Primordial Black Holes in Inflation

Kin-Wang Ng (吳建宏)

Institute of Physics &
Institute of Astronomy and Astrophysics
Academia Sinica, Taiwan

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Collaborators: Wolung Lee (NTNU)
Shu-Lin Cheng (NTNU)

Primordial Black Holes

- Formed at high-density contrasts (*$\delta\rho/\rho \sim 0.5$*) over a wide range of scales or masses in the radiation-dominated Universe
- There have been stringent astrophysical and cosmological constraints on M_{PBH}
- $10M_{\odot}$ PBHs could be the binary BHs observed by aLIGO gravity-wave detectors

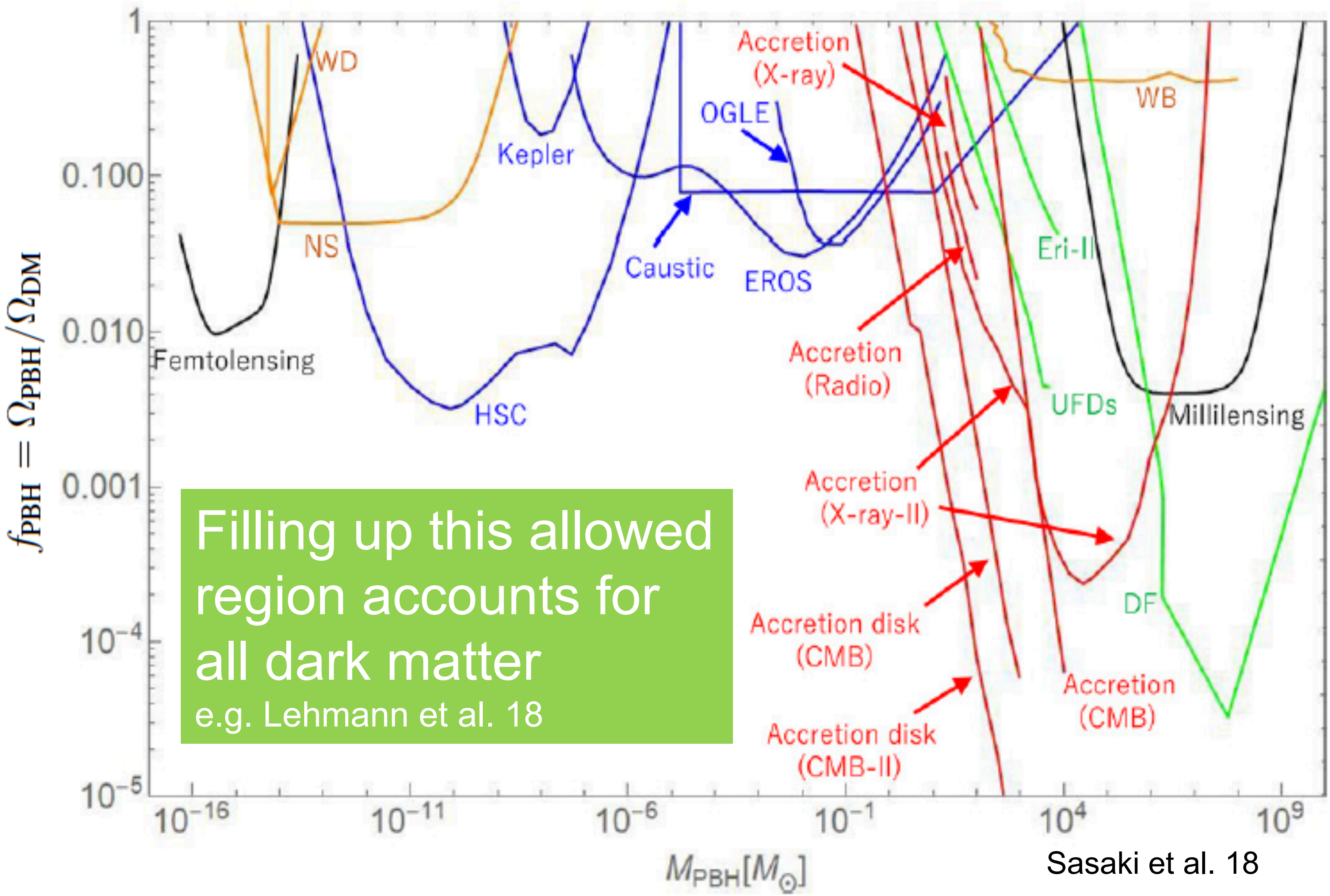
  Bird et al. 16., Clesse et al. 16, Sasaki et al. 16

- PBHs behave like cold dark matter

  García-Bellido, Linde, Wands 96

- They, although being of baryonic origin, do not participate in big-bang nucleosynthesis

Astrophysical and Cosmological Constraints on PBHs

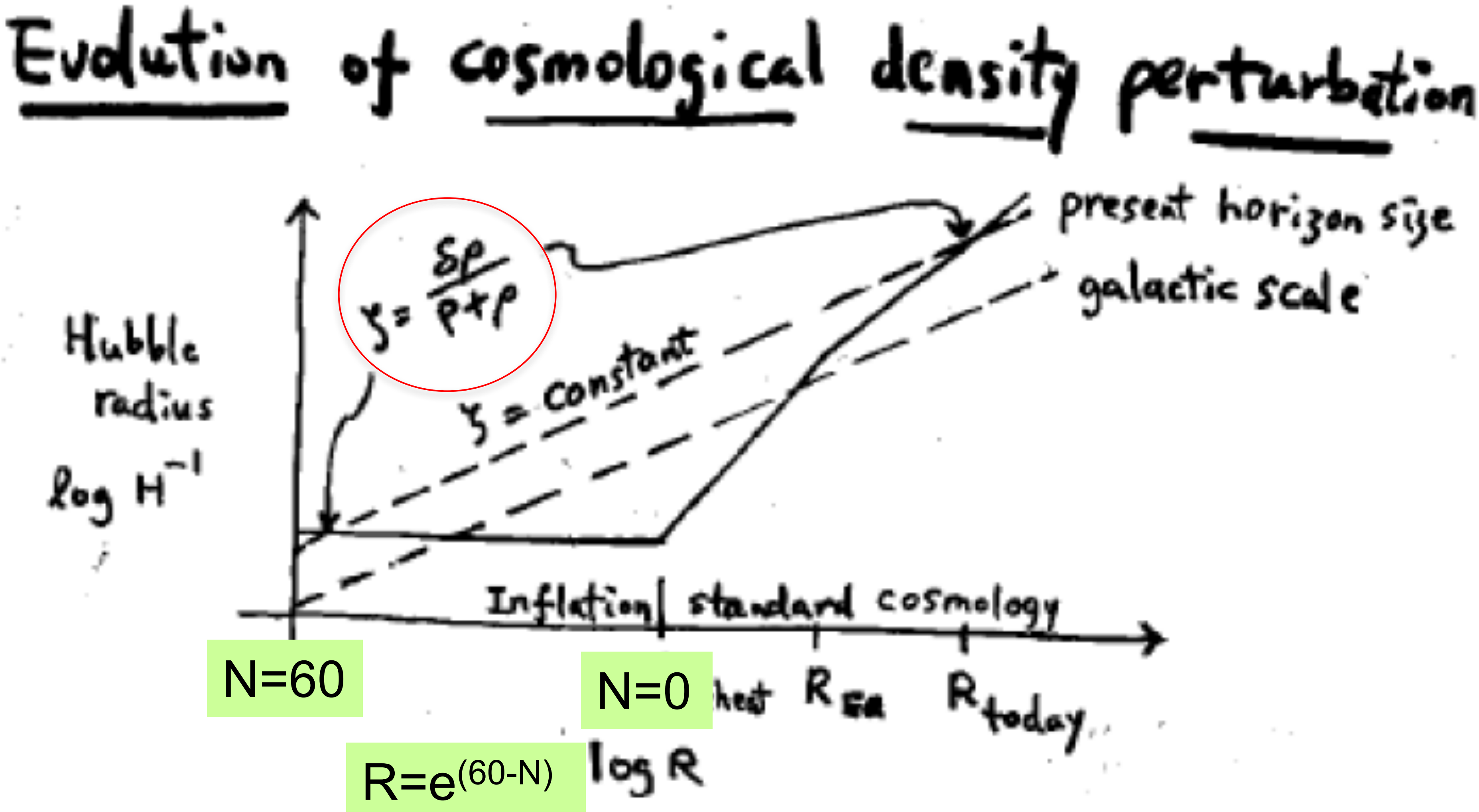


Filling up this allowed region accounts for all dark matter

e.g. Lehmann et al. 18

Sasaki et al. 18

Production of PBHs in inflation



$$M_{\rm BH} = \frac{4\pi M_p^2}{H} e^{2N} = 2.74 \times 10^{-38} e^{2N} \left(\frac{M_p}{H}\right) M_\odot$$

N is e-foldings before the end of inflation

PBH Production in Inflation

- Single-field slow-roll inflation models, matter density perturbation ($\delta\rho/\rho \sim 10^{-5}$) too small
- Modified inflation potential to achieve blue-tilted matter power spectra or running spectral indices, leading to large density perturbation at the end of inflation, but mostly $M_{PBH} << M_{\odot}$
  García-Bellido, Linde, Wands 96
- To boost M_{PBH} , hybrid inflation, double inflation, curvaton models by inflating small-scale density perturbation to the size of a stellar-mass to supermassive PBH Kawasaki, Kohri, Yokoyama, Yanagida….
- Inflation with an inflecton point Garcia-Bellido, Morales,…
- Trapped inflation Peloso, Unal, Ng+,…

Trapped axion Inflation

We consider a version of the trapped inflation driven by a pseudoscalar φ that couples to a $U(1)$ gauge field A_μ:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\alpha}{4f}\varphi\tilde{F}^{\mu\nu}F_{\mu\nu}\right], \quad (3)$$

$$\varphi = \phi(\eta) + \delta\varphi(\eta, \vec{x})$$

$d\eta = dt/a$

Under the temporal gauge, $A_\mu = (0, \vec{A})$, we decompose $\vec{A}(\eta, \vec{x})$ into its right and left circularly polarized Fourier modes, $A_\pm(\eta, \vec{k})$, whose equation of motion is then given by

$k/(aH) < 2|\xi|$

Spinoidal instability

$$\left[\frac{d^2}{d\eta^2} + k^2 \mp 2aHk\xi\right]A_\pm(\eta, k) = 0, \quad \xi \equiv \frac{\alpha}{2fH}\frac{d\phi}{dt}. \quad (5)$$

Background

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} + \frac{dV}{d\phi} = \frac{\alpha}{f}\langle \vec{E}\cdot\vec{B}\rangle,$$

$$3H^2 = \frac{1}{M_p^2}\left[\frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + V(\phi) + \frac{1}{2}\langle \vec{E}^2 + \vec{B}^2\rangle\right]$$

$$\langle \vec{E}\cdot\vec{B}\rangle \simeq -2.4\cdot 10^{-4}\frac{H^4}{\xi^4}\,e^{2\pi\xi},$$

$$\left\langle\frac{\vec{E}^2+\vec{B}^2}{2}\right\rangle \simeq 1.4\cdot 10^{-4}\frac{H^4}{\xi^3}\,e^{2\pi\xi}.$$

$$\frac{1}{2}\langle \vec{E}^2 + \vec{B}^2\rangle = \int \frac{dk\,k^2}{4\pi^2 a^4}\sum_{\lambda=\pm}\left(\left|\frac{dA_\lambda}{d\eta}\right|^2 + k^2|A_\lambda|^2\right),$$

$$\langle \vec{E}\cdot\vec{B}\rangle = -\int \frac{dk\,k^3}{4\pi^2 a^4}\frac{d}{d\eta}\left(|A_+|^2 - |A_-|^2\right).$$

Perturbation

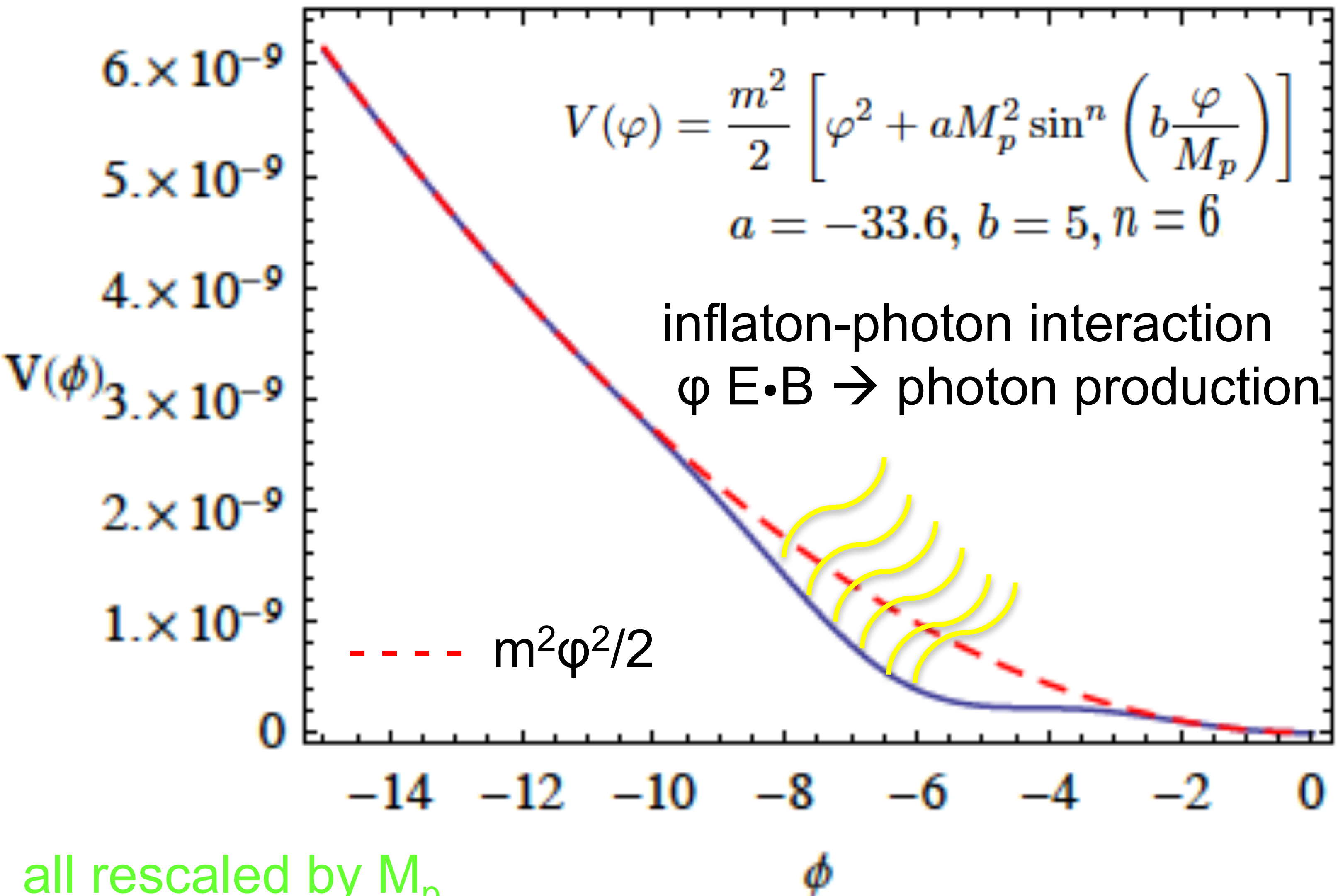
$$\left[\frac{\partial^2}{\partial t^2} + 3\beta H\frac{\partial}{\partial t} - \frac{\vec{\nabla}^2}{a^2} + \frac{d^2V}{d\phi^2}\right]\delta\varphi(t,\vec{x}) = \frac{\alpha}{f}\left(\vec{E}\cdot\vec{B} - \langle \vec{E}\cdot\vec{B}\rangle\right)$$

$$\delta\varphi = \frac{\alpha}{3\beta f H^2}\left(\vec{E}\cdot\vec{B} - \langle \vec{E}\cdot\vec{B}\rangle\right)$$

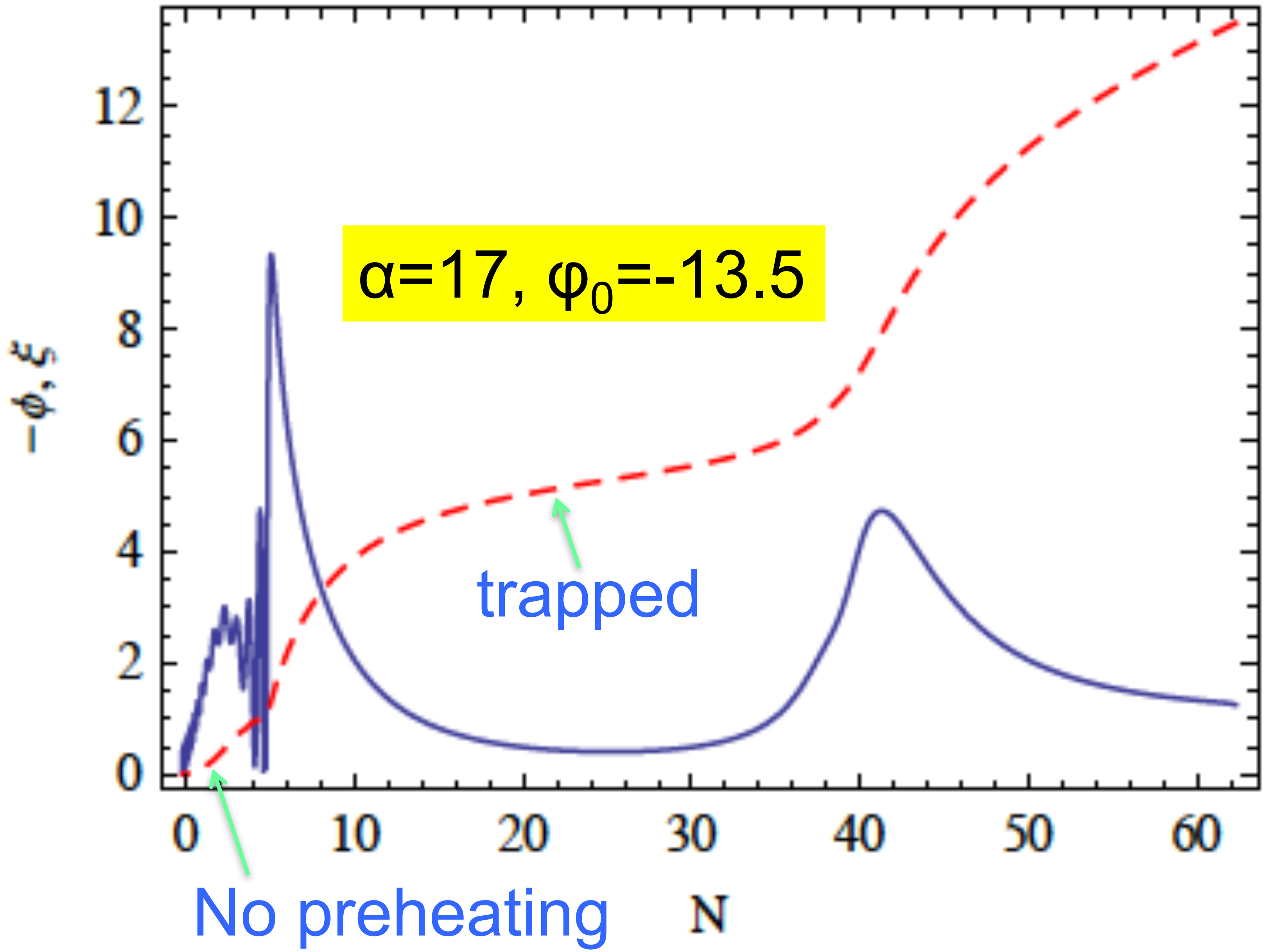
$$\beta \equiv 1 - 2\pi\xi\frac{\alpha}{f}\frac{\langle \vec{E}\cdot\vec{B}\rangle}{3H(d\phi/dt)}$$

$$\Delta_\zeta^2(k) = \langle\zeta(x)^2\rangle = \frac{H^2\langle\delta\varphi^2\rangle}{(d\phi/dt)^2} = \left[\frac{\alpha\langle \vec{E}\cdot\vec{B}\rangle}{3\beta f H(d\phi/dt)}\right]^2$$

e.g. Trapped axion inflation with a steep potential Cheng, Lee, Ng 16

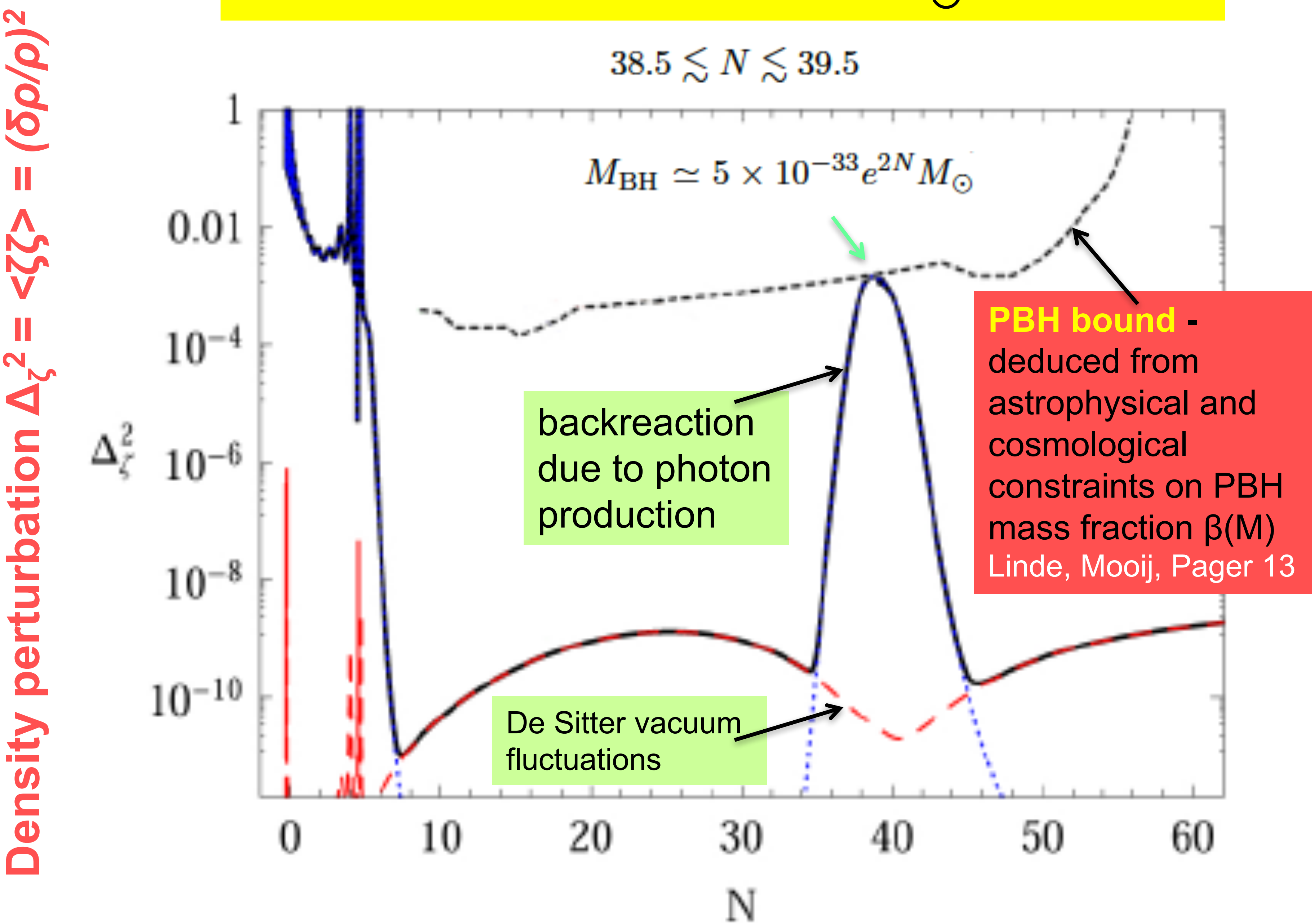


all rescaled by M_p



Note that inflation starts at N~60

Production of 10-100$M_\odot$ PBHs



PBH Associated Gravitational Waves in Inflation

$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta} - \vec{\nabla}^2\right] h_{ij} = 0$$

Free gravitational wave equation

De Sitter vacuum fluctuations during inflation lead to almost scale-invariant primordial gravitational waves

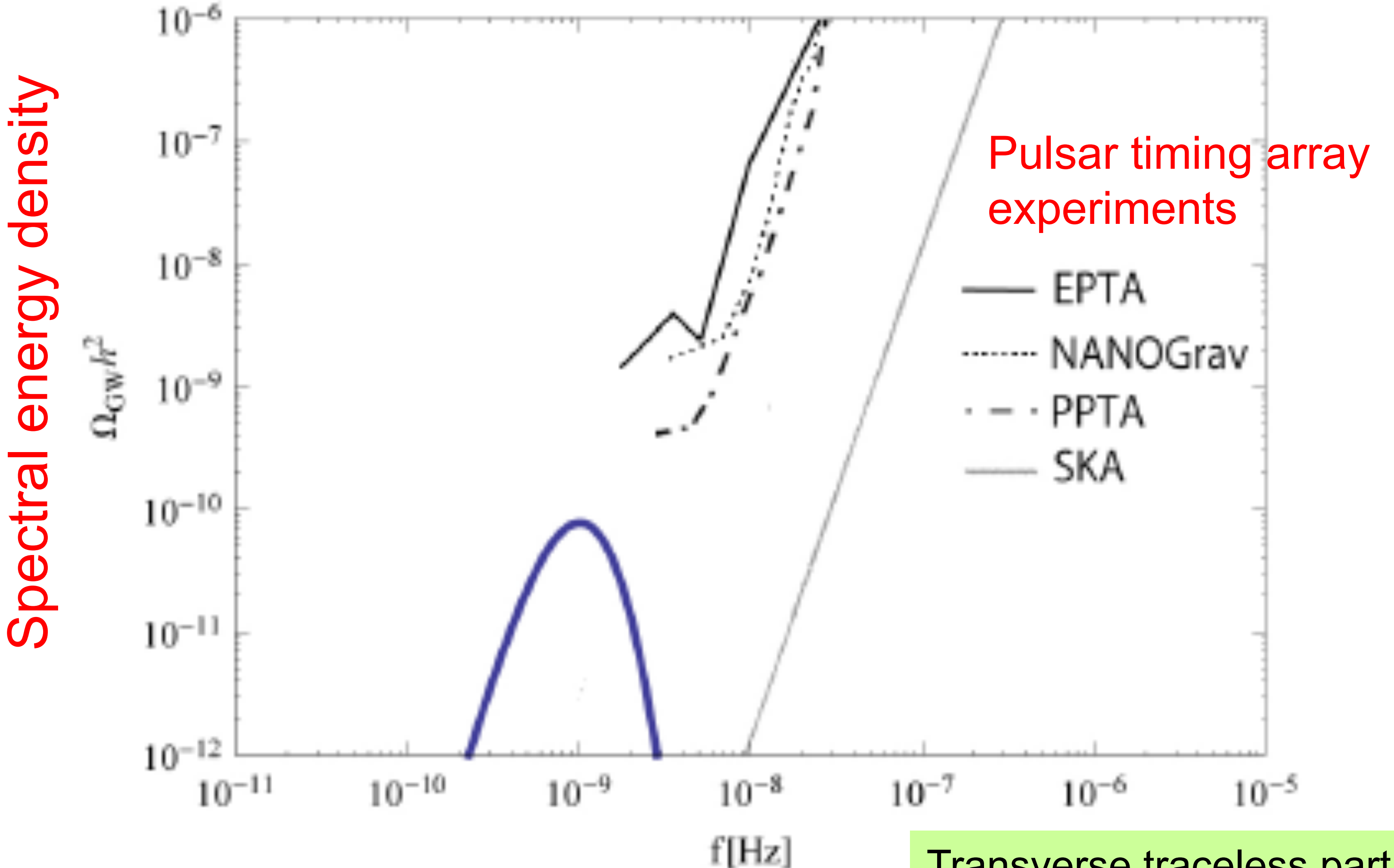
$\Delta_\zeta^2 = \langle\zeta\zeta\rangle = (\delta\rho/\rho)^2 \sim 10^{-10}$

$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta} - \vec{\nabla}^2\right] h_{ij} = GT_{ij}$$

Sources due to transverse traceless part of 2nd density perturbation $\zeta\zeta$

$\Delta_\zeta^2 = \langle\zeta\zeta\rangle = (\delta\rho/\rho)^2 \sim 10^{-3}$

Associated Gravitational Waves in Trapped Inflation



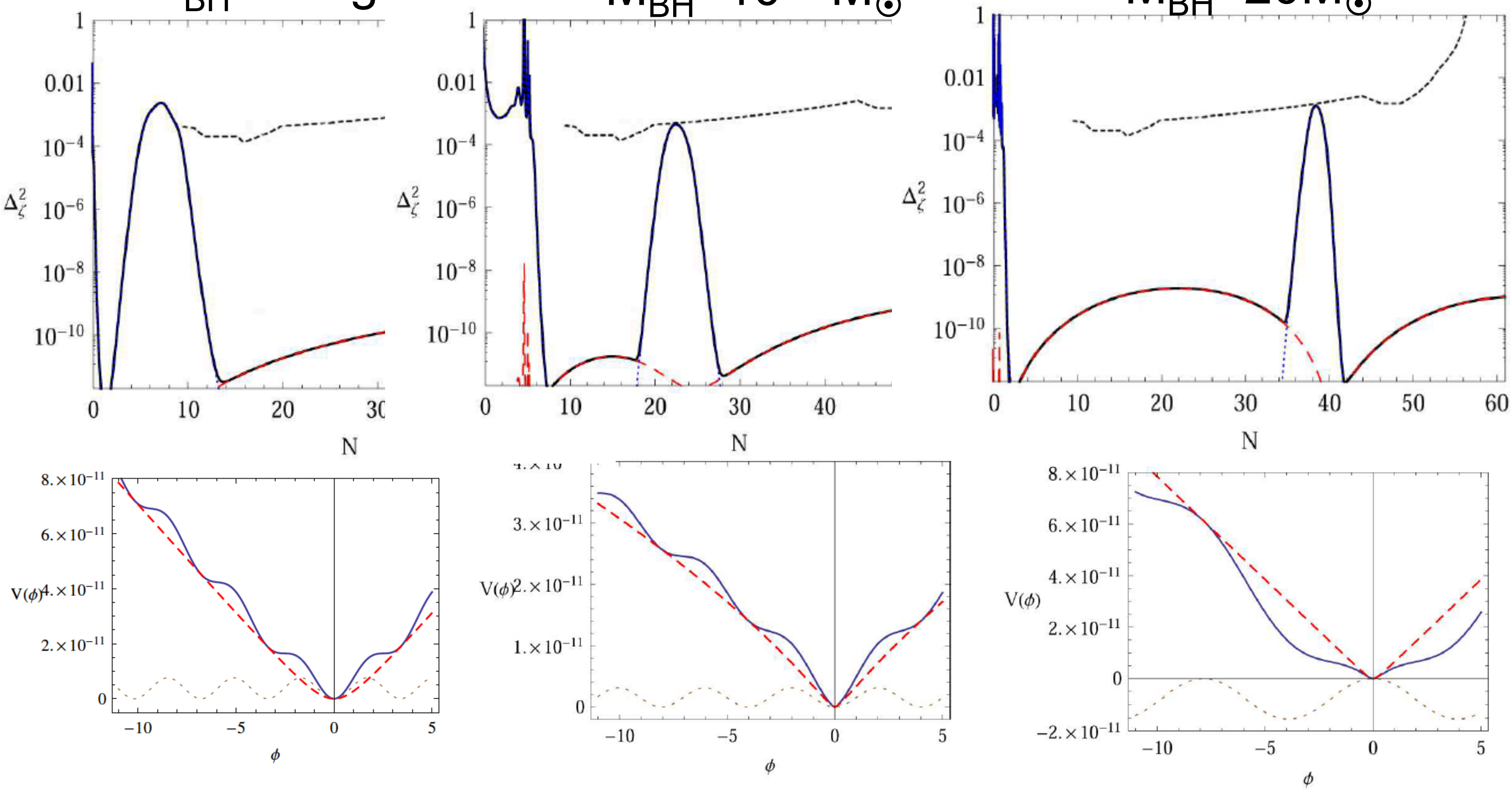
$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta} - \vec{\nabla}^2\right] h_{ij} = \frac{2a^2}{M_p^2}\left(-E_iE_j - B_iB_j\right)^{TT}$$

Transverse traceless part of photon energy-momentum

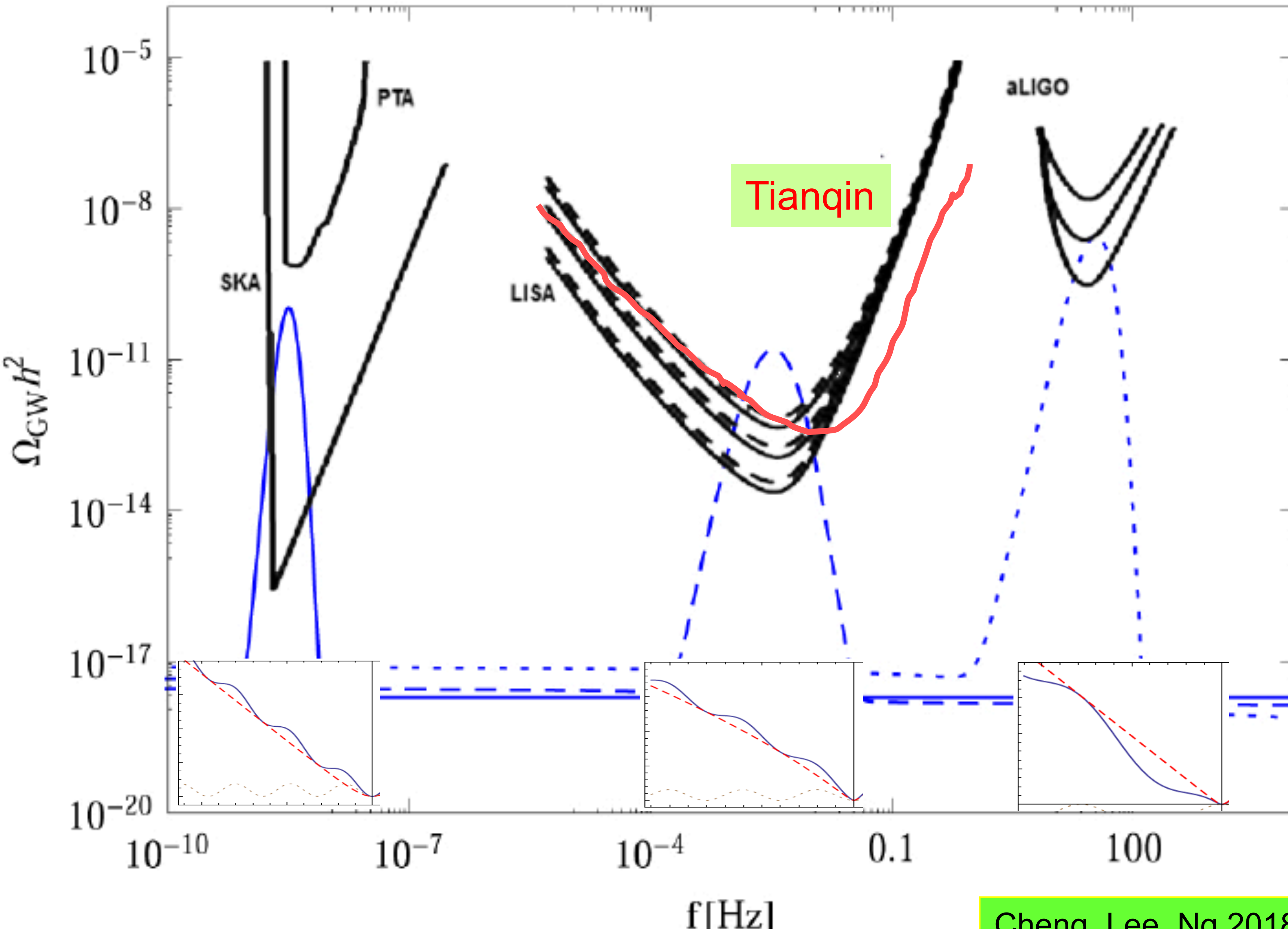
<E·E> ~ <E·B> ~ ζ

Production of PBHs realized in axion monodromy inflation with sinusoidal modulations

M_{BH}~10^8 g　　M_{BH}~$10^{-13}M_\odot$　　M_{BH}~$20M_\odot$

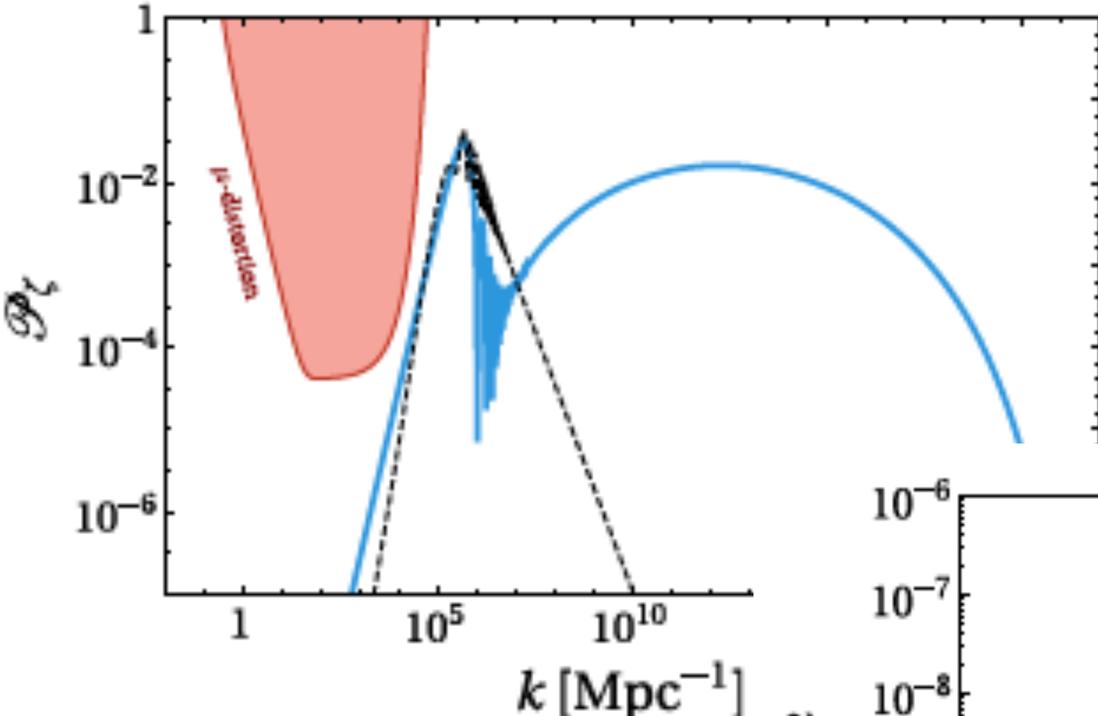


GWs associated with PBHs in modulated axion inflation

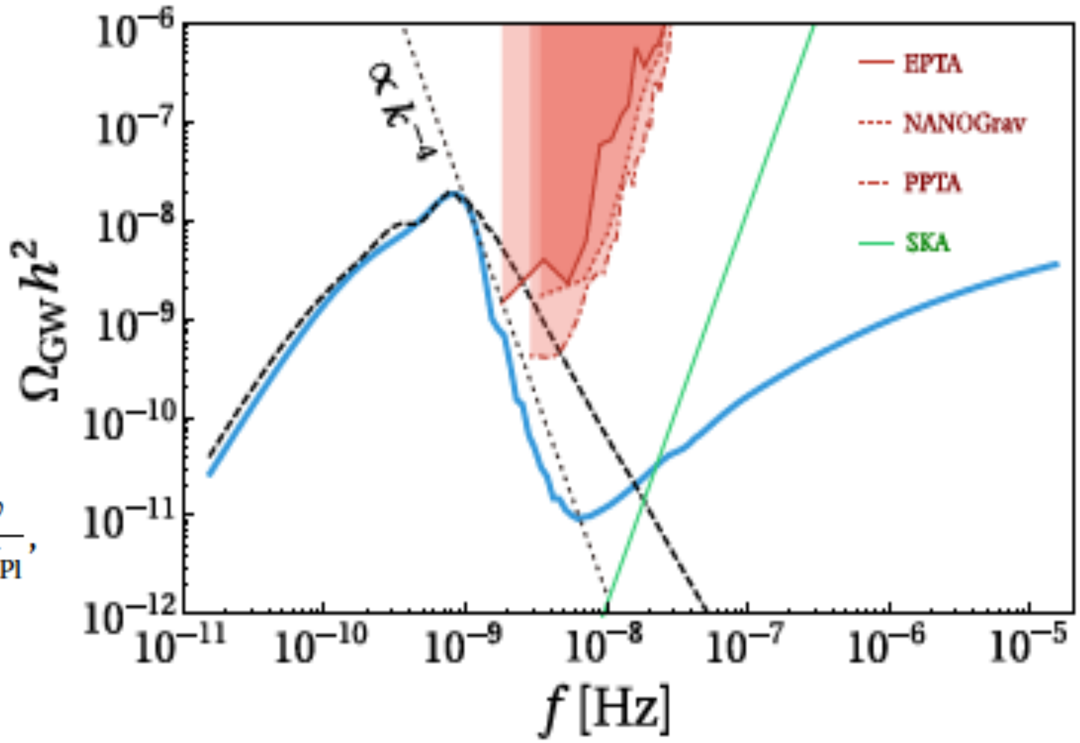


Cheng, Lee, Ng 2018

Broader spectrum of PBHs and GWs in other inflation models



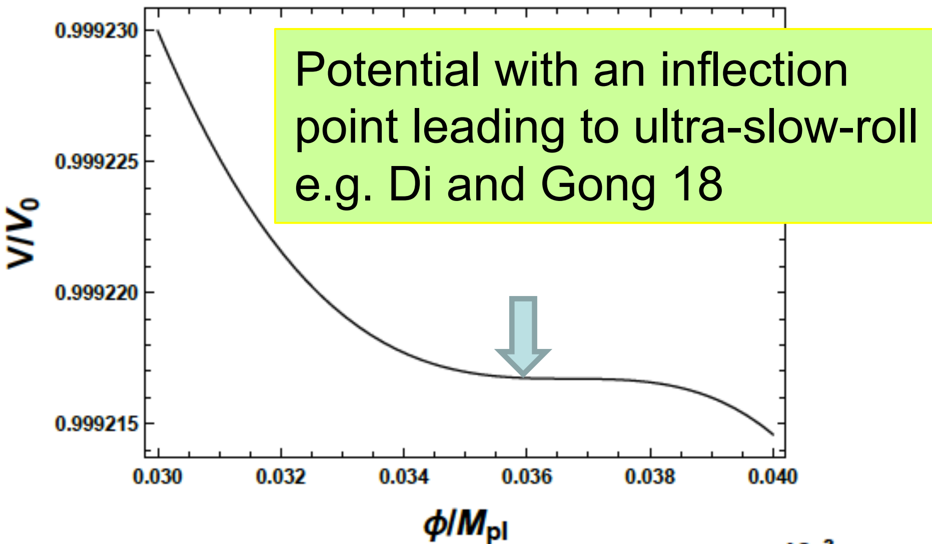
Double inflation
e.g. Inomata et al. 16

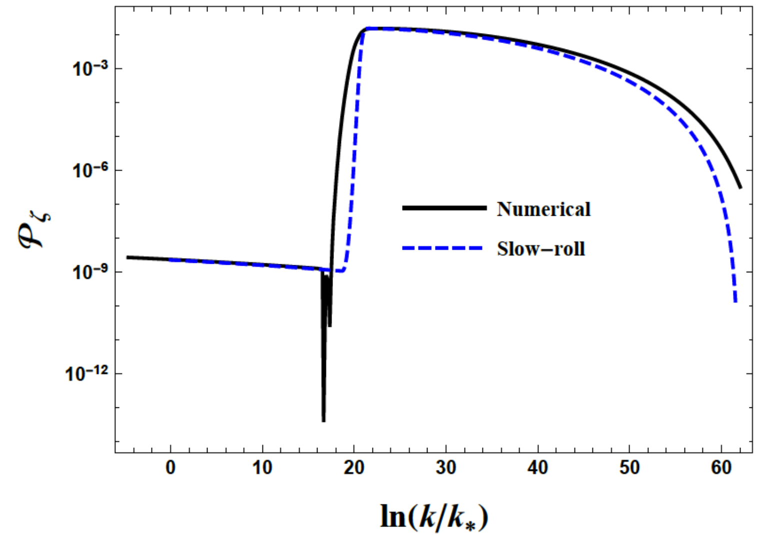


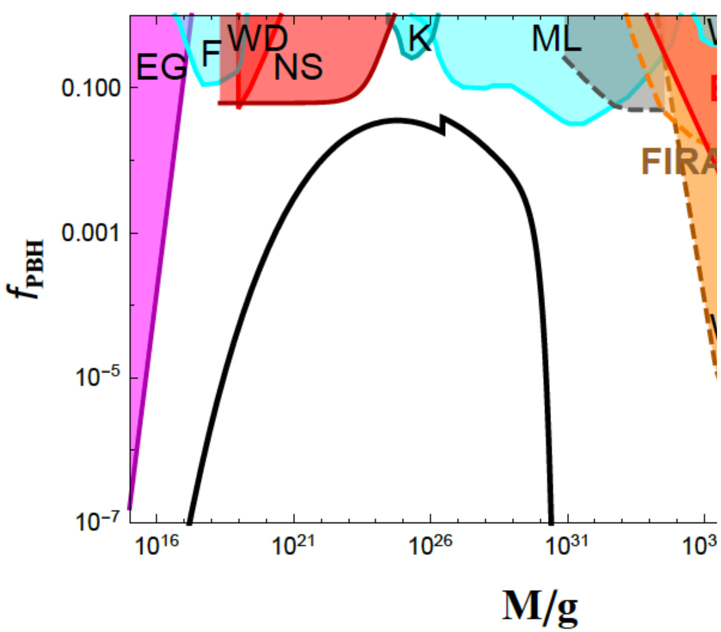
$$V(\phi,\varphi)=V_{\rm pre}(\phi)+V_{\rm stb}(\phi,\varphi)+V_{\rm new}(\varphi),$$

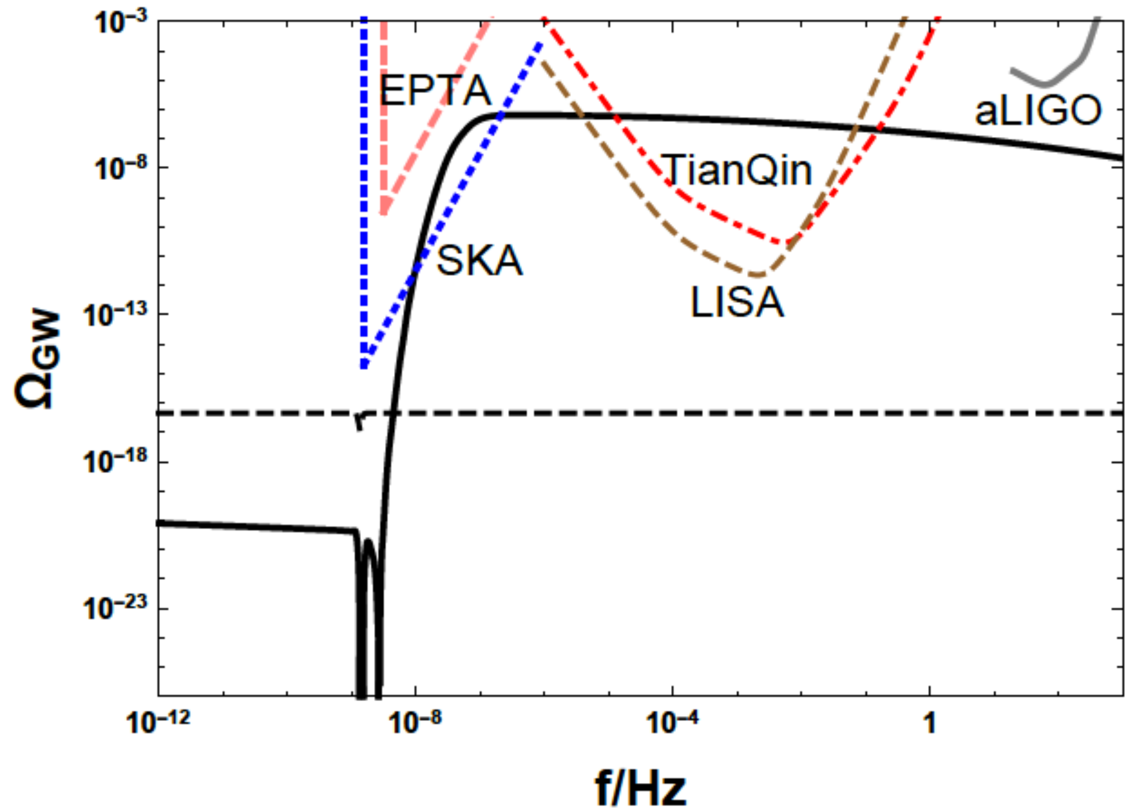
$$V_{\rm new}(\varphi)=\left(v^2-g\frac{\varphi^n}{M_{\rm Pl}^{n-2}}\right)^2-\kappa\frac{\varphi^2}{2M_{\rm Pl}^2}-\varepsilon v^4\frac{\varphi}{M_{\rm Pl}},$$

$$V_{\rm stb}(\phi,\varphi)=c_{\rm pot}\frac{V_{\rm pre}(\phi)}{2M_{\rm Pl}^2}\varphi^2,$$

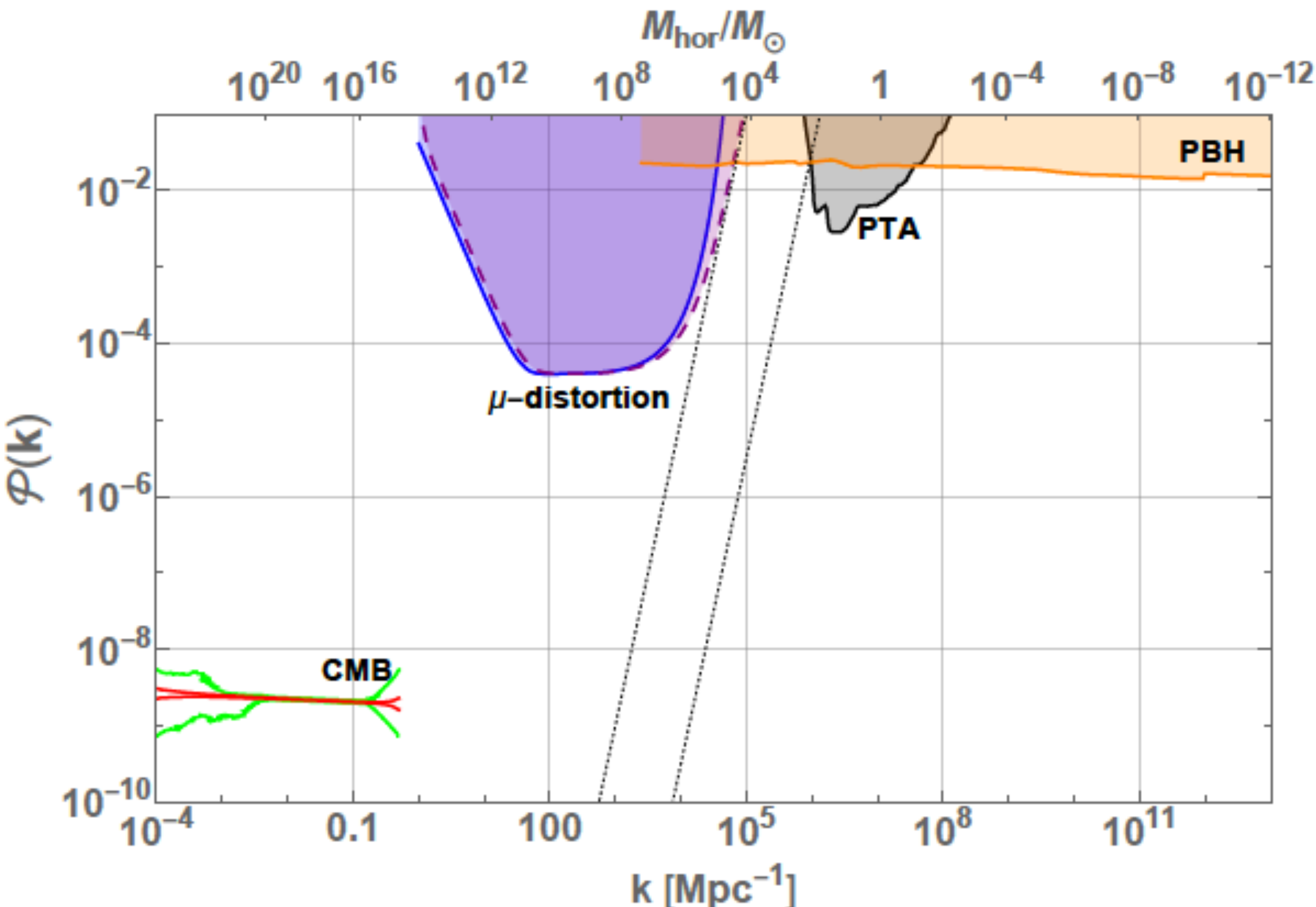

Potential with an inflection point leading to ultra-slow-roll e.g. Di and Gong 18
V/V0
0.999230
0.999225
0.999220
0.999215
0.030
0.032
0.034
0.036
0.038
0.040
ϕ/Mpl


Numerical
Slow–roll
10^-3
10^-6
10^-9
10^-12
0
10
20
30
40
50
60
ln(k/k*)


EG
F
WD
NS
K
ML
FIRA
0.100
0.001
10^-5
10^-7
10^16
10^21
10^26
10^31
fPBH
M/g

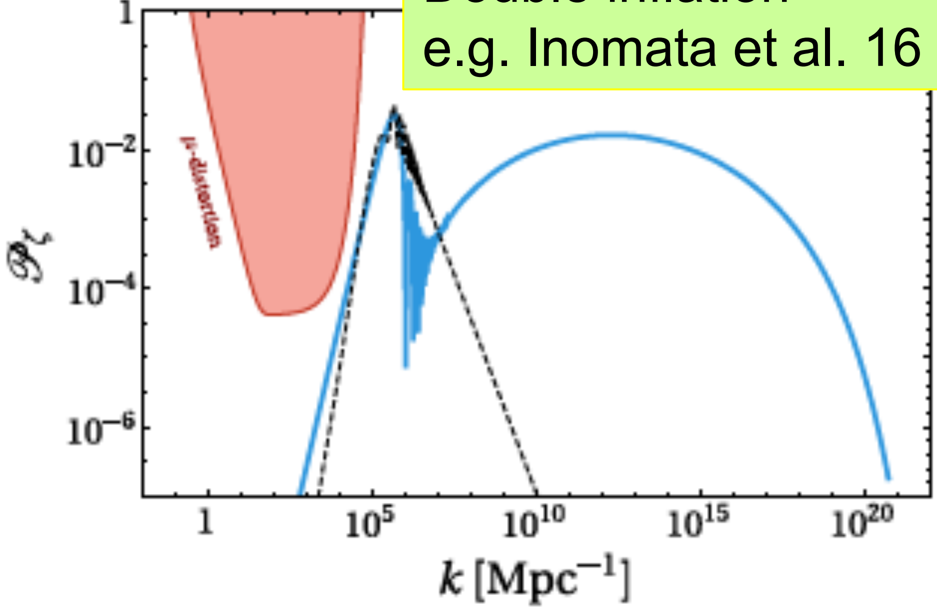

EPTA
SKA
TianQin
LISA
aLIGO
10^-3
10^-8
10^-13
10^-18
10^-23
10^-12
10^-8
10^-4
1
ΩGW
f/Hz

How many and how big PBHs from Inflation

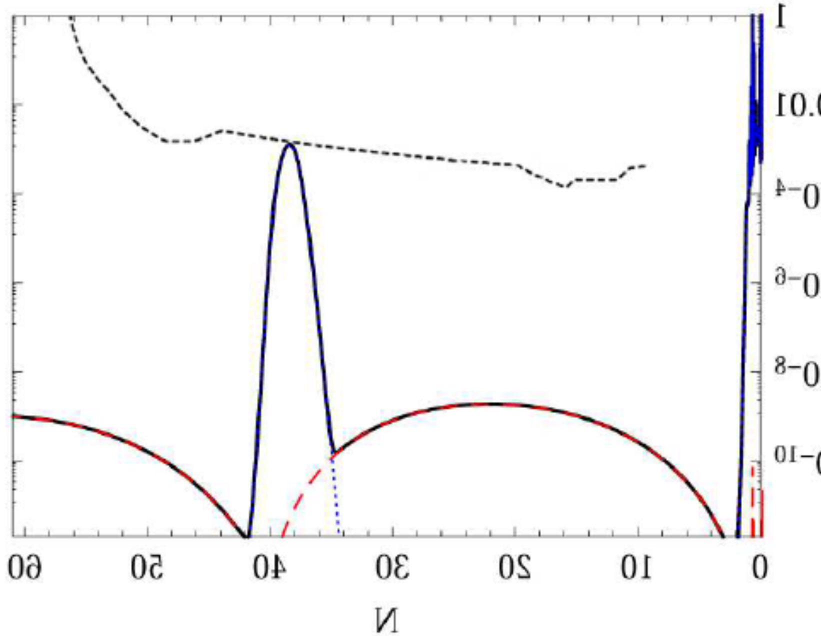


Byrnes+ 18

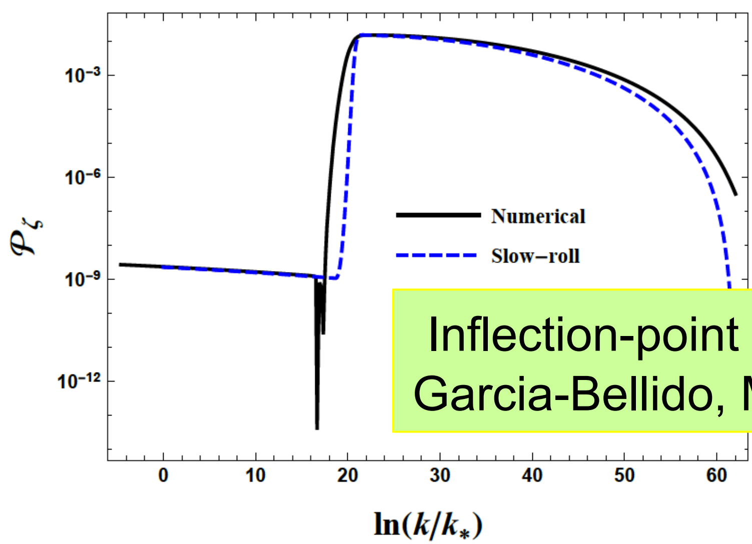
Broad vs narrow spectrum



Double inflation
e.g. Inomata et al. 16



Trapped inflation
e.g. Cheng et al.. 16



Inflection-point inflation
Garcia-Bellido, Morales 16

Super-horizon curvature perturbation growth

$$\Delta^2_{\zeta_k} = \frac{H^2}{8\pi^2\epsilon} e^{-2(3+\eta)\Delta N}$$

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\dot{\epsilon}}{\epsilon H}.$$

ΔN duration of super-horizon growth

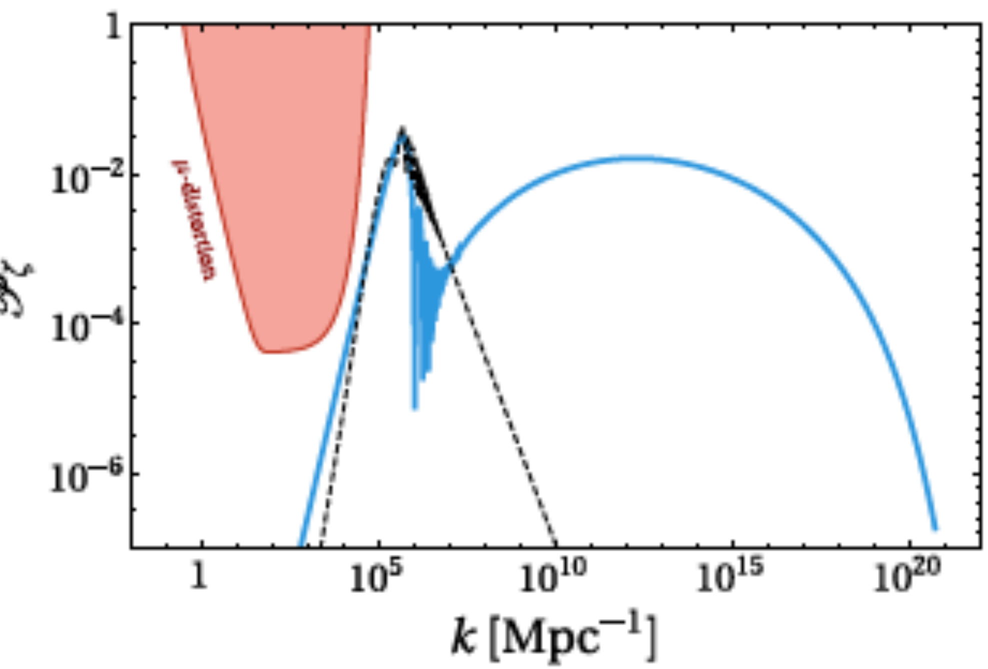
At horizon-crossing, limited by a successful inflation

Super-horizon growth

Casuality ⟹ η >-6

Cheng+ 18

Biggest PBH with $M_{BH} \sim 10\text{-}100\ M_{\odot}$



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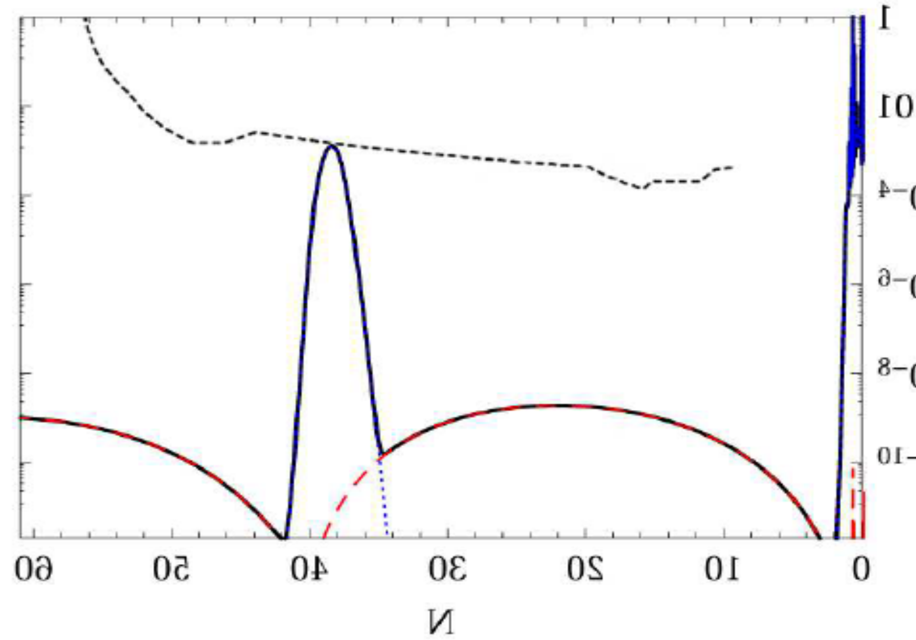

Trapped inflation
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