기초과학연구원
Institute for Basic Science

hhh-EWPT correlation beyond 1-loop order

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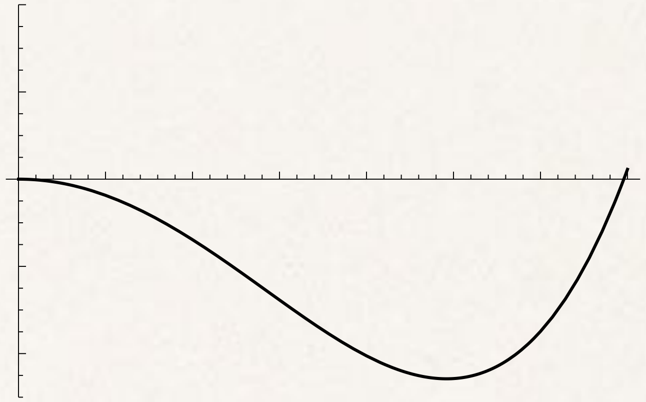
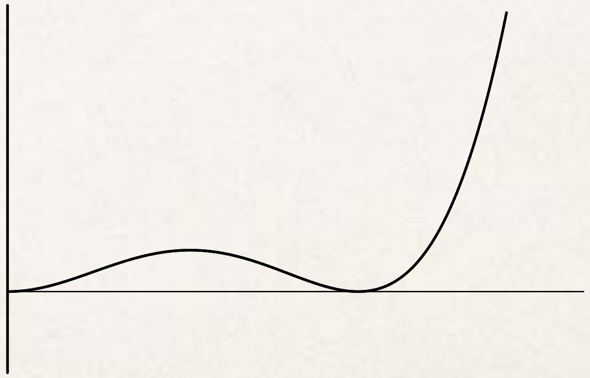
based on arXiv: 1811.00336

Outline

- Introduction
- 2-loop corrections to λ_{hhh}



- 1^{st}-order EW phase transition (EWPT) and its correlation with λ_{hhh}



- Summary

Introduction

λ_{hhh}-EWPT correlation

- Triple Higgs coupling (λ_{hhh}) can be modified if EWPT is strong 1st order.

[S.Kanemura, Y.Okada, E.S., PLB606 (2005) 361;
C.Grojean, G.Servant, J.Wells, PRD71 (2005) 036001]

- O(1) quartic couplings in Higgs potential play an essential role.

Question

How large is the 2-loop effect on (1) λ_{hhh} and (2) 1st-order EWPT?

As an example, we consider Inert Doublet Model (IDM).

Inert Doublet Model (IDM)

particle content

SM + Z₂-odd doublet η

tree-level potential w/ Z₂ $\Phi \to \Phi$ and $\eta \to -\eta$

$$V_0(\Phi,\eta) = \mu_1^2 \Phi^\dagger \Phi + \mu_2^2 \eta^\dagger \eta + \frac{\lambda_1}{2}(\Phi^\dagger\Phi)^2 + \frac{\lambda_2}{2}(\eta^\dagger\eta)^2 + \lambda_3(\Phi^\dagger\Phi)(\eta^\dagger\eta)$$
$$+ \lambda_4(\Phi^\dagger\eta)(\eta^\dagger\Phi) + \left[\frac{\lambda_5}{2}(\Phi^\dagger\eta)^2 + \text{h.c}\right]$$

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix}, \quad \eta = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix}.$$

H or A (lightest Z₂ odd particle) can be a dark matter (DM).

Model parameters

Original parameters: $\mu_1^2,\ \mu_2^2,\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5.$

Converted parameters: $v,\ \mu_2^2,\ \lambda_2,\ m_h,\ m_H,\ m_A,\ m_{H^\pm},$

At tree level

$$\lambda_1 = \frac{m_h^2}{v^2}, \quad \lambda_3 = \frac{2}{v^2}(m_{H^\pm}^2 - \mu_2^2),$$

$$\lambda_4 = \frac{1}{v^2}(m_H^2 + m_A^2 - 2m_{H^\pm}^2), \quad \lambda_5 = \frac{1}{v^2}(m_H^2 - m_A^2).$$

In our study, H is DM, and $M_H \simeq M_h/2$ is taken.

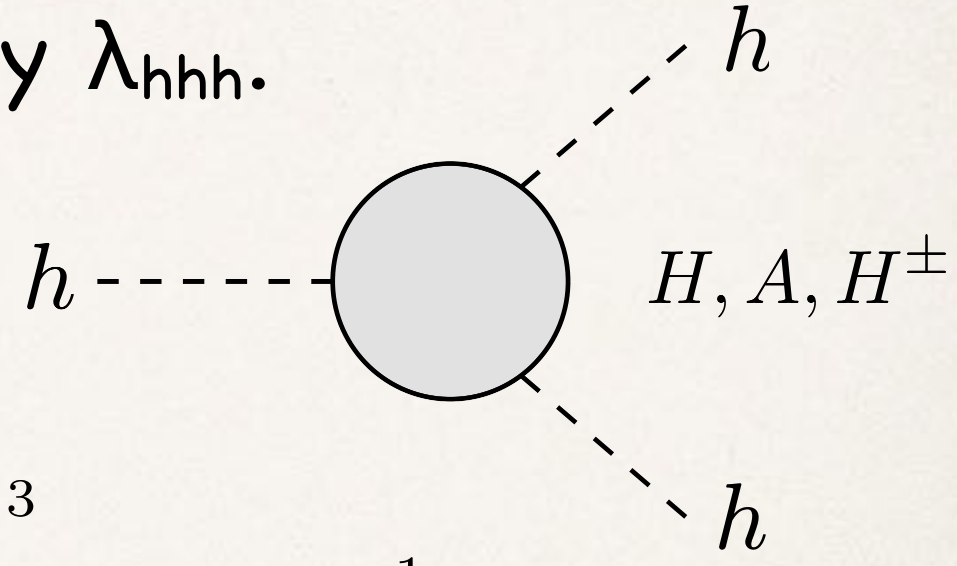
DM physics point of view, $\mu_2^2 \longrightarrow \bar{\lambda}_{hHH} \equiv \lambda_3 + \lambda_4 + \lambda_5$

Also, we take $M_A = M_{H+}$ to avoid ρ-parameter constraint.

At 1-loop

[S.Kanemura, M. Kikuchi, K. Sakurai, PRD94,115011(2016)]

- Extra Higgs boson loops can modify λ_{hhh}.



$$\lambda_{hhh}^{\mathrm{IDM}} = \frac{3m_h^2}{v} + \Delta^{(1)}\lambda_{hhh}^{\mathrm{IDM}},$$

$$\Delta^{(1)}\lambda_{hhh}^{\mathrm{IDM}} = \sum_{\phi=H,A,H^{\pm}} n_\phi \frac{4m_\phi^4}{16\pi^2 v^3}\left(1-\frac{\mu_2^2}{m_\phi^2}\right)^3 \qquad m_\phi^2 = \mu_2^2 + \frac{1}{2}\bar{\lambda}_{h\phi\phi}v^2$$

$$n_H = n_A = 1,\ n_{H^{\pm}} = 2$$

$$= \begin{cases} \displaystyle\sum_\phi n_\phi \frac{4m_\phi^4}{16\pi^2 v^3} & \text{for} \quad \mu_2^2 \ll \dfrac{1}{2}\bar{\lambda}_{h\phi\phi}v^2, \\ \displaystyle\sum_\phi n_\phi \frac{4}{16\pi^2 v^3}\frac{(\bar{\lambda}_{h\phi\phi}v^2/2)^3}{m_\phi^2} & \text{for} \quad \mu_2^2 \gg \dfrac{1}{2}\bar{\lambda}_{h\phi\phi}v^2. \end{cases}$$

$\mu_2^2 \ll \bar{\lambda}_{h\phi\phi}v^2/2$ is necessary for 1st-order EWPT (see later).

* H is DM, so A and H+ are the dominant contributors.

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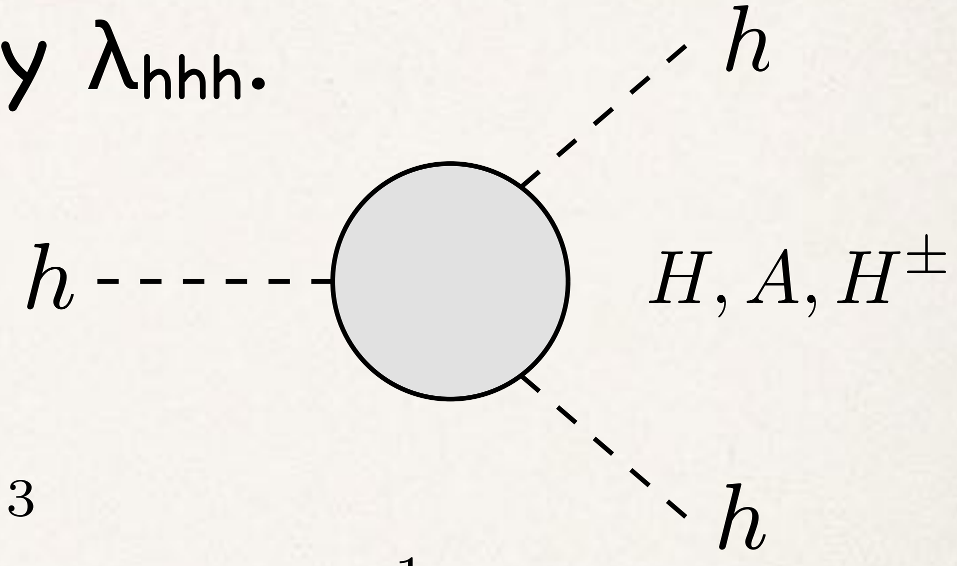

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power correction!!

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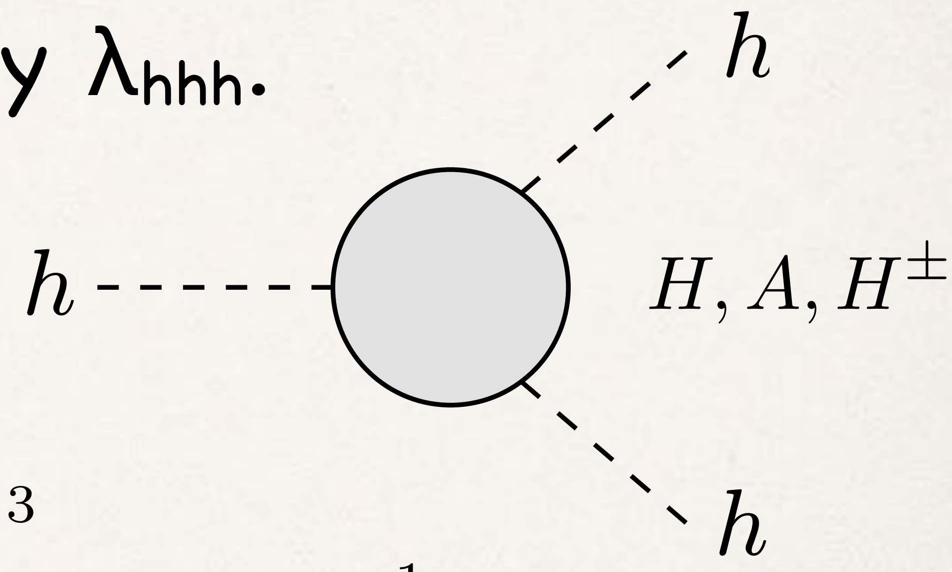

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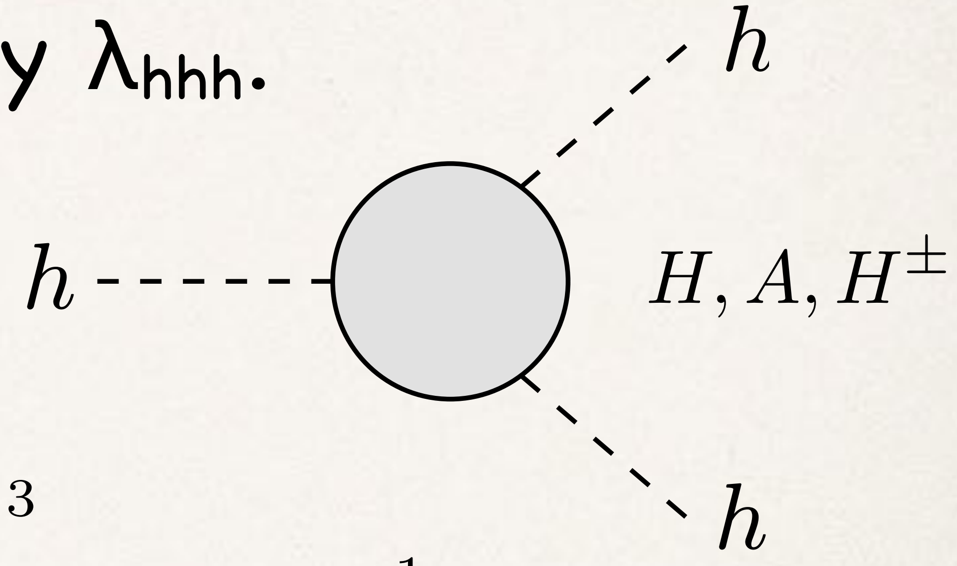

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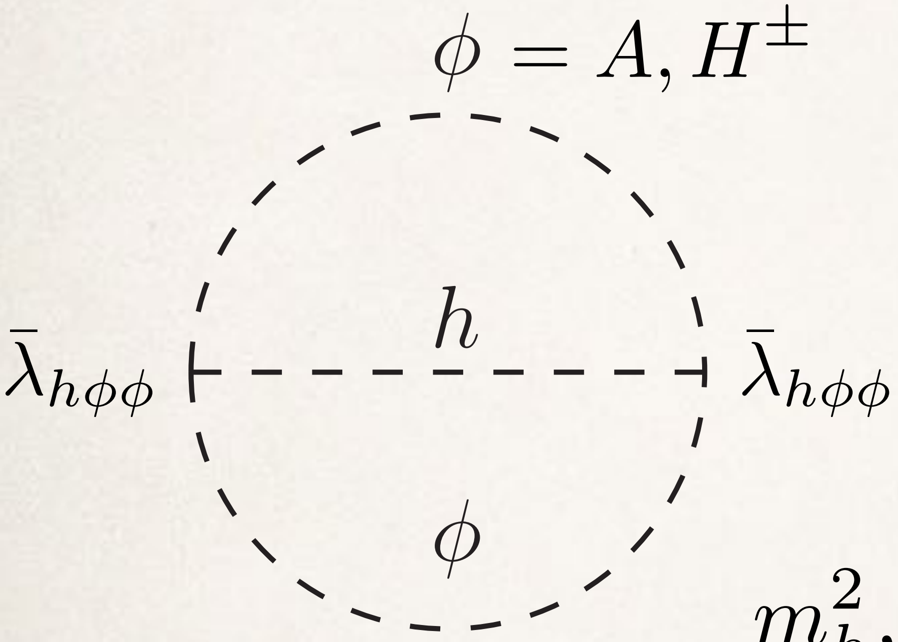
At 2-loop

Dominant 2-loop diagrams



$$\simeq \sum_{\phi=A,H^{\pm}} \frac{8n_\phi \bar{\lambda}^2_{h\phi\phi}}{(16\pi^2)^2} \frac{m_\phi^2}{v} \left(\ln \frac{m_\phi^2}{\bar{\mu}^2} - \frac{1}{2} \right) + \cdots$$

$$m_h^2, \mu_2^2 \ll m_A^2, m_{H^\pm}^2$$

Combining with 1-loop contributions,

$$\Delta^{(1)} \lambda_{hhh}^{\mathrm{IDM}} + \Delta^{(2)} \lambda_{hhh}^{\mathrm{IDM}}$$

$$\simeq \sum_{\phi=A,H^{\pm}} \frac{4n_\phi}{16\pi^2 v^3} \left[m_\phi^4(m_\phi) - \frac{4m_\phi^6}{16\pi^2 v^2} \right] + \cdots .$$

Leading 2-loop effect comes through RG running of m$_\phi$.

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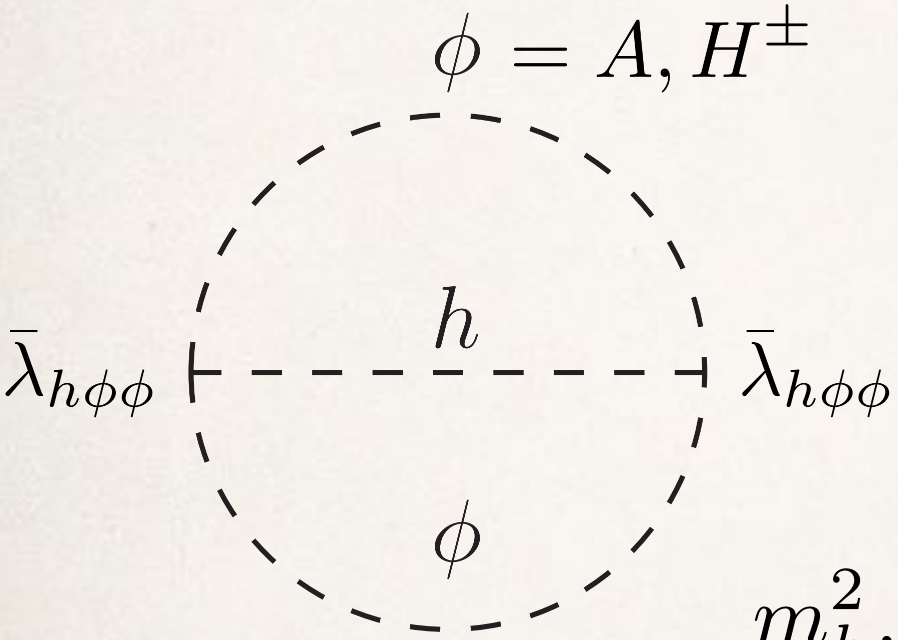

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Numerical results

$$M_H = 62.7\ \mathrm{GeV},\ \lambda_2 = 0.02\ \text{and}\ \bar{\lambda}_{hHH} = 4.6 \times 10^{-3}\ \text{at}\ M_Z.$$

$$M_A = M_{H^\pm}$$

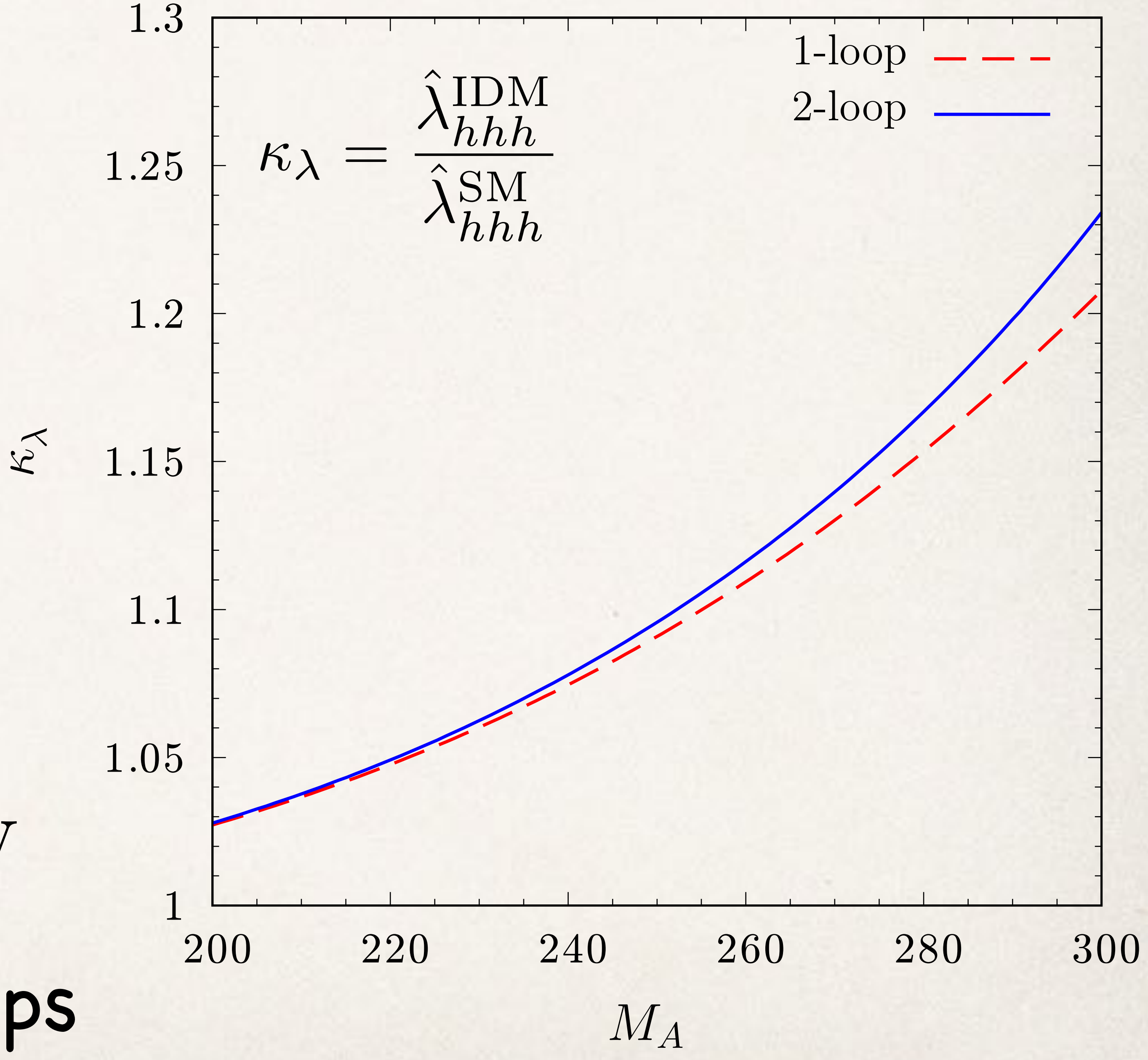
- Both 1 and 2-loops corrs. grow w/ increasing M_A.

  1 loop corr. is consistent with H-COUP [Kanemura, Kikuchi, Sakurai, Yagyu, 1710.04603].

- M_A has the upper bound.

$$\mu_2^2 < 0 \longleftarrow M_A \gtrsim 300\ \mathrm{GeV}$$

- Difference btw 1 and 2-loops is less than about 2%.



EW baryogenesis (EWBG)

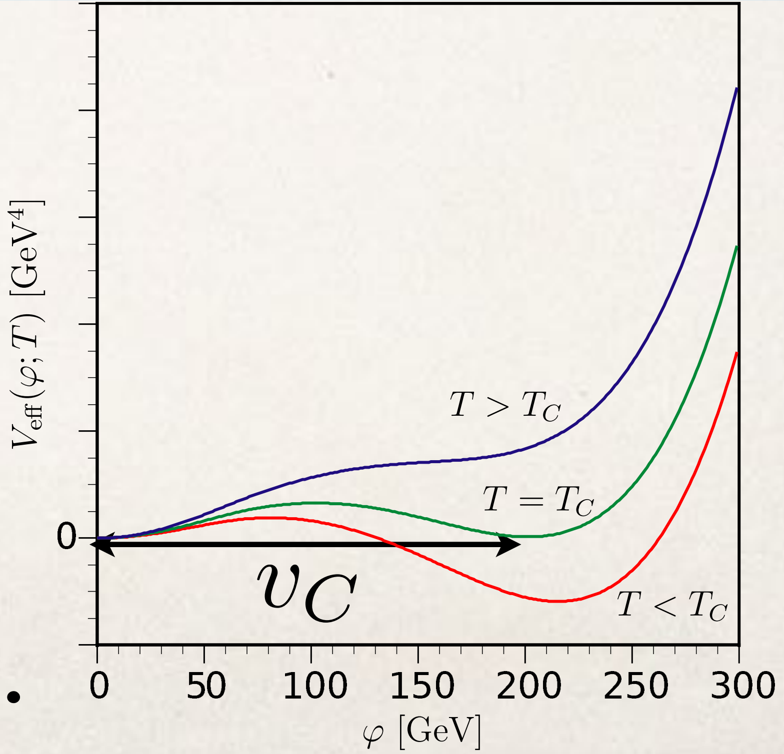
Sakharov's conditions

[Kuzmin, Rubakov, Shaposhnikov, PLB155,36 ('85)]

- B violation: anomalous (sphaleron) process
- C violation: chiral gauge interaction
- CP violation: KM phase and/or other sources in beyond the SM
- Out of equilibrium: 1st-order EW phase transition (EWPT) with expanding bubble walls

Baryon number preservation criterion

$$\boxed{\frac{v_C}{T_C} \gtrsim 1}$$



T_C: critical temperature; V_C: VEV at T_C.

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Related to Higgs physics

- Out of equilibrium: 1st-order EW phase transition (EWPT) with expanding bubble walls

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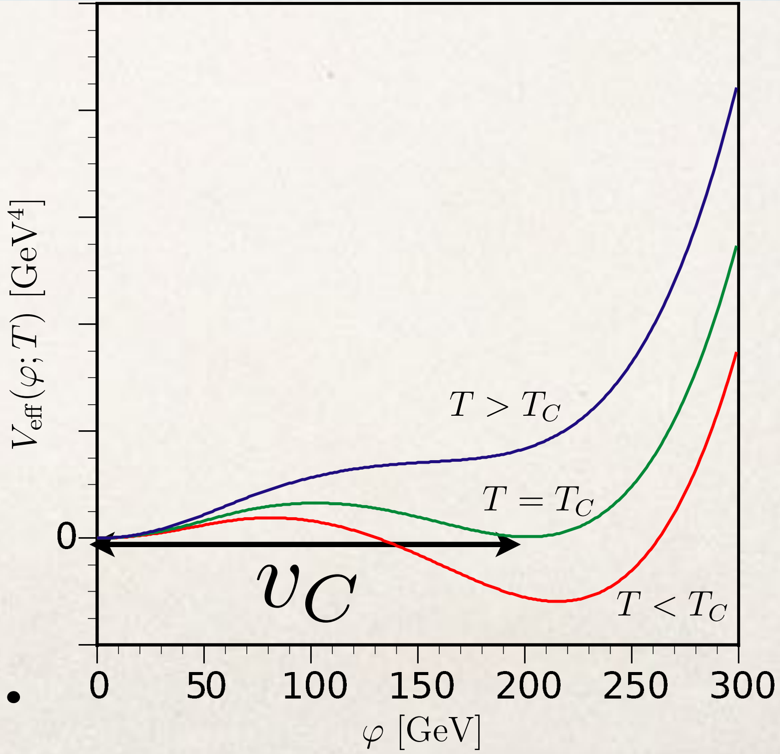
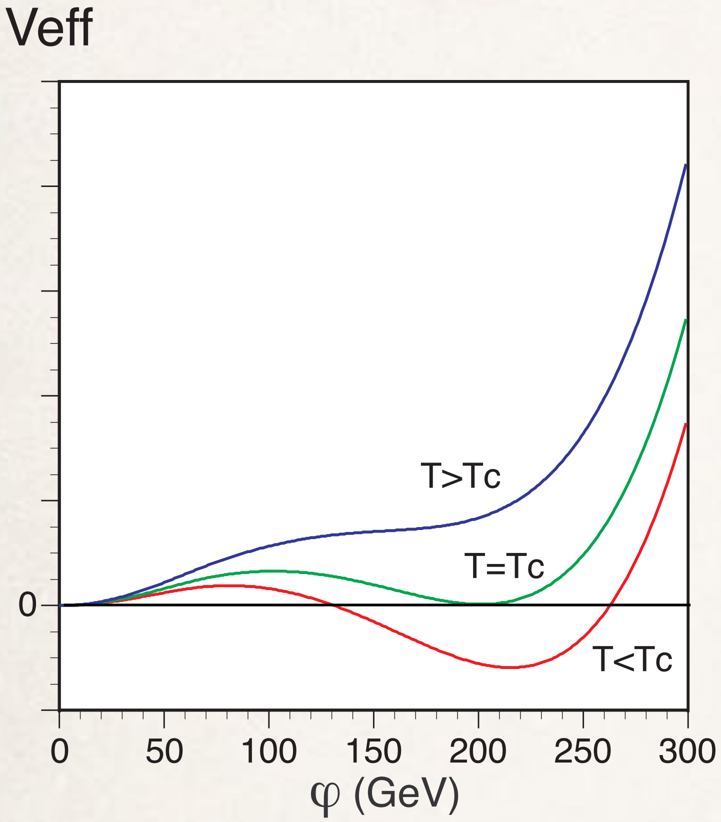


1st-order phase transition

$$V_{\rm eff} \simeq D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4 \xrightarrow[T=T_C]{} \frac{\lambda_{T_C}}{4}\varphi^2(\varphi - v_C)^2$$



$$v_C = \frac{2ET_C}{\lambda_{T_C}} \quad \Rightarrow \quad \frac{v_C}{T_C} = \frac{2E}{\lambda_{T_C}}$$

Extra Higgs loops can enhance E.

$$\bar{m}_\phi^2 = \mu_2^2 + \frac{1}{2}\bar{\lambda}_{h\phi\phi}\varphi^2$$

At 1-loop

$$V_{\rm eff} \ni \begin{cases} -(\bar{\lambda}_{h\phi\phi}/2)^{3/2}T\left(1+\frac{\mu_2^2}{\bar{\lambda}_{h\phi\phi}\varphi^2/2}\right)^{3/2}\varphi^3 & \text{for} \quad \mu_2^2 \ll \bar{\lambda}_{h\phi\phi}\varphi^2/2, \\ -(\mu_2^2)^{3/2}T\left(1+\frac{\bar{\lambda}_{h\phi\phi}\varphi^2}{\mu_2^2}\right)^{3/2} & \text{for} \quad \mu_2^2 \gg \bar{\lambda}_{h\phi\phi}\varphi^2/2. \end{cases}$$

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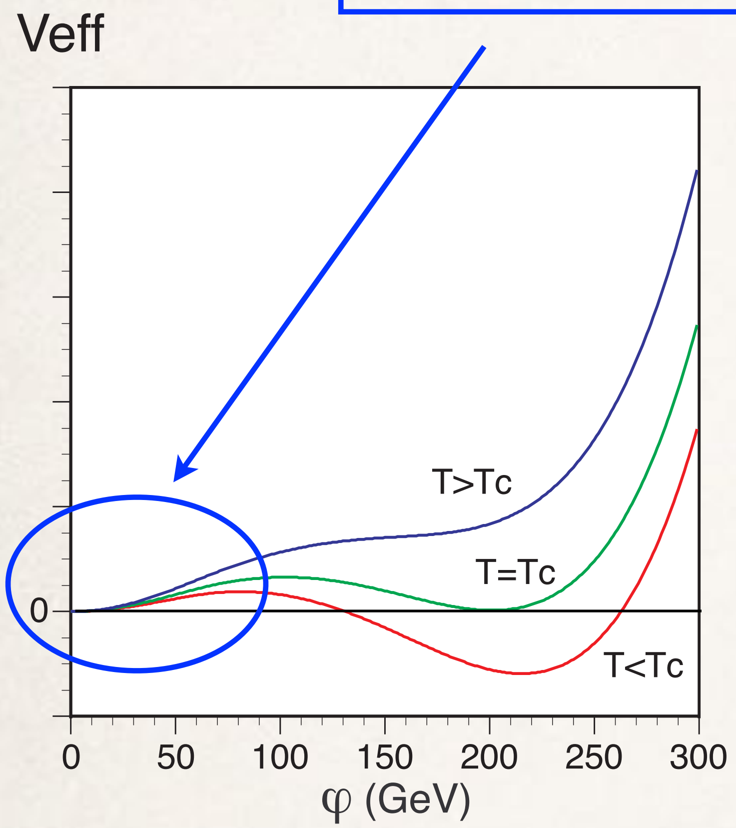

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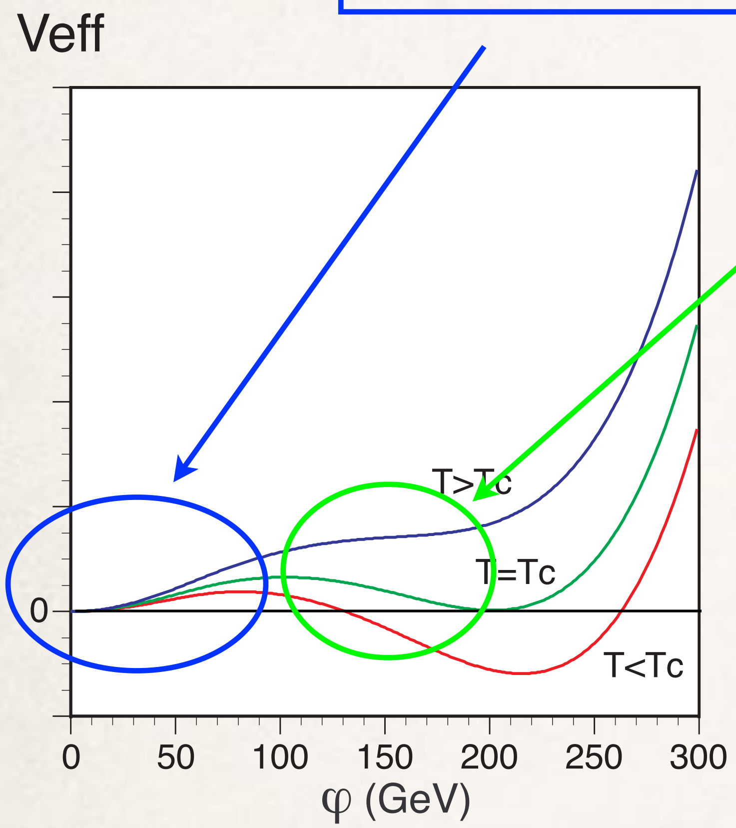

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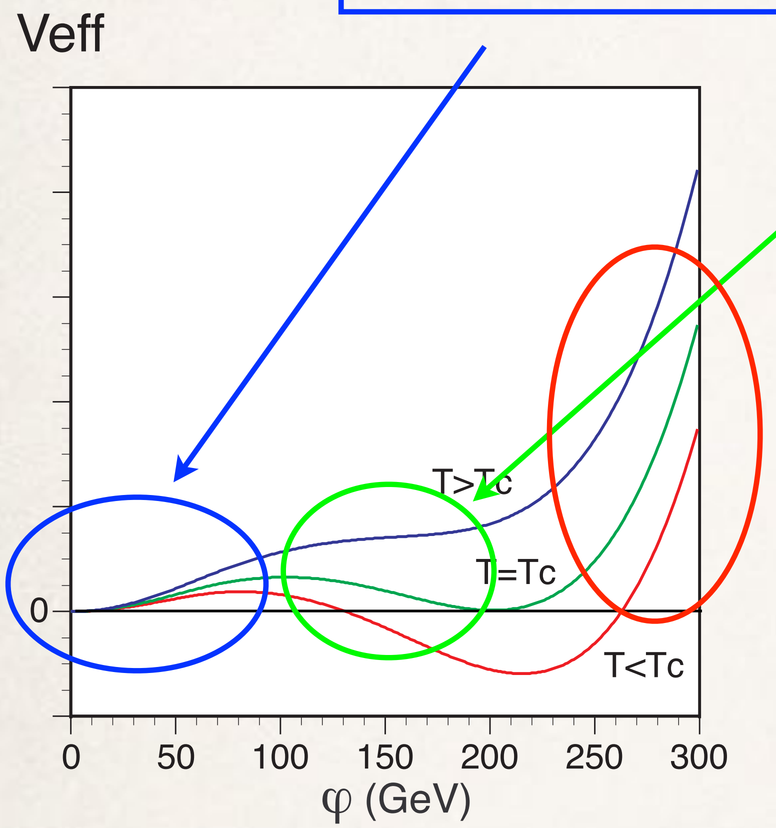

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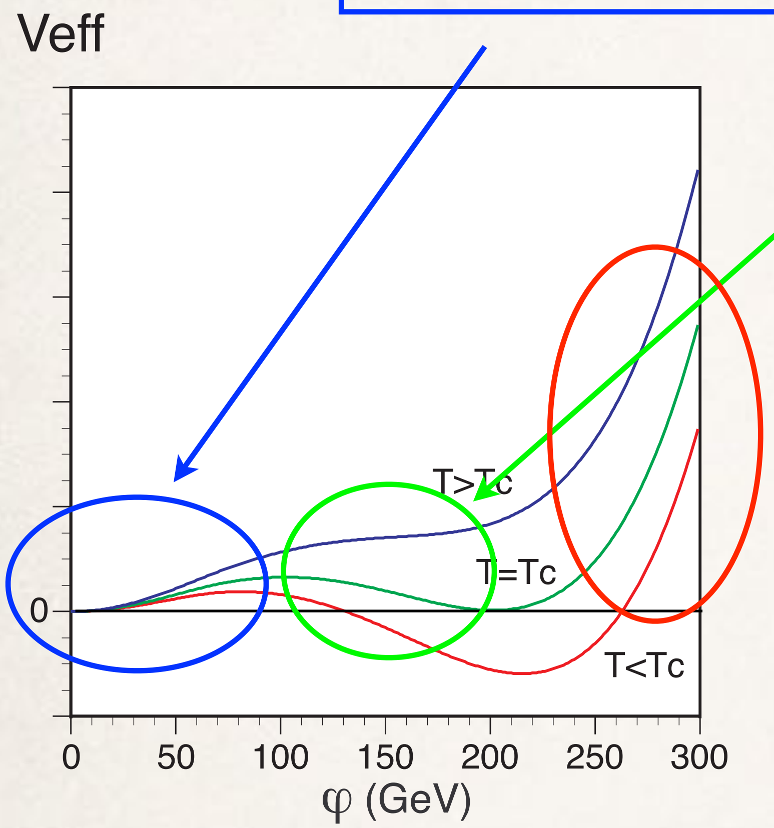

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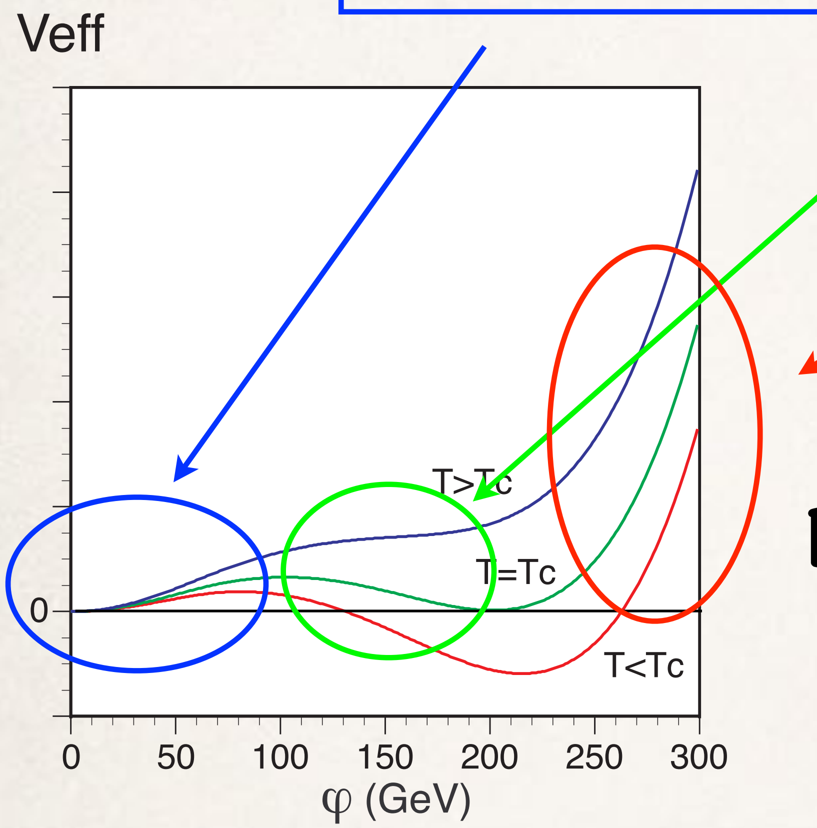

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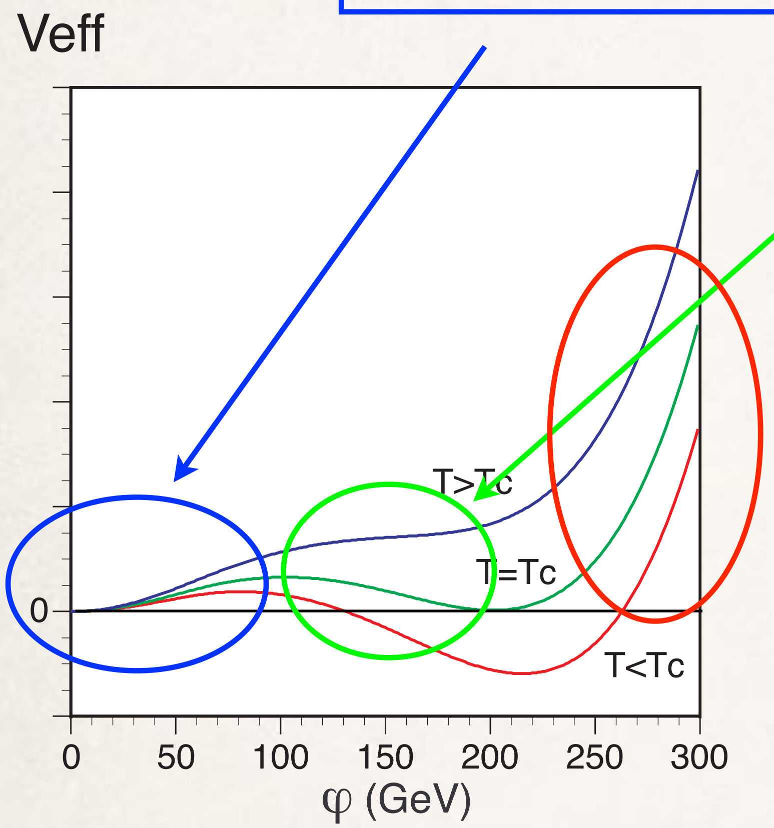

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$$\bar{m}_\phi^2 = \mu_2^2 + \frac{1}{2}\bar{\lambda}_{h\phi\phi}\varphi^2$$

At 1-loop

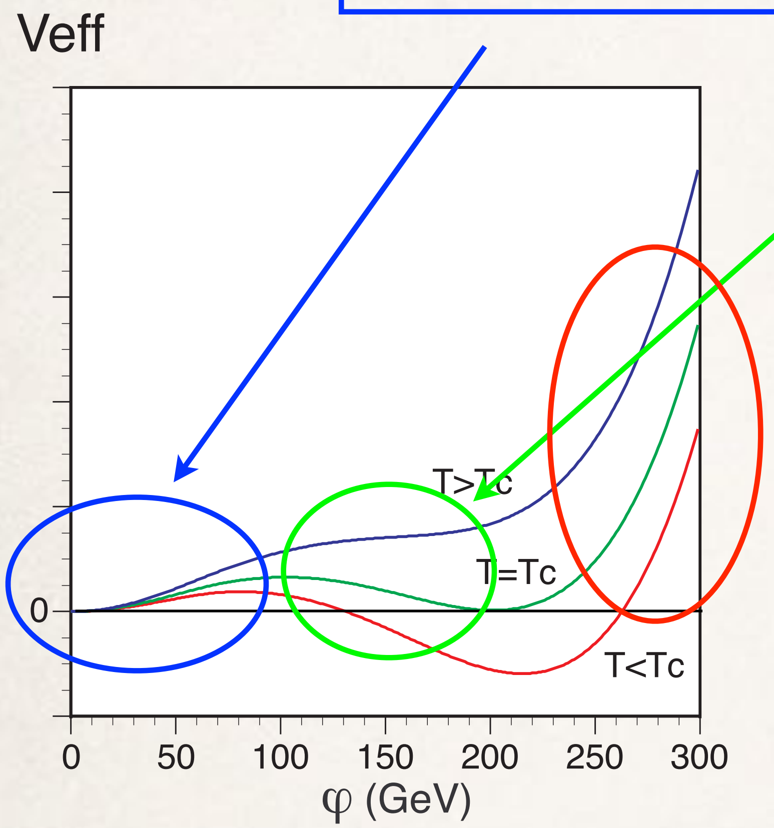
non-decoupling

$$V_{\rm eff} \ni \begin{cases} -(\bar{\lambda}_{h\phi\phi}/2)^{3/2}T\left(1+\dfrac{\mu_2^2}{\bar{\lambda}_{h\phi\phi}\varphi^2/2}\right)^{3/2}\varphi^3 & \text{for} \quad \mu_2^2 \ll \bar{\lambda}_{h\phi\phi}\varphi^2/2, \\ -(\mu_2^2)^{3/2}T\left(1+\dfrac{\bar{\lambda}_{h\phi\phi}\varphi^2}{\mu_2^2}\right)^{3/2} & \text{for} \quad \mu_2^2 \gg \bar{\lambda}_{h\phi\phi}\varphi^2/2. \end{cases}$$

decoupling

1st-order phase transition

$$V_{\rm eff} \simeq \boxed{D(T^2-T_0^2)\varphi^2}\,\boxed{-ET\varphi^3}+\boxed{\frac{\lambda_T}{4}\varphi^4}\;\underset{T=T_C}{\rightarrow}\;\frac{\lambda_{T_C}}{4}\varphi^2(\varphi-v_C)^2$$



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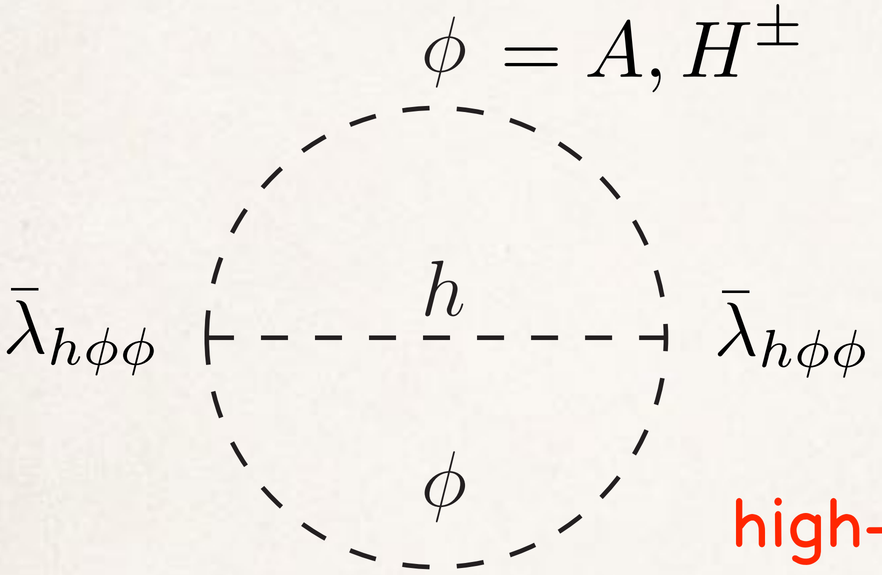
decoupling

Non-decoupling heavy Higgs bosons play a central role in enhancing E.

At 2-loop

Dominant 2-loop diagrams



$$\simeq \sum_{\phi} n_{\phi} \frac{T^2 \bar{\lambda}^2_{h\phi\phi} \varphi^2}{128\pi^2} \ln \frac{\bar{m}^2_{\phi}}{T^2},$$

high-T expansion

$$m_h^2 \ll m_A^2, m_{H^{\pm}}^2 \lesssim T^2.$$

Known fact: [J.E.Bagnasco and M.Dine, PLB303,308(1993)]

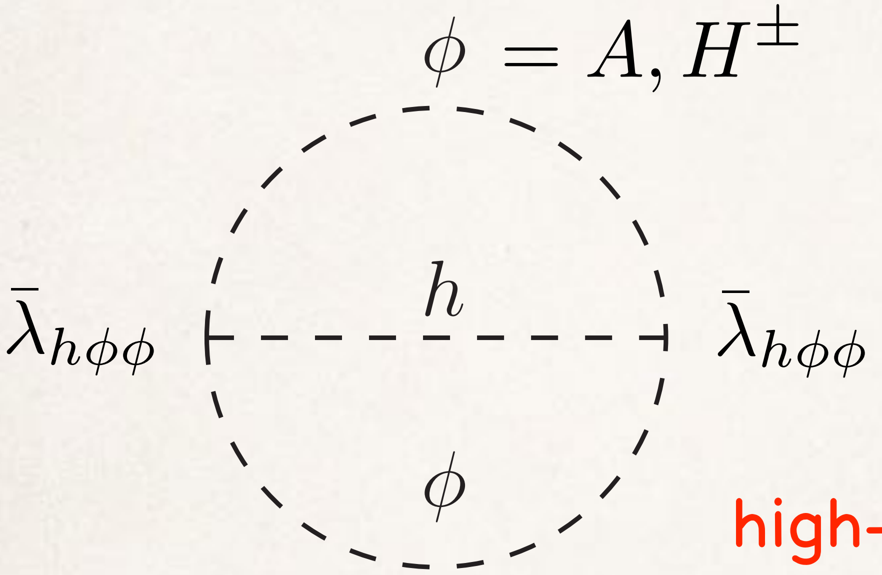
$+\varphi^2 \ln\left(\frac{\bar{m}^2}{T^2}\right) \longrightarrow$ 1st-order EWPT is weakened.

$-\varphi^2 \ln\left(\frac{\bar{m}^2}{T^2}\right) \longrightarrow$ 1st-order EWPT is strengthened.

At 2-loop, v_C/T_C would be weakened by extra Higgs bosons.

At 2-loop

Dominant 2-loop diagrams



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vc/Tc

$M_H = 62.7\ \mathrm{GeV},\ \lambda_2 = 0.02$ and $\bar{\lambda}_{hHH} = 4.6 \times 10^{-3}$ at M_Z.

$M_A = M_{H^\pm} \qquad \bar{\mu} = M_A$

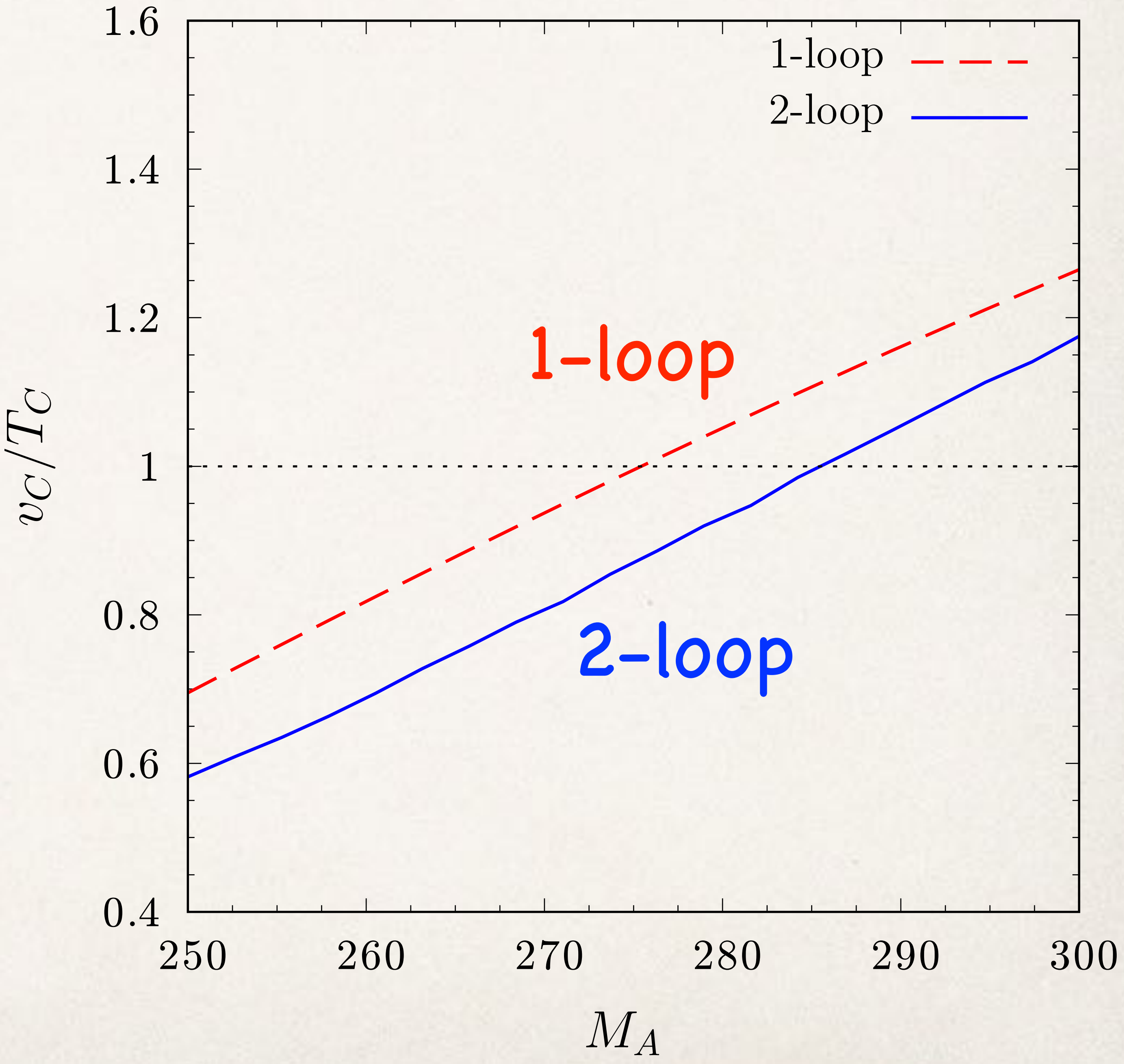
@ 2-loop

- vc/Tc is weakened by about (7-16)%.

consistent with Ref. [M.Laine, M.Meyer, G. Nardini, NPB920,565(2017).]

- Larger MA is needed to realize vc/Tc>1.

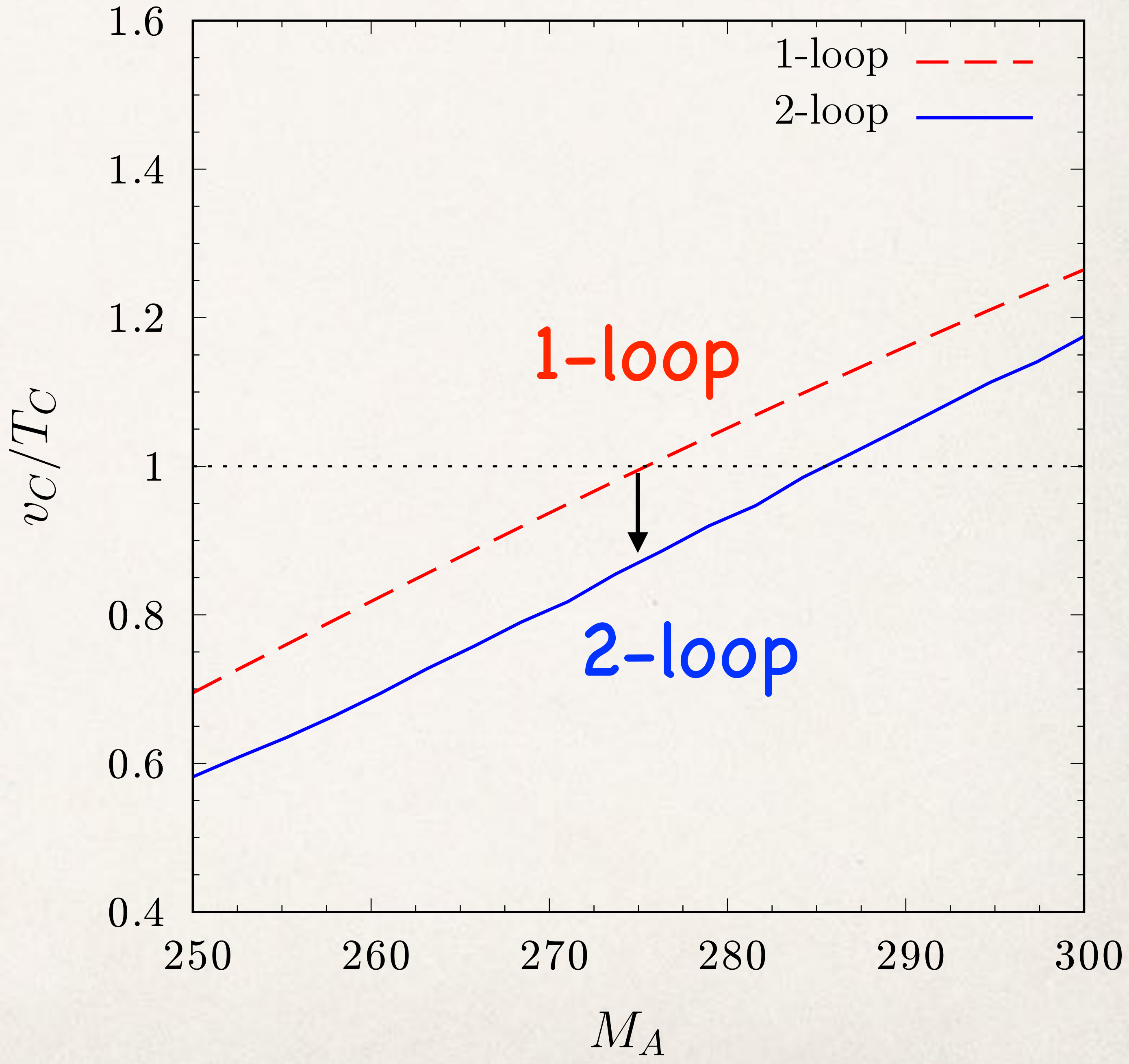
$\longrightarrow \quad \kappa_\lambda \nearrow$



v_C/T_C

$$M_H = 62.7\ \mathrm{GeV},\ \lambda_2 = 0.02 \text{ and } \bar{\lambda}_{hHH} = 4.6 \times 10^{-3} \text{ at } M_Z.$$

$$M_A = M_{H^\pm} \qquad \bar{\mu} = M_A$$



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- Larger M_A is needed to realize v_C/T_C>1.

$$\longrightarrow \quad \kappa_\lambda \nearrow$$

κ_λ-v_C/T_C correlations

$$M_H = 62.7 \text{ GeV}, \ \lambda_2 = 0.02 \text{ and } \bar{\lambda}_{hHH} = 4.6 \times 10^{-3} \text{ at } M_Z.$$

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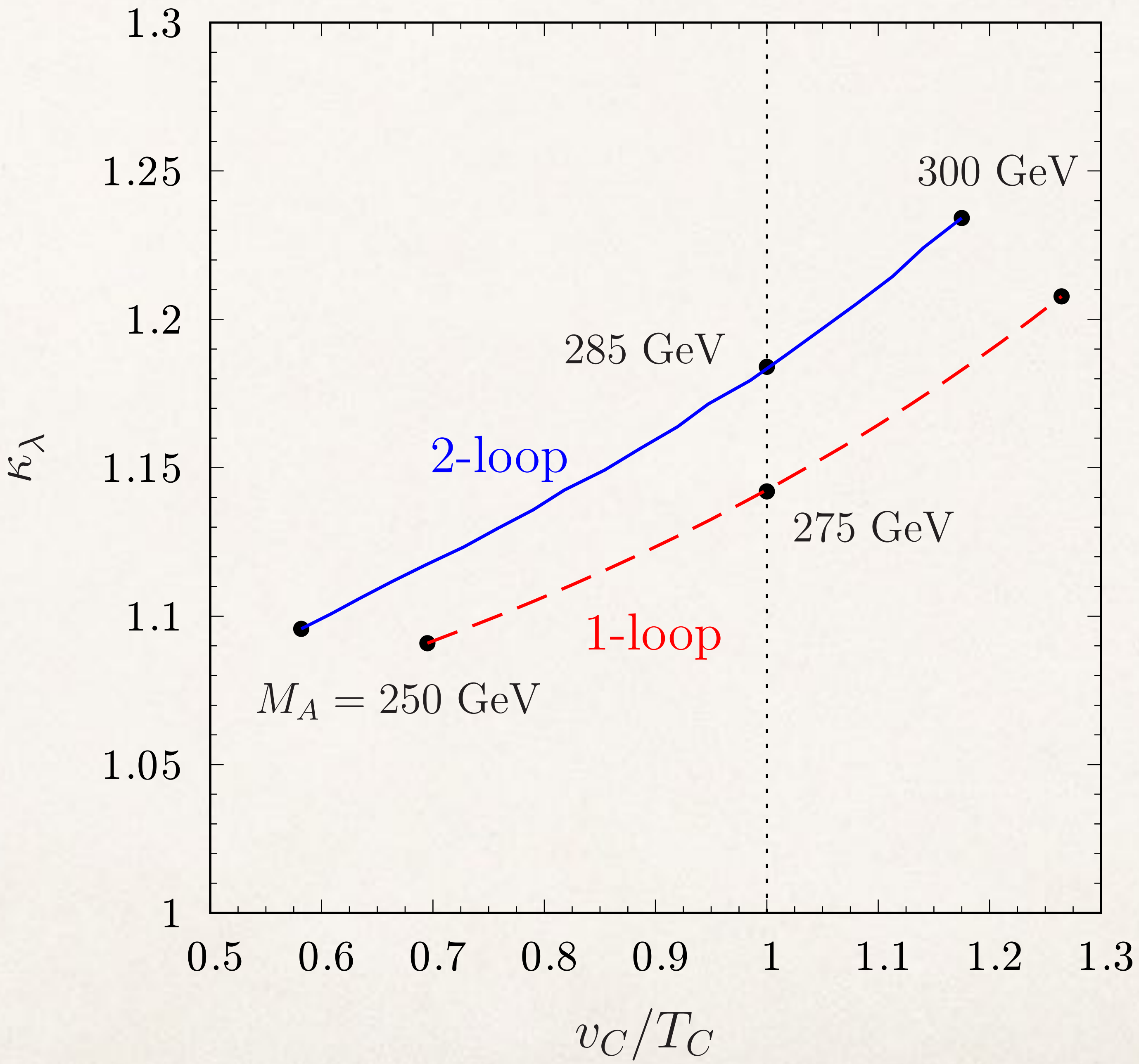


At 2-loop κ_λ is enhanced by about 4% for $v_C/T_C > 1$.

Summary

- We have discussed the hhh coupling and 1st-order EWPT at 2-loop level in the IDM.

@2-loop level

- κ_λ gets larger by at most 2%.
- 1st-order EWPT gets weaken by (7-16)%
- κ_λ-v_C/T_C correlation at 2-loop level is modified by about 4%.

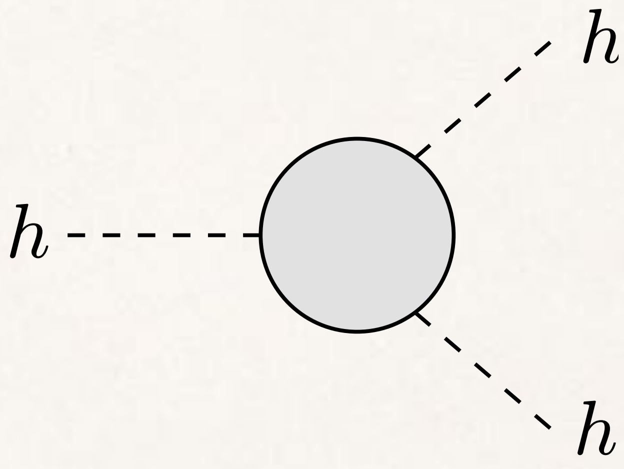
$\kappa_\lambda \gtrsim 1.18$ for $v_C/T_C \gtrsim 1$ @2-loop

This can be tested at future colliders!

Backup

Effective hhh vertex

We will evaluate effective hhh vertex



$$= \hat{\Gamma}_{hhh}(p_1^2, p_2^2, p_3^2)$$

Even though $\hat{\Gamma}_{hhh}(p_1^2, p_2^2, p_3^2) \neq \hat{\Gamma}_{hhh}(0,0,0)$, the ratio can be approximate by

$$\kappa_\lambda \equiv \frac{\hat{\Gamma}_{hhh}^{\mathrm{IDM}}(p_1^2, p_2^2, p_3^2)}{\hat{\Gamma}_{hhh}^{\mathrm{SM}}(p_1^2, p_2^2, p_3^2)} \simeq \frac{\hat{\Gamma}_{hhh}^{\mathrm{IDM}}(0,0,0)}{\hat{\Gamma}_{hhh}^{\mathrm{SM}}(0,0,0)}$$

*At 1-loop, this is the good approximation ($\lesssim$ 1% err. in regions of our interest). [S. Kanemura, Y. Okada, E.S. C.-P. Yuan, PRD70,115002(04)]

Effective potential method

$$-\hat{\Gamma}_{hhh}(0,0,0) \equiv \hat{\lambda}_{hhh} = \hat{Z}_h^{3/2}\lambda_{hhh},$$

on-shell

MS-bar

where

$$\hat{Z}_h = \frac{Z_h^{\rm OS}}{Z_h^{\overline{\rm MS}}} \qquad \lambda_{hhh} = \left.\frac{\partial^3 V_{\rm eff}}{\partial\varphi^3}\right|_{\varphi=v}$$

$V_{\rm eff}$ is the MS-bar regularized effective potential, which is expanded as

$$V_{\rm eff}(\varphi) = V_0(\varphi) + V_1(\varphi) + V_2(\varphi)$$

We first consider the SM.

λ_{hhh} in the SM

$$V_0(\Phi) = -\mu_\Phi^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger\Phi)^2, \quad \Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG^0) \end{pmatrix},$$

$$\boxed{\begin{aligned} m_h^2 &= \left.\frac{\partial^2 V_{\text{eff}}}{\partial\varphi^2}\right|_{\varphi=v} = 2\lambda v^2 + \mathcal{D}_m \Delta V_{\text{eff}}(\varphi), \\ \lambda_{hhh}^{\text{SM}} &= \left.\frac{\partial^3 V_{\text{eff}}}{\partial\varphi^3}\right|_{\varphi=v} = \frac{3m_h^2}{v} + \mathcal{D}_\lambda \Delta V_{\text{eff}}(\varphi), \end{aligned}}$$

where

$$\mathcal{D}_m = \left[\frac{\partial^2}{\partial\varphi^2} - \frac{1}{v}\frac{\partial}{\partial\varphi}\right]_{\varphi=v}, \quad \mathcal{D}_\lambda = \left[\frac{\partial^3}{\partial\varphi^3} - \frac{3}{v}\left(\frac{\partial^2}{\partial\varphi^2} - \frac{1}{v}\frac{\partial}{\partial\varphi}\right)\right]_{\varphi=v},$$

$$\Delta V_{\text{eff}}(\varphi) = V_1(\varphi) + V_2(\varphi)$$

NOTE: Higgs pole mass (M_h)

$$M_h^2 = m_h^2 + \text{Re}\Sigma_h(M_h) - \text{Re}\Sigma_h(0).$$

At 1-loop

MS-bar regularized 1-loop effective potential

$$V_1(\varphi) = \sum_i c_i \frac{\bar{m}_i^4}{4(16\pi^2)} \left(\ln \frac{\bar{m}_i^2}{\bar{\mu}^2} - c_i \right)$$

$\bar{m}_i$: field-dependent masses of particle i

$\bar{\mu}$: renormalization scale

c=3/2 (scalars, fermions)

c=5/6 (gauge bosons)

Dominant 1-loop correction comes from the top loop.

[W. Hollik and S. Penaranda, EPJC23,163(2002)]

$$\lambda_{hhh}^{\rm SM} = \frac{3m_h^2}{v}\left[1 + \frac{1}{16\pi^2}\left(-\frac{16m_t^4}{m_h^2 v^2}\right)\right]$$

Power correction!! (*log corr. are absorbed into m$_h$.)

New physics effects can also be power like (see later).

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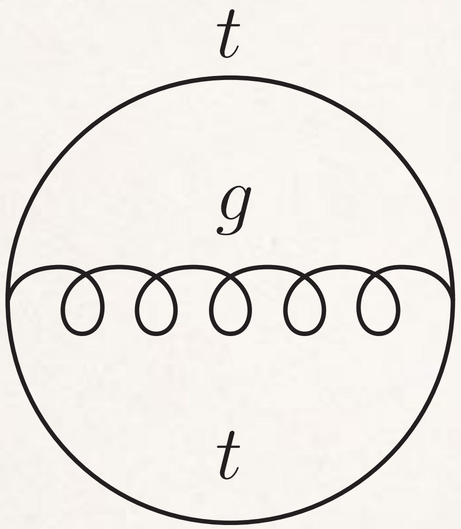
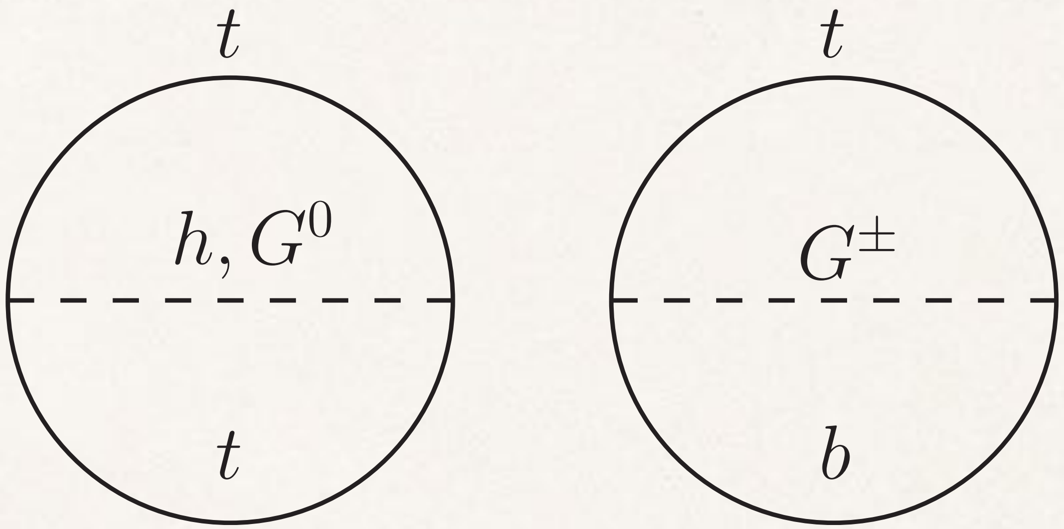
At 2-loop

Dominant diagrams



$\mathcal{O}(g_3^2 y_t^4)$



$\mathcal{O}(y_t^6)$

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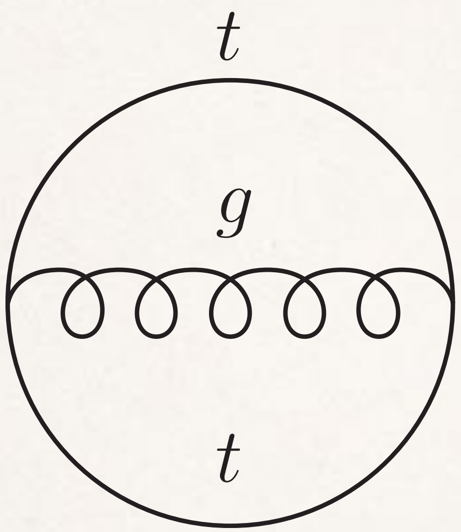
where $\ell_t = \ln\frac{m_t^2}{\bar{\mu}^2}$ (*All parameters are MS-bar variables.)

At 2-loop

Dominant diagrams



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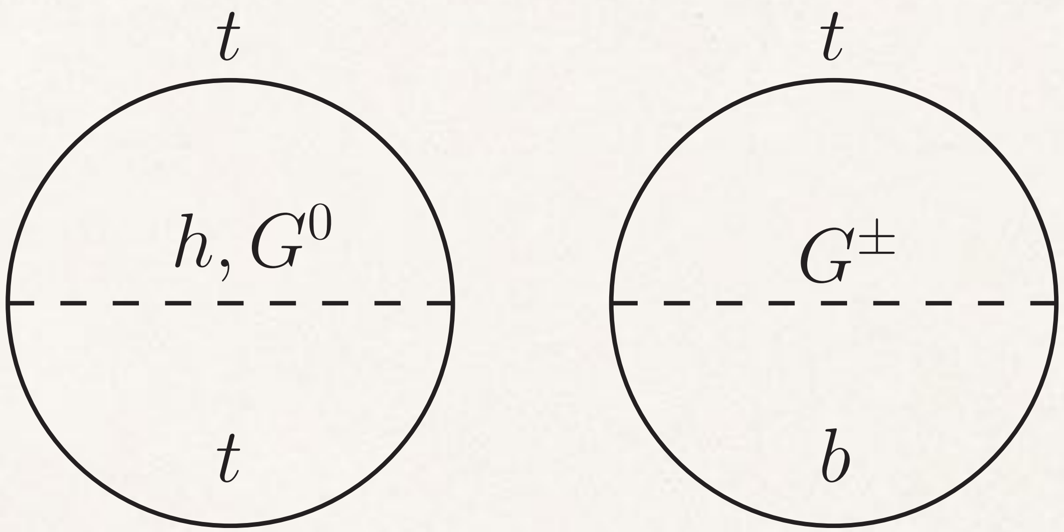

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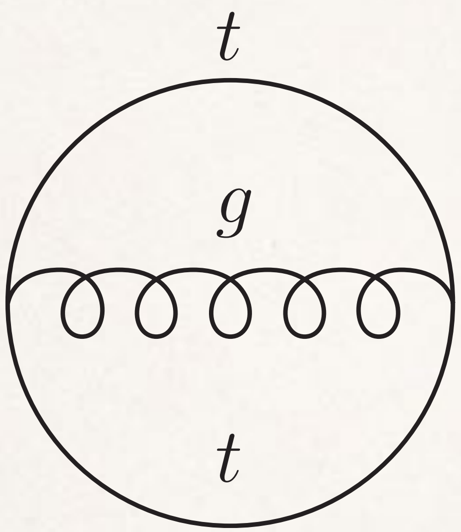

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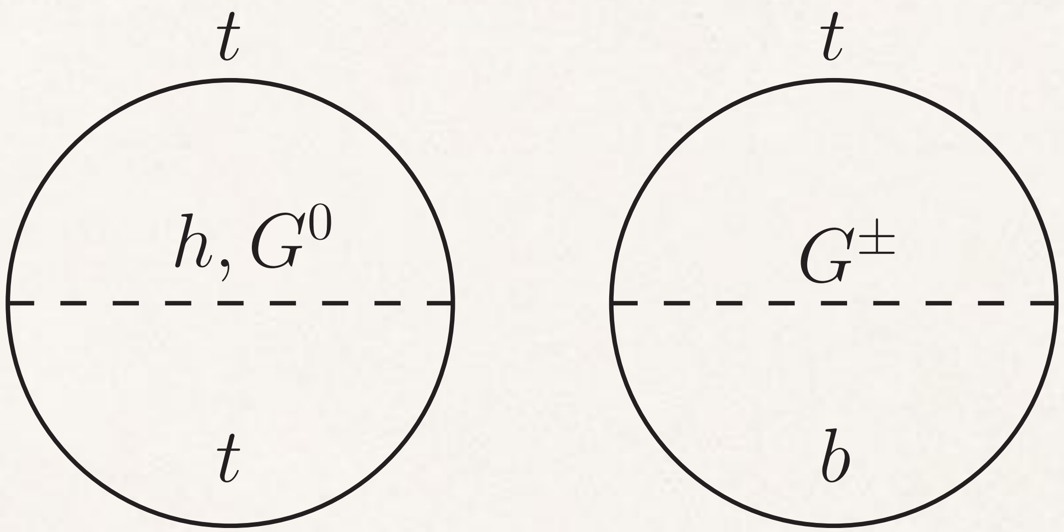

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$$\lambda_{hhh}^{\rm SM} = \frac{3m_h^2}{v}\left[1 + \frac{1}{16\pi^2}\left(-\frac{16m_t^4(m_t)}{m_h^2 v^2}\right) + \frac{1}{(16\pi^2)^2}\frac{m_t^4}{m_h^2 v^2}\left(\frac{128g_3^2}{3} + 56y_t^2\right)\right],$$

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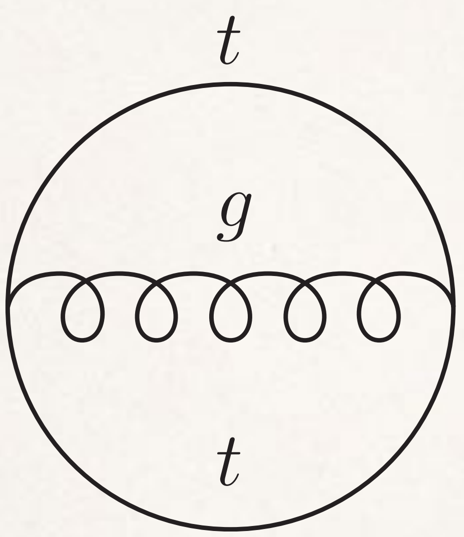

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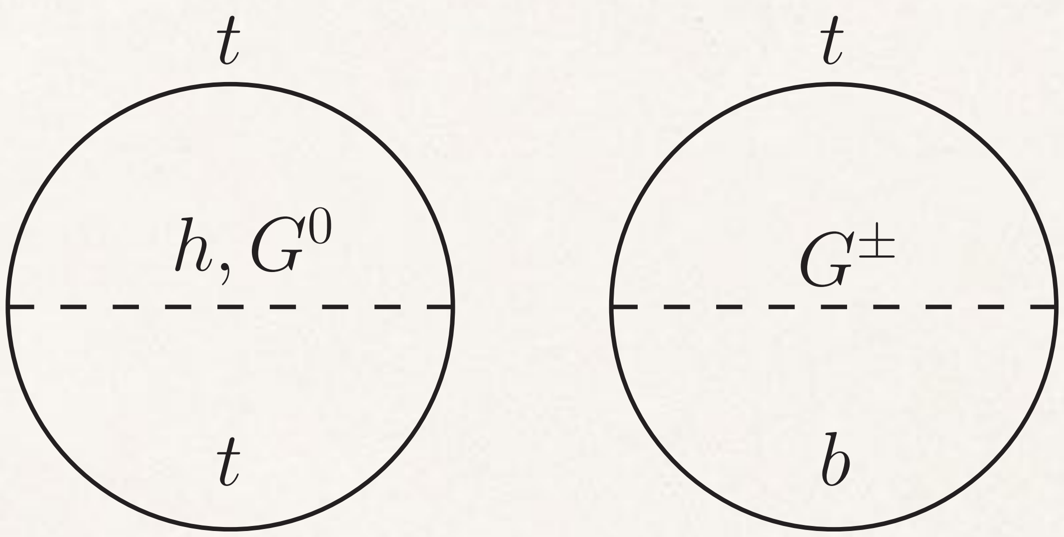

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log terms are absorbed by running top mass.

At 2-loop

After expressing the MS-bar parameters with OS ones, one gets

$$\hat{\lambda}_{hhh}^{\text{SM}} \simeq \frac{3M_h^2}{v_{\text{phys}}} \left[1 + \frac{1}{16\pi^2} \left(-\frac{16M_t^4}{M_h^2 v_{\text{phys}}^2} + \frac{7}{2}\frac{M_t^2}{v_{\text{phys}}^2} \right) + \frac{1}{(16\pi^2)^2} \frac{16M_t^4}{M_h^2 v_{\text{phys}}^2} \left(24g_3^2 + \frac{7M_t^2}{v_{\text{phys}}^2} \right) \right]$$

$$= (190.4 \text{ GeV}) \times \left[1 - 8.5\% + 2.2\% \right] = 178.4 \text{ GeV},$$

$$v_{\text{phys}}^2 = 1/(\sqrt{2}G_F) = (246.22 \text{ GeV})^2$$

2-loop contribution is about 1/4 of 1-loop top effect.

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$$=(190.4\ \text{GeV})\times\Big[1-8.5\%+2.2\%\Big]=178.4\ \text{GeV},$$

$$v_{\text{phys}}^2=1/(\sqrt{2}G_F)=(246.22\ \text{GeV})^2$$

2-loop contribution is about 1/4 of 1-loop top effect.

Higgs masses

$$
\begin{aligned}
m_h^2 &= \lambda_1 v^2, \\
m_H^2 &= \mu_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)v^2, \\
m_A^2 &= \mu_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5)v^2, \\
m_{H^\pm}^2 &= \mu_2^2 + \frac{1}{2}\lambda_3 v^2.
\end{aligned}
$$

- Heavy Higgs masses come from both μ_2^2 and symmetry breaking term (λv^2).

- We may know which part is dominant by measuring Higgs couplings precisely.