Parton physics from Euclidean current-current correlator with a valence heavy quark: pion light-cone distribution amplitude as an example

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W. Detmold and CJDL, PRD73 (2006), 014501

W. Detmold, I. Kanamori, CJDL, S. Mondal and Y. Zhao, arXiv:1810.12194

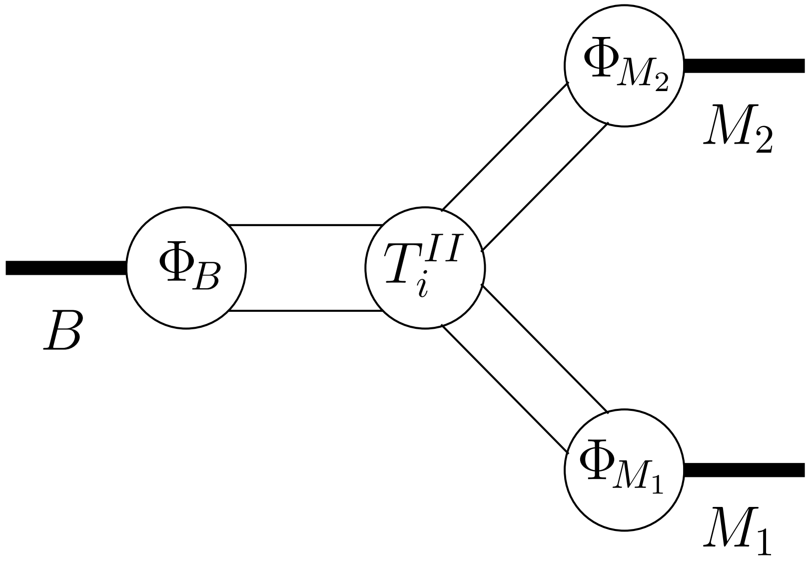
Outline

- Motivation and general strategy.
- Lattice OPE and the pion light-cone wavefunction.
- Exploratory numerical result.
- Outlook.

Motivation and general strategy

Importance of parton physics is beyond doubt

- ★ Hadron structure and QCD are interesting for their own sake
- ★ Inputs for other branches of HEP



Parton distribution from lattice QCD

The “traditional” approach

- Hadronic tensor

$$W_S^{\mu\nu}(p,q) = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle p,S|\,[J^\mu(x), J^\nu(0)]\,|p,S\rangle$$

optical theorem

Imaginary part → challenging in Euclidean QCD

$$T_S^{\mu\nu}(p,q) = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle p,S|T\,[J^\mu(x) J^\nu(0)]\,|p,S\rangle$$

- Light-cone OPE

$$T[J^\mu(x)J^\nu(0)] = \sum_{i,n} \mathcal{C}_i\left(x^2,\mu^2\right)\, x_{\mu_1}\ldots x_{\mu_n}\, \mathcal{O}_i^{\mu\nu\mu_1\ldots\mu_n}(\mu)$$

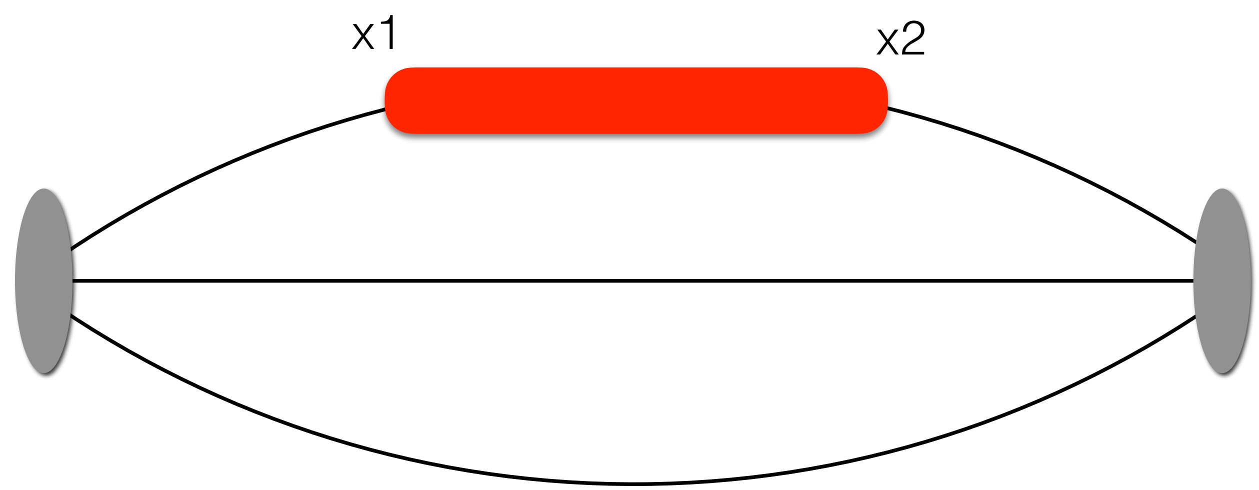
local operators, issue of operator mixing

leading moments in practice

Power divergences arising from Lorentz symmetry breaking

Parton distribution from lattice QCD

The "new" approach



- ★ Y.-Q. Ma and J.-W. Qiu, PRL 120 (2018) (CC)
- ★ A.V. Radyushkin, PRD 96 (2017) (WL)
- ★ A.J. Chambers *et al.*, PRL 118 (2017) (CC)
- ★ X. Ji, PRL 110 (2013) (WL)
- ★ Z. Davoudi and M. Savage, PRD 86 (2012) (SmOp)
- ★ V. Braun and D. Mueller, EPJC 55 (2008) (CC)
- ★ W. Detmold and CJDL, PRD 73 (2006) (CC)
- ★ Liu, K.-F., PRD 62 (2000) (CC)
- ★ U. Aglietti *et al.*, PLB 411 (1998) (CC)

Most involve input of a large scale

Introducing the valence heavy quark

W.Detmold and CJDL, 2006

- Valence → not in the action.
- The "heavy quark" is relativistic.

  → propagating in both space and time

- The current for computing the even moments of the PDF

$$J^{\mu}_{\Psi,\psi}(x) = \overline{\Psi}(x)\gamma^{\mu}\psi(x) + \overline{\psi}(x)\gamma^{\mu}\Psi(x)$$

↓ Compton tensor

$$T^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_{S} \langle p,S| t^{\mu\nu}_{\Psi,\psi}(q) |p,S\rangle = \sum_{S} \int d^4x \; \mathrm{e}^{iq\cdot x} \langle p,S| T\left[J^{\mu}_{\Psi,\psi}(x) J^{\nu}_{\Psi,\psi}(0) \right] |p,S\rangle$$

Strategy for extracting the moments

$$T^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_S \langle p,S|t^{\mu\nu}_{\Psi,\psi}(q)|p,S\rangle = \sum_S \int d^4x\ \mathrm{e}^{iq\cdot x}\langle p,S|T\left[J^\mu_{\Psi,\psi}(x)J^\nu_{\Psi,\psi}(0)\right]|p,S\rangle$$

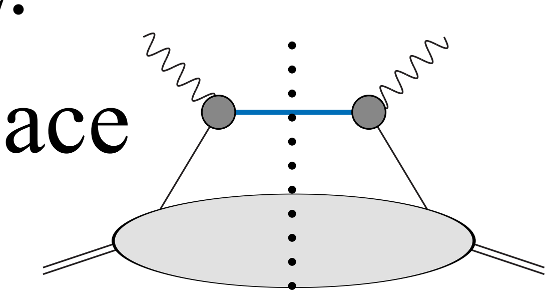
$$J^\mu_{\Psi,\psi}(x) = \overline{\Psi}(x)\gamma^\mu\psi(x) + \overline{\psi}(x)\gamma^\mu\Psi(x)$$

- Simple renormalisation for quark bilinears.

- Work with the hierarchy of scales $\Lambda_{\mathrm{QCD}} << \sqrt{q^2} \leq m_\Psi << \frac{1}{a}$

  → Heavy scales for short-distance OPE.

  → Avoid branch point in Minkowski space at $(q+p)^2 \sim (m_N + m_\Psi)^2$

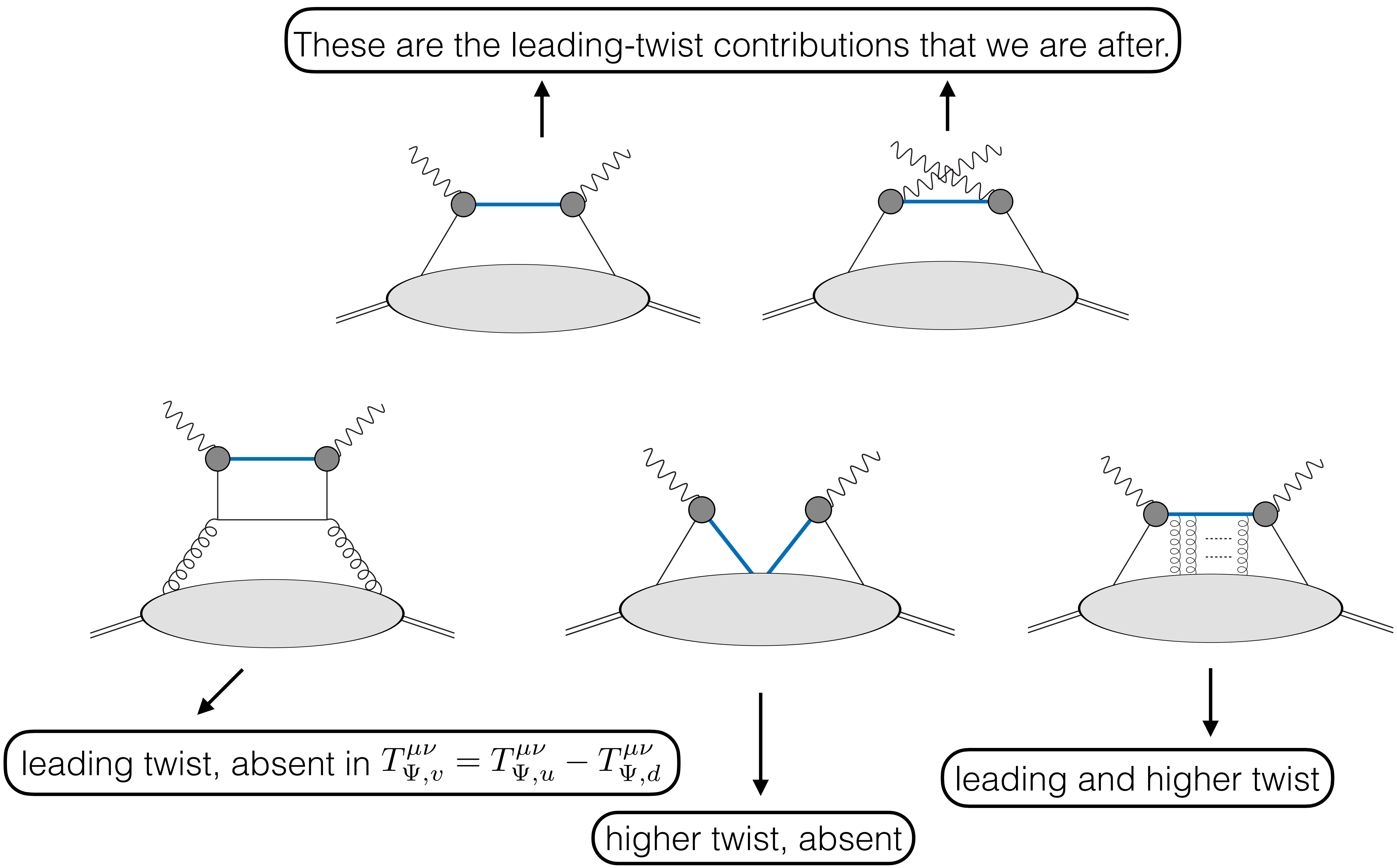


- Extrapolate $T^{\mu\nu}_{\Psi,\psi}(p,q)$ to the continuum limit first.

  → Then match to the short-distance OPE results.

  → Extract the moments without power divergence.

Short-distance OPE & valence heavy quark



Lattice OPE
and pion light-cone distribution amplitude

W.Detmold, I.Kanamori, CJDL, S.Mondal, Y.Zhao, work in progress and arXiv:1810.12194

Pion light-cone wavefunction

Important input for flavour physics

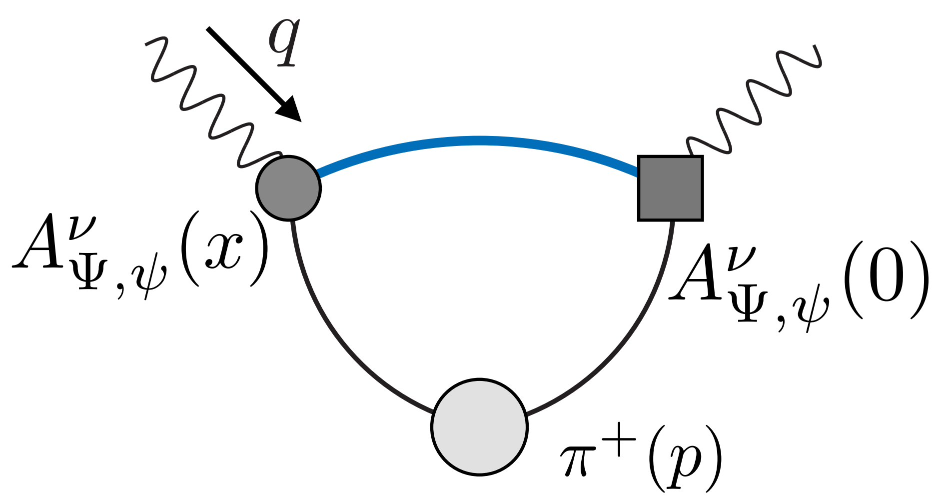
$$\langle 0|\bar{d}(-z)\gamma_\mu\gamma_5 \mathscr{W}[-z,z]u(z)|\pi^+(p)\rangle = i p_\mu f_\pi \int_{-1}^{1} d\xi \; \mathrm{e}^{-i\xi p\cdot z}\phi_\pi(\xi)$$

OPE

$$a_n = \int_0^1 d\xi \; \xi^n \; \phi_\pi(\xi)$$

$$f_\pi a_{n-1} [p^{\mu_1} \dots p^{\mu_n} - \text{Traces}] = -i\langle 0|\bar{d}\gamma^{\{\mu_1}\gamma^5 (iD^{\mu_2}) \dots (iD^{\mu_n\}})u - \text{Traces}|\pi^+(p)\rangle$$

Lattice OPE, AA amplitude



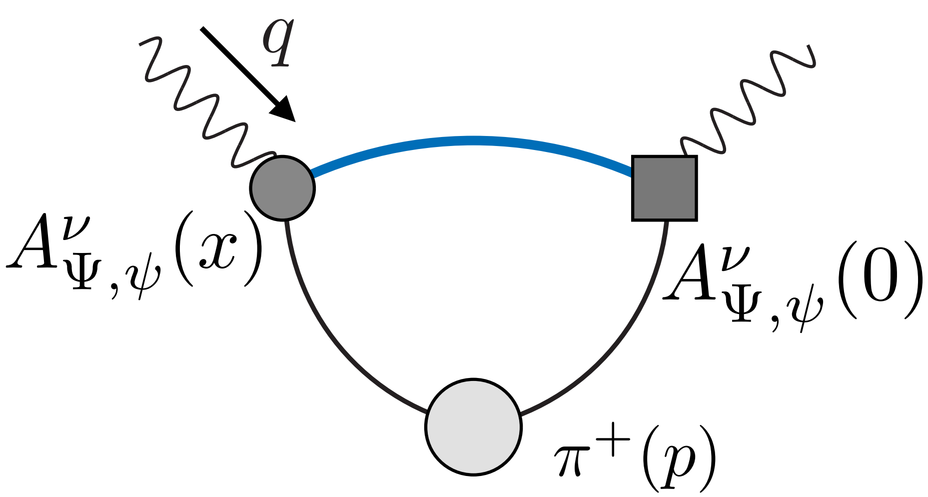
$$+ \left(\begin{array}{c} \mu \leftrightarrow \nu \\ x \leftrightarrow 0 \end{array} \right)$$

$$A^{\mu}_{\Psi,\psi} = \overline{\Psi}\gamma^{\mu}\gamma_5\psi + \overline{\psi}\gamma^{\mu}\gamma_5\Psi$$

$$U^{\mu\nu}_{A}(q,p) = \int d^4x \; e^{iqx} \; \langle 0|T[A^{\mu}_{\Psi,\psi}(x)A^{\nu}_{\Psi,\psi}(0)]|\pi^+(p)\rangle$$

μ and ν anti-symmetrised

Lattice OPE, AA amplitude result



$$+ \begin{pmatrix} \mu \leftrightarrow \nu \\ x \leftrightarrow 0 \end{pmatrix}$$

$$U_A^{[\mu\nu]}(q,p) = \left(\frac{-2i\epsilon^{\mu\nu\rho\sigma} \; p^{\rho} q^{\sigma} \; f_{\pi}}{\tilde{Q}^2} \right) \sum_{n=0,\text{even}}^{\infty} \frac{\zeta^n \; \mathcal{C}_n^2(\eta) \; C_n(q, m_{\Psi}, \mu)}{n+1} a_n(\mu)$$

$$\boxed{\begin{aligned} \mu &= 1 \\ \nu &= 2 \end{aligned}}$$

$$\boxed{\zeta = \frac{\sqrt{p^2 q^2}}{\tilde{Q}^2}, \qquad \eta = \frac{p \cdot q}{\sqrt{p^2 q^2}}}$$

$$\boxed{C_n = 1}$$

$$U_A^{[12]}(q,p) = \left[\frac{-2i \; \left(p^3 q^4 - p^4 q^3\right) \; f_{\pi}}{\tilde{Q}^2} \right] \sum_{n=0,\text{even}}^{\infty} F_n\left(q^2, p \cdot q, m_{\Psi}\right) a_n$$

Exploratory numerical result

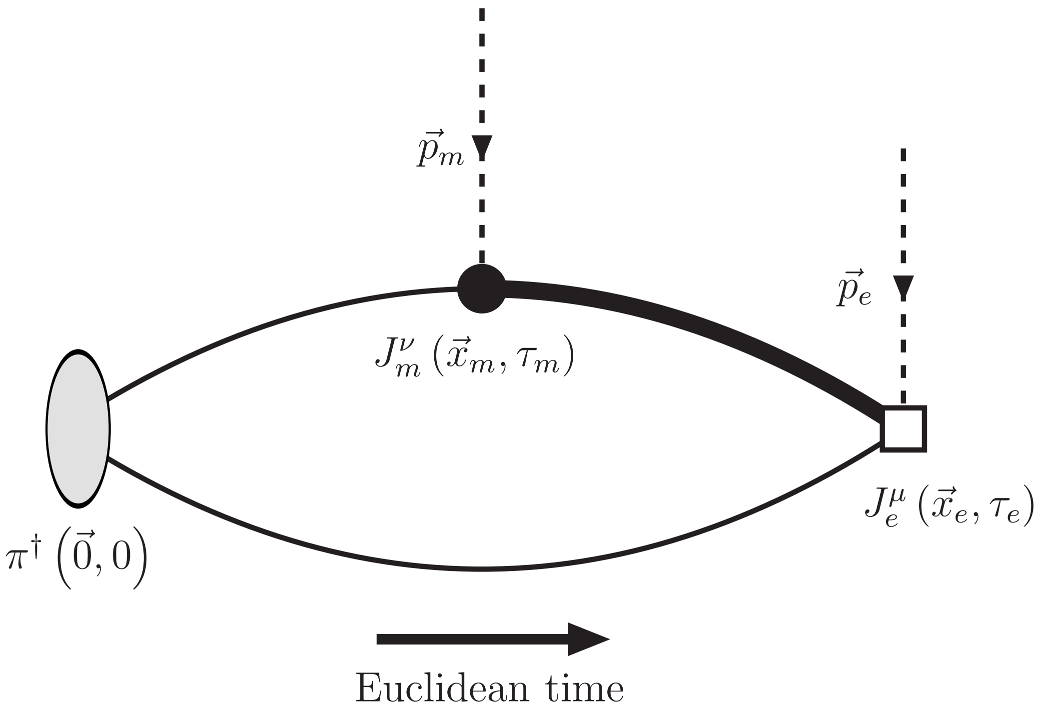
W.Detmold, I.Kanamori, CJDL, S.Mondal, Y.Zhao, work in progress

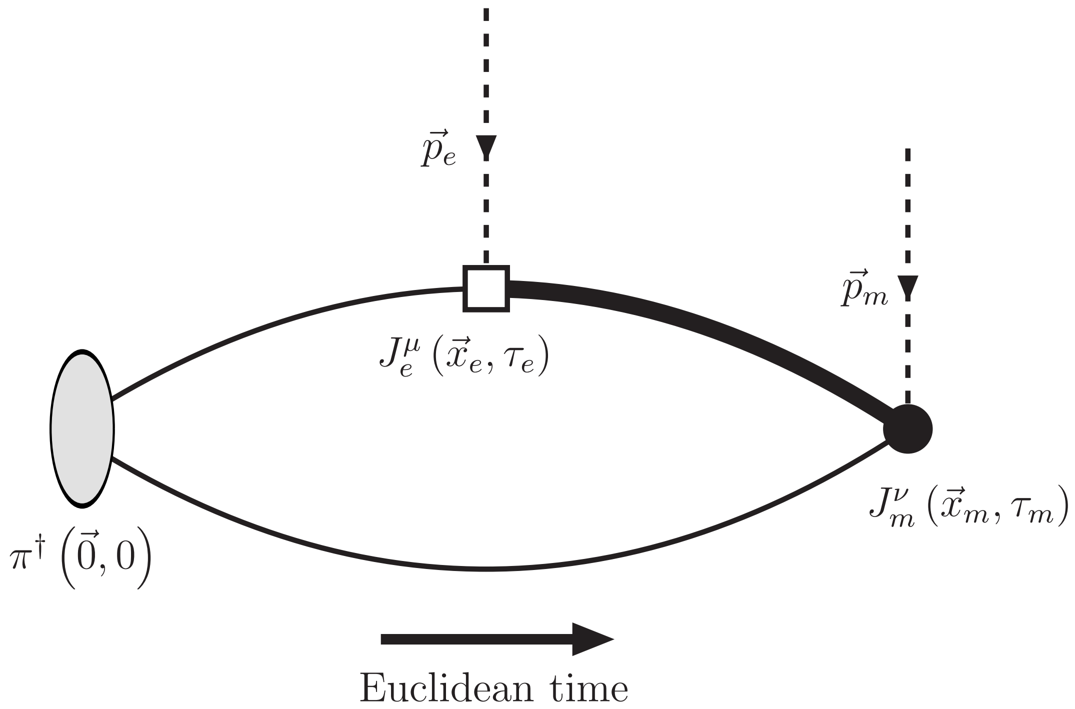
Simulation details

Testing our approach with fine quenched lattices

- In this talk: exploratory study with ~300 configs:
  - → $a^{-1} \sim 4$ GeV, $L^3 \times T = 48^3 \times 96$
  - → $a^{-1} \sim 3.34$ GeV, $L^3 \times T = 40^3 \times 80$
  - → $a^{-1} \sim 2.67$ GeV, $L^3 \times T = 32^3 \times 64$
  - → $m_\Psi \sim 1.3$ and 2 GeV, $M_\pi \sim 450$ MeV
  - → NP-improved clover valence fermion
- Available (> 200 each) configs at $L = 32, 40, 48, 64, 96$
  - → Cut-off scale as high as 8 GeV

The correlators





$$C_3^{\mu\nu}(\tau_e, \tau_m; \vec{p}_e, \vec{p}_m) = \int \mathrm{d}^3x_e \int \mathrm{d}^3x_m \ \mathrm{e}^{i\vec{p}_e\cdot\vec{x}_e}\mathrm{e}^{i\vec{p}_m\cdot\vec{x}_m} \left\langle 0 \middle| \mathrm{T}\left[J_e^\mu(\vec{x}_e, \tau_e)\, J_m^\nu(\vec{x}_m, \tau_m)\, \mathcal{O}_\pi^\dagger(\vec{0}, 0)\right] \middle| 0 \right\rangle$$

$$C_\pi(\tau_\pi; \vec{p}) = \int \mathrm{d}^3x \ \mathrm{e}^{i\vec{p}\cdot\vec{x}} \left\langle 0 \middle| \mathcal{O}_\pi(\vec{x}, \tau_\pi)\mathcal{O}_\pi^\dagger(\vec{0}, 0) \middle| 0 \right\rangle \xrightarrow{\tau_\pi\to\infty} \frac{\left|\left\langle \pi(\vec{p}) \middle| \mathcal{O}_\pi^\dagger(\vec{0}, 0) \middle| 0 \right\rangle\right|^2}{2E_\pi} \times \mathrm{e}^{-E_\pi\tau_\pi}$$

Analysis strategy

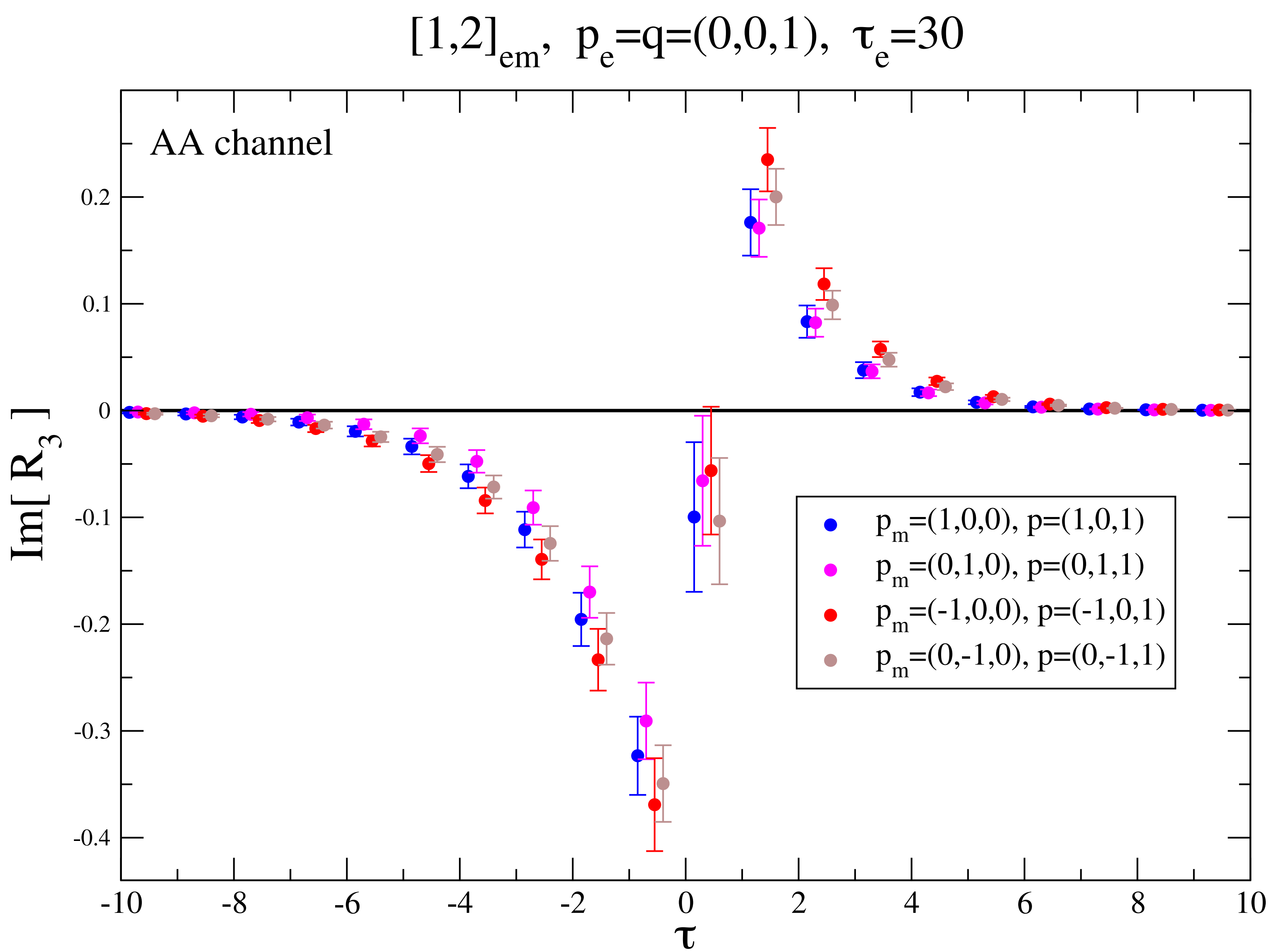
$$R_3^{\mu\nu}(\tau, \vec{q}, \vec{p}) \equiv \int \mathrm{d}^3x \; \mathrm{e}^{i\vec{q}\cdot\vec{x}} \left\langle 0 \left| \mathrm{T}\left[J_e^{\mu}(\vec{x}, \tau)\, J_m^{\nu}(\vec{0}, 0) \right] \right| \pi(\vec{p}) \right\rangle$$

$$U^{\mu\nu}(p, q) = i \int_{\tau_{\mathrm{min}}}^{\tau_{\mathrm{max}}} \mathrm{d}\tau \; \mathrm{e}^{iq_4\tau} \; R_3^{\mu\nu}(\tau, \vec{q}, \vec{p})$$

real for purely imaginary q_4

purely imaginary

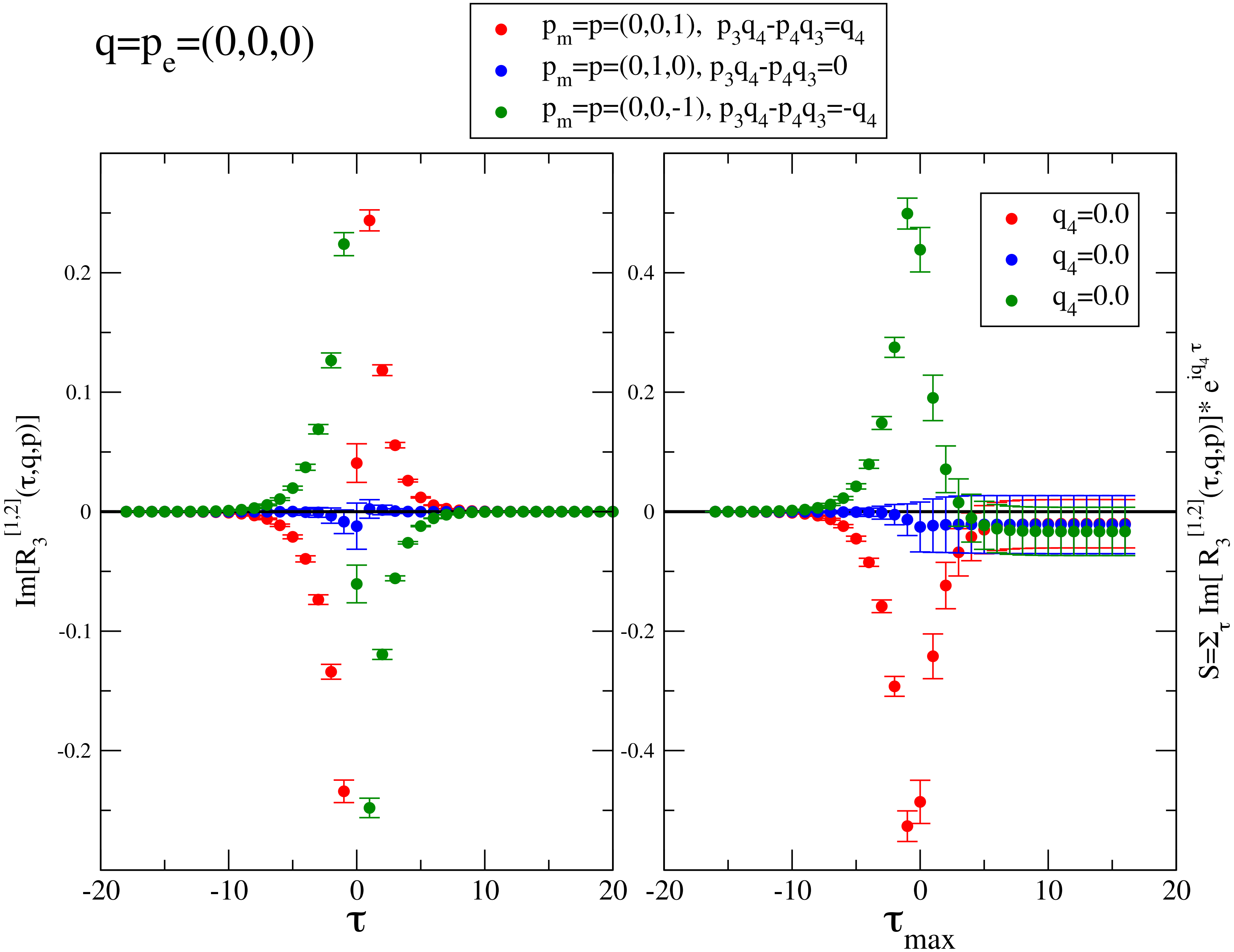
Exploratory numerical results



All have same $p^3q^4 - p^4q^3$, $p \cdot q$ and q^2

$$U_A^{[12]}(q,p) = \left[\frac{2\ \left(p^3q^4 - p^4q^3\right)\ f_\pi}{\tilde{Q}^2}\right] \sum_{n=0,\text{even}}^{\infty} F_n\left(q^2, p\cdot q, m_\Psi\right) a_n$$

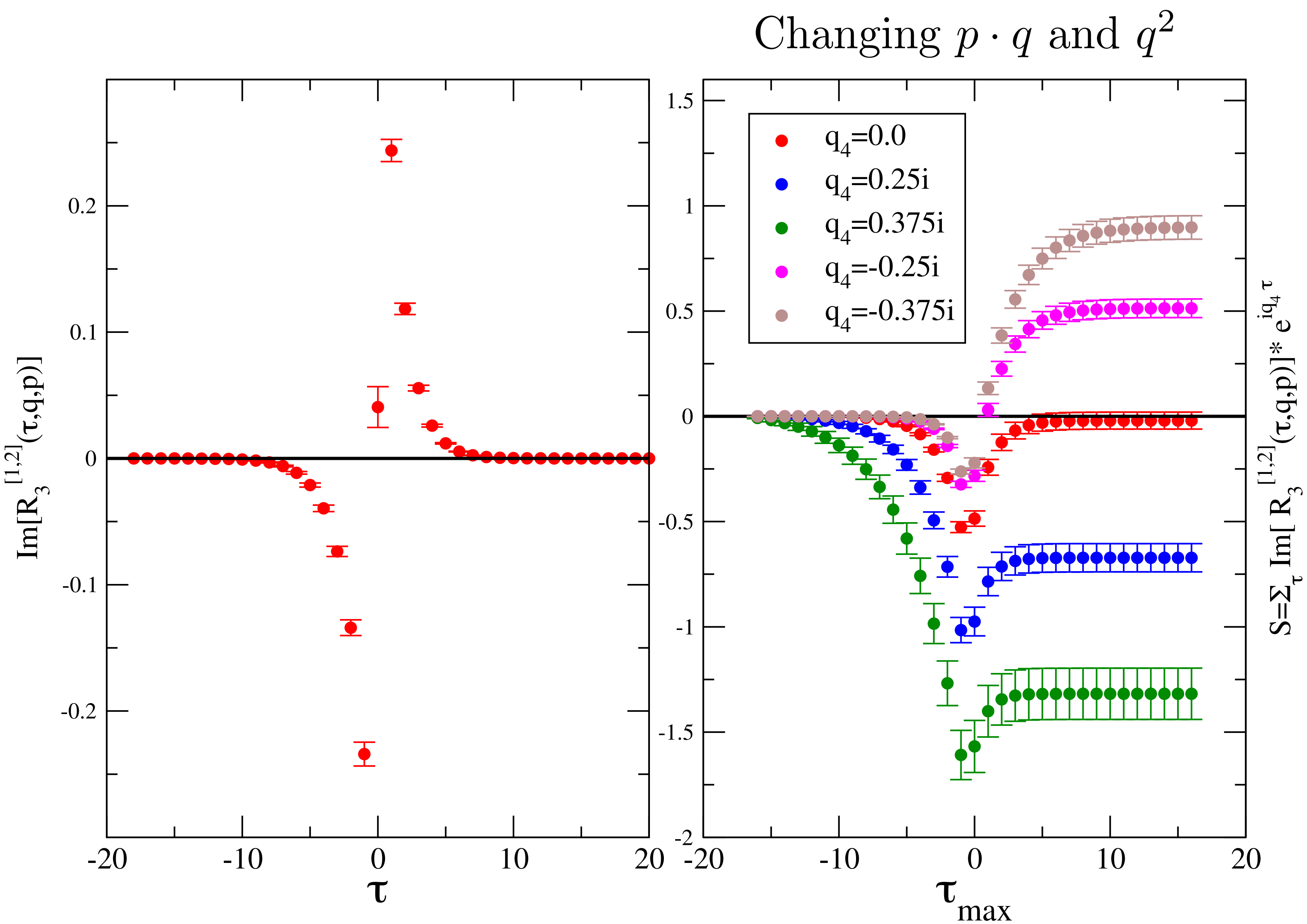
Exploratory numerical results



$$U_A^{[12]}(q,p) = \left[\frac{2\left(p^3 q^4 - p^4 q^3\right) f_\pi}{\tilde{Q}^2} \right] \sum_{n=0,\mathrm{even}}^{\infty} F_n\left(q^2, p\cdot q, m_\Psi\right) a_n$$

Exploratory numerical results

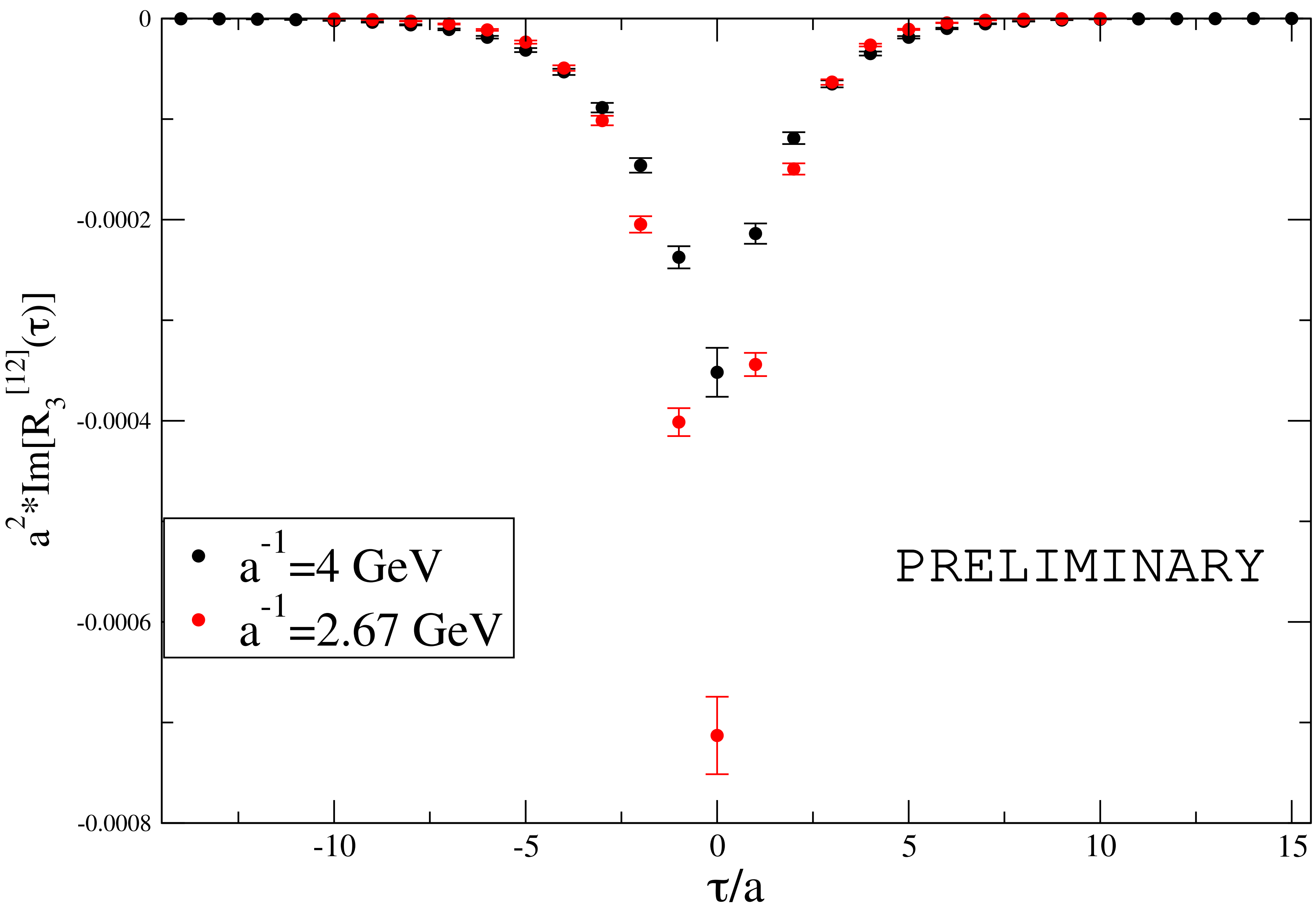
q=p_e=(0,0,0), p=p_m=(0,0,1)



$$U_A^{[12]}(q,p) = \left[\frac{2\ \left(p^3q^4 - p^4q^3\right)\ f_\pi}{\tilde{Q}^2}\right] \sum_{n=0,\mathrm{even}}^{\infty} F_n\left(q^2, p\cdot q, m_\Psi\right) a_n$$

Exploratory numerical results

p=(0,0,0), q=(0,0,1), m_Ψ=1.3 GeV, L~ 2.4 fm

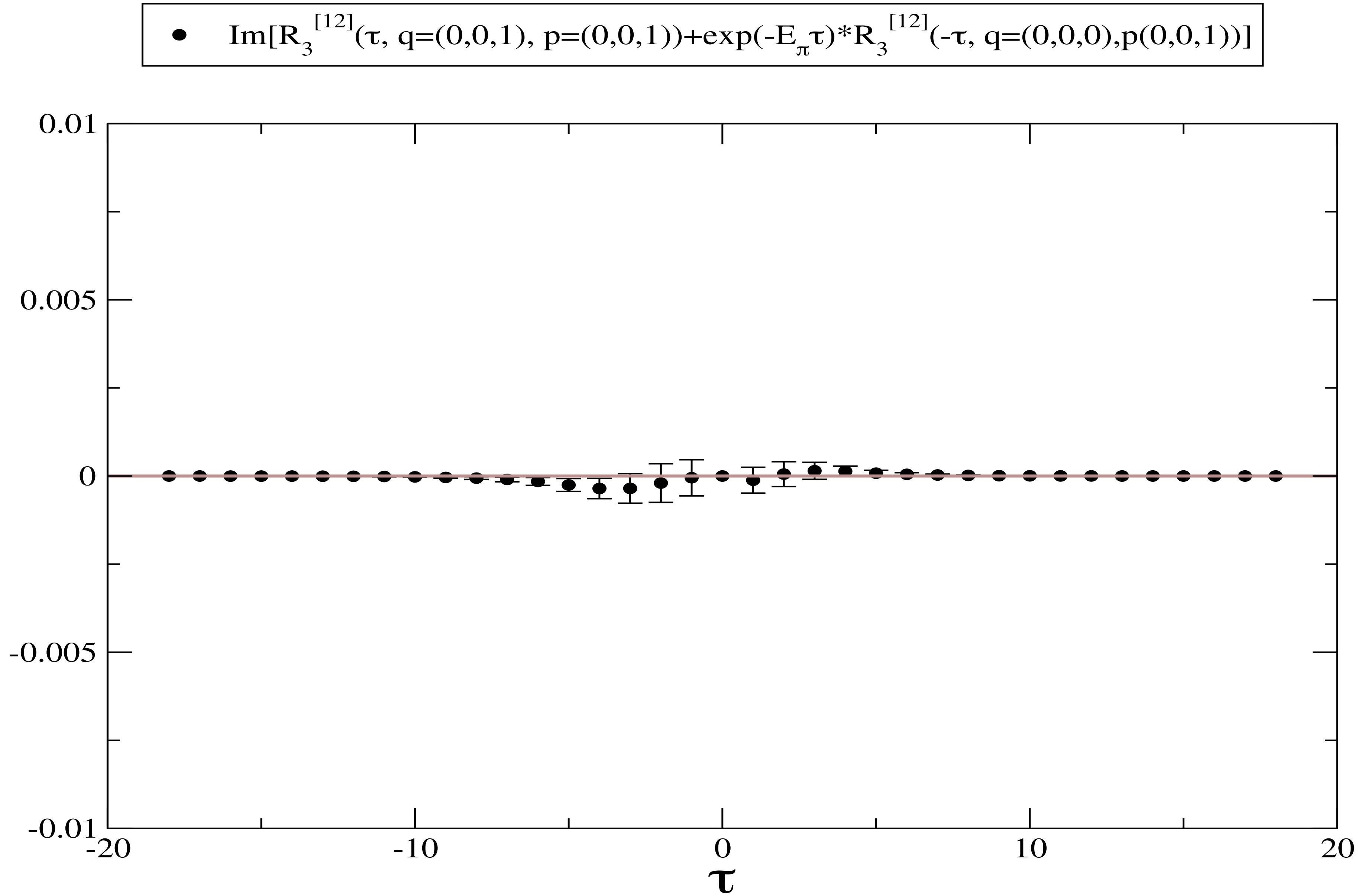


$$U_A^{[12]}(q,p) = \left[\frac{2\ \left(p^3 q^4 - p^4 q^3\right)\ f_\pi}{\tilde{Q}^2}\right] \sum_{n=0,\mathrm{even}}^{\infty} F_n\left(q^2, p\cdot q, m_\Psi\right) a_n$$

Exploratory numerical results

Translational invariance implies

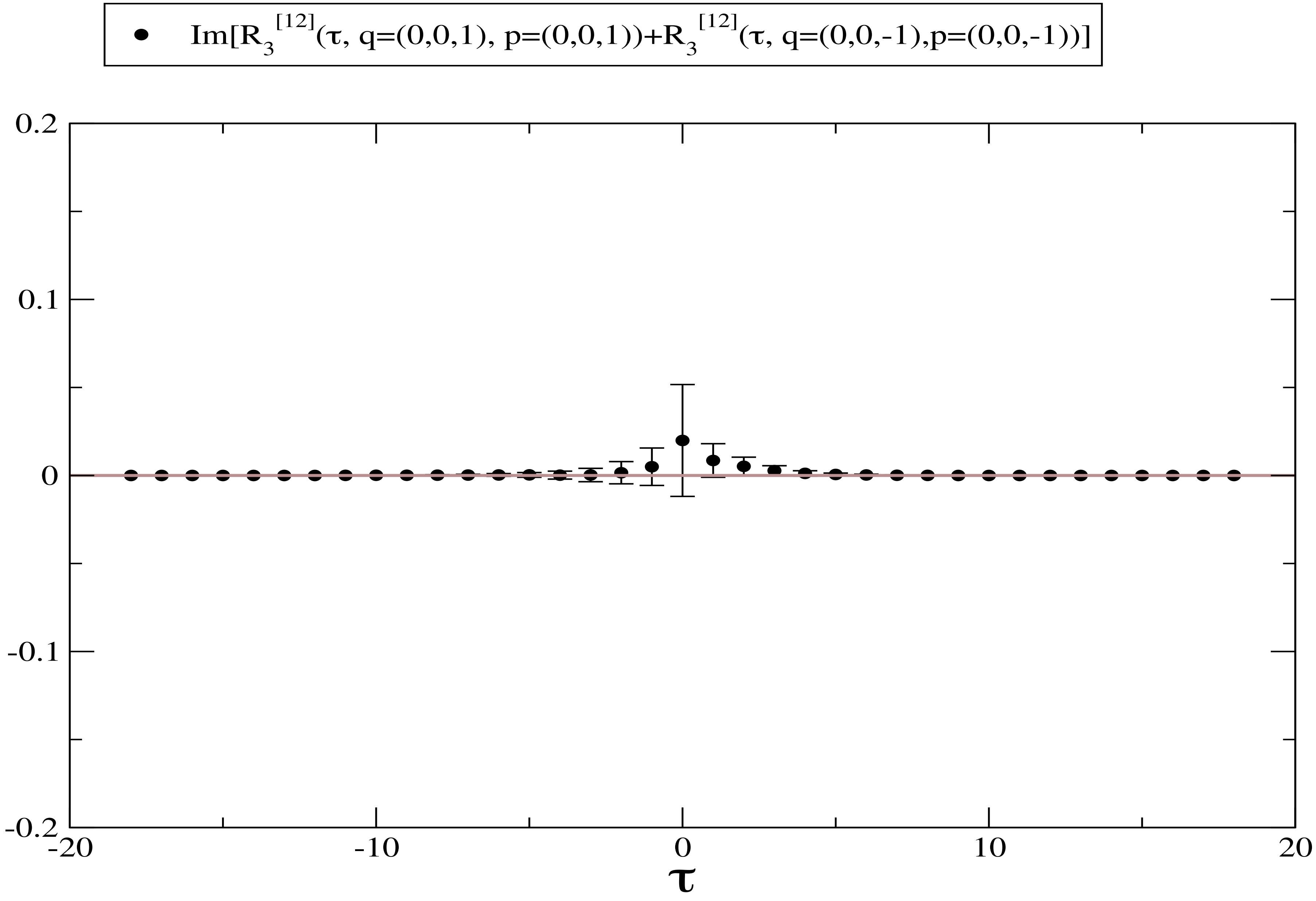
$$R_3^{[\mu\nu]}(\tau,\vec{q},\vec{p}) + \mathrm{e}^{-E_\pi(\vec{p})\tau} R_3^{[\mu\nu]}(-\tau,\vec{p}-\vec{q},\vec{p}) = 0$$



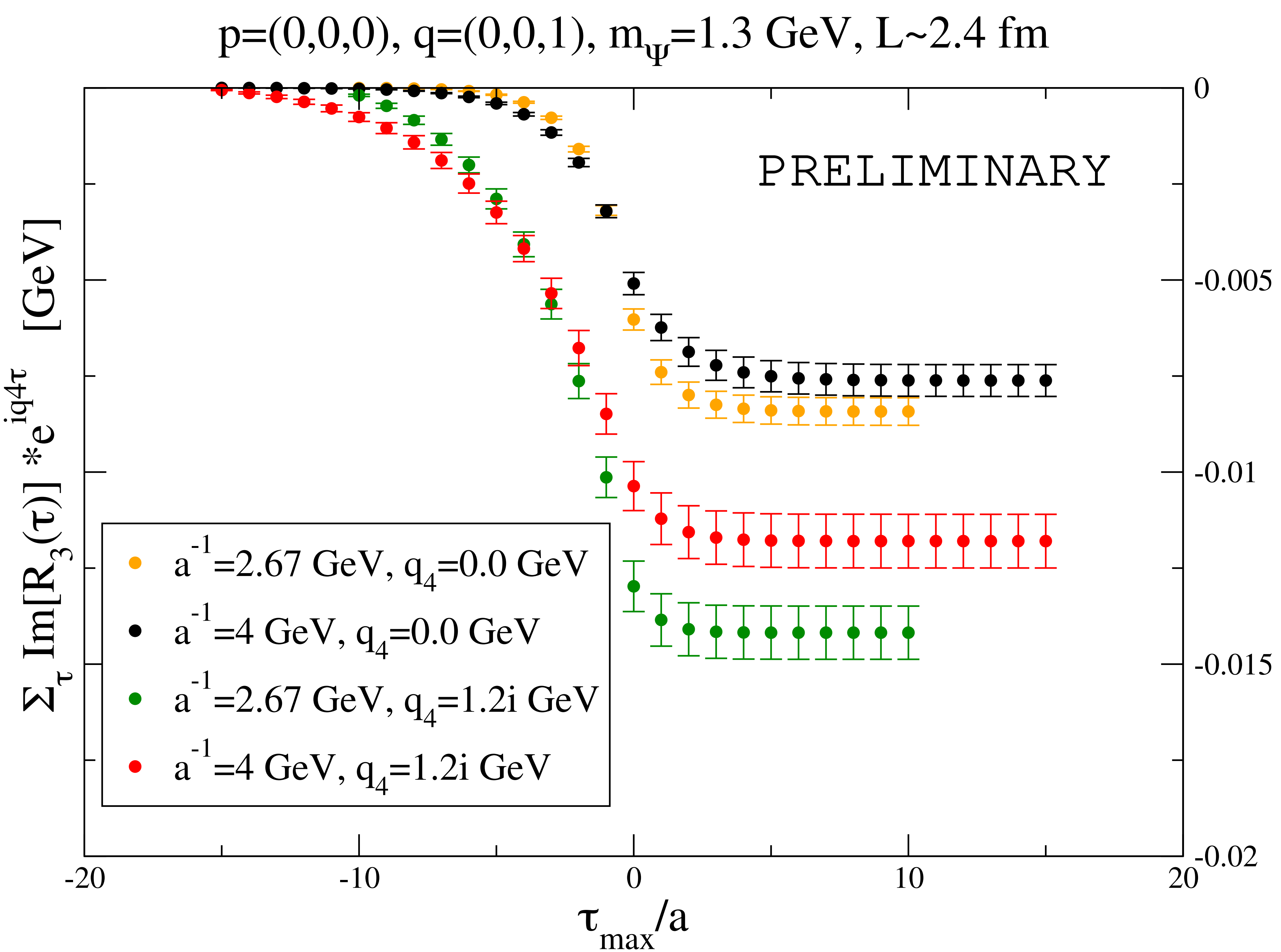
Exploratory numerical results

Parity implies

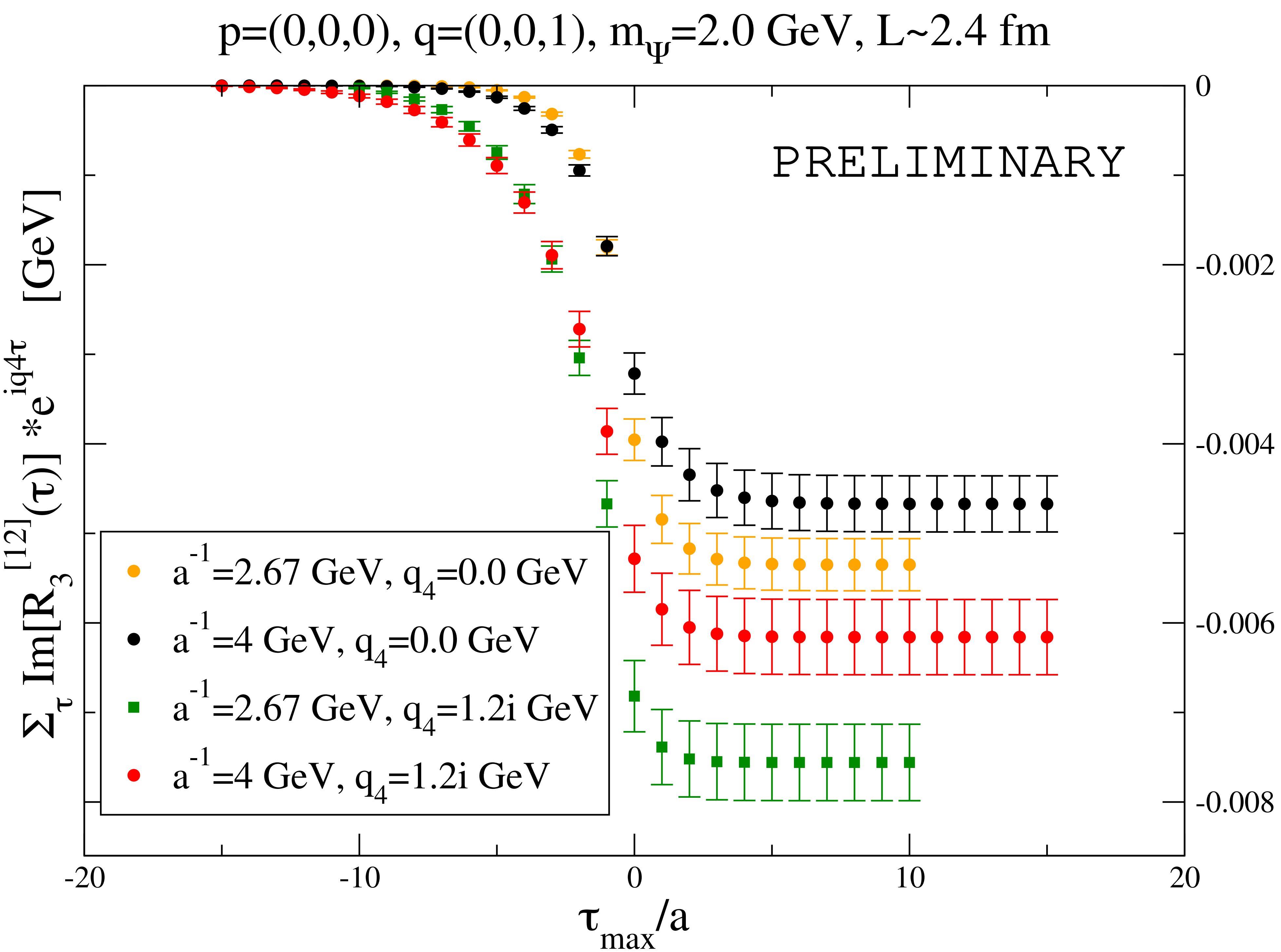
$$R_3^{\mu\nu}(\tau,\vec{q},\vec{p}) + g^{\mu\mu}g^{\nu\nu}R_3^{\mu\nu}(\tau,-\vec{q},-\vec{p}) = 0$$



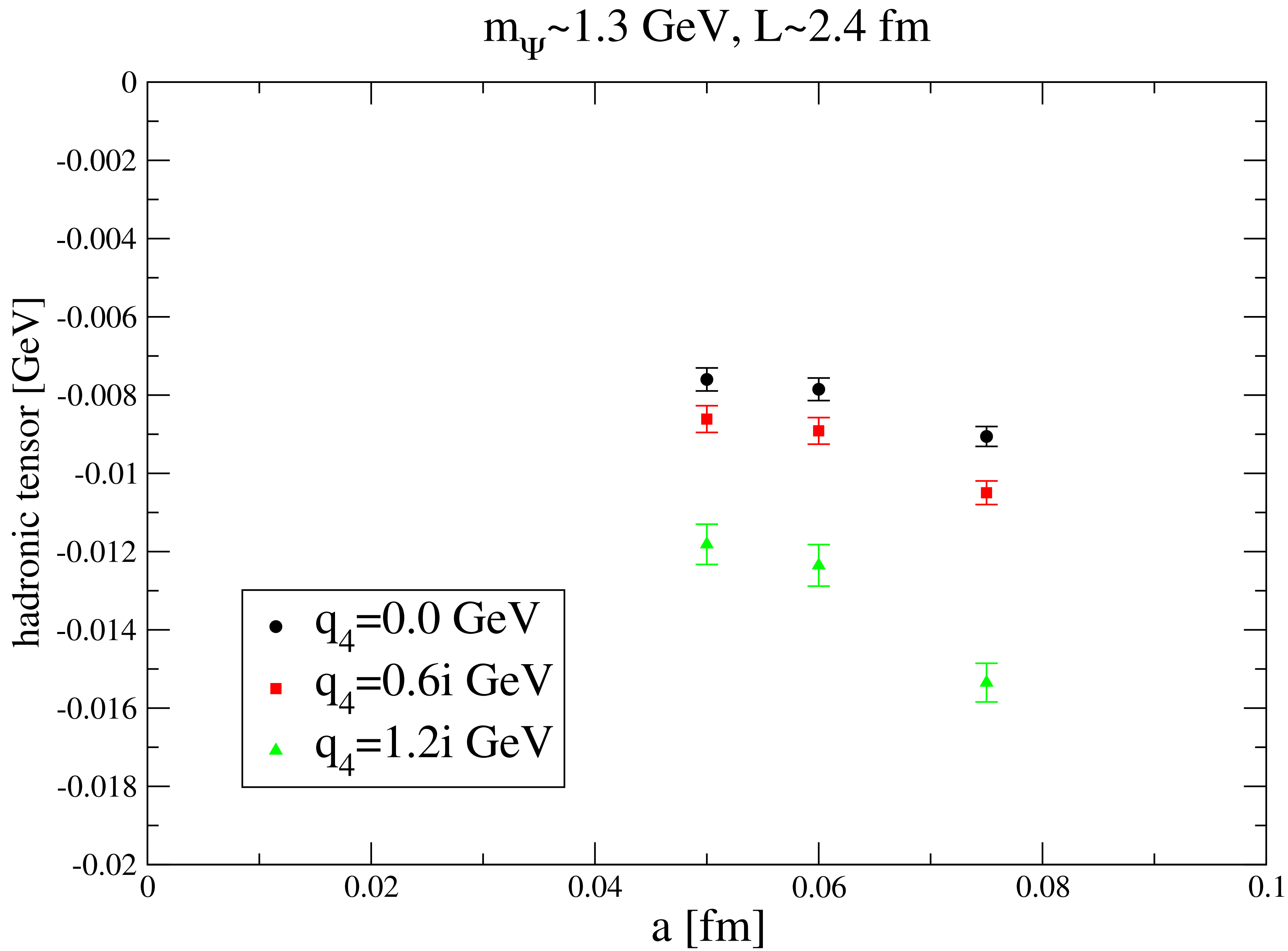
Exploratory numerical results



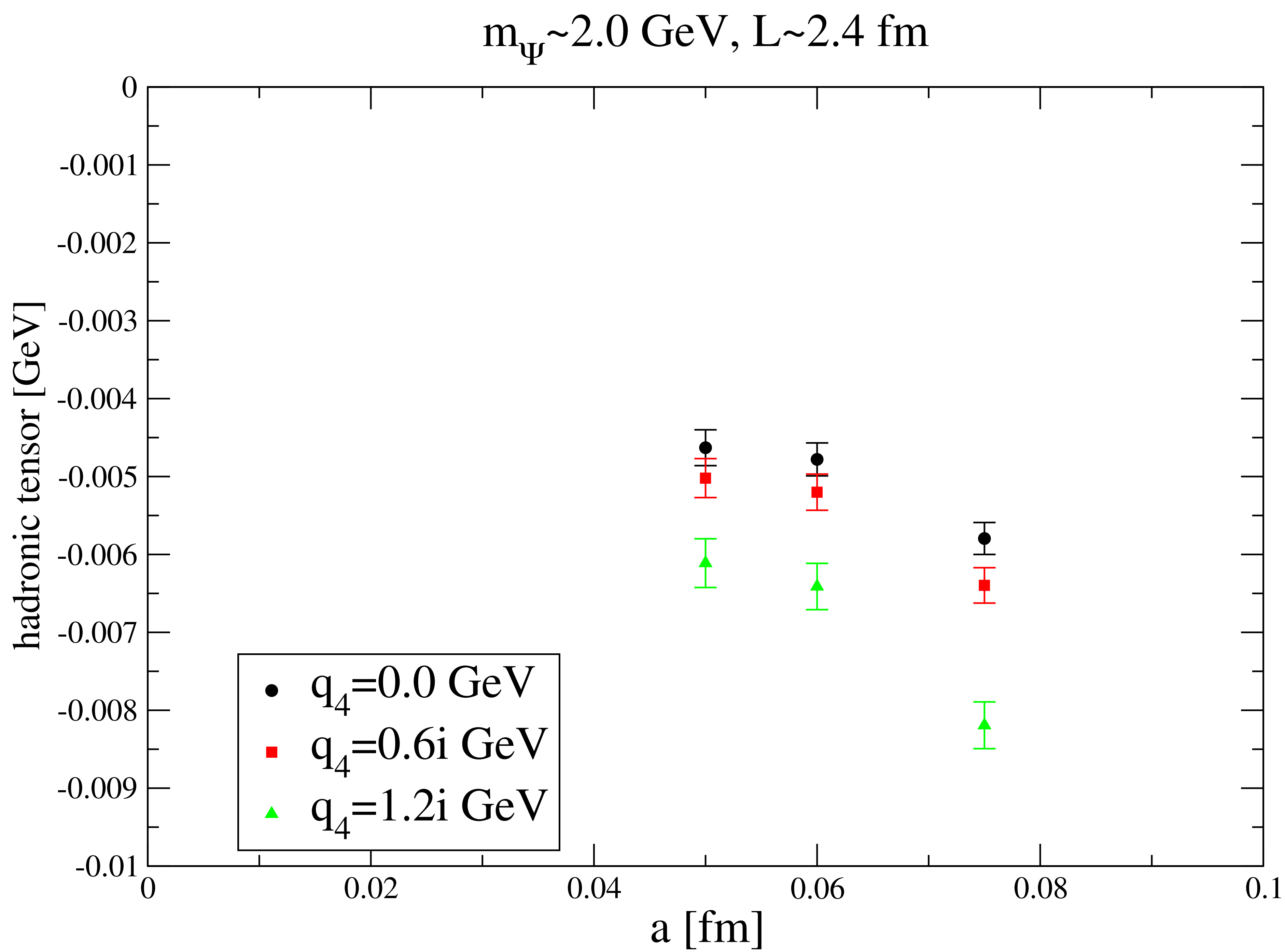
Exploratory numerical results



$$\vec{p} = \vec{q} = \vec{0}$$



$$\vec{p} = \vec{q} = \vec{0}$$



Outlook

- Our method has the potential to give viable information relevant to partonic physics.
- Thorough understanding of issues in numerical implementation, including ***the continuum limit***.
- Attempt the extraction of the moments soon.
- Investigate the relation to other methods.
- Many opportunities ahead.

Backup slides

Analysis details

$$R^{\mu\nu}_{3,\pi\to m\to e}(\tau_{3,m},\vec{q}_e,\vec{p}) = \theta(\tau_e-\tau_m)\times\frac{C_3^{\mu\nu}(\tau_e,\tau_m;\vec{p}_e,\vec{p}_m)}{C_\pi(\tau_m;\vec{p})}\times\left\langle\pi(\vec{p})\middle|\mathcal{O}^\dagger_\pi(\vec{0},0)\middle|0\right\rangle$$

$$= \int \mathrm{d}^3x\ \mathrm{e}^{i\vec{q}_e\cdot\vec{x}}\left\langle 0\middle|J_e^\mu(\vec{x},\tau_{3,m})\,J_m^\nu(\vec{0},0)\middle|\pi(\vec{p})\right\rangle_{\tau_{3,m}=\tau_e-\tau_m\geq 0;\ \vec{p}=\vec{p}_e+\vec{p}_m}$$

$$R^{\mu\nu}_{3,\pi\to e\to m}(\tau_{3,e},\vec{q}_m,\vec{p}) = \theta(\tau_m-\tau_e)\times\frac{C_3^{\mu\nu}(\tau_e,\tau_m;\vec{p}_e,\vec{p}_m)}{C_\pi(\tau_e;\vec{p})}\times\left\langle\pi(\vec{p})\middle|\mathcal{O}^\dagger_\pi(\vec{0},0)\middle|0\right\rangle$$

$$= \int \mathrm{d}^3x\ \mathrm{e}^{i\vec{q}_m\cdot\vec{x}}\left\langle 0\middle|J_m^\nu(\vec{x},\tau_{3,e})\,J_e^\mu(\vec{0},0)\middle|\pi(\vec{p})\right\rangle_{\tau_{3,e}=\tau_m-\tau_e\geq 0;\ \vec{p}=\vec{p}_e+\vec{p}_m}$$

$$R_3^{\mu\nu}(\tau,\vec{q},\vec{p}) \equiv \int \mathrm{d}^3x\ \mathrm{e}^{i\vec{q}\cdot\vec{x}}\left\langle 0\middle|\mathrm{T}\left[J_e^\mu(\vec{x},\tau)\,J_m^\nu(\vec{0},0)\right]\middle|\pi(\vec{p})\right\rangle$$

$$= \theta(\tau)\times R^{\mu\nu}_{3,\pi\to m\to e}(\tau,\vec{q},\vec{p}) + \theta(-\tau)\times \mathrm{e}^{-E_\pi(\vec{p})\tau}\times R^{\mu\nu}_{3,\pi\to e\to m}(-\tau,\vec{q},\vec{p})$$

$$U^{\mu\nu}(p,q) = i\int \mathrm{d}\tau\ R_3^{\mu\nu}(\tau,\vec{q},\vec{p})$$