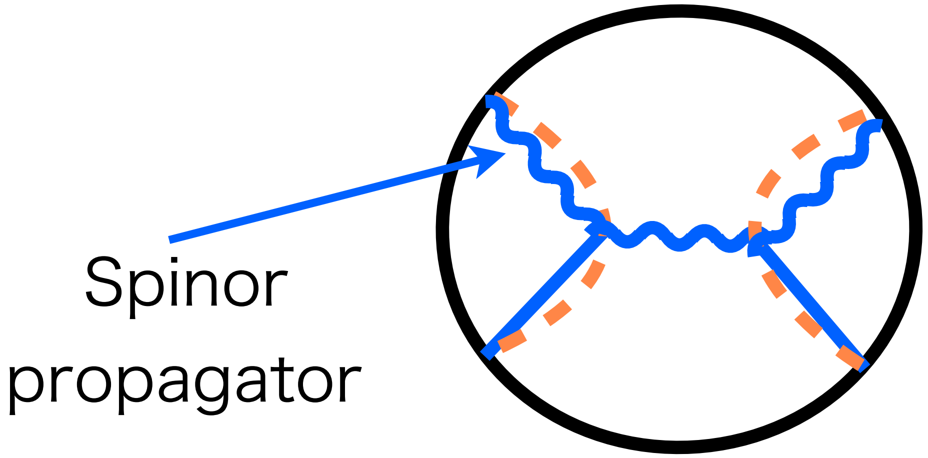
- Conformal block
  for CFT with fermion

- Towards
  super conformal block



[E. Hijano, P. Kraus,
E. Perlmutter, R. Sniverly, 2015],⋯

our target



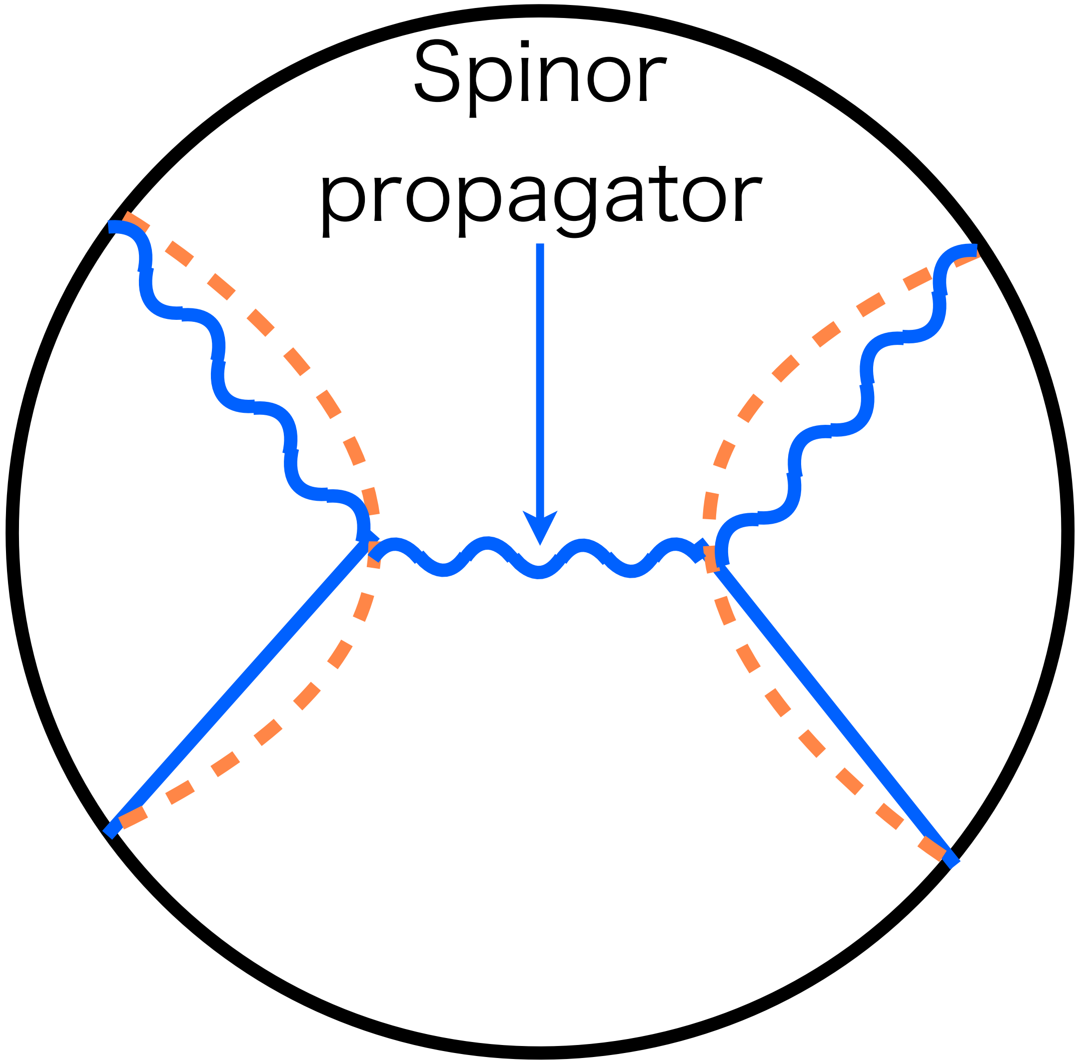
Our result

Geodesic Witten diagrams
with spinor fields
in odd dimensional AdS

- Embedding formalism for spinor propagators in odd dimensional AdS

- Checking that 4-pt GWD with fermion exchange satisfies the conformal Casimir equation for 4-pt conformal block

- GWD expansion of fermion exchange 4-point Witten diagram



Outline

1. Introduction

2. Detail of our result

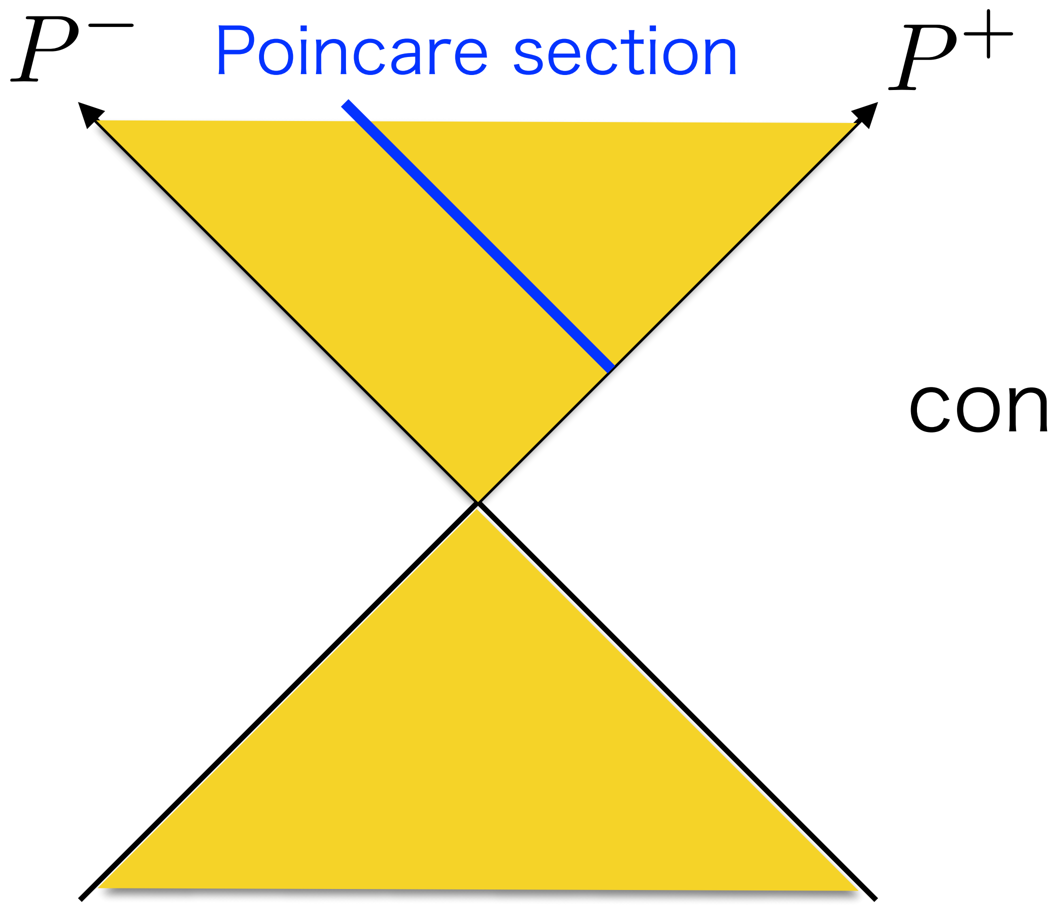
Embedding formalism

[Dirac, 1936], ⋯, [Costa, Penedones, Poland, Rychkov, 2011],⋯

In the embedding formalism, d+1-dim AdS bulk and d-dim boundary coordinates can be described by d+2-dim Minkowski spacetime.

Bulk coordinate $$X^A = \frac{1}{z}(1, z^2 + x^2, x^a)$$

Boundary coordinate $$P^A = (1, x^2, x^a)$$



In the embedding formalism,
conformal symmetry and AdS's isometry
can be represented by
simple Lorentz symmetry.

AdS spinor propagators in the embedding formalism

We consider odd dim AdS and even dim CFT.

Spinor bulk-boundary propagator

$$G_{b\partial}^{\Delta,\frac{1}{2}}(X,\bar{S}_b;P,S_\partial)=\mathcal{C}_{\Delta,\frac{1}{2}}\frac{\langle\bar{S}_b\Pi_-S_\partial\rangle}{(-2X\cdot P)^{\Delta+\frac{1}{2}}}$$

Spinor bulk-bulk propagator

$$G_{bb}^{\Delta,\frac{1}{2}}(X,\bar{S}_b;Y,T_b)=\langle\bar{S}_b\,\Pi_+T_b\rangle\left(\frac{d}{du}G_{bb}^{\Delta_+,0}(u)\right)+\langle\bar{S}_b\,\Pi_-T_b\rangle\left(\frac{d}{du}G_{bb}^{\Delta_-,0}(u)\right)$$

They are consistent with the expressions
without the embedding formalism
in [M. Henningson, K. Sfetsos, 1998], [T. Kawano, K. Okuyama, 1999].

We construct (geodesic) Witten diagram
by using these propagators.

Conformal Casimir equation and equation of AdS propagator

Conformal block $W(\Delta, \Delta_i; P_i)$

satisfies the conformal Casimir equation.

$$-\frac{1}{2}(L_1 + L_2)^2 W_4(\Delta, \Delta_i; P_i) = C_{\Delta,s} W_4(\Delta, \Delta_i; P_i)$$

In the view point of GWD,

the conformal Casimir equation is related to the equation of the bulk-bulk propagator $(X \neq Y)$**.**

$$\left[(\Gamma^A \nabla_A)^2 - m^2\right] \Psi(X) = (-\frac{1}{2} L^{AB} L_{AB} - C_{\Delta,\frac{1}{2}}) \Psi(X) = 0$$

$$m = \Delta - \frac{d}{2}$$

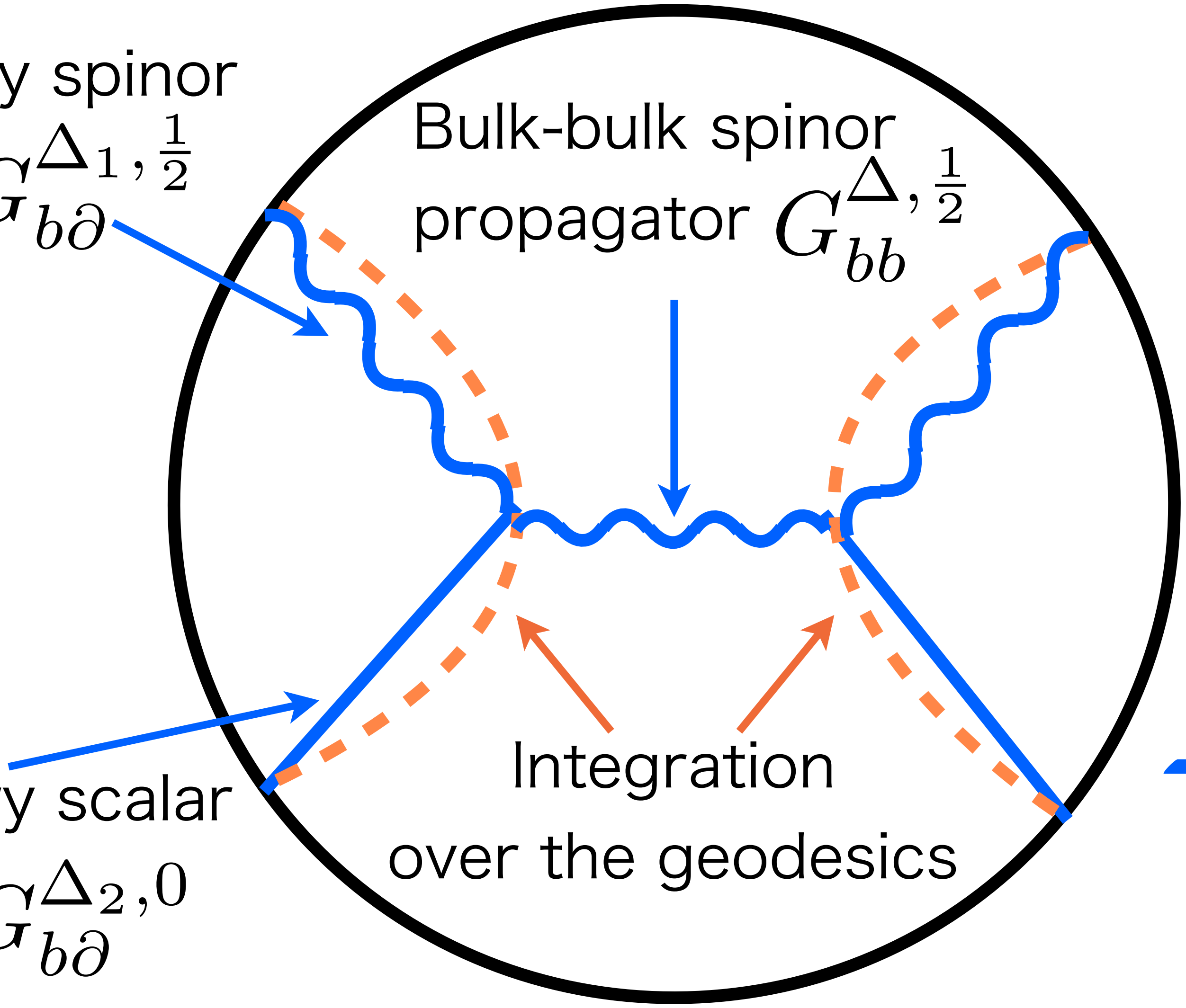
Amplitude of 4-point GWD with fermion exchange

$$\mathcal{W}_4(\Delta, \Delta_i) = \int_{-\infty}^{\infty} d\lambda F_\Delta[P_1, P_2, Y(\lambda), \bar{S}_{1\partial}, T_b]$$

$$\times(\overleftarrow{\partial}_{T_b}(1 + Y(\lambda))\overrightarrow{\partial}_{\bar{T}_b})G_{b\partial}^{\Delta_4,\frac{1}{2}}(Y(\lambda), \bar{T}_b; P_4, S_{4\partial})G_{b\partial}^{\Delta_3,0}(Y(\lambda); P_3)$$

$$F_\Delta[P_1, P_2, Y, \bar{S}_{1\partial}, T_b]$$

$$:= \int_{-\infty}^{\infty} d\lambda\, \bar{G}_{b\partial}^{\Delta_1,\frac{1}{2}}(X(\lambda), S_b; P_1, \bar{S}_{1\partial})G_{b\partial}^{\Delta_2,0}(X(\lambda); P_2)(\overleftarrow{\partial}_{S_b}(1 + X(\lambda))\overrightarrow{\partial}_{\bar{S}_b})G_{bb}^{\Delta,\frac{1}{2}}(X(\lambda), \bar{S}_b; Y, T_b)$$



Bulk-boundary spinor propagator $G_{b\partial}^{\Delta_1,\frac{1}{2}}$

Bulk-bulk spinor propagator $G_{bb}^{\Delta,\frac{1}{2}}$

Bulk-boundary scalar propagator $G_{b\partial}^{\Delta_2,0}$

Integration over the geodesics

F_Δ is invariant under the rotation.

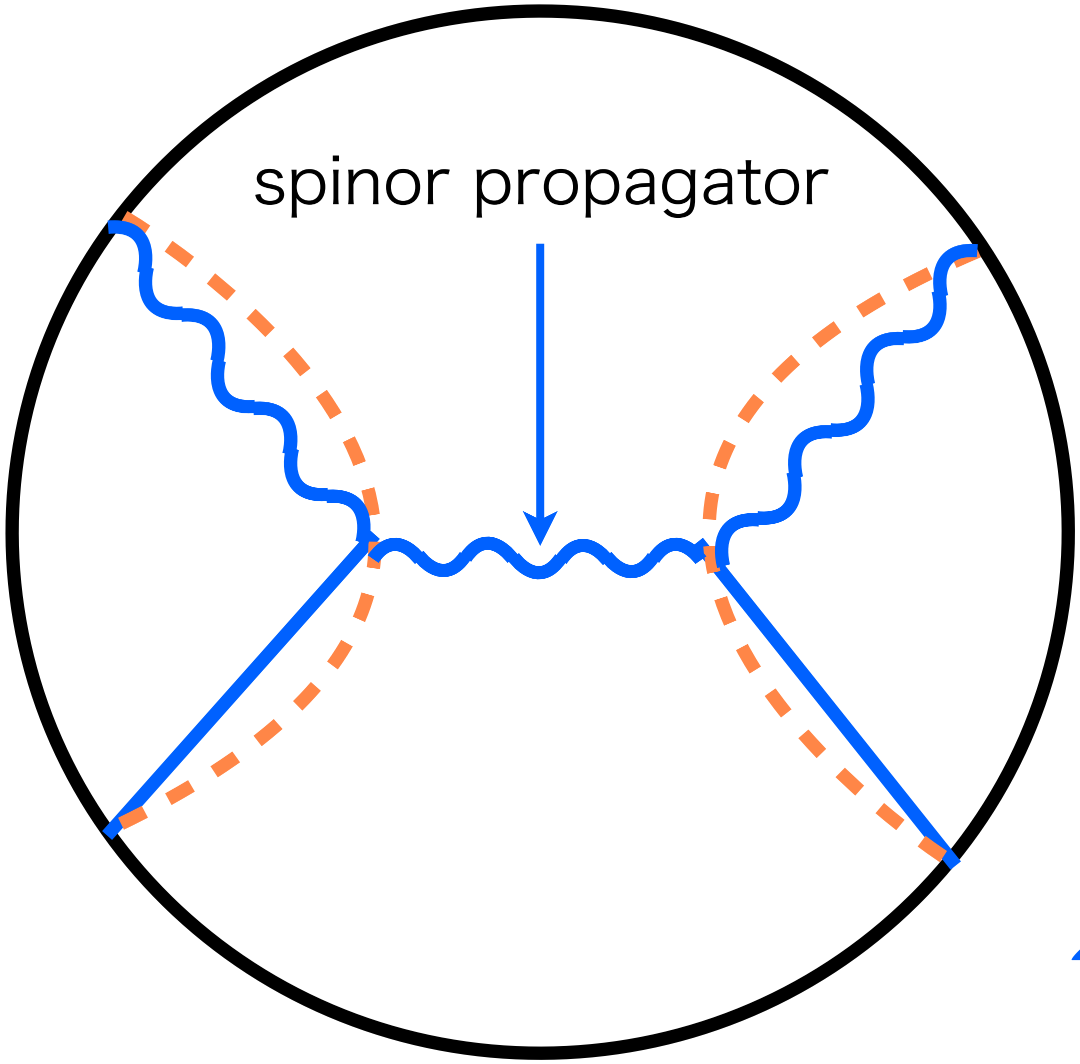
$$(L_1 + L_2)F_\Delta = F_\Delta \overleftarrow{L}_Y$$

4-point GWD satisfies the conformal Casimir equation.

Amplitude of 4-point GWD with fermion exchange

$$\mathcal{W}_4(\Delta,\Delta_i)=\int_{-\infty}^{\infty}d\lambda F_\Delta[P_1,P_2,Y(\lambda),\bar{S}_{1\partial},T_b](\overleftarrow{\partial}_{T_b}(1+Y(\lambda))\overrightarrow{\partial}_{\bar{T}_b})G_{b\partial}^{\Delta_4,\frac{1}{2}}(Y(\lambda),\bar{T}_b;P_4,S_{4\partial})G_{b\partial}^{\Delta_3,0}(Y(\lambda);P_3)$$

$$\begin{aligned}&F_\Delta[P_1,P_2,Y,\bar{S}_{1\partial},T_b]\\ :=&\int_{-\infty}^{\infty}d\lambda\,\bar{G}_{b\partial}^{\Delta_1,\frac{1}{2}}(X(\lambda),S_b;P_1,\bar{S}_{1\partial})G_{b\partial}^{\Delta_2,0}(X(\lambda);P_2)(\overleftarrow{\partial}_{S_b}(1+X(\lambda))\overrightarrow{\partial}_{\bar{S}_b})G_{bb}^{\Delta,\frac{1}{2}}(X(\lambda),\bar{S}_b;Y,T_b)\end{aligned}$$

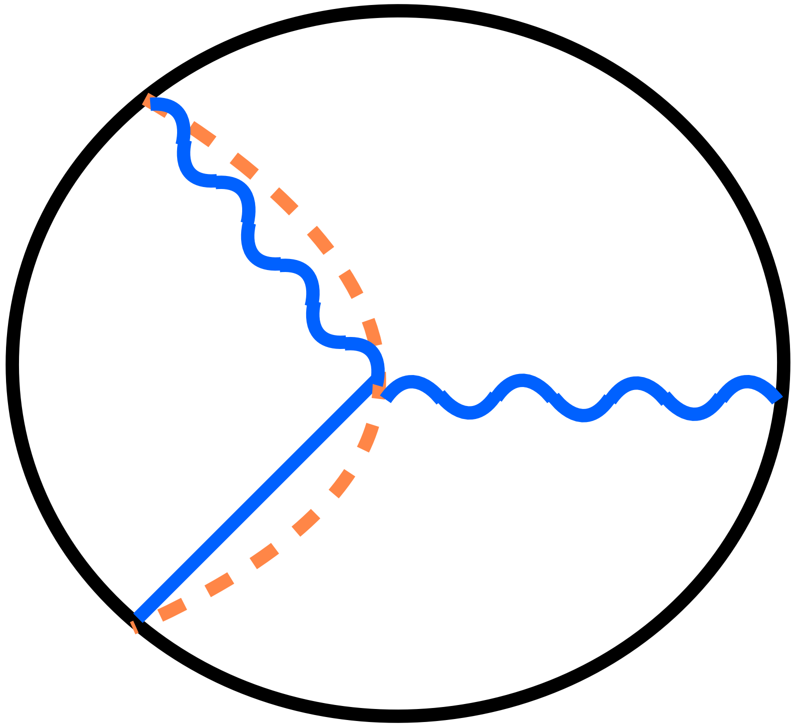


Because of the rotation invariance of F_Δ and the equation of the bulk-bulk propagator, the 4-point GWD satisfies the conformal Casimir equation.

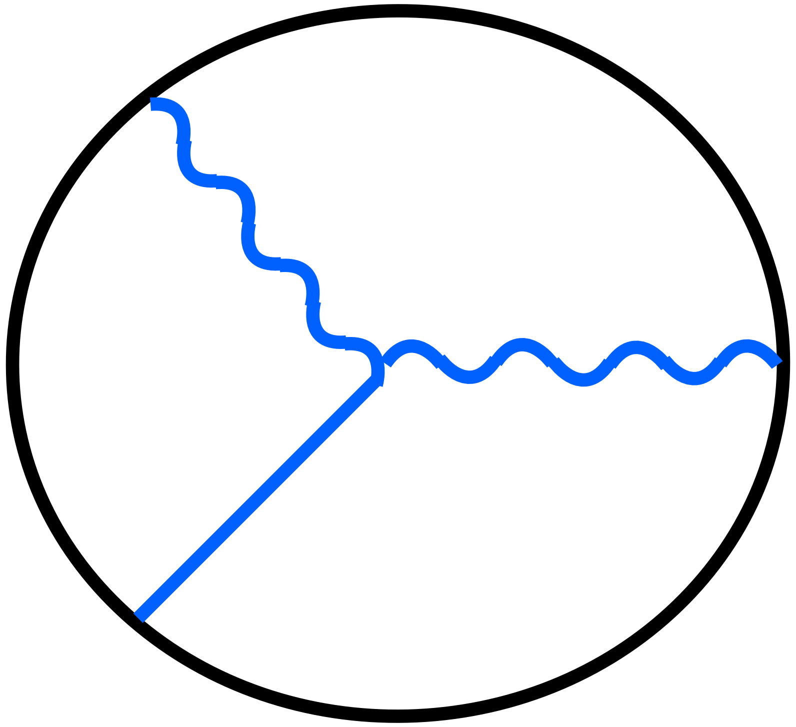
$$-\frac{1}{2}(L_1+L_2)^2\mathcal{W}_4(\Delta,\Delta_i)=C_{\Delta,\frac{1}{2}}\mathcal{W}_4(\Delta,\Delta_i)$$

Ratio between 3-point GWD and Witten diagram

Geodesic Witten diagram $\mathcal{W}_3$
(Integration over the geodesic)



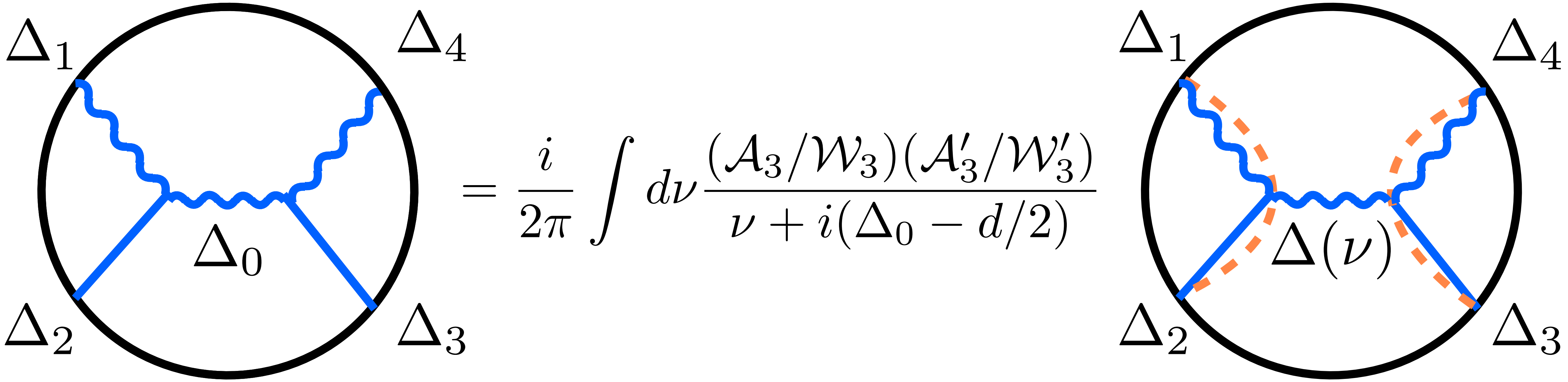
Witten diagram A_3
(Integration over the whole AdS)



The ratio is useful for GWD expansion of the Witten diagram.

$$\frac{\mathcal{A}_3}{\mathcal{W}_3(\gamma_{31})} = \pi^{d/2}\Gamma\left(\frac{1}{2}\left(-d+\Delta_1+\Delta_2+\Delta_3+1\right)\right)\frac{\Gamma\left(\frac{1}{2}(\Delta_3+\Delta_1-\Delta_2)\right)}{\Gamma(\Delta_3)\Gamma(\Delta_1+1/2)}$$

GWD expansion of fermion exchange 4-point Witten diagram



$$= \frac{i}{2\pi}\int d\nu \frac{(\mathcal{A}_3/\mathcal{W}_3)(\mathcal{A}_3'/\mathcal{W}_3')}{\nu + i(\Delta_0 - d/2)}$$

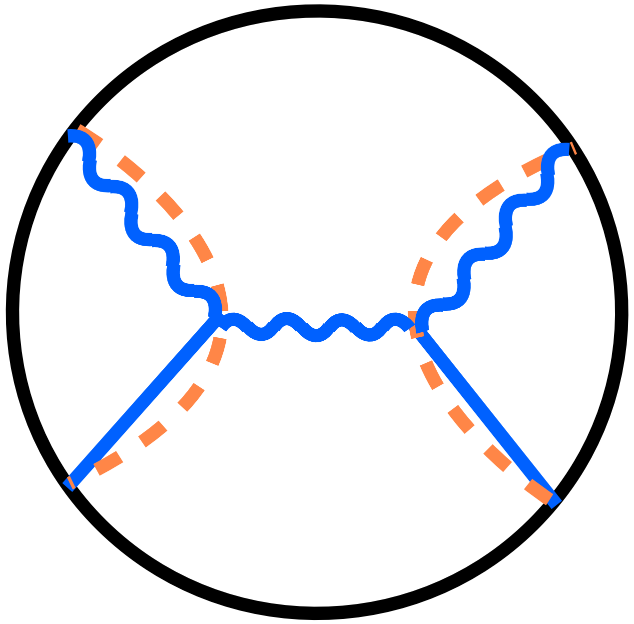
Poles of $\frac{(\mathcal{A}_3/\mathcal{W}_3)(\mathcal{A}_3'/\mathcal{W}_3')}{\nu + i(\Delta_0 - d/2)}$ **determine conformal dimensions of intermediate states in GWD (conformal block) expansion.**

$d/2 + i\nu = \Delta_0,$ (Single trace operator)

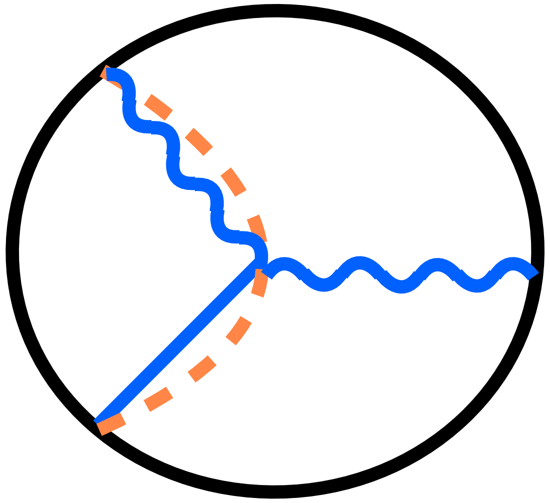
$d/2 + i\nu = \Delta_1 + \Delta_2 + 2m,\ d/2 + i\nu = \Delta_3 + \Delta_4 + 2m,\ (m = 0, 1, \cdots)$

(Double trace operators)

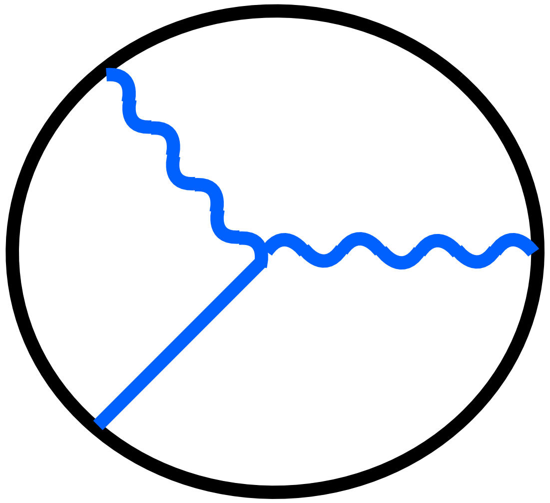
Summary



- We study geodesic Witten diagram with spinor fields in odd dimensional AdS.



- We show that 4-point geodesic Witten diagram with fermion exchange satisfies the conformal Casimir equation for conformal block.



- We compute the ratio between 3-point GWD and Witten diagram and consider GWD expansion of the 4-point fermion exchange Witten diagram.