GENERALIZED PARTON DISTRIBUTIONS, OPE, AND HOLOGRAPHY

The 2nd Workshop on Parton
Distribution Functions

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Institute of Physics, Academia Sinica
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References

- Early work:
  R.C. Brower, J. Polchinski, M. J.Strassler and C-I Tan, "The Pomeron and Gauge-String Duality", JHEP 12 (2007) 005, (arXiv:hepth/063115).
  M.Costa, V. Goncalves and J. Penedones, "Conformal Regge Theory", JHEP 12 (2012).
  R. C. Brower, M. S. Costa, M. Djuric, T. Raben, and C-I Tan, "Strong coupling expansion for conformal Pomeron/Odderon Trajectories", JHEP 02 (2015) 104.
- Phenomenological Applications:
  R. C. Brower, M. Djuric, I. Sarcevic, and C-I Tan, String-gauge dual description of DIS at small-x", JHEP 11 (2010) 051,
  M. S. Costa and M. Djuric, "DVCS from gauge/gravity duality", Phys. Rev D **86** 016009 (2012),
  M. Djuric and N. Evans, "Vector meson production at low-x from gauge/gravity duality", JHEP 09 (2013) 084.
  R. Nishio and T. Wateri, "Investigating Generalized Parton Distribution in Gravity Dual", Phys. Lett. B707, 362 (2012),"HE Photon-HadronScattering in Holographic QCD", Phys. Rev. D **84**, 075025 (2011),
  Many others.
- Other Related Applications:
  R. Nally, T. Raben, and C-I Tan, "Inclusive production through AdS/CFT", JHEP 11 (2017) 075.
  A. Watanabe and H-N Li, "Photon Structure Functions at small-x in Holographic QCD", Phys. Lett. B751 (2015) 321-325
  A. Watanabe and M. Huang, "Total hadronic cross section at high e=nergies in holographic QCD",
  Many others.
- Recent Applications to B-H Physics:
  Timothy Raben and Chung-I Tan, "Minkowski Conformal Blocks and the Regge Limit of SYK-like Models", Phys. Rev. D **98**, 086009 (2018), (arXiv:1801:04208).
  Simone Caron-Huot, "Analyticity in Spin in Conformal Theories", arXiv: 1703.00278v2.
  Simmons-Duff, D. Stanford, E. Witten, "A Spacetime derivation of the Lorentsian OPE inversion formula", arXiv: 1711.03816.

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Outline

- Generalized PDF and HE Scattering in QCD
  - Conformal Invariance and O(4,2) symmetry
- OPE and Conformal Block Expansion
- AdS/CFT and Holographic QCD
  - Pomeron as Graviton in AdS
- Holographic Treatment of DIS
- TOTEM at LHC: Size and Shape of Proton
  - Pion and Odderon
- Pomeron/Odderon Intercept in QCD

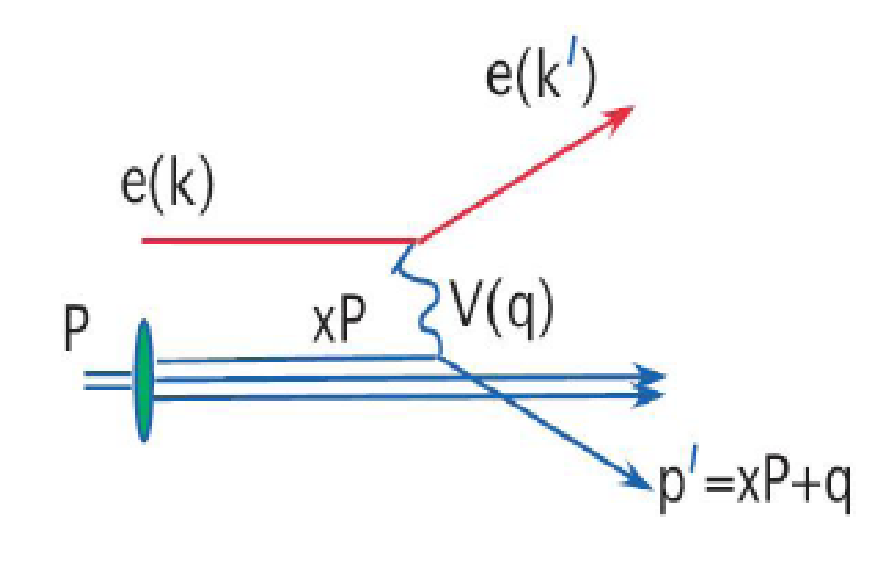
Outline

- **Generalized PDF as HE Scattering:**

  *** Unification of UV and IR physics**

  *** Non-Perturbative Treatment and Conformal Invariance**

  ***** $O(2,2) = O(1,1) \otimes O(1,1)$ **vs** $O(4,2) = O(3,1) \otimes O(1,1)$**:**



$$F_2(x, Q2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \left[\sigma_T(\gamma^* p) +_L (\gamma^* p)\right]$$

$$Small\ x : \ \frac{Q^2}{s} \to 0$$

Optical Theorem

$$\sigma_{total}(s, Q^2) = (1/s)\mathrm{Im}\ A(s, t = 0; Q^2)$$

Partonic vs Confinement?

UV vs IR?

$$\gamma(1) + \text{proton}(2) \to \gamma(3) + \text{proton}(4)$$

$$T^{\mu\nu}(p,q;p',q') = \langle p'|\mathbf{T}\{J^\mu(x)J^\nu(0)\}|p\rangle$$

$$\text{At } t=0; \quad T^{\mu\nu} = W_1(x,Q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + W_2(x,Q^2)\left(p_\mu + \frac{q_\mu}{2x}\right)\left(p_\nu + \frac{q_\nu}{2x}\right).$$

$$\text{DIS}: \quad \langle p|[J^\mu(x), J^\nu(0)]|p\rangle = \mathcal{F}_1(x,Q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \mathcal{F}_2(x,Q^2)\left(p_\mu + \frac{q_\mu}{2x}\right)\left(p_\nu + \frac{q_\nu}{2x}\right)$$

$$\mathcal{F}_\alpha(x,q) = 2\pi \, \mathrm{Im}\, W_\alpha(x,q)$$

$$\mathcal{F}_2(x,q) = (q^2/4\pi^2\alpha_{em})(\sigma_T + \sigma_L)$$

Application of Minkowski $d > 1$ CFT for Scattering:

Lorentz boost and dilatation consist of $O(1,1) \times O(1,1)$ subgroup of the full conformal transformations, $O(4,2)$.

It has long been known that approximate $O(2,2)$ symmetry is an important feature of QCD near-forward scattering at high energies

Treat the case of deep inelastic scattering (DIS) as a realization of $O(2,2)$ invariance for near forward scattering.

$$\gamma(1) + \text{proton}(2) \to \gamma(3) + \text{proton}(4)$$

$$T^{\mu\nu}(p,q;p',q') = \langle p'|\mathbf{T}\{J^\mu(x)J^\nu(0)\}|p\rangle$$

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$$\text{DIS}: \quad \langle p|[J^\mu(x), J^\nu(0)]|p\rangle = \mathcal{F}_1(x,Q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \mathcal{F}_2(x,Q^2)\left(p_\mu + \frac{q_\mu}{2x}\right)\left(p_\nu + \frac{q_\nu}{2x}\right)$$

$$\mathcal{F}_\alpha(x,q) = 2\pi\,\mathrm{Im}\,W_\alpha(x,q)$$

$$\mathcal{F}_2(x,q) = (q^2/4\pi^2\alpha_{em})(\sigma_T + \sigma_L)$$

For $t \neq 0$,

$$\boxed{\gamma(q_1) + \text{proton}(p_1) \to \gamma(q_2) + \text{proton}(p_2)}$$

$$\boxed{T^{\mu\nu}(p_1, p_2, q_1, q_2) = \text{F.T.}\ \langle p_2 | \mathbf{T}\{J^\mu(x_2) J^\nu(x_1)\} | p_1 \rangle}$$

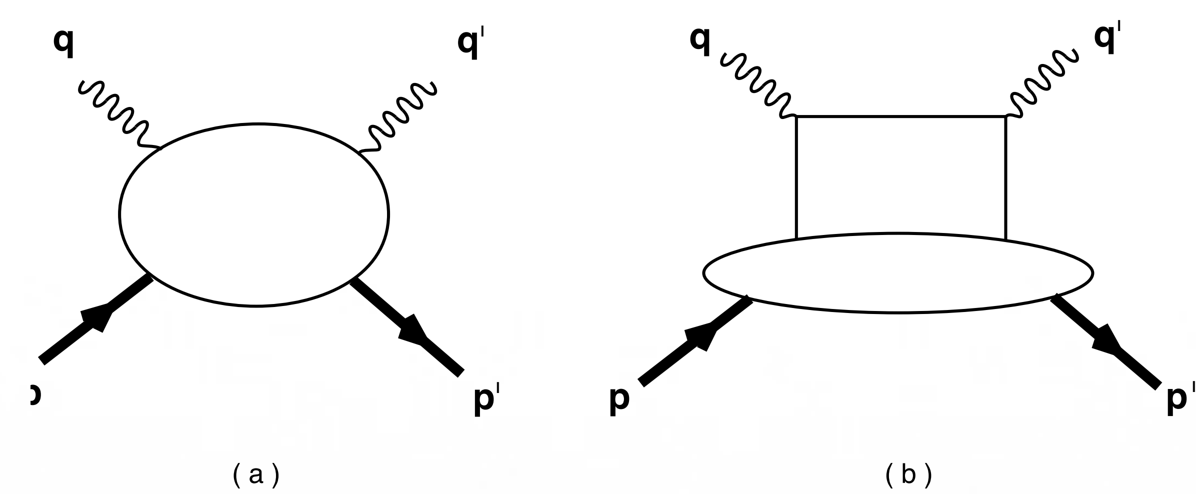


Figure 2. (*a*) Compton scattering; (*b*) leading Feynman diagrams for DVCS.

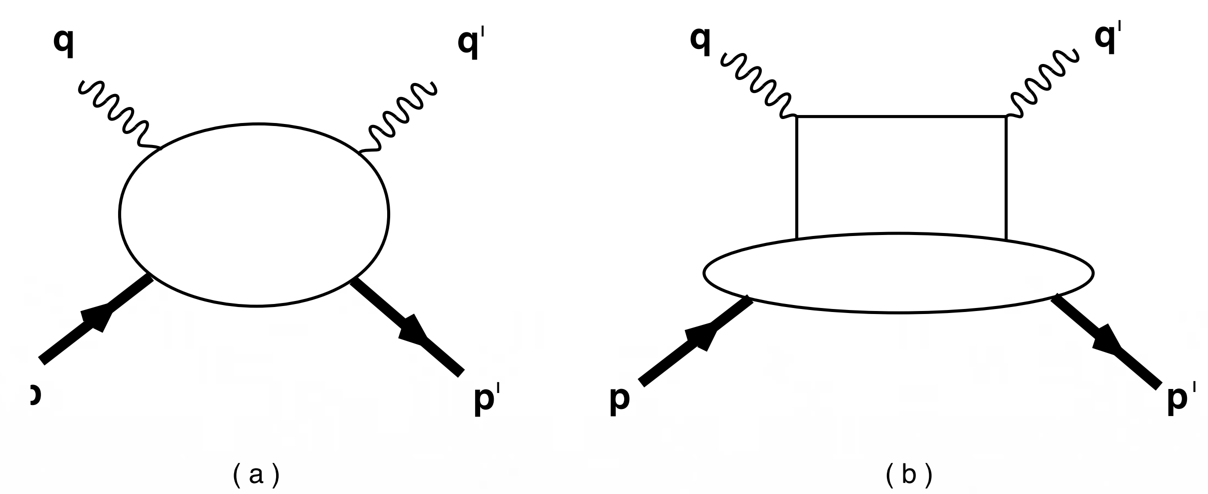
$$\boxed{\begin{aligned} T^{\mu\nu}(p_i, q_j) = {} & V_1(p_i, q_j)\bar{g}_1^{\mu\rho}\bar{g}_{2,\rho}^{\nu} + V_2(p_i, q_j)(p\cdot\bar{g}_1)^\mu (p\cdot\bar{g}_2)^\nu \\ & + V_3(p_i, q_j)(q_2\cdot\bar{g}_1)^\mu (q_1\cdot\bar{g}_2)^\nu + V_4(p_i, q_j)(p\cdot\bar{g}_1)^\mu (q_1\cdot\bar{g}_2)^\nu \\ & + V_5(p_i, q_j)(q_2\cdot\bar{g}_1)^\mu (p\cdot\bar{g}_2)^\nu + A^{\mu\nu\rho\sigma}(p_i, q_j) q_{1\rho} q_{2\sigma} \epsilon^{\sigma\rho\mu\nu} \end{aligned}}$$

$$\bar{g}^{\mu\nu} = \eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \qquad\qquad p = (p_1 + p_2)/2$$

For $t \neq 0$, Full conformal transformation $O(4,2) \simeq O(3,1) \otimes O(1,1)$:

$$\gamma(q_1) + \text{proton}(p_1) \to \gamma(q_2) + \text{proton}(p_2)$$

$$T^{\mu\nu}(p_1, p_2, q_1, q_2) = \text{F.T.}\ \langle p_2 | \mathbf{T}\{J^\mu(x_2) J^\nu(x_1)\} | p_1 \rangle$$



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Lorentz boost and dilatation consist of $O(1,1) \times O(1,1)$

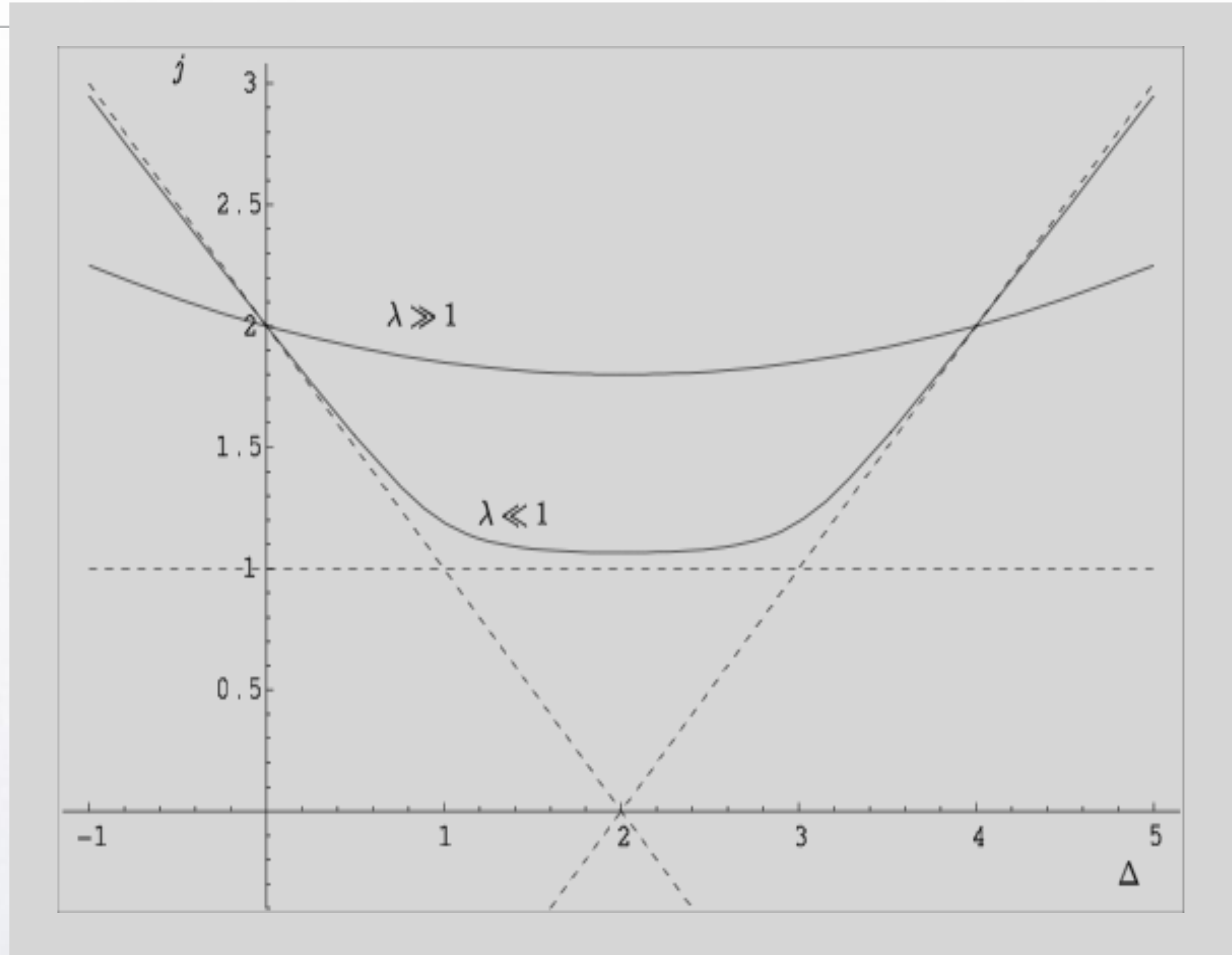
- Moments, Dilatation, and Anomalous Dimensions:

$$M_n(Q^2) = \int_0^1 dx x^{n-2} F_2(x, Q^2), \quad M_n(Q^2) \to (Q)^{-\gamma_n}$$

"Anomalous dimension"

$$\gamma_j = \Delta - j - \tau, \qquad \text{dimension} = \Delta, \quad \text{"spin"} = j$$

for the twist-two, $\tau = 2$, operators, j even, appropriate for the DIS.



- $x \to 0$ limit, Lorentz Boost and Effective Spin:

$$F_2(x, Q^2) \to x^{\delta}, \quad \delta = 1 - j_{eff}$$

for flavor-non-singlet, $\delta \simeq 1/2$,
($j_{eff} \equiv \rho -$ intercept $\simeq 1/2$)
for flavor-singlet, (gluon), $\delta \simeq -\varepsilon < 0$,
($j_{eff} \equiv$ tensor $-$ gluonball $-$ intercept $\simeq 1 + \varepsilon$).

- $x \to 1$ and/or $Q \to O(1)$

IR physics – Confinement effects.

- Unification:

$\Delta - j$ curve.

Theoretical Challenge: Calculate $\gamma(j)$ "non-perturbatively".

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- **Holographic Treatment of DIS:**

  ***Anomalous Dimension, DGLAP, etc.**

  *** Confinement, Saturation, etc.**

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- **Pomeron/Odderon intercept and AdS/CFT**

$$\boxed{\gamma^*(1) + \gamma^*(3) \to \gamma^*(2) + \gamma^*(4)}$$

$$\langle 0|T(\mathcal{J}_1(x_1)\mathcal{J}_2(x_2)\mathcal{J}_4(x_4)\mathcal{J}_3(x_3))|0\rangle = \frac{1}{(x_{12}^2)^{\Delta_1}(x_{34}^2)^{\Delta_3}} F^{(M)}(u,v)$$

$$\boxed{u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \qquad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2}}$$

$$\boxed{F^{(M)}(u,v) = \sum_\alpha a_\alpha^{(12;34)} G_\alpha^{(M)}(u,v)}$$

New Variables:

$$u = x\bar{x}, \quad v = (1-x)(1-\bar{x})$$

$$q \equiv \frac{2-x}{x}, \quad \text{and} \quad \bar{q} \equiv \frac{2-\bar{x}}{\bar{x}}$$

$$\boxed{w = \sqrt{q\bar{q}} \simeq \sqrt{u}^{-1} \to \infty \qquad \sigma = (\sqrt{q/\bar{q}} + \sqrt{\bar{q}/q})/2 \to \infty}$$

Minkowski OPE and Scattering

$$\boxed{F(w,\sigma)=\sum_\alpha\sum_\ell a^{(12),(34)}_{\ell,\alpha}\,G(w,\sigma;\ell,\Delta_{\ell,\alpha})}$$

$$\mathcal{D}\;G_{\Delta,\ell}(u,v)=C_{\Delta,\ell}\;G_{\Delta,\ell}(u,v)$$

$\mathcal{D}=(1-u-v)\partial_v(v\partial_v)+u\partial_u(2u\partial_u-d)-(1+u-v)(u\partial_u+v\partial_v)(u\partial_u+v\partial_v)\,,$
with $\Delta_{12}=0$, $\Delta_{34}=0$, and $\Delta_{ij}=\Delta_i-\Delta_j$

$$C_{\Delta,\ell}=\Delta(\Delta-d)/2+\ell(\ell+d-2)/2$$

$$C_{\Delta,\ell}=(\widetilde{\Delta}^2+\widetilde{\ell}^2)/2-(\epsilon^2+\epsilon+1/2) \qquad \epsilon=(d-2)/2$$

$$\widetilde{\Delta}=\Delta-(\epsilon+1)\,,\;\widetilde{\ell}=\ell+\epsilon$$

$$C_{\Delta,\ell}=\lambda_+(\lambda_+-1)+(\lambda_-(\lambda_--1)+2\epsilon\lambda_-) \qquad \lambda_\pm=(\Delta\pm\ell)/2$$

Unitary Representation of $O(5,1)$

$$F(w,\sigma)=\sum_\ell\int_{-\infty}^{\infty}\frac{d\nu}{2\pi}\,a(\ell,\nu)\,\mathcal{G}(\ell,\nu;w,\sigma)$$

$$a(\ell,\nu)=\sum_\alpha\frac{r_\alpha(\ell)}{\nu^2+\widetilde{\Delta}_\alpha(\ell)^2}=\sum_\alpha\frac{r_\alpha(\ell)}{2\nu}\Big(\frac{1}{\nu+i\widetilde{\Delta}_\alpha(\ell)}+\frac{1}{\nu-i\widetilde{\Delta}_\alpha(\ell)}\Big)$$

$$\widetilde{\Delta}\equiv i\nu=\Delta-d/2$$

$$F(w,\sigma)=\sum_\alpha\sum_\ell a^{(12),(34)}_{\ell,\alpha}\,G(w,\sigma;\ell,\Delta_{\ell,\alpha})$$

$\mathcal{G}(\ell,\nu;w,\sigma)=\mathcal{G}^{(+)}(\ell,\nu;w,\sigma)+\mathcal{G}^{(-)}(\ell,\nu;w,\sigma)$, where $\mathcal{G}^{(+)}(\ell,\nu;w,\sigma)=\mathcal{G}^{(-)}(\ell,-\nu;w,\sigma)$, with $\mathcal{G}^{(+)}$ leading to convergence in the lower ν-plane and $\mathcal{G}^{(-)}$ in the upper ν-plane

Dynamics

$$a_j(\Delta)\sim\frac{1}{\Delta-\Delta_j}\quad\to\quad\frac{1}{\Delta-\Delta(j)}$$

Single Trace Gauge Invariant Operators of $\mathcal{N}=4$ SYM,

$$Tr[F^2],\quad Tr[F_{\mu\rho}F_{\rho\nu}],\quad Tr[F_{\mu\rho}D^S_\pm F_{\rho\nu}],\quad Tr[Z^\tau],\quad Tr[D^S_\pm Z^\tau],\cdots$$

Super-gravity in the $\lambda\to\infty$:

$$Tr[F^2]\leftrightarrow\phi,\quad Tr[F_{\mu\rho}F_{\rho\nu}]\leftrightarrow G_{\mu\nu},\quad\cdots$$

Symmetry of Spectral Curve:

$$\Delta(j)\leftrightarrow 4-\Delta(j)$$

Euclidean CFT

$$SO(5,1)=SO(1,1)\times SO(4)$$

$$\mathcal{A}(u,v)\leftrightarrow\int_{d/2-i\infty}^{d/2+i\infty}\frac{d\Delta}{2\pi i}\sum_j\ a_j(\Delta)\,G_{\Delta,j}(u,v)$$

Minkowski CFT: $SO(4,2)=SO(1,1)\times SO(3,1)$

Unitary Representation of $O(4,2)$

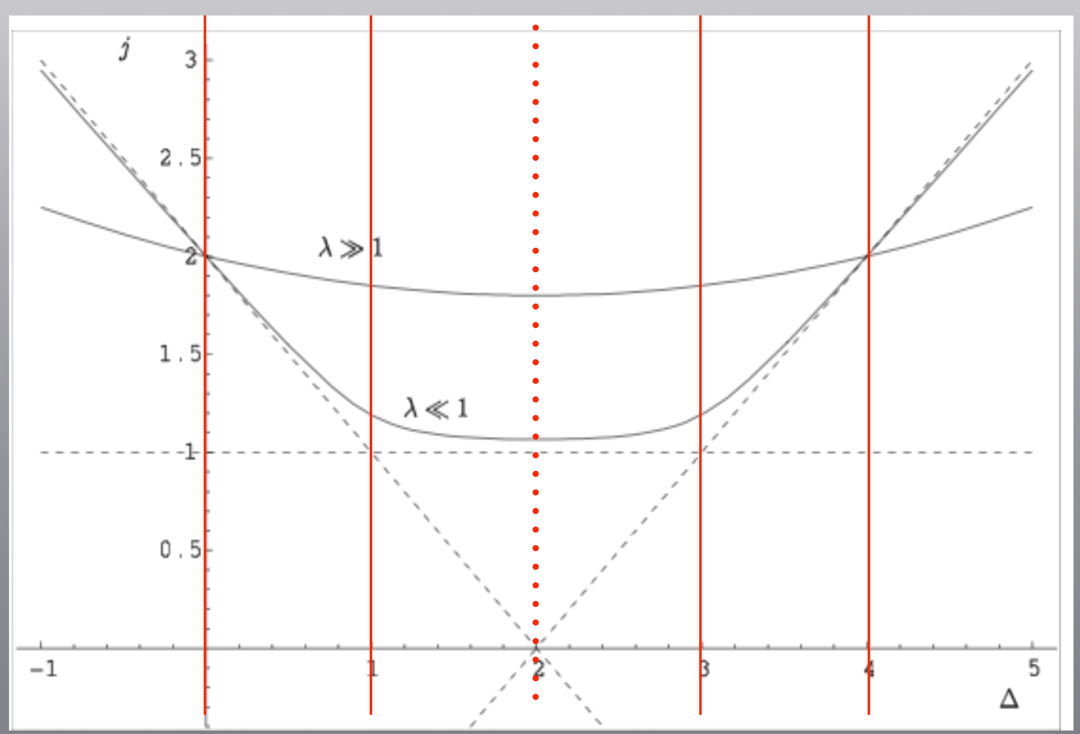
$$\mathcal{A}(u,v)=\int_{d/2-i\infty}^{d/2+i\infty}\frac{d\Delta}{2\pi i}\int_{-\epsilon-i\infty}^{-\epsilon+i\infty}\frac{d\ell}{2\pi i}\ a(\Delta,\ell)\ \mathcal{G}(u,v,\Delta,\ell)$$

Conformal Regge theory $\Leftrightarrow$ meromorphic representation in the $\nu-\ell$ plane

$$a(\ell,\nu)=\sum_\alpha\frac{r_\alpha(\ell)}{\nu^2+\widetilde{\Delta}_\alpha(\ell)^2}=\sum_\alpha\frac{r_\alpha(\ell)}{2\nu}\Big(\frac{1}{\nu+i\widetilde{\Delta}_\alpha(\ell)}+\frac{1}{\nu-i\widetilde{\Delta}_\alpha(\ell)}\Big)$$

ANOMALOUS DIMENSIONS:

$$\gamma(j,\lambda)=\Delta(j,\lambda)-j-2$$



$$\gamma_2=0$$

$$\Delta(j)=2+\sqrt{2}\sqrt{\sqrt{g^2N_c}(j-j_0)}$$

$$\gamma_n=2\sqrt{1+\sqrt{g^2N}(n-2)/2}-n$$

Energy-Momentum Conservation built-in automatically.

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II. Gauge-String Duality: AdS/CFT

Weak Coupling:

Gluons and Quarks: $A_\mu^{ab}(x), \psi_f^a(x)$

Gauge Invariant Operators: $\bar\psi(x)\psi(x),\ \ \bar\psi(x)D_\mu\psi(x)$

$S(x) = TrF^2_{\mu\nu}(x),\ \ O(x) = TrF^3(x)$

$T_{\mu\nu}(x) = TrF_{\mu\lambda}(x)F_{\lambda\nu}(x),\ \ etc.$

$$\mathcal{L}(x) = -TrF^2 + \bar\psi \not{D}\psi + \cdots$$

Strong Coupling:

Metric tensor: $G_{mn}(x) = g^{(0)}_{mn}(x) + h_{mn}(x)$

Anti-symmetric tensor (Kalb-Ramond fields): $b_{mn}(x)$

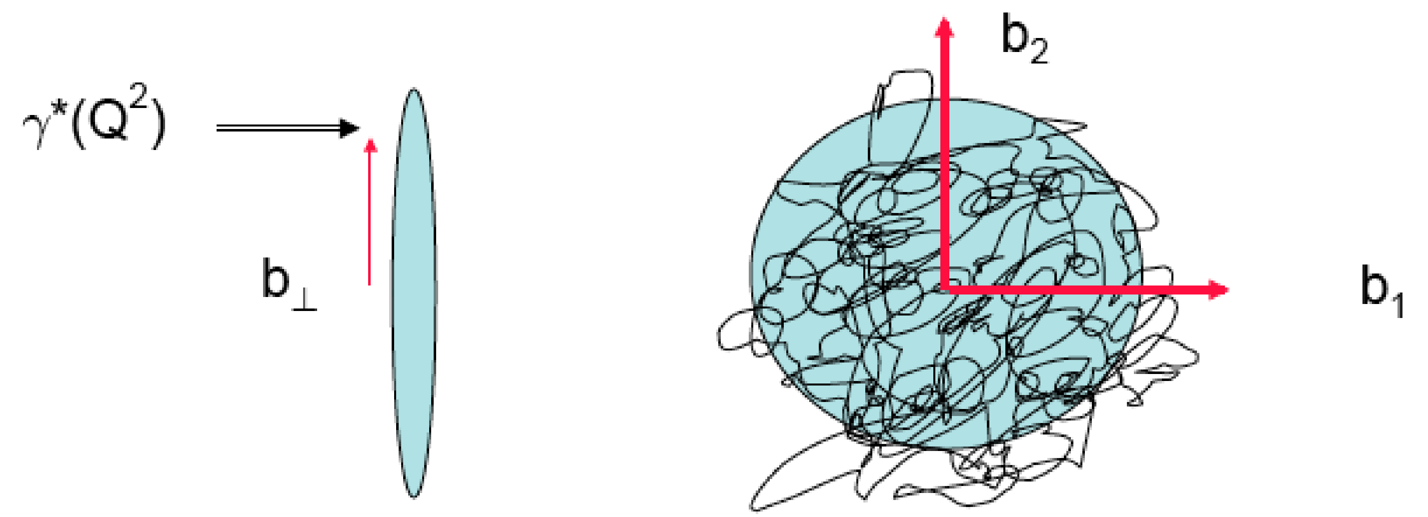
Dilaton, Axion, etc. $\phi(x),\ a(x),\ etc.$

Other differential forms: $C_{mn\cdots}(x)$

$$\mathcal{L}(x) = \mathcal{L}(G(x), b(x), C(x), \cdots)$$

QCD EMERGENCE OF 5-DIM ADS

"Fifth" co-ordinate is size z / z' of proj/target



5 kinematical Parameters:

2-d Longitudinal	$p^\pm = p^0 \pm p^3 \simeq \exp[\pm\log(s/\Lambda_{qcd})]$
2-d Transverse space:	$x'_\perp - x_\perp = b_\perp$
1-d Resolution:	$z = 1/Q$ (or $z' = 1/Q'$)

$\mathcal{N} = 4$ SYM Scattering at High Energy

$$\langle e^{\int d^4x \phi_i(x)\mathcal{O}_i(x)}\rangle_{CFT} = \mathcal{Z}_{string}\left[\phi_i(x,z)|_{z\sim 0} \to \phi_i(x)\right]$$

Bulk Degrees of Freedom from type-IIB Supergravity on AdS5:

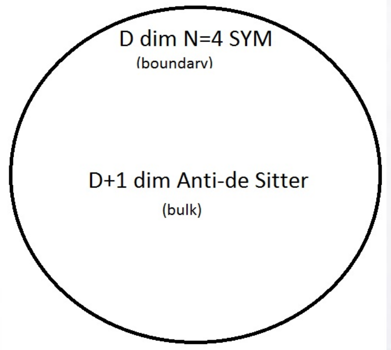
- metric tensor: G_{MN}
- Kalb-Ramond 2 Forms: B_{MN}, C_{MN}
- Dilaton and zero form: ϕ and C_0

$$\lambda = g^2 N_c \to \infty$$

Supergravity limit

- Strong coupling
- Conformal
- Pomeron as Graviton in AdS

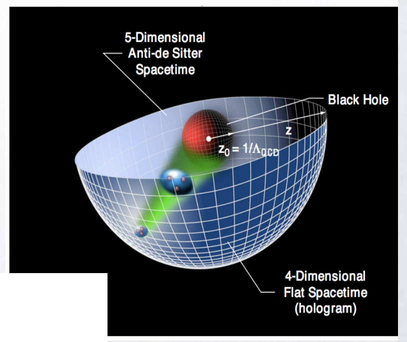
Background and Motivation



The AdS/CFT is a holographic duality that equates a string theory (gravity) in high dimension with a conformal field theory (gauge) in 4 dimensions. Specifically, compactified 10 dimensional super string theory is conjectured to correspond to $\mathcal{N} = 4$ Super Yang Mills theory in 4 dimensions in the limit of large 't Hooft coupling:

$\lambda = g_s N = g^2_{ym} N_c = R^4/\alpha'^2 >> 1.$

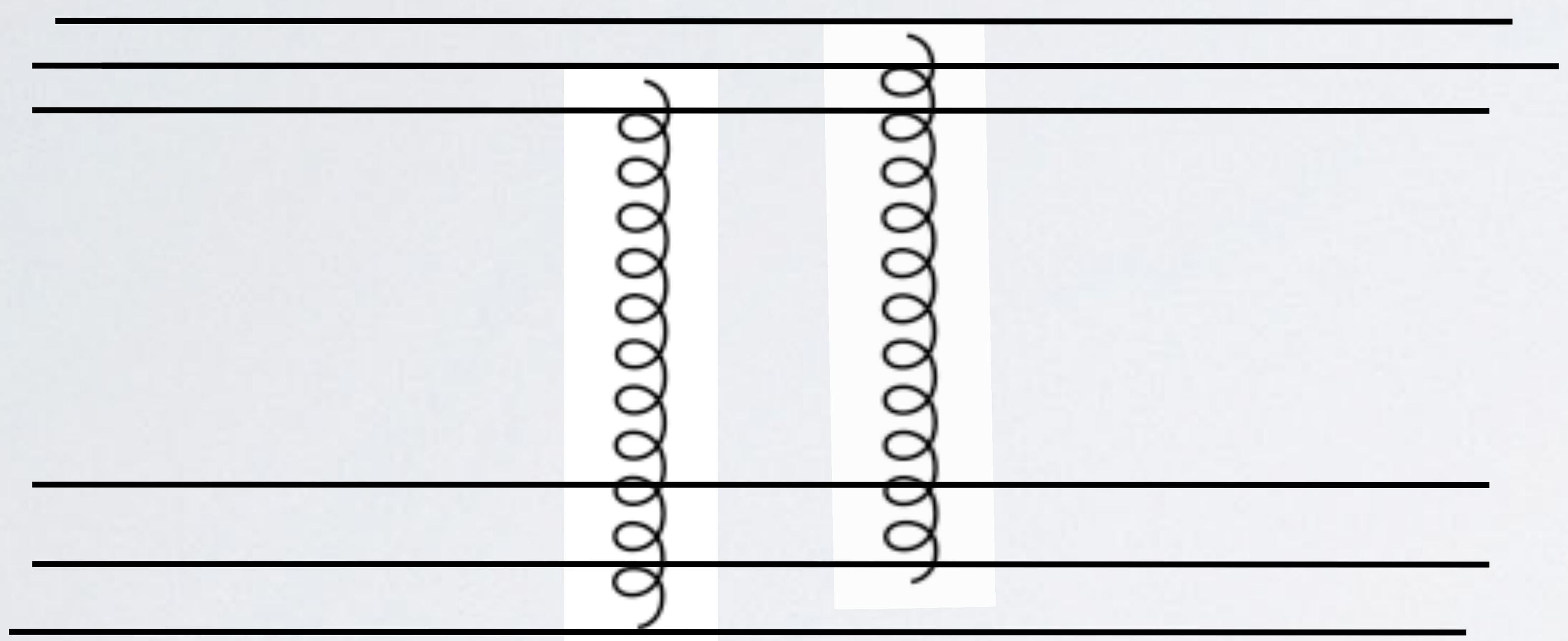
$$ds^2 = \frac{R^2}{z^2}\left[dz^2 + dx\cdot dx\right] + R^2 d\Omega_5 \to e^{2A(z)}\left[dz^2 + dx\cdot dx\right] + R^2 d\Omega_5$$



HIGH ENERGY SCATTERING <=> POMERON

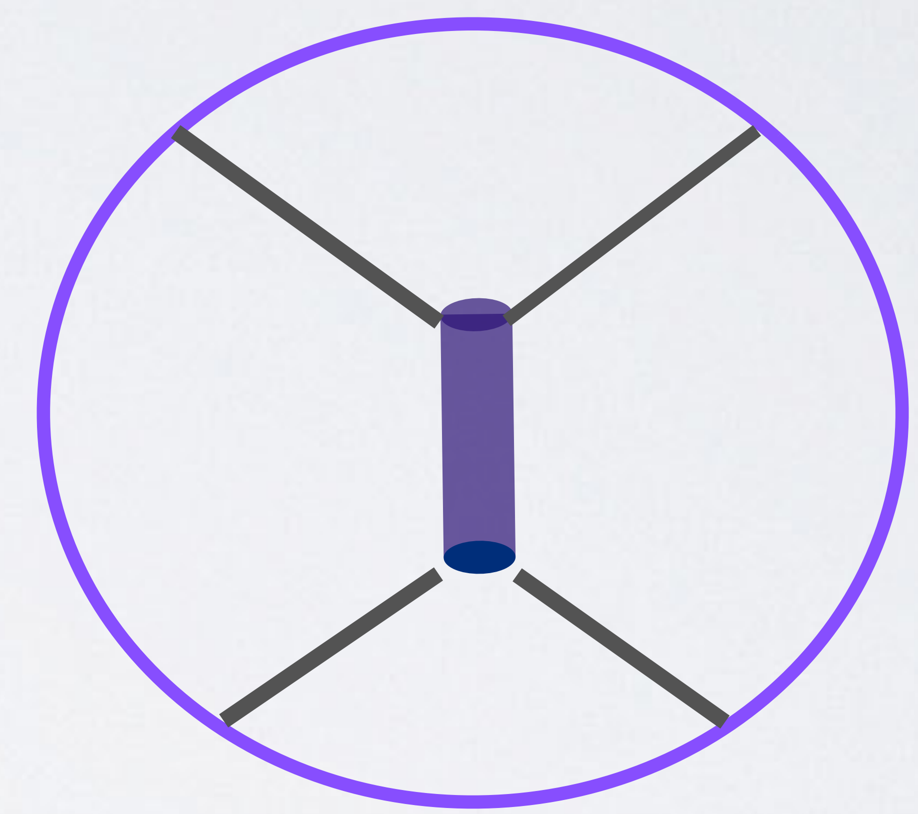
WHAT IS THE POMERON ?

WEAK: TWO-GLUON <=> STRONG: ADS GRAVITON



$$J_{cut} = 1 + 1 - 1 = 1$$

F.E. Low. Phys. Rev. D 12 (1975), p. 163.
S. Nussinov. Phys. Rev. Lett. 34 (1975), p. 1286.



$$J = 2$$

$$S = \frac{1}{2\kappa^2} \int d^4x dz \sqrt{-g(z)} \Big(-\mathcal{R} + \frac{12}{R^2} + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi \Big)$$

AdS Witten Diagram: Adv. Theor. Math. Physics 2 (1998)253

Unification and Universality:

Gauge/String Duality (AdS/CFT) $\longrightarrow$ 2-GLUONS $\simeq$ GRAVITON

- Weak-Strong Duality

Unification and Universality:

Gauge/String Duality (AdS/CFT) $\longrightarrow$ 2-GLUONS $\simeq$ GRAVITON

- Gauge Dynamics — Geometry of Space-Time
- Weak-Strong Duality

Unification and Universality:

Gauge/String Duality (AdS/CFT) $\longrightarrow$ 2-GLUONS $\simeq$ GRAVITON

- Unification of Soft and Hard Physics in High Energy Collision
- Gauge Dynamics — Geometry of Space-Time
- Weak-Strong Duality

- Improved phenomenology based on "Large Pomeron intercept", e.g., DIS at small-x: (DGLAP vs Pomeron), Elastic/Total Cross sections, DVCS, Central Diffractive Higgs Production, etc.

Soft Pomeron trajectory [Donnachie, Landshoff]

- Trajectory selected by exchanged quantum numbers. For elastic scattering these are the vacuum quantum numbers.

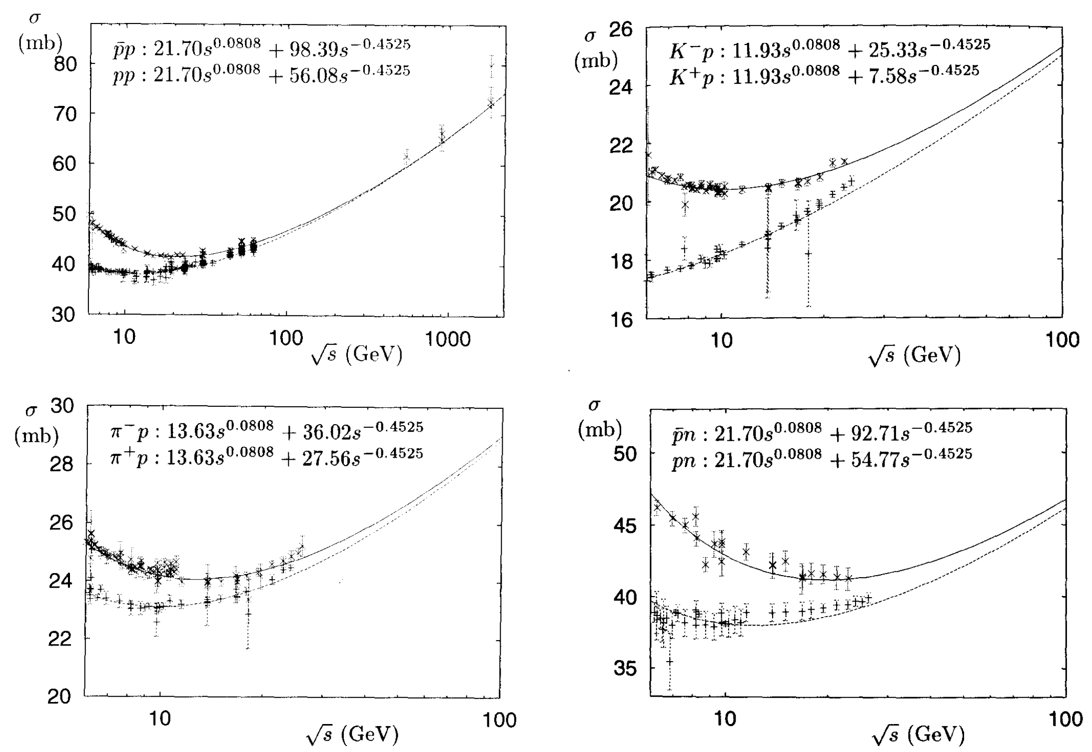
$j_P(t) \approx 1.08 + 0.25t \quad \text{(GeV units)}$

Elastic cross sections in QCD

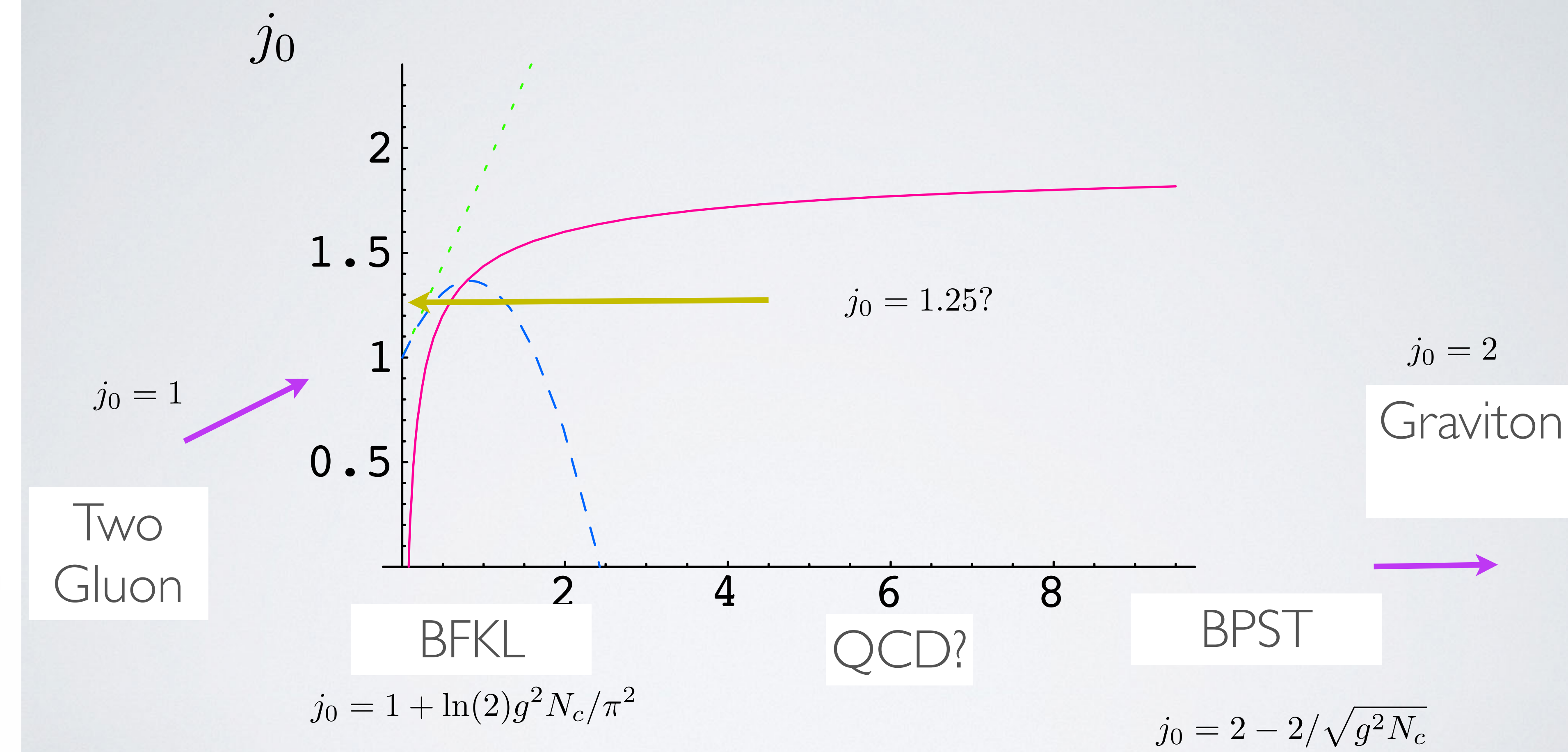
$\sigma \sim s^{j_P(0)-1} \sim s^{0.08}$

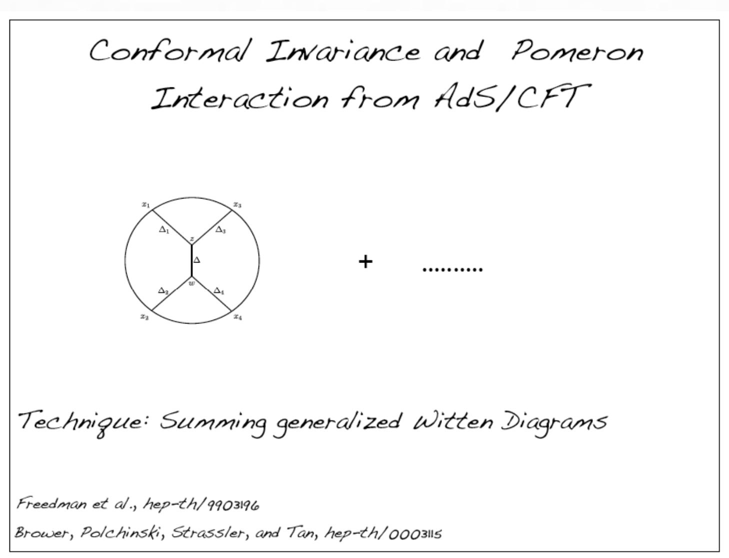
Exchange of even spin glueballs $(J \geq 2)$

$\mathcal{O}_J \sim F_{\alpha[\beta_1} D_{\beta_2} \dots D_{\beta_{J-1}} F_{\beta_J]}{}^{\alpha}$



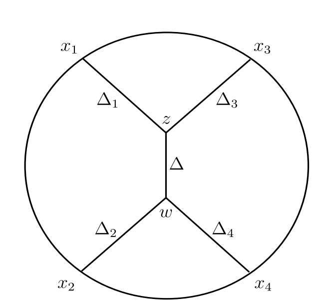
$\mathcal{N} = 4$ Strong vs Weak $g^2 N_c$





- Draw all "Witten-Feynman" Diagrams in AdS_5,
- High Energy Dominated by Spin-2 Exchanges:

$p_1 + p_2 \to p_3 + p_4$



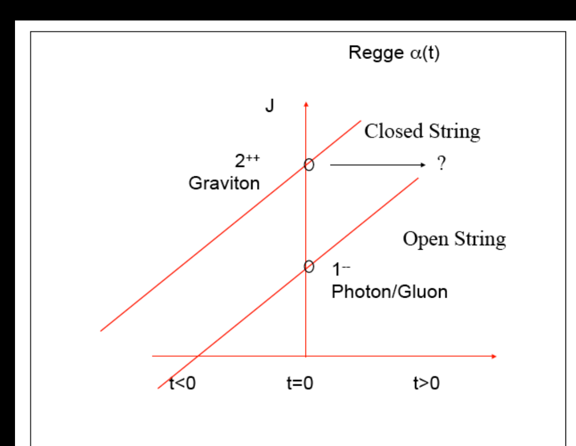
One Graviton Exchange at High Energy

$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5} \tilde{\Phi}_\Delta(p_1^2,z)\tilde{\Phi}_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde{\Phi}_\Delta(p_2^2,z')\tilde{\Phi}_\Delta(p_4^2,z')$$

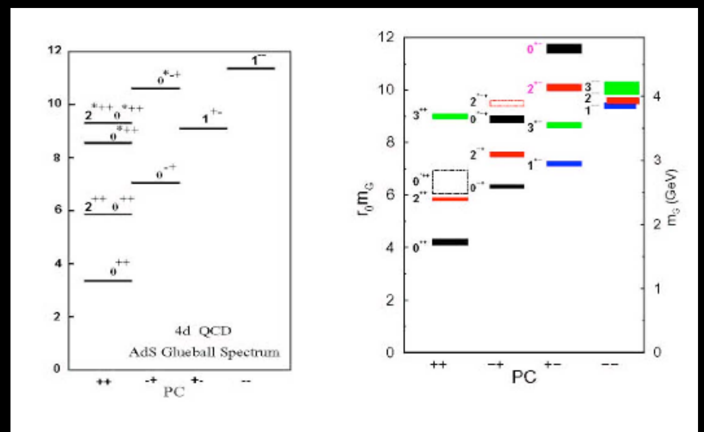
$$\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')$$

QCD Pomeron <===> Graviton (metric) in AdS

Flat-space String



Confinement



Conformal Invariance

Fixed cut in J-plane:

j_0

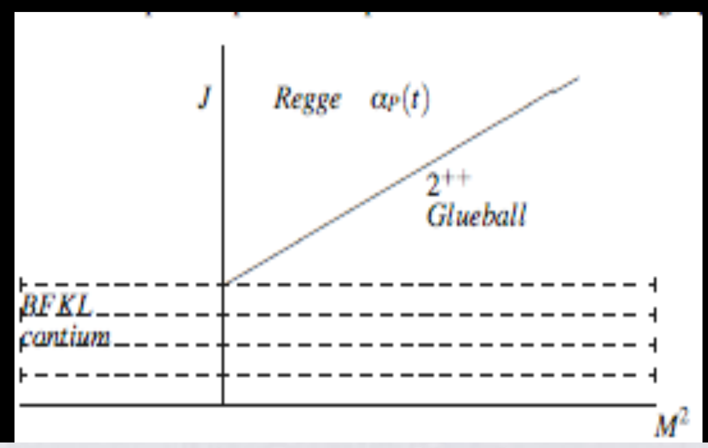
Weak coupling:
(BFKL)

$$j_0 = 1 + \frac{4 \ln 2}{\pi} \alpha N$$

Strong coupling:

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

Pomeron in AdS Geometry



Outline

- **Generalized PDF as HE Scattering:**

  *** Unification of UV and IR physics**

  *** Non-Perturbative Treatment and Conformal Invariance**

  ***** $O(2,2) = O(1,1) \otimes O(1,1)$ **vs** $O(4,2) = O(3,1) \otimes O(1,1)$**:**

- **OPE and Conformal Block Expansion**
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  ***Anomalous Dimension, DGLAP, etc.**

  *** Confinement, Saturation, etc.**

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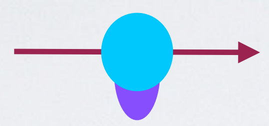
- **Pomeron/Odderon intercept and AdS/CFT**

BASIC BUILDING BLOCK

- Elastic Vertex:



- Pomeron/Graviton Propagator:

$$\mathcal{K}(s,b,z,z') = -\left(\frac{(zz')^2}{R^4}\right)\int \frac{dj}{2\pi i}\left(\frac{1+e^{-i\pi j}}{\sin \pi j}\right)\hat{s}^j\, G_j(z,x^\perp,z',x'^\perp;j)$$

conformal:

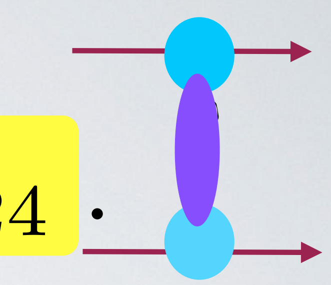
$$G_j(z,x^\perp,z',x'^\perp) = \frac{1}{4\pi z z'}\frac{e^{(2-\Delta(j))\xi}}{\sinh\xi}\,,$$

$$\Delta(j) = 2+\sqrt{2}\,\lambda^{1/4}\sqrt{(j-j_0)}$$

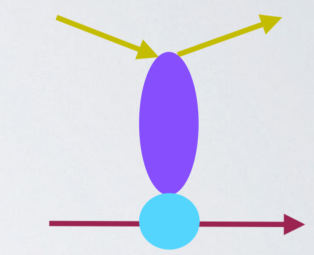
ADS BUILDING BLOCKS BLOCKS

For 2-to-2

$$A(s,t) = \Phi_{13} * \widetilde{\mathcal{K}}_P * \Phi_{24}\,.$$



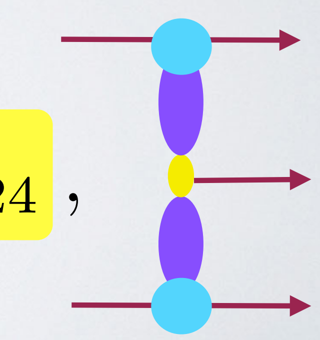
$$A(s,t) = g_0^2\int d^3\mathbf{b}d^3\mathbf{b}'\, e^{i\mathbf{q}_\perp\cdot(\mathbf{x}-\mathbf{x}')}\,\Phi_{13}(z)\,\mathcal{K}(s,\mathbf{x}-\mathbf{x}',z,z')\,\Phi_{24}(z')$$



$$d^3\mathbf{b} \equiv dz d^2x_\perp\sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

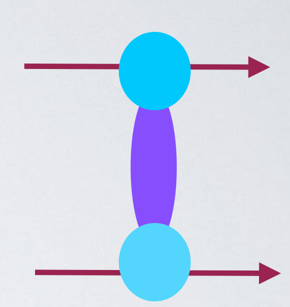
For 2-to-3

$$A(s,s_1,s_2,t_1,t_2) = \Phi_{13} * \widetilde{\mathcal{K}}_P * V * \widetilde{\mathcal{K}}_P * \Phi_{24}\,,$$



ELASTIC VS DIS ADS BUILDING BLOCKS

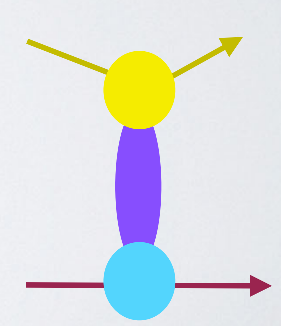
$$A(s,x_\perp - x'_\perp) = g_0^2\int d^3\mathbf{b}d^3\mathbf{b}'\Phi_{12}(z)G(s,x_\perp - x'_\perp,z,z')\Phi_{34}(z')$$



$$\sigma_T(s) = \frac{1}{s}Im A(s,0)$$

for $F_2(x,Q)$

$$\Phi_{13}(z) \to \Phi_{\gamma^*\gamma^*}(z,Q) = \frac{1}{z}[Qz)^4(K_0^2(Qz)+K_1^2(Qz)]$$



$$d^3\mathbf{b} \equiv dz d^2x_\perp\sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

Deep-Inelastic Scattering as Minkowski CFT

Reduction to $d=2$:
Discontinuity:
Mellon Representation:

$$W_2(w,\sigma_2) = \sum_\alpha \sum_{\ell\ even} a_\alpha(\ell)\mathcal{K}_\alpha(w,\sigma_2;\ell)$$

$$W(w,\sigma_2) = W_0(w,\sigma_2) - \sum_\alpha \int_{L_0-i\infty}^{L_0+i\infty}\frac{d\ell}{2\pi i}\frac{1+e^{-i\pi\ell}}{\sin\pi\ell}a_\alpha^{(12),(34)}(\ell)K_\alpha(w,\sigma_0;\ell)$$

$$ImW(w,\sigma_2) = \sum_\alpha \int_{L_0-i\infty}^{L_0+i\infty}\frac{d\ell}{2i}a_\alpha^{(12),(34)}(\ell)K_\alpha(w,\sigma_2;\ell)$$

Dilatation: $\to \frac{dM(\sigma_2,2n)}{d\log\sigma_2} \simeq -(\Delta(2n)-2)A(\sigma_2,2n) \to$ **DGLAP**

Lorentz Boost $\to \mathcal{F}_2(w,\sigma) \simeq w^{\ell_{eff}-1} \to$ **Effective Spin**, ℓ_{eff}

Graviton Spectral Curve: $a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j} \quad \to \quad \frac{1}{\Delta - \Delta(j)}$

Single Trace Gauge Invariant Operators of $\mathcal{N} = 4$ SYM,

$$Tr[F_{\pm\perp} D_{\pm}^{j-2} F_{\perp\pm}], \quad j = 2, 4, \cdots$$

Super-gravity in the $\lambda \to \infty$:

$$\Delta(2) = 4; \quad \Delta(j) = O(\lambda^{1/4}) \to \infty, \quad j > 2$$

Symmetry of Spectral Curve:

$$\Delta(j) \leftrightarrow 4 - \Delta(j)$$

Spectral Curve:

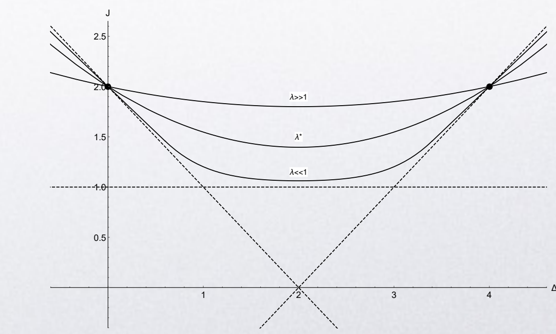
$$Im\,F(w,\sigma) = \pm \sum_{\alpha} \int_{L_0 - i\infty}^{L_0 + i\infty} \frac{d\ell}{2i} a^{(12),(34)}(\ell, \Delta_\alpha(\ell)) G(w,\sigma;\ell,\Delta_\alpha(\ell))$$

$$\Delta(\ell)(\Delta(\ell) - d) = m^2_{AdS}(\ell) \qquad d = 4, \quad m^2_{AdS}(\ell) = \sum_{n=1} \beta_n (\ell - 2)^n$$

B.Basso, 1109.3154v2

$$\Delta_P(\ell) \simeq 2 + B(\ell)\sqrt{\ell - \ell_{eff}}$$

$$\mathrm{Im}\,F(w,\sigma) \sim \frac{w^{\ell_{eff}-1}}{|\ln w|^{3/2}}$$



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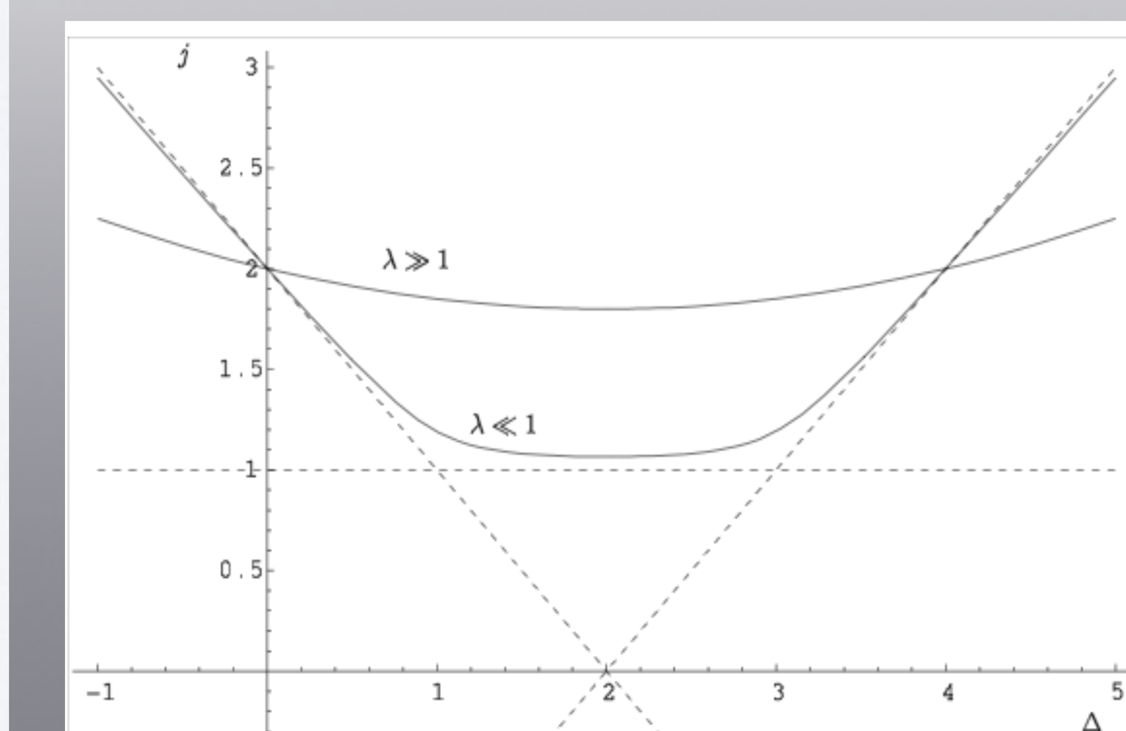
HE scattering after AdS/CFT

MOMENTS AND ANOMALOUS DIMENSION

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2} F_2(x,Q^2) \to Q^{-\gamma_n}$$



$\gamma_2 = 0$

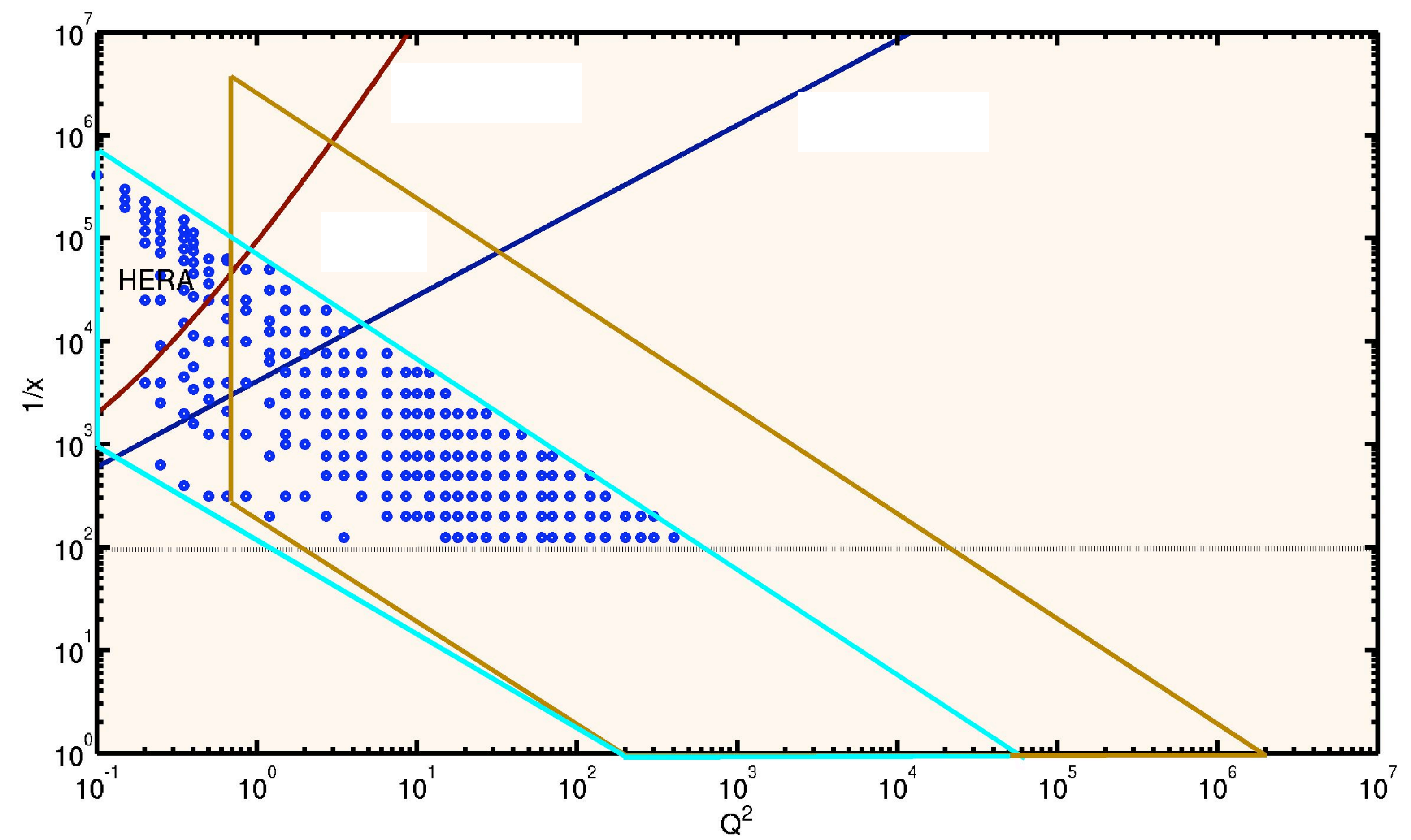
$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

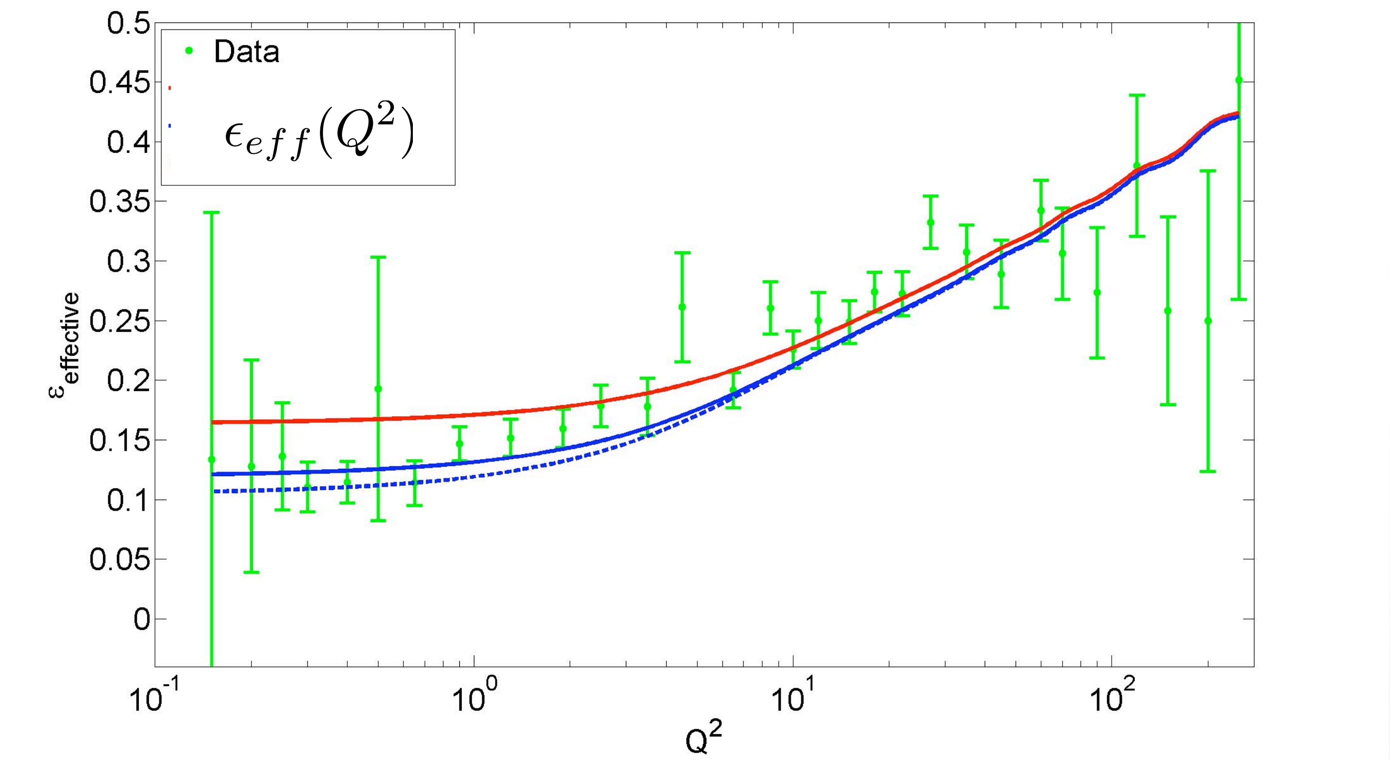
Simultaneous compatible large Q^2 and small x evolutions!

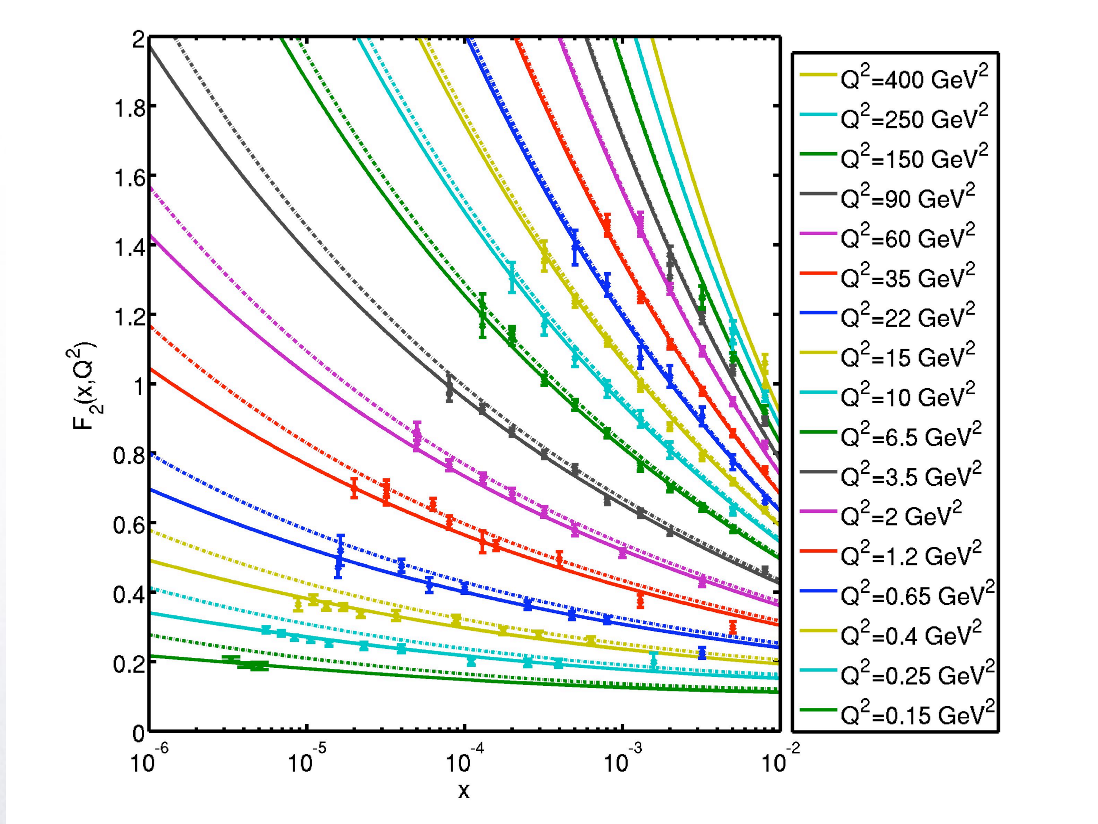
Energy-Momentum Conservation built-in automatically.

dots are H1-ZEUS small-x data points



$F_2(x, Q^2) \sim (1/x)^{\epsilon_{effective}}$



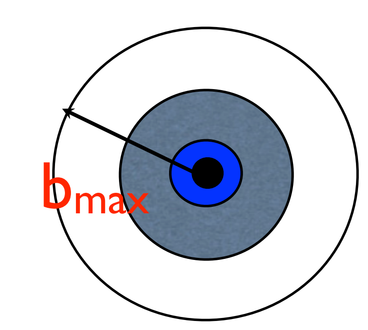


Saturation of Froissart Bound

- The Confinement deformation gives an exponential cutoff for $b > b_{max} \sim c \log(s/s_0)$,
- Coefficient $c \sim 1/m_0$, m_0 being the mass of lightest tensor glueball.
- Froissart is respected and saturated.

$$\Delta b \sim \log(s/s_0)$$

Disk picture



b_{max} determined by confinement.

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Size and Shape of Hadrons

TOTEM Elastic cross section at LHC

- new structure at $-0.05 < t < 0$
  Importance of "two-Pion shadow".

L. Jenkovszky, I. Szany and C-I Tan, "Shape of Hadron and Pion Cloud", Eur. Phys. J. A (2018) 54:116, arXiv 1710.10594PJ-C,

- establishing dip structure at $-t \simeq 1-2$
  Strong evidence of Odderon.

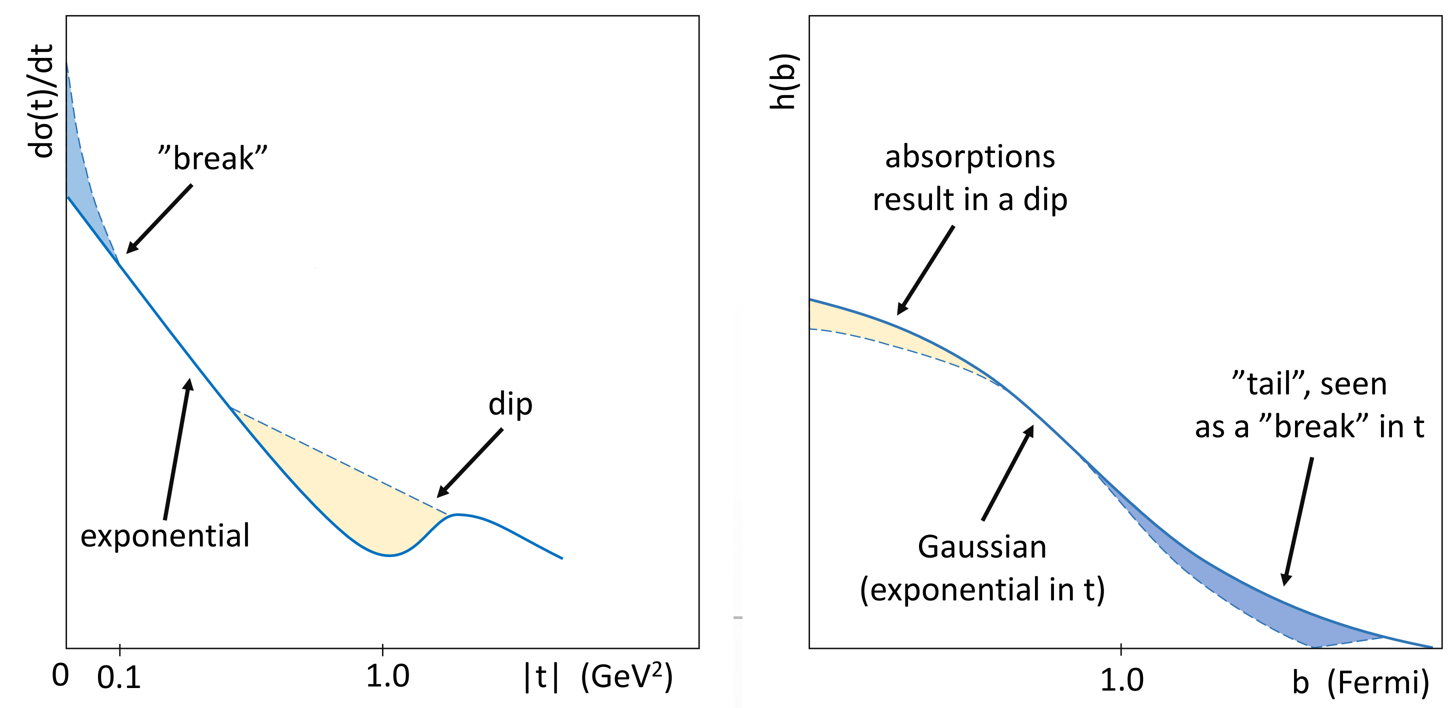
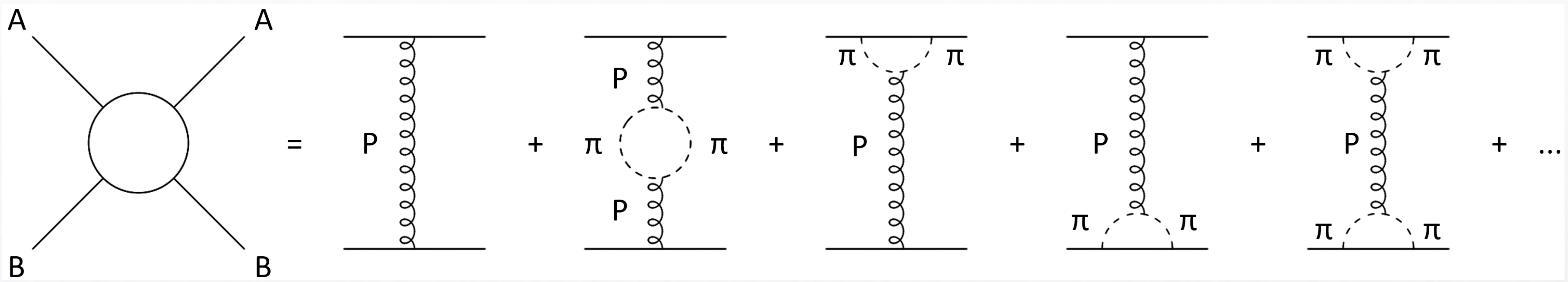


Fig. 1 Schematic (qualitative) view of the "break", followed by the diffraction minimum ("dip"), shown both as function in t and its Fourier transform (impact parameter representation), in b. While the "break" reflects the presence of the pion cloud around the nucleon at the outer edge of an expanding disk in b, the dip results from absorption corrections, suppressing the impact parameter amplitude at small b.



Expect sharp-structure at $x \to 1$.

$$\boxed{F(x) \simeq x^{\delta}(1-x)^{\beta}}$$

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IV: Pomeron in the conformal Limit, OPE, and Anomalous Dimensions

$G_{mn} = g^0_{mn} + h_{mn}$

Massless modes of a closed string theory:

Need to keep higher string modes

As CFT, equivalence to OPE in strong coupling: using AdS

MOMENTS AND ANOMALOUS DIMENSION

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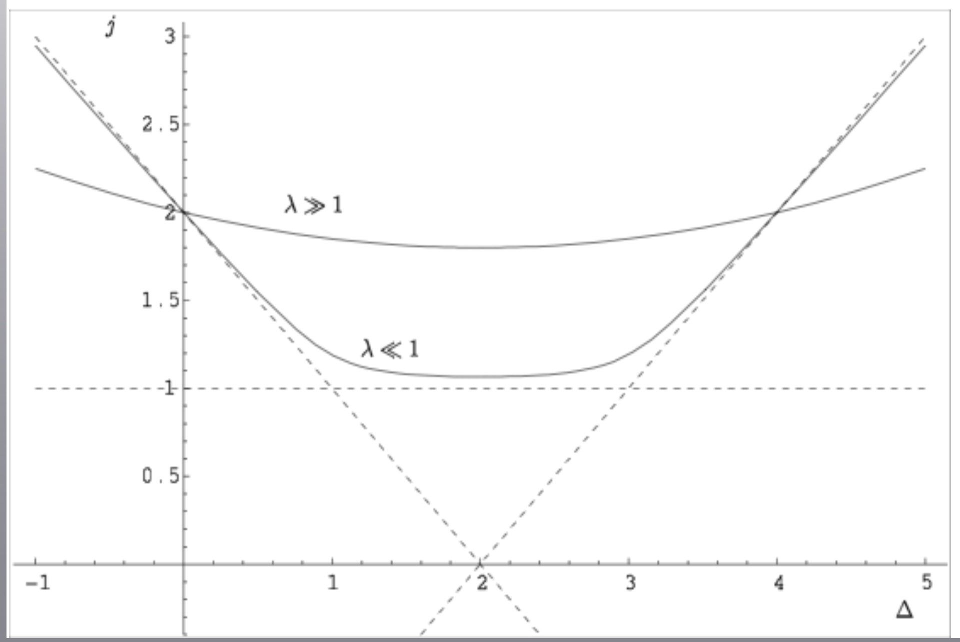


$\gamma_2 = 0$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

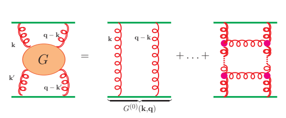
$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

Simultaneous compatible large Q^2 and small x evolutions!

Energy-Momentum Conservation built-in automatically.

BFKL

Balitsky, Fadin, Kuraev, Lipatov (BFKL): perturbative Pomeron. Large logs get in the way of usual perturabtion theory: resum $\alpha_s\, log(s)$ to all orders. Bfkl equation – integral equation for Green's function in Mellin space



$$G(\mathbf{k},\mathbf{k}',\mathbf{q},Y) = \int_{-i\infty}^{+i\infty} \frac{d\omega}{2\pi i} e^{Y\omega} f_\omega(\mathbf{k},\mathbf{k}',\mathbf{q}) \to \int_{-i\infty}^{+i\infty} \frac{d\omega}{2\pi i} e^{Y\omega} \sum_{n\in\mathcal{Z}} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \frac{E_{\gamma,n}(k) E^*_{\gamma,n}(k')}{\omega - \bar{\alpha}_s \chi(\gamma, n)}$$

where in Leading Log (LL)

$$\chi(\gamma,n) = 2\psi(1) - \psi(\gamma + \frac{|n|}{2}) - \psi(1 - \gamma + \frac{|n|}{2}) \quad and \quad \omega_0 = \frac{4\alpha_s N_c}{\pi} ln(2)$$

Surprising conformal symmetry greatly simplifies things in coordinate space

Brower, Costa, Djuric, Nally, TR, Tan (KU) 9/15/17 4 / 19

Full $O(4,2)$ Conformal Group

$$SO(4,2) = SO(1,1) \times SO(3,1)$$

Anomalous dimensions

$$\gamma(j) = \Delta(j) - j - \tau_{twist}$$

DGLAPP

BFKL

BPST Pomeron

Graviton/Pomeron Regge trajectory [Brower, Polchinski, Strassler, Tan 06]

- Operators that contribute are the twist 2 operators

$$\mathcal{O}_J \sim F_{\alpha[\beta_1} D_{\beta_2} \ldots D_{\beta_{J-1}} F_{\beta_J]}{}^{\alpha}$$

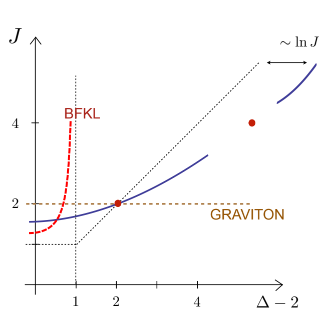
- Dual to string theory spin J field in leading Regge trajectory

$$\left(D^2 - m^2\right) h_{a_1 \ldots a_J} = 0$$

$$m^2 = \Delta(\Delta - 4) - J\,, \quad \Delta = \Delta(J)$$

- Diffusion limit

$$J(\Delta) = J_0 + \mathcal{D}(\Delta - 2)^2 \Rightarrow m^2 = \frac{2}{\alpha'}(J-2) - \frac{J}{L^2}$$



HE scattering after AdS/CFT

Formal Treatment via World-Sheet OPE

$$(L_0 - 1)V_P = (\bar{L}_0 - 1)V_P = 0$$

- Flat Space Pomeron Vertex Operator

$$\mathcal{V}^{\pm}_P = (2\partial X^{\pm}\bar{\partial}X^{\pm}/\alpha')^{1+\alpha' t/4} e^{\mp ik\cdot X}\,.$$

- Flat Space Odderon Vertex Operator

$$\mathcal{V}^{\pm}_O = (2\epsilon_{\pm,\perp}\partial X^{\pm}\bar{\partial}X^{\perp}/\alpha')(2\partial X^{\pm}\bar{\partial}X^{\pm}/\alpha')^{\alpha' t/4} e^{\mp ik\cdot X}$$

- Pomeron Vertex Operator in AdS

$$\mathcal{V}_P(j,\nu,k,\pm) \sim (\partial X^{\pm}\overline{\partial}X^{\pm})^{\frac{j}{2}} e^{\mp ik\cdot X} e^{(j-2)u} K_{\pm 2i\nu}(|t|^{1/2}e^{-u})$$

- Odderon Vertex Operator in AdS

$$\mathcal{V}_O(j,\nu,k,\pm) \sim (\partial X^{\pm}\overline{\partial}X^{\perp} - \partial X^{\perp}\overline{\partial}X^{\pm})(\partial X^{\pm}\overline{\partial}X^{\pm})^{\frac{j-1}{2}} e^{\mp ik\cdot X} e^{(j-1)u} K_{\pm 2i\nu}(|t|^{1/2}e^{-u})$$

13

POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

B.Basso, 1109.3154v2

POMERON

$$\alpha_p = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\lambda^{3/2}} + \frac{6\zeta(3)+2}{\lambda^2} + \frac{18\zeta(3)+\frac{361}{64}}{\lambda^{5/2}} + \frac{39\zeta(3)+\frac{447}{32}}{\lambda^3} + \cdots$$

Brower, Polchinski, Strassler, Tan

Gromov et al.

ODDERON

Kotikov, Lipatov, et al.

Costa, Goncalves, Penedones (1209.4355)
Kotikov, Lipatov (1301.0882)

Solution-a: $\alpha_O = 1 - \frac{8}{\lambda^{1/2}} -$

Solution-b: $\alpha_O = 1 - \frac{0}{\lambda^{1/2}} -$

Brower, Djuric, Tan
Avsar, Hatta, Matsuo

Brower, Costa, Djuric, Raben, Tan

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Solution-a:

$$\alpha_O = 1 - \frac{8}{\lambda^{1/2}} - \frac{4}{\lambda} + \frac{13}{\lambda^{3/2}} + \frac{96\zeta(3)+41}{\lambda^2} + \frac{288\zeta(3)+\frac{1823}{16}}{\lambda^{5/2}} + \frac{720\zeta(5)+1344\zeta(3)-\frac{3585}{4}}{\lambda^3} -$$

Solution-b:

$$\alpha_O = 1 - \frac{0}{\lambda^{1/2}} -$$

Brower, Djuric, Tan

Avsar, Hatta, Matsuo

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POMERON

$$\alpha_p = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\lambda^{3/2}} + \frac{6\zeta(3)+2}{\lambda^2} + \frac{18\zeta(3)+\frac{361}{64}}{\lambda^{5/2}} + \frac{39\zeta(3)+\frac{447}{32}}{\lambda^3} + \cdots$$

Brower, Polchinski, Strassler, Tan

Gromov et al.

ODDERON

Kotikov, Lipatov, et al.

Costa, Goncalves, Penedones (1209.4355)

Kotikov, Lipatov (1301.0882)

Solution-a:

$$\alpha_O = 1 - \frac{8}{\lambda^{1/2}} - \frac{4}{\lambda} + \frac{13}{\lambda^{3/2}} + \frac{96\zeta(3)+41}{\lambda^2} + \frac{288\zeta(3)+\frac{1823}{16}}{\lambda^{5/2}} + \frac{720\zeta(5)+1344\zeta(3)-\frac{3585}{4}}{\lambda^3}$$

Solution-b:

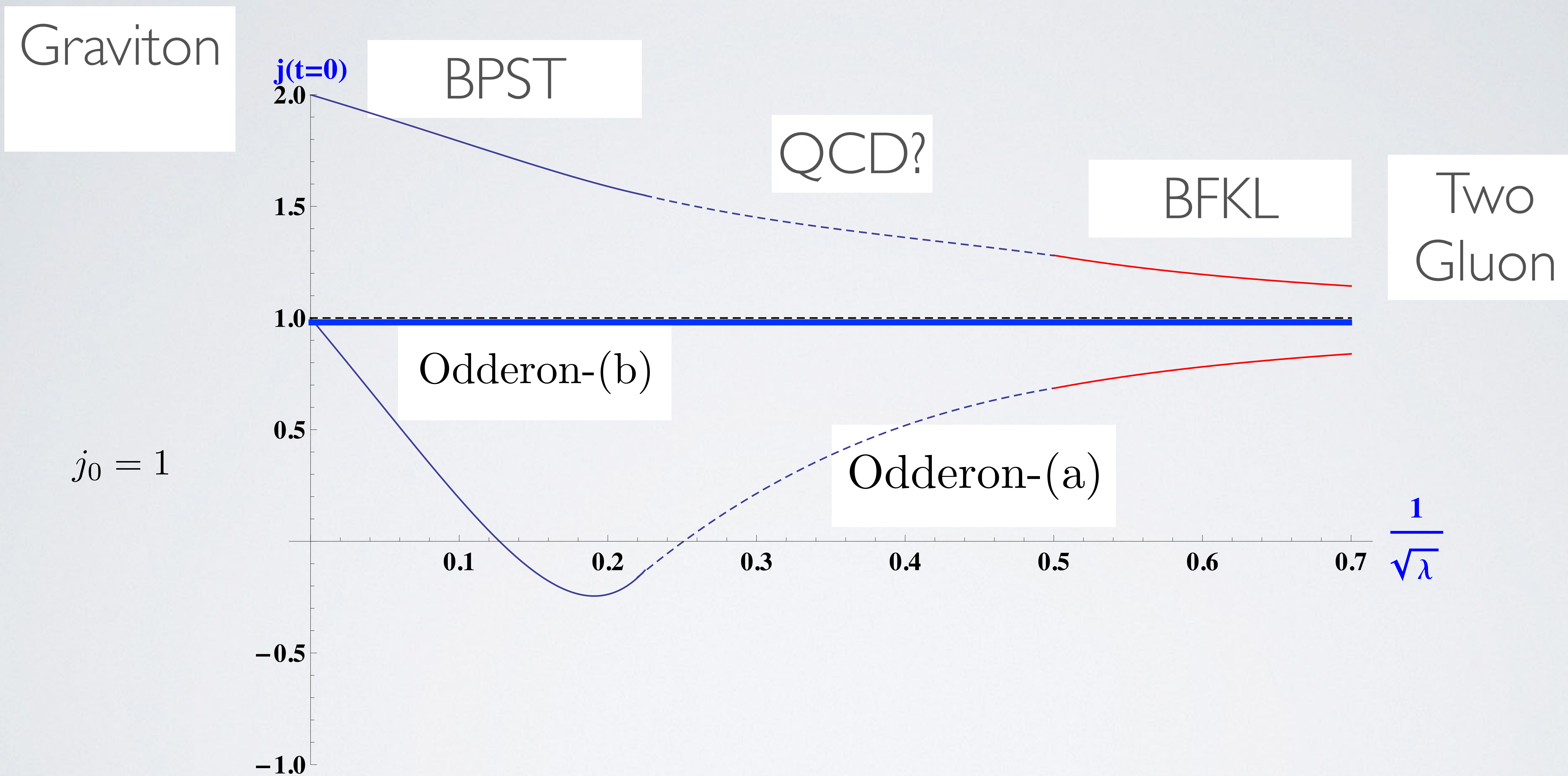
$$\alpha_O = 1 - \frac{0}{\lambda^{1/2}} - \frac{0}{\lambda} + \frac{0}{\lambda^{3/2}} + \frac{0}{\lambda^2} + \frac{0}{\lambda^{5/2}} + \frac{0}{\lambda^3} + \cdots$$

Brower, Djuric, Tan

Avsar, Hatta, Matsuo

Brower, Costa, Djuric, Raben, Tan

$\mathcal{N} = 4$ Strong vs Weak $g^2 N_c$



Summary and Outlook for AdS-CFT for QCD

- Provide meaning for Pomeron non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- First principle description of elastic/total cross sections, DIS at small-x, Central Diffractive Glueball production at LHC, etc.
- New starting point for unitarization, saturation, etc.
- Inclusive Production and Dimensional Scalings.
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