The Axidental Universe

Matthew Kleban
Center for Cosmology and Particle Physics
New York University

NCTS Annual Meeting, NTHU Taiwan

December 2018

Based on:

- Work with T. Bachlechner, K. Eckerle, O. Janssen
  - 1703.00453,1709.01080, 1810.02822
  - One more paper nearly complete

1982

Large numbers

- We live in an enormous universe - one Hubble volume is 10^{60} Planck lengths across, or 10^{40} Fermi
- Why is the universe so much larger than the length scales of particle physics?
- Why do we live when it is just becoming dominated by dark energy (~10Gyrs = 10^{60} Planck times)?
- Why is it so smooth and isotropic to few/100,000 on large scales (but has structure on smaller scales), and why so flat (so close to critical density given the expansion rate)

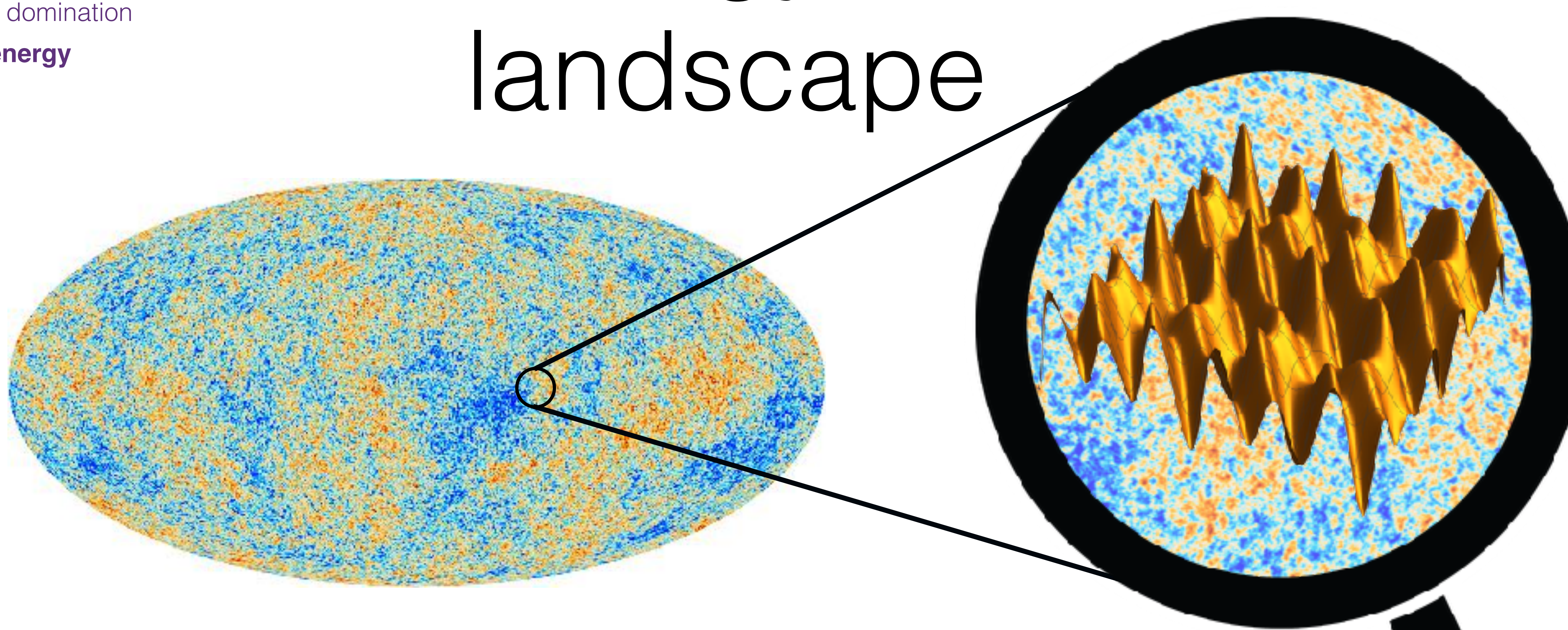
Large numbers

- The Hubble volume would be tiny if dark energy took its natural (particle physics scale) value
- Explaining large size requires solving the cosmological constant (CC) problem
- Tied up with the age question as well
- Smoothness, isotropy, and flatness can be explained by a period of inflation, but *why* did it happen?
- Can there be one explanation for all these features?

Expansion history

- big bang
- inflation (>~ 60 efolds)
- radiation domination
- matter domination (CDM, cold dark matter)
- dark energy ($\sim 10^{-123}\ M_P^4$)

Dark energy and the landscape



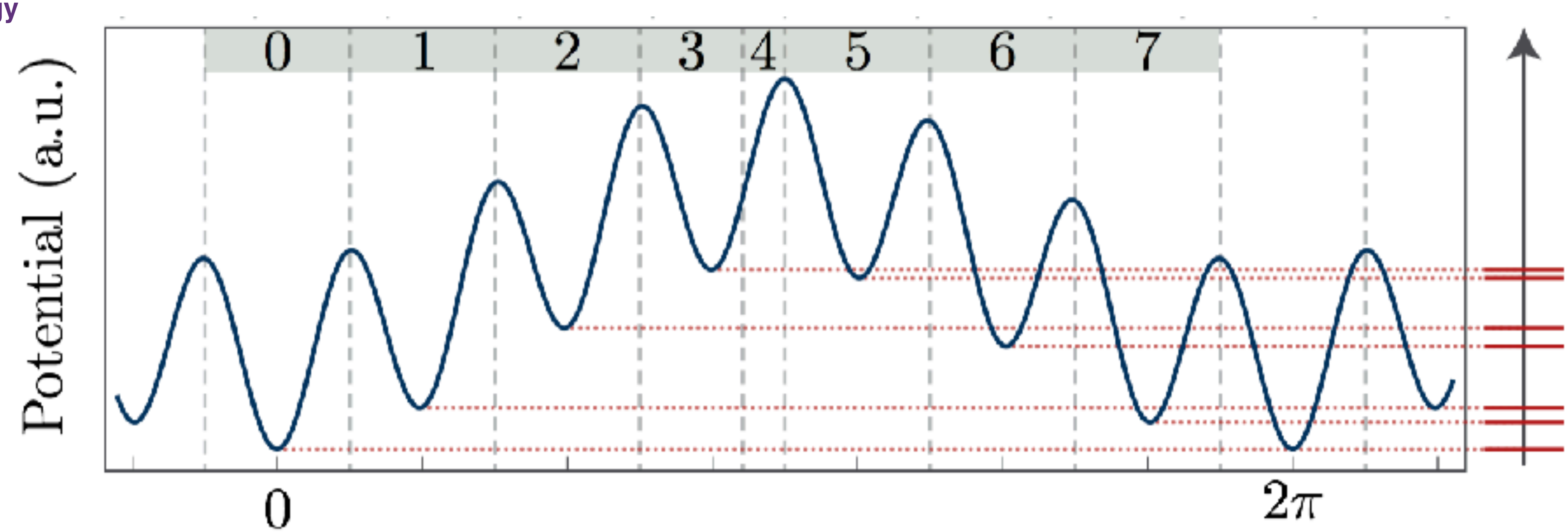
- The simplest explanation for dark energy is a tiny vacuum energy density Λ
- This seems to require a “discretuum” of meta-stable phases (“vacua”) with energies distributed over the fundamental range from negative to positive

Weinberg, Bousso+Polchinski

- We live in a phase with small CC because that’s where structure/life is

Linde, Banks, Weinberg

Discretuum



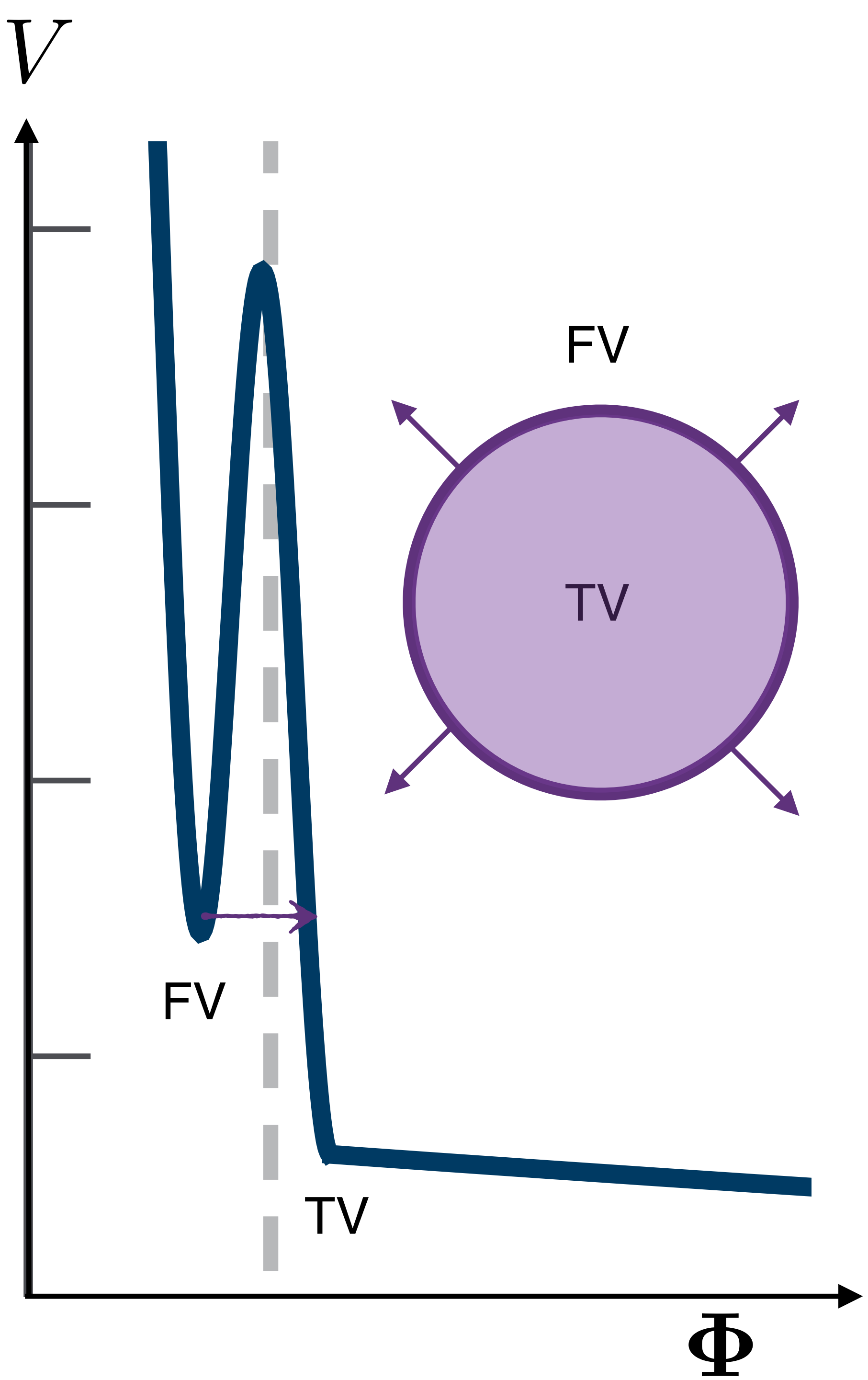
Requirement:

$$N_{phases} > 1/\Lambda = 10^{123}$$

Anthropics

- Weinberg's argument was that Λ is small because if it was larger, structure would be exponentially rare
- Perturbations only grow during the matter dominated epoch, so Λ must dominate *after* matter (but not very much after)
- If Λ dominates *before* matter perturbations $\delta\rho/\rho$ have time to grow to O(1) on some length scale, matter will never collapse and stars will never ignite
- But while having small Λ is *necessary* for structure to form, it is not *sufficient*

The "big bang"



- In a universe with many phases, the typical state has large Λ, and the small Λ regions are created by tunneling
- Tunneling produces an FRW cosmology that is dominated by negative curvature that *prevents structures from forming*, like large Λ
- Roughly *60 efolds* of *post-tunneling inflation* are needed to reduce the curvature, else no structures form!

Freivogel, Kleban, Rodriguez-Martinez, Susskind

Inflation

- So: need lots of slow-roll inflation to follow tunneling from a false vacuum, because the bubble universe is dominated by negative curvature at birth, and curvature (like Λ) must dominate *after* matter does, else no structure
- Another issue - Weinberg assumed $\delta\rho/\rho \sim$ few x 10^{-5}, but if the landscape has such an enormous number of phases, maybe $\delta\rho/\rho$ can vary ("scan")
- If $\delta\rho/\rho \sim .5$, say, structure will not be very rare even if Λ is large and one loses the explanation for small Λ

Dark matters

- Lastly, the constraints on Λ and curvature depend on matter/radiation ratio in the early universe after inflation
- Structure barely grows during radiation domination, and it grows during matter domination proportionally to the scale factor
- So if matter dominates very early, at higher energy density, Λ can be much larger

Intractable?

- To test this, should study all cosmological histories in the theory and focus on those in which structure forms, then ask what the typical Λ, curvature, etc is within that set

- But a final problem is that any theory with $N_{phases} > 10^{123}$ that can accommodate small Λ is very complex and seems intractable

- It has been argued that finding *any minimum* with vacuum energy in the range $-10^{-123} < \rho < +10^{-123}$ is an NP hard problem

Denef+Douglas

- If finding local minima is that hard, good luck with dynamics like cosmological histories

Plan of this talk

- I will study an axion landscape, with a very large number of local energy minima - more than enough to allow small enough values of dark energy
- All parameters can be taken random at GUT/string scale - no tuning
- Tractable - in a random instance with 10^{200} minima I can find many with vacuum energy in any given range in a few minutes on a laptop, and study dynamics
- Selecting histories with structure not only selects small CC, but produces cosmologies very much like ours (flat and homogeneous, small $\delta\rho/\rho$, structure forms when DE dominates)
- Most of my axions will be very heavy (~GUT scale mass), but at the end I'll discuss ultralight ("fuzzy") axions

Axions

- In string theory compactifications one often has N~O(100s) of axions

Axion Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial\boldsymbol{\theta}^\top \boldsymbol{K}\partial\boldsymbol{\theta} - \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\boldsymbol{\mathcal{Q}}^I\boldsymbol{\theta} + \delta^I\right)\right] - V_0$$

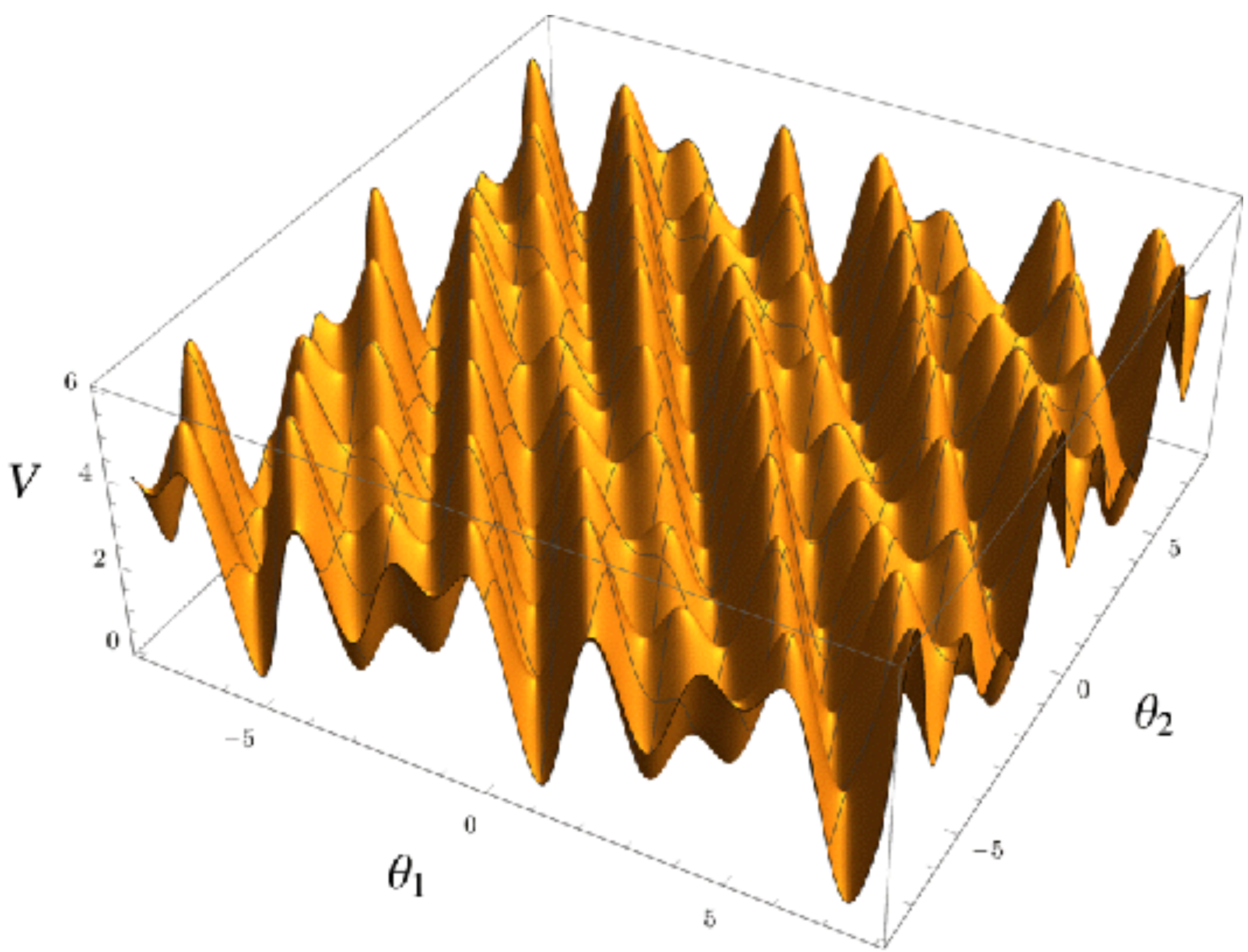
Q is a full-rank P-rows x N-columns *integer* matrix, ***ϴ*** are N scalars, and ***Q^I*** is the **I**th row (so the argument of each cos is an integer linear combination the N fields), ***δ*** are phases, and ***K*** is a positive NxN kinetic matrix

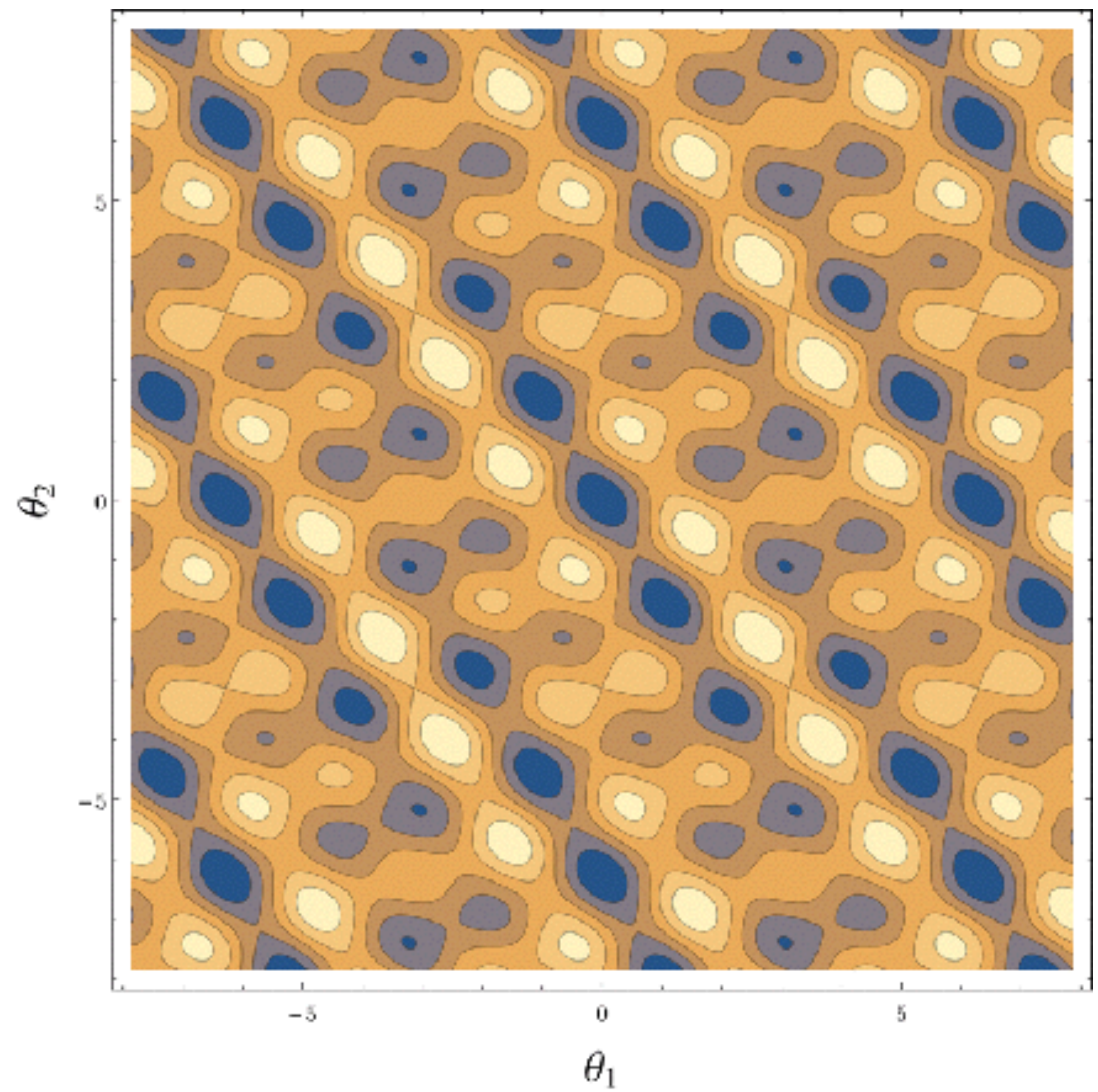
I'll focus on the regime 2N > P > N >> 1
P <= N has only one distinct minimum
P >> 2N tends towards Gaussian

$$V=\sum_{I=1}^{P}\Lambda_I^4\left[1-\cos\left(\boldsymbol{\mathcal{Q}}^I\cdot\boldsymbol{\theta}\right)\right]$$

An example:

$$N=2,P=3:\ V=3-\cos\left(\theta_1-3\theta_2\right)-\cos\left(2\theta_1+\theta_2\right)-\cos\left(3\theta_1+2\theta_2\right)$$





$$V = \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\boldsymbol{\mathcal{Q}}^I \cdot \boldsymbol{\theta}\right)\right]$$

Shift symmetries

- Since ***Q*** has integer entries there are N exact periodicities (e.g. shifting any of the N components of ***θ*** by 2 pi)
- These exact symmetries define a rank-N lattice on which the potential is exactly periodic, so one should restrict to a single fundamental cell of that lattice
- There are also P-N *approximate* shift symmetries!
- One implication is that all P phases **δ** can be made exponentially small (in N) and will be ignored from now on

$$V = \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\boldsymbol{\mathcal{Q}}^I \cdot \boldsymbol{\theta}\right)\right]$$

Landscape

- For P>N there are (*super)exponentially many* (in P) distinct minima in each fundamental cell (~10^P for P~100) and they can be easily located and studied
- To see this, first define a set of P auxiliary fields $\boldsymbol{\phi}$ equal to the arguments of the cosines: $\boldsymbol{\phi} \simeq \boldsymbol{\mathcal{Q}}\,\boldsymbol{\theta}$
- The potential is then a simple sum of cosines, plus a constraint that forces the fields to lie on the constraint surface where the $\boldsymbol{\phi}$ satisfy the appropriate relations

Auxiliary potential

$$V = \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\boldsymbol{\mathcal{Q}}^I \cdot \boldsymbol{\theta}\right)\right]$$

$$V_{\mathrm{NP}} = \underbrace{\sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos(\phi^I)\right]}_{\equiv V_{\mathrm{aux}}} + \sum_{a=1}^{P-N} \nu_a \boldsymbol{\mathcal{R}}^a \boldsymbol{\phi}\,.$$

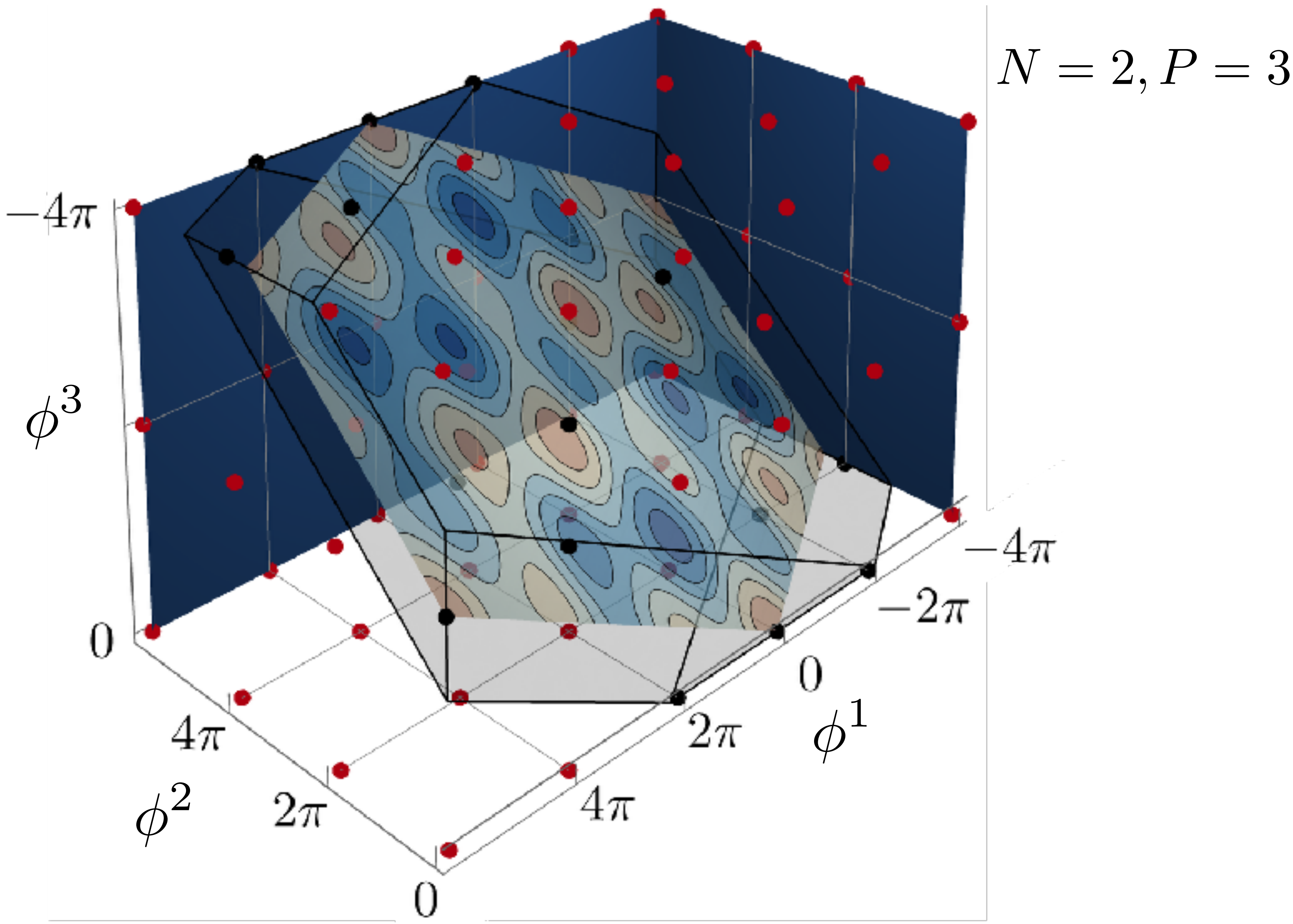
The P fields $\boldsymbol{\phi}$ span a P-dimensional auxiliary space, and the physical potential is just the auxiliary potential V_{aux} evaluated on the N-dimensional column space of ***Q***

The row space of the (P-N)xP matrix ***R*** must be the P-N dimensional space orthogonal to the column space of ***Q***

Then the equations of motion for the P-N Lagrange multipliers ***v*** enforce that the $\boldsymbol{\phi}$ is constrained to this N-dimensional plane

$$V_{\rm NP} = \underbrace{\sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos(\phi^I)\right]}_{\equiv V_{\rm aux}} + \sum_{a=1}^{P-N} \nu_a \boldsymbol{\mathcal{R}}^a \boldsymbol{\phi}$$

Auxiliary space



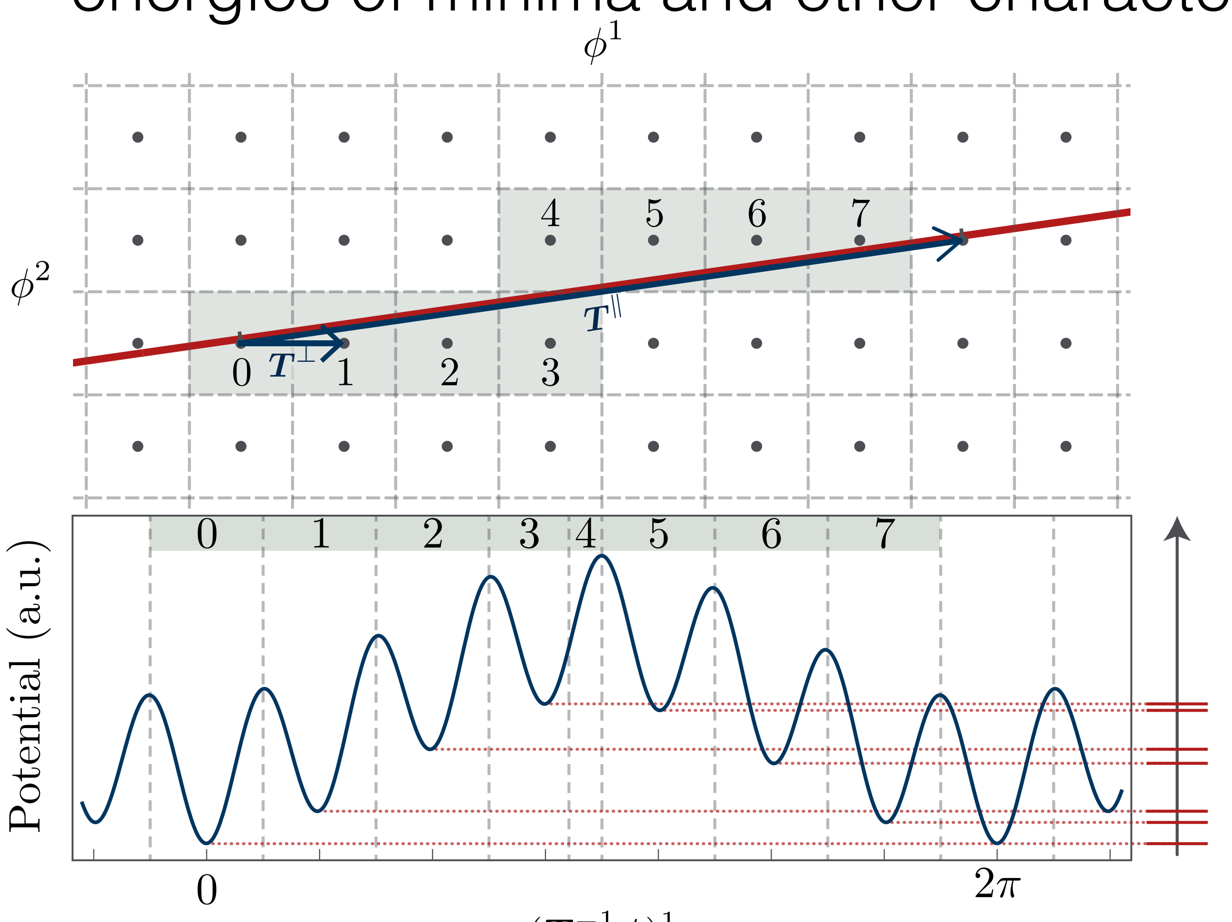
$$V = \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\boldsymbol{\mathcal{Q}}^I \cdot \boldsymbol{\theta}\right)\right]$$

Well-aligned

- In the regime 2N >> P > N >> 1 with ***Q*** random, theories like this turn out to be generically extremely "well-aligned"
- This means that the approximate shift symmetries are very close to exact, which makes it possible to locate very many minima and determine their characteristics with extreme accuracy, and very efficiently (for instance, in a few minutes you can locate 10^{200} minima on a laptop)
- The technical reason is that $D^2=\det(\boldsymbol{Q}^T\boldsymbol{Q})$ is factorially large when N,P are large, and D determines the number of nearby lattice points

Very well-aligned

- The constraint surface passes very close to exponentially many lattice points
- This has several implications:
  - Any non-zero phases can be made very small by choosing the origin of field space near the lattice point closest to the constraint surface (10^{-10} for N~100 - a rational version of the “irrational axion”)
  - When the constraint surface is close to a lattice point, cosines can be expanded to quadratic order to a good approximation, so finding the locations and vacuum energies of minima and other characteristics of the potential is very easy



$$\mathcal{L} = \frac{1}{2}\partial\boldsymbol{\theta}^{\top}\boldsymbol{K}\partial\boldsymbol{\theta} - \sum_{I=1}^{P}\Lambda_I^4\left[1-\cos\left(\boldsymbol{\mathcal{Q}}^I\boldsymbol{\theta}+\delta^I\right)\right]$$

$$V = \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\mathcal{Q}^I \cdot \boldsymbol{\theta}\right)\right]$$

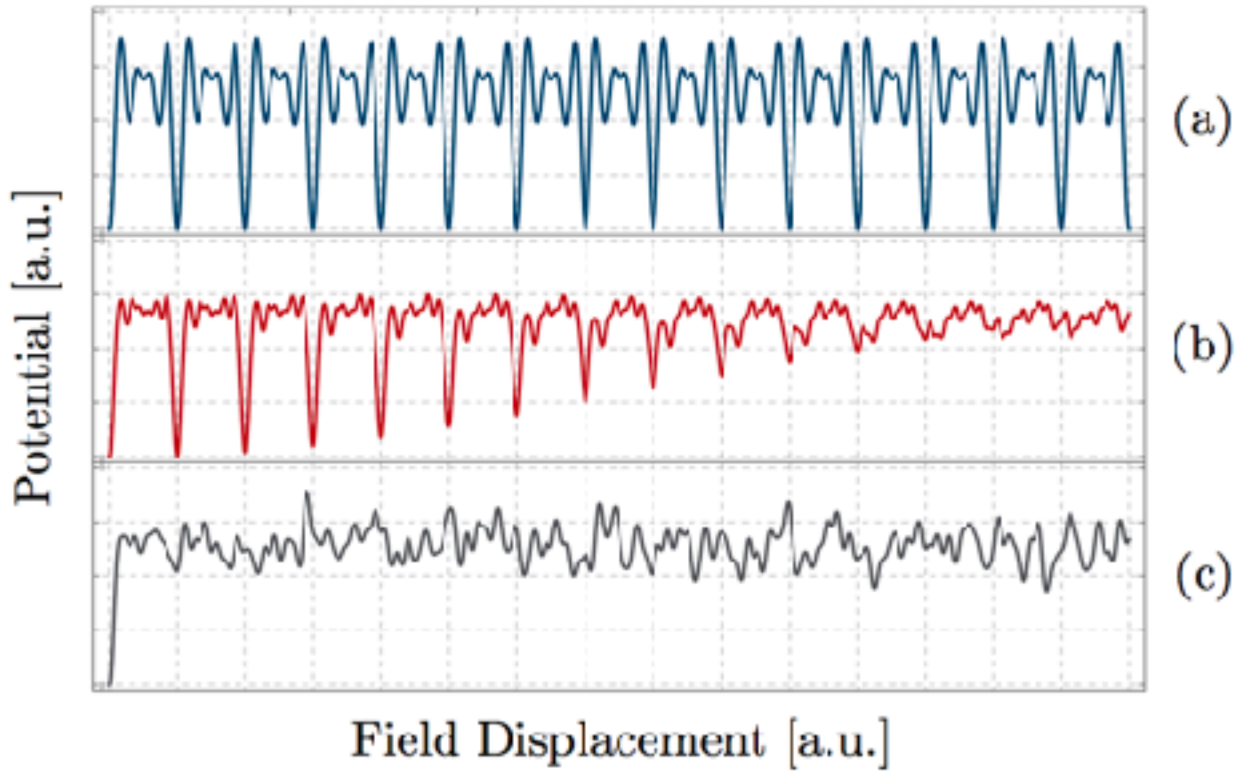
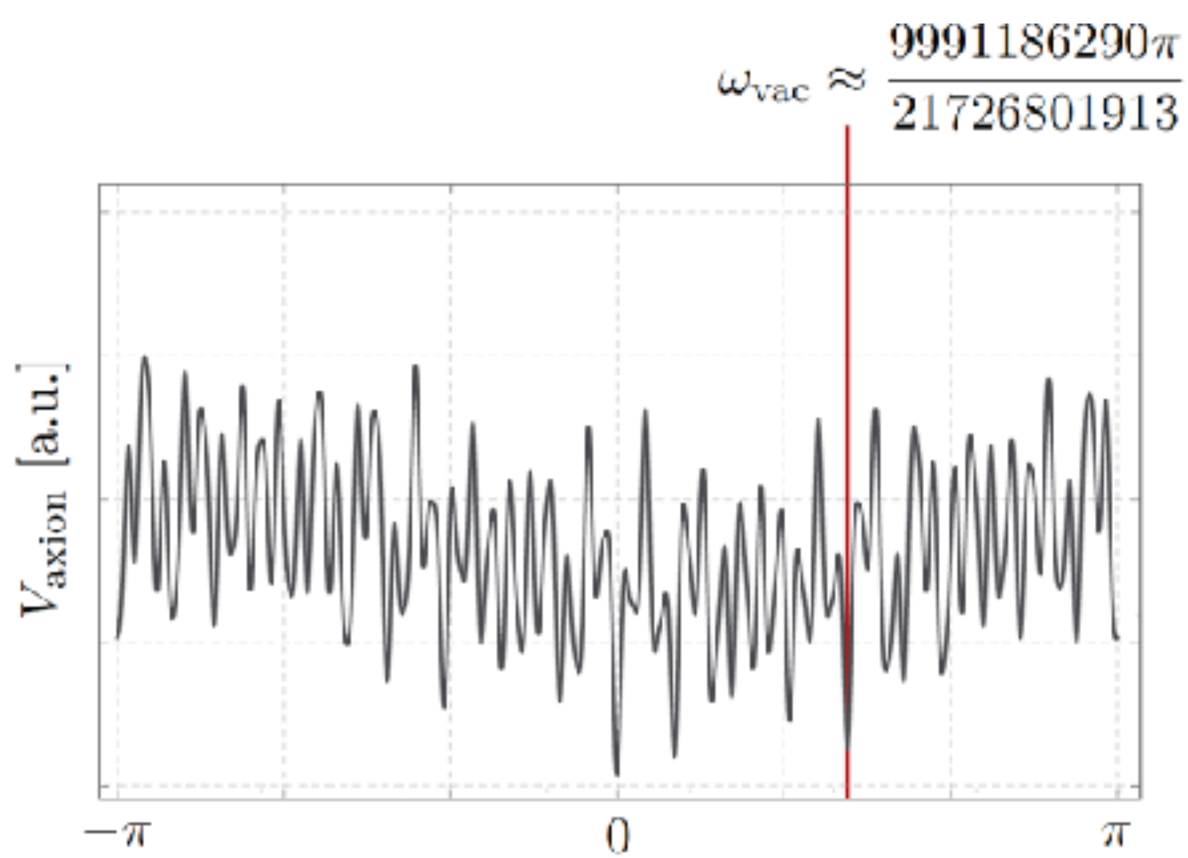


Figure 1. The potential plotted along three different rays through field space (starting at the global minimum), for an example of the potential in (1.1) with $N = 23, P = 40$. Top pane: a line oriented along an exact symmetry direction. Middle pane: a line oriented along an approximate symmetry direction. Bottom pane: a random direction.



$$V = \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\boldsymbol{\mathcal{Q}}^I \cdot \boldsymbol{\theta}\right)\right]$$

Random ensembles

- $\boldsymbol{Q}$ can be quite sparse - it's OK to have only a fraction ~few/N of its entries be non-zero and O(1) - or very dense

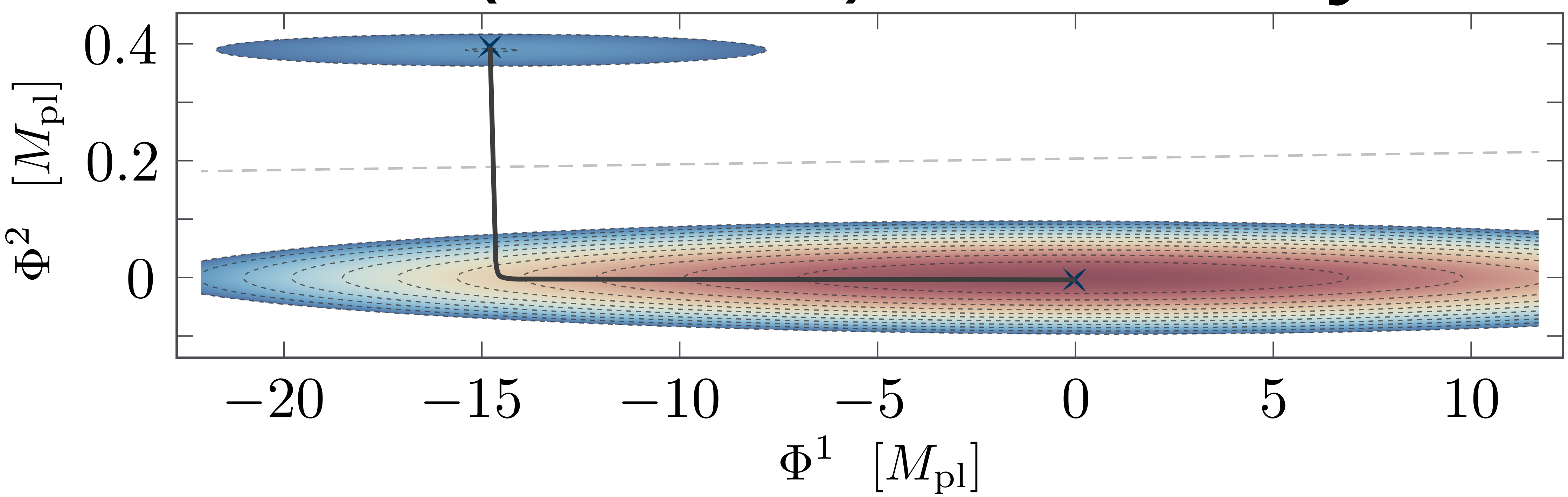
- The distribution of vacuum energies is fairly smooth and there are very many vacua:

$$\mathcal{N}_{\rm vac} \lesssim \left(\frac{\sqrt{P}}{2\ell_P}\right)^{P-N} \sqrt{\det \boldsymbol{\mathcal{Q}}^\top \boldsymbol{\mathcal{Q}}} \approx \left(\frac{\sqrt{P}}{2\ell_P}\right)^{P-N} \sigma_{\mathcal{Q}}^N \sqrt{\frac{P!}{(P-N)!}}$$

For instance, suppose the entries of $\boldsymbol{Q}$ are +/-1 or 0 and P=N+1=500; then $N_{vac} \sim 10^{524}$

$$\mathcal{L} = \frac{1}{2}\partial\boldsymbol{\theta}^\top \boldsymbol{K}\partial\boldsymbol{\theta} - \sum_{I=1}^{P}\Lambda_I^4\left[1-\cos\left(\boldsymbol{\mathcal{Q}}^I\boldsymbol{\theta}+\delta^I\right)\right] - V_0$$

Vacuum (meta)stability



$$B \gtrsim \frac{27\pi^2}{2}\frac{\sigma_{\rm min}^4}{\epsilon^3} \gtrsim \left(\frac{900}{N}\right)^2 \times \left(\frac{\Lambda^4}{V_0}\right)^3 \times \left(\frac{f}{\sigma_{\mathcal{Q}}\Lambda}\right)^4$$

- These vacua are metastable, and one can analyze the tunneling rates and paths analytically

- With N~100s, random parameters at the GUT scale, and $V_0 \sim \Lambda^4$, small CC minima have lifetimes that greatly exceed 10 gigayears

$$\mathcal{L} = \frac{1}{2}\partial\boldsymbol{\theta}^\top \boldsymbol{K}\partial\boldsymbol{\theta} - \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\boldsymbol{\mathcal{Q}}^I \boldsymbol{\theta} + \delta^I\right)\right]$$

Inflation

- Inflation requires long, flat regions of the potential - usually regarded as a fine-tuning
- For instance, $\varepsilon = .5\,(M_P V'/V)^2 \sim (M_P/f)^2 \sim 10^4 >> 1$, where $\boldsymbol{K} = 1/f^2$, $f \sim M_{GUT} \sim .01 M_P$, and $\boldsymbol{Q} \sim 1$.
- So is inflation even possible in this theory?

$$\mathcal{L} = \frac{1}{2}\partial\boldsymbol{\theta}^\top \boldsymbol{K}\partial\boldsymbol{\theta} - \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\boldsymbol{\mathcal{Q}}^I\boldsymbol{\theta} + \delta^I\right)\right]$$

Field range and axion alignment

- Consider multiple axions
- In the case of two axions, the field space is a parallelogram
- The diagonal is longer than either side
- With N fields this is the idea of N-flation, and there have been many related ideas since then (KNP/clockwork, kinetic alignment…)

Dimopoulos, Kachru, Mcgreevy, Wacker,
Kim, Niles, Peloso,
Bachlechner, Dias, Frazer, McAllister …

- This analysis generalizes all of these mechanisms

$$\mathcal{L} = \frac{1}{2}\partial\boldsymbol{\theta}^\top \boldsymbol{K}\partial\boldsymbol{\theta} - \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\boldsymbol{\mathcal{Q}}^I \boldsymbol{\theta} + \delta^I\right)\right]$$

Inflation

The typical field diameter is $\mathcal{D} \geq \frac{\pi}{\|\boldsymbol{\mathcal{Q}}\,\hat{\psi}_1\|_\infty} \approx \frac{\pi}{\ell_P}\frac{\sqrt{P}}{\sigma_{\mathcal{Q}}}\frac{1}{\sqrt{\lambda_1}} \sim N^{3/2} f$

- The three factors of $N^{1/2}$ come from (1) the sparsity of Q, (2) diagonals of a cube, (3) the hierarchy of eigenvalues
- The $N^{3/2}$ allows large field inflation even if f << M_P - that is important because the potential is roughly quadratic near minima so one needs superPlankian range
- With extra terms (wait a few slides) this is compatible with weak gravity, but not field-range/dS swampland
- (The potential is very diverse away from minima, and there are also small field trajectories)

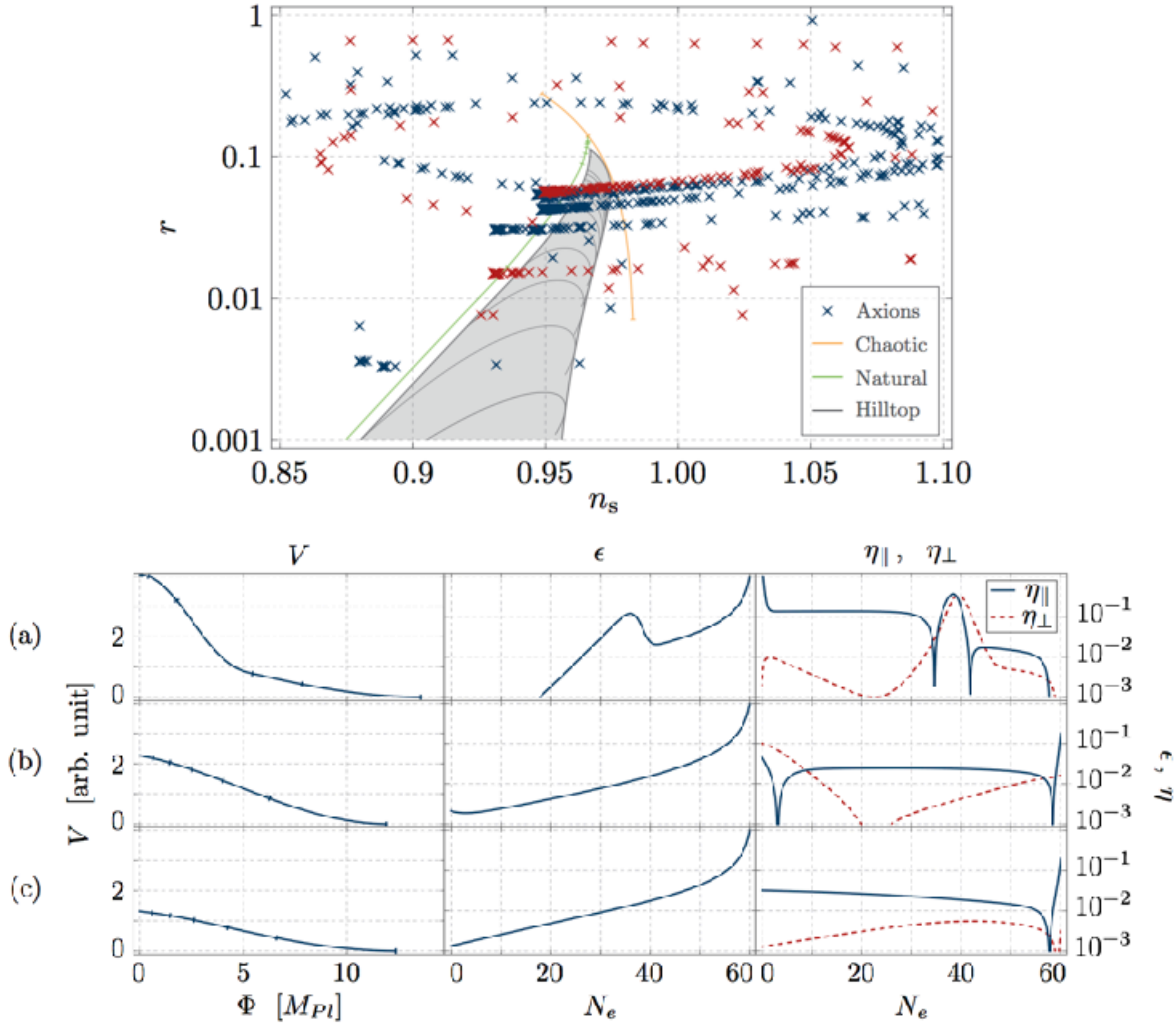


Figure 3. Effective potential and slow roll parameters during the last 60 efolds of three particular trajectories. The observables in the single field, slow roll approximation are: (a) $n_s = 0.73$, $r = 5 \times 10^{-4}$; (b) $n_s = 0.96$, $r = 0.03$; (c) $n_s = 0.93$, $r = 0.03$. Ticks along the potential mark $\Delta N_e = 10$ intervals.

Inflation after tunneling

- Thin wall CdL tunneling requires V''/V >> 1
- Slow-roll requires V''/V << 1
- Incompatible? Not really - not when there are multiple directions in field space!

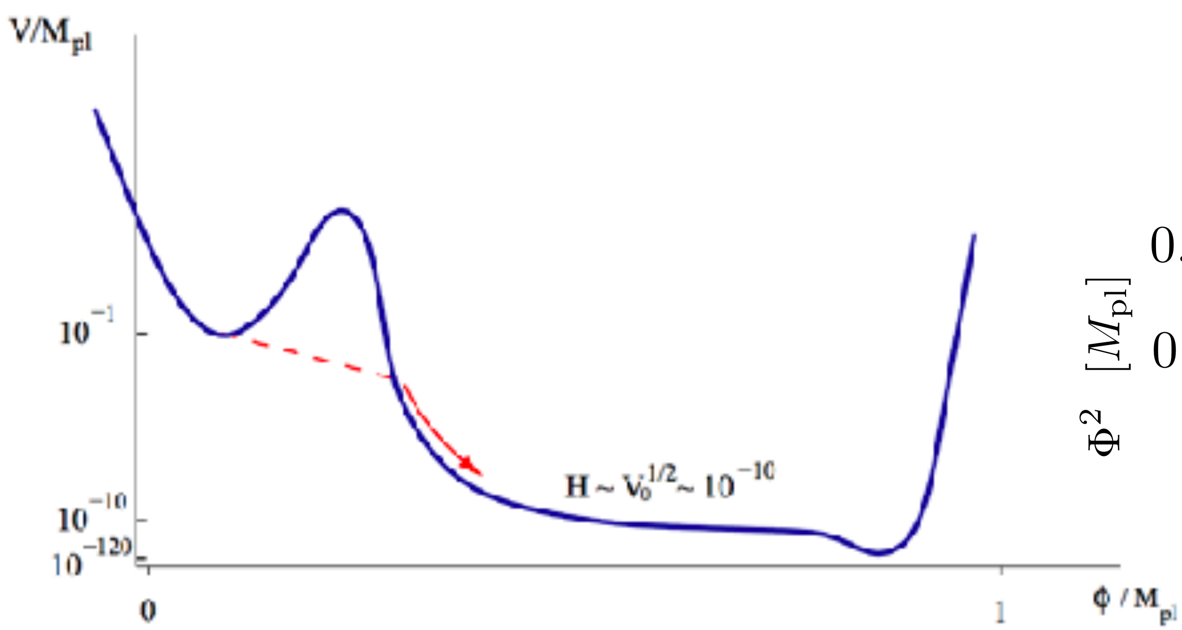
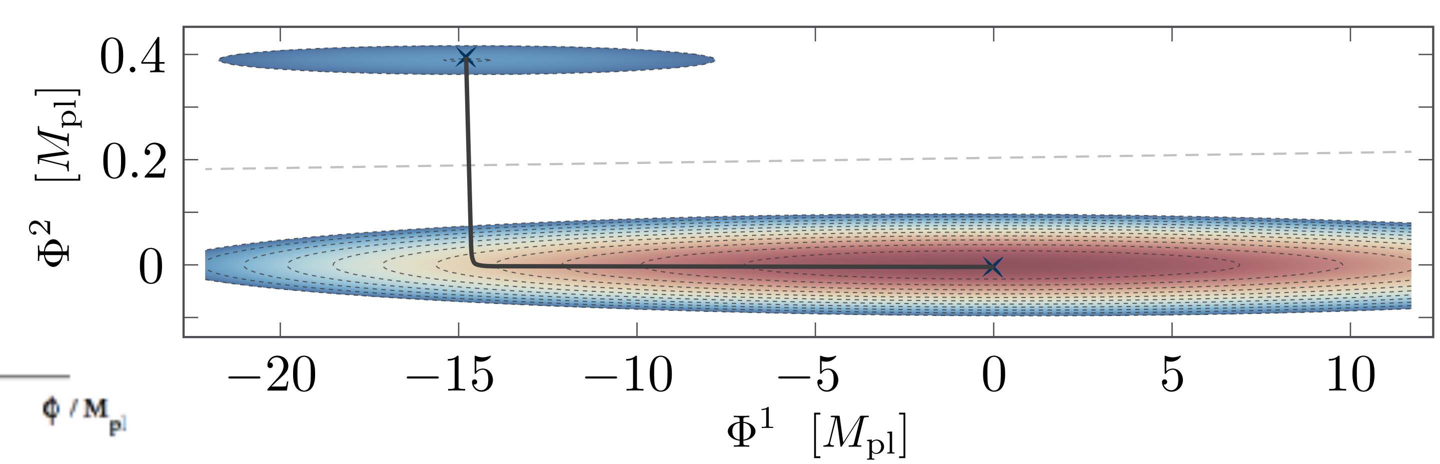
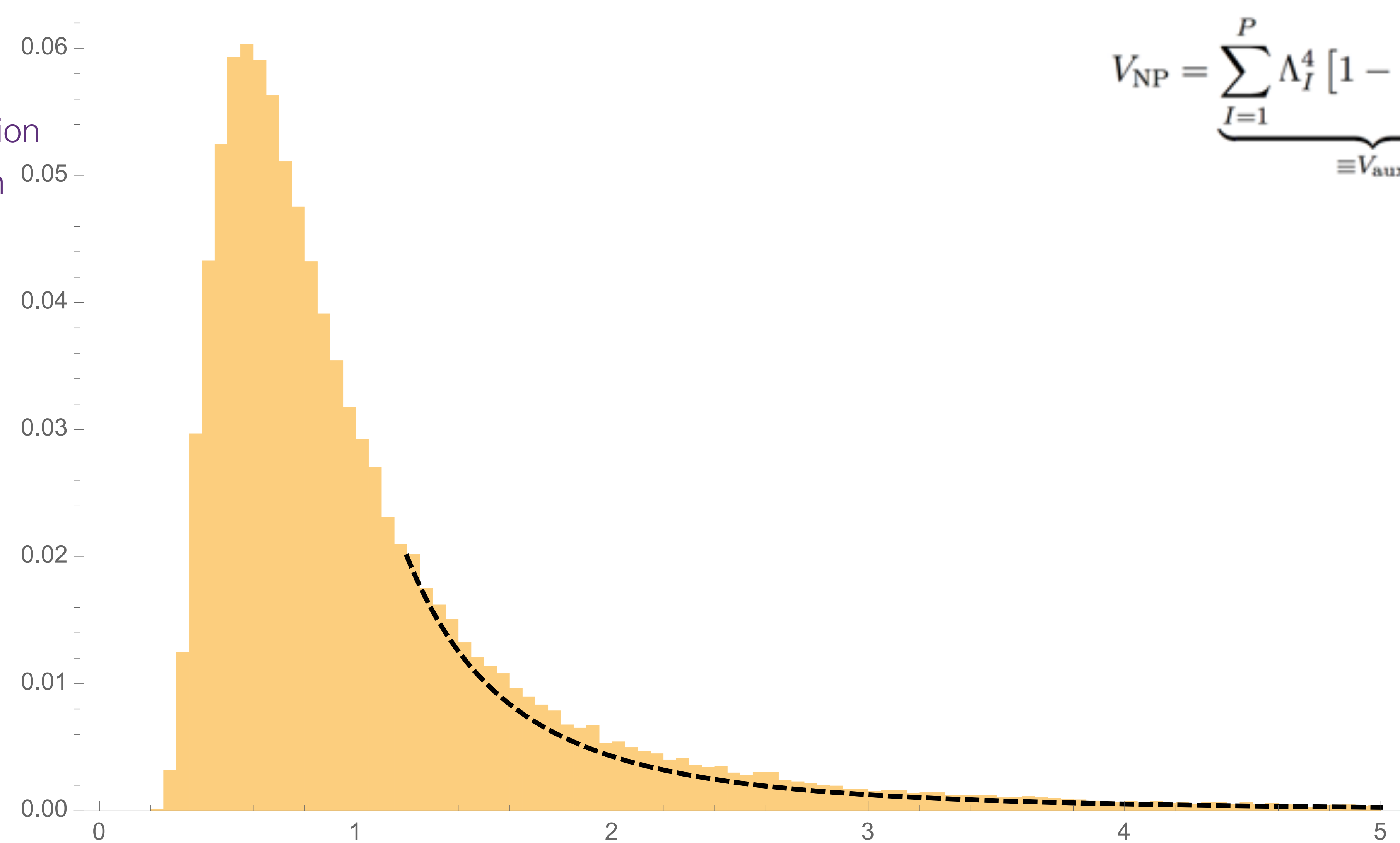


Figure 1: A highly unusual potential which is a typical example of the ones we will consider.



big bang
inflation
radiation domination
matter domination
dark energy



$$V_{\mathrm{NP}} = \underbrace{\sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos(\phi^I)\right]}_{\equiv V_{\mathrm{aux}}} + \sum_{a=1}^{P-N} \nu_a \boldsymbol{\mathcal{R}}^a \boldsymbol{\phi}$$

- In fact, tunnelings are sometimes followed by a substantial amount of slow roll inflation (in contrast to the old lore) in the "quadratic domain"

- The distribution on the number of efolds post-tunneling is peaked at $\langle N_e \rangle \sim (9f/M_P\,\sigma_Q)^2 \sim 2N(f/M_P)^2$, and for P=N+1 it has a heavy tail that falls of as $1/N_e^2$

Perturbations

- Post-tunneling inflation takes place fairly near a minimum, so the inflationary potential is between quadratic and linear (at least for cosines)
- All such trajectories have nearly identical $\delta\rho/\rho$!
- With $\Lambda=f=.01\ M_P$ and N~100s, $\delta\rho/\rho \sim 10^{-4}$
- So at least for inflation post-tunneling, $\delta\rho/\rho$ doesn't scan - and tunneling is how the landscape is populated

$$V = \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\boldsymbol{\mathcal{Q}}^I \cdot \boldsymbol{\theta}\right)\right]$$

Dark matter?

- If the charge matrix ***Q*** is not full-rank, there are would-be flat directions and exactly massless particles
- One expects these are lifted by non-perturbative gravity effects at scale $\Lambda^4 \sim (M_P)^4 e^{-S}$ with $S = \text{few}^* M_P/f$, making these axions very light but not massless
- Such potentials are also be required by the weak gravity conjecture extended to axions

Fuzzy miracle

$$m_{\rm DM} = \frac{M_{\rm pl}^2}{f_{\rm DM}} e^{-S^1/2} \qquad S^1 \approx \mathcal{S}\sqrt{N}\frac{M_{\rm pl}}{f_{\rm DM}}$$

- Plugging in GUT-scale f~0.01 M_P and N~100 gives an interesting mass and the right abundance!

$$\Omega_{\rm axion} \sim 0.2 \left(\frac{f_{\rm DM}}{.04\, M_{\rm pl}}\right)^2 \left(\frac{m_{\rm DM}}{10^{-22}\,{\rm eV}}\right)^{1/2}$$

c.f. Hui, Ostriker, Tremaine, Witten
Arvanitaki, Dimopoulos, Dubovsky, Kaloper, March-Russell

$$V = \sum_{I=1}^{P} \Lambda_I^4 \left[1 - \cos\left(\boldsymbol{\mathcal{Q}}^I \cdot \boldsymbol{\theta}\right)\right]$$

Strong CP and QCD axion dark matter

- A linear combination **a** of the axions can couple as **a•*θ*** G^G, where the components of **a** are e.g. random O(1) dimensionless numbers

- One of the components of ***θ*** is the inflaton, so this coupling will reheat the universe to gluons at the end of inflation

- Another is the would-be massless direction(s), which will get a potential from QCD instantons, hence solving the strong CP problem

- It will also provide dark matter, but then one needs f~10^{12} GeV for the abundance to be about right with O(1) initial vev, or anthropics might select regions with smaller misalignment if f is larger

Freivogel

- However, if inflation is high-scale, too much isocurvature to be consistent with observation

Summary

Given a random axion theory with N ~ 100s and all other parameters O(1) at the GUT scale, and two simple assumptions:

(1) The starting point is a generic inflating false vacuum in a generic landscape theory in this class

(2) We require that structure forms somewhere at some time (meaning matter perturbations reach order 1)

With (1) and (2) generic cosmological histories in these landscapes look **very** similar to that of our universe!

They are big, ~10 billion years old, nearly flat, have small Gaussian primordial perturbations with nearly correct amplitude, are just becoming dominated by dark energy with correct density, plus radiation and dark matter…

Conclusions

- big bang (tunneling, open curvature) ✓
- inflation ($>\sim$ 60 efolds with $\delta\rho/\rho \sim 10^{-4}$) ✓
- radiation domination (couple to QCD or a U(1)) ✓
- matter domination ($\Omega_{CDM} \sim 1$ and fuzzy) ✓
- dark energy ($\sim 10^{-123}\ M_P^4$) ✓

Open questions

- Is metastable de Sitter/inflation consistent with quantum gravity?
- If so, what kind of axion landscapes emerge from string theory?
- How sparse is $\boldsymbol{Q}$, what are the kinetic matrices $\boldsymbol{K}$?
- What about couplings to other moduli?
- This was the first model of its kind, and it works surprisingly well in describing our universe - was that luck, or is it a generic feature?