# Lepton Angular Distribution in the Drell-Yan Process

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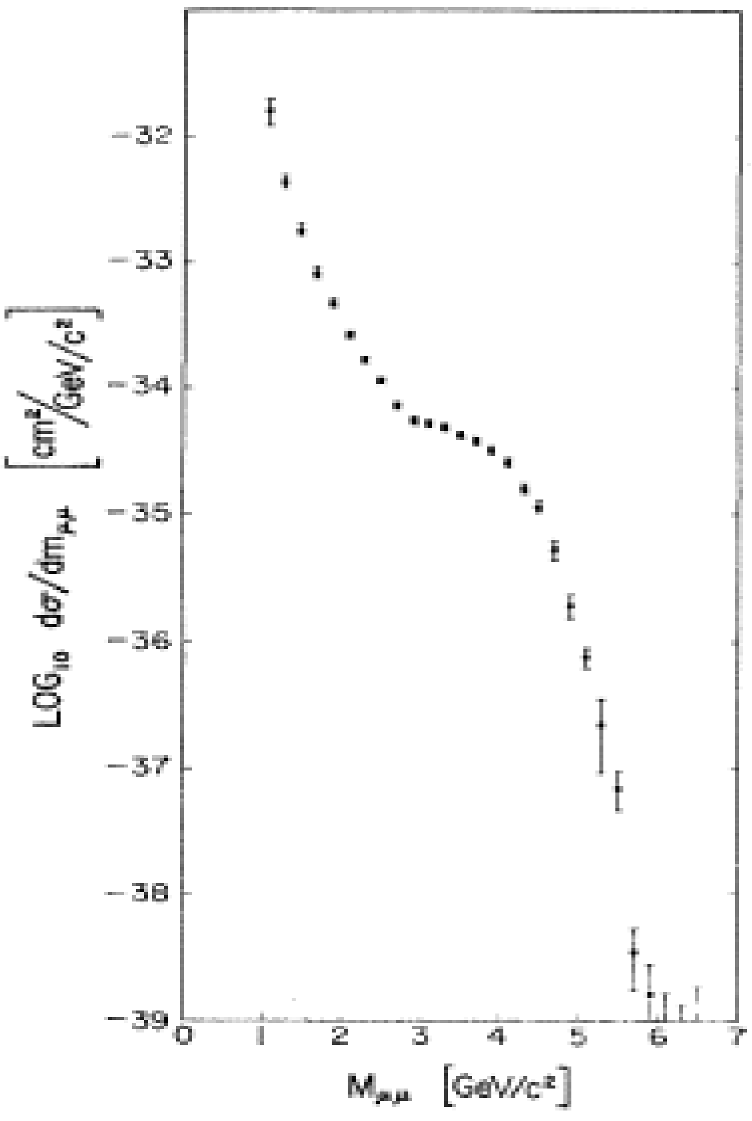
Based on the papers of Wen-Chen Chang, Evan McClellan, Oleg Teryaev and JCP:

Phys. Lett. B758 (2016) 384; PRD 96 (2017) 054020;
and two preprints (arXiv: 1808.04398 and 1811.03256)

# First Dimuon Experiment




$p + U \rightarrow \mu^{+} + \mu^{-} + X$ 29 GeV proton

Lederman et al. PRL 25 (1970) 1523

Experiment originally designed to search for neutral weak boson ($Z^0$)

Missed the J/Ψ signal !

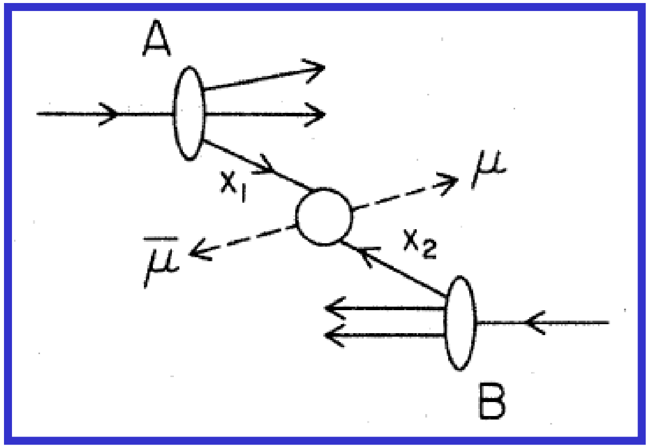
"Discovered" the Drell-Yan process

# The Drell-Yan Process

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s \to \infty$, $Q^2/s$ finite, $Q^2$ and $s$ being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as $Q^2/s \to 1$ is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function $\nu W_2$ near threshold.




$$\left(\frac{d^2\sigma}{dx_1 dx_2}\right)_{D.Y.} = \frac{4\pi\alpha^2}{9 s x_1 x_2} \sum_a e_a^2 \left[ q_a(x_1)\bar{q}_a(x_2) + \bar{q}_a(x_1) q_a(x_2) \right]$$

# Naive Drell-Yan and Its Successor*

T-M. Yan
Floyd R. Newman Laboratory of Nuclear Studies
Cornell University
Ithaca, NY 14853

February 1, 2008

Abstract

We review the development in the field of lepton pair production since proposing parton-antiparton annihilation as the mechanism of massive lepton pair production. The basic physical picture of the Drell-Yan model has survived the test of QCD, and the predictions from the QCD improved version have been confirmed by the numerous experiments performed in the last three decades. The model has provided an active theoretical arena for studying infrared and collinear divergences in QCD. It is now so well understood theoretically that it has become a powerful tool for new phyiscs information such as precision measurements of the W mass and lepton and quark sizes.

“… our original crude fit did not even remotely resemble the data. Sid and I went ahead to publish our paper because of the model’s simplicity…”

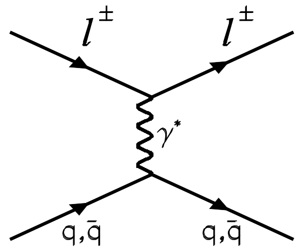
“… the successor of the naïve model, the QCD improved version, has been confirmed by the experiments…”

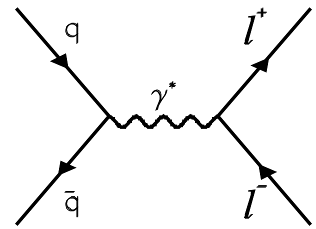
“The process has been so well understood theoretically that it has become a powerful tool for precision measurements and new physics.”

*Talk given at the Drell Fest, July 31, 1998, SLAC on the occasion of Prof. Sid Drell's retirement.

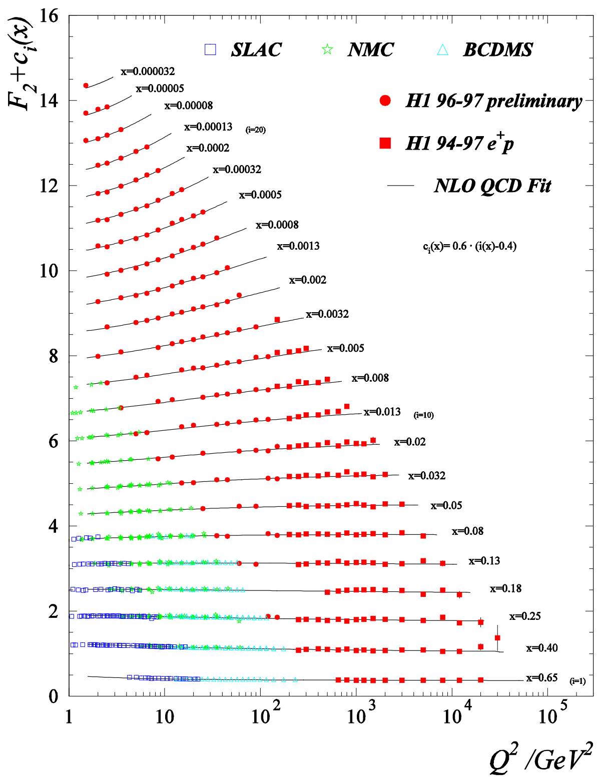
# Complementarity between DIS and Drell-Yan

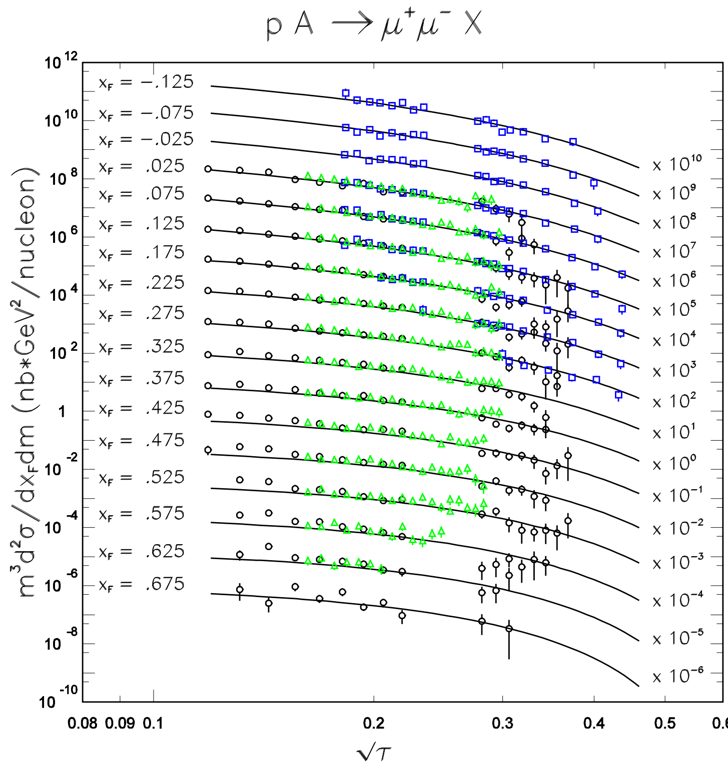
DIS




Drell-Yan




Ann.Rev.Nucl. Part. Sci. 49 (1999) 217;

Peng and Qiu, Prog. Part. Nucl. Phys. 76 (2014)43

Both DIS and Drell-Yan process are tools to probe the quark and antiquark structure in hadrons (factorization, universality)

# Angular Distribution in the "Naïve" Drell-Yan

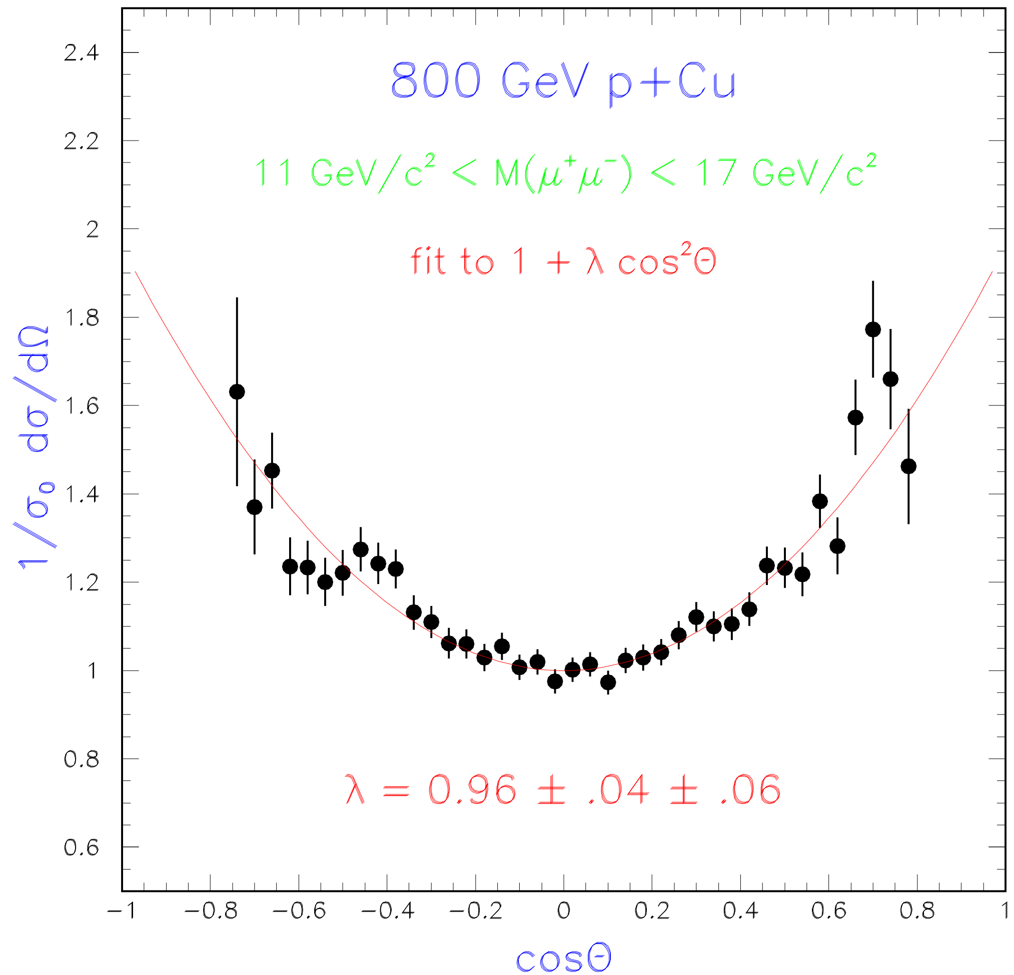
VOLUME 25, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1970

(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin-$\frac{1}{2}$ parton-antiparton pairs. This means a distribution in the di-muon rest system varying as $(1+\cos^2\theta)$ rather than $\sin^2\theta$ as found in Sakurai's[10] vector-dominance model, where $\theta$ is the angle of the muon with respect to the time-like photon momentum. The model used in Fig.

# Drell-Yan angular distribution

Lepton Angular Distribution of “naïve” Drell-Yan:

$$\frac{d\sigma}{d\Omega} = \sigma_0 (1 + \lambda \cos^2 \theta); \quad \lambda = 1$$

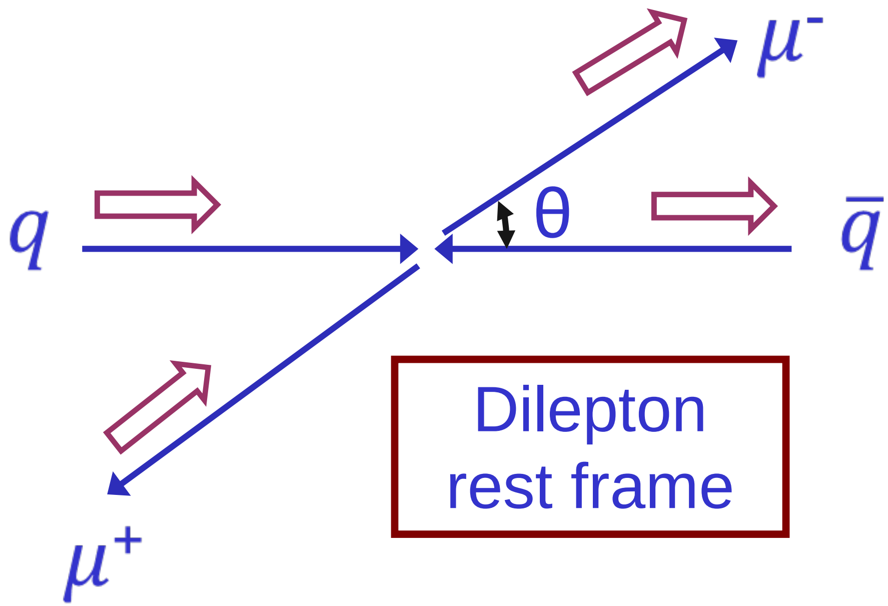



Data from Fermilab E772

**(Ann. Rev. Nucl. Part. Sci. 49 (1999) 217-253)**

# Why is the lepton angular distribution $1 + \cos^2\theta$ ?

**Helicity conservation and parity**




$RL \to RL$

$$d\sigma \sim (1+\cos\theta)^2$$

$RL \to LR$

$$d\sigma \sim (1-\cos\theta)^2$$

$LR \to LR$

$$d\sigma \sim (1+\cos\theta)^2$$

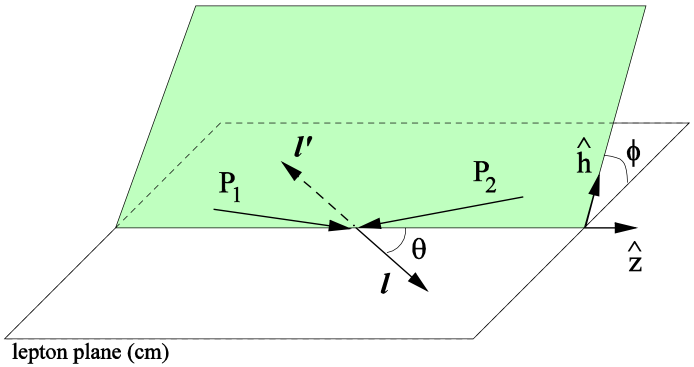
$LR \to RL$

$$d\sigma \sim (1-\cos\theta)^2$$

Adding all four helicity configurations:

$$d\sigma \sim 1+\cos^2\theta$$

# Drell-Yan lepton angular distributions




Θ and Φ are the decay polar and azimuthal angles of the $\mu^-$ in the dilepton rest-frame

Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

Lam-Tung relation: $1-\lambda=2\nu$

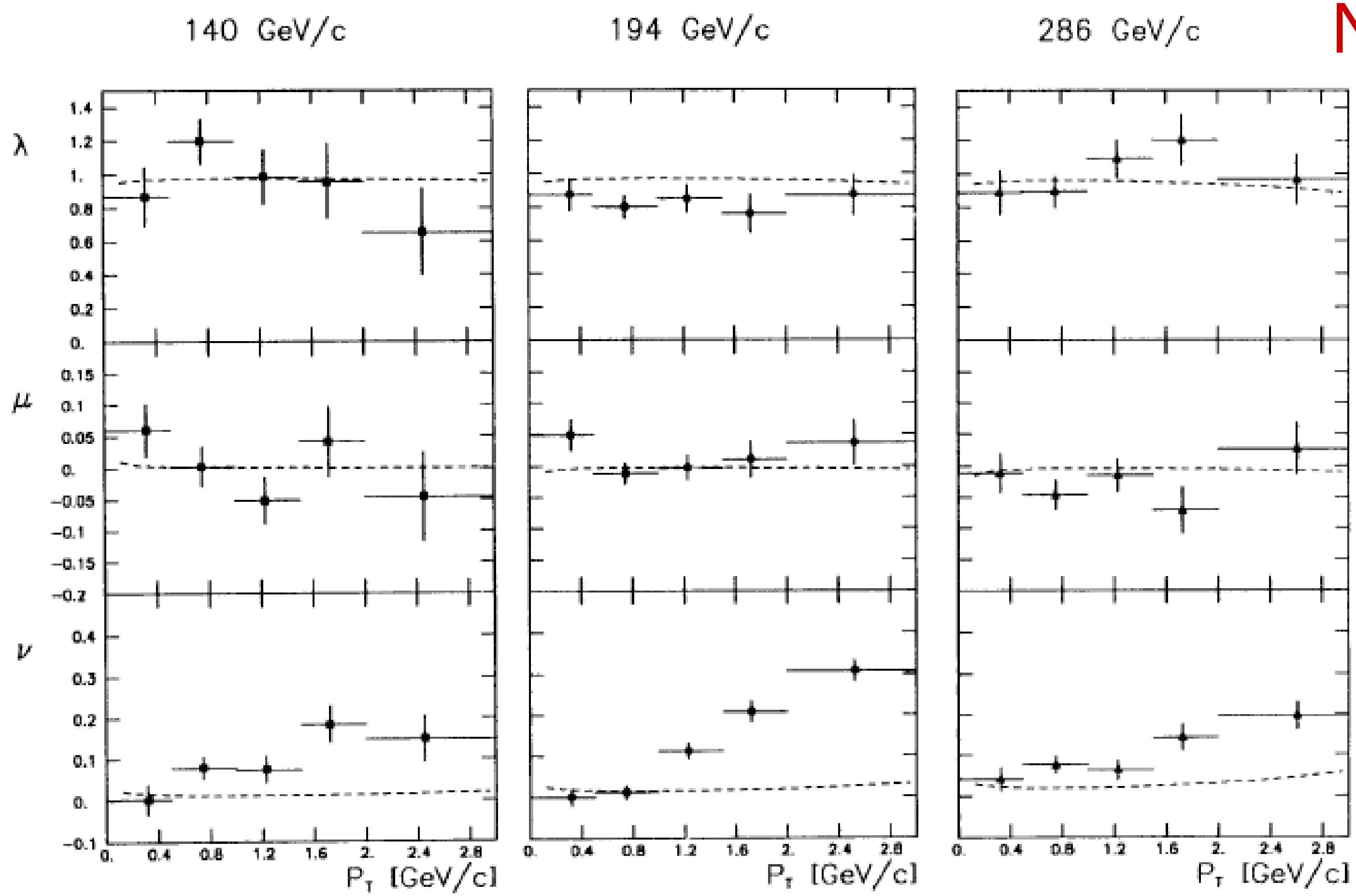
– Reflect the spin-1/2 nature of quarks (analog of the Callan-Gross relation in DIS)

– Insensitive to QCD - corrections

# Decay angular distributions in pion-induced Drell-Yan

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos2\phi\right]$$

NA10 $\pi^- + W$




Z. Phys. 37 (1988) 545

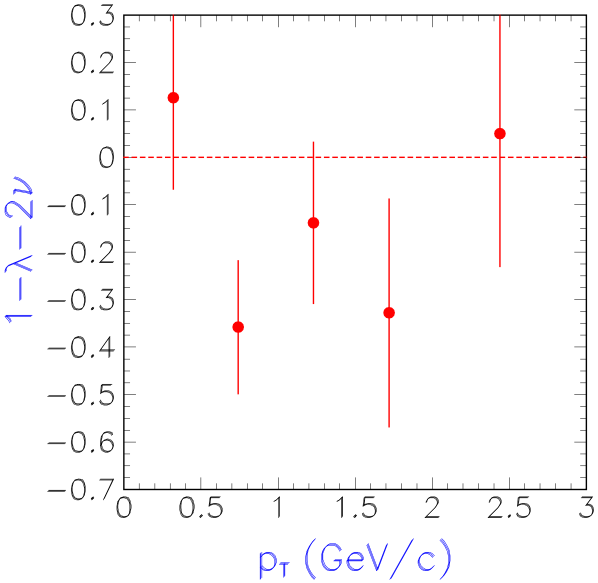
Dashed curves are from pQCD calculations

$\nu \neq 0$ and $\nu$ increases with $p_T$

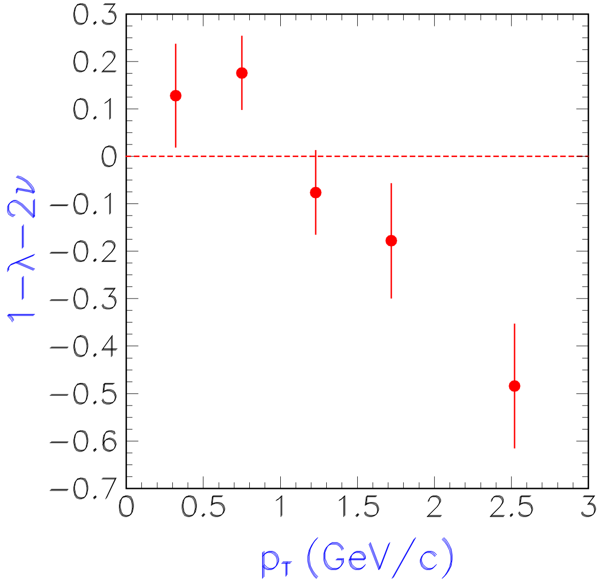
# Decay angular distributions in pion-induced Drell-Yan

**Is the Lam-Tung relation (1-λ-2ν=0) violated?**

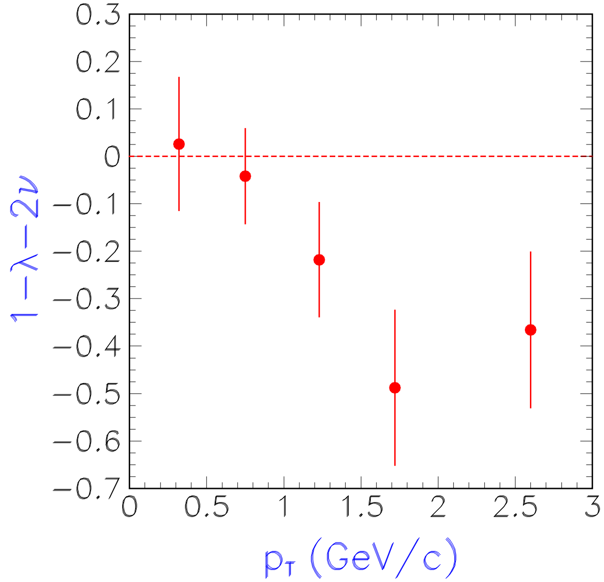
140 GeV/c




194 GeV/c




286 GeV/c




Data from NA10 (Z. Phys. 37 (1988) 545)

Violation of the Lam-Tung relation suggests interesting new origins (Brandenburg, Nachtmann, Mirkes, Brodsky, Khoze, Müller, Eskolar, Hoyer,Väntinnen, Vogt, etc.)

Physics Letters B 726 (2013) 262–265

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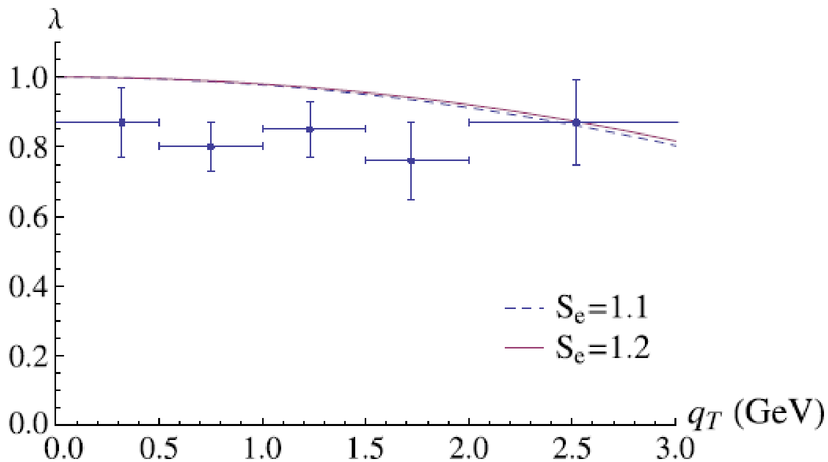
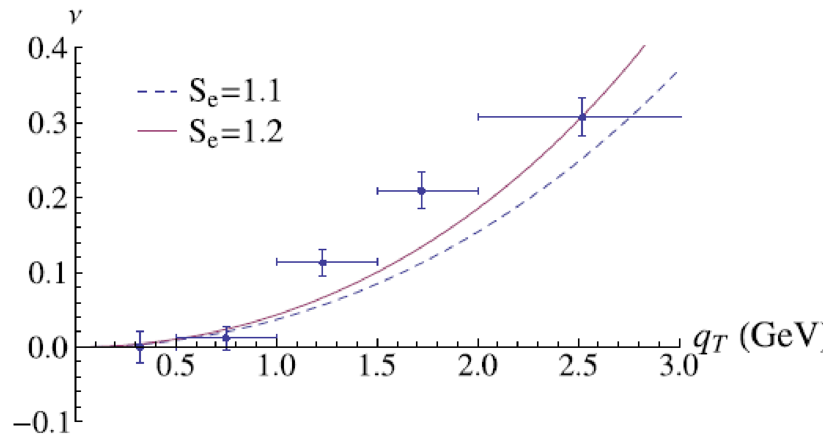
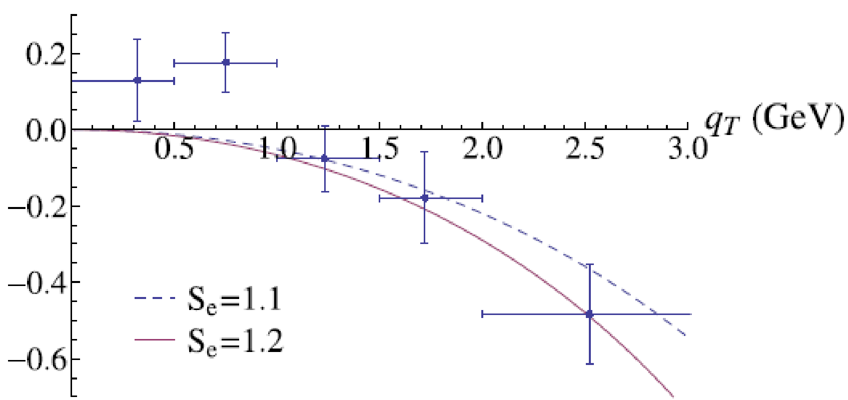
# Glauber gluons in pion-induced Drell–Yan processes

Chun-peng Chang[a,b], Hsiang-nan Li[a,b,c,*]

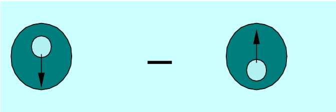



$1 - \lambda - 2\nu$

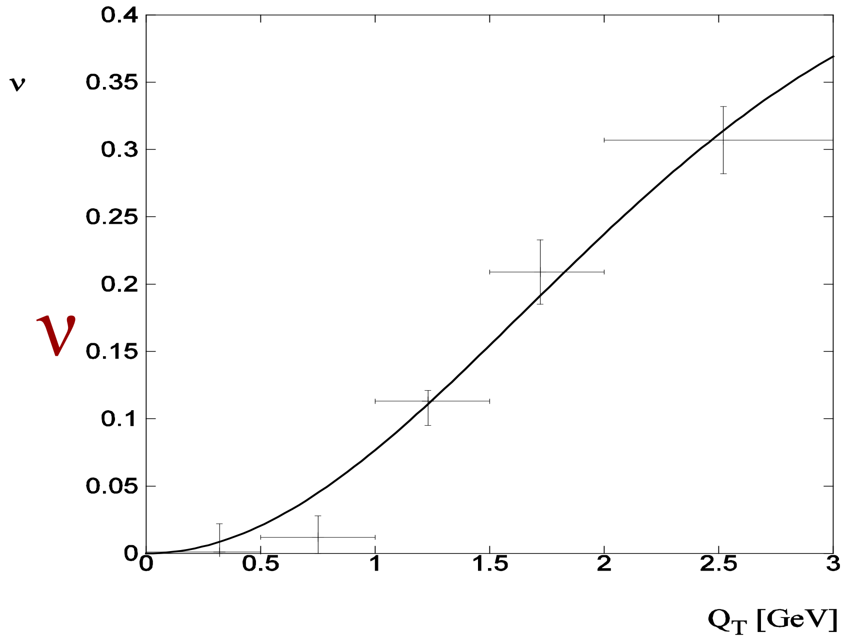


# Boer-Mulders function $h_1^{\perp}$



- Boer pointed out that the cos2ϕ dependence can be caused by the presence of the Boer-Mulders function.

- $h_1^{\perp}$ can lead to an azimuthal dependence with $\nu \propto \left(\frac{h_1^{\perp}}{f_1}\right)\left(\frac{\bar{h}_1^{\perp}}{\bar{f}_1}\right)$




Boer, PRD 60 (1999) 014012

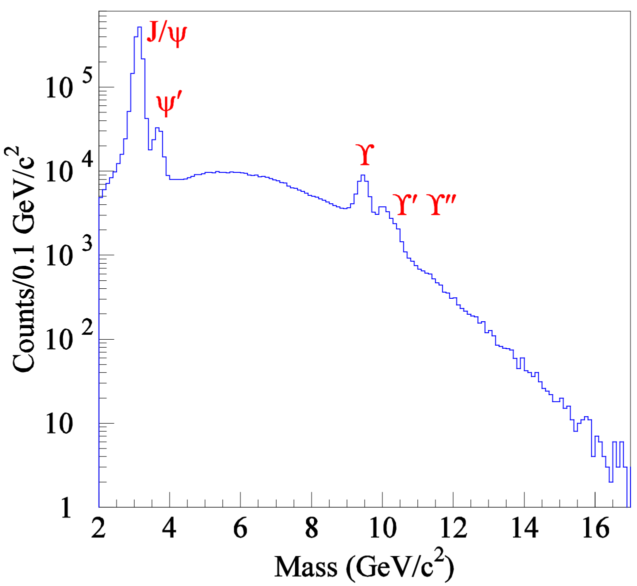
The violation of the Lam-Tung relation is due to the presence of the Boer-Mulders TMD function

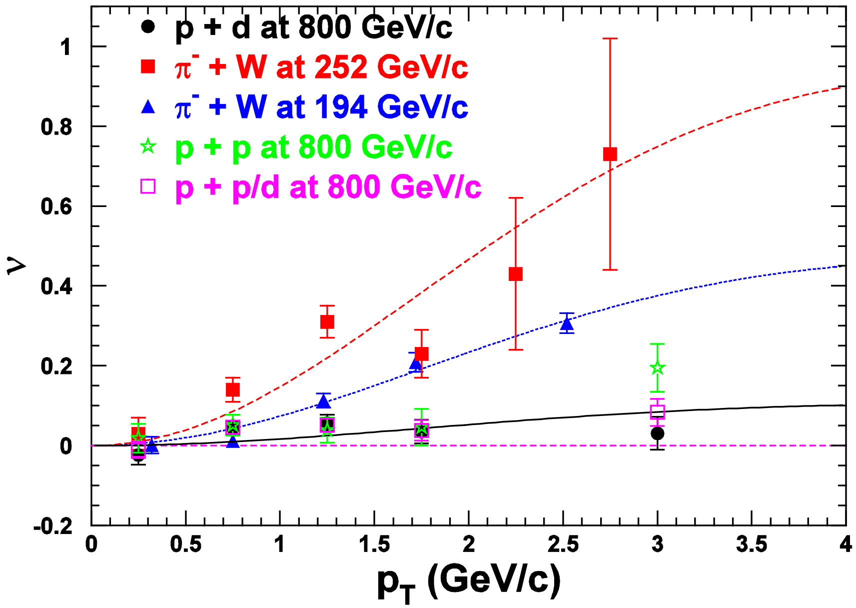
**The puzzle is resolved. It also leads to the first extraction of the Boer-Mulders function**

# Azimuthal angular distribution data from proton-induced Drell-Yan data (Fermilab E866)

Fermilab E866

Lingyan Zhu et al., PRL 99 (2007) 082301; PRL 102 (2009) 182001




With Boer-Mulders function $h_1^\perp$:

$\nu(\pi^- W \to \mu^+\mu^- X) \sim$ [valence $h_1^\perp(\pi)$] * [valence $h_1^\perp(p)$]

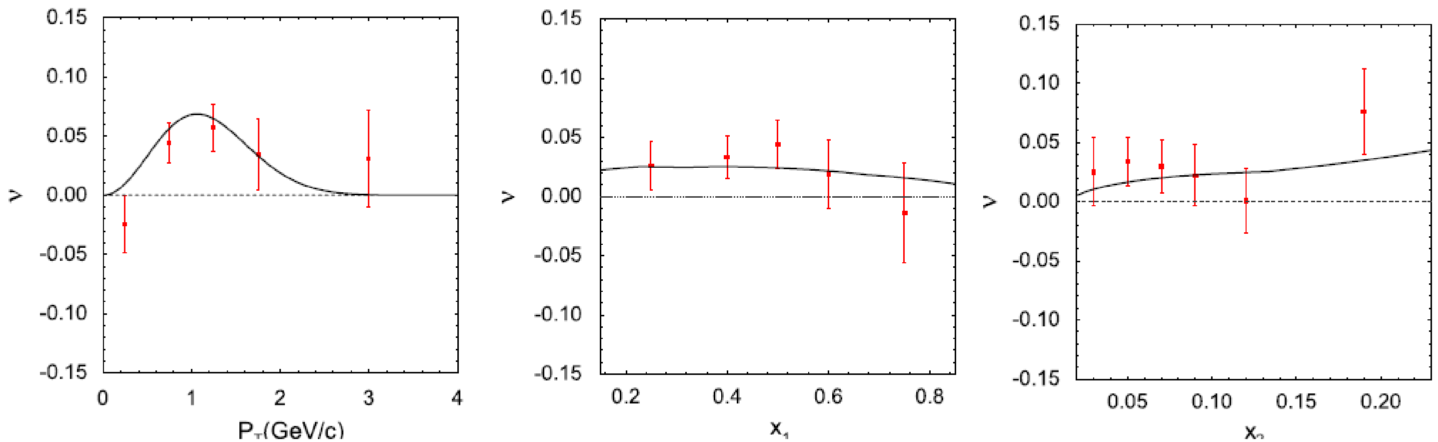
$\nu(pd \to \mu^+\mu^- X) \sim$ [valence $h_1^\perp(p)$] * [sea $h_1^\perp(p)$]

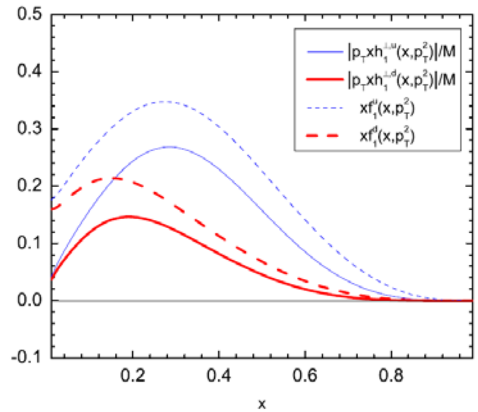
Sea-quark BM function is much smaller than valence BM function

# Extraction of Boer-Mulders functions from p+d Drell-Yan

(B. Zhang, Z. Lu, B-Q. Ma and I. Schmidt, arXiv:0803.1692)

## Fit to the p+d Drell-Yan data



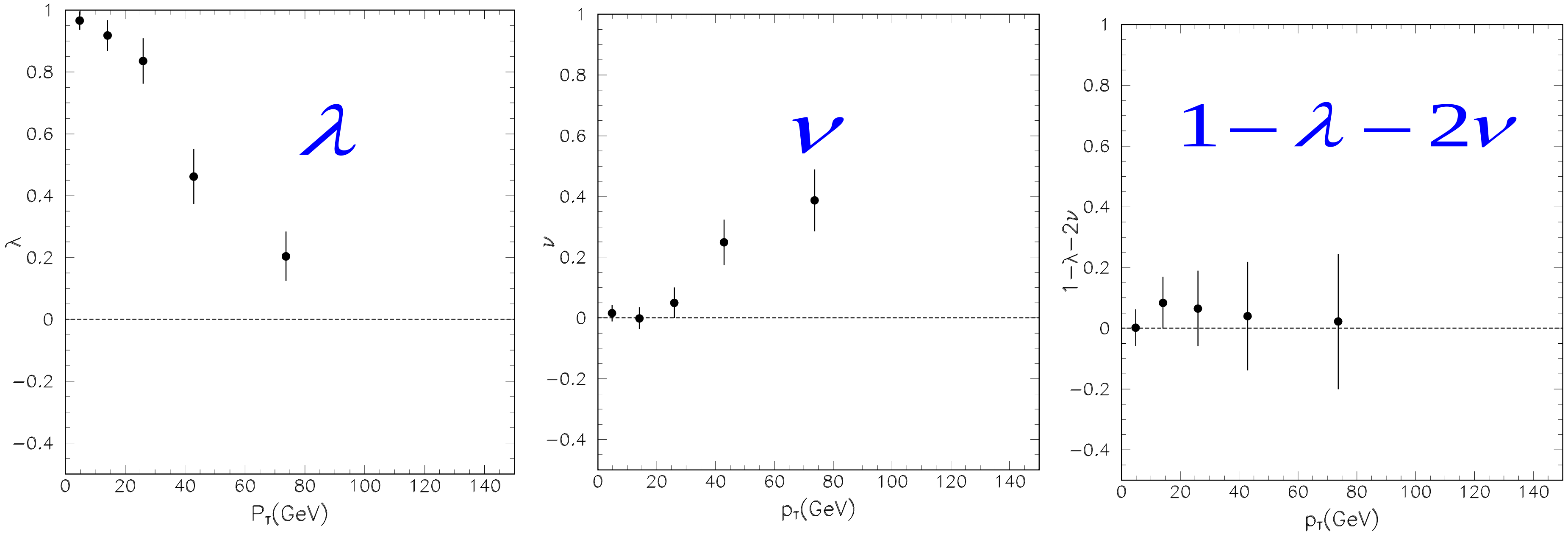




← Satisfy the positivity bound

# Angular distribution data from CDF Z-production

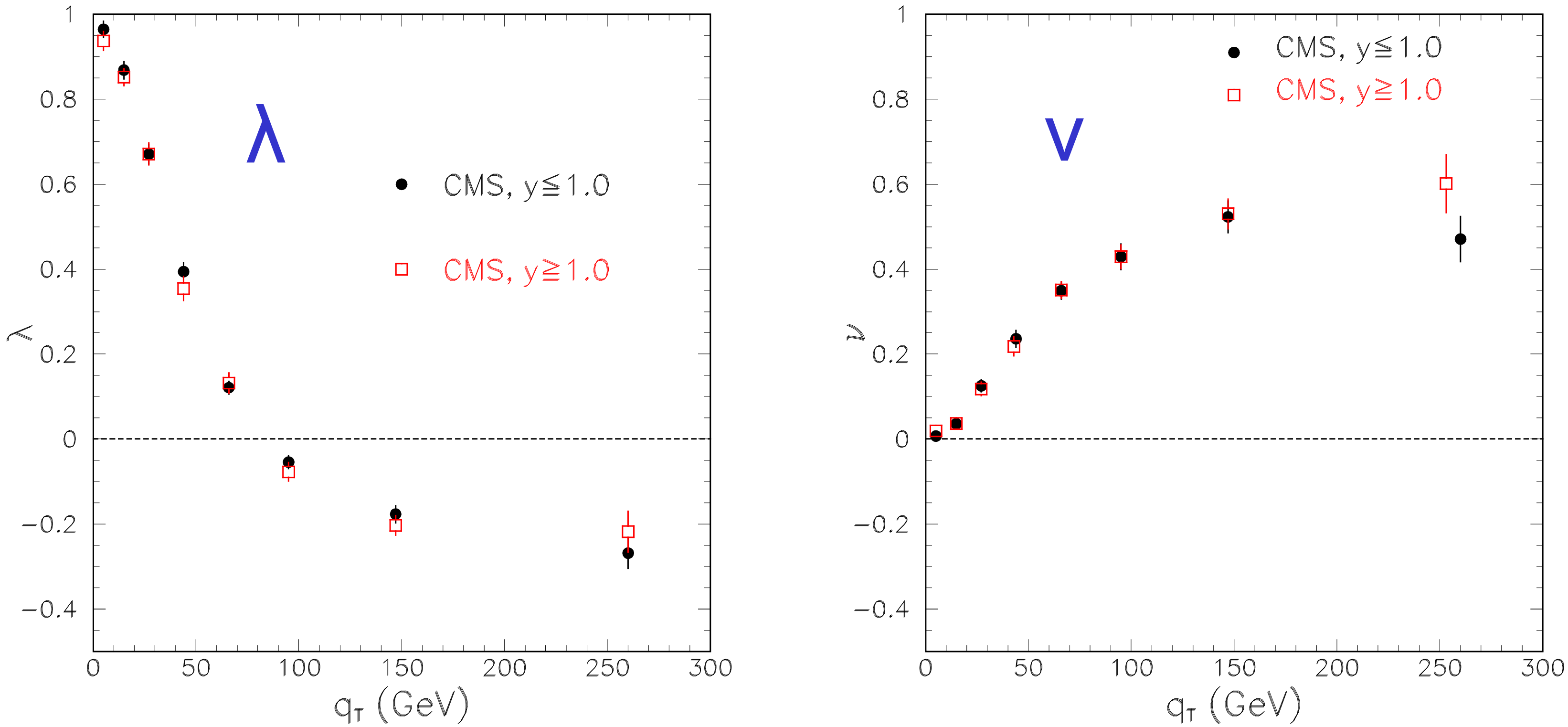
$$p+\overline{p}\rightarrow e^{+}+e^{-}+X \quad \text{at} \sqrt{s}=1.96\,\text{TeV}$$

arXiv:1103.5699 (PRL 106 (2011) 241801)




- Strong $p_T$ ($q_T$) dependence of λ and ν
- Lam-Tung relation (1-λ = 2ν) is satisfied within experimental uncertainties (TMD is not expected to be important at large $p_T$)

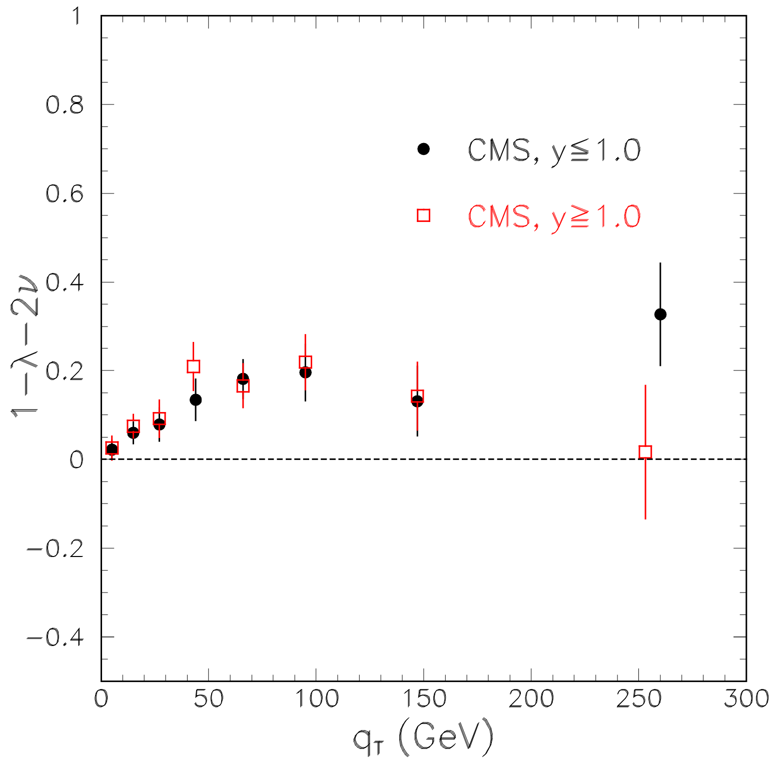
# Recent CMS (ATLAS) data for *Z*-boson production in *p*+*p* collision at 8 TeV




(arXiv:1504.03512, PL B 750 (2015) 154)

- Striking $q_T$ ($p_T$) dependencies for λ and ν were observed at two rapidity regions
- Is Lam-Tung relation violated?

# Recent data from CMS for *Z*-boson production in *p*+*p* collision at 8 TeV




- Yes, the Lam-Tung relation is violated ($1-\lambda > 2\nu$)!
- Can one understand the origin of the violation of the Lam-Tung relation (It cannot be due to the Boer-Mulders function)?

# Interpretation of the CMS Z-production results

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1 \sin 2\theta \cos\phi$$
$$+ \frac{A_2}{2}\sin^2\theta \cos 2\phi + A_3 \sin\theta \cos\phi + A_4 \cos\theta$$
$$+ A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta \sin\phi$$

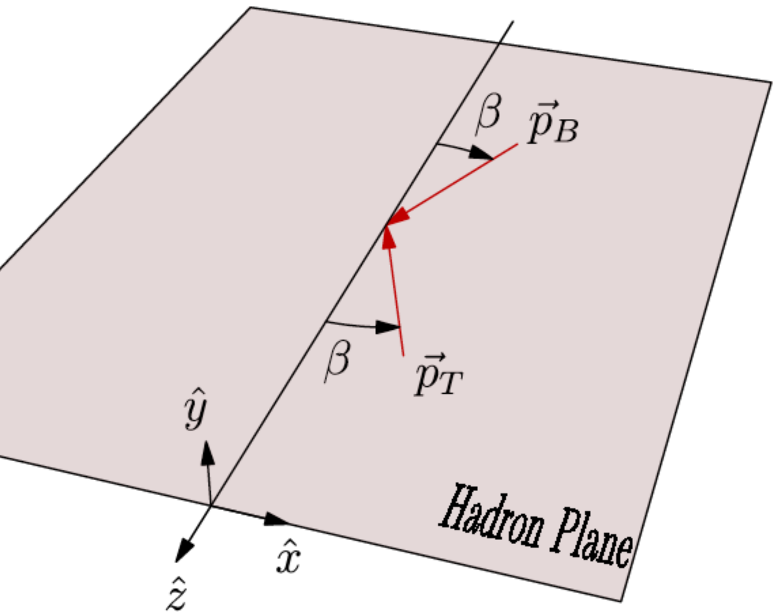
Questions:

- How is the above expression derived?
- Can one express $A_0 - A_7$ in terms of some quantities?
- Can one understand the $q_T$ dependence of $A_0, A_1, A_2,$ etc?
- Can one understand the origin of the violation of Lam-Tung relation?

$\lambda = \frac{2-3A_0}{2+A_0}$; $\nu = \frac{2A_2}{2+A_0}$; L-T relation, $1-\lambda = 2\nu$, becomes $A_0 = A_2$

# How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame




1) Hadron Plane

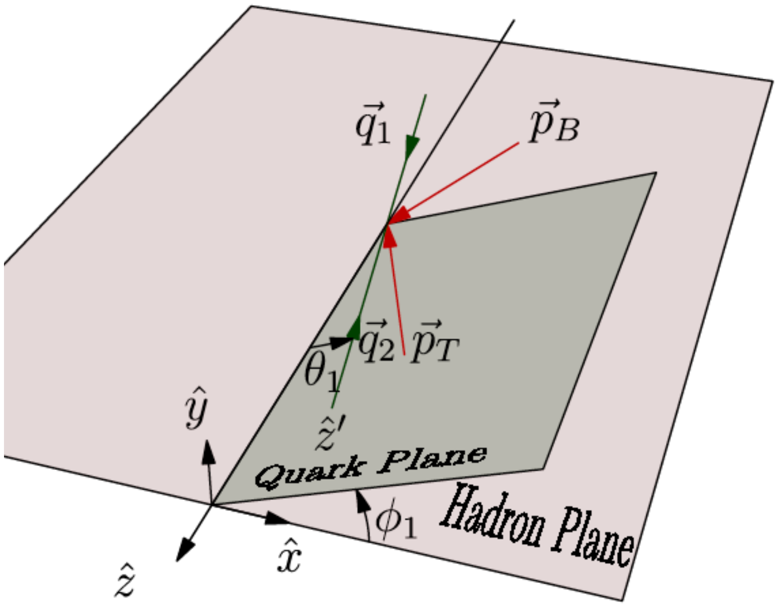
- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle $\beta$ satisfies the relation $\tan\beta = q_T / Q$

- $Q$ is the mass of the dilepton ($Z$)
- when $q_T \to 0,\ \beta \to 0^\circ$;

  when $q_T \to \infty,\ \beta \to 90^\circ$

# How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame




1) Hadron Plane

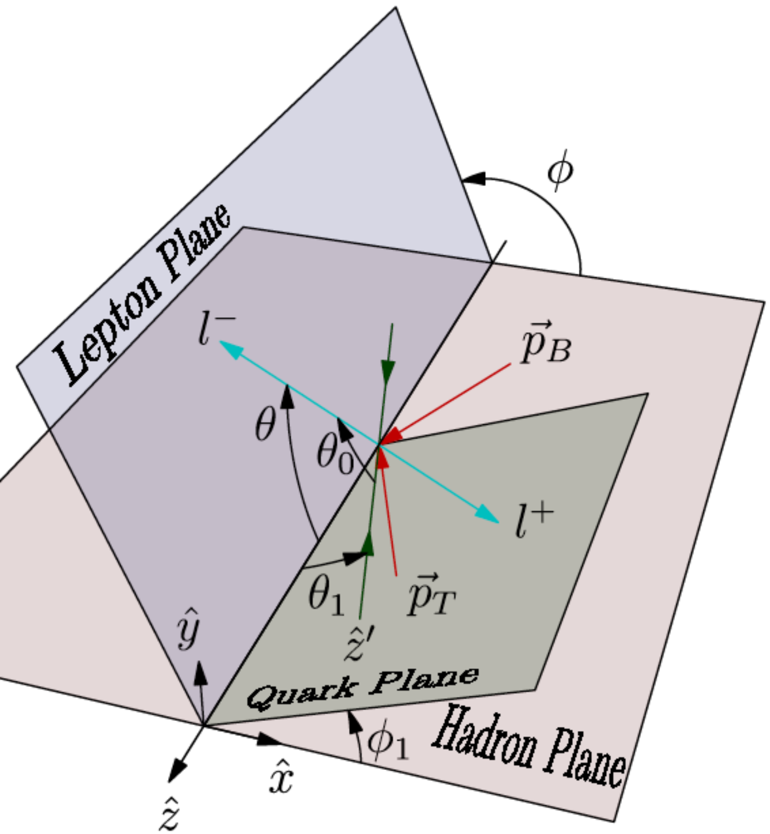
- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle $\beta$ satisfies the relation $\tan\beta = q_T / Q$

2) Quark Plane

- $q$ and $\overline{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ and $\hat{z}$ axes form the quark plane
- $\hat{z}'$ axis has angles $\theta_1$ and $\phi_1$ in the C-S frame

# How is the angular distribution expression derived?

## Define three planes in the Collins-Soper frame




1) Hadron Plane

- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle $\beta$ satisfies the relation $\tan\beta = q_T / Q$

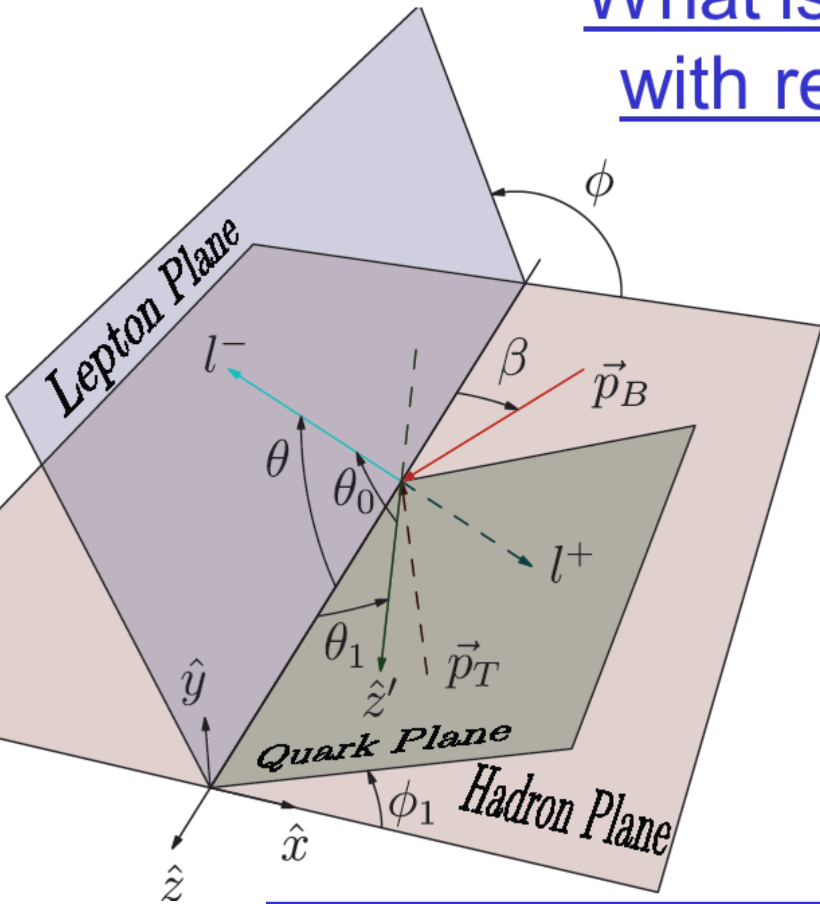
2) Quark Plane

- $q$ and $\overline{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ axis has angles $\theta_1$ and $\phi_1$ in the C-S frame

3) Lepton Plane

- $l^-$ and $l^+$ are emitted back-to-back with equal $|\vec{P}|$
- $l^-$ and $\hat{z}$ form the lepton plane
- $l^-$ is emitted at angle $\theta$ and $\phi$ in the C-S frame

# How is the angular distribution expression derived?




What is the lepton angular distribution with respect to the $\hat{z}'$ (natural) axis?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

Azimuthally symmetric !

How to express the angular distribution in terms of θ and ϕ?

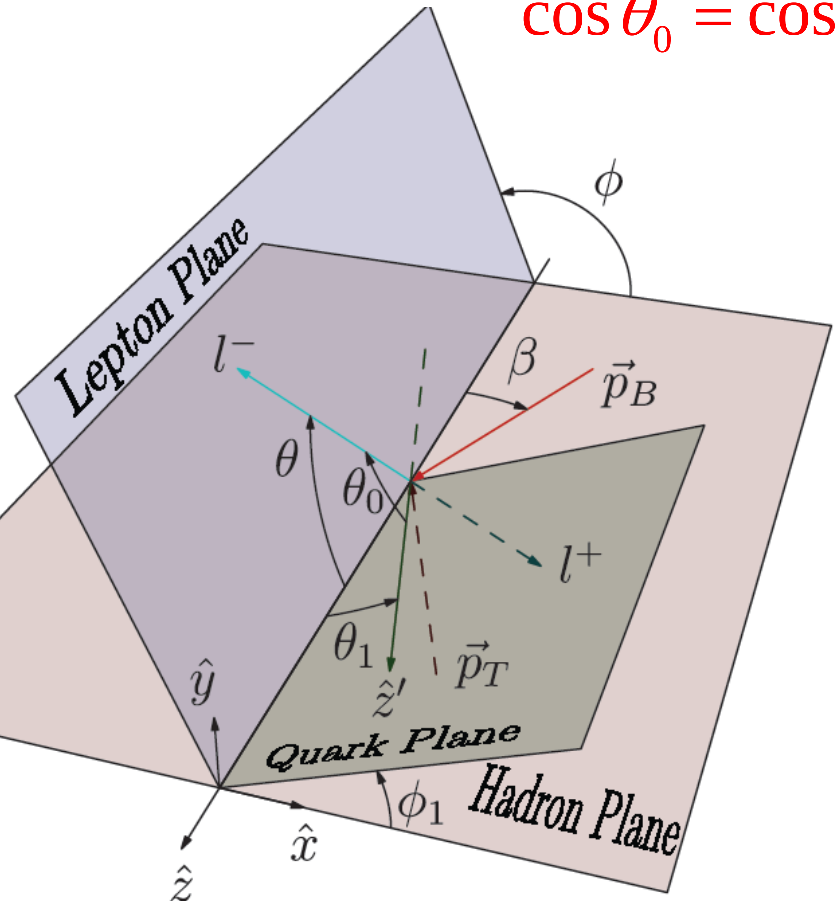
Use the following relation:

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$

# How is the angular distribution expression derived?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$




$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto\ & (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta) \\ & + (\frac{1}{2}\sin 2\theta_1\cos\phi_1)\sin 2\theta\cos\phi \\ & + (\frac{1}{2}\sin^2\theta_1\cos 2\phi_1)\sin^2\theta\cos 2\phi \\ & + (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta \\ & + (\frac{1}{2}\sin^2\theta_1\sin 2\phi_1)\sin^2\theta\sin 2\phi \\ & + (\frac{1}{2}\sin 2\theta_1\sin\phi_1)\sin 2\theta\sin\phi \\ & + (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi.\end{aligned}$$

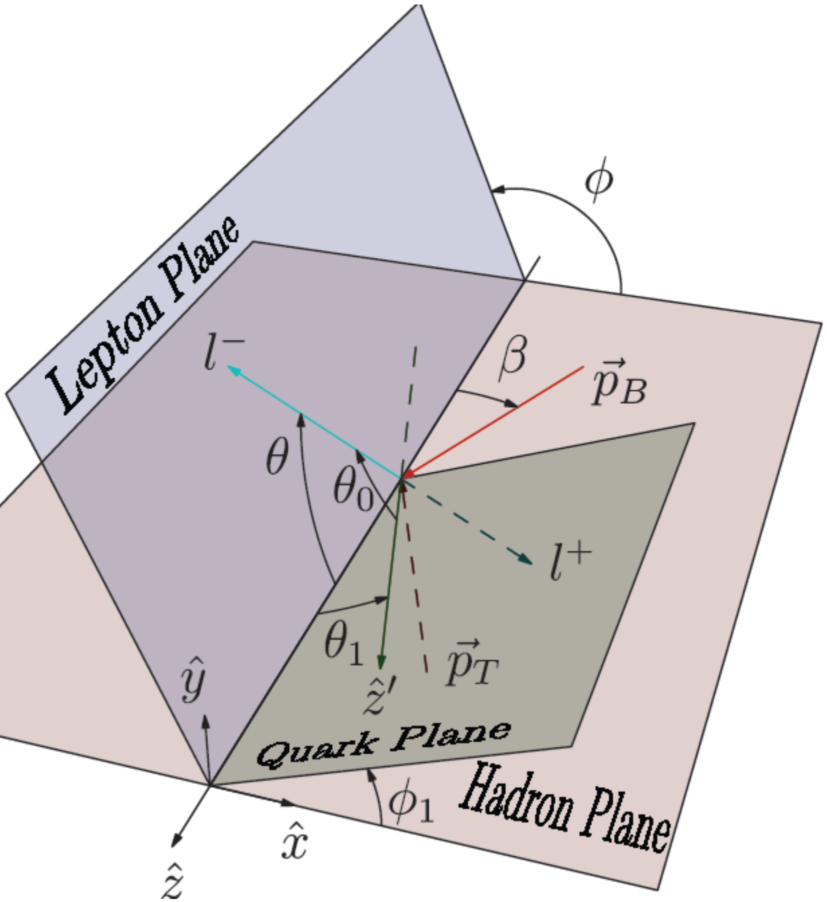
# All eight angular distribution terms are obtained!

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta)$$
$$+ (\frac{1}{2}\sin 2\theta_1 \cos\phi_1)\sin 2\theta\cos\phi$$
$$+ (\frac{1}{2}\sin^2\theta_1 \cos 2\phi_1)\sin^2\theta\cos 2\phi$$
$$+ (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta$$
$$+ (\frac{1}{2}\sin^2\theta_1 \sin 2\phi_1)\sin^2\theta\sin 2\phi$$
$$+ (\frac{1}{2}\sin 2\theta_1 \sin\phi_1)\sin 2\theta\sin\phi$$
$$+ (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi.$$

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta)$$
$$+ A_1\sin 2\theta\cos\phi$$
$$+ \frac{A_2}{2}\sin^2\theta\cos 2\phi$$
$$+ A_3\sin\theta\cos\phi + A_4\cos\theta$$
$$+ A_5\sin^2\theta\sin 2\phi$$
$$+ A_6\sin 2\theta\sin\phi$$
$$+ A_7\sin\theta\sin\phi$$

$A_0 - A_7$ are entirely described by $\theta_1, \phi_1$ and $a$

# Angular distribution coefficients $A_0 - A_7$




$$A_0 = \langle \sin^2 \theta_1 \rangle$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle$$

$$A_2 = \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a \langle \sin \theta_1 \cos \phi_1 \rangle$$

$$A_4 = a \langle \cos \theta_1 \rangle$$

$$A_5 = \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle$$

$$A_7 = a \langle \sin \theta_1 \sin \phi_1 \rangle$$

# Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

- $A_0 \geq A_2$ (or $1 - \lambda - 2\nu \geq 0$)
- Lam-Tung relation ($A_0 = A_2$) is satisfied when $\phi_1 = 0$
- Forward-backward asymmetry, $a$, is reduced by a factor of $\langle \cos\theta_1 \rangle$ for $A_4$
- $A_5, A_6, A_7$ are odd function of $\phi_1$ and must vanish from symmetry consideration
- Some equality and inequality relations among $A_0 - A_7$ can be obatined

# Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

Some bounds on the coefficients can be obtained

$$0 < A_0 < 1$$

$$-1/2 < A_1 < 1/2$$

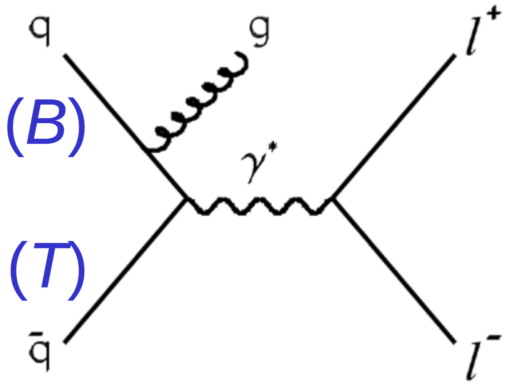
$$-1 < A_2 < 1$$

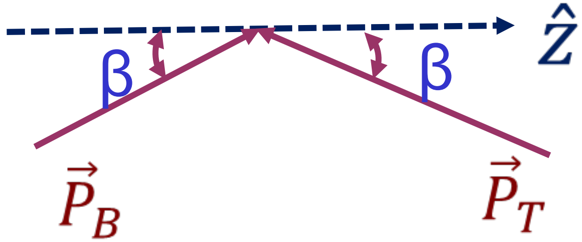
$$-a < A_3 < a$$

$$-a < A_4 < a$$

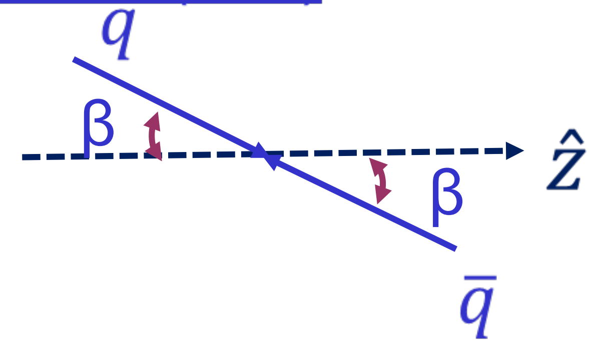
# What are the values of $\theta_1$ and $\phi_1$ at order $\alpha_s$?

1) $q\bar{q} \to \gamma^*(Z^0)g$

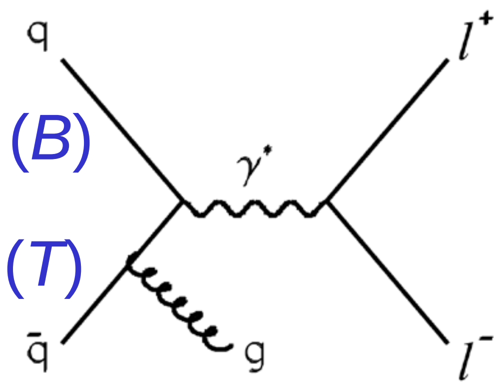


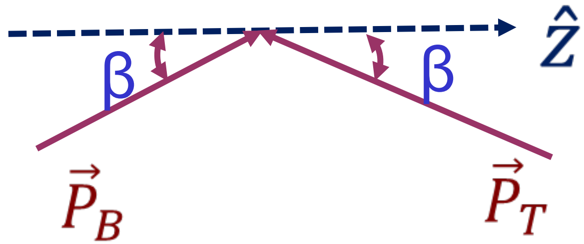



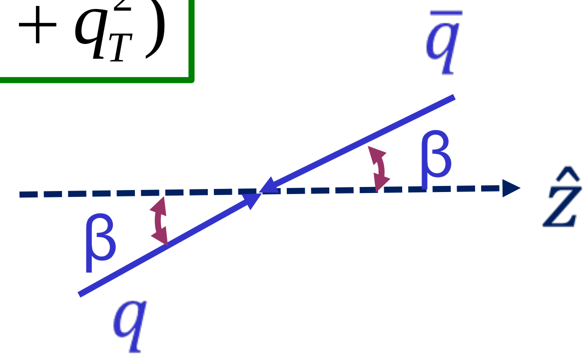

In $\gamma^*$ rest frame (C-S)




$$\sin^2\beta = q_T^2/(Q^2 + q_T^2)$$



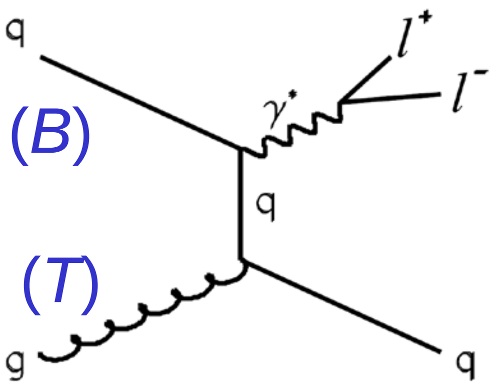

$\theta_1 = \beta$ and $\phi_1 = 0$; $A_0 = A_2 = \sin^2\beta$

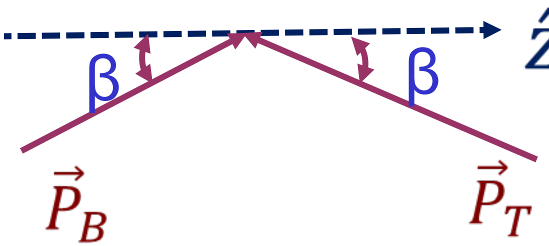
$$\lambda = \frac{2-3A_0}{2+A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}; \quad \nu = \frac{2A_2}{2+A_0} = \frac{2q_T^2}{2Q^2+3q_T^2}$$

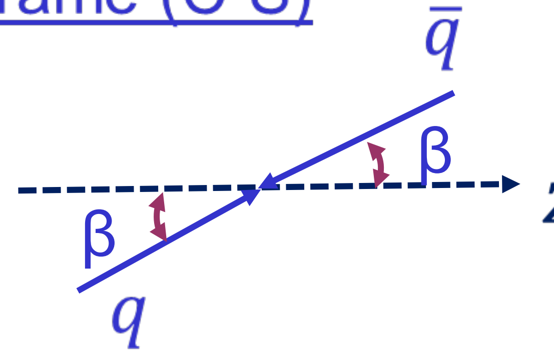
# What are the values of $\theta_1$ and $\phi_1$ at order $\alpha_s$?

2) $qg \to \gamma^*(Z^0)q$

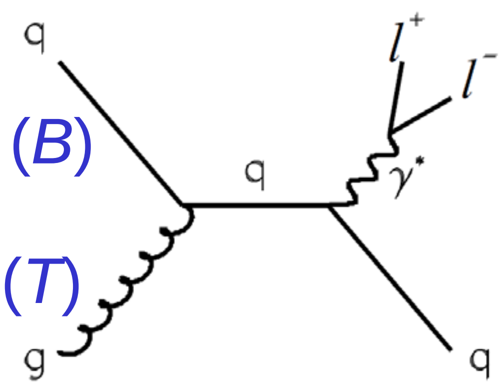
<u>In $\gamma^*$ rest frame (C-S)</u>

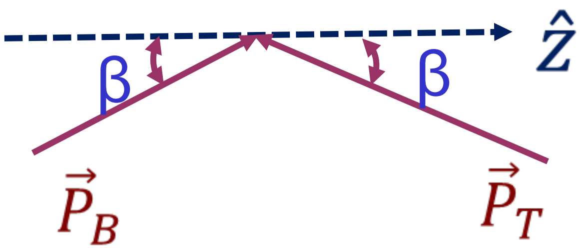


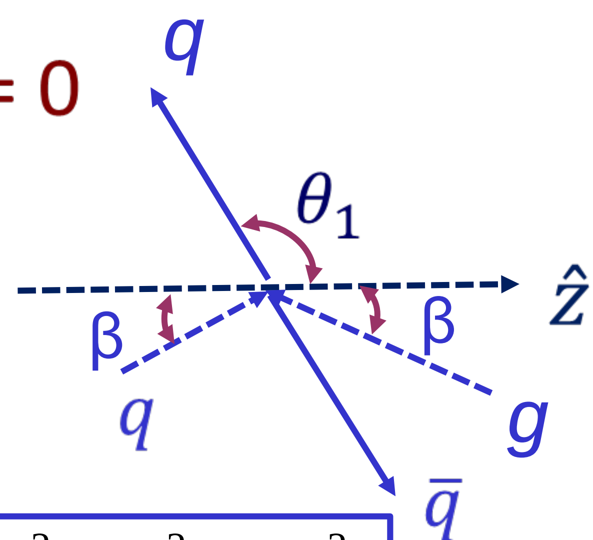




$\theta_1 = \beta$ and $\phi_1 = 0$



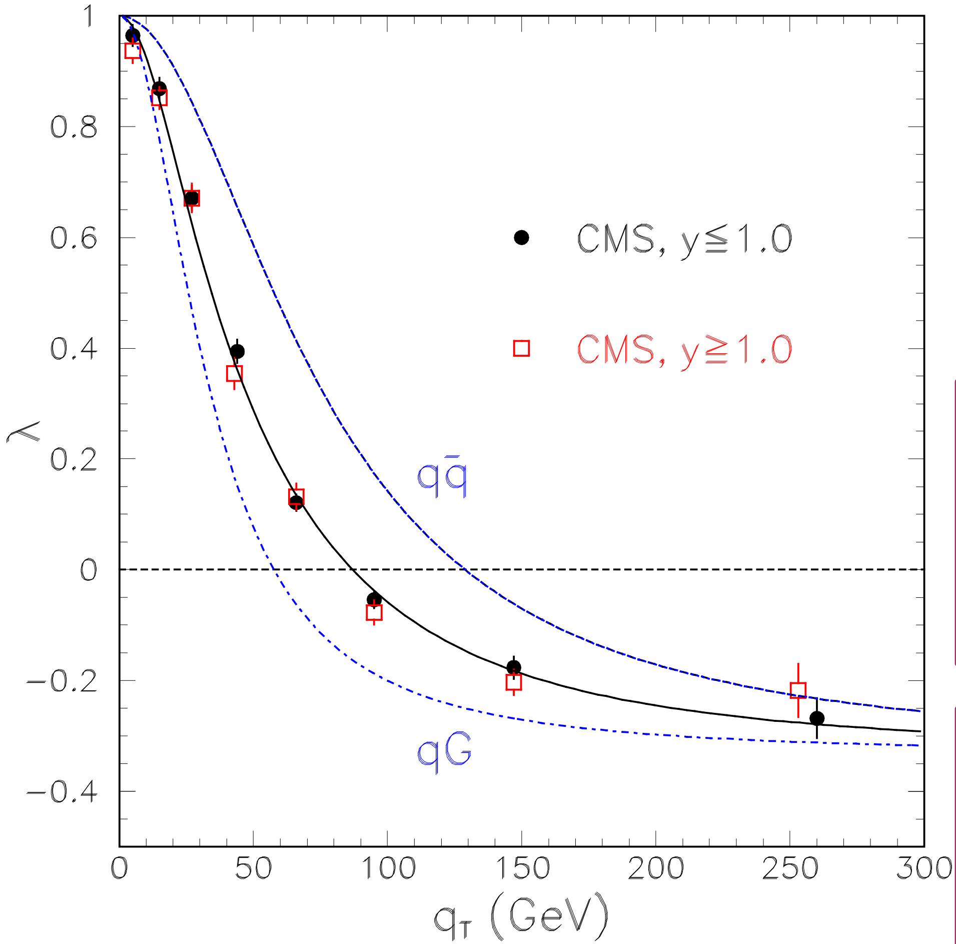




$\theta_1 > \beta$ and $\phi_1 = 0$; $A_0 = A_2 \approx 5q_T^2/(Q^2 + 5q_T^2)$

$$\lambda = \frac{2-3A_0}{2+A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}; \qquad \nu = \frac{2A_2}{2+A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$

# Compare with CMS data on λ

## (*Z* production in *p*+*p* collision at 8 TeV)




$$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \to Zg$$

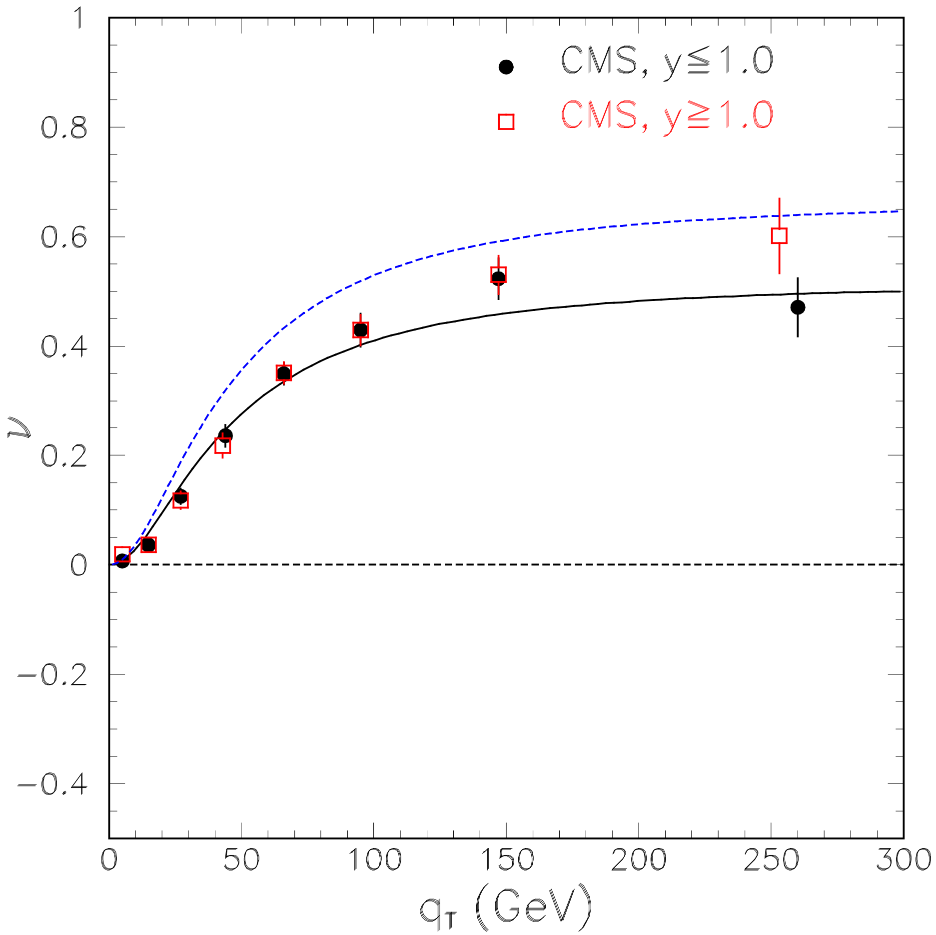
$$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \to Zq$$

For both processes
$\lambda = 1$ at $q_T = 0$ ($\theta_1 = 0^o$)
$\lambda = -1/3$ at $q_T = \infty$ ($\theta_1 = 90^o$)

Data can be well described with a mixture of 58.5% $qG$ and 41.5% $q\bar{q}$ processes

# Compare with CMS data on $\nu$

## ($Z$ production in $p$+$p$ collision at 8 TeV)




$$\nu = \frac{2q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \to Zg$$

$$\nu = \frac{10q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \to Zq$$

Dashed curve corresponds to a mixture of 58.5% $qG$ and 41.5% $q\bar{q}$ processes

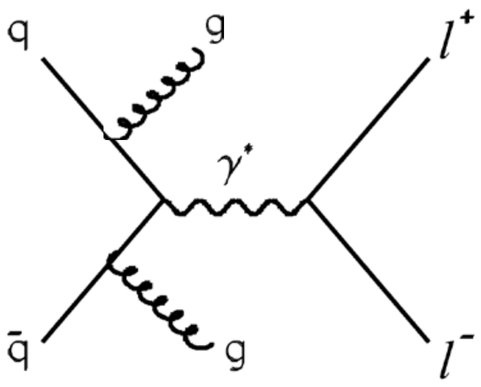
Solid curve corresponds to

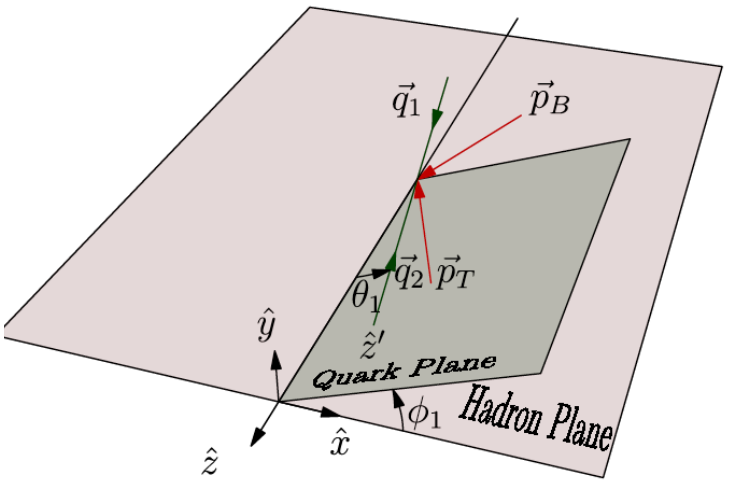
$$\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.77$$

$q - \bar{q}$ axis is non-coplanar relative to the hadron plane

# Origins of the non-coplanarity

1) Processes at order $\alpha_s^2$ or higher

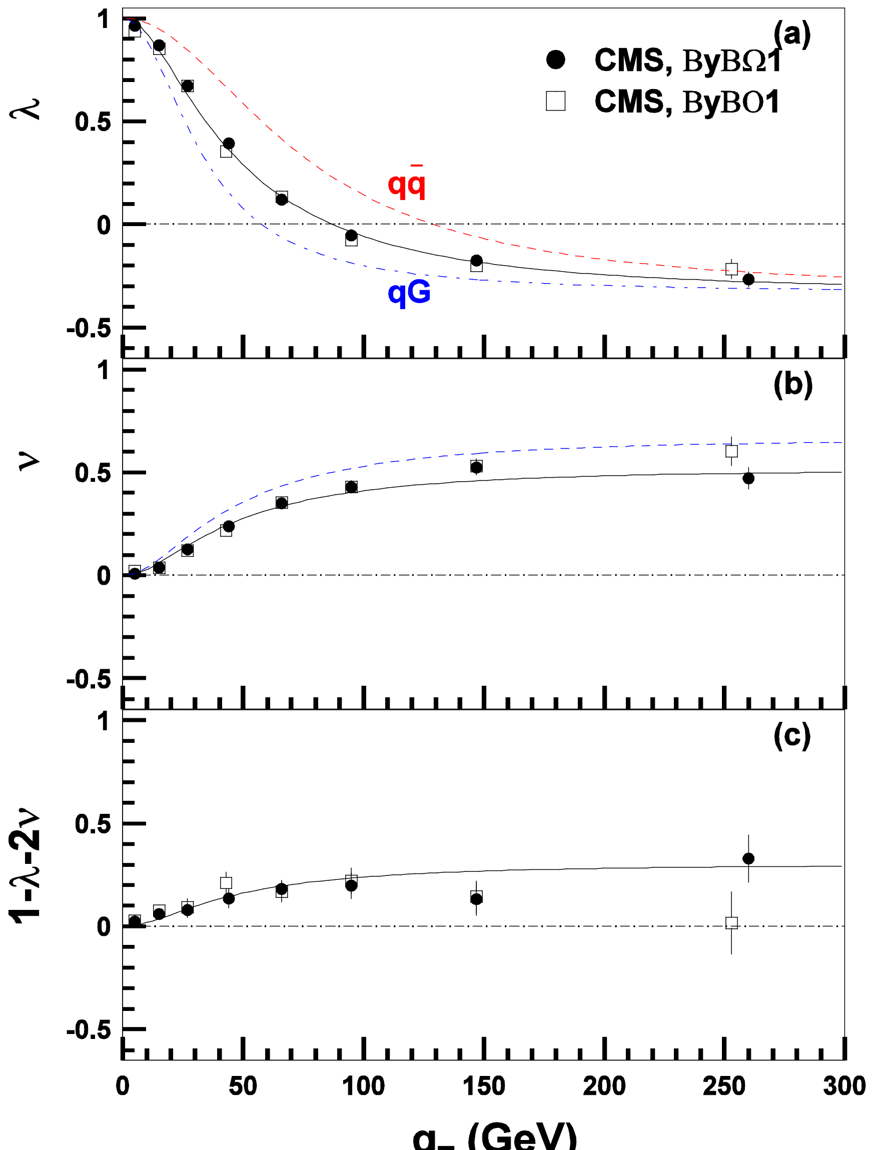



2) Intrinsic $k_T$ from interacting partons

(Boer-Mulders functions in the beam and target hadrons)

# Compare with CMS data on Lam-Tung relation

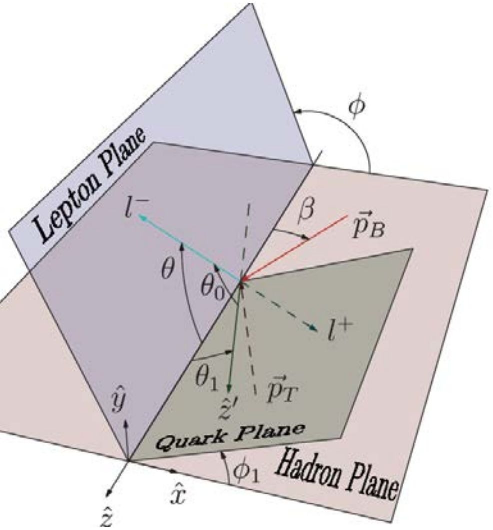



Solid curves correspond to a mixture of 58.5% $qG$ and 41.5% $q\bar{q}$ processes, and

$$\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.77$$

Violation of Lam-Tung relation is well described

# From LO to NLO to NNLO




What happens at LO?

$$A_0 = \sin^2\theta_1$$
$$A_1 = \sin\theta_1\cos\theta_1\cos\phi_1$$
$$A_2 = \sin^2\theta_1\cos 2\phi_1$$
$$A_3 = a\sin\theta_1\cos\phi_1$$
$$A_4 = a\cos\theta_1$$
$$A_5 = \sin^2\theta_1\sin\phi_1\cos\phi_1$$
$$A_6 = \sin\theta_1\cos\theta_1\sin\phi_1$$
$$A_7 = a\sin\theta_1\sin\phi_1$$

At Leading-order,

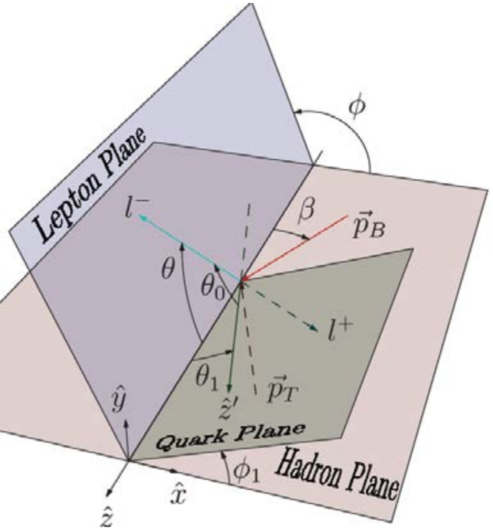
$\theta_1 = 0$ and $\phi_1 = 0$.

Hence, we have

1) All $A_i$ vanishes, except $A_4$

2) $A_4 = a\cos\theta_1$

# From LO to NLO to NNLO




$$A_0 = \sin^2\theta_1$$
$$A_1 = \sin\theta_1 \cos\theta_1 \cos\phi_1$$
$$A_2 = \sin^2\theta_1 \cos 2\phi_1$$
$$A_3 = a \sin\theta_1 \cos\phi_1$$
$$A_4 = a \cos\theta_1$$
$$A_5 = \sin^2\theta_1 \sin\phi_1 \cos\phi_1$$
$$A_6 = \sin\theta_1 \cos\theta_1 \sin\phi_1$$
$$A_7 = a \sin\theta_1 \sin\phi_1$$

What happens at NLO?

At Next-to-leading-order,

$\theta_1 \neq 0$ and $\phi_1 = 0$.

Hence, we have

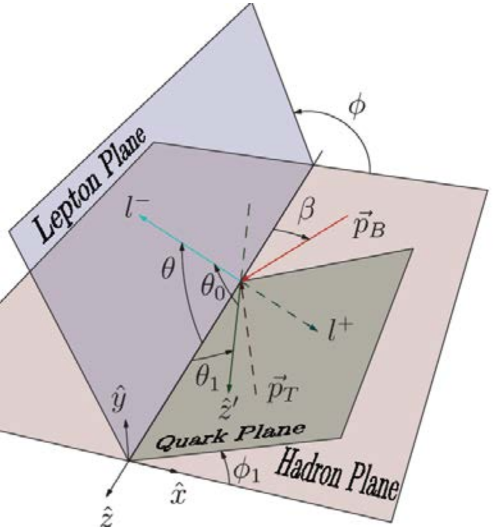
1) $A_0 = A_2$

2) $A_0 / A_1 = A_3 / A_4$

3) $a^2 = A_3 A_4 / A_1$

4) $A_5 = A_6 = A_7 = 0$

# From LO to NLO to NNLO




$$A_0 = \sin^2\theta_1$$
$$A_1 = \sin\theta_1 \cos\theta_1 \cos\phi_1$$
$$A_2 = \sin^2\theta_1 \cos 2\phi_1$$
$$A_3 = a \sin\theta_1 \cos\phi_1$$
$$A_4 = a \cos\theta_1$$
$$A_5 = \sin^2\theta_1 \sin\phi_1 \cos\phi_1$$
$$A_6 = \sin\theta_1 \cos\theta_1 \sin\phi_1$$
$$A_7 = a \sin\theta_1 \sin\phi_1$$

What happens at NNLO?

At NNLO,

$\theta_1 \neq 0$ and $\phi_1 \neq 0$.

Hence, we have

1) $A_0 \geq A_2$

2) $|A_0 / A_1| \geq |A_3 / A_4|$

3) $a^2 = A_3 A_4 / A_1$

4) $A_5 = A_6 = A_7 = 0$

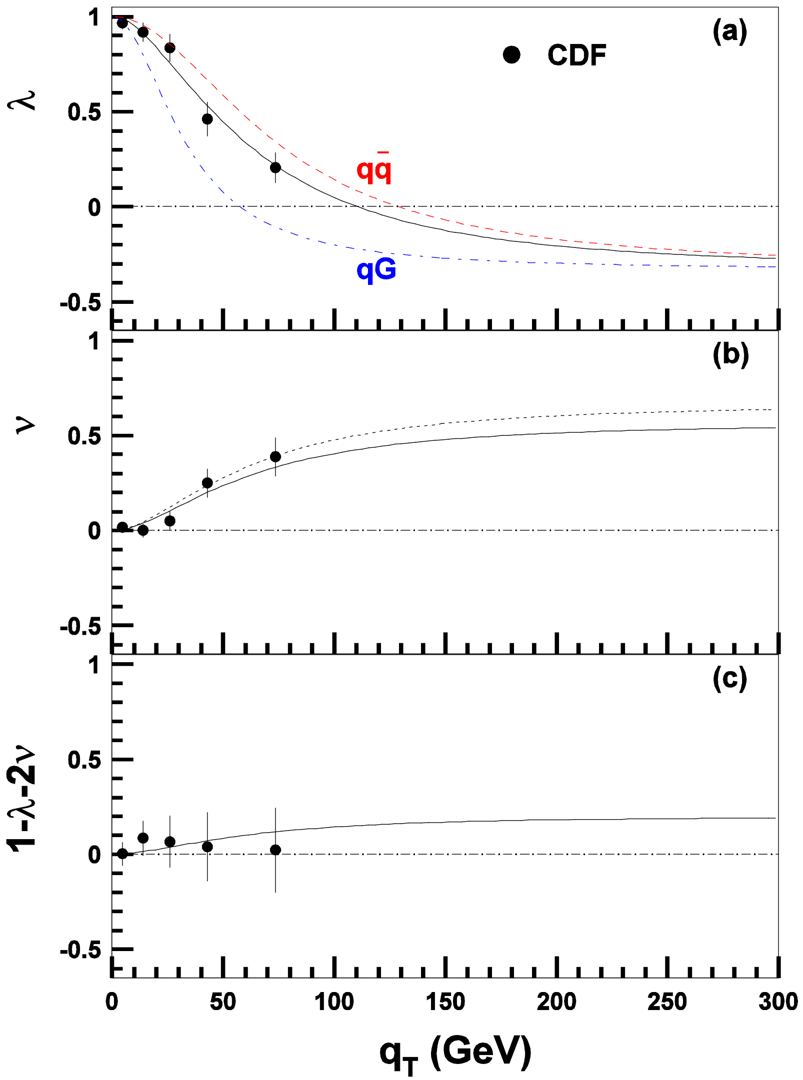
(if $\theta_1$ and $\phi_1$ are uncorrelated)

$A_5, A_6, A_7 \neq 0$

(if $\theta_1$ and $\phi_1$ are correlated)

# Compare with CDF data

## (Z production in $p + \bar{p}$ collision at 1.96 TeV)

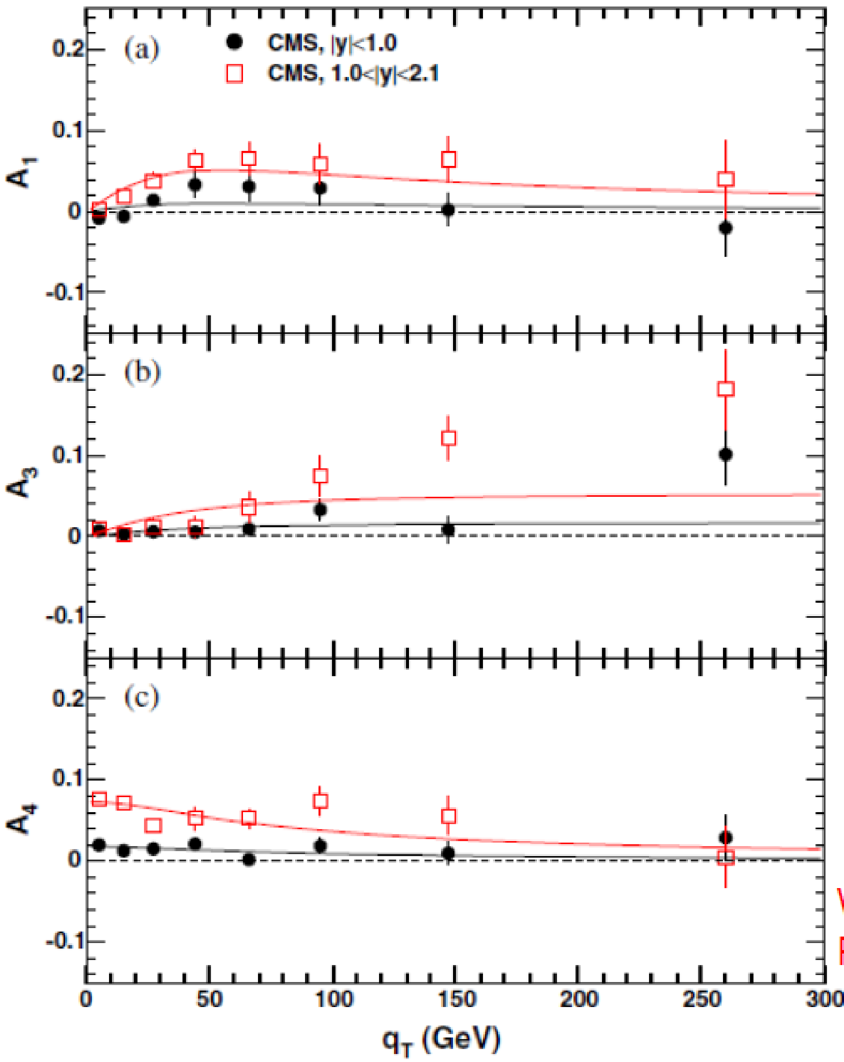



Solid curves correspond to a mixture of 27.5% $qG$ and 72.5% $q\bar{q}$ processes, and

$$\left\langle \sin^2\theta_1 \cos 2\phi_1 \right\rangle / \left\langle \sin^2\theta_1 \right\rangle = 0.85$$

Violation of Lam-Tung relation is not ruled out

# Compare CMS data on $A_1$, $A_3$ and $A_4$ with calculations




$$A_1 = r_1 \left[ f \frac{q_T Q}{Q^2 + q_T^2} + (1 - f) \frac{\sqrt{5} q_T Q}{Q^2 + 5 q_T^2} \right]$$

$$A_3 = r_3 \left[ f \frac{q_T}{\sqrt{Q^2 + q_T^2}} + (1 - f) \frac{\sqrt{5} q_T}{\sqrt{Q^2 + 5 q_T^2}} \right]$$

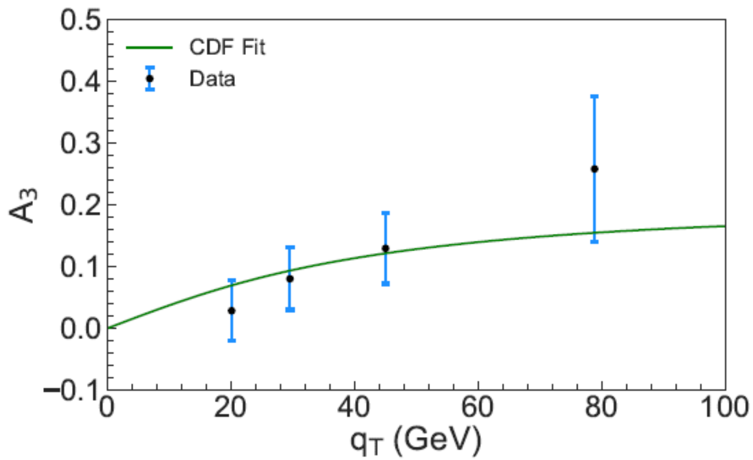
$$A_4 = r_4 \left[ f \frac{Q}{\sqrt{Q^2 + q_T^2}} + (1 - f) \frac{Q}{\sqrt{Q^2 + 5 q_T^2}} \right]$$

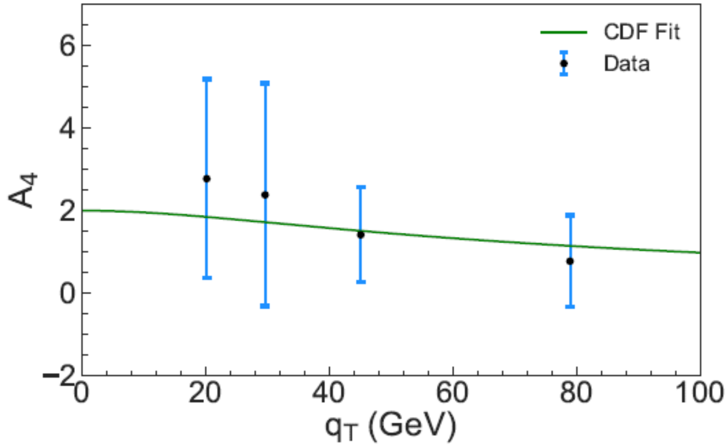
**Rapidity of $A_1$, $A_3$ and $A_4$ are well described**

W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev
Phys. Rev. D 96, 054020 (2017)

# Future prospects

- Extend this study to W-boson production
  - Preliminary results show that the data can be well described




W-boson production in p-pbar collision from CDF

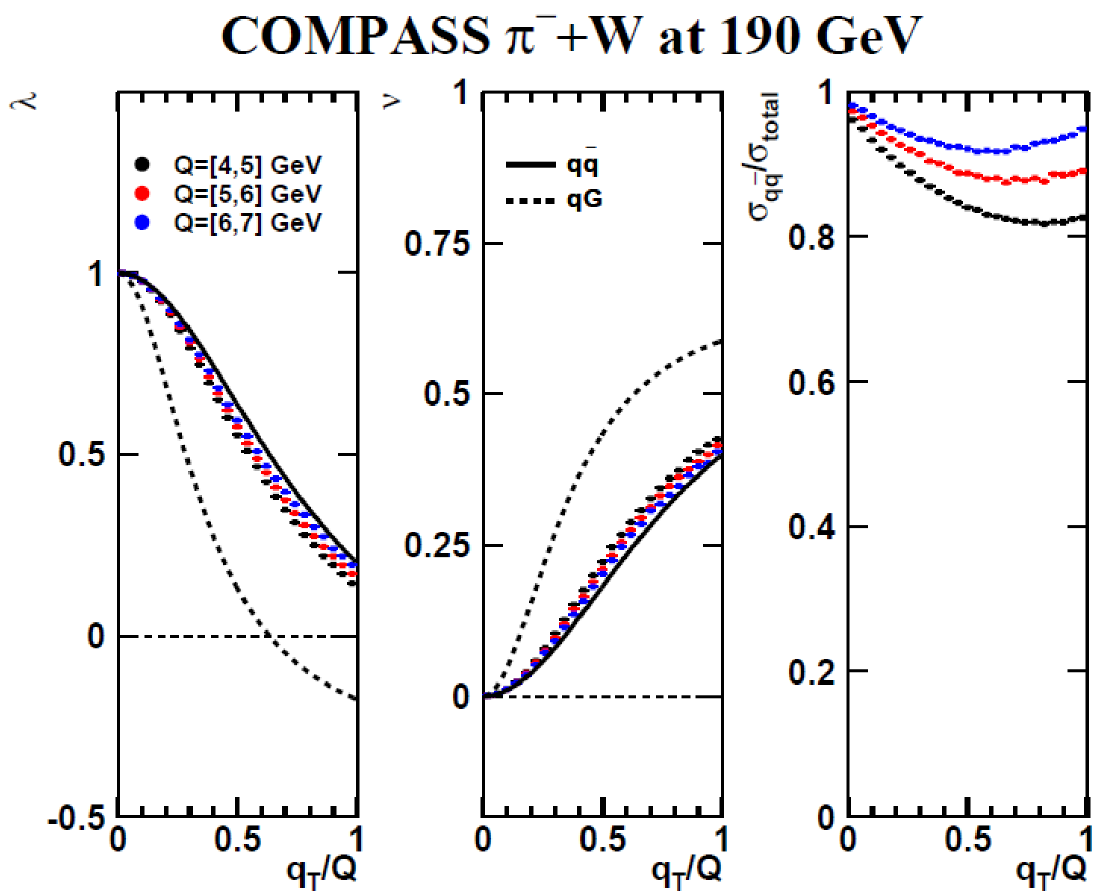
Yan Lyu, W.C. Chang, E. McClellan, JCP, and O. Teryaev

# Future prospects

- Extend this study to fixed-target Drell-Yan data

**Lepton Angular Distributions of Fixed-target Drell-Yan Experiments in Perturbative QCD and a Geometric Approach**

Wen-Chen Chang,[1] Randall Evan McClellan,[2,3] Jen-Chieh Peng,[3] and Oleg Teryaev[4]




arXiv:1811.03256v1

# Future prospects

- Extend this study to semi-inclusive DIS at high $p_T$ (involving two hadrons and two leptons)
  - Relevant for EIC measurements
- Rotational invariance, equality, and inequality relations formed by various angular distribution coefficients
  - See preprint arXiv: 1808.04398
- Comparison with pQCD calculations
  - See preprint arXiv: 1811.03256
  - Lambertson and Vogelsang, PRD 93 (2016) 114013

# Future prospects

On the Rotational Invariance and Non-Invariance of Lepton Angular Distributions in Drell-Yan and Quarkonium Production

Jen-Chieh Peng[a], Daniël Boer[b], Wen-Chen Chang[c], Randall Evan McClellan[a,d], Oleg Teryaev[e]

arXiv:1808.04398

Quantities invariant under rotations along the y-axis (Faccioli et al.)

$$\mathcal{F} = \frac{1+\lambda+\nu}{3+\lambda}$$

$$\mathcal{F} = \frac{1+\lambda_0 - 2\lambda_0 \sin^2\theta_1 \sin^2\phi_1}{3+\lambda_0} = \frac{1+\lambda_0 - 2\lambda_0 y_1^2}{3+\lambda_0}$$

$$\tilde{\lambda} = \frac{2\lambda+3\nu}{2-\nu}$$

$$\tilde{\lambda} = \frac{\lambda_0 + 3\lambda_0 \sin^2\theta_1 \sin^2\phi_1}{1+\lambda_0 \sin^2\theta_1 \sin^2\phi_1} = \frac{\lambda_0 + 3\lambda_0 y_1^2}{1+\lambda_0 y_1^2}$$

$$\tilde{\lambda}' = \frac{(\lambda-\nu/2)^2 + 4\mu^2}{(3+\lambda)^2}$$

$$\tilde{\lambda}' = \frac{\lambda_0^2 (z_1^2 + x_1^2)^2}{(3+\lambda_0)^2} = \frac{\lambda_0^2 (1-y_1^2)^2}{(3+\lambda_0)^2}$$

# Future prospects

- Extend this study to Z plus jets data at LHC
  - Lam-Tung relation is expected to be satisfied by Z plus single jet events, but badly violated by Z plus multi-jet events.
  - $\lambda$ for Z plus quark-jet events would be different from that of Z plus gluon-jet events
  - Hence data on $\lambda$ can test various algorithms proposed for distinguishing a quark-jet from a gluon-jet
  - Would be great to have these data from LHC!

# Summary

- The lepton angular distribution coefficients $A_0 - A_7$ can be described in terms of the polar and azimuthal angles of the $q-\bar{q}$ axis
- Violation of the Lam-Tung relation is due to the acoplanarity of the $q-\bar{q}$ axis and the hadron plane. This can come from order $\alpha_s^2$ or higher processes or from intrinsic $k_T$
- This approach can be extended to fixed-target Drell-Yan and many other hard-processes
- Extraction of the Boer-Mulders function in the Drell-Yan process must take into account of the pQCD effects