Light-cone parton distribution functions from lattice QCD simulations at the physical point

Krzysztof Cichy
Adam Mickiewicz University, Poznań, Poland

in collaboration with:

Constantia Alexandrou (Univ. of Cyprus, Cyprus Institute)
Martha Constantinou (Temple University, Philadelphia)
Kyriakos Hadjiyiannakou (Univ. of Cyprus)
Karl Jansen (DESY Zeuthen)
Aurora Scapellato (Univ. of Cyprus, Univ. of Wuppertal)
Fernanda Steffens (Univ. of Bonn)

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Outline of the talk

1. Introduction
   - Basics
   - PDFs from the lattice
   - Quasi-PDFs
   - Procedure
2. Results
   - Lattice setup
   - Bare ME
   - Renormalized ME
   - Matching
   - Final results
3. Conclusions and prospects

Based on:

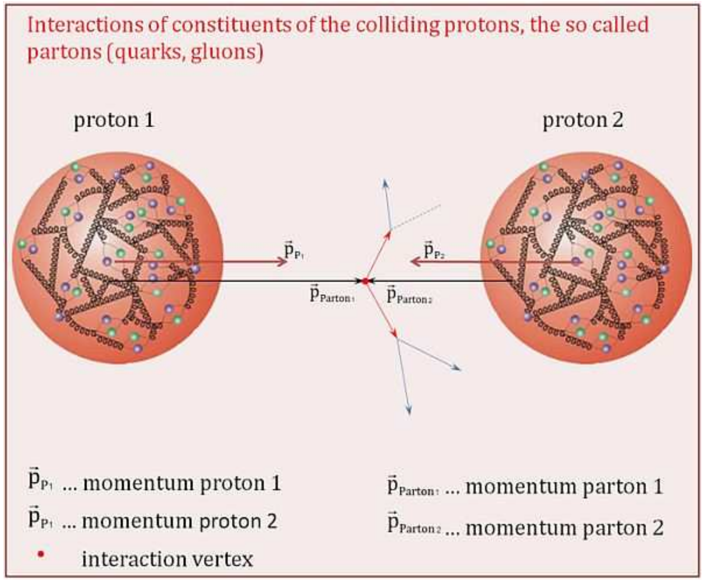
- **C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Reconstruction of light-cone parton distribution functions from lattice QCD simulations at the physical point", Phys. Rev. Lett. 121 (2018) 112001**
- **C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Transversity parton distribution functions from lattice QCD", Phys. Rev. D (Rapid Communications), in press, arXiv: 1807.00232 [hep-lat]**
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", Nucl. Phys. B923 (2017) 394-415 (invited Frontiers Article)
- K. Cichy, M. Constantinou, "A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results", invited review article for a special issue of Advances in High Energy Physics, arXiv: 1811.07248 [hep-lat]

Introduction

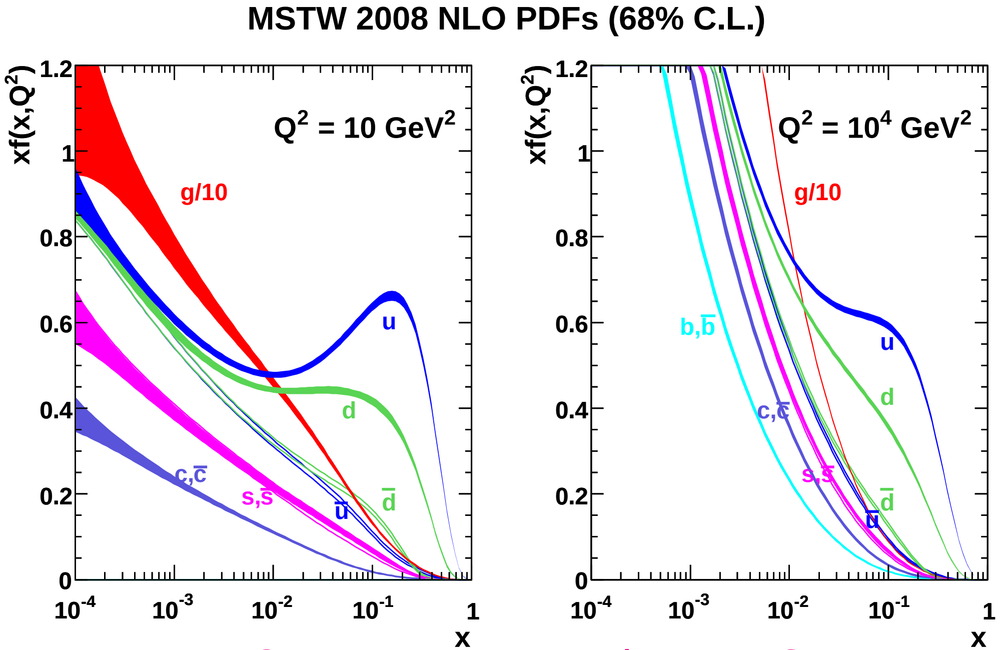
PDFs

- Hadrons are complicated systems with properties resulting from the strong dynamics of quarks and gluons inside them.
- This dynamics is characterized in terms of, among others, parton distribution functions (PDFs).
- PDFs are essential in making predictions for collider experiments.

$$\sigma_{AB} = \sum_{a,b=q,g} \sigma_{ab} \otimes f_{a|A}(x_1, Q^2) \otimes f_{b|B}(x_2, Q^2)$$



Source: LHC, CERN



MSTW2008, Eur. Phys. J. C63, 189

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$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

where: $\xi^- = \frac{\xi^0-\xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-,0)$ is the Wilson line from 0 to ξ^-.

- This expression is light-cone dominated – needs $\xi^2 = \vec{x}^2 + t^2 \sim 0$ – very hard due to non-zero lattice spacing!

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- This expression is light-cone dominated – needs $\xi^2 = \vec{x}^2 + t^2 \sim 0$ – very hard due to non-zero lattice spacing!
- Accessible on the lattice – moments of the distributions, but:
  - ⋆ higher derivatives noisy,
  - ⋆ operator mixing.

- The common feature of all the approaches is that they rely to some extent on the factorization framework:

$$\underbrace{Q(x,\mu_R)}_{\text{some lattice observable}} = \int_{-1}^{1} \frac{dy}{y} C\left(\frac{x}{y}, \mu_F, \mu_R\right) q(y, \mu_F),$$

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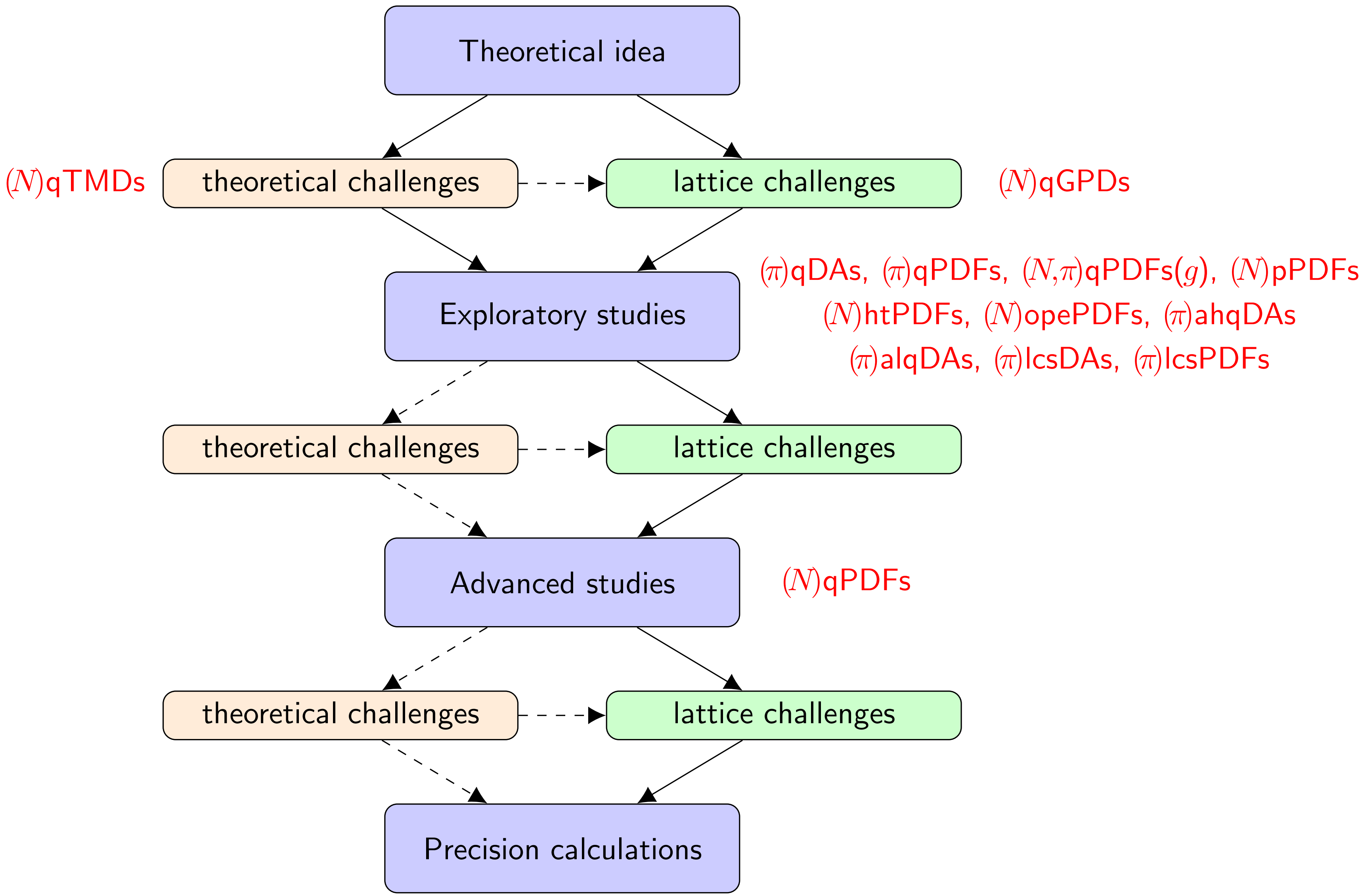
- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$ with different choices of Γ,Γ' Dirac structures and objects $F(z)$.
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014, 2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

Approaches to light-cone PDFs



A guide to light-cone PDFs from lattice QCD: an overview of approaches, techniques and results

Krzysztof Cichy[1], Martha Constantinou[2] [a]

[1] *Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*

[2] *Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

- 97 pages, **arXiv: 1811.07248 [hep-lat]**
- invited review article for a special issue of Advances in High Energy Physics
- discusses in detail quasi-distributions:
  nucleon: **non-singlet quark qPDFs**, qGPDs, qTMDs, singlet qPDFs, gluon qPDFs; pion: qPDFs, qDAs
- reviews also other approaches:
  hadronic tensor, auxiliary scalar quark, auxiliary heavy quark, auxiliary light quark, pseudo-distributions, “OPE without OPE”, lattice cross sections

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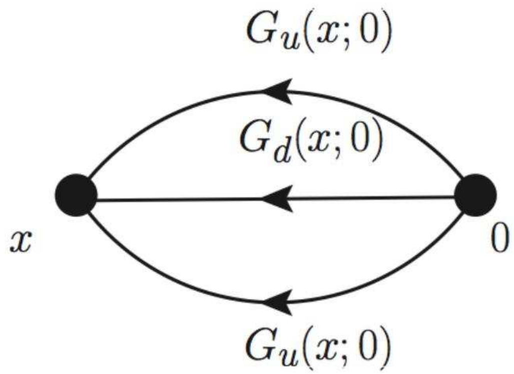

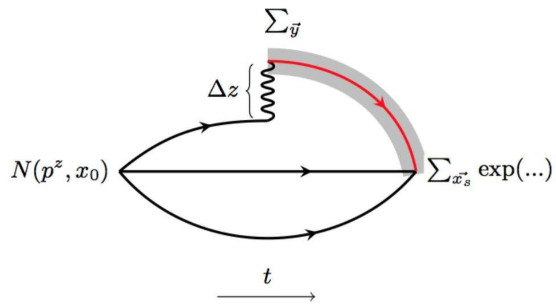

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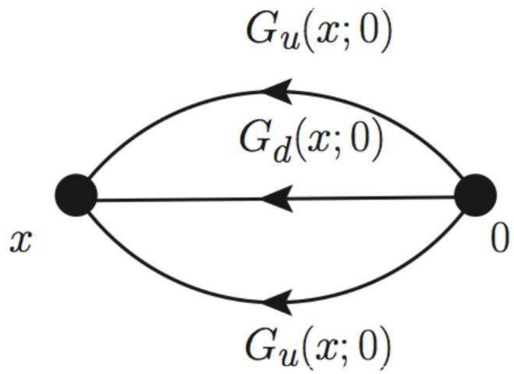

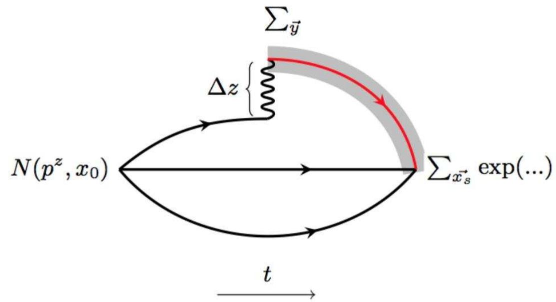

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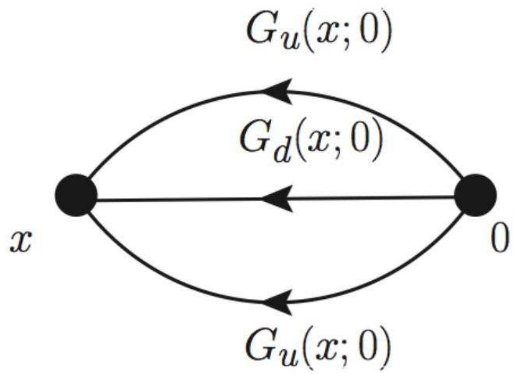

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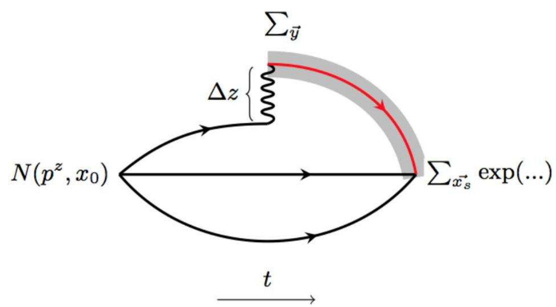
- The highly non-trivial aspect:
  how to relate $\tilde{q}(x,\mu^2,P_3)$ to the light-front PDF $q(x,\mu^2)$ (infinite momentum frame)
  $\Rightarrow$ **Large Momentum Effective Theory (LaMET)**



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Non-perturbative renormalization scheme: RI'-MOM.

G. Martinelli et al., Nucl. Phys. B445 (1995) 81

RI'-MOM renormalization conditions (for cases without mixing):
for the operator:

$$Z_q^{-1}\,Z_{\mathcal{O}}(z)\,\frac{1}{12}\mathrm{Tr}\left[\mathcal{V}(p,z)\left(\mathcal{V}^{\mathrm{Born}}(p,z)\right)^{-1}\right]\Big|_{p^2=\bar{\mu}_0^2}=1\,,$$

for the quark field:

$$Z_q=\frac{1}{12}\mathrm{Tr}\left[(S(p))^{-1}\,S^{\mathrm{Born}}(p)\right]\Big|_{p^2=\bar{\mu}_0^2}\,.$$

- momentum p in the vertex function is set to the RI′ renormalization scale $\bar{\mu}_0$
- $\mathcal{V}(p,z)$ – amputated vertex function of the operator,
- $\mathcal{V}^{\mathrm{Born}}$ – its tree-level value, $\mathcal{V}^{\mathrm{Born}}(p,z){=}i\gamma_3\gamma_5\,e^{ipz}$ for helicity,
- $S(p)$ – fermion propagator ($S^{\mathrm{Born}}(p)$ at tree-level).

This prescription handles all divergences that are present and applies the necessary finite renormalization related to the lattice regularization.

To relate the quasi-PDFs to the usual PDFs, one uses the fact that the IR region of the distributions is untouched when going from a finite to an infinite momentum. In other words, if $q(x,\mu)$ is the usual PDF defined through light-cone correlations, then one should have:

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Explicit formulae for 1-loop perturbative matching:

- transverse momentum cutoff scheme to $\overline{\mathrm{MS}}$ matching
  X. Xiong et al., PRD **90** (2014) 014051
- $\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$ matching W. Wang, S. Zhao, R. Zhu, 1708.02458
- RI$\to \overline{\mathrm{MS}}$ matching I.W. Stewart, Y. Zhao, 1709.04933, Y.-S. Liu et al., 1807.06566
- treatment of the UV log divergence in wave function corrections T. Izubuchi et al., 1801.03917, C. Alexandrou et al., 1803.02685, 1807.00232

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7. Apply target mass corrections to eliminate residual m_N/P_3 effects.

Results

Lattice setup

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$

β=2.10,	$c_{\rm SW}$=1.57751, a=0.0938(3)(2) fm
$48^3 \times 96$	$a\mu = 0.0009$ $\quad m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV $\quad m_\pi L = 2.98(1)$

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001
C. Alexandrou et al., Phys. Rev. D (Rapid Communications), in press, arXiv: 1807.00232

The Wilson twisted mass fermion action for the 2 light (u, d quarks) is given in the so-called twisted basis by: [R. Frezzotti, P. Grassi, G.C. Rossi, S. Sint, P. Weisz, 2000-2004]

$$S_l[\psi,\bar{\psi},U] = a^4 \sum_x \bar{\chi}_l(x)\big(D_W + m_{0,l} + i\mu_l\gamma_5\tau_3\big)\chi_l(x),$$

- D_W – Wilson-Dirac operator,
- $m_{0,l}$ and μ_l – bare untwisted and twisted light quark masses,
- $\chi_l = (\chi_u,\, \chi_d)$ – 2-component vector in flavor space; chiral rotation of standard one: $\psi = e^{i\gamma_5\tau_3\omega/2}\chi$

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- $\chi_l = (\chi_u,\, \chi_d)$ – 2-component vector in flavor space; chiral rotation of standard one: $\psi = e^{i\gamma_5\tau_3\omega/2}\chi$
- Maximal twist: $\omega = \pi/2$ by tuning the PCAC mass to zero $\Rightarrow$ automatic $\mathcal{O}(a)$-improvement.

Momentum smearing

$$S_{\text{mom}}\psi(x) = \frac{1}{1+6\alpha}\left(\psi(x) + \alpha\sum_{j=\pm1}^{\pm3} U_j(x)e^{i\xi\hat{j}}\psi(x+\hat{j})\right)$$

G. Bali et al., Phys. Rev. D93, 094515 (2016)

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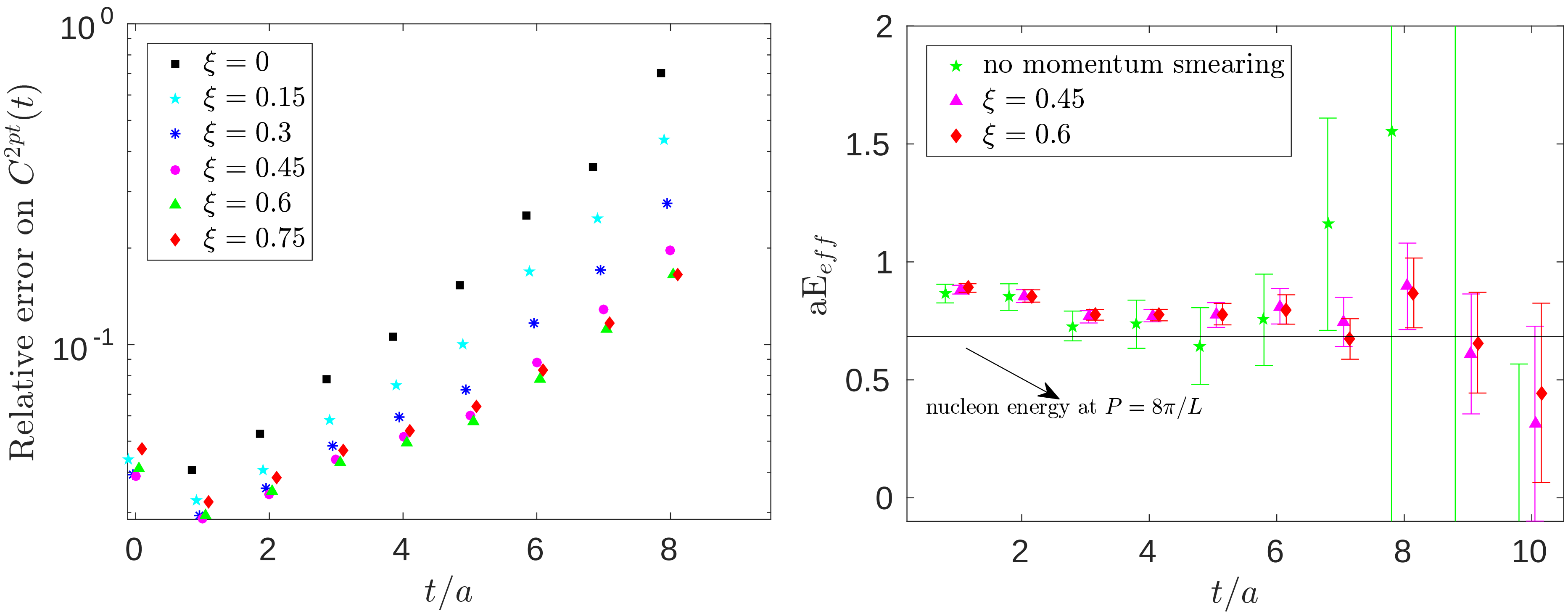

50 iterations of (Gaussian) momentum smearing, $\alpha = 4$

Computation setup

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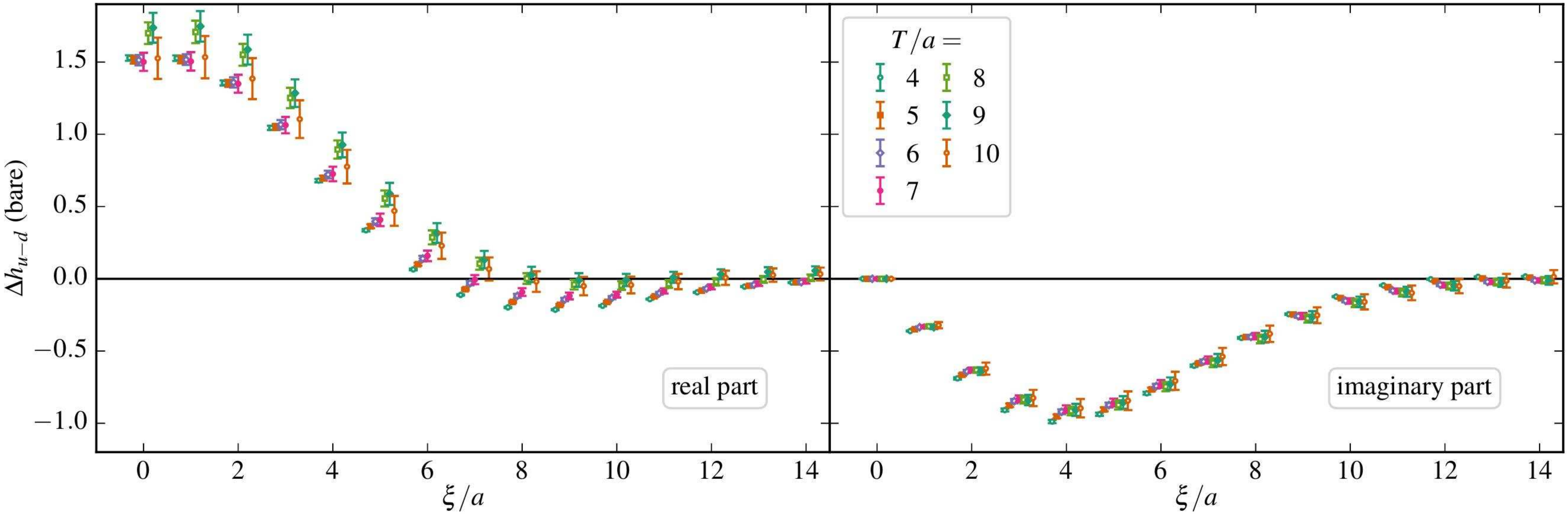
- Bias from the LP inversions corrected using the Covariant Approximation Averaging technique (CAA)
  E. Shintani et al., Phys. Rev. D91, 114511 (2015)

Step 1

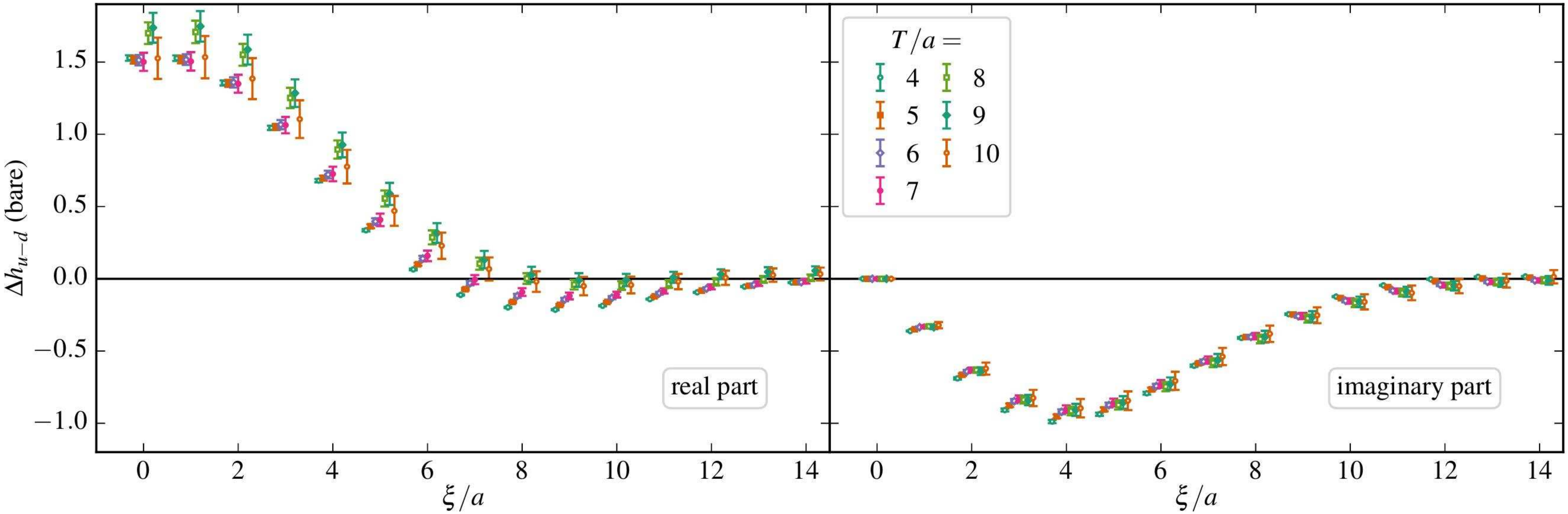
The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$
2. Compute vertex functions and the resulting renormalization functions in the intermediate RI'-MOM scheme $Z^{\mathrm{RI}'}(z,\mu)$.
3. Convert the renormalization functions to the $\overline{\mathrm{MS}}$ scheme and evolve to $\bar{\mu}=2$ GeV.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\overline{\mathrm{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}(x,\mu^2,P_3)=\int\frac{dz}{4\pi}\,e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle.$$
6. Relate quasi-PDFs to light-cone PDFs via a matching procedure.
7. Apply target mass corrections to eliminate residual m_N/P_3 effects.



Plot by Jeremy Green, talk in the Lattice PDFs workshop, Maryland, April 2018

ETMC $N_f = 2+1+1$ ensemble, $a \approx 0.094$ fm, $M_\pi \approx 363$ MeV, $24^3 \times 48$



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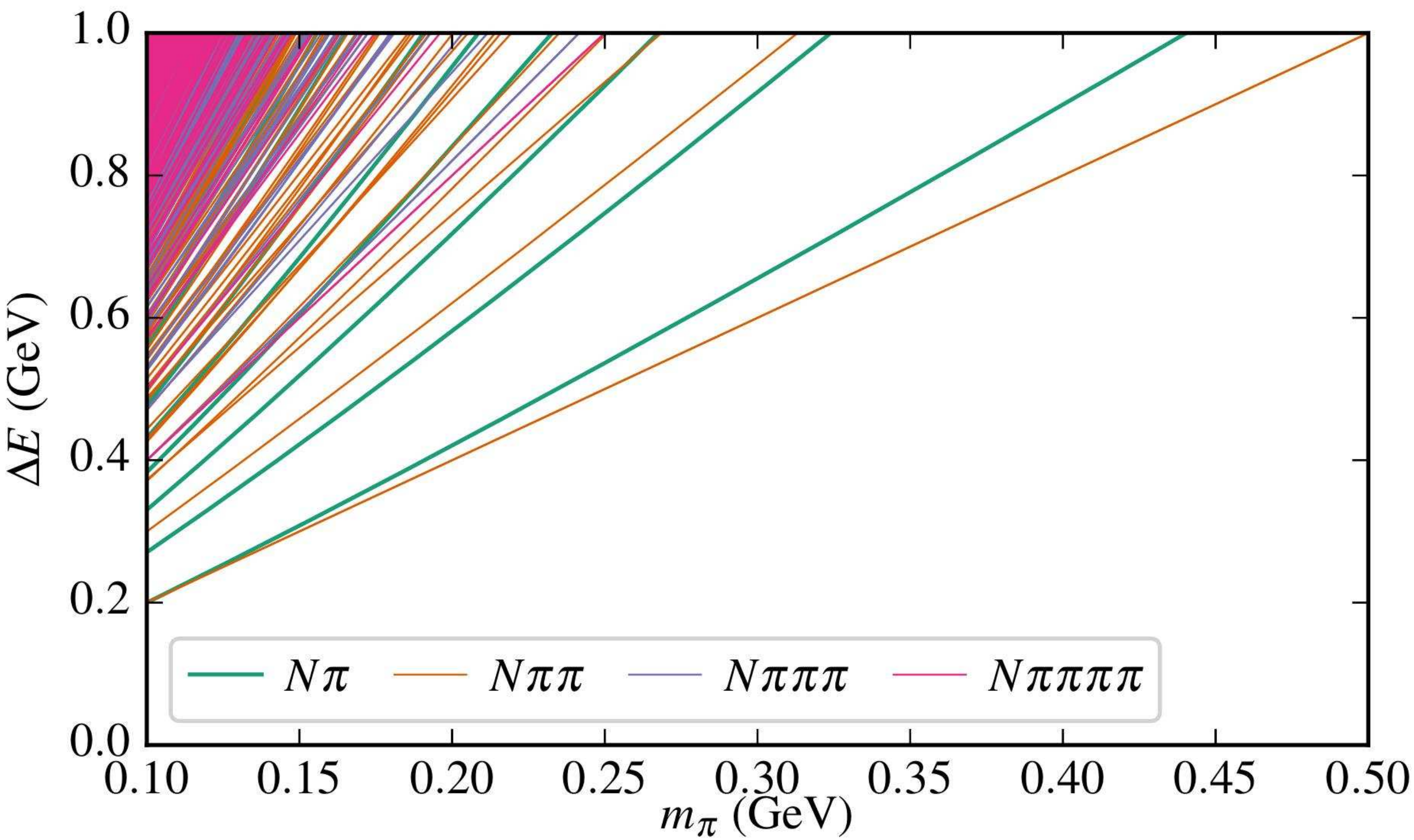
Excited states clearly manifested as bare ME going below zero.

Excited states



Plot by Jeremy Green, plenary talk in LATTICE 2018, July 2018

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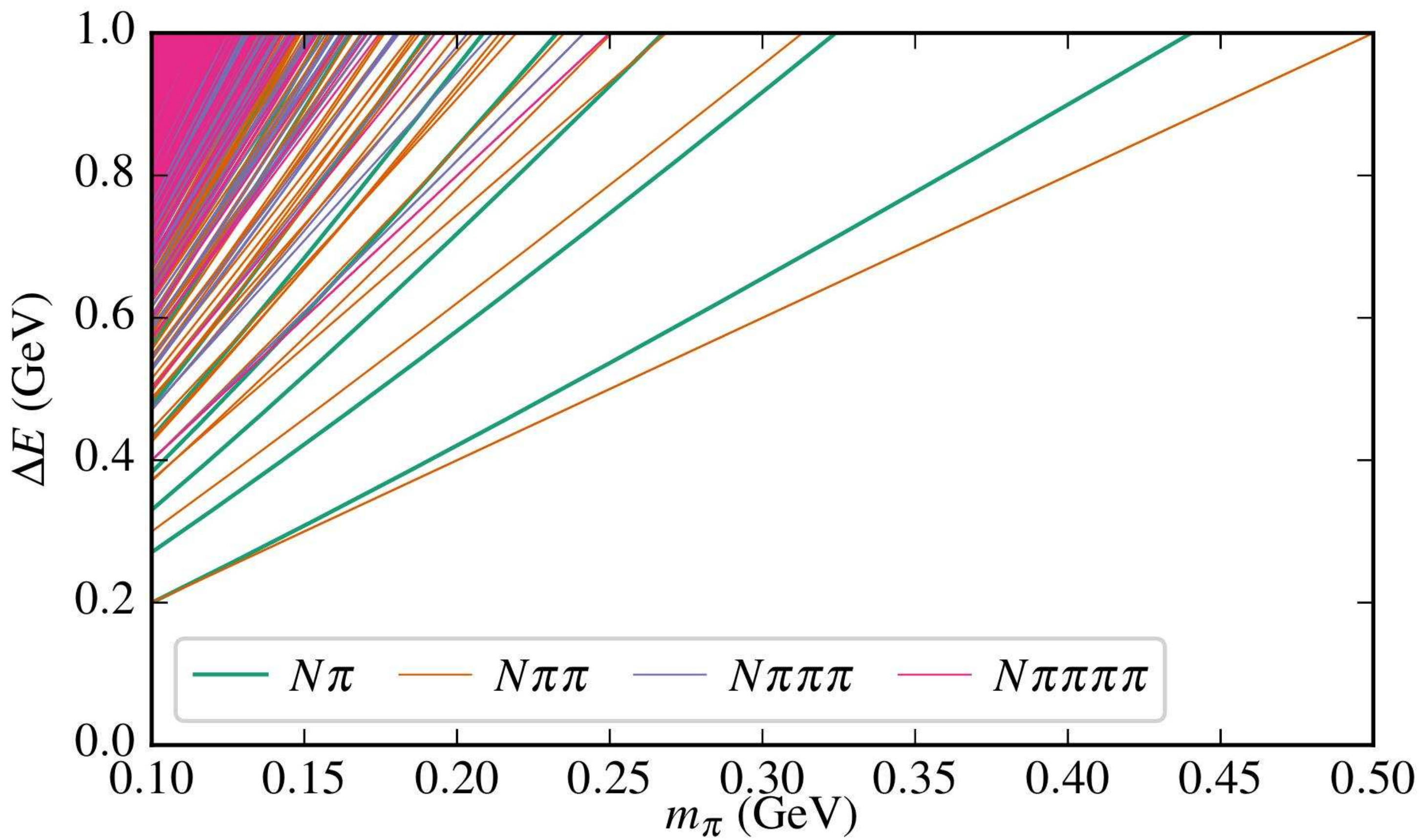
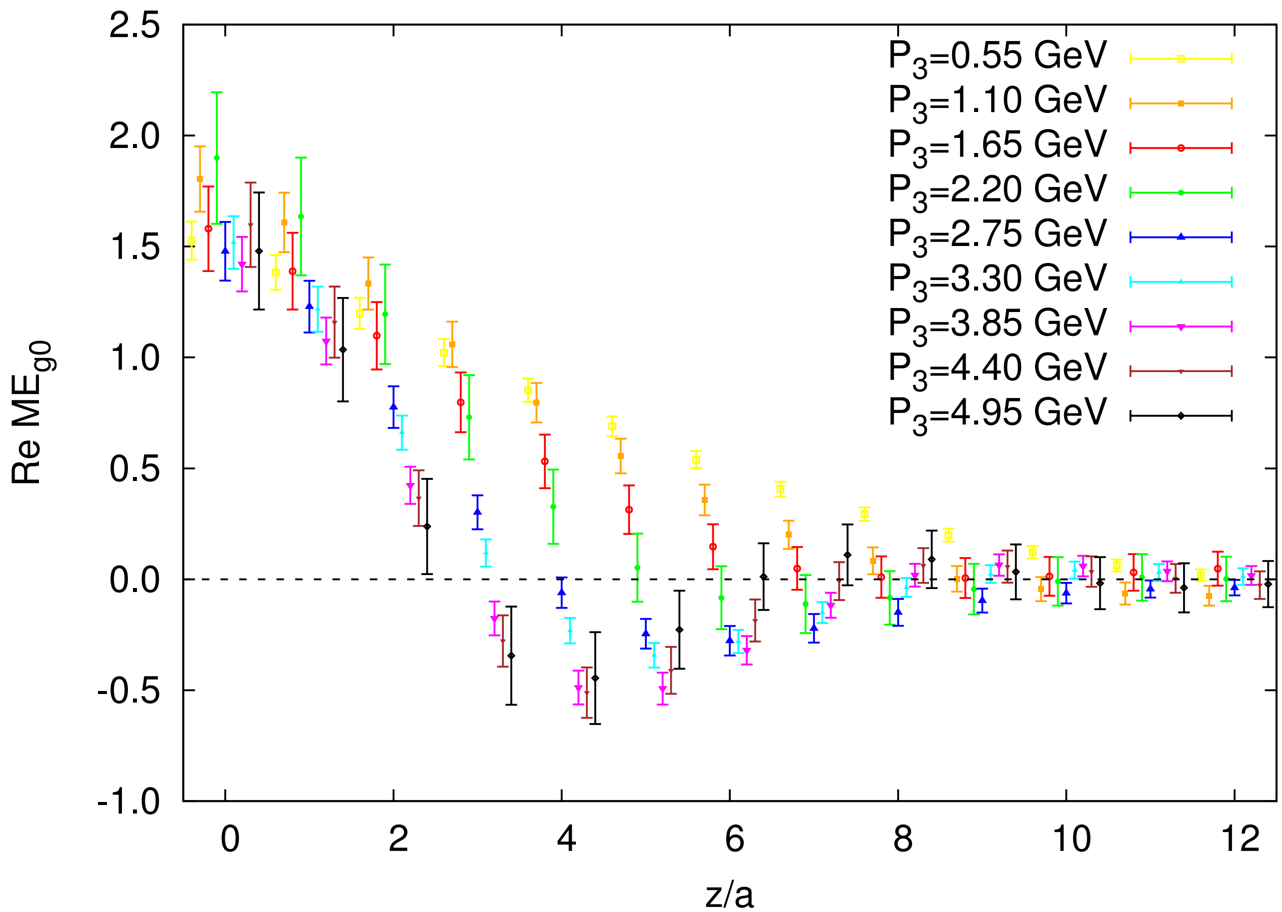


Plot by Jeremy Green, plenary talk in LATTICE 2018, July 2018

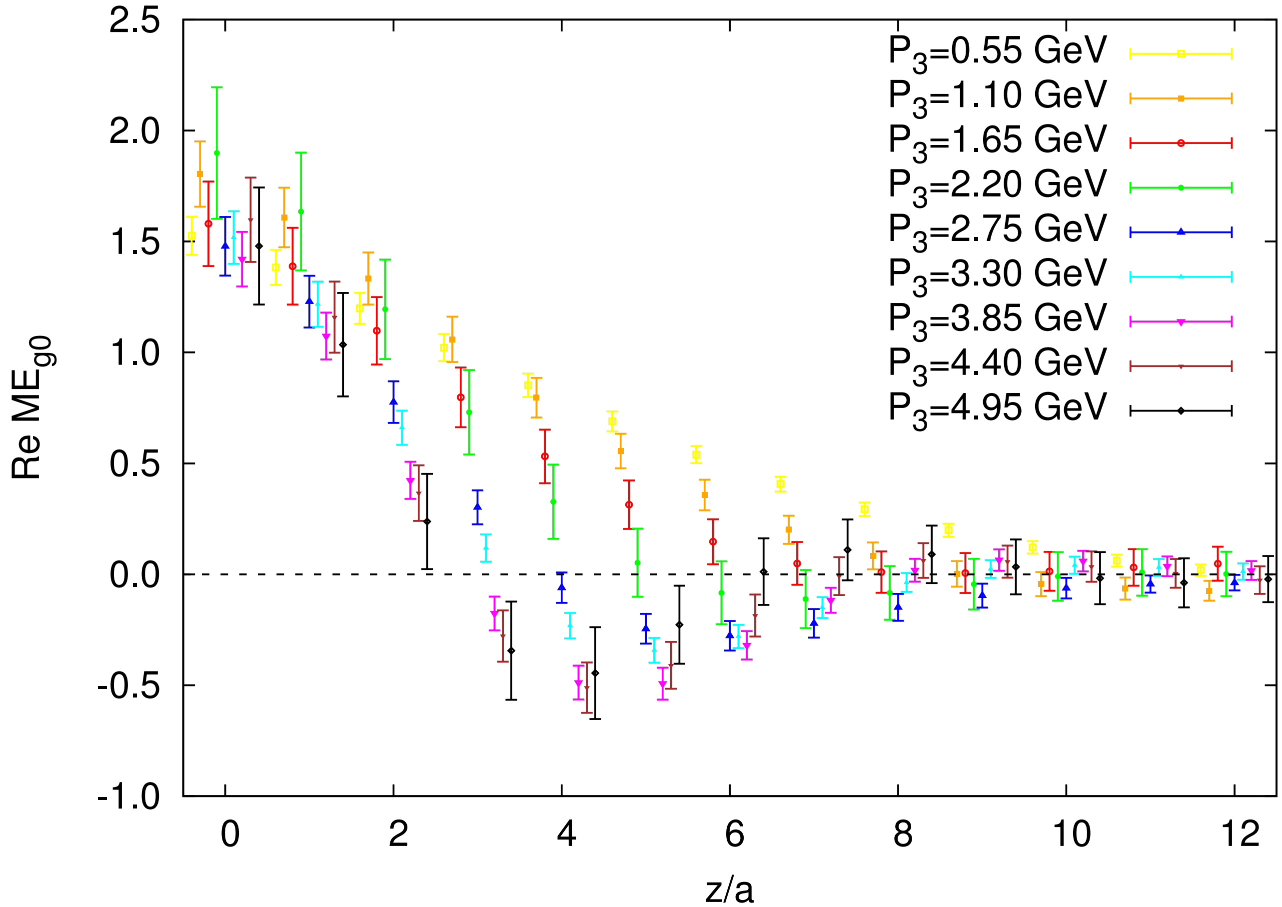
Excited states clearly enhanced at smaller pion masses.

Many excited states at the physical point!
Need to be suppressed by a source-sink separation for which one-state and two-state fits agree.



ETMC $N_f = 2+1+1$ ensemble,
$a \approx 0.094$ fm, $M_\pi \approx 298$ MeV, $24^3 \times 48$

Excited states



ETMC $N_f = 2+1+1$ ensemble,
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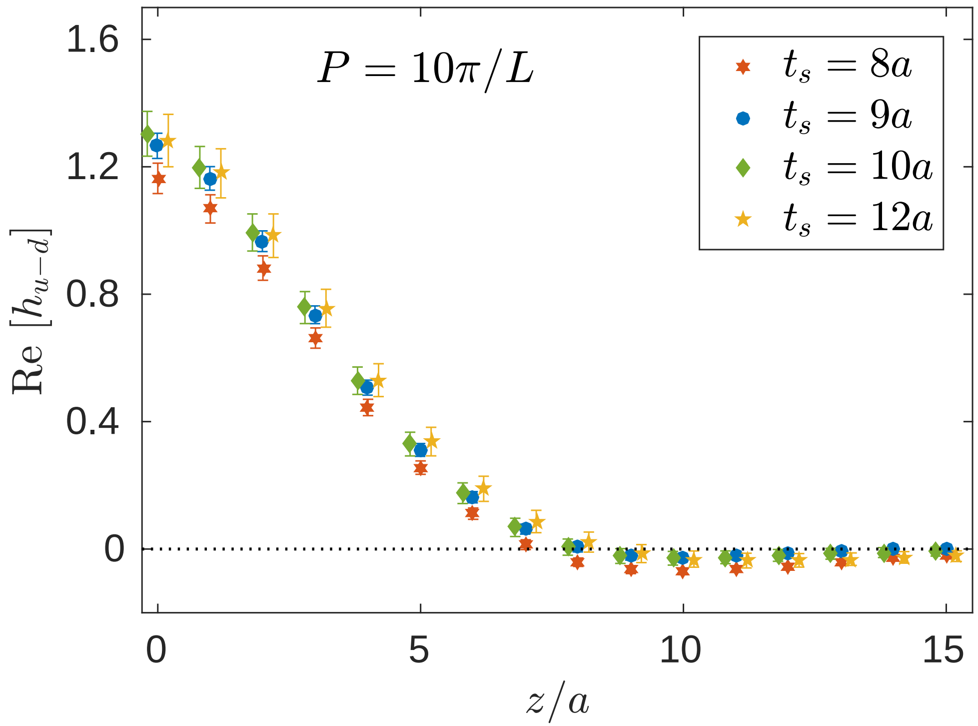
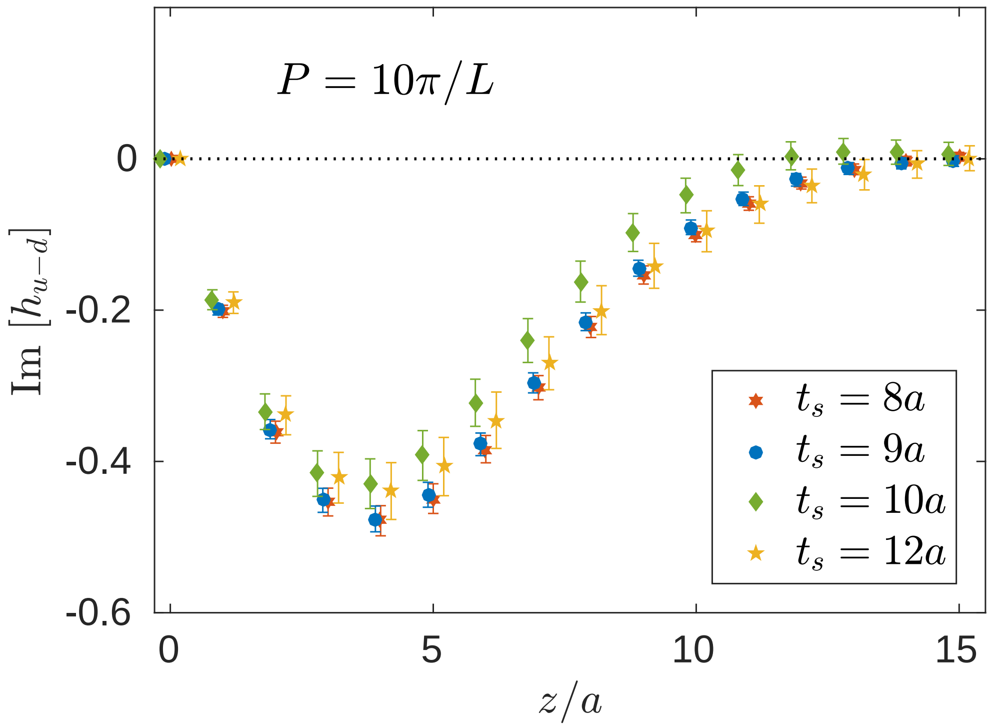
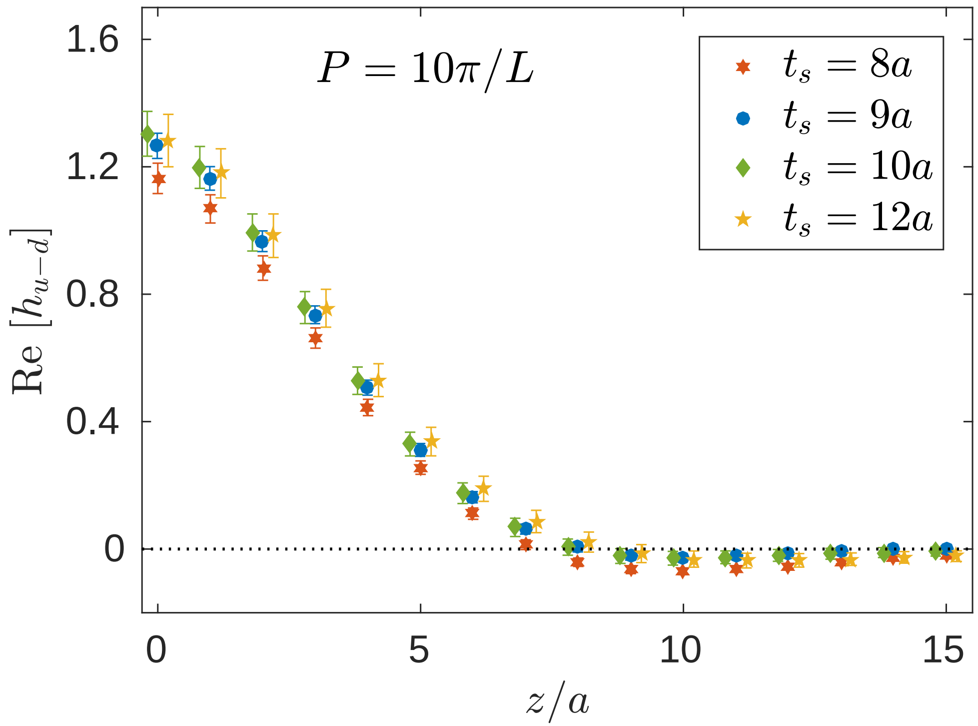
Excited states clearly enhanced at larger boosts.

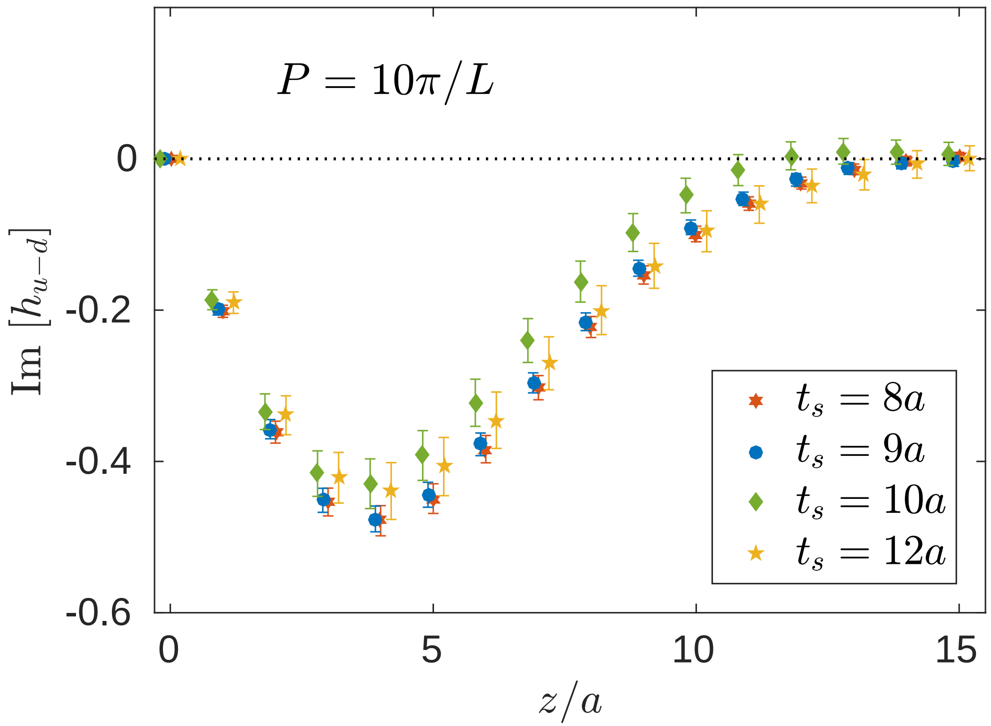




Statistics:

- $t_s = 8a$ – 4320 measurements,
- $t_s = 9a$ – 8820 measurements,
- $t_s = 10a$ – 9000 measurements,
- $t_s = 12a$ – 72990 measurements.





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Increasing t_s by 1 lattice spacing worsens the signal by a factor 2-3!

Excited states – summation method

$$\mathcal{R}(P;t_s) \equiv \sum_{\tau=a}^{t_s-a} \frac{C^{3\mathrm{pt}}(P;t_s,\tau)}{C^{2\mathrm{pt}}(P_i;t_s)} =$$

$$= C + \mathcal{M}\, t_s + \mathcal{O}\left(e^{-(E_1-E_0)t_s}\right)$$

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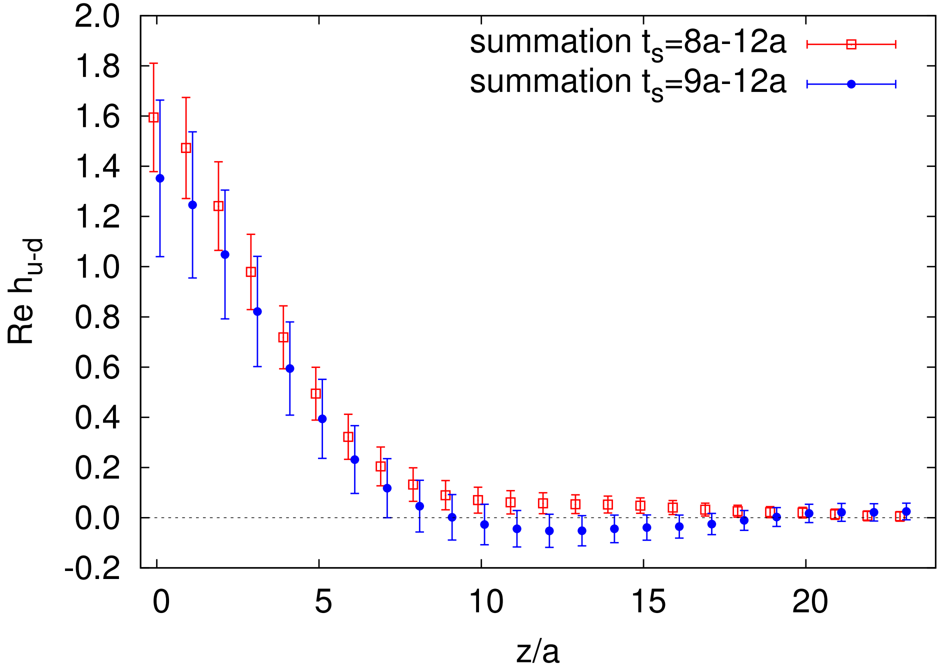

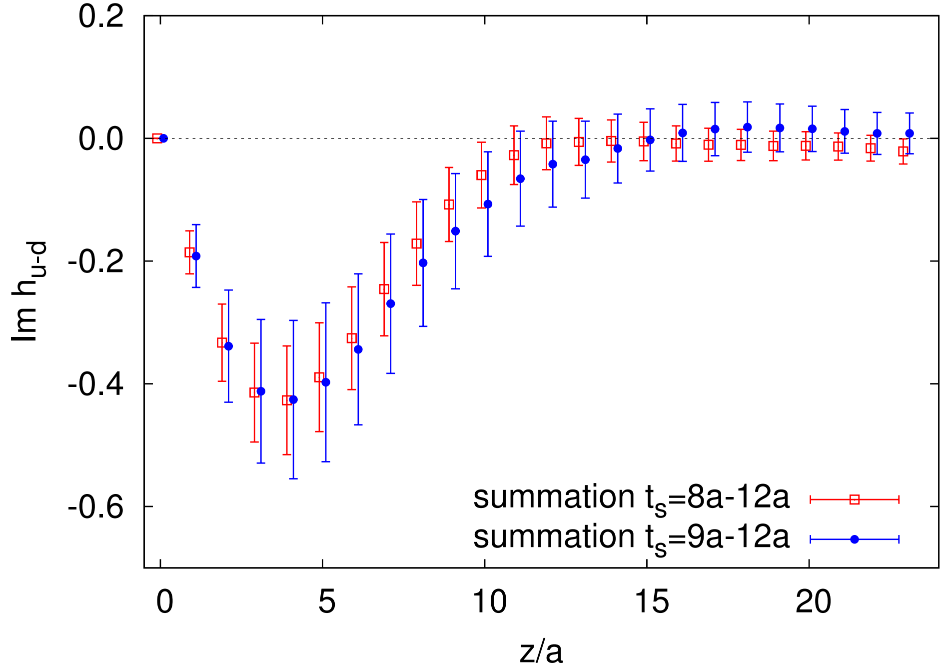

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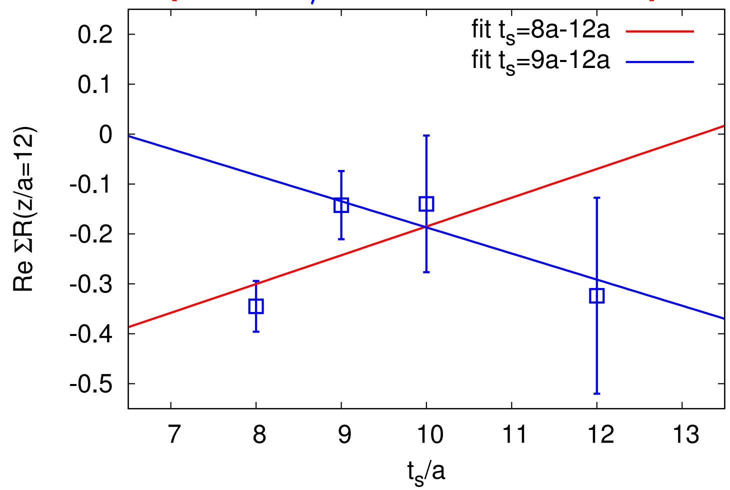
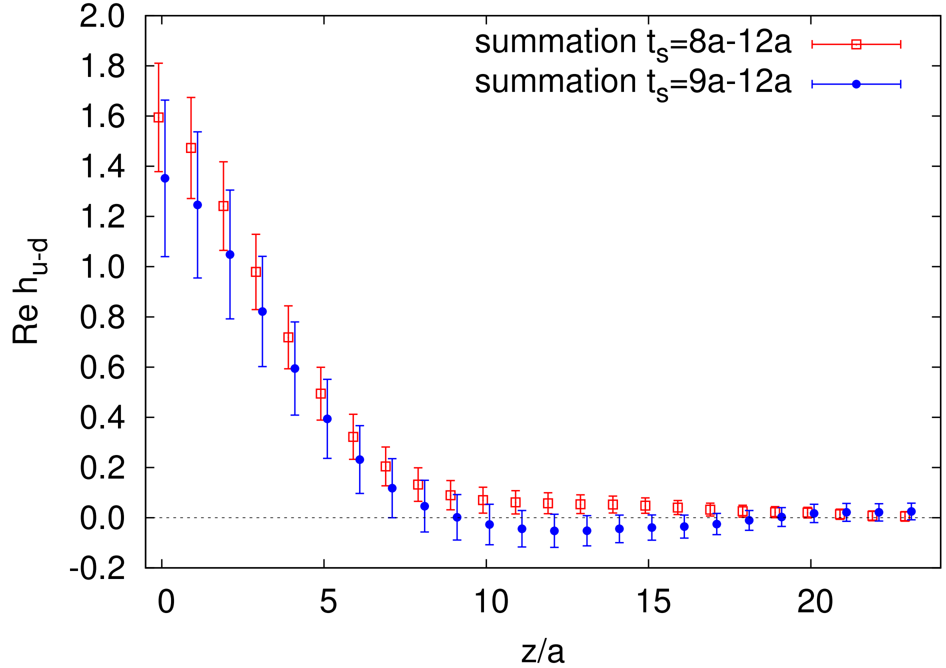
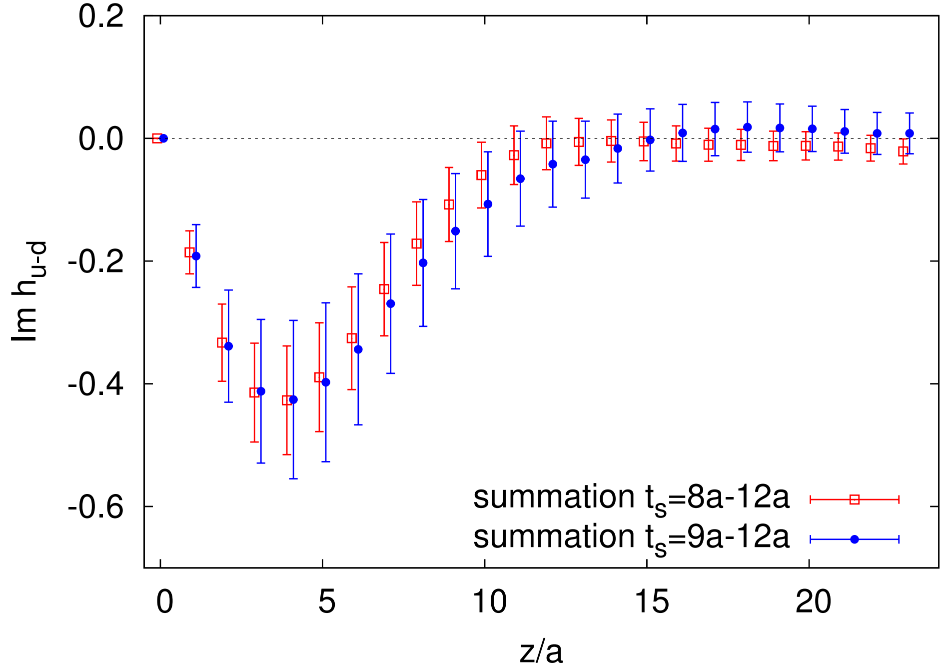
Example: $z/a = 12$, real part







Excited states – 2-state fits

$$C^{2\mathrm{pt}}(P;t) = |A_0|^2 e^{-E_0 t} + |A_1|^2 e^{-E_1 t}$$

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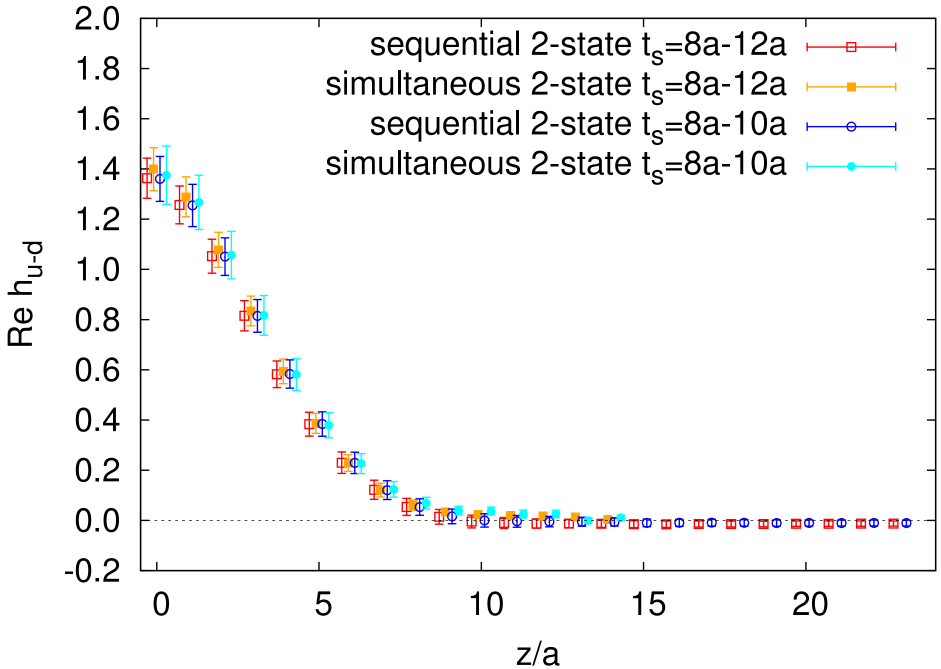
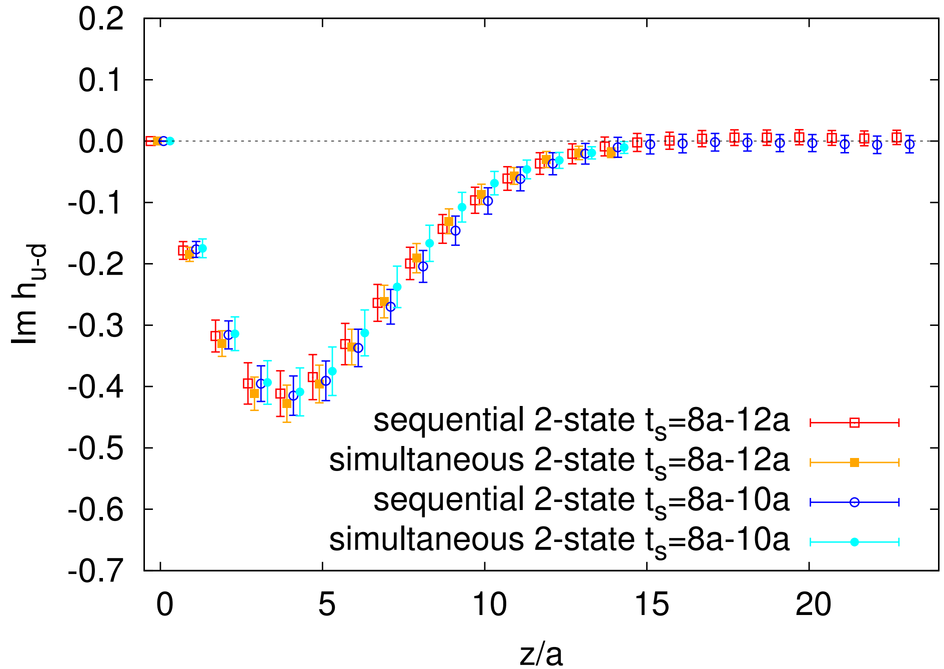
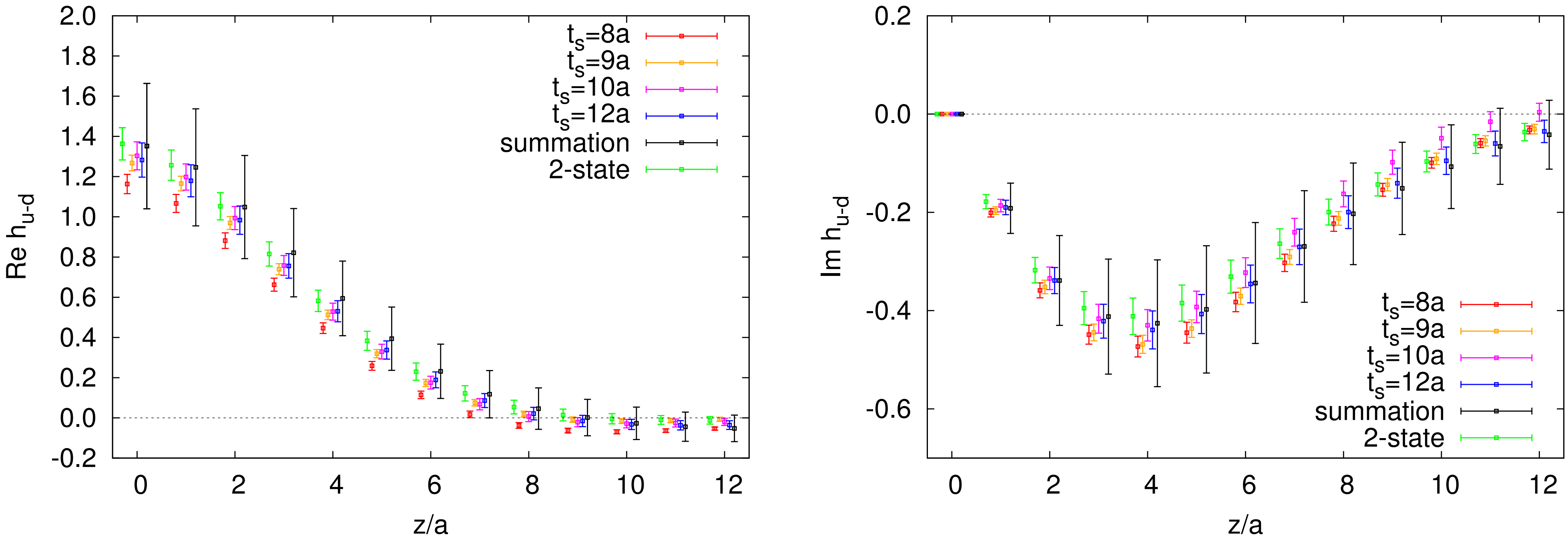




Excited states – comparison



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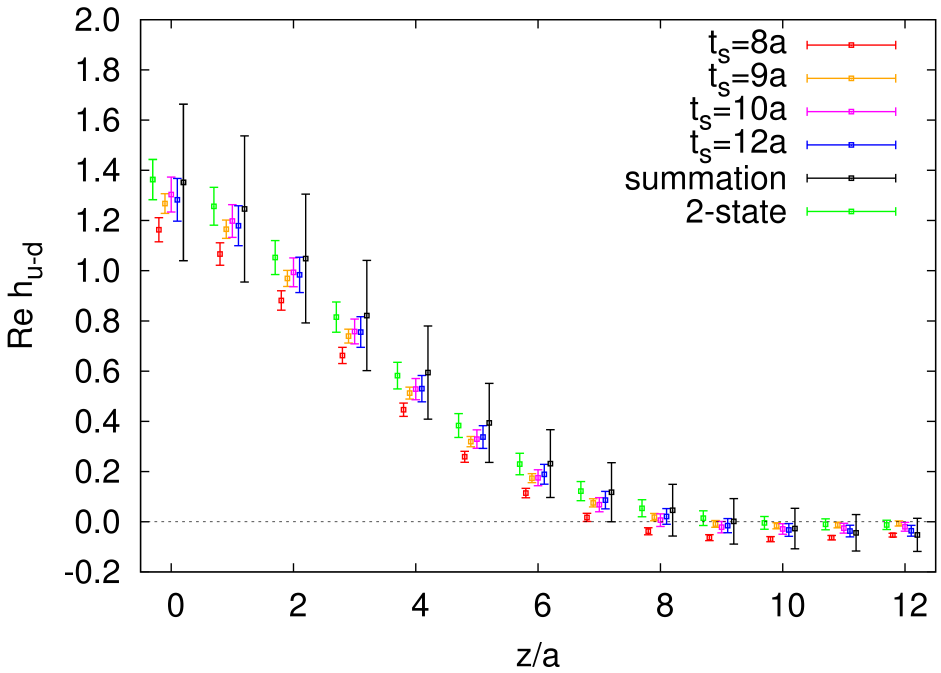

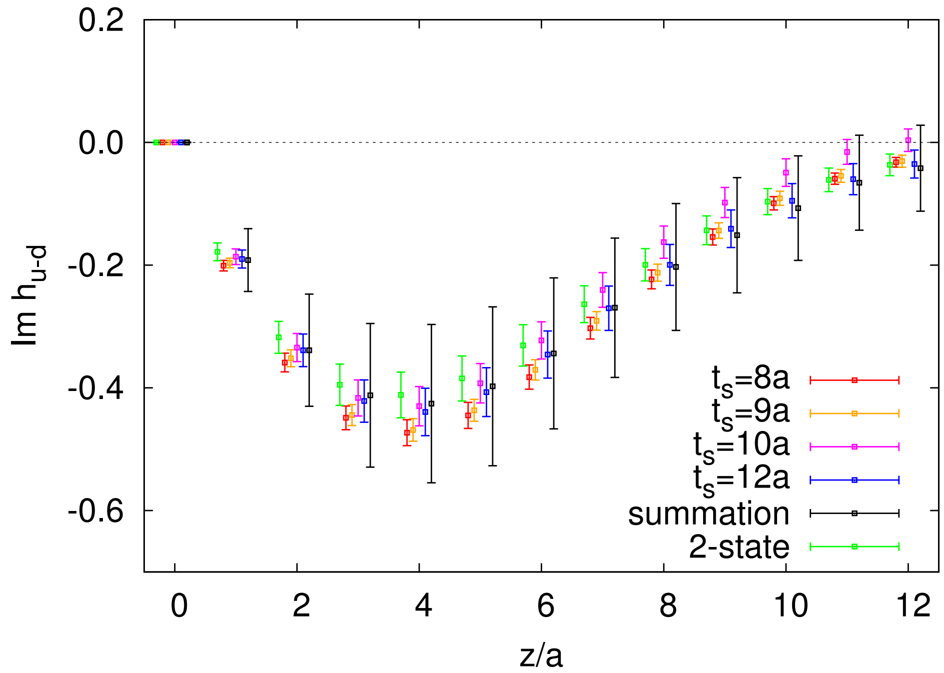

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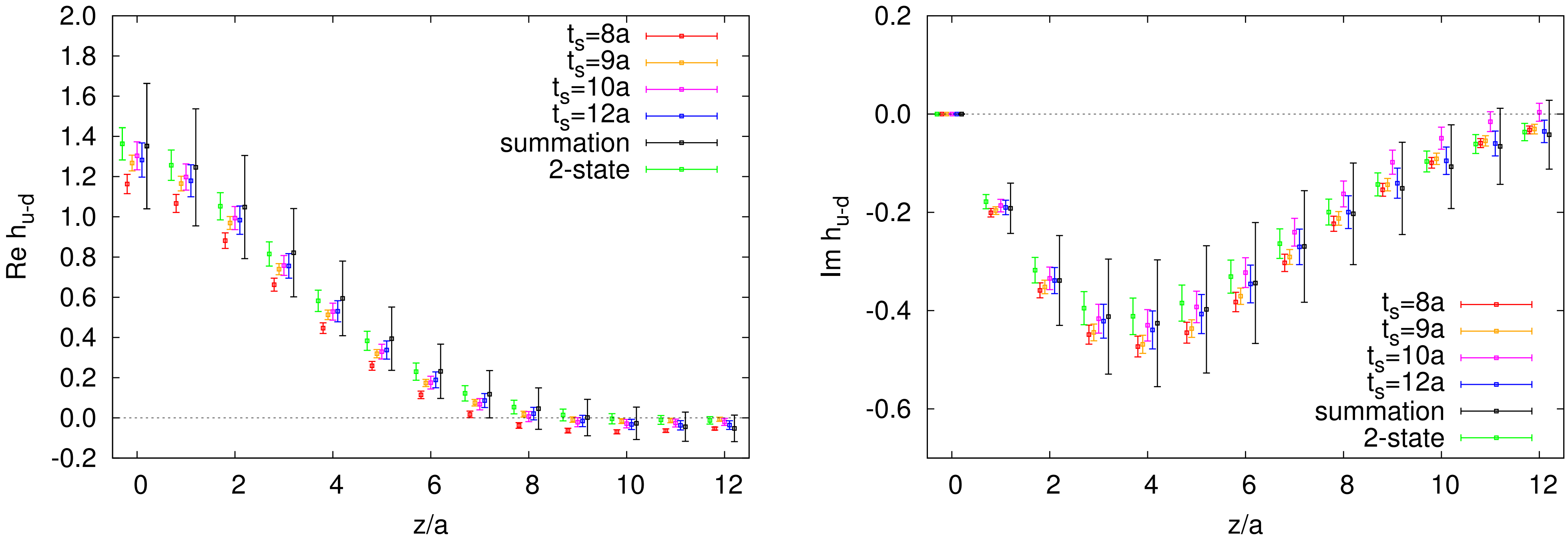

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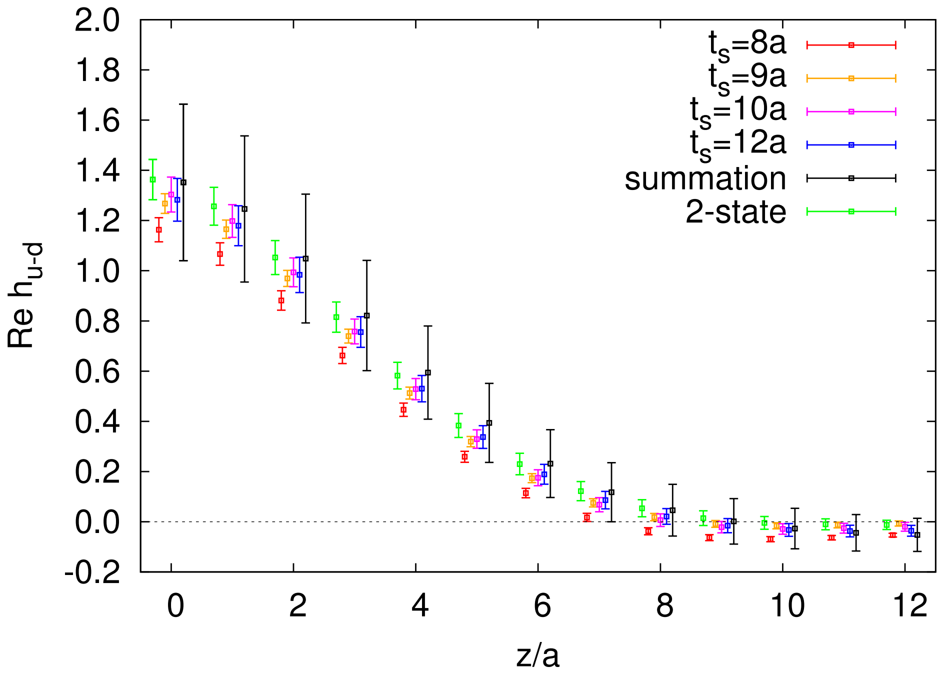

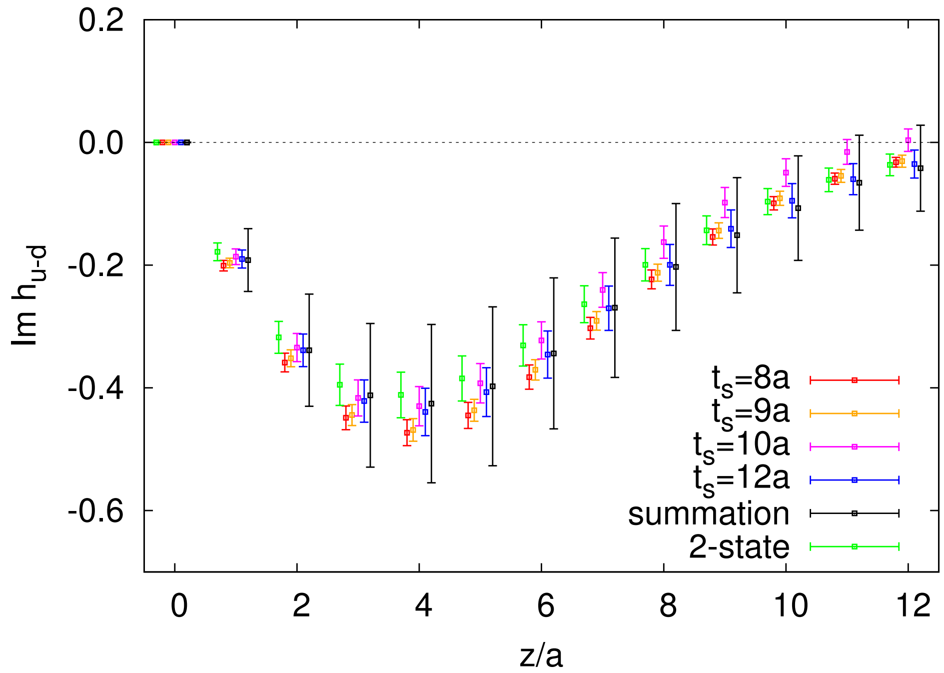

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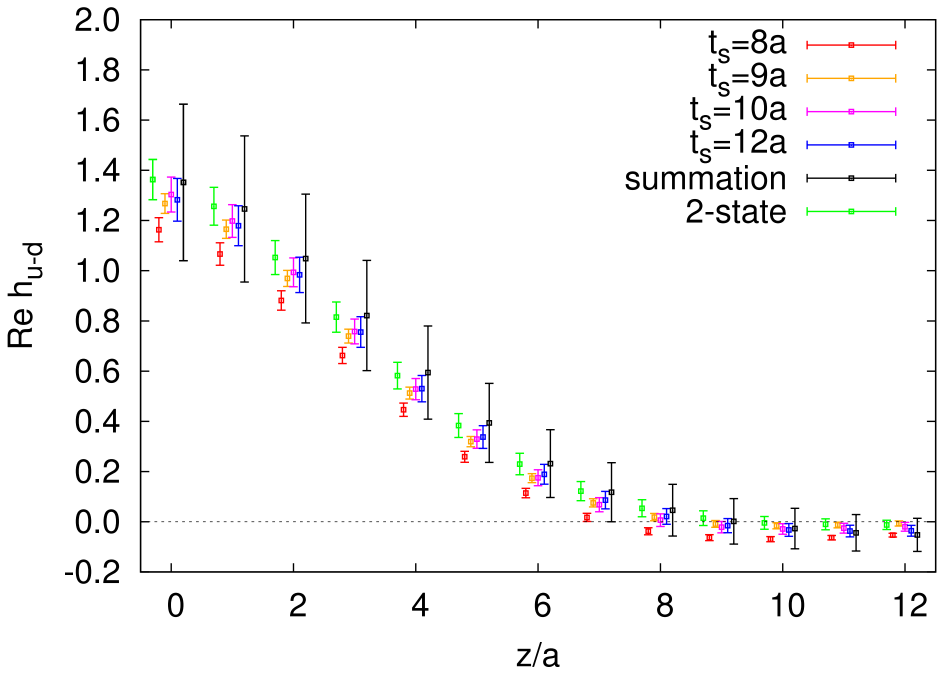

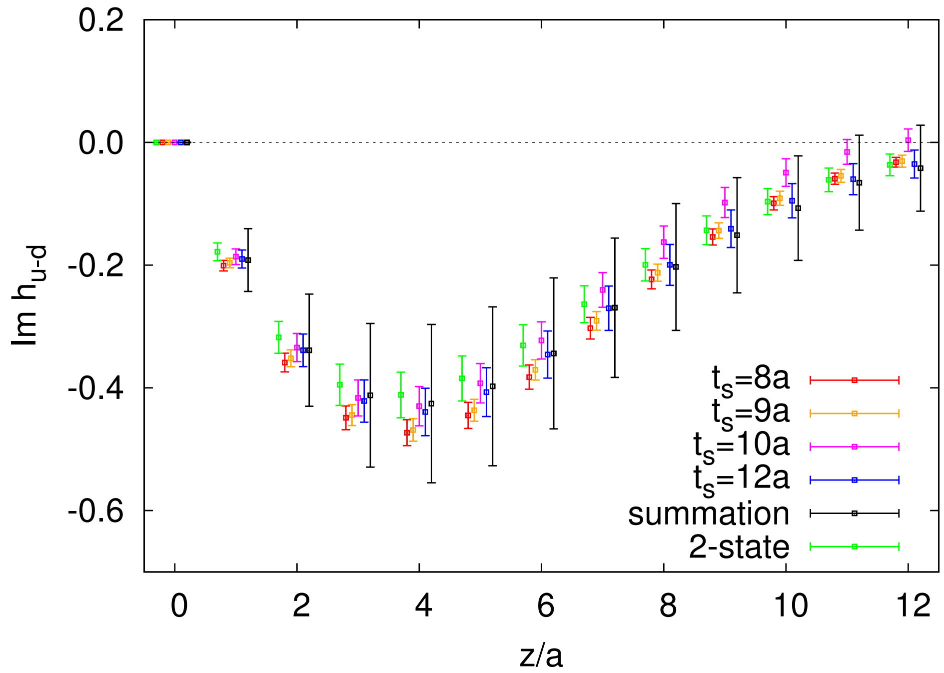

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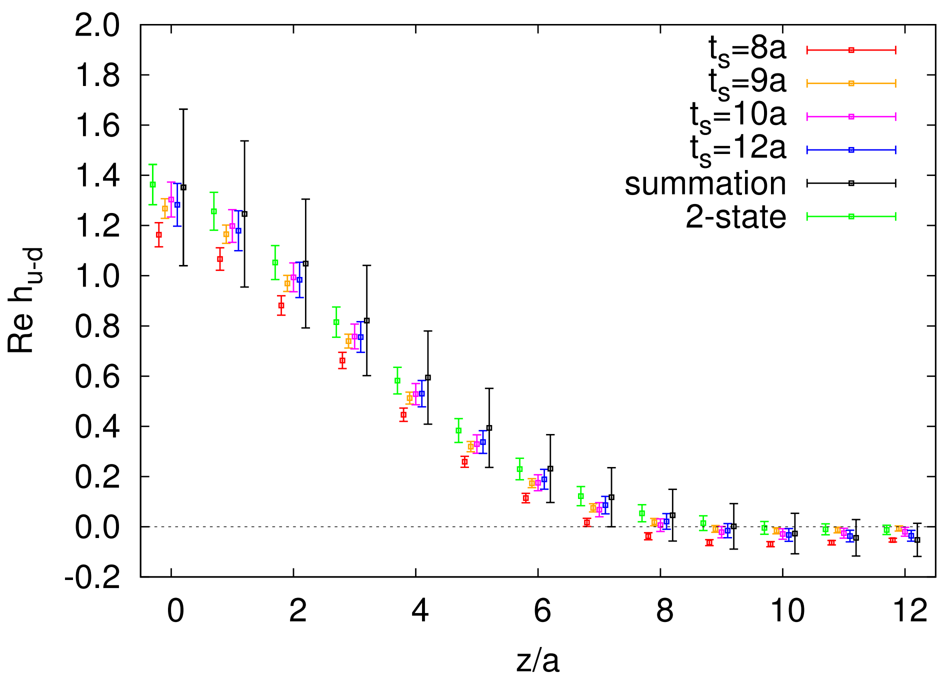

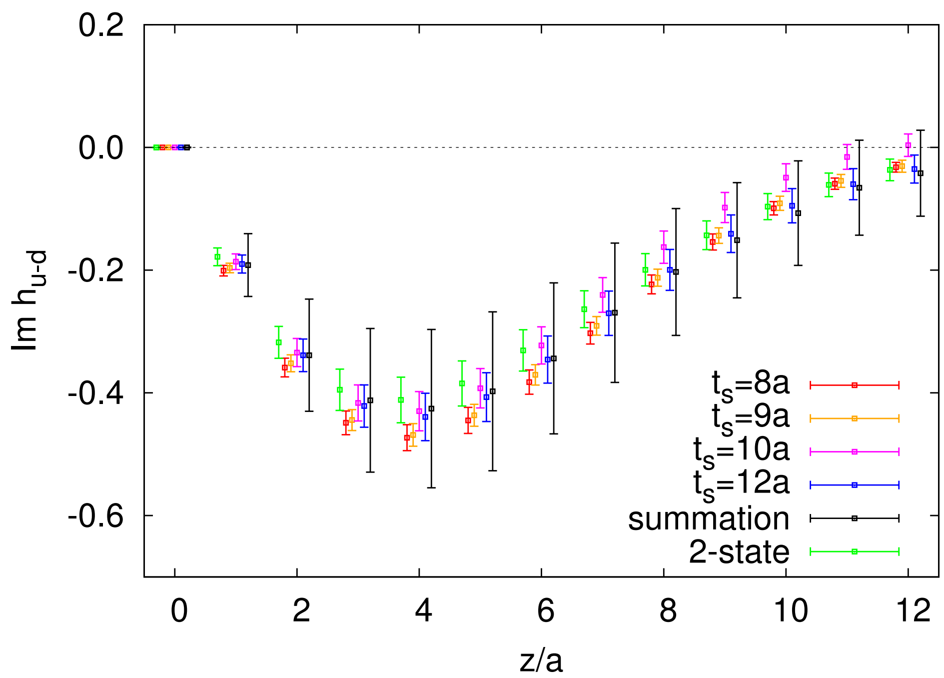

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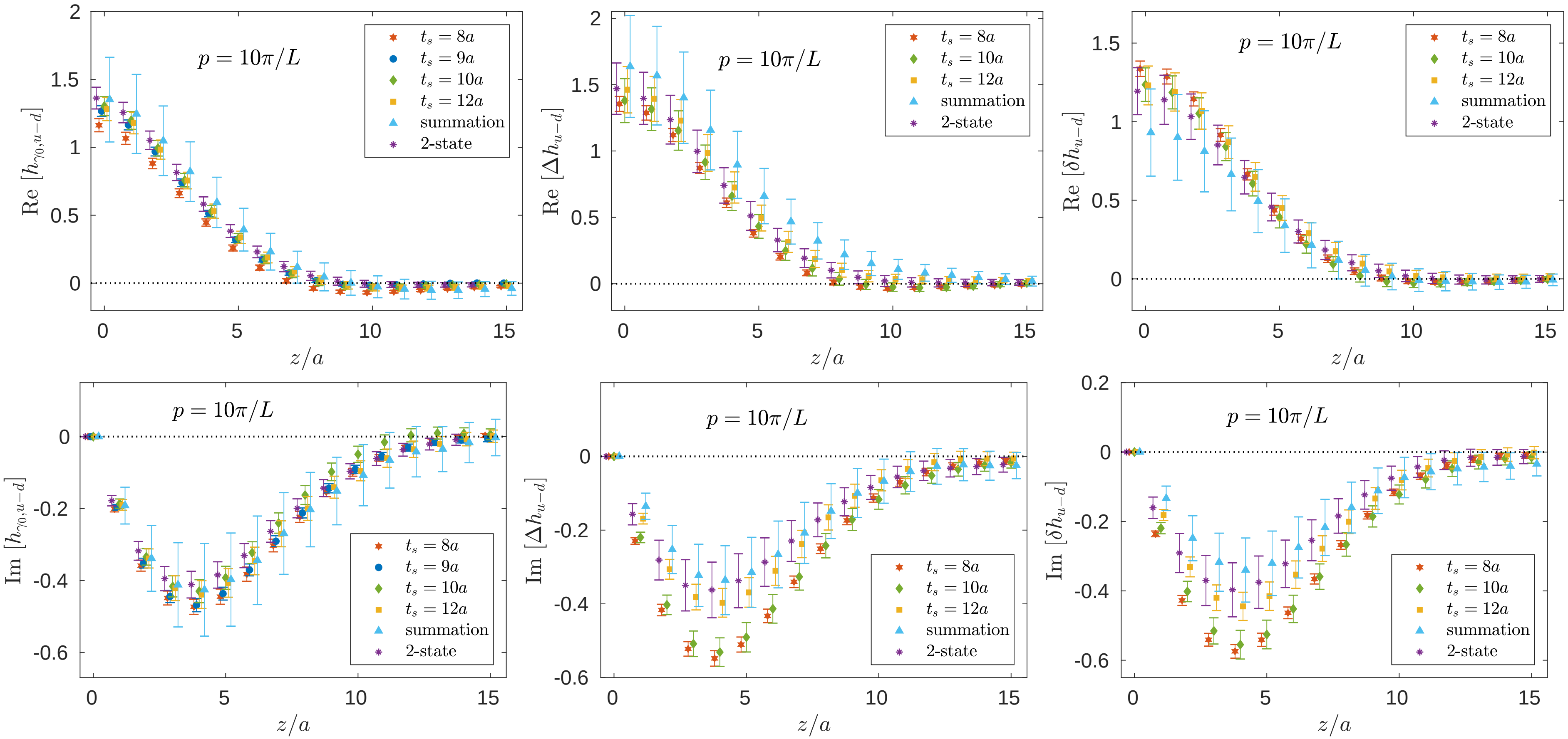

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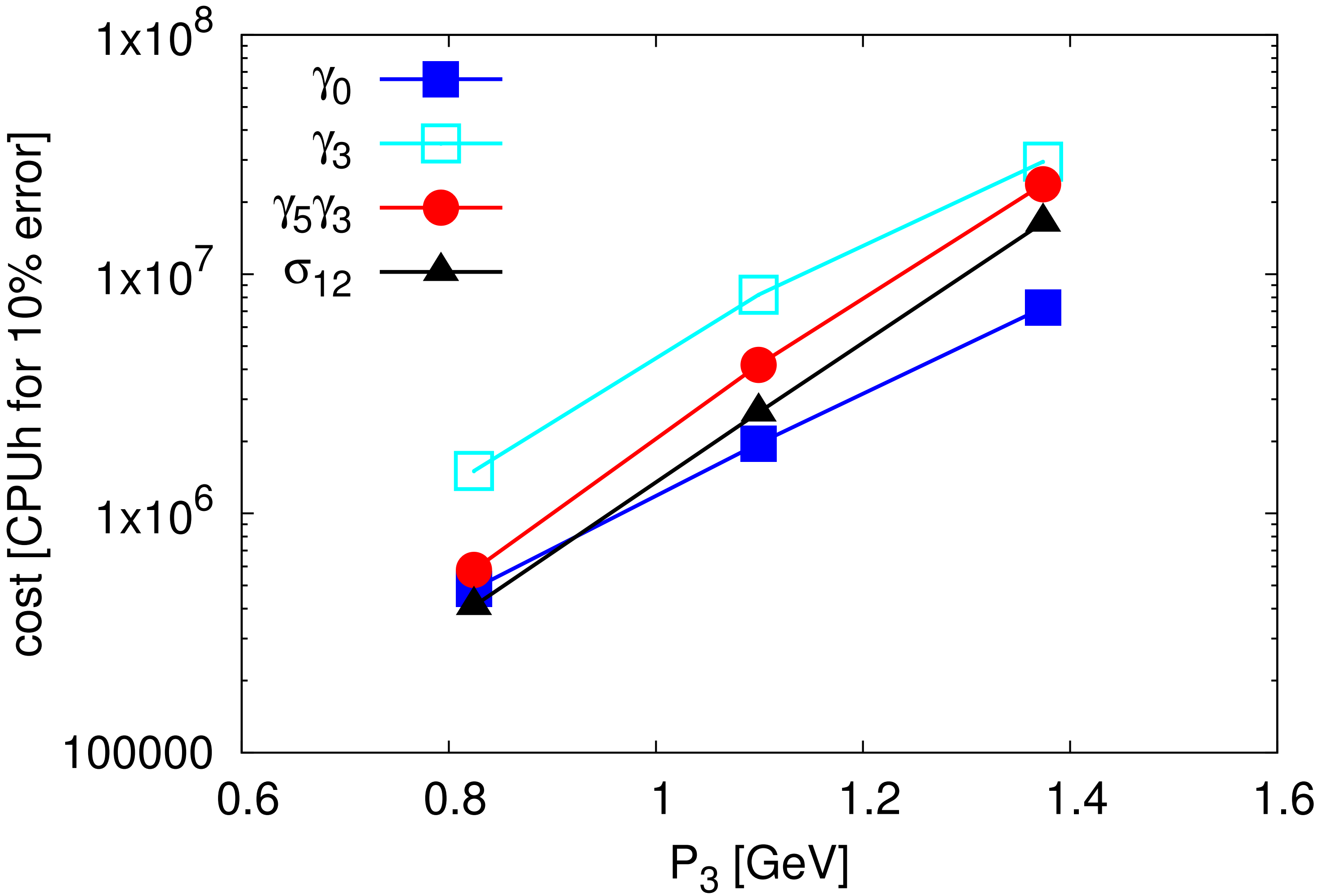
Cost of the computation

Source-sink separation $T_s = 12a \approx 1.13$ fm



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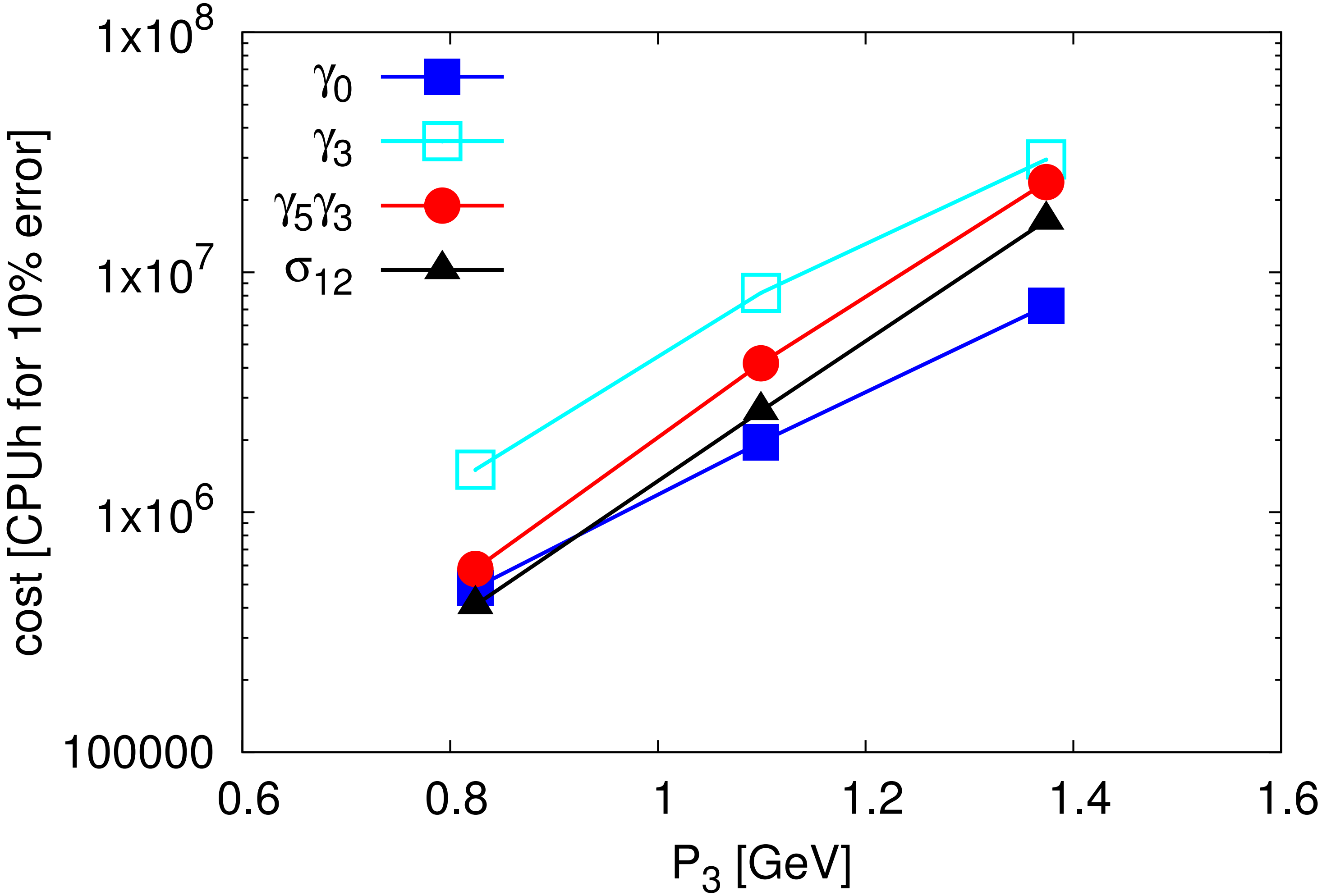

Reaching 1.5 GeV @ $T_s \approx 1.1$ fm needs already $\mathcal{O}(20)$ million CPUh

Cost extrapolation to 3 GeV

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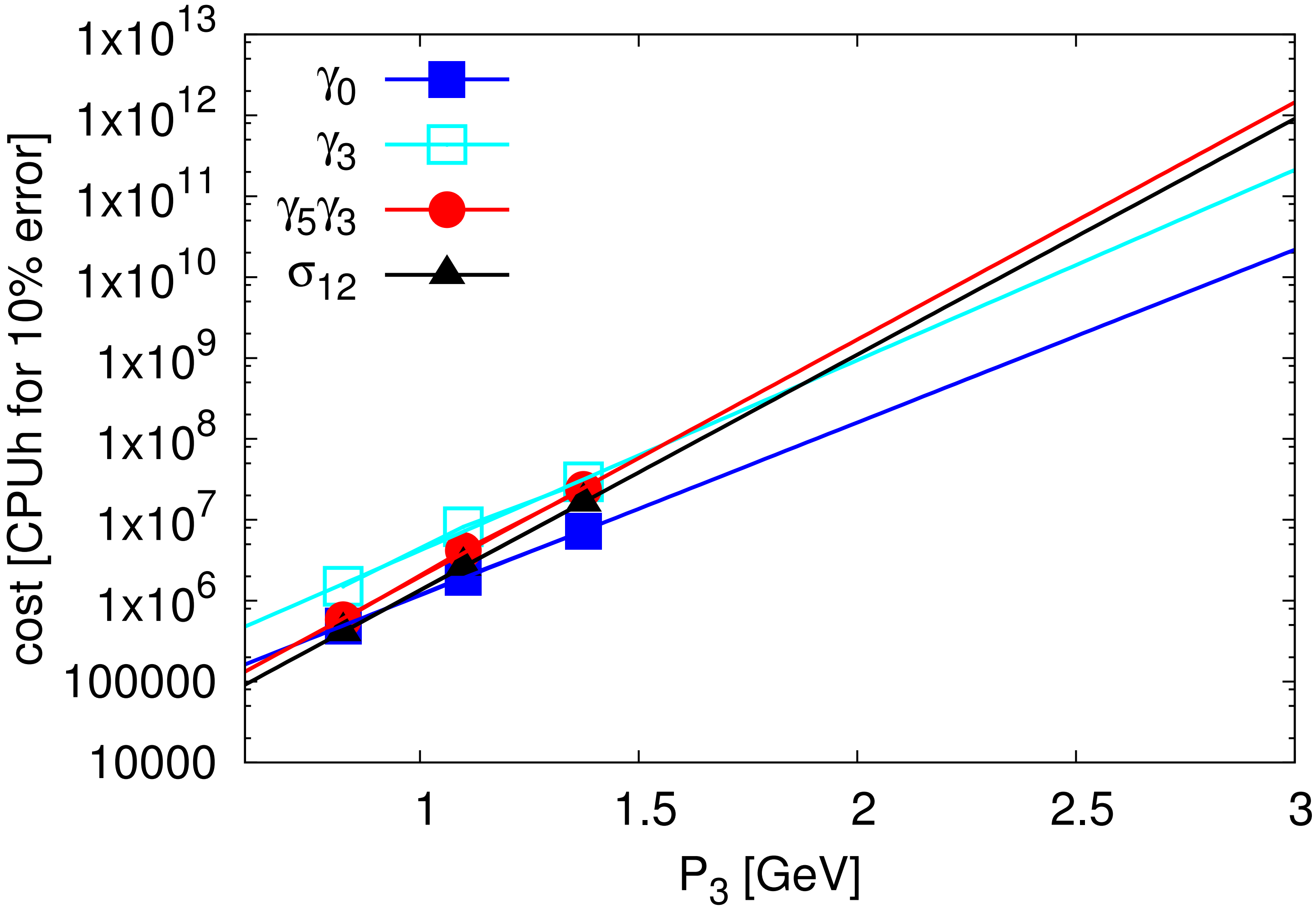

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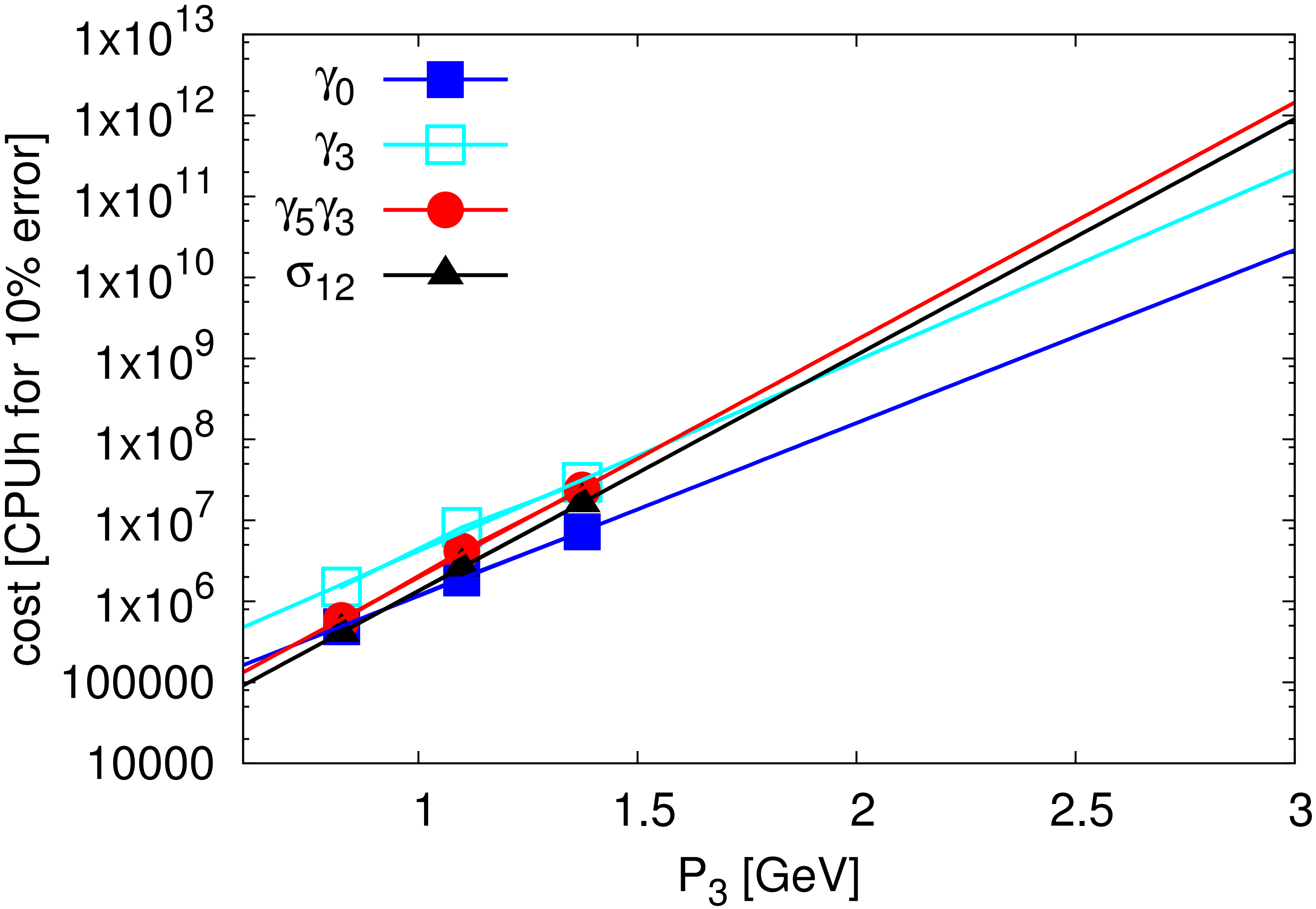
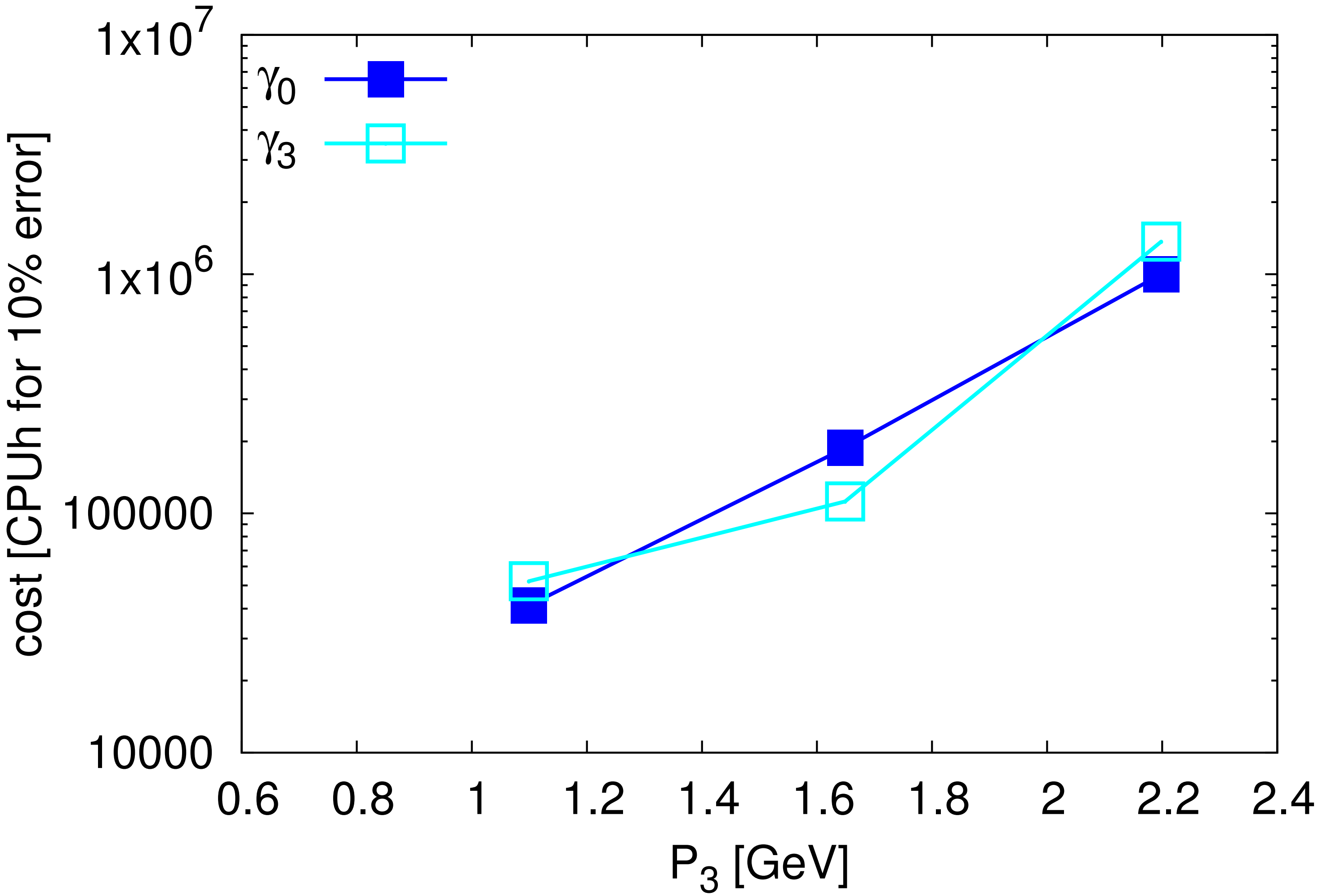


Going to 3 GeV @ $T_s \approx 1.1$ fm needs already
TENS/HUNDREDS THOUSAND million CPUh

Source-sink separation $T_s = 8a \approx 0.75$ fm



Cost of the computation – lower T_s

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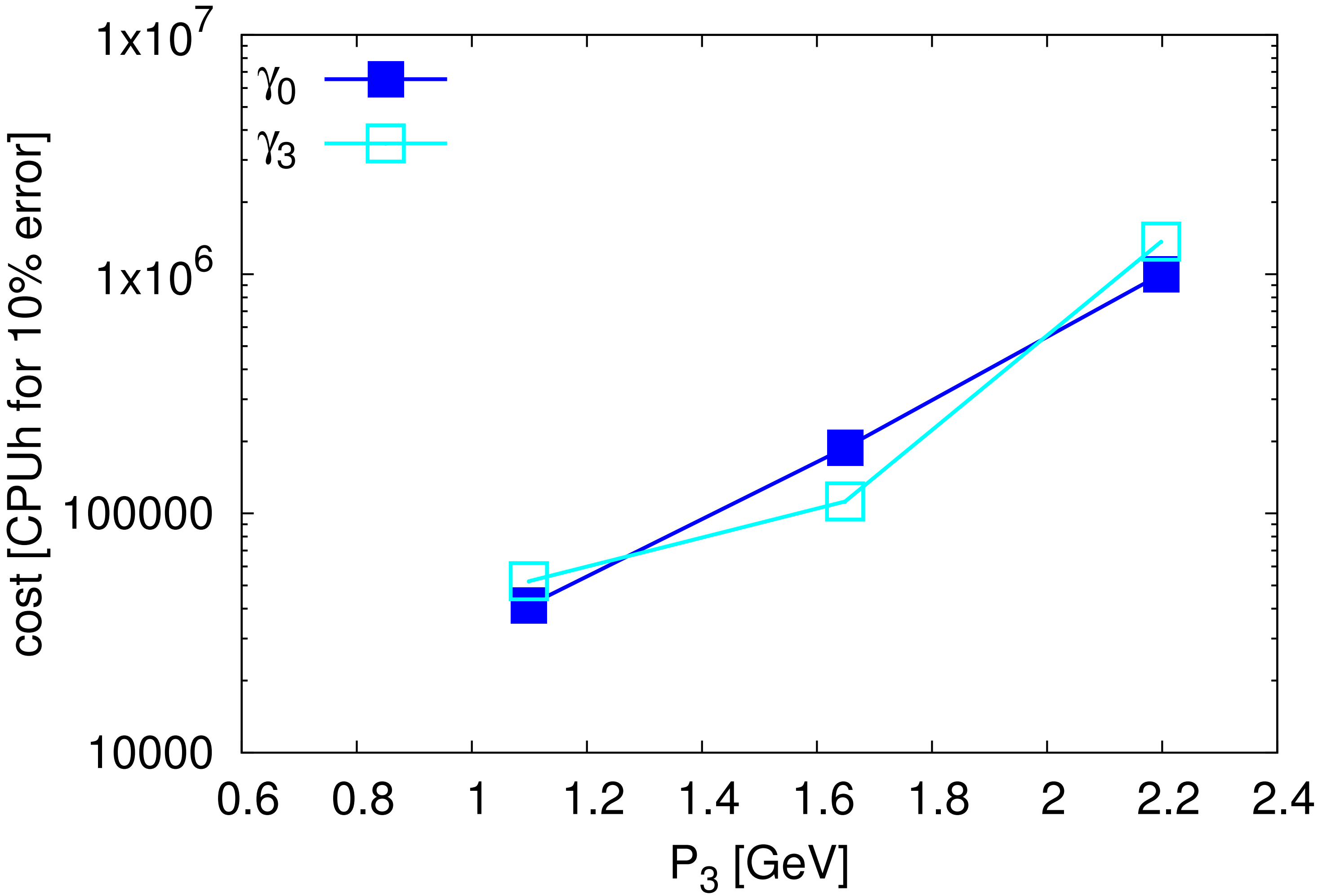

Reaching 2.2 GeV @ $T_s \approx 0.75$ fm pretty cheap – $\mathcal{O}(1)$ million CPUh

Cost extrapolation to 3 GeV – lower T_s

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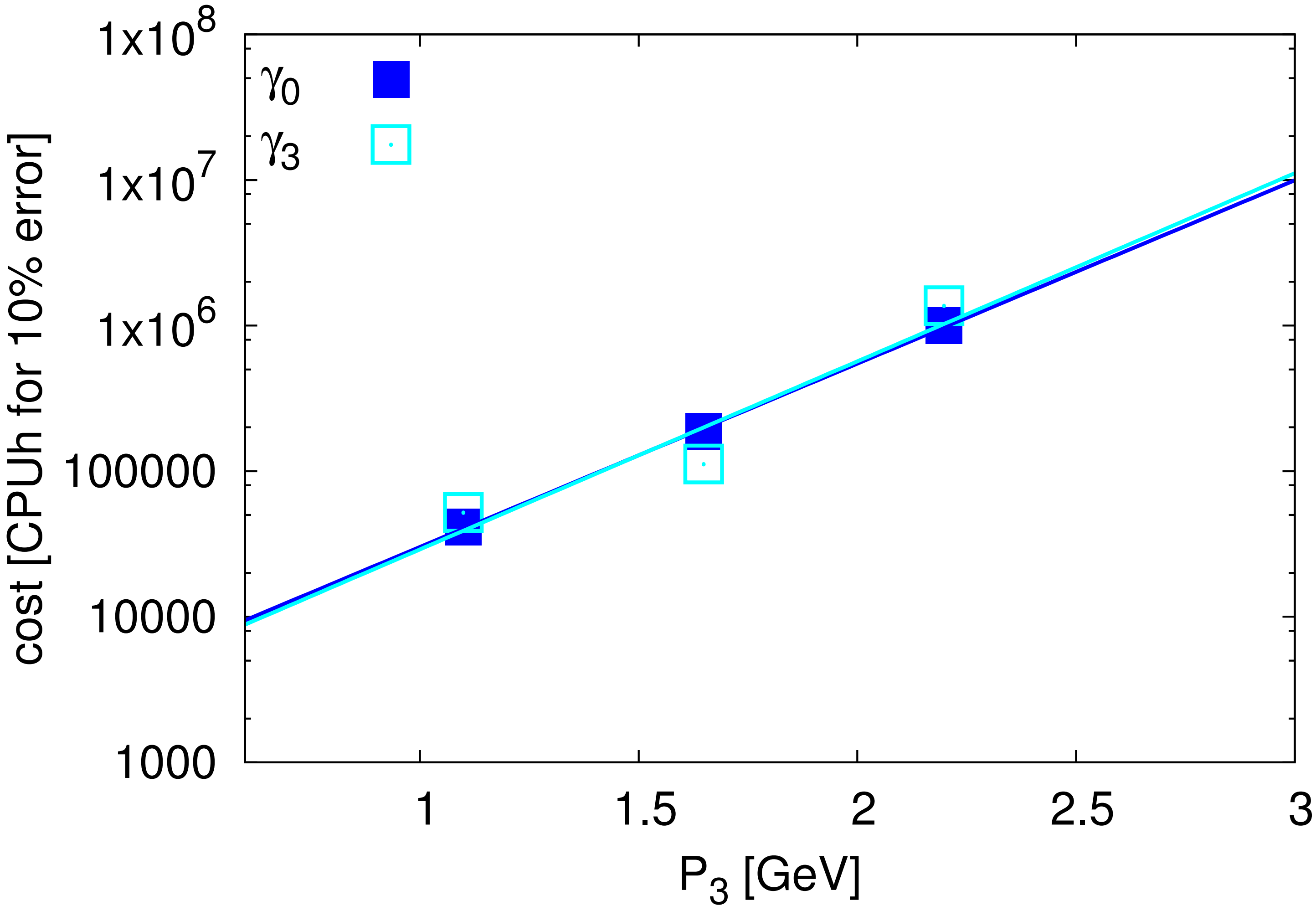

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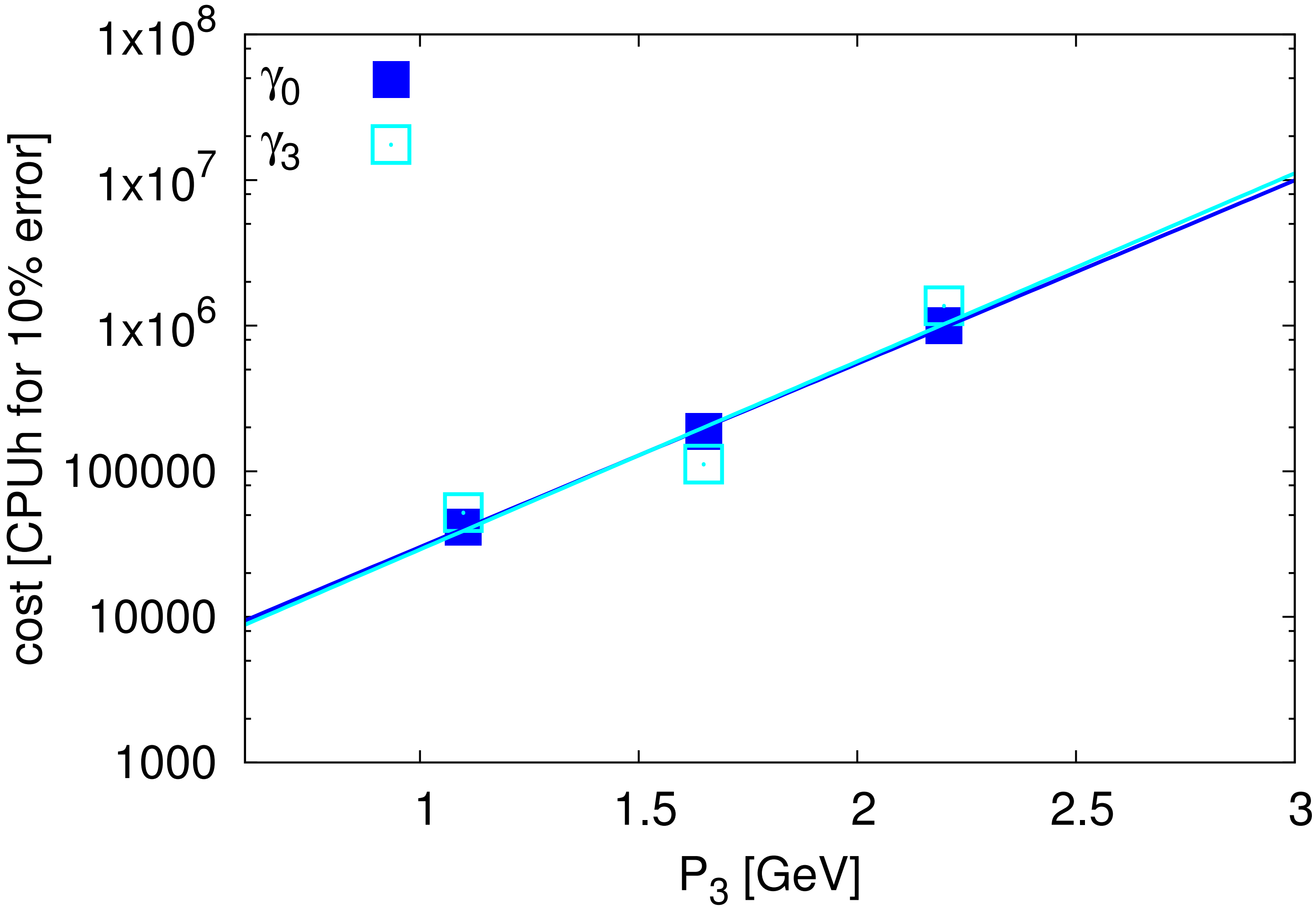

Going to 3 GeV @ $T_s \approx 0.75$ fm feasible – $\mathcal{O}(10)$ million CPUh.

BUT: definitely too large excited states contamination

Conclusion from this

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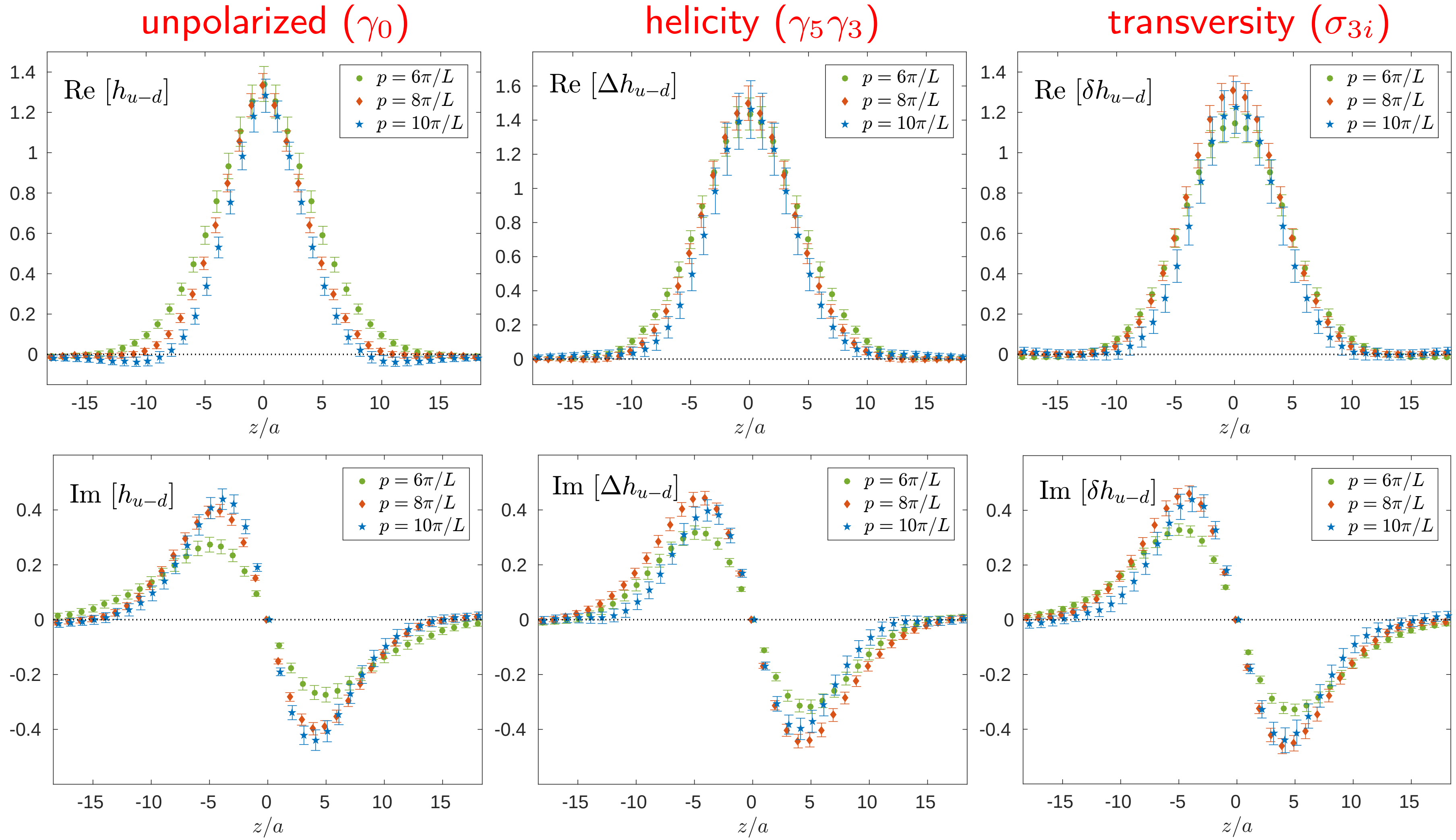
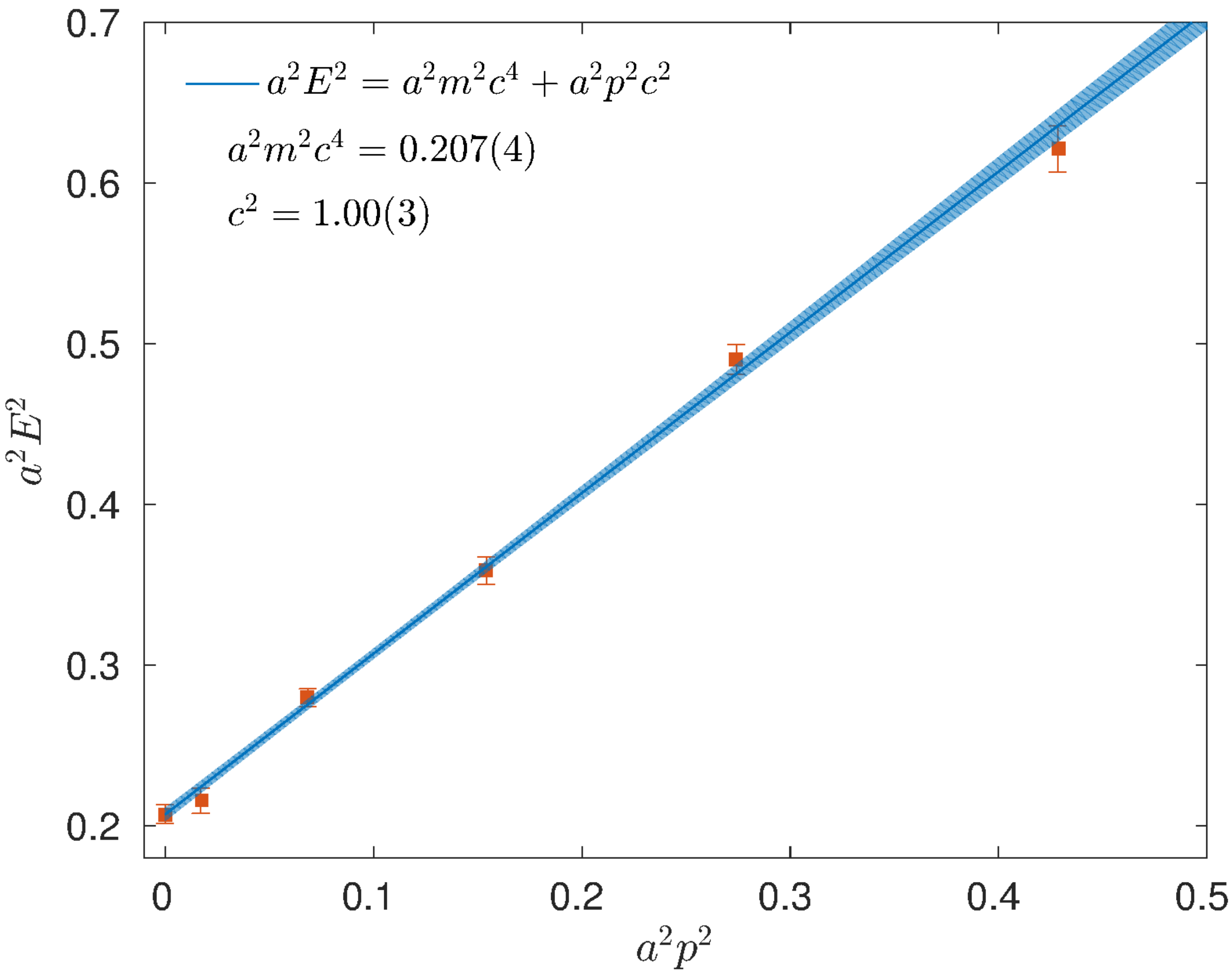
- Key aspect for the future: how to tackle the signal-to-noise problem at safe source-sink separations.

Bare matrix elements at $t_s = 12a$



STATISTICS:

unpolarized	helicity	transversity
$P_3 = \frac{6\pi}{L}$ – 4800 meas.	$P_3 = \frac{6\pi}{L}$ – 6240 meas.	$P_3 = \frac{6\pi}{L}$ – 9600 meas.
$P_3 = \frac{8\pi}{L}$ – 38250 meas.	$P_3 = \frac{8\pi}{L}$ – 38250 meas.	$P_3 = \frac{8\pi}{L}$ – 38250 meas.
$P_3 = \frac{10\pi}{L}$ – 72990 meas.	$P_3 = \frac{10\pi}{L}$ – 72990 meas.	$P_3 = \frac{10\pi}{L}$ – 72990 meas.



$a^2E^2 = a^2m^2c^4 + a^2p^2c^2$
$a^2m^2c^4 = 0.207(4)$
$c^2 = 1.00(3)$
0.7
0.6
0.5
0.4
0.3
0.2
a^2E^2
0
0.1
0.2
0.3
0.4
0.5
a^2p^2

Steps 2-4

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$
2. Compute vertex functions and the resulting renormalization functions in the intermediate RI'-MOM scheme $Z^{\mathrm{RI}'}(z,\mu)$.
3. Convert the renormalization functions to the $\overline{\mathrm{MS}}$ scheme and evolve to $\bar{\mu}=2$ GeV.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\overline{\mathrm{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}(x,\mu^2,P_3)=\int\frac{dz}{4\pi}\,e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle.$$
6. Relate quasi-PDFs to light-cone PDFs via a matching procedure.
7. Apply target mass corrections to eliminate residual m_N/P_3 effects.

Stout smearing

- The power divergence related to the Wilson line makes the values of Z-factors very large at large lengths.

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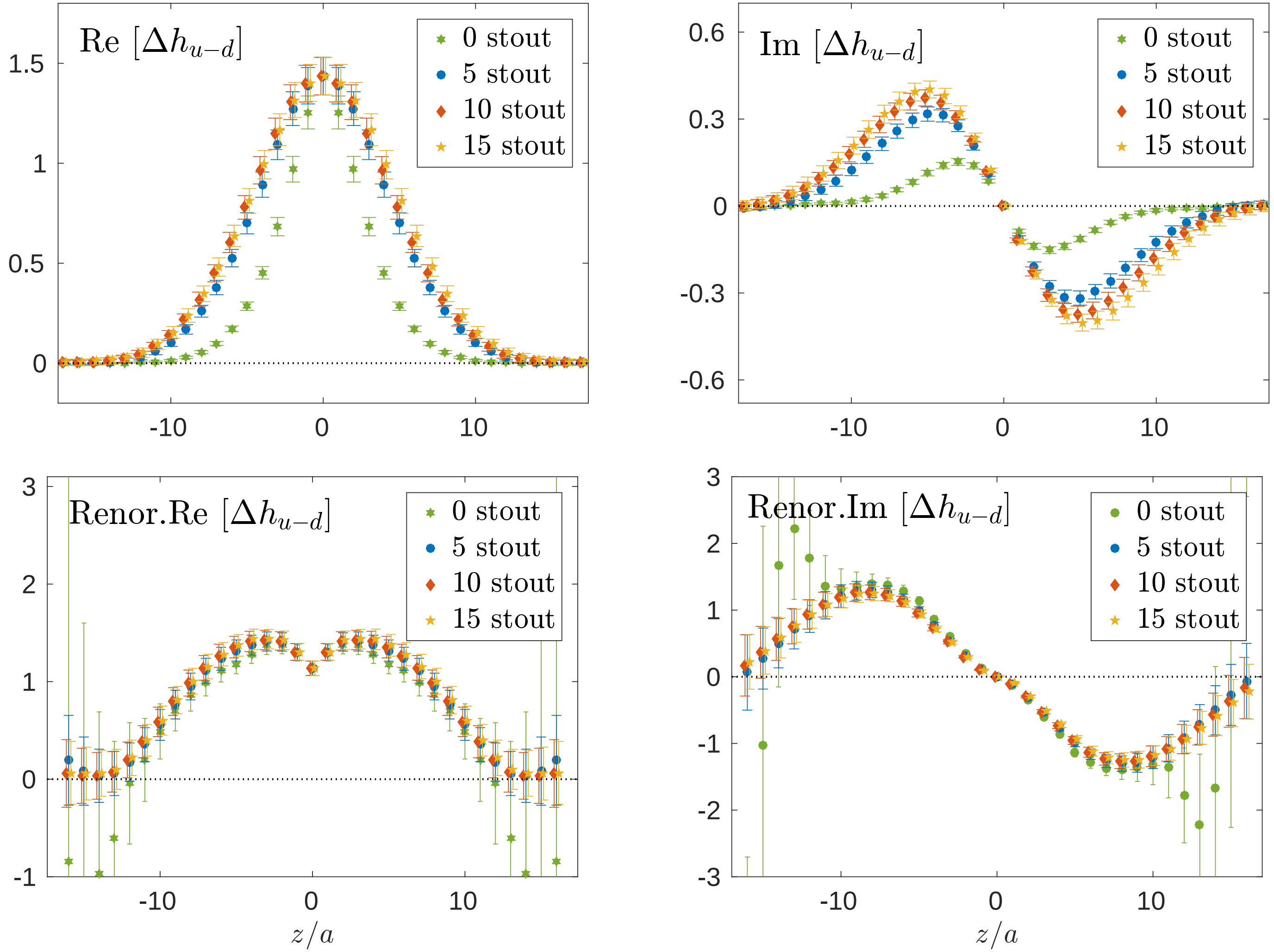
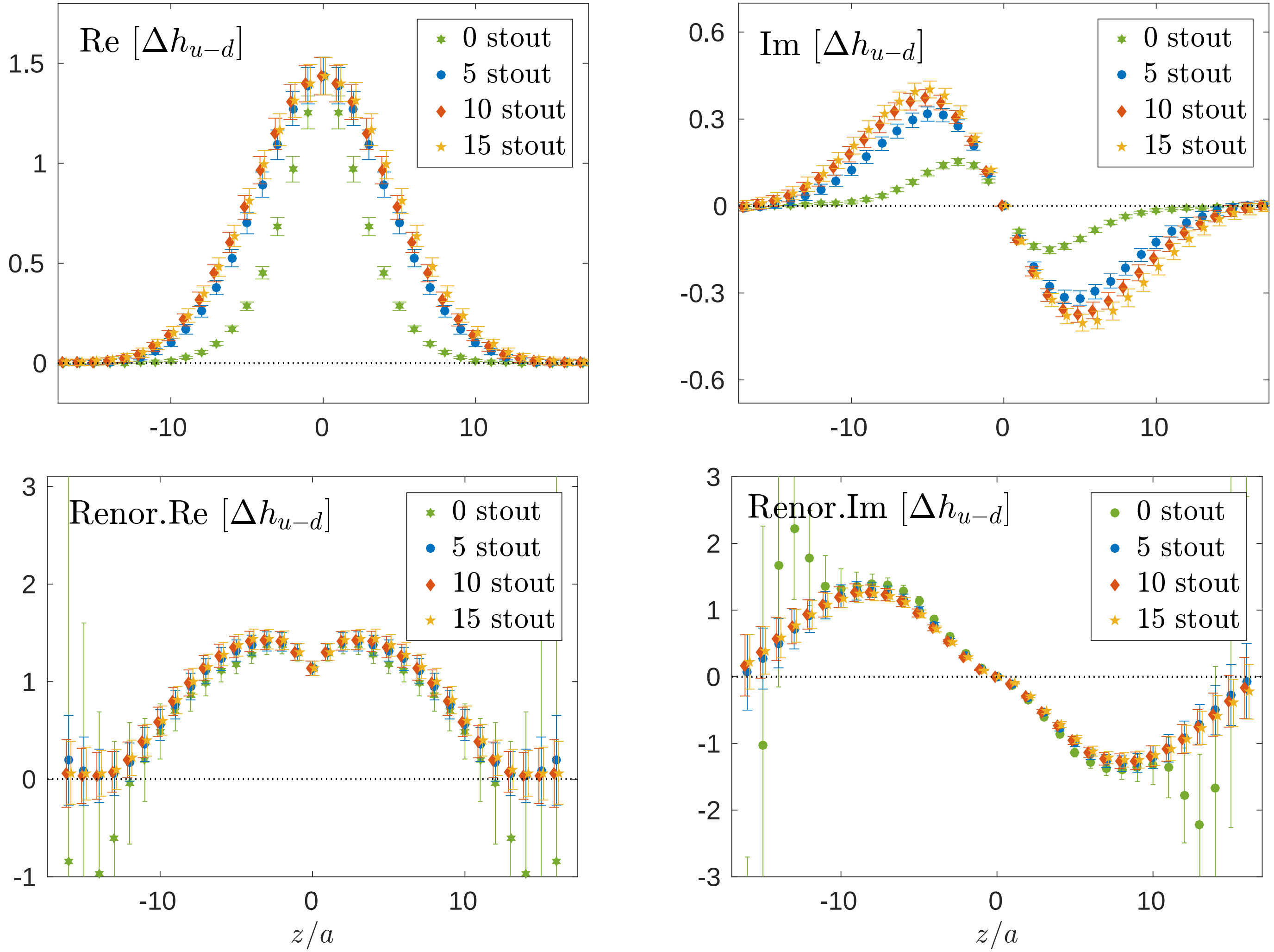
- **But:** renormalized matrix elements should be **independent** of the number of stout steps!
- Note: we do not apply it to the Dirac operator – potentially dangerous procedure and difficult to check.

Nucleon momentum $\frac{6\pi}{48}$



Renormalized matrix elements for helicity PDFs

Nucleon momentum $\frac{6\pi}{48}$



Important self-consistency check for the renormalization procedure!

Steps 5-6

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

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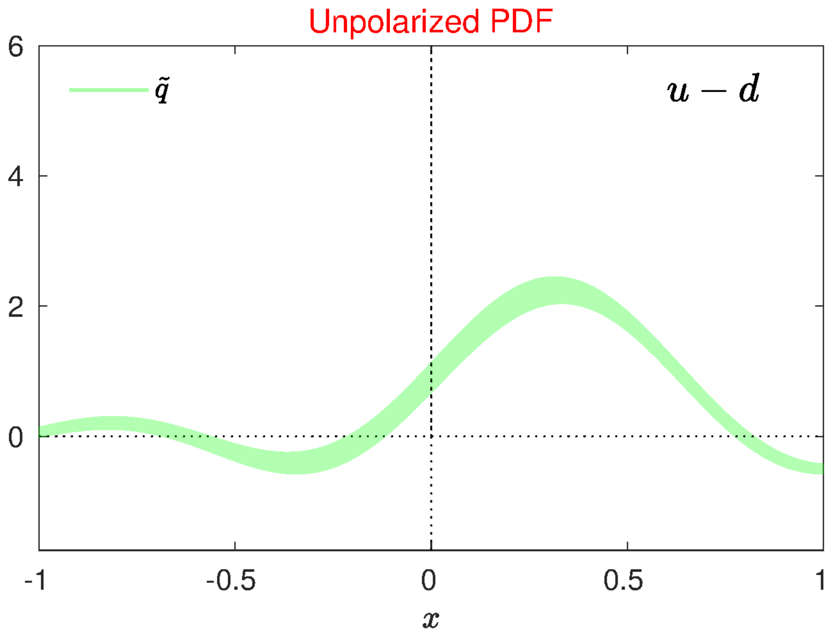
Nucleon momentum $\frac{10\pi}{48}$



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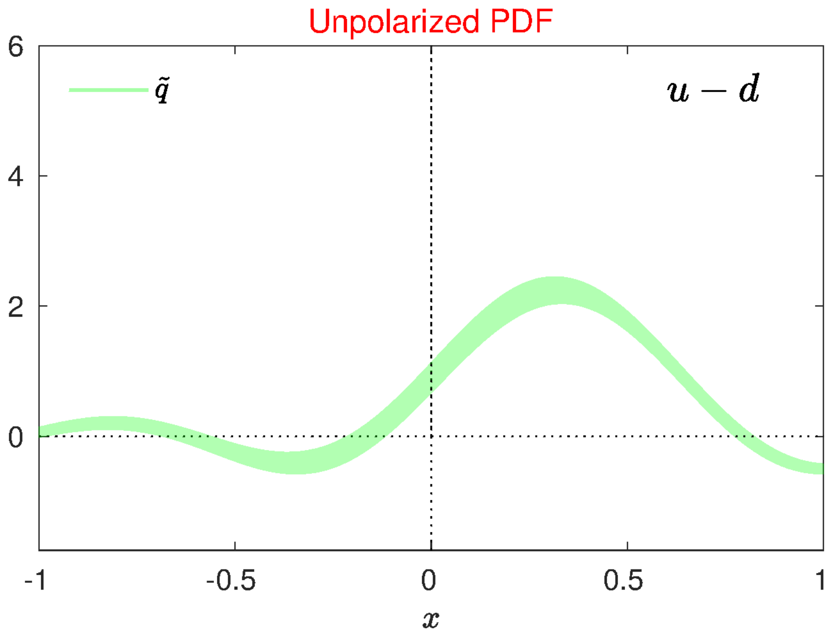

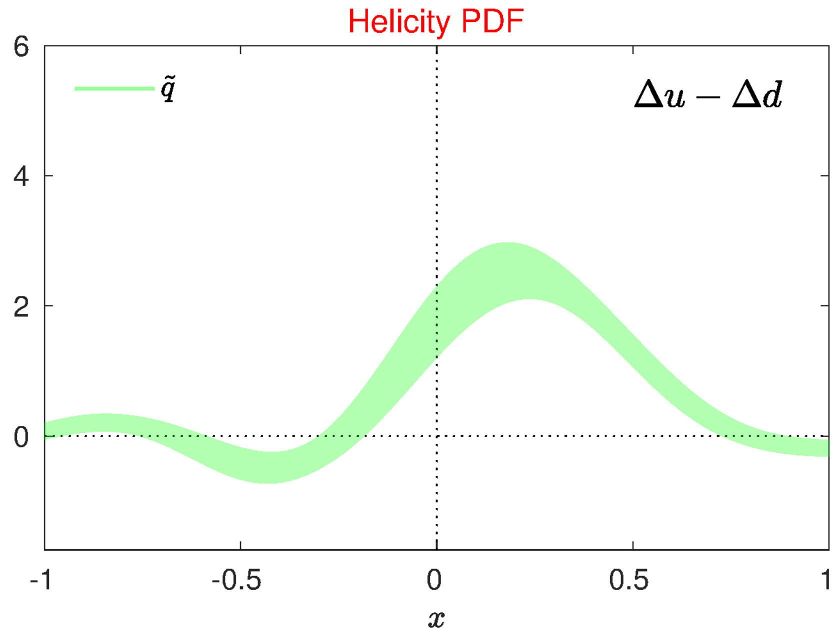

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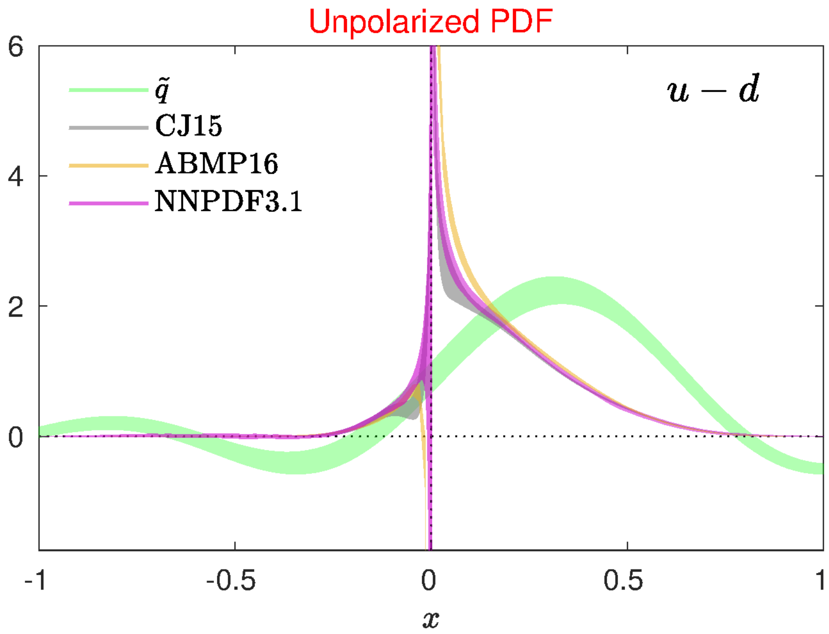

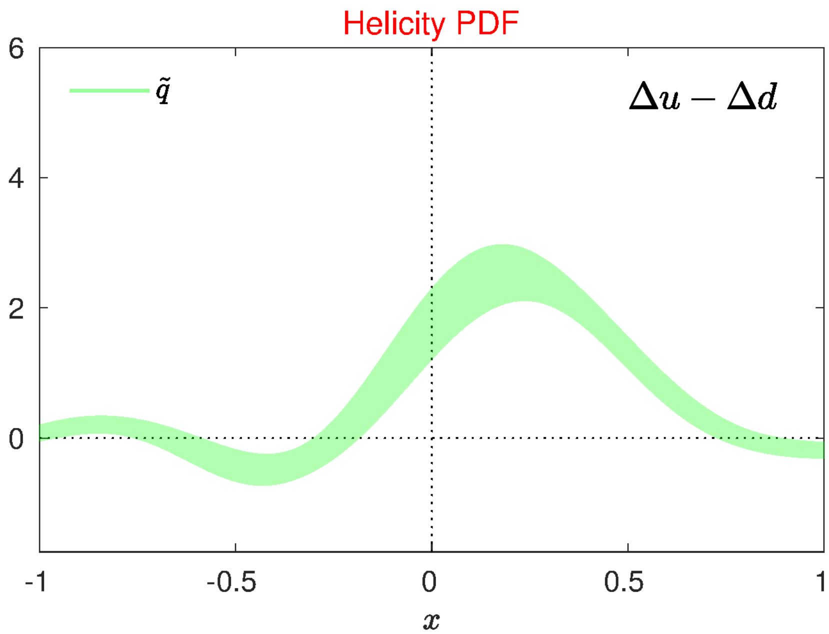

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Quasi-PDFs + pheno

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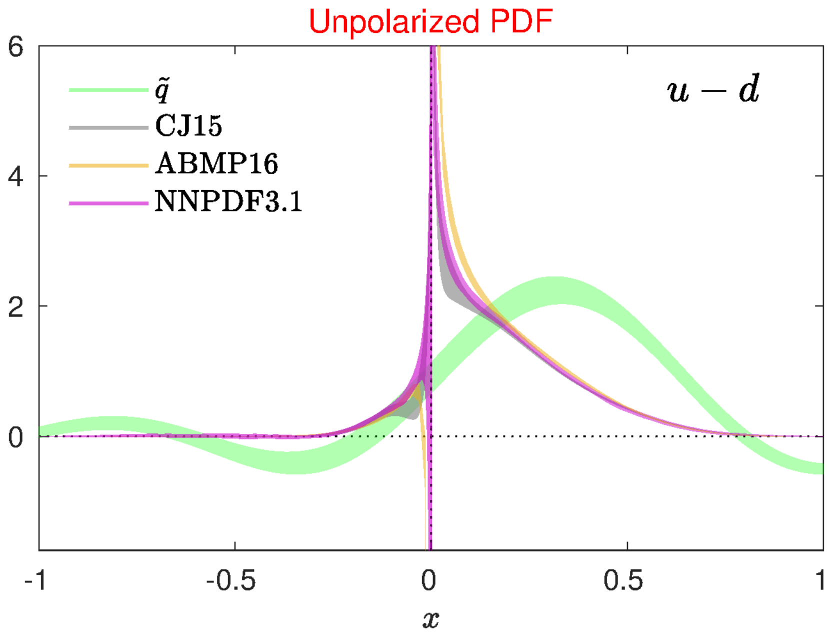

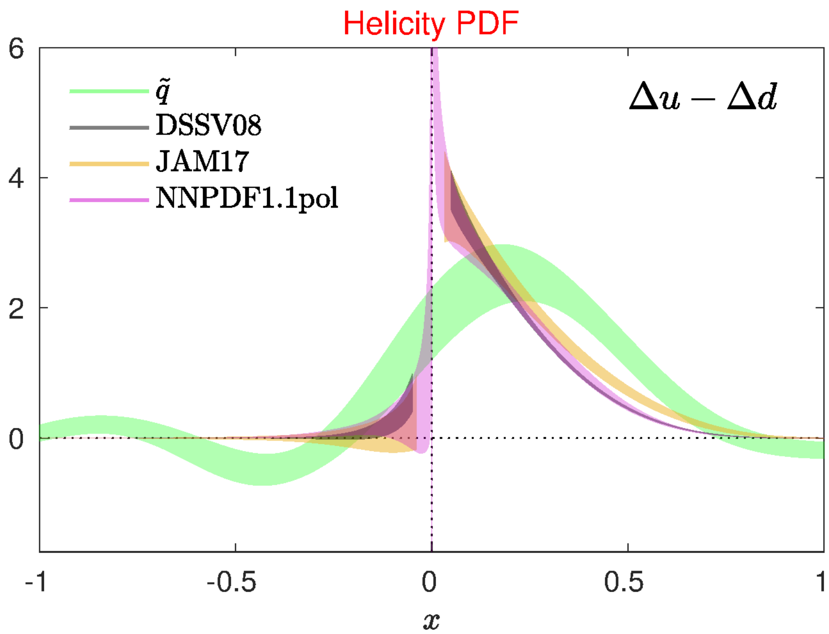

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Matching to light-front PDFs (unpolarized, helicity)

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|}\, C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C – matching kernel: [C. Alexandrou et al., arXiv:1803.02685 [hep-lat]]

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ι=0 for γ_0 and ι=1 for $\gamma_3/\gamma_5\gamma_3$.

Plus prescription at ξ=1:

$$\int \frac{d\xi}{|\xi|} \left[C\left(\xi, \frac{\xi\mu}{xP_3}\right)\right]_+ \tilde{q}\left(\frac{x}{\xi}\right) = \int \frac{d\xi}{|\xi|} C\left(\xi, \frac{\xi\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}\right) - \tilde{q}(x) \int d\xi\, C\left(\xi, \frac{\mu}{xP_3}\right).$$

Alternative matching: [T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]

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$$+ \frac{\alpha_s C_F}{2\pi} \delta(1-\xi) \left(\frac{3}{2} \ln \frac{\mu^2}{4y^2 P_3^2} + \frac{5}{2}\right)$$

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violates particle number conservation:

$$\int_{-\infty}^{\infty} dx\, q(x,\mu) \neq \int_{-\infty}^{\infty} dx\, \tilde{q}(x,\mu,P_3) \qquad \text{and} \qquad \int_{-\infty}^{\infty} d\xi\, C(\xi, \xi\mu/xP_3) \neq 1,$$

which **increases** with growing P_3 (around 8% at $P_3 = 10\pi/48$).

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In our procedure, particle number is **conserved**. This amounts to a modification of the $\overline{\mathrm{MS}}$ scheme; modification **decreases** with growing P_3.

Modification of the $\overline{\mathrm{MS}}$ scheme

We introduce a modified $\overline{\mathrm{MS}}$ scheme ($\mathrm{M}\overline{\mathrm{MS}}$) with an extra subtraction made outside the physical region of the unintegrated vertex corrections.
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In z-space:

$$\begin{aligned} Z^{M\overline{MS}}_{\Gamma_{\gamma^0}}(z\mu) &= 1 - \frac{\alpha_s}{2\pi} C_F \left(\frac{3}{2}\ln\left(\frac{1}{4}\right) + \frac{5}{2}\right) \\ &+ \frac{3}{2}\frac{\alpha_s}{2\pi} C_F \left(i\pi\frac{|z\mu|}{2z\mu} - Ci(z\mu) + \ln(z\mu) - \ln(|z\mu|) - iSi(z\mu)\right) \\ &- \frac{3}{2}\frac{\alpha_s}{2\pi} C_F e^{iz\mu}\left(\frac{2Ei(-iz\mu) - \ln(-iz\mu) + \ln(iz\mu) + i\pi Sign(z\mu)}{2}\right). \end{aligned}$$

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The above has to modify the conversion factor, i.e. the conversion will be RI$\to \overline{\mathrm{MS}} \to \mathrm{M}\overline{\mathrm{MS}}$.

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Consistency check: $z \to 0$ limit:

$$Z^{M\overline{MS}}_{\Gamma_{\gamma^0}}(z\to 0) = 1 - \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\ln\left(\frac{\mu^2 z^2 e^{2\gamma_E}}{4}\right) + \frac{5}{2}\right) = Z^{Ratio}_{\Gamma_{\gamma^0}}(z\mu)$$

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Exactly cancels the divergence in $\ln(z)$ present in $\overline{\mathrm{MS}}$!
(consistency with: M. Constantinou, H. Panagopoulos, Phys. Rev. D96 (2017) 054506
and with the "Ratio" scheme of T. Izubuchi et al., Phys. Rev. D98 (2018) 056004)

Recently we derived the matching formula for transversity PDFs ($\overline{\mathrm{MS}} \longrightarrow \overline{\mathrm{MS}}$):
[C. Alexandrou et al., arXiv:1807.00232 [hep-lat]]

$$\delta C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{2\xi}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{2}{\xi}\right]_+ & \xi > 1, \\ \left[\frac{2\xi}{1-\xi}\left(\ln\frac{x^2P_3^2}{\xi^2\mu^2}(4\xi(1-\xi))\right) - \frac{2\xi}{1-\xi}\right]_+ & 0 < \xi < 1, \\ \left[-\frac{2\xi}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{2}{1-\xi}\right]_+ & \xi < 0, \end{cases}$$

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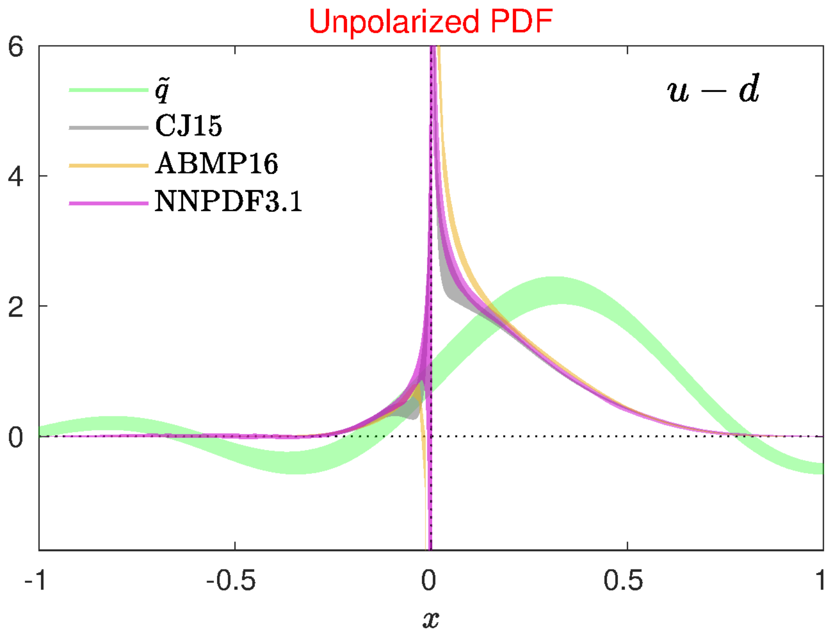
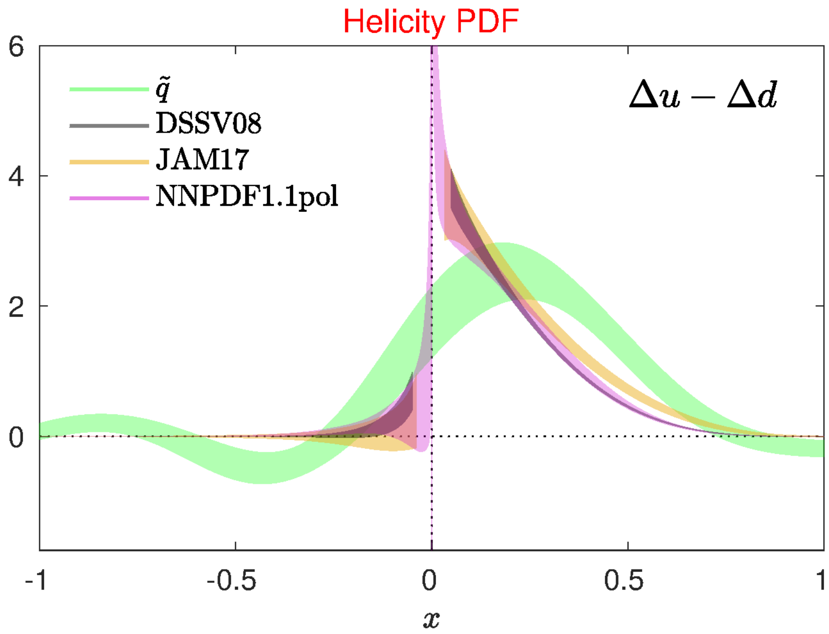
Similar modification of the conversion factor as above, i.e.:

- the conversion is RI$\rightarrow \overline{\mathrm{MS}} \rightarrow \mathrm{M}\overline{\mathrm{MS}}$,
- the matching is $\mathrm{M}\overline{\mathrm{MS}} \rightarrow \overline{\mathrm{MS}}$.

Nucleon momentum $\frac{10\pi}{48}$





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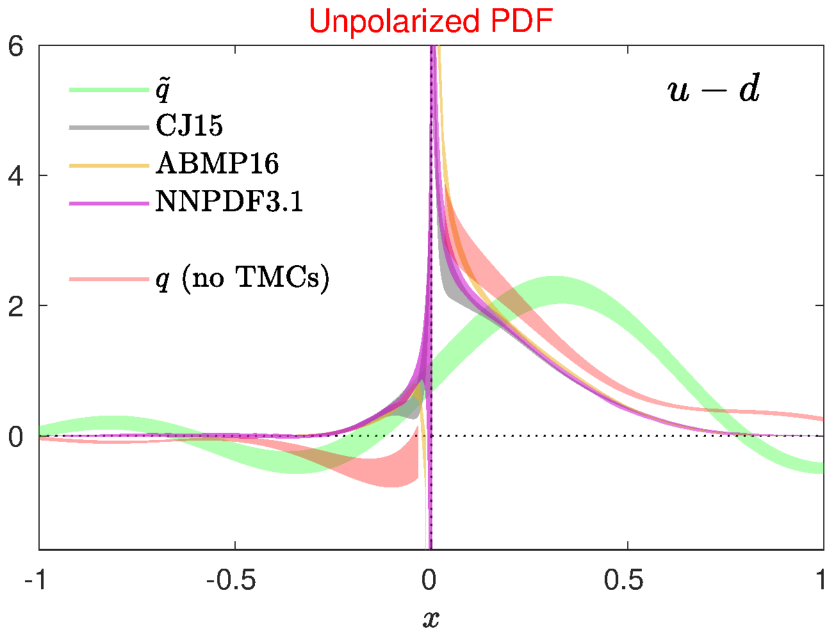

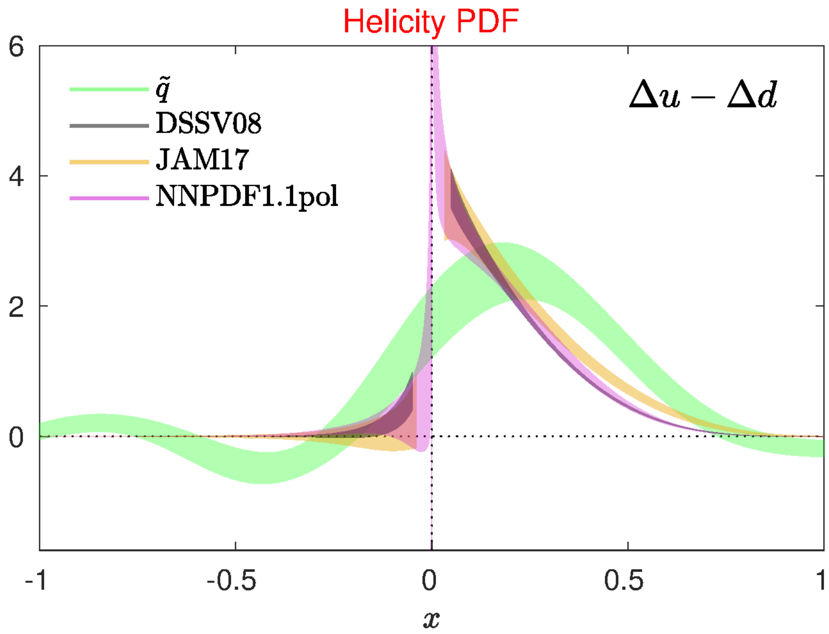

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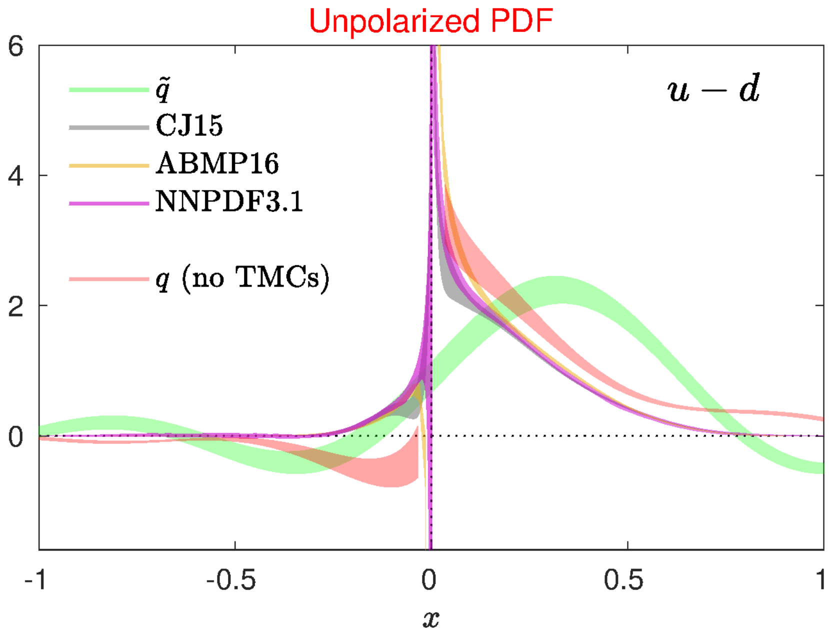

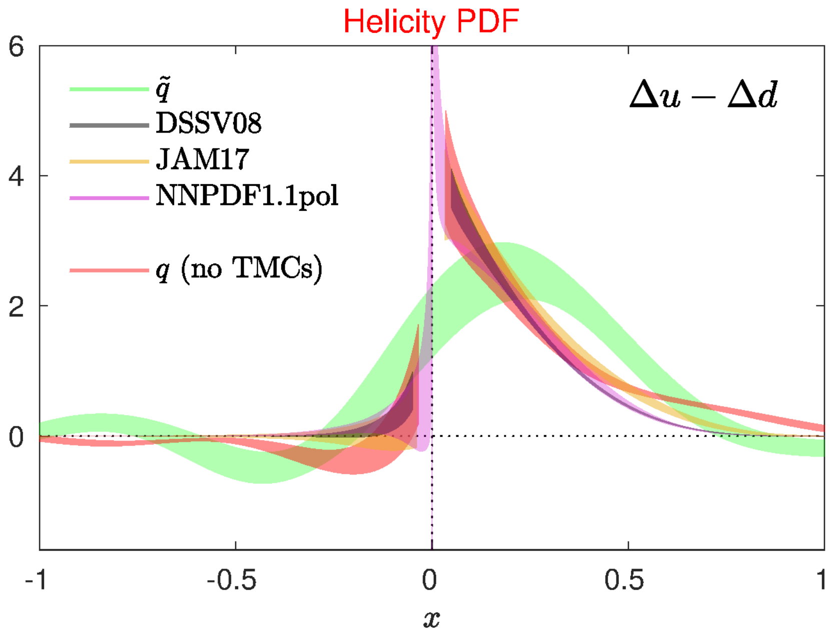

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Step 7

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$
2. Compute vertex functions and the resulting renormalization functions in the intermediate RI'-MOM scheme $Z^{\mathrm{RI}'}(z,\mu)$.
3. Convert the renormalization functions to the $\overline{\mathrm{MS}}$ scheme and evolve to $\bar{\mu}=2$ GeV.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\overline{\mathrm{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}(x,\mu^2,P_3)=\int\frac{dz}{4\pi}\,e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle.$$
6. Relate quasi-PDFs to light-cone PDFs via a matching procedure.
7. Apply target mass corrections to eliminate residual m_N/P_3 effects.

Target mass corrections

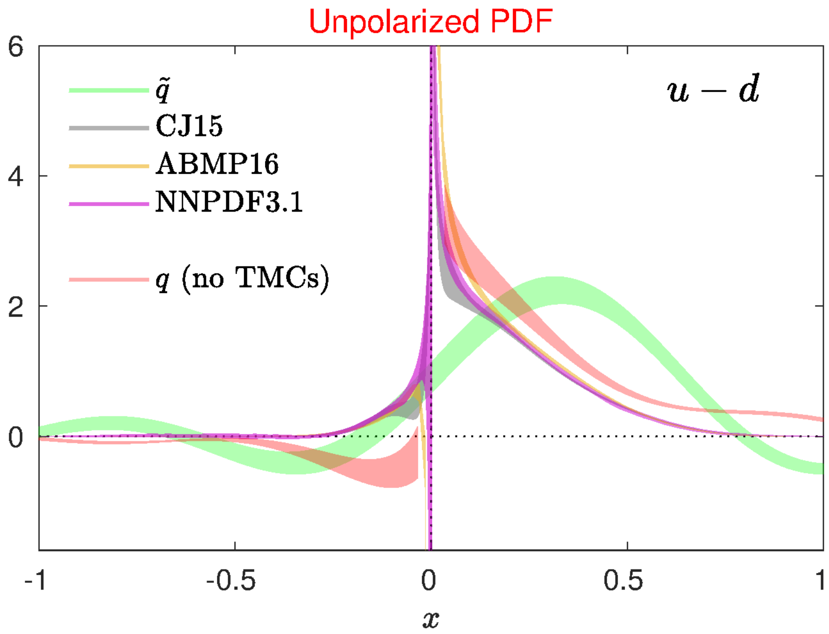
In the infinite momentum frame, nucleon mass does not matter, i.e. $m_N/P_3 = 0$.

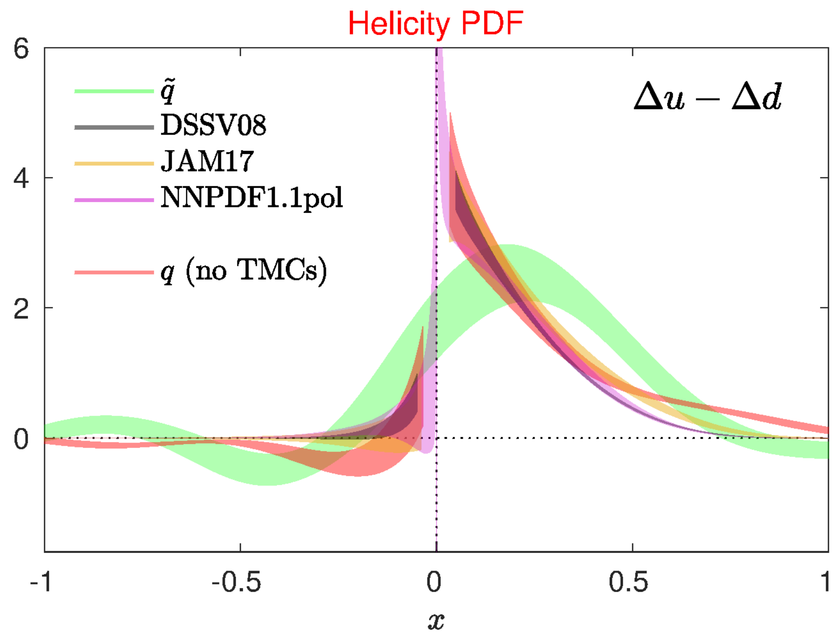
Here, we work with nucleon boosted to finite momentum P_3 and we need to correct for $m_N/P_3 \neq 0$.

We use formulae derived in:
[J.W. Chen et al., Nucl.Phys. B911 (2016) 246-273, arXiv:1603.06664 [hep-ph]]

Important feature: particle number is conserved in nucleon mass corrections.

Nucleon momentum $\frac{10\pi}{48}$





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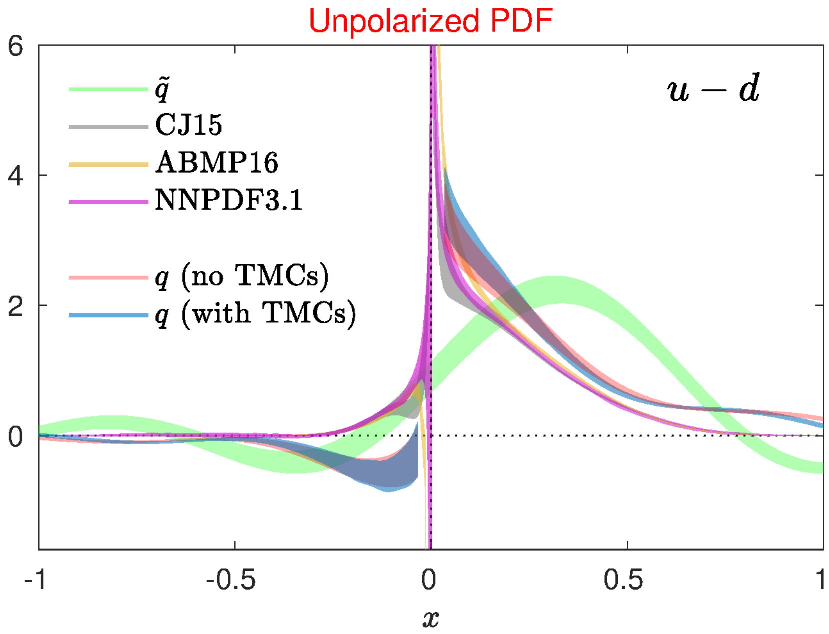

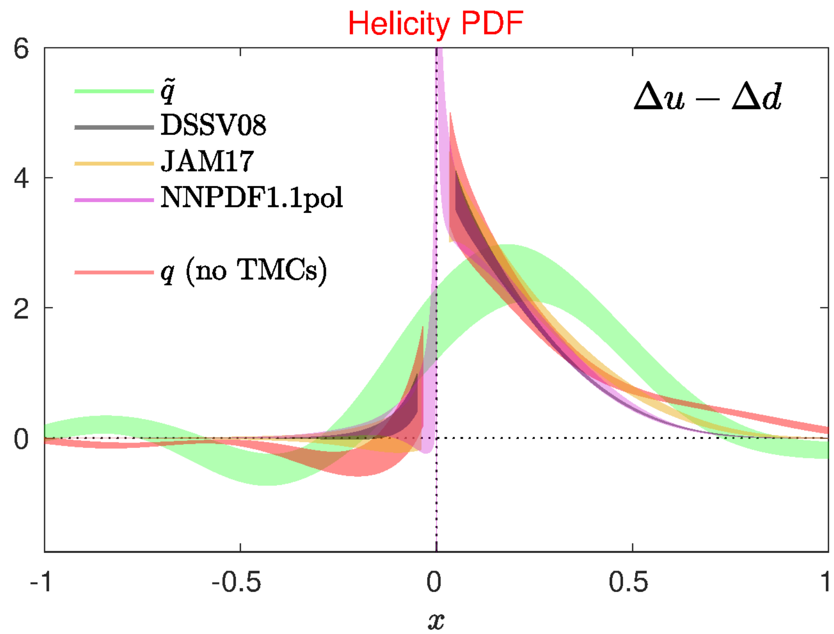

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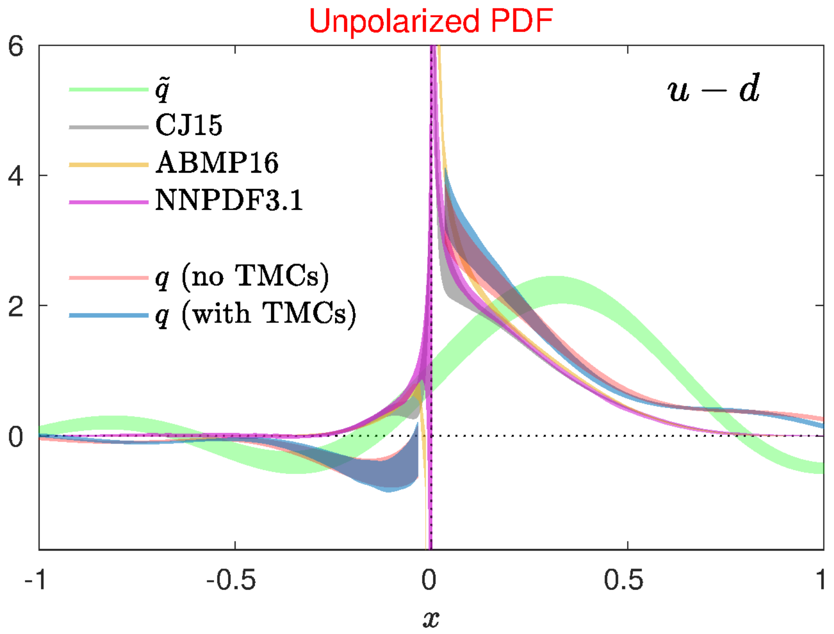

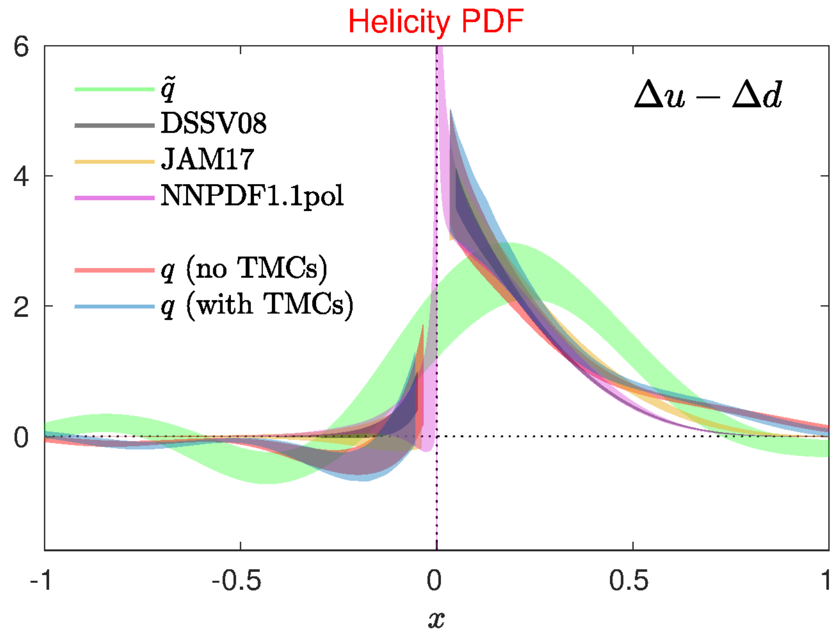

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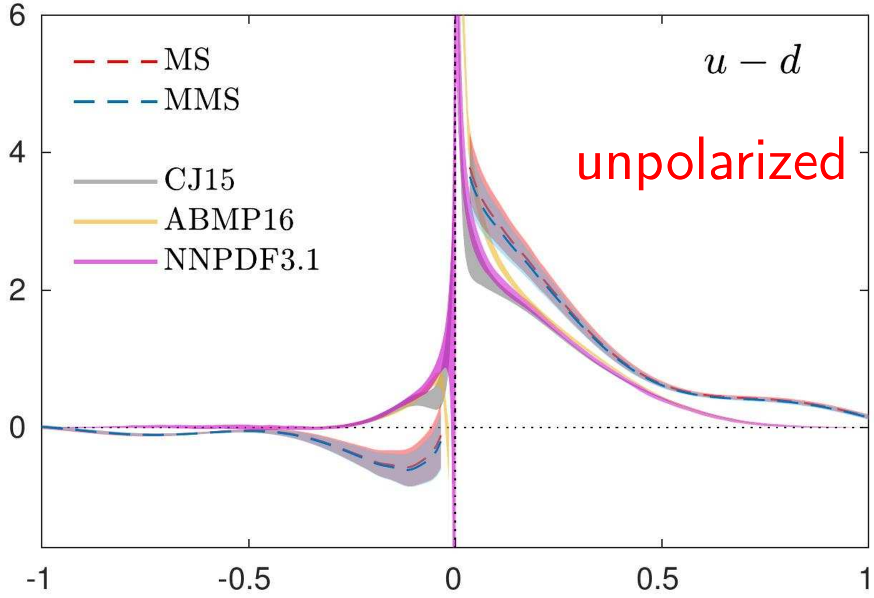
Effect from $\mathrm{M\overline{MS}}$



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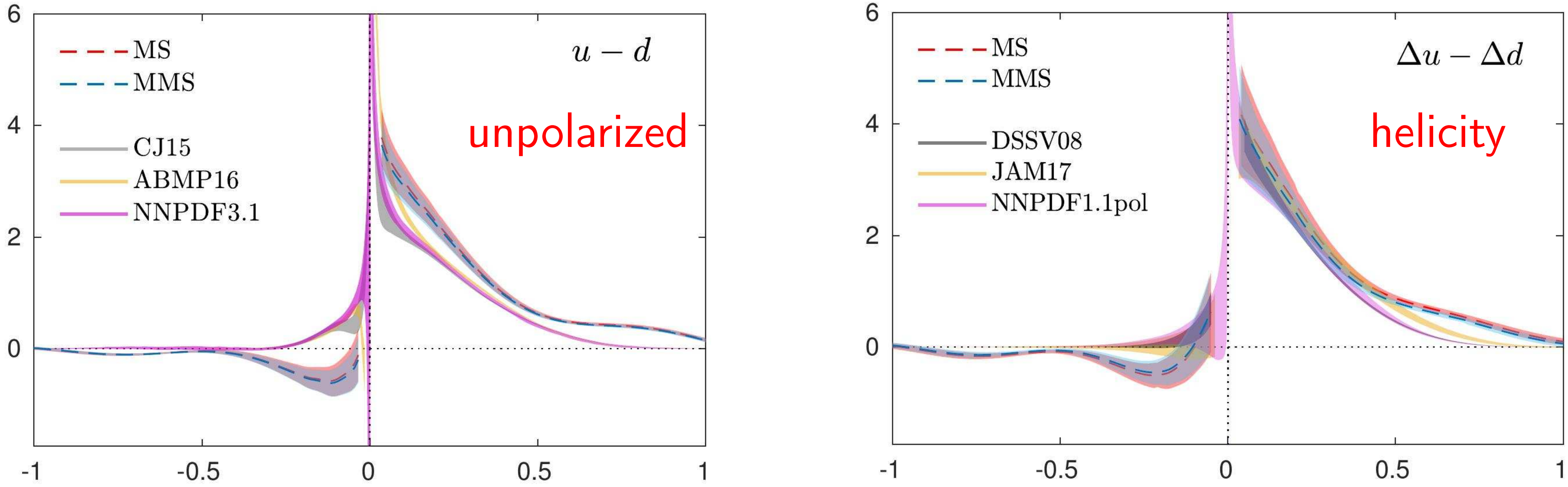

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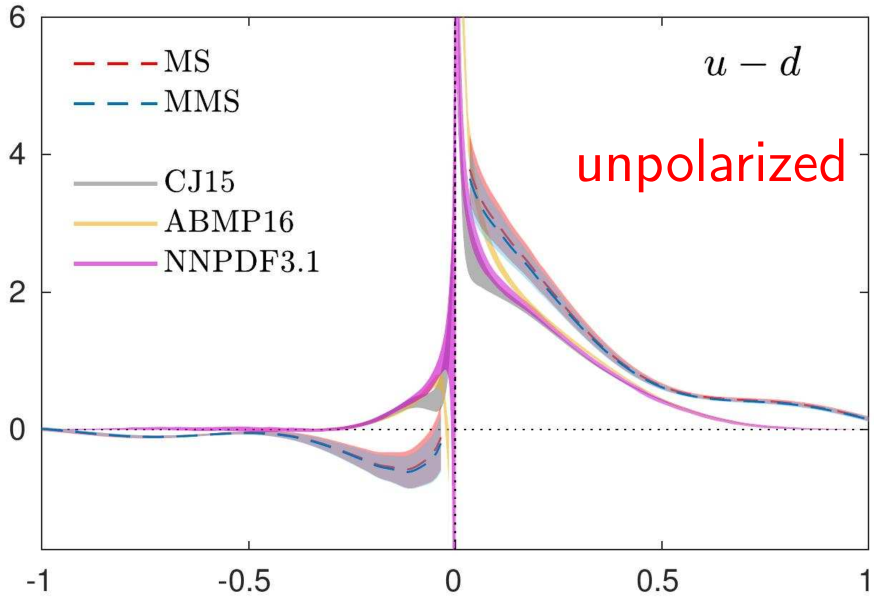

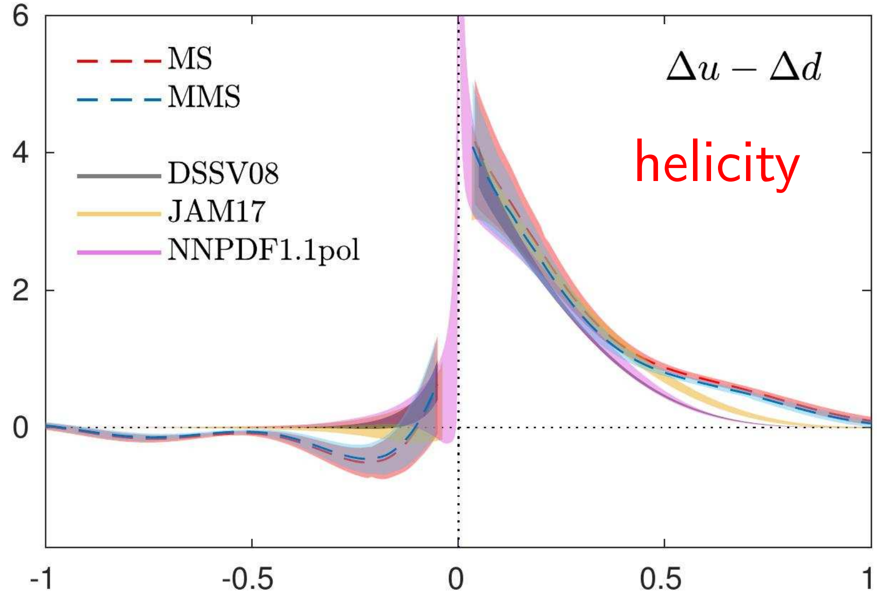

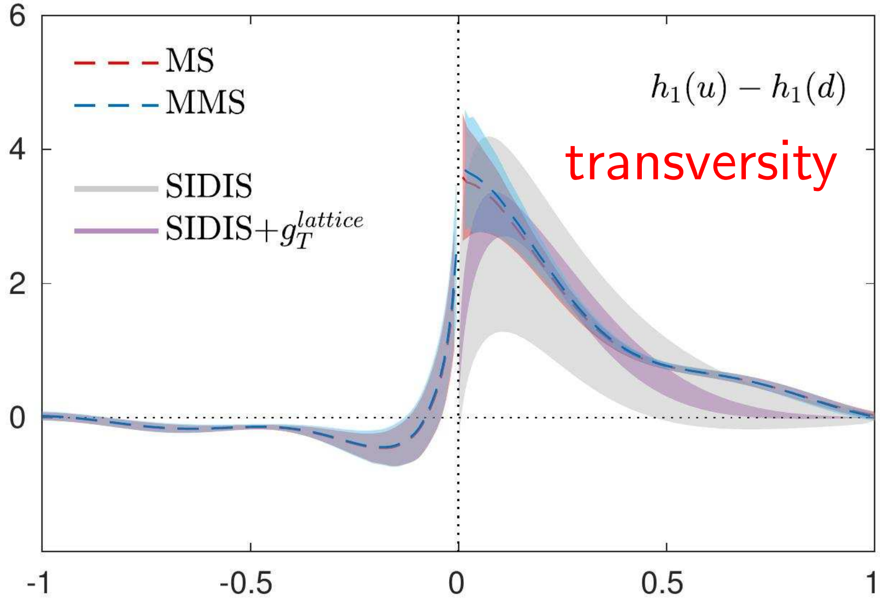

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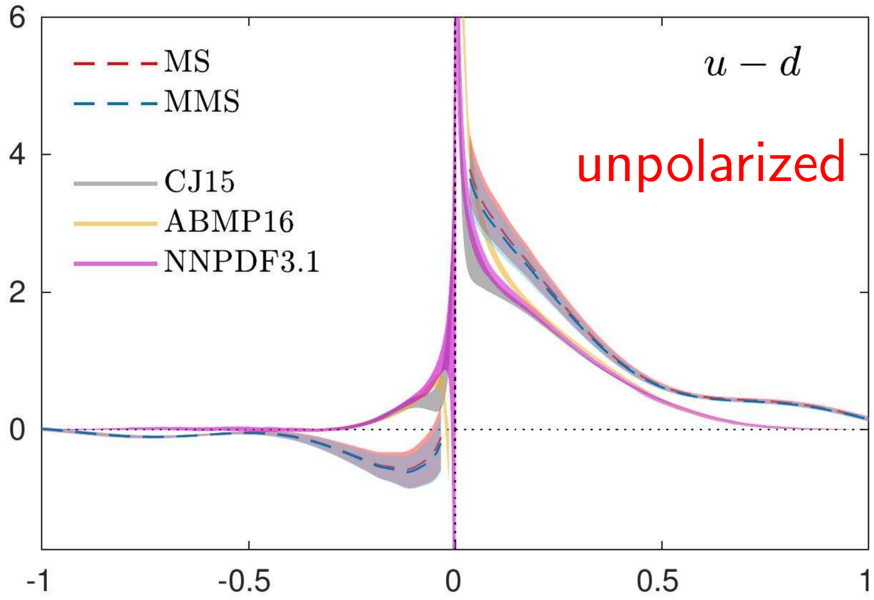
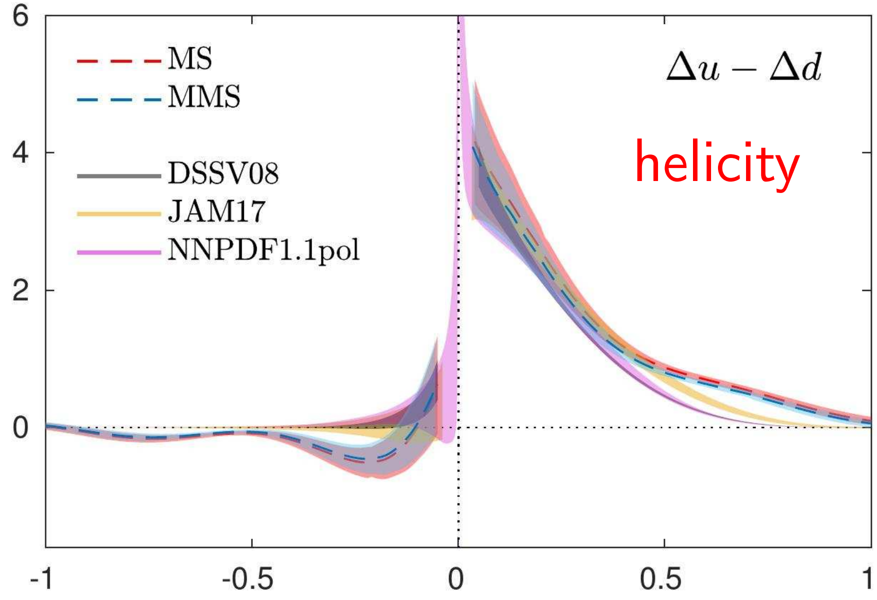
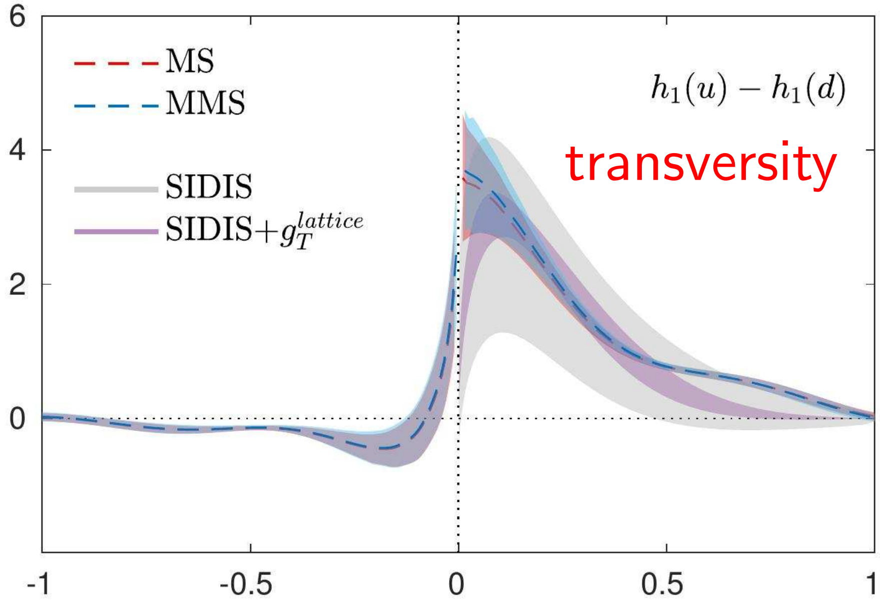


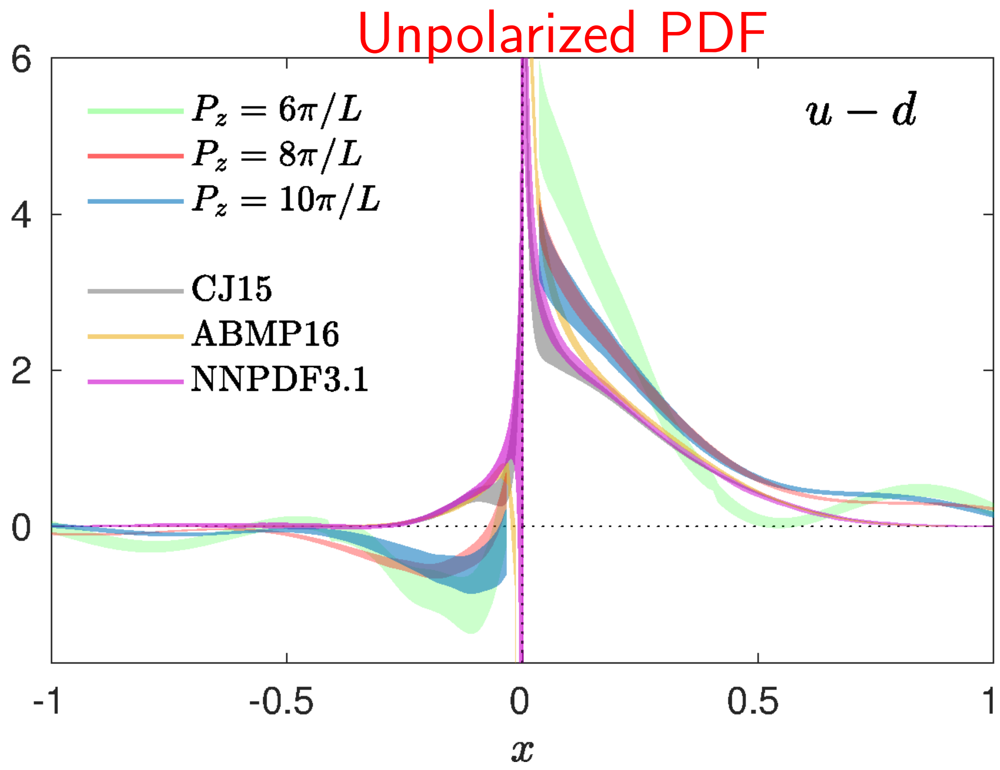




Nucleon momentum $\frac{10\pi}{48}$

As expected, the effect is very small (modification of $\overline{\mathrm{MS}}$ only in unphysical regions)

Nucleon momenta $\frac{\{6,8,10\}\pi}{48}$



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+ effect from $\overline{\text{MMS}}$

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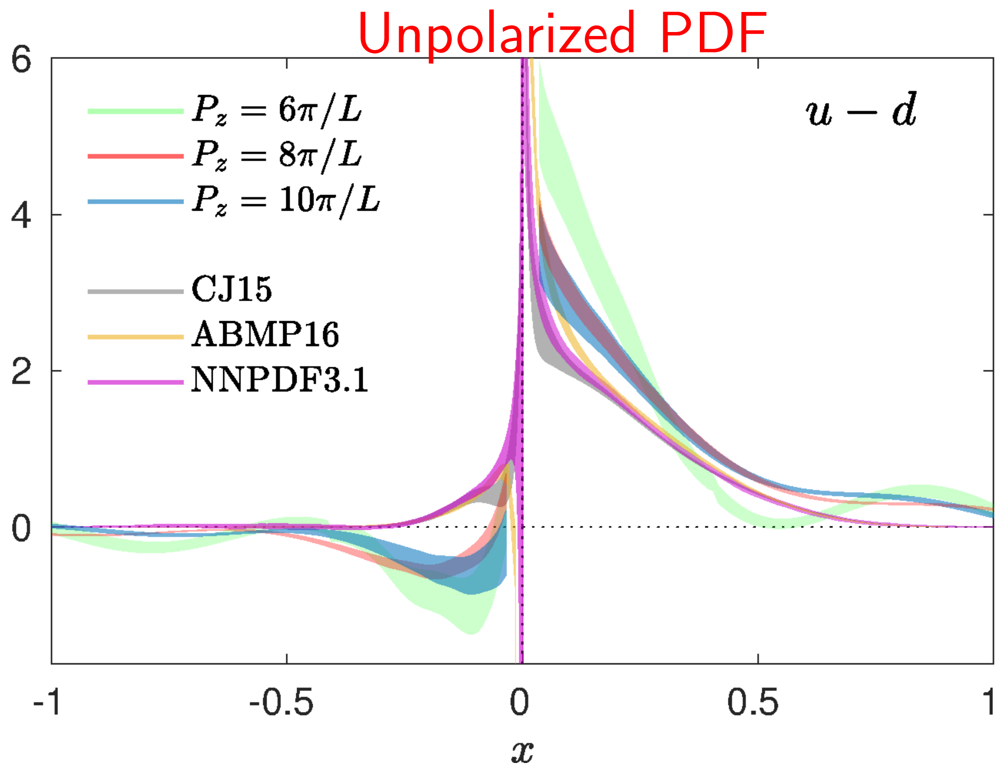

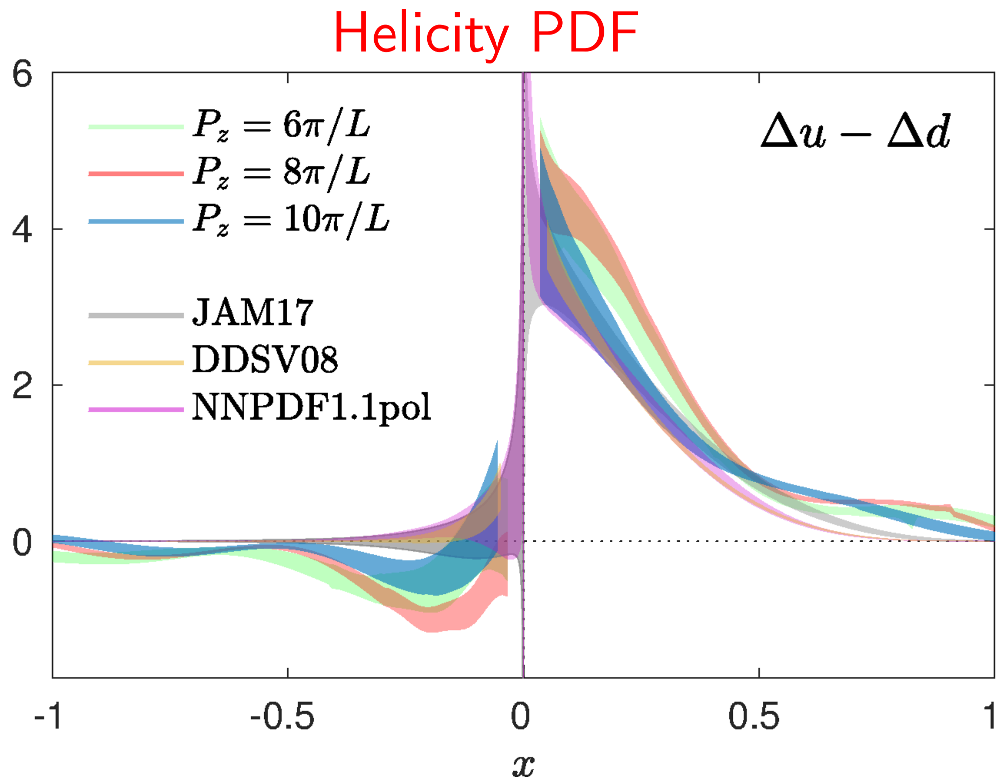

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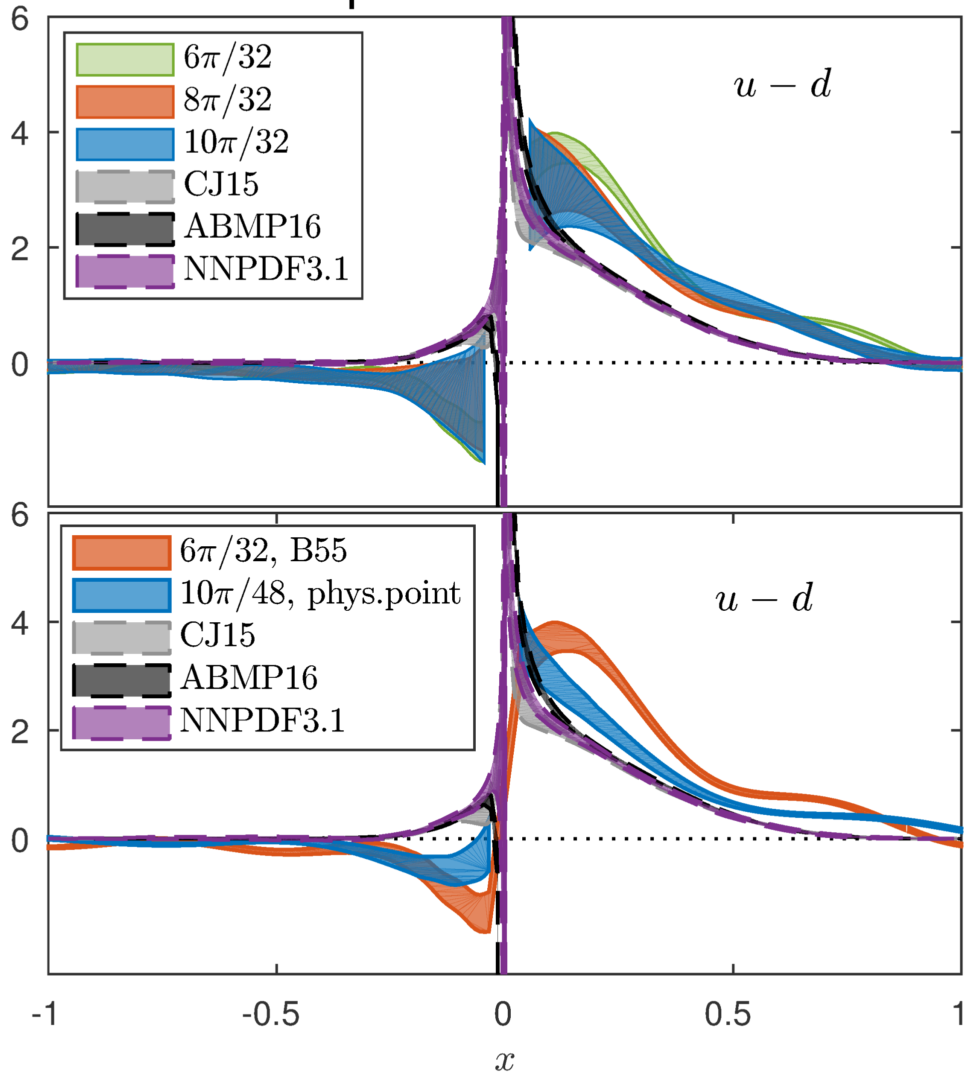
Physical vs. non-physical pion mass – 135 vs. 375 MeV
unpolarized PDF



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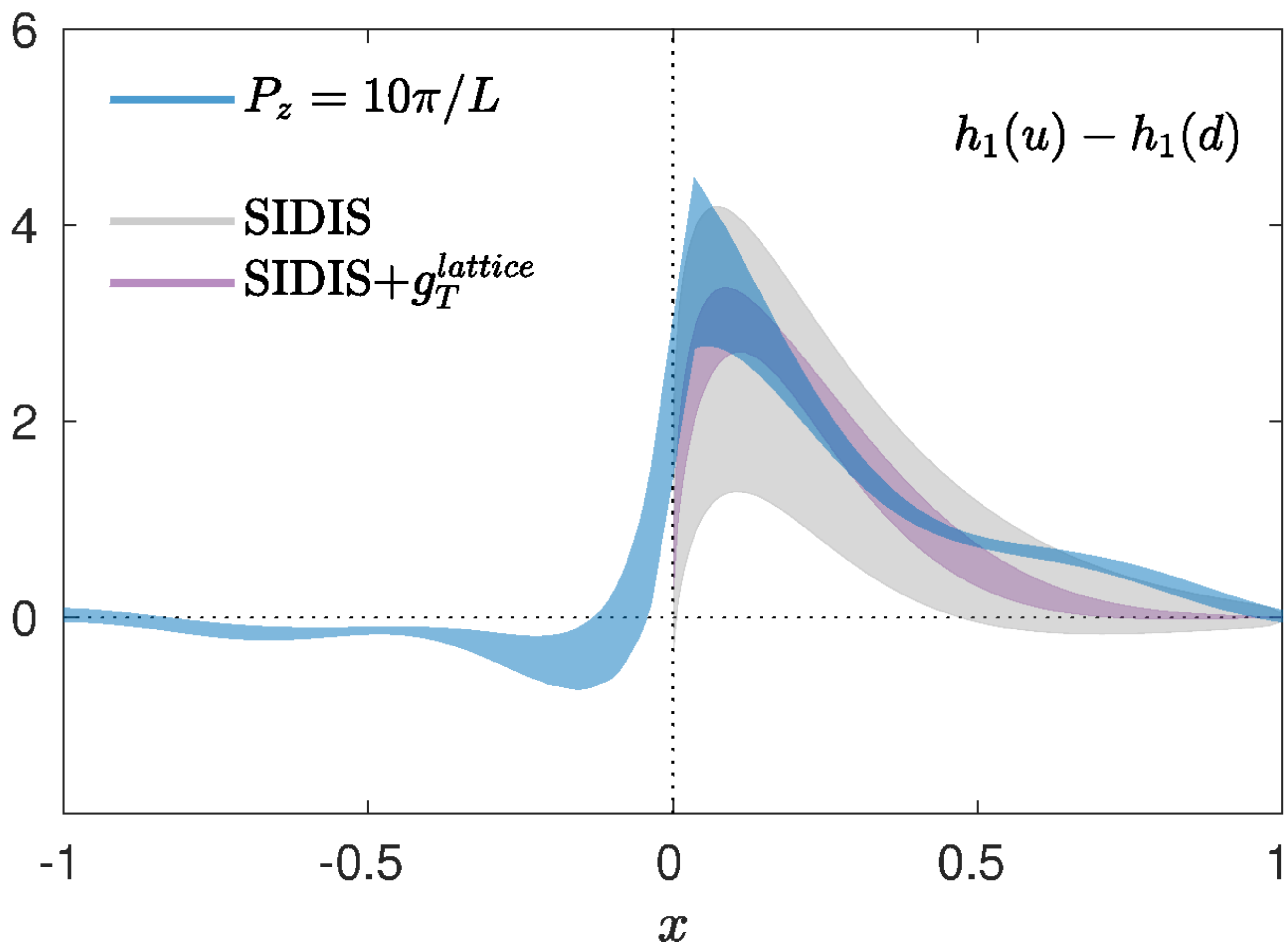

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Systematics

Different systematic effects still need to be addressed:

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Biggest challenge:
Reach large momenta at large source-sink separations

Conclusions and prospects

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001
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Thank you for your attention!

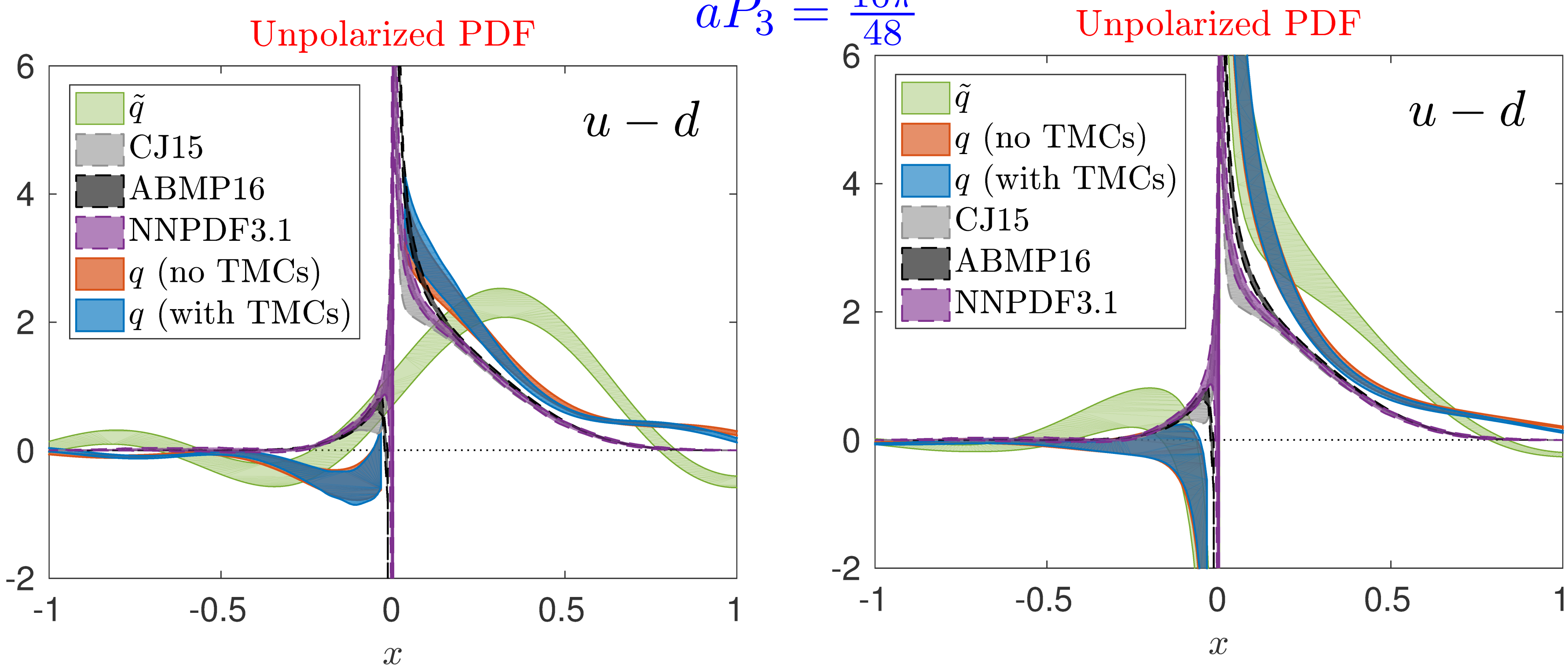
Standard Fourier transform defining qPDFs: $\tilde{q}(x) = 2P_3 \int_{-z_{\max}}^{z_{\max}} \frac{dz}{4\pi} e^{ixzP_3} h(z)$

can be rewritten using integration by parts as: [H.W. Lin et al., arXiv:1708.05301]

$$\tilde{q}(x) = h(z)\frac{e^{ixzP_3}}{2\pi i x}\Bigg|_{-z_{\max}}^{z_{\max}} - \int_{-z_{\max}}^{z_{\max}} \frac{dz}{2\pi}\frac{e^{ixzP_3}}{ix} h'(z).$$

Truncation: $h(|z| \geq z_{\max}) = 0$ is equivalent to neglecting the surface term.

$$aP_3 = \frac{10\pi}{48}$$



Oscillations ✓ Small-x ✗