Recent developments in fragmentation function analysis and applications

Alberto Accardi
Hampton U. and Jefferson Lab

2nd Workshop on Parton Distribution Functions

Academia Sinica, Taipei (Taiwan)
Nov 26th, 2018

Overview

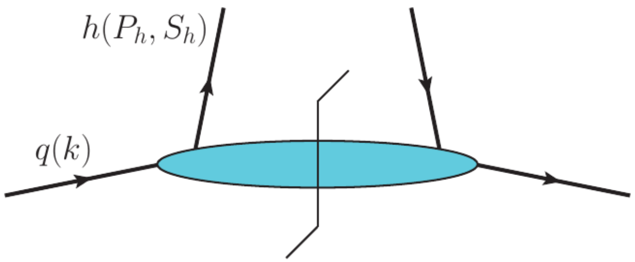
- **Intro:**
- **FF analysis:**
  - (Global) fits
  - Gluon and strange quark puzzles
- **Beyond global:**
  - Universal fits of FFs and PDFs
- **Beyond Leading-Twist**
  - Hadron Mass Corrections
- **Novel FF sum rules**
  - FF vs. "current jet correlators"
  - Novel phenomenology… and one more universal fit
- **Final thoughts**

Introduction

Fragmentation Functions

Metz, Vossen, Prog.Part.Nucl.Phys. 91 (2016) 136-202
Nocera, Lectures at HUGS 2017 (www.jlab.org/hugs)

- Rich field:
  - **Collinear, transverse momentum dependent**
    - Flavor separation of PDFs
    - Quark-gluon angular momentum
    - In-medium: confinement effects



At leading twist:

$$\begin{aligned}
\Delta^{h/q}(z; P_h, S_h) \;=\; z\Big(&\Delta^{h/q\,[\gamma^-]}\,\gamma^+ - \Delta^{h/q\,[\gamma^-\gamma_5]}\,\gamma^+\gamma_5 + \Delta^{h/q\,[i\sigma^{i-}\gamma_5]}\,i\sigma^{i+}\gamma_5 \\
&+ \Delta^{h/q\,[\mathbb{1}]}\,\mathbb{1} - \Delta^{h/q\,[i\gamma_5]}\,i\gamma_5 - \Delta^{h/q\,[\gamma^i]}\,\gamma^i \\
&+ \Delta^{h/q\,[\gamma^i\gamma_5]}\,\gamma^i\gamma_5 - \tfrac{1}{2}\,\Delta^{h/q\,[i\sigma^{ij}\gamma_5]}\,i\sigma^{ij}\gamma_5 + \Delta^{h/q\,[i\sigma^{-+}\gamma_5]}\,i\sigma^{-+}\gamma_5 \\
&+ \Delta^{h/q\,[\gamma^+]}\,\gamma^- - \Delta^{h/q\,[\gamma^+\gamma_5]}\,\gamma^-\gamma_5 + \Delta^{h/q\,[i\sigma^{i+}\gamma_5]}\,i\sigma^{i-}\gamma_5\Big)\,,
\end{aligned}$$

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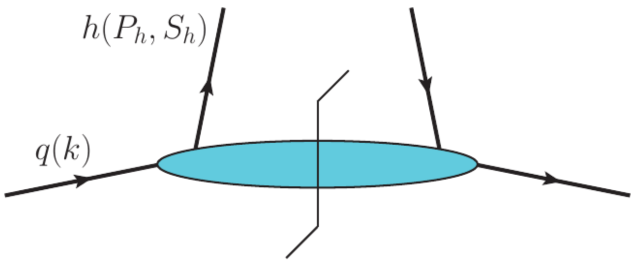

Quark polarization →

Hadron pol. ↓

H \ q	U	L	T
U	$D_1^{h/q}$		$H_1^{\perp\, h/q}$
L		$G_1^{h/q}$	$H_{1L}^{\perp\, h/q}$
T	$D_{1T}^{\perp\, h/q}$	$G_{1T}^{h/q}$	$H_1^{h/q}$ $H_{1T}^{\perp\, h/q}$

Survive k_T integration: only 3 collinear FFs

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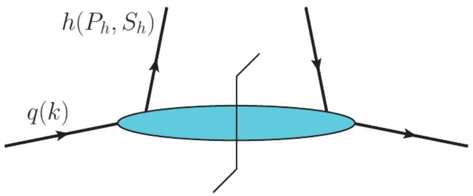
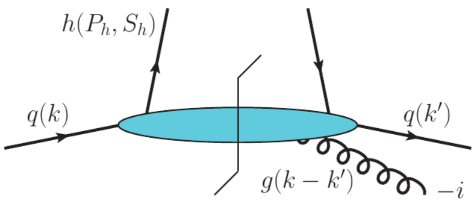


  - **Twist 2, Twist 3, …**
    - Multi-parton correlations
    - Single-spin asymmetries
    - Other quantum interference effects



    - collinear tw-3 FFs: $\hat{D}_{FT}^{h/q}$ $\hat{G}_{FT}^{h/q}$ $\hat{H}_{FU}^{h/q}$ $\hat{H}_{FL}^{h/q}$
    - twist-3 TMD-FF: *Bacchetta at al, JHEP 0702 (2007) 093*

Fragmentation Functions

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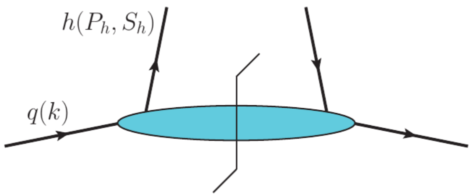

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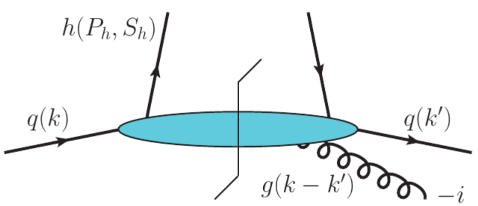
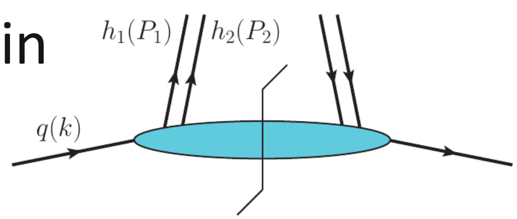


  - **Di-hadrons, too**
    - Relative momentum instead of hadron spin
    - Alternative access to PDF, TMDs



$D_1^{h_1 h_2/q}$ $G_1^{\perp\, h_1 h_2/q}$ $H_1^{\sphericalangle\, h_1 h_2/q}$ $H_1^{\perp\, h_1 h_2/q}$

Observables

Metz, Vossen, Prog.Part.Nucl.Phys. 91 (2016) 136-202
Nocera, Lectures at HUGS 2017 (www.jlab.org/hugs)

Process	Quantity	Remarks
	Integrated FF $D_1(z)$	
$e^+e^- \to hX$	$\sum_q e_q^2 D_1^{h/q}(z)$	
$e^+e^- \to h_a h_b X$	$\sum_q e_q^2 D_1^{h_a/q}(z_a) D_1^{h_b/\bar{q}}(z_b) + \{q \leftrightarrow \bar{q}\}$	back-to-back production of hadron pair
$\ell p \to \ell h X$	$\sum_q e_q^2 f_1^{q/p}(x) D_1^{h/q}(z)$	
$pp \to hX$	$\sum_{i,j,k} f_1^{i/p_a}(x_a) \otimes f_1^{j/p_b}(x_b) \otimes D_1^{h/k}(z)$	cannot access z
$pp \to \gamma h X$	$\sum_{i,j,k} f_1^{i/p_a}(x_a) \otimes f_1^{j/p_b}(x_b) \otimes D_1^{h/k}(z)$	back-to-back production of hadron with direct γ
$pp \to (h, \text{jet})X$	$\sum_{i,j,k} f_1^{i/p_a}(x_a)\, f_1^{j/p_b}(x_b)\, D_1^{h/k}(z)$	hadron in jet: can access z
	TMD FF $D_1(z, k_T)$	
$e^+e^- \to h_a h_b X$	$\sum_q e_q^2 D_1^{h_a/q}(z_a, k_{aT}) \otimes D_1^{h_b/\bar{q}}(z_b, k_{bT})$ $+\{q \leftrightarrow \bar{q}\}$	back-to-back production of hadron pair
$pp \to h_a h_b X$	$\sum_{i,j,k,l} f_1^{i/p_a}(x_a, p_{aT}) \otimes f_1^{j/p_b}(x_b, p_{bT})$ $\otimes D_1^{h_a/k}(z_a, k_{aT}) \otimes D_1^{h_b/l}(z_b, k_{bT})$	back-to-back production of hadron pair
$pp \to \gamma h X$	$\sum_{i,j,k} f_1^{i/p_a}(x_a, p_{aT}) \otimes f_1^{j/p_b}(x_b, p_{bT})$ $\otimes D_1^{h/k}(z, k_T)$	back-to-back production of hadron with direct γ
$e^+e^- \to$ $(h, \text{jet/thrust axis})X$	$\sum_q e_q^2 D_1^{h/q}(z, k_T)$	can access z, k_T
$\ell p \to \ell h X$	$\sum_q e_q^2 f_1^{q/p}(x, p_T) \otimes D_1^{h/q}(z, k_T)$	
$pp \to (h, \text{jet})X$	$\sum_{i,j,k} f_1^{i/p_a}(x_a)\, f_1^{j/p_b}(x_b)\, D_1^{h/k}(z, k_T)$	hadron in jet: can access z, k_T
	TMD FF $H_1^{\perp}(z, k_T)$	
$e^+e^- \to h_a h_b X$	$\sum_q e_q^2 H_1^{\perp\, h_a/q}(z_a, k_{aT}) \otimes H_1^{\perp\, h_b/\bar{q}}(z_b, k_{bT})$ $+\{q \leftrightarrow \bar{q}\}$	back-to-back production of hadron pair
$\ell p^{\uparrow} \to \ell h X$	$\sum_q e_q^2 h_1^{q/p}(x, p_T) \otimes H_1^{\perp\, h/q}(z, k_T)$	
$p^{\uparrow} p \to (h, \text{jet})X$	$\sum_{i,j,k} h_1^{i/p_a}(x_a)\, f_1^{j/p_b}(x_b)\, H_1^{\perp\, h/k}(z, k_T)$	hadron in jet: can access z, k_T

Observables

Metz, Vossen, Prog.Part.Nucl.Phys. 91 (2016) 136-202
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Twist-3 FFs		
$\ell p^\uparrow \to hX$	$\sum_q e_q^2 h_1^{q/p}(x)\,\{H^{h/q},\ H_1^{\perp(1)\,h/q},\hat{H}_{FU}^{h/q,\Im}\} + \ldots$	
$\vec{\ell} p^\uparrow \to hX$	$\sum_q e_q^2 h_1^{q/p}(x)\,E^{h/q} + \ldots$	
$p^\uparrow p \to hX$	$\sum_{i,j,k} h_1^{i/p_a}(x_a) \otimes f_1^{j/p_b}(x_b) \otimes \{H^{h/k},\ H_1^{\perp(1)\,h/k},\hat{H}_{FU}^{h/k,\Im}\} + \ldots$	
Di-hadron FFs		
$e^+e^- \to (h_1, h_2)X$	$\sum_q e_q^2 D_1^{h_1 h_2/q}(z, M_h)$	for large M_h also $D_1^{h/i}(z)$ contribute
$e^+e^- \to (h_{a1}, h_{a2})\,(h_{b1}, h_{b2})X$	$\sum_q e_q^2 D_1^{h_{a1} h_{a2}/q}(z_a, M_{ha})\,D_1^{h_{b1} h_{b2}/\bar{q}}(z_b, M_{hb}) + \{q \leftrightarrow \bar{q}\}$ $\sum_q e_q^2 H_1^{\sphericalangle\,h_{a1} h_{a2}/q}(z_a, M_{ha})\,H_1^{\sphericalangle\,h_{b1} h_{b2}/\bar{q}}(z_b, M_{hb}) + \{q \leftrightarrow \bar{q}\}$ $\sum_q e_q^2 G_1^{\perp\,h_{a1} h_{a2}/q}(z_a, M_{ha})\,G_1^{\perp\,h_{b1} h_{b2}/\bar{q}}(z_b, M_{hb}) + \{q \leftrightarrow \bar{q}\}$	back-to-back production of di-hadron pair
$\ell p \to \ell(h_1, h_2)X$	$\sum_q e_q^2 f_1^{q/p}(x)\,D_1^{h_1,h_2/q}(z, M_h)$	
$\ell p^\uparrow \to \ell(h_1, h_2)X$	$\sum_q e_q^2 h_1^{q/p}(x)\,H_1^{\sphericalangle\,h_1,h_2/q}(z, M_h)$	
$pp \to (h_1, h_2)X$	$\sum_{i,j,k} f_1^{i/p_a}(x_a) \otimes f_2^{j/p_b}(x_b) \otimes D_1^{h_1 h_2/k}(z, M_h)$	
$p^\uparrow p \to (h_1, h_2)X$	$\sum_{i,j,k} h_1^{i/p_a}(x_a) \otimes f_2^{j/p_b}(x_b) \otimes H_1^{\sphericalangle\,h_1 h_2/k}(z, M_h)$	

Many distributions, single "global" fits

Jimenez-Delgado, Melnitchouk, Owens, JPG 40 (2013)
Forte and Watt – Ann.Rev.Nucl.Part.Sci. 63 (2013) 291
Metz, Vossen, Prog.Part.Nucl.Phys. 91 (2016) 136-202

nucl PDF

PDF

→ *Del Debbio*

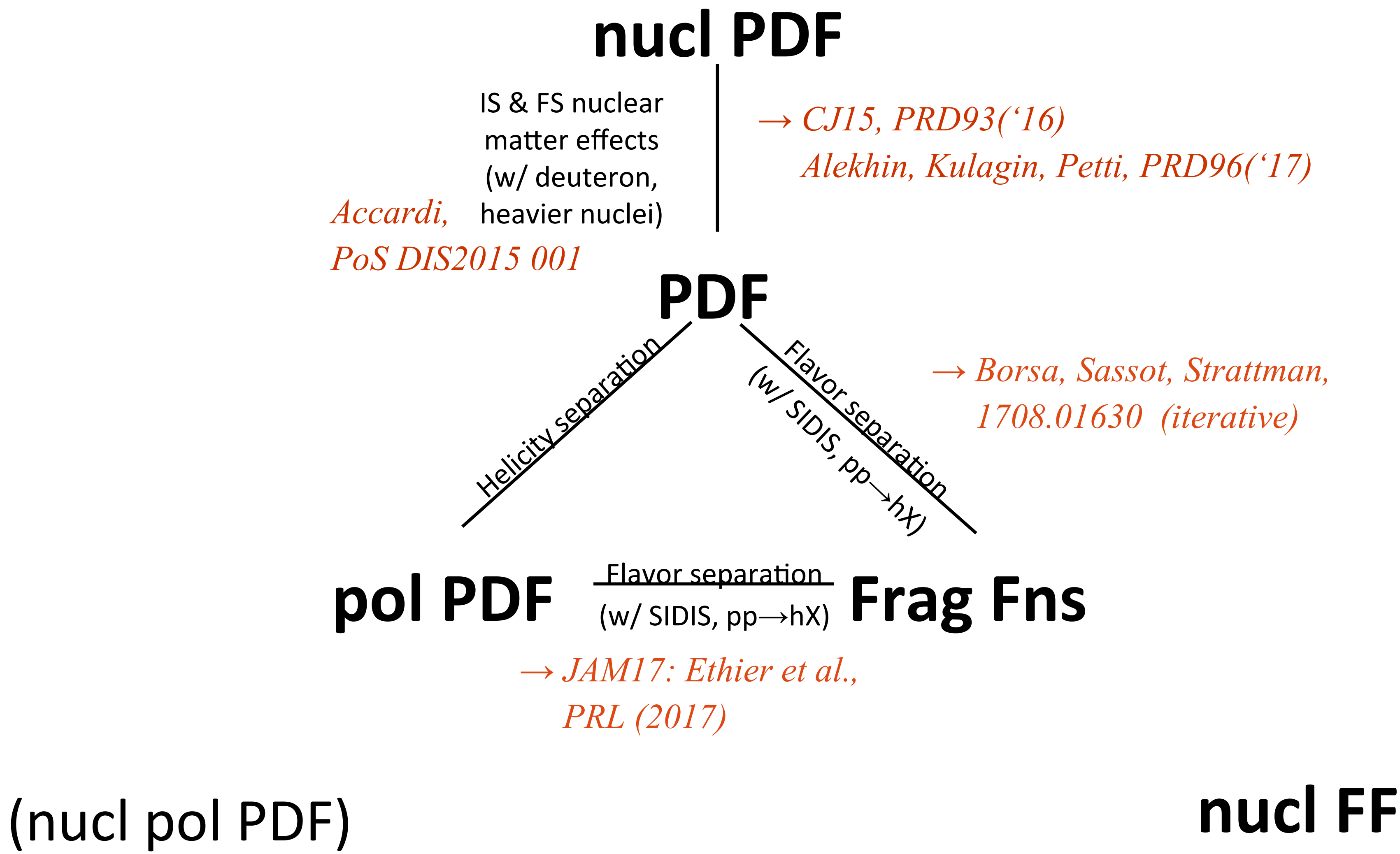
pol PDF

Frag Fns

(nucl pol PDF)

nucl FF

But physics is not isolated: need "universal" fits



New fitting methods, capabilities

- More computing power, efficient implementations
  - **New fitting, analysis methods**

- In traditonal fits:
  - Detailed χ^2 scans, refined statistical analysis

- Monte Carlo fitting methods:
  - **NNPDF**: bootstrap + neural network fit
  - **JAM**: bootstrap + Iterative Monte Carlo (IMC) approach

  Large number of parameters, trustable uncertainties, sophisticated statistics analysis

- Machine learning methods
  - e.g., self-organizing maps *[Liuti et al.]*

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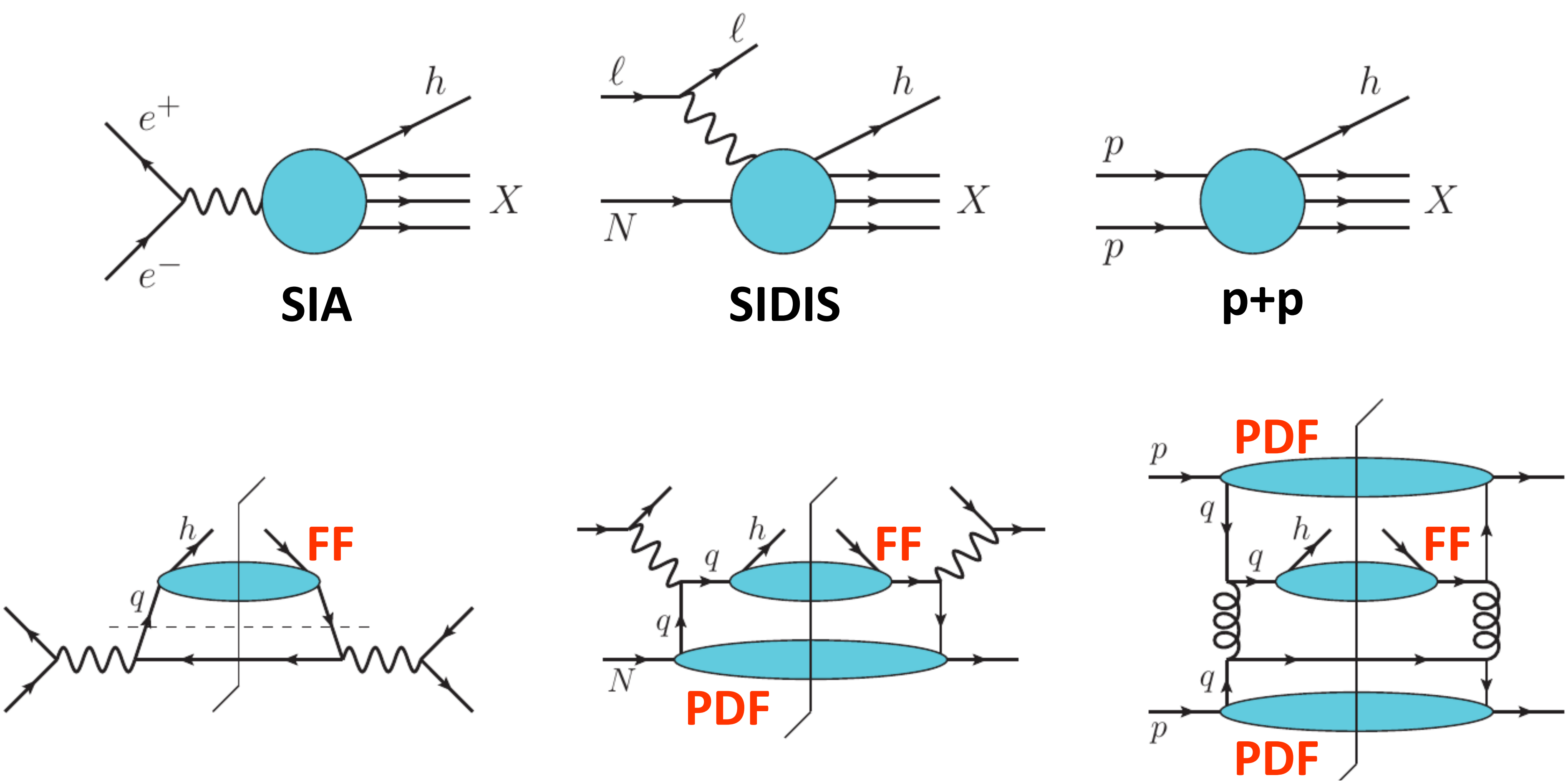
Universal fits era
(and more...)
is within reach !!

- Machine learning methods
  - e.g., self-organizing maps *[Liuti et al.]*

QCD analysis of Fragmentation Functions - the D_1 case -

20 years of unpolarized collinear FF fits



Cross-talk: PDFs and FFs do not live un isolation!

20 years of unpolarized collinear FF fits

Slide adapted from E.Nocera, HUGS 2017

	DHESS	HKNS	JAM	NNFF1.0
SIA	☑	☑	☑	☑
SIDIS	☑	☒	☒	☒
PP	☑	☒	☒	☑ ← NNFF1.1
statistical treatment	Iterative Hessian 68% - 90%	Hessian $\Delta\chi^2 = 15.94$	Monte Carlo	Monte Carlo
parametrisation	standard	standard	standard	neural network
HF scheme	ZM-VFN	ZM-VFN	ZM-VFN	ZM-VFN
hadron species	$\pi^\pm$, $K^\pm$, $p/\bar{p}$, $h^\pm$	$\pi^\pm$, $K^\pm$, $p/\bar{p}$	$\pi^\pm$, $K^\pm$	$\pi^\pm$, $K^\pm$, $p/\bar{p}$
latest update	PRD 91 (2015) 014035 PRD 95 (2017) 094019	PTEP 2016 (2016) 113B04	PRD 94 (2016) 114004	EPJA xxxx (2017) arXiv:1807.03310

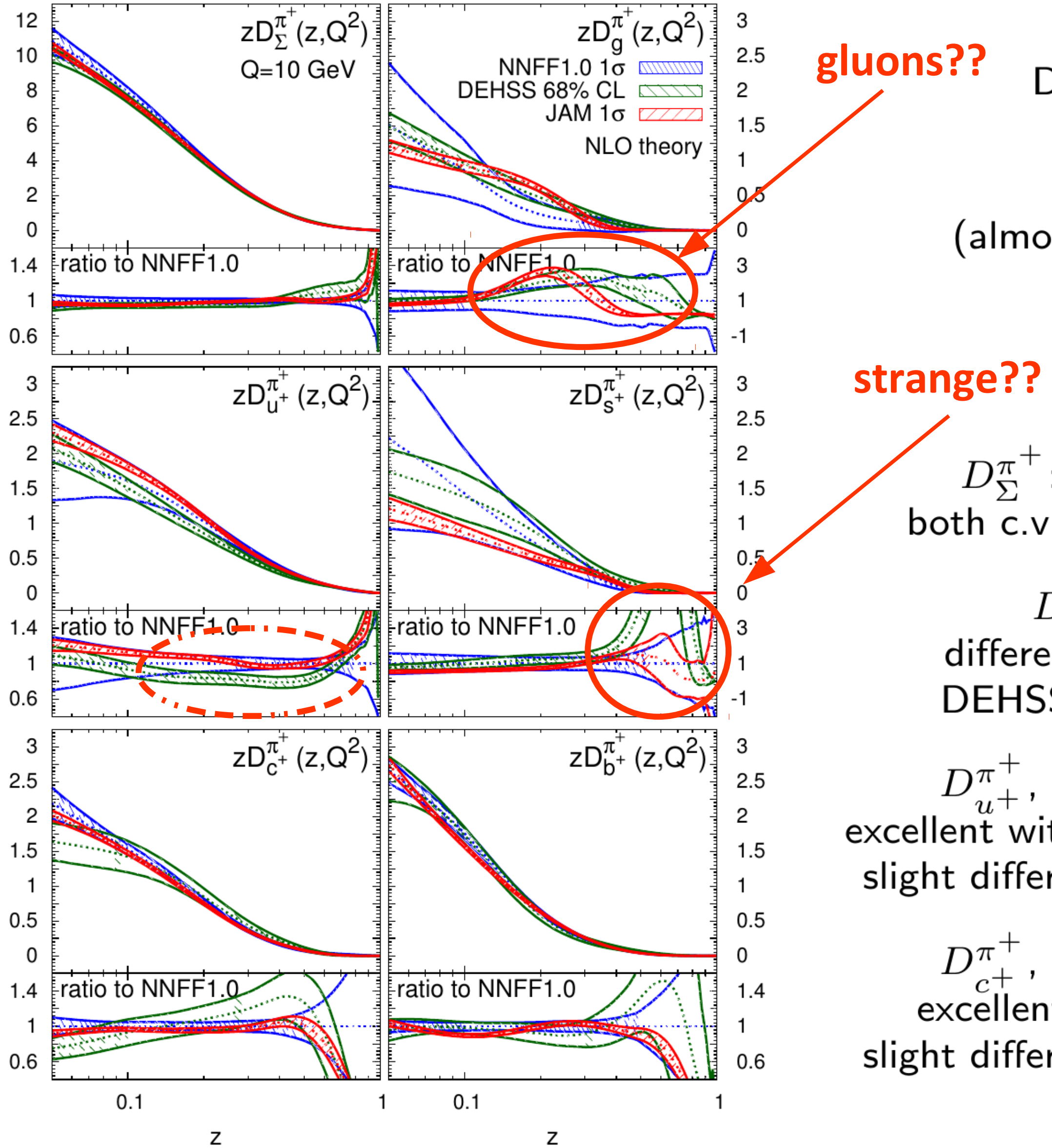
$+$ some others (including analyses for specific hadrons)

BKK95 [ZPB 65 (1995) 471] $\pi^\pm, K^\pm$
BKK96 [PRD 53 (1996) 3553] K^0
DSV97 [PRD 57 (1998) 5811] Λ^0
BFGW00 [EPJ C19 (2001) 89] $h^\pm$
SGK18 [PRD96 (2017) 034028] $h^\pm$
AKK06 [NPB 734 (2008) 42] Λ

AESS11 [PRD 83 (2011) 034002] η
SKMNA13 [PRD 88 (2013) 054019] $\pi^\pm, K^\pm$
LSS15 [PRD 96 (2016) 074026] SIDIS only
SKM18 [PRD96 (2017) 034028] $D*$
AKSRV17 [PRD96 (2017) 034028] $D*$

Comparisons: Pions

Slide adapted from E.Nocera, HUGS 2017



DEHSS [PRD 91 (2015) 014035]
(+SIDIS +PP)

JAM [PRD 94 (2016) 114004]
(almost same dataset as NNFF1.0)

$D_\Sigma^{\pi^+}$: excellent mutual agreement
both c.v. and unc. (bulk of the dataset)

$D_g^{\pi^+}$: slight disagreement
different shapes, larger uncertainties
DEHSS: data; JAM: parametrisation

$D_{u^+}^{\pi^+}$, $D_{s^+}^{\pi^+}$: good overall agreement
excellent with JAM, though larger uncertainties
slight different shape w.r.t. DHESS (dataset)

$D_{c^+}^{\pi^+}$, $D_{b^+}^{\pi^+}$: good overall agreement
excellent with JAM, same uncertainties
slight different shape w.r.t. DHESS (dataset)

Comparisons: Kaons

Slide adapted from E.Nocera, HUGS 2017

DEHSS [PRD 95 (2017) 094019]
(+SIDIS +PP)

JAM [PRD 94 (2016) 114004]
(almost same dataset as NNFF1.0)

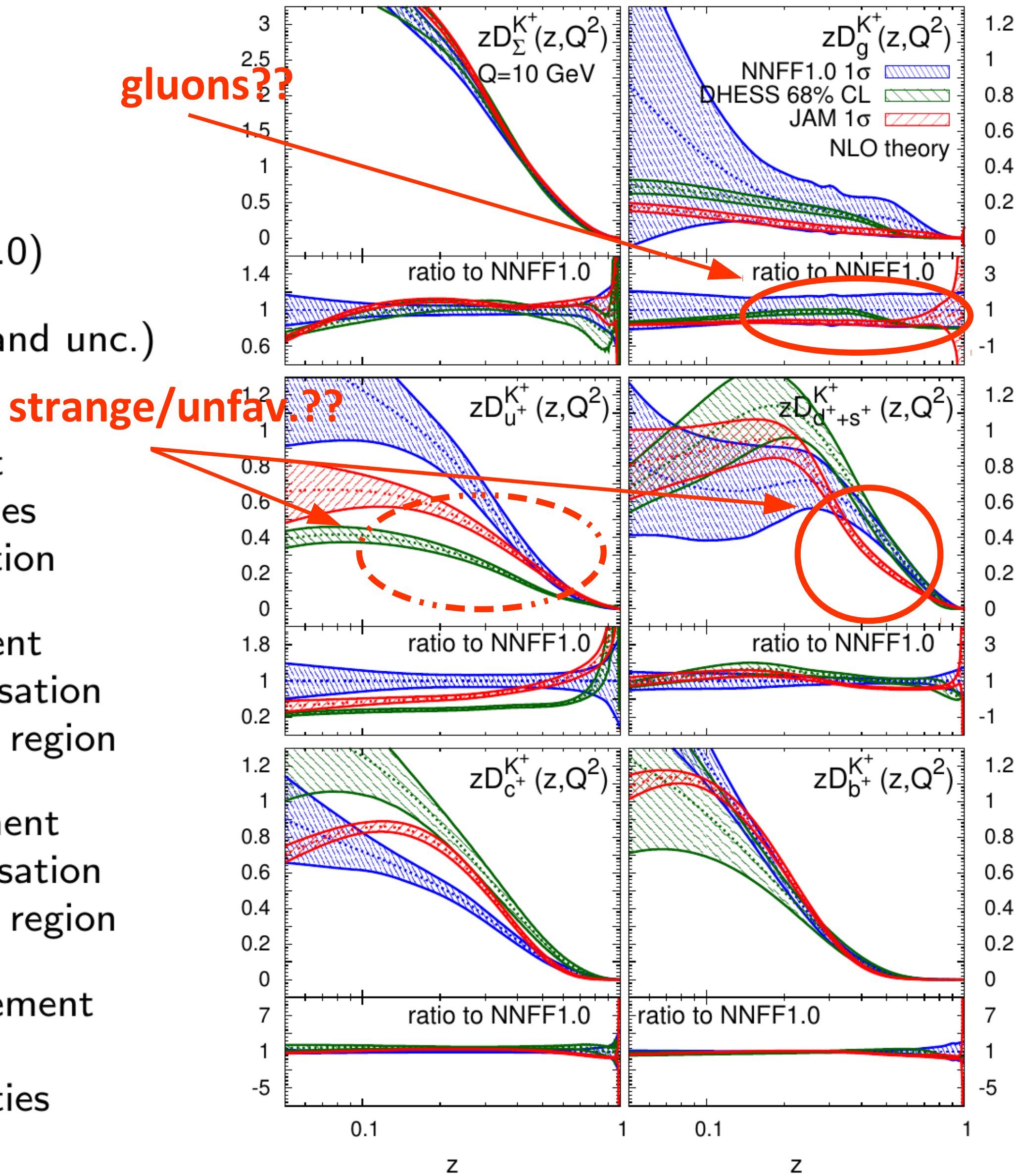
$D_{\Sigma}^{K^+}$: excellent agreement (both c.v. and unc.)
bulk of the dataset

$D_{g}^{K^+}$: good mutual agreement
similar shapes, larger uncertainties
DEHSS: data; JAM: parametrisation

$D_{u^+}^{K^+}$: mutual sizable disagreement
differences in dataset and parametrisation
comparable uncertainties in the data region

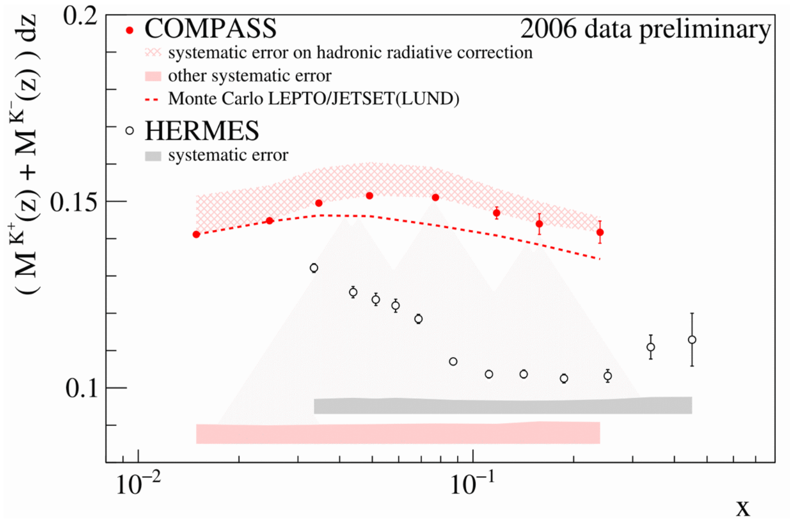
$D_{d^+}^{K^+} + D_{s^+}^{K^+}$: fair mutual agreement
differences in dataset and parametrisation
comparable uncertainties in the data region

$D_{c^+}^{K^+}$, $D_{b^+}^{K^+}$: excellent mutual agreement
uncertainties similar to JAM
DHESS shows inflated uncertainties



Strange strange quarks… or strange Kaons?

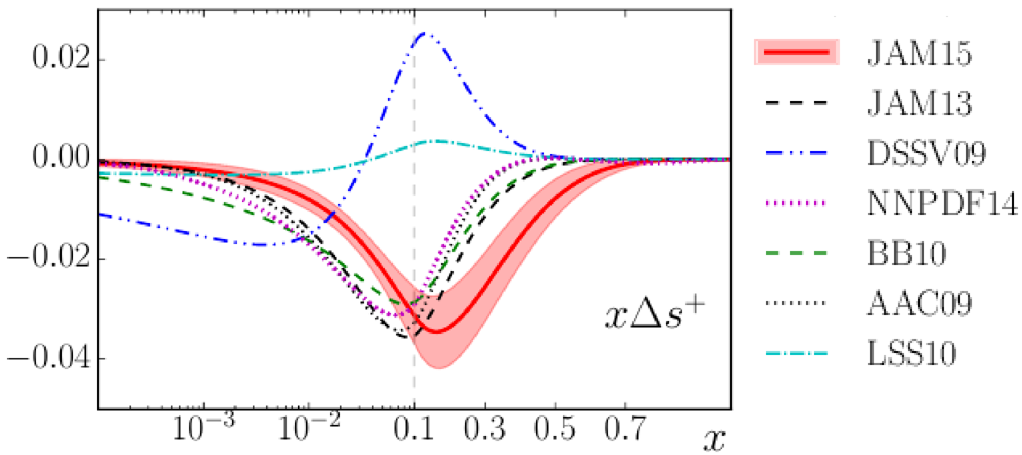
- **s : large or small?**
  - Possibly, large Hadron Mass effects
    Guerrero, Accardi, PRD 97 (2018) 114012
  - Extraction of s(x) strongly affected by **kaon <u>systematic</u> uncertainty**



- **Δs : positive or negative?**
  - Depends on **kaon FF used in SIDIS calculations!**
    LSS, PRD 84&91

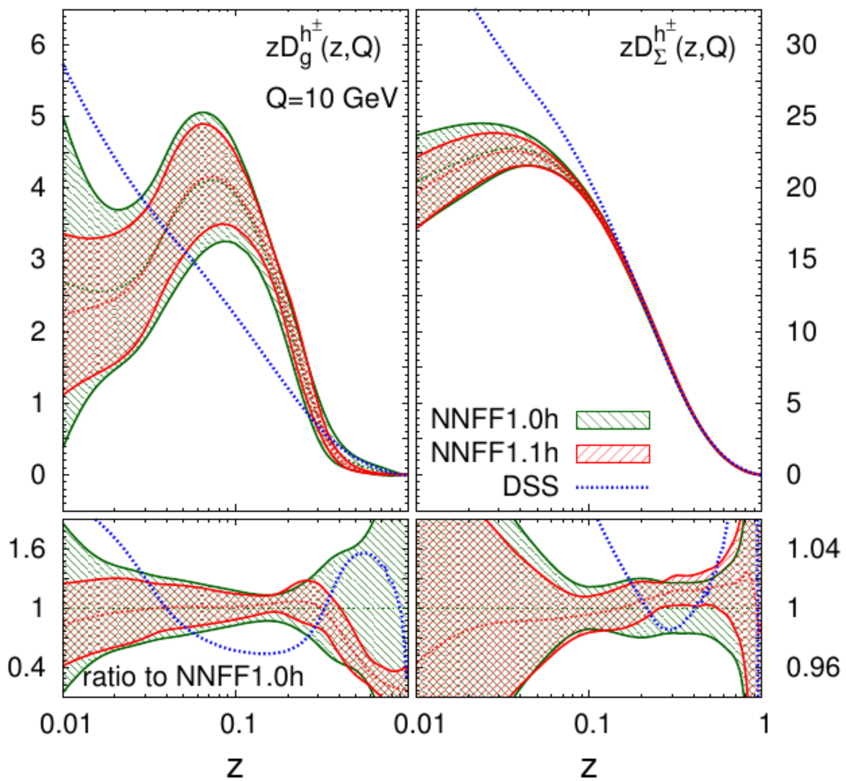
$$A_1^h(x,z,Q^2) = \frac{g_1^h(x,z,Q^2)}{F_1(x,z,Q^2)}$$

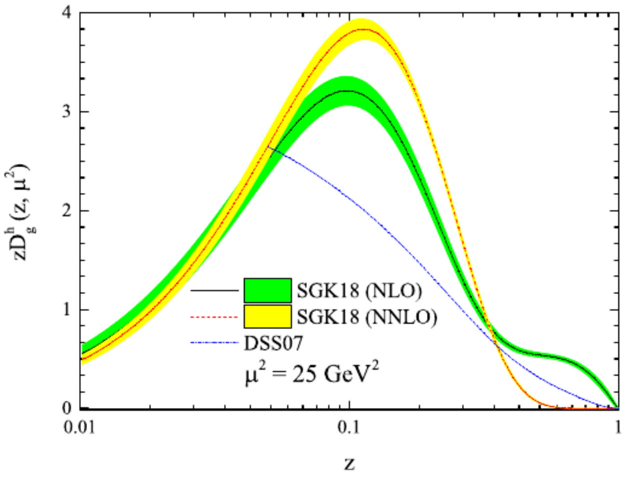
  - What about the unpol s ?



More comparisons: unidentified hadrons

- More sensitivity to gluons:
  - Longitudinal SIA cross section (data for h+ only)
  - Hadron production in p+p at LHC
    - (cf. hadrons at RHIC as in DEHSS)





Gluons are softer, valence-like:
- Why not in DEHSS?
- Is RHIC in tension with LHC?
- Is SIDIS in tension with rest?

More comparisons: unidentified hadrons

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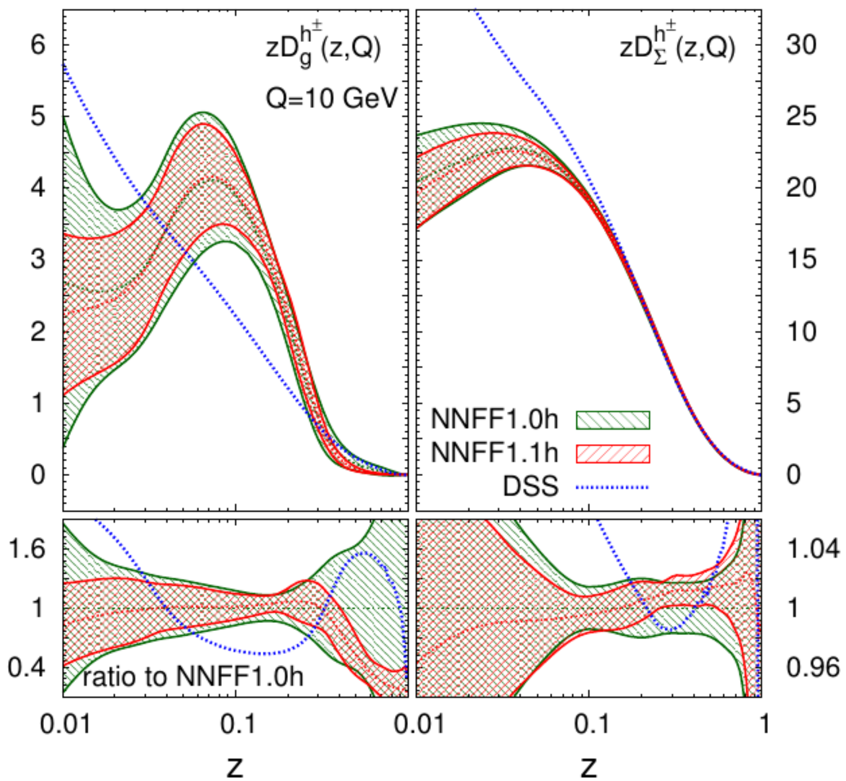
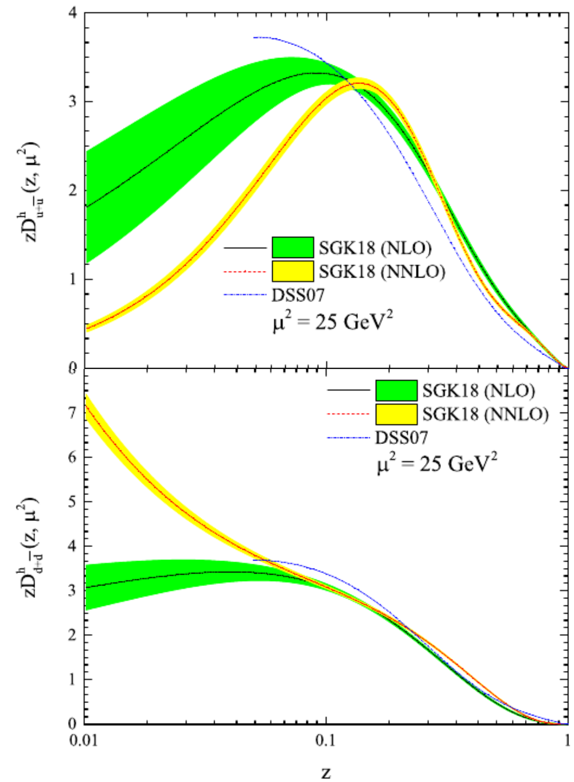


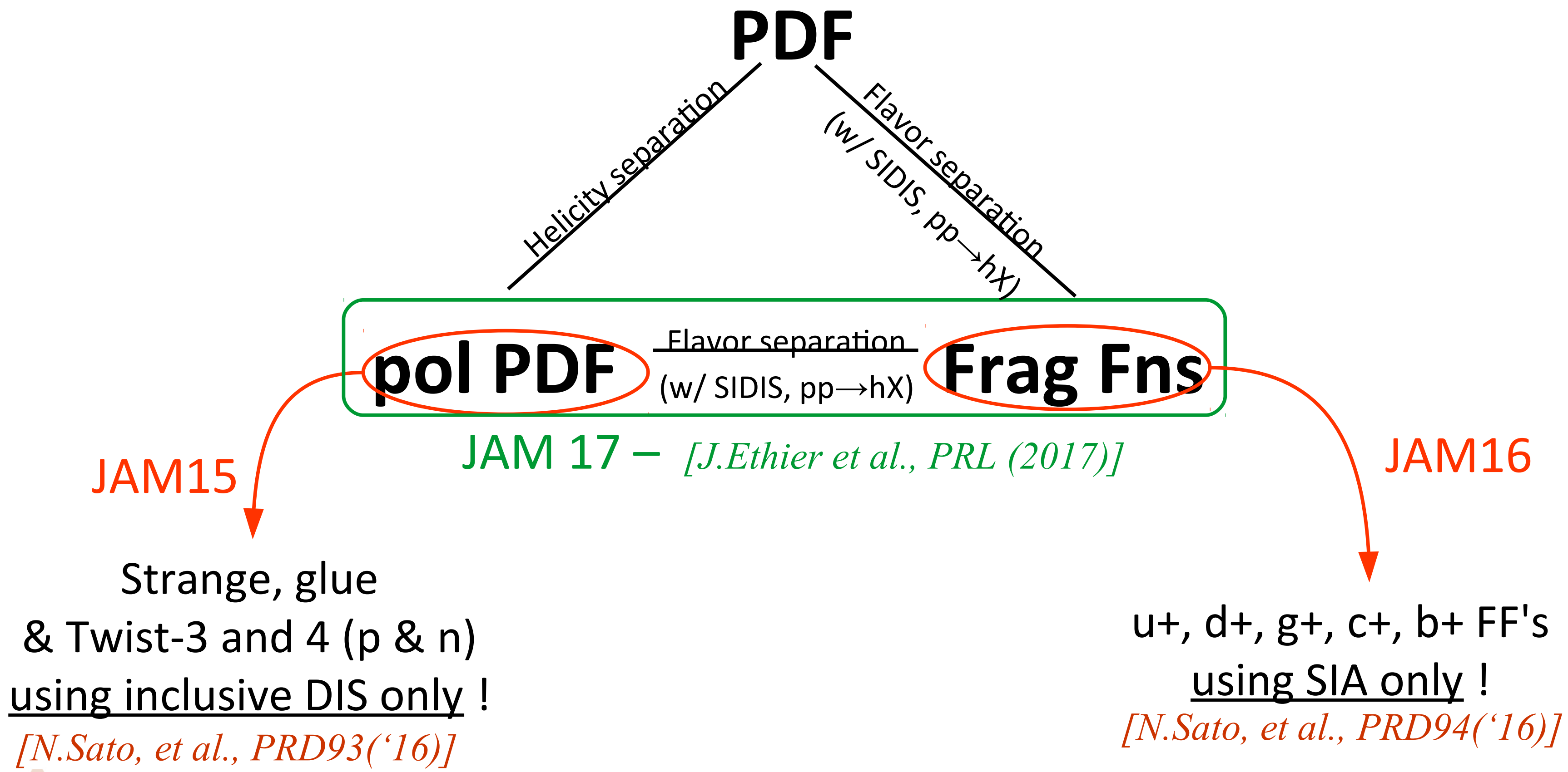


Perturbative instability at small z?
Do we need resummation?

Beyond global:
Universal fits of FFs and PDFs

Polarized strange puzzle: needs "universal" fit!

- Idea: fit FF and PDFs at the same time
- Lots of parameters, non-trivial correlations
  - Needs flexible, robust method: **JAM's Iterative Monte Carlo!**



Polarized strange puzzle: needs "universal" fit!

First simultaneous extraction of spin-dependent parton distributions and fragmentation functions from a global QCD analysis

J. J. Ethier,[1,2] N. Sato,[3] and W. Melnitchouk[2]

[1] *College of William and Mary, Williamsburg, Virginia 23187, USA*
[2] *Jefferson Lab, Newport News, Virginia 23606, USA*
[3] *University of Connecticut, Storrs, Connecticut 06269, USA*

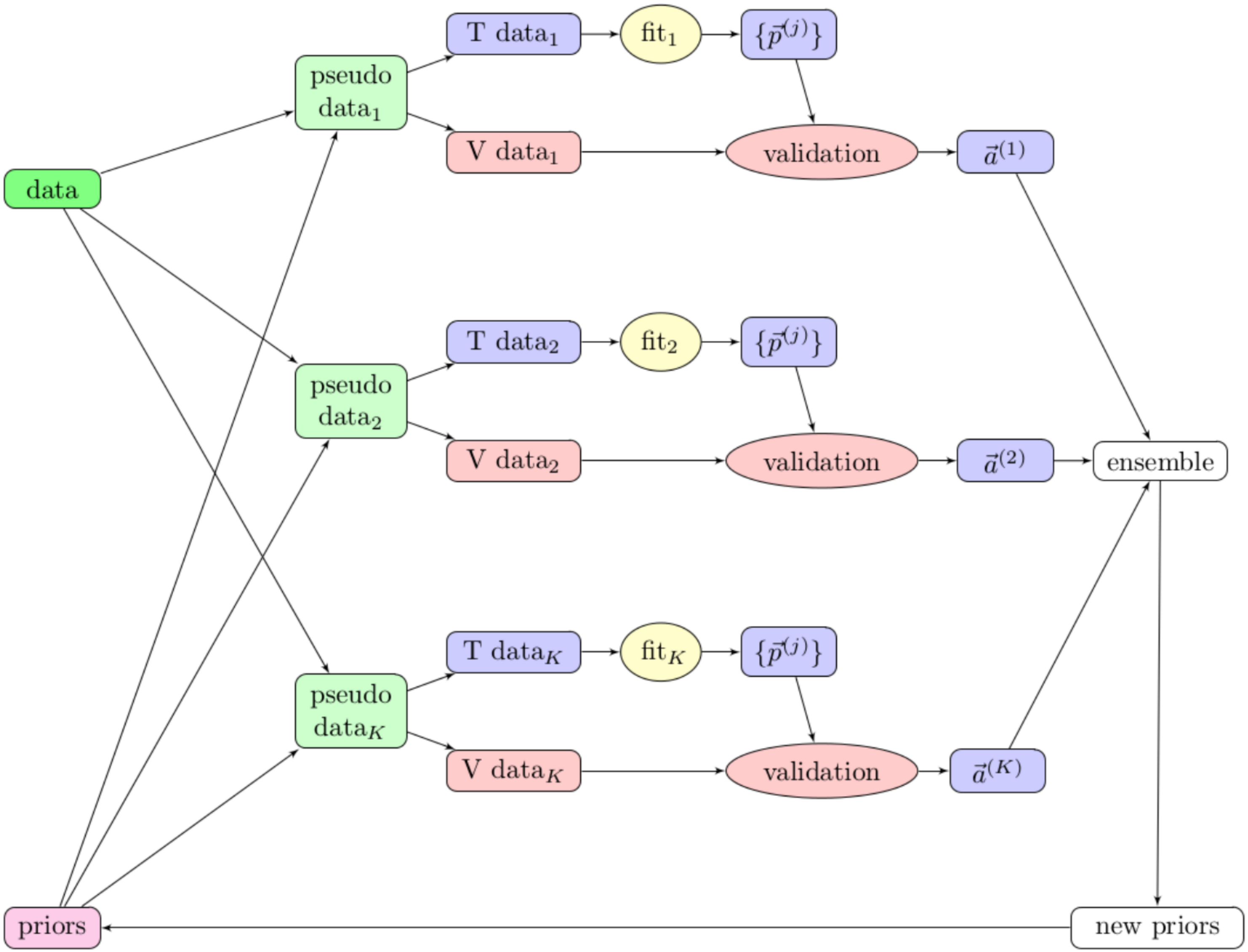
Jefferson Lab Angular Momentum (JAM) Collaboration

(Dated: May 18, 2017)

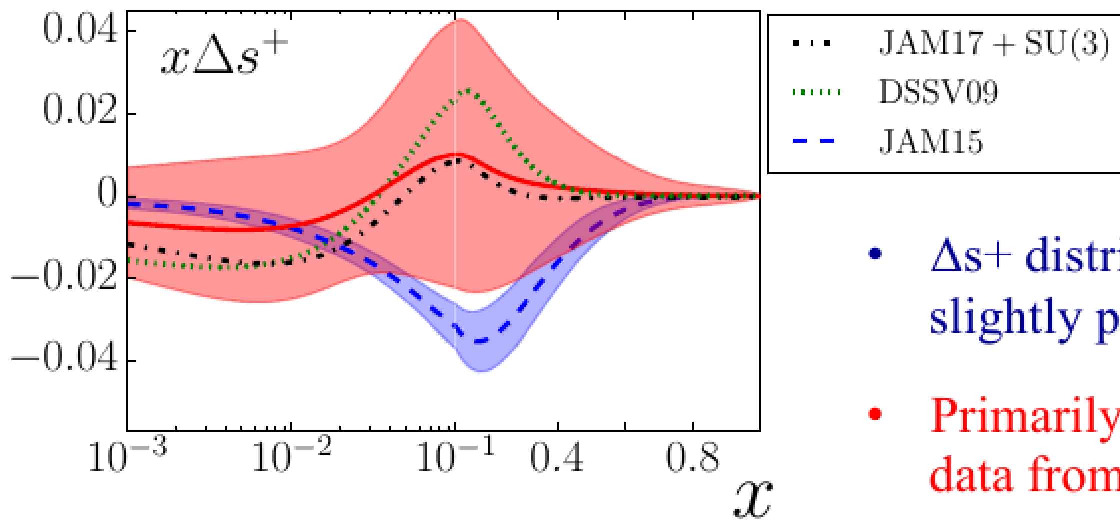
We perform the first global QCD analysis of polarized inclusive and semi-inclusive deep-inelastic scattering and single-inclusive e^+e^- annihilation data, fitting simultaneously the parton distribution and fragmentation functions using the iterative Monte Carlo method. Without imposing SU(3) symmetry relations, we find the strange polarization to be very small, consistent with zero for both inclusive and semi-inclusive data, which provides a resolution to the strange quark polarization puzzle. The combined analysis also allows the direct extraction from data of the isovector and octet axial charges, and is consistent with a small SU(2) flavor asymmetry of the polarized sea.

Most slides by J.Ethier – mistakes, misinterpretations are all on me

The Iterative Monte Carlo procedure



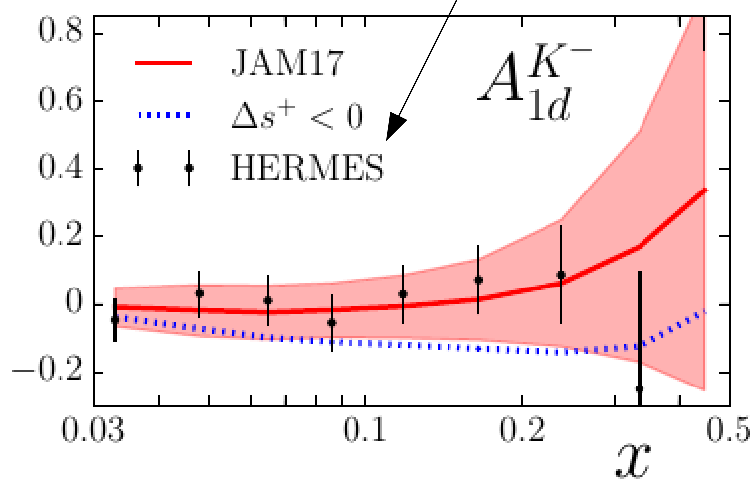
Strange polarization



- Δs+ distribution consistent with zero, slightly positive in intermediate *x* range
- Primarily influenced by HERMES K- data from deuterium target

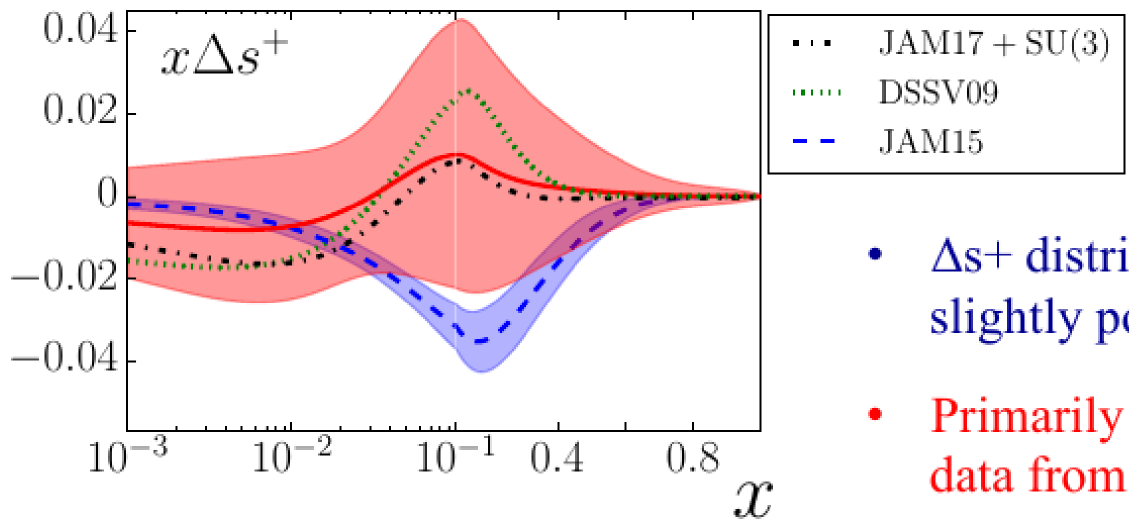
process	target	N_{dat}	χ^2
DIS	$p, d, {}^3\text{He}$	854	854.8
SIA $(\pi^\pm, K^\pm)$		850	997.1
SIDIS $(\pi^\pm)$			
HERMES [15]	d	18	28.1
HERMES [15]	p	18	14.2
COMPASS [16]	d	20	8.0
COMPASS [17]	p	24	18.2
SIDIS $(K^\pm)$			
HERMES [15]	d	27	18.3
COMPASS [16]	d	20	18.7
COMPASS [17]	p	24	12.3
Total:		**1855**	**1969.7**

Hadron mass corrections marginally lower the data



Similar sensitivity from COMPASS K⁻(d)

Strange polarization

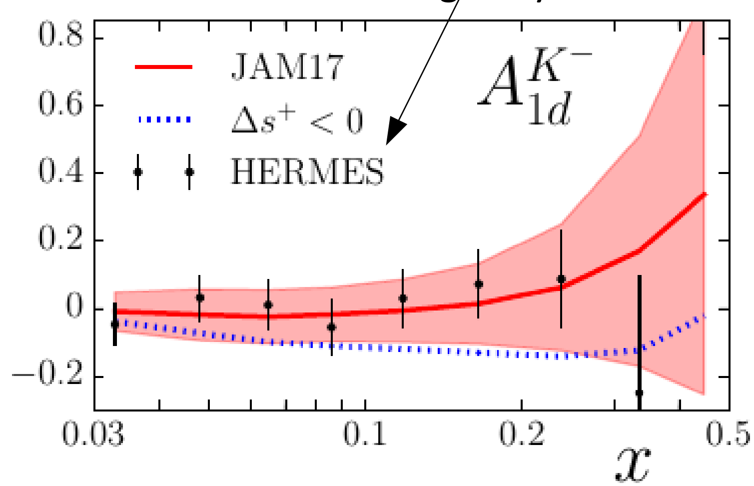


- Δs+ distribution consistent with zero, slightly positive in intermediate x range
- Primarily influenced by HERMES K- data from deuterium target

Why does DIS+SU(3) give large negative Δs+?

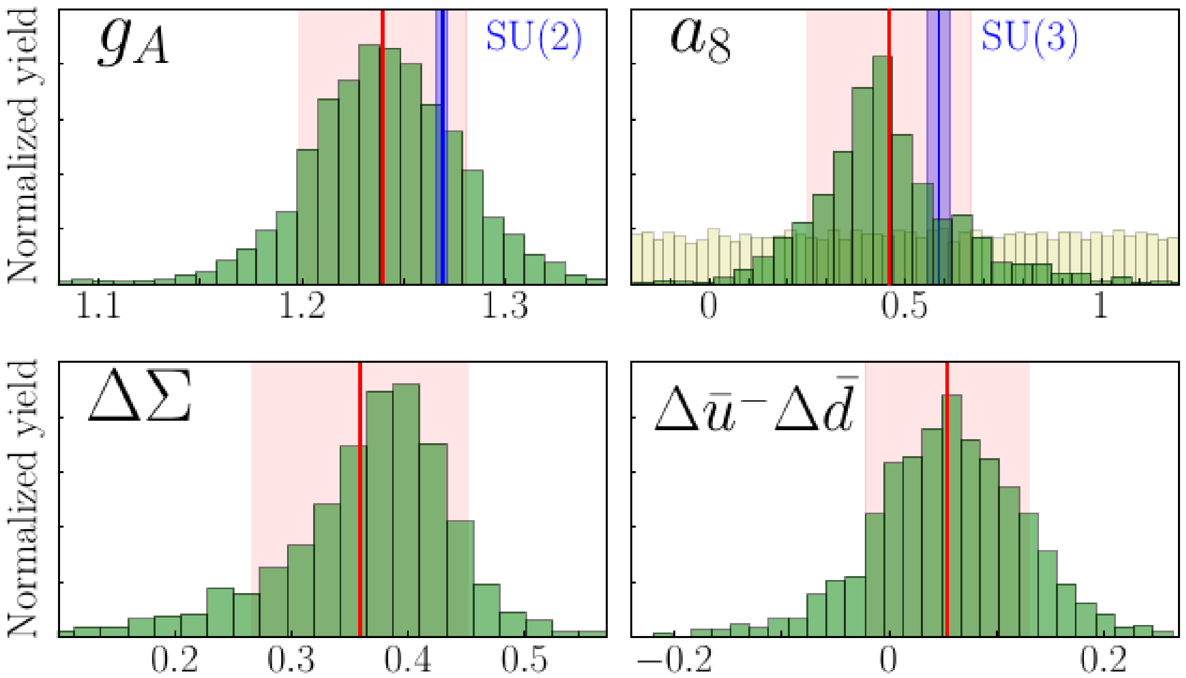
- Low x DIS deuterium data from COMPASS prefers small negative Δs+
- Needs to be more negative in intermediate region to satisfy SU(3) constraint
- b parameter for Δs+ typically fixed to values ~6-10, producing a peak at x~0.1

Hadron mass corrections marginally lower the data



Similar sensitivity from COMPASS K⁻(d)

Moments



$g_A = 1.24 \pm 0.04$ Confirmation of SU(2) symmetry to ~2%

$a_8 = 0.46 \pm 0.21$ ~20% SU(3) breaking ± ~20%; large uncertainty

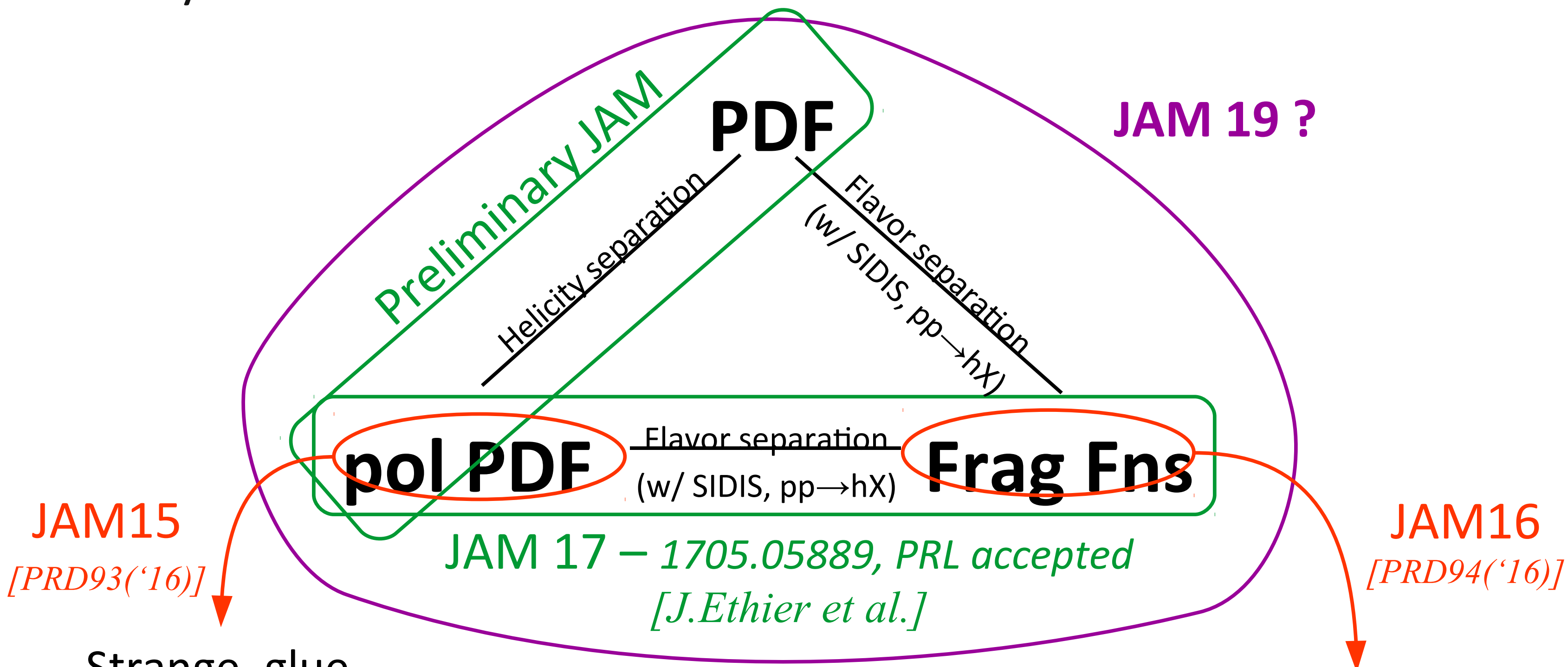
- Need better determination of $\Delta s+$ moment to reduce a_8 uncertainty!

$$\Delta \mathbf{s}^+ = -0.03 \pm 0.09$$

JAM - Iterative Monte Carlo approach

N.Sato et al [JAM], ***PRD93 (2016) 074005*** *and* ***PRD94 (2016) 114004***

- Provides control over large number of parameters
- Maximizes extraction of physics information from data
- Statistically robust uncertainties



Strange, glue
& Twist-3 and 4 (p & n)
using inclusive DIS only !

u+, d+, g+, c+, b+ FF's
using SIA only !

Beyond "leading twist" fits: hadron mass corrections

In collaboration with J.Guerrero

Guerrero, Accardi, PRD 97 (2018) 114012

Guerrero, Ethier, Accardi, Melnitchouk, Casper, JHEP 1509 (2015) 169

Accardi, Hobbs, Melnitchouk, JHEP 0911 (2009) 084

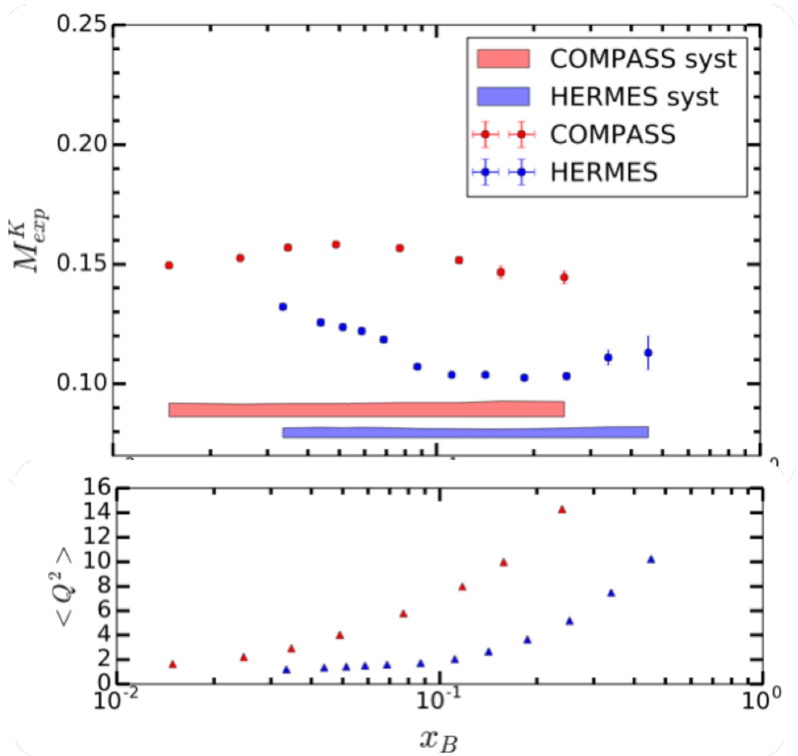
Kaons at HERMES and COMPASS

HERMES:

- Claim very different s-quark shape compared to CTEQ6L
- the strange PDF may not be what we think!

But COMPASS:

- Different x_B dependence
- Overall values higher



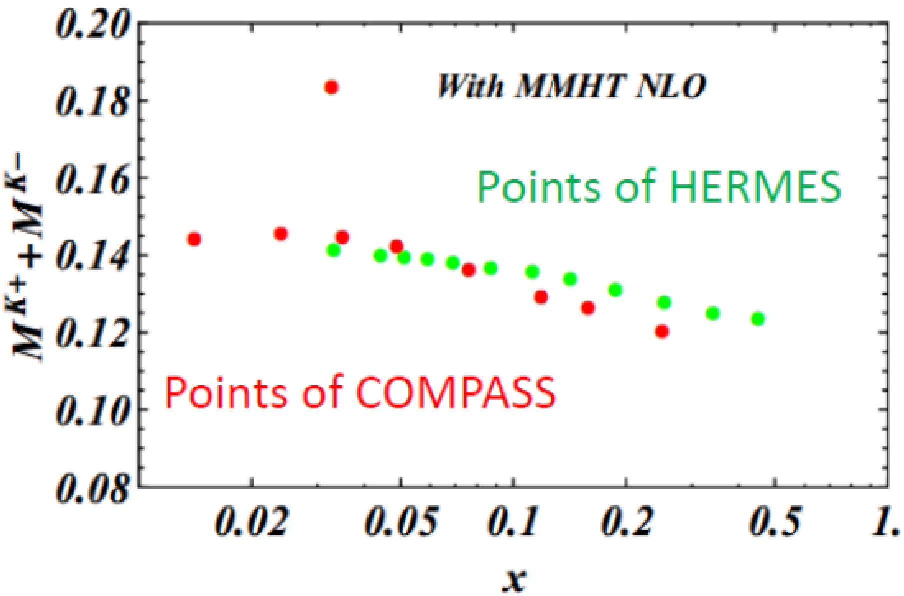
Where does this difference come from?

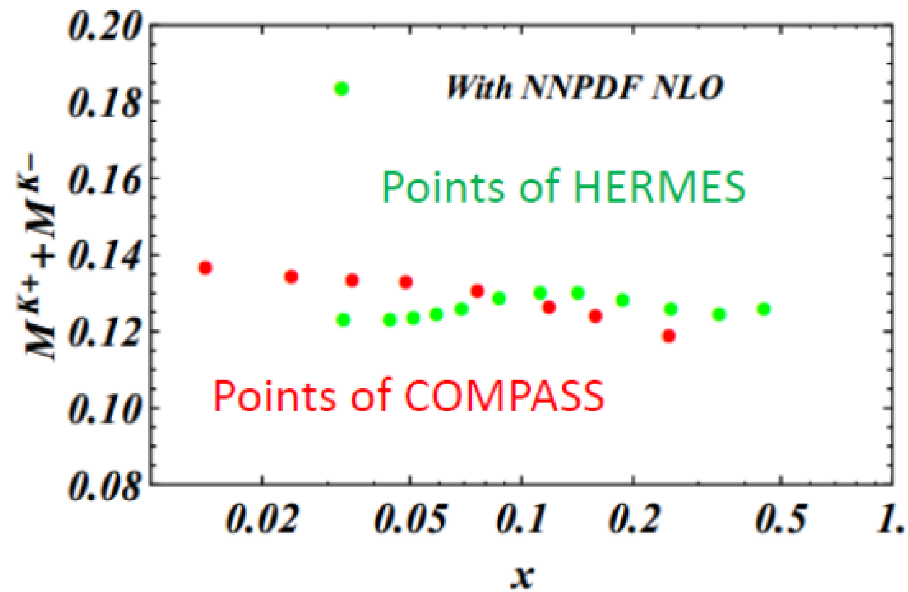
Is it real or apparent?

NLO, Q2 evolution?

- NLO theory at HERMES and compass kinematics:

Plots from Chung-Wen Kao, talk at DIS 2018





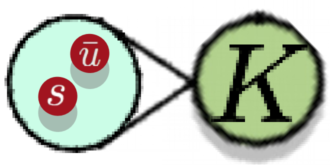
- MMHT+DSS17
- NNPDF +DSS17

→

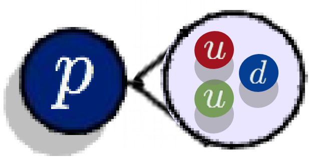
- Expect H & C similar
- Small Q^2 evolution
- Theory shapes =/= data
- Other effects?

Hadron Mass Corrections?

- In pQCD, the mass of proton and detected hadron are usualy neglected

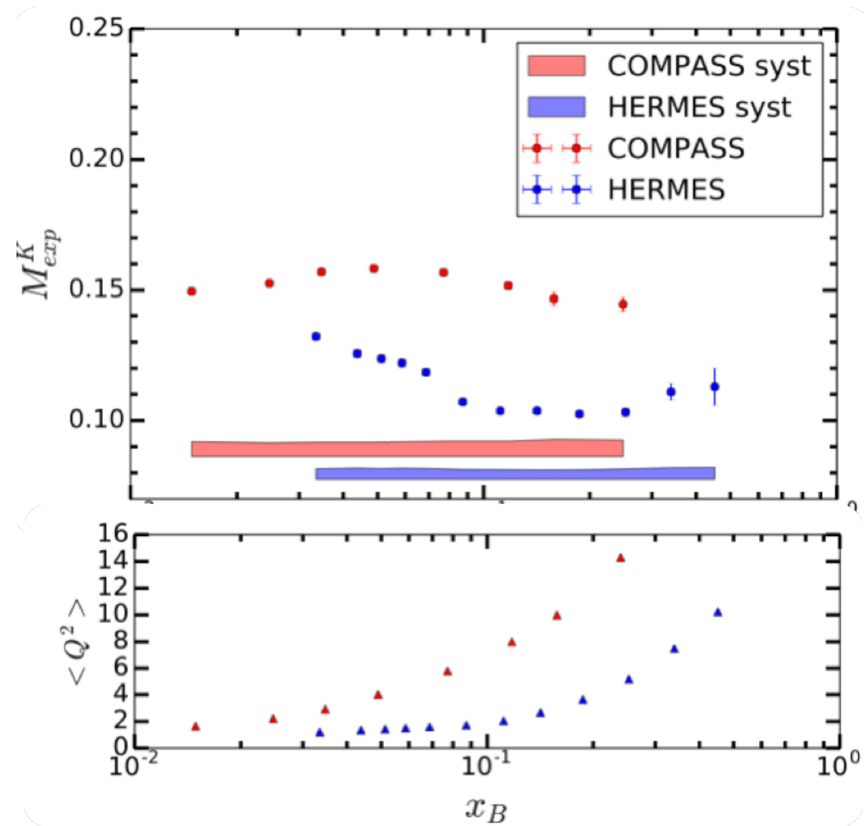


$m_K \simeq 0.5$ GeV



$m_p \simeq 1$ GeV

$\overline{Q^2}_{\rm C} \gtrsim \overline{Q^2}_{\rm H} \simeq 1 - 10\ {\rm GeV}^2$



Maybe masses are not so negligible!

Hadron Mass Effects

- Kinematic mass effects can be captured by suitable scaling variables

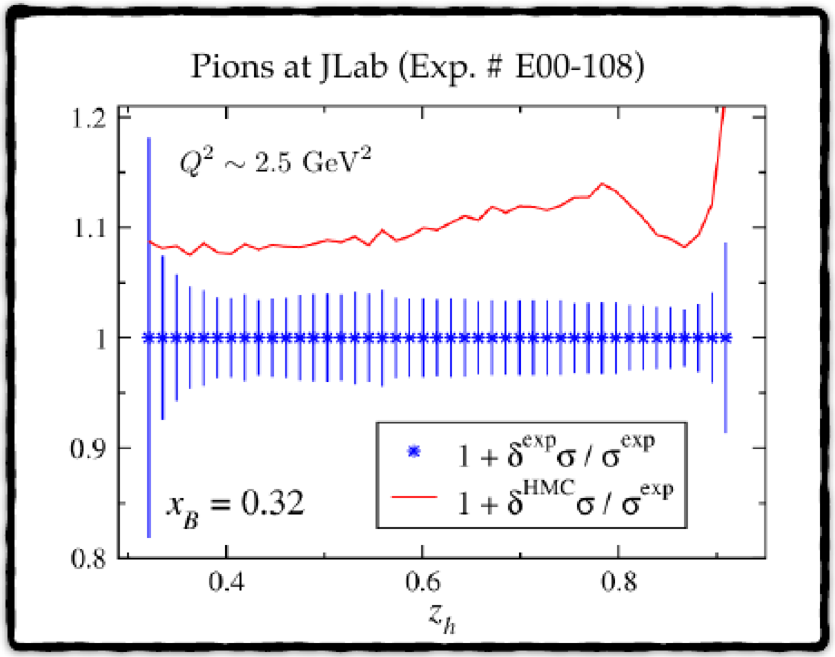
Guerrero et al JHEP 1509, 169 (2015)

$$x_B \longrightarrow \xi_h \equiv \xi\Big(1 + \frac{m_h^2}{\zeta_h Q^2}\Big)$$

$$z_h \longrightarrow \zeta_h = \frac{z_h}{2}\frac{\xi}{x_B}\left(1 + \sqrt{1 - \frac{4x_B^2 M^2 m_h^2}{zh z_h^2\, Q^4}}\right)$$

$$\xi = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 M^2/Q^2}}$$

Accardi et al JHEP 0911, 084 (2009)



$m_\pi \sim 0.14$ GeV

- These allow one to match the internal partonic kinematics to the external, measurable hadronic kinematics

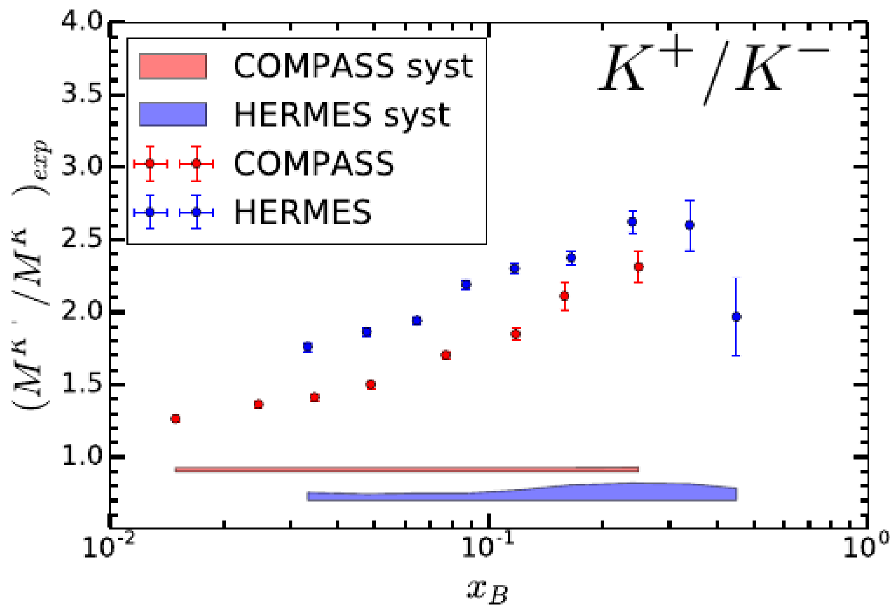
Direct data comparison: K^+/K^- ratios

Guerrero, Accardi PRD 97 (2018) 114012

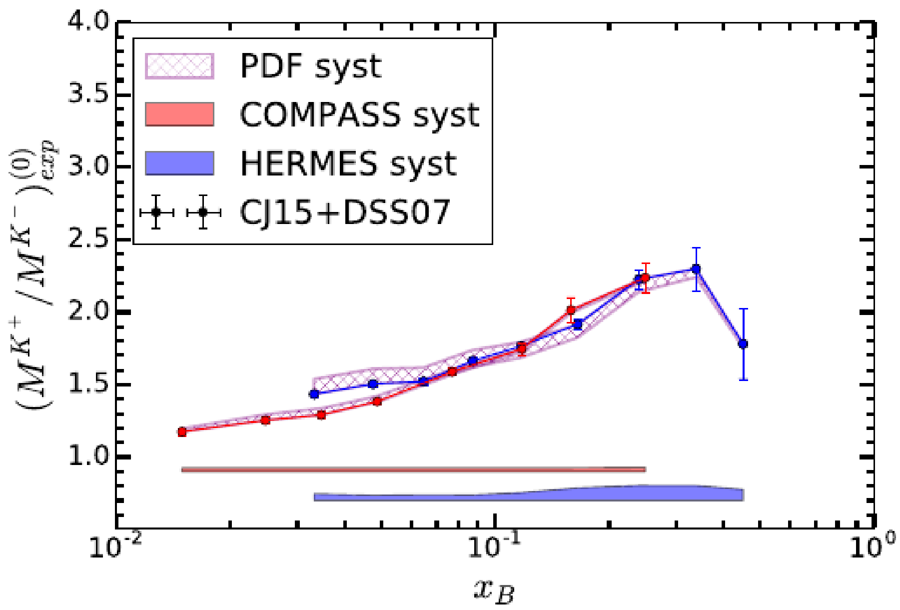
$$r_{\exp} = \left(\frac{M^{K^+}}{M^{K^-}}\right)_{\exp}$$

- COMPASS: $r^{(0)}_{\exp} \equiv r_{\exp} \times R_{HMC} \times R^{H\to C}_{evo}$
- HERMES: $r^{(0)}_{\exp} \equiv r_{\exp} \times R_{HMC}$

Experimental Data



"Massless data" at same Q²



- HERMES & COMPASS fully compatible after removing HMCs.

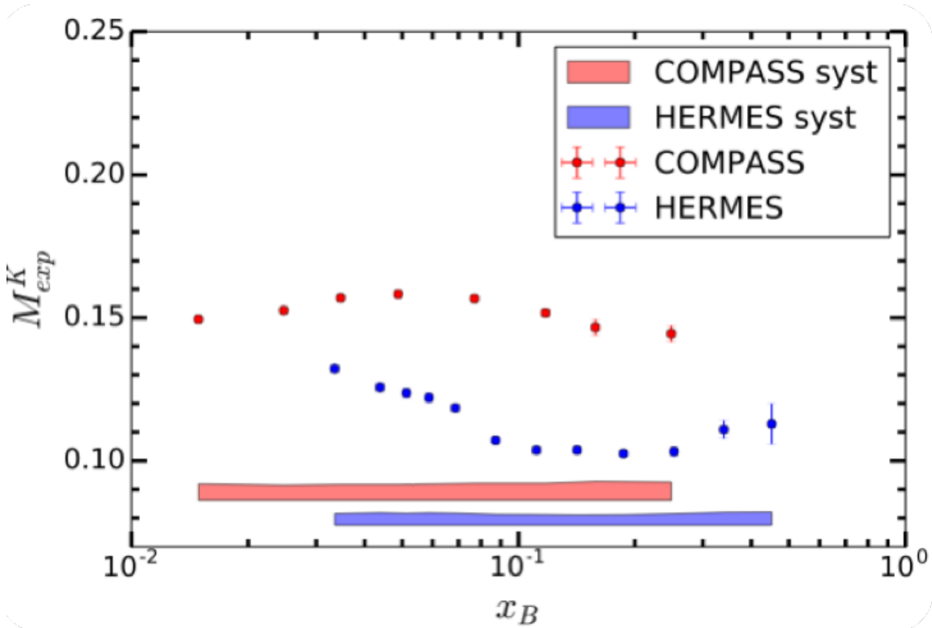
Direct data comparison: $K^+ + K^-$ multiplicities

Guerrero, Accardi PRD 97 (2018) 114012

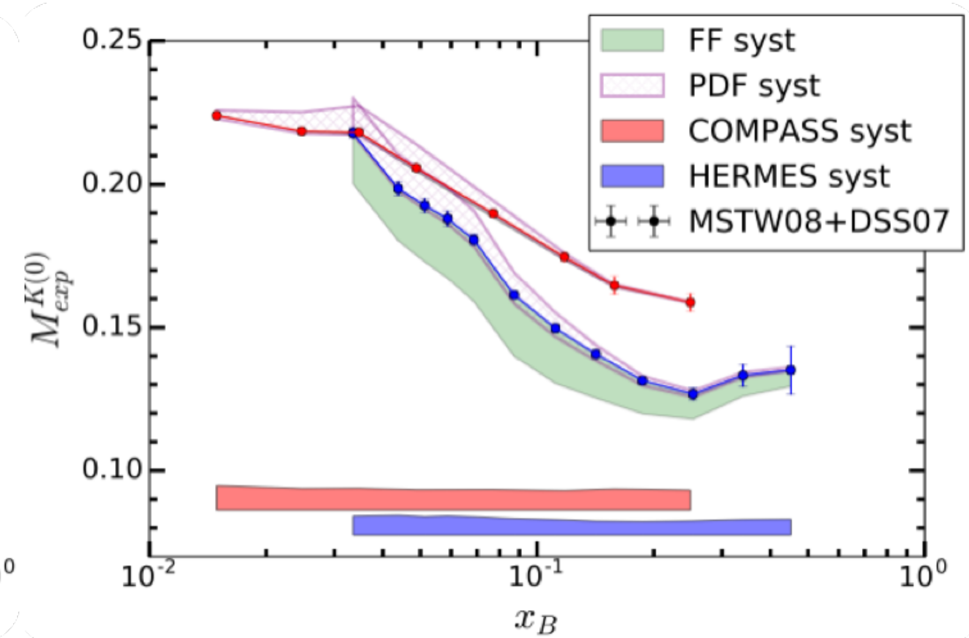
$$r_{\rm exp} = \left(\frac{M^{K^+}}{M^{K^-}}\right)_{\rm exp}$$

- COMPASS: $r^{(0)}_{\rm exp} \equiv r_{\rm exp} \times R_{HMC} \times R^{H\to C}_{evo}$
- HERMES: $r^{(0)}_{\rm exp} \equiv r_{\rm exp} \times R_{HMC}$

Experimental Data



"Massless data" at same Q^2



Remaining small difference in (now negative) slope: NLO effects? Refinements in scaling variables?

New TMD-FF sum rules

Accardi, Signori, in preparation
& PoS (DIS 2018) proceedings

Accardi, Bacchetta, PLB 773 (2017) 632

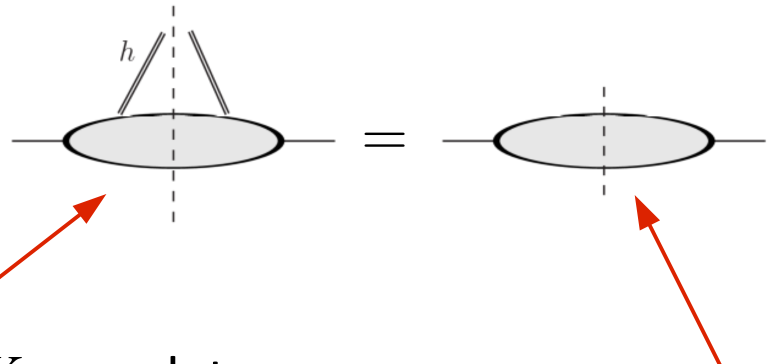
Semi-inclusive vs. inclusive correlators - 1

- Idea: generalize quark sum rule *AA, Bachetta '17 (Collins, Soper '82 ; Meissner, Metz, Pitonyak '10)*

$$\sum_{h,S_h} \int d^2 p_{hT} \, \frac{dp_h^-}{2p_h^-} \, p_h^- \, \Delta^h(l, p_h) = l^- \, \Xi(l)$$

Graphically:

$$\sum_{h,S_h} \int d^2 p_{hT} dz$$



SIDIS $q \to h{+}X$ correlator

"Jet" $q \to X$ correlator
(dressed & cut quark prop.)

- Dirac projections $\Longrightarrow$ TMD-level sum rules
- Use $p_{hT}{}^{\alpha}$ $\Longrightarrow$ further sum rules

Semi-inclusive vs. inclusive correlators - 2

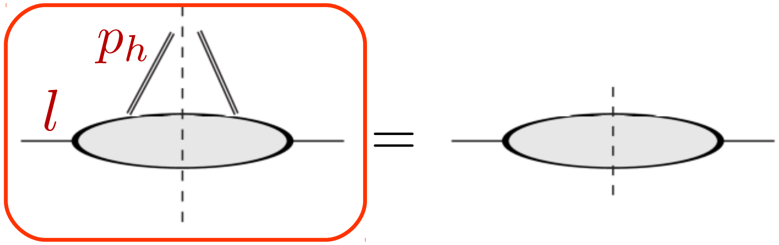
- Idea: generalize quark sum rule

AA, Bachetta '17
(Collins, Soper '82 ; Meissner, Metz, Pitonyak '10)

$$\sum_{h,S_h} \int d^2 p_{hT} \frac{dp_h^-}{2p_h^-} p_h^- \Delta^h(l,p_h) = l^- \, \Xi(l)$$

Graphically:

$$\frac{1}{2} \sum_{h,S_h} \int d^2 p_{hT} dz$$



Semi-inclusive $q \to h + X$ fragmentation correlator:

$$\Delta_{ij}(l,p_h;n_+) = \sum_X F.T. \langle 0 | \mathcal{U}^{n_+}_{(+\infty,\eta)} \, \psi_i(\eta) | h, X \rangle \langle h, X | \bar{\psi}_j(0) \, \mathcal{U}^{n_+}_{(0,+\infty)} \, | 0 \rangle$$

Semi-inclusive vs. inclusive correlators - 2

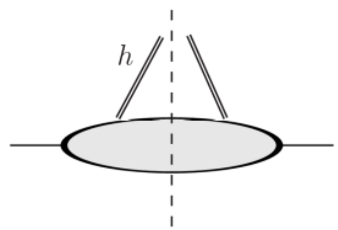
- Idea: generalize quark sum rule

AA, Bachetta '17
(Collins, Soper '82 ; Meissner, Metz, Pitonyak '10)

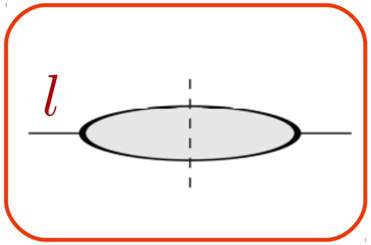
$$\sum_{h,S_h} \int d^2 p_{hT} \frac{dp_h^-}{2p_h^-} p_h^- \Delta^h(l, p_h) = l^- \, \Xi(l)$$

Graphically:

$$\frac{1}{2} \sum_{h,S_h} \int d^2 p_{hT} dz$$



$=$



Semi-inclusive $q \to h{+}X$ fragmentation correlator:

$$\Delta_{ij}(l, p_h; n_+) = \sum_X F.T. \langle 0 | \mathcal{U}^{n_+}_{(+\infty,\eta)} \psi_i(\eta) | h, X \rangle \langle h, X | \bar{\psi}_j(0) \mathcal{U}^{n_+}_{(0,+\infty)} | 0 \rangle$$

Inclusive $q \to X$ "jet" correlator

$$\Xi_{ij}(l, n_+) = F.T. \langle 0 | \mathcal{U}^{n_+}_{(+\infty,\eta)} \psi_i(\eta) \bar{\psi}_j(0) \mathcal{U}^{n_+}_{(0,+\infty)} | 0 \rangle$$

Spectral representation of the quark-jet correlator

AA, Signori '18

- **Convolution representation of the correlator**

$$\Xi(l,n_+) = \int d^4p\, \widetilde{S}(p)\widetilde{W}(l-p;n_+)$$

$$\widetilde{S}_{ij}(p) = F.T.\Big[\psi_i(\xi)_j(0)\Big]$$

$$\widetilde{W}(p;n_+) = F.T.\Big[W(\xi,0;n_+)\Big]$$

- **Spectral decomposition of quark operator**

$$\widetilde{S}(p) = \hat{s}_3(p^2)\not{p} + \sqrt{p^2}\,\hat{s}_1(p^2)\mathbb{I}$$

- p^2 interpreted as
  - mass of the particles inside the jet
  - Controlled by the **quark's spectral functions ρ_1 and ρ_2**

$$\langle\Omega|\hat{s}_{1,3}(p^2)\,|\Omega\rangle = \frac{1}{(2\pi)^3}\rho_{1,3}(p^2)\ .$$

$$\rho_3(\mu^2) \geq |\rho_1(\mu^2)| \quad \text{and} \quad \int_0^\infty d\mu^2 \rho_3(\mu^2) = 1$$

- Wilson line effects retained, show up at twist-4 level

TMD jet correlator in full glory

AA, Signori '18

- Current jet TMD correlator

$$J(l^-, l_T; n_+) \equiv \frac{1}{2} \int dl;^+ \, \Xi(l; n_+)$$

Expand in Dirac structures, take traces, use spectral representation:

$$J(l^-, l_T; n_+) = \frac{1}{2} \not{n}_+ + \frac{1}{2k^-} \not{l}_T + \frac{M_{jet}}{2l^-} \mathbb{I} + \frac{P_{jet}^2 + l_T^2 + T_{jet}^2}{2(l^-)^2} \not{n}_-$$

where

$$M_{jet} = \int_0^\infty d\mu^2 \, \mu \, \rho_1(\mu^2)$$

Jet "mass" ~ chiral condensate ~ O(100 MeV)

$$P_{jet}^2 = \int_0^\infty d\mu^2 \, \mu^2 \, \rho_3(\mu^2)$$

Jet's "virtuality" (inv. Mass of final state)

$$T_{jet}^2 = \langle \Omega | F.T. \Big[-\partial_T^2 W_+(\xi; 0)|_{\xi^- = \xi_T = 0} \Big] \otimes \hat{s}_3(\mu^2) | \Omega \rangle$$

Jet's intrinsic tr. mom. broadening: rescattering on Wilson gluons

TMD jet correlator in full glory

AA, Signori '18

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$$J(l^-, l_T; n_+) \equiv \frac{1}{2} \int dl;^+ \, \Xi(l; n_+)$$

Expand in Dirac structures, take traces, use spectral representation:

$$J(l^-, l_T; n_+) = \frac{1}{2} \not{n}_+ + \frac{1}{2k^-} \not{l}_T + \frac{M_{jet}}{2l^-} \mathbb{I} + \frac{P_{jet}^2 + l_T^2 + T_{jet}^2}{2(l^-)^2} \not{n}_-$$

"Perturbatively": $|\Omega\rangle \longrightarrow |0\rangle$

$$M_{jet}^{pert} = m_q$$ Current quark mass ~ O(1 MeV) << M_{jet}

$$P_{jet}^2{}^{pert} = m_q^2$$ On-shell quark

$$T_{jet}^2 = \langle 0 | F.T. \Big[-\partial_T^2 W_+(\xi; 0)|_{\xi^- = \xi_T = 0} \Big] \, |0\rangle \otimes \rho_3(\mu^2)$$

Wilson line decouples, but still provides intrinsics broadening

Quark-quark TMD sum rules

AA, Signori '18

- General jet correlator sum rule:

$$\sum_{h,S_h} \int d^2 p_{hT} \frac{dp_h^-}{2p_h^-} \, p_h^\mu \, \Delta^h(l, p_h) = F.T. \langle 0 | \, \mathcal{U}^{n_+}_{(+\infty,\eta)} \, \psi_i(\eta) \, \hat{\boldsymbol{P}} \, \bar{\psi}_j(0) \, \mathcal{U}^{n_+}_{(0,+\infty)} \, | 0 \rangle$$

Quark-quark TMD sum rules

AA, Signori '18

- General jet correlator sum rule:

$$\sum_{h,S_h} \int d^2 p_{hT} \frac{dp_h^-}{2p_h^-} p_h^\mu \Delta^h(l,p_h) = \begin{cases} l^\mu \, \Xi(l) & \mu = - \quad \text{longitudinal} \\ 0 & \mu = 1,2 \quad \text{transverse} \end{cases}$$

- For TMDs, take suitable traces:

	Longitudinal	**Transverse**
Twist-2	*Collins-Soper* $\sum_{h,S_h} \int dz \, z \, D_{1h}(z) = 1$	*Schaefer-Teryaev* $\sum_{h,S_h} \int dz \, z \, H_{1h}^{\perp(1)}(z) = 0$

(see Collins, Soper '82 ; Meissner, Metz, Pitonyak '10)

$$f^{(1)}(z) \equiv \int d^2 P_{hT} \frac{P_{hT}^2}{2\Lambda^2} f(z, P_{hT})$$

Quark-quark TMD sum rules

AA, Signori '18

- General jet correlator sum rule:

$$\sum_{h,S_h} \int d^2 p_{hT} \frac{dp_h^-}{2p_h^-} p_h^\mu \Delta^h(l,p_h) = \begin{cases} l^\mu \, \Xi(l) & \mu = - \quad \text{longitudinal} \\ 0 & \mu = 1,2 \quad \text{transverse} \end{cases}$$

- For TMDs, take suitable traces:

	Longitudinal	**Transverse**
Twist-2	*Collins-Soper* $\sum_{h,S_h} \int dz\, z\, D_{1h}(z) = 1$	*Schaefer-Teryaev* $\sum_{h,S_h} \int dz\, z\, H_{1h}^{\perp(1)}(z) = 0$
Twist-3	$\sum_{h,S_h} \int dz\, E_h(z, p_{hT}) = \frac{M_q}{\Lambda}$ NEW!	$\sum_{h,S_h} \int dz\, D_h^{\perp(1)}(z) = 0$ NEW!
	$\sum_{h,S_h} \int dz\, H_h(z) = 0$ NEW!	$\sum_{h,S_h} \int dz\, G_h^{\perp(1)}(z) = 0$ NEW!

$$f^{(1)}(z) \equiv \int d^2 P_{hT} \frac{P_{hT}^2}{2\Lambda^2} f(z, P_{hT})$$

Quark-gluon-quark TMD sum rules

AA, Signori '18

❑ Using Equation of Motion relations in q-q sume rules:

Longitudinal

$$\sum_{h,S_h} \int dz\, \widetilde{E}_h(z) = \frac{M_q - m_q}{\Lambda} \quad \text{NEW!} \quad \Longrightarrow \quad \text{Transversity in DIS!}$$

$$\sum_{h,S_h} \int dz\, \widetilde{H}_h(z) = 0 \quad \text{NEW!} \quad \Longrightarrow \quad \sum_{h,S_h} \int dz z F_{UT}^{\sin\phi_S}(x,z) = 0$$

Diehl-Sapeta

NEW: proven at correlator level

Transverse

$$\sum_{h,S_h} \int dz\, \widetilde{D}^{\perp(1)}(z) = \frac{\langle p_{hT}^2/z^2 \rangle}{2\Lambda^2} \; * \quad \text{NEW!}$$

$$\sum_{h,S_h} \int dz\, \widetilde{G}^{\perp(1)}(z) = 0 \quad \text{NEW!}$$

$M_q^{\text{pert}} = m_q \Rightarrow$ Old sum rule

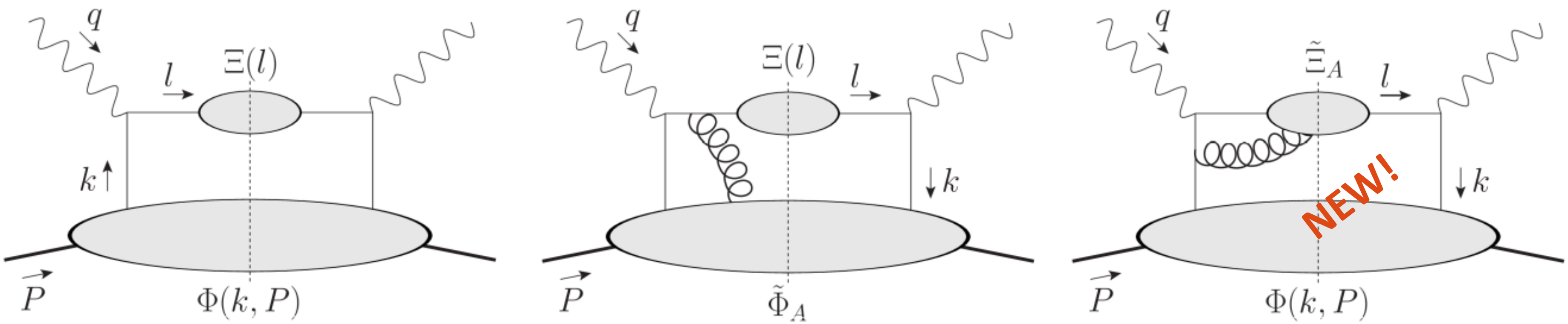
* in the "parton frame", l_T=0
($= \langle l_T^2 \rangle$ in "hadron frame")

Novel phenomenology!

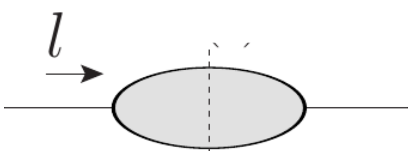
In collaboration with A.Bacchetta
Physics Letters B 773 (2017) 632
+ A.Signori, M.Radici

Inclusive DIS with jet correlators

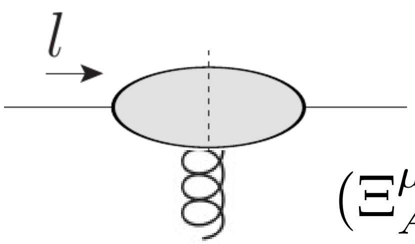
AA, Bacchetta, PLB 773 ('17) 632



Jet correlators: → non-asymptotic quark states



$$\Xi_{ij}(l,n_+) = F.T.\langle 0|\,\mathcal{U}^{n_+}_{(+\infty,\eta)}\,\psi_i(\eta)\bar{\psi}_j(0)\,\mathcal{U}^{n_+}_{(0,+\infty)}\,|0\rangle$$



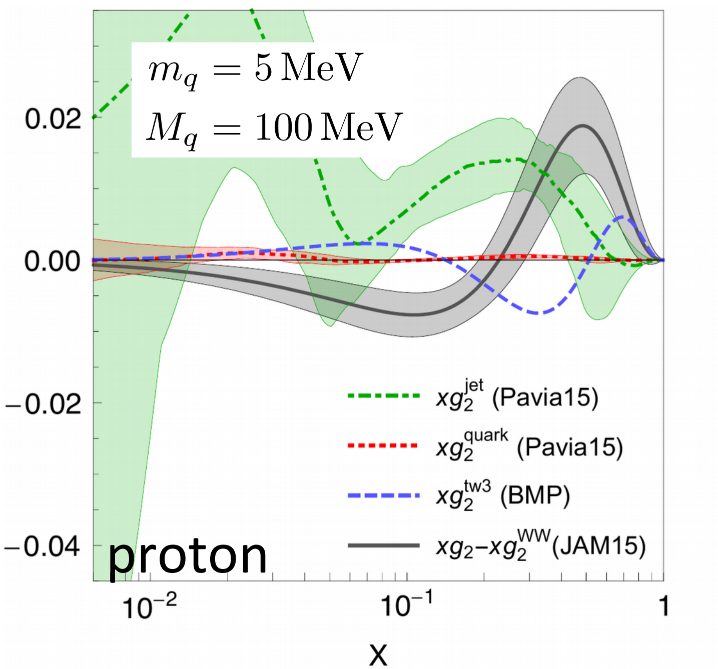
$$(\Xi^\mu_A)_{ij} = F.T.\,\langle 0|\,\mathcal{U}^{n_+}_{(+\infty,\eta)}\,gA^\mu(\eta)\;\psi_i(\eta)\;\bar{\psi}_j(0)\,\mathcal{U}^{n_+}_{(0,+\infty)}\,|0\rangle$$

g2 structure function revisited

AA, Bacchetta, PLB 773 ('17) 632

- Integrating SIDIS, and using EOM, Lorentz Invariance Relations:

$$g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2}\sum_a e_a^2 \left(g_2^{q,\mathrm{tw3}}(x_B) + \underbrace{\frac{m_q}{M}\left(\frac{h_1^q}{x}\right)^\star(x_B)}_{\equiv\, g_2^{quark}} + \underbrace{\frac{M_q - m_q}{M}\frac{h_1^q(x_B)}{x_B}}_{\equiv\, g_2^{jet}} \right)$$



Consequences:

- h1 accessible in inclusive DIS
  - ↔ Potentially large signal
- Burkardt-Cottingham sum rule broken

$$\int_0^1 g_2(x) = M_{\text{“jet”}} \int_0^1 dx\, \frac{h_1(x)}{x}$$

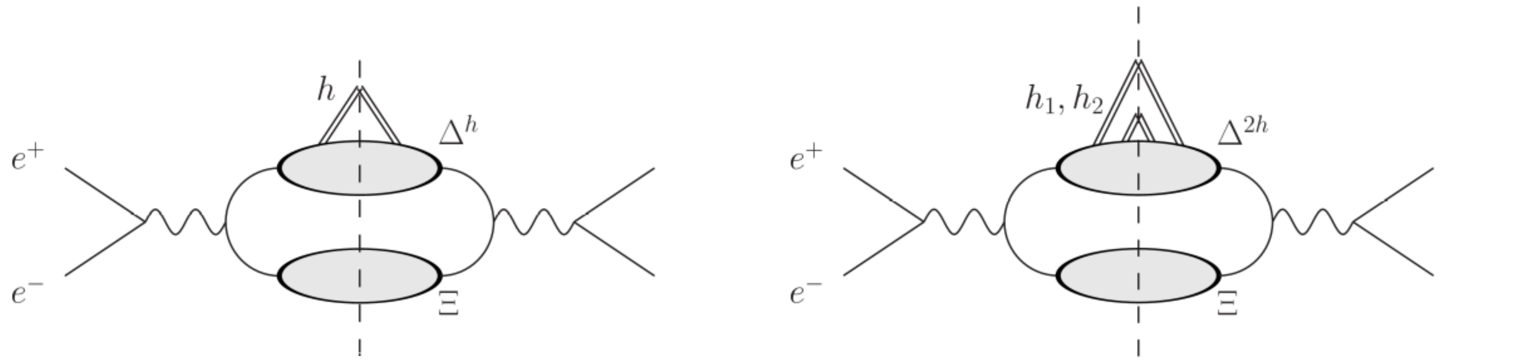
- ETL: novel way to measure tensor charge

$$\int_0^1 x g_2^{q-\bar{q}}(x) = 2\, M_{\text{“jet”}} \int_0^1 dx\, h_1^{q-\bar{q}}(x)$$

Measuring the jet correlator

Accardi, Bacchetta, Signori, Radici, in prep.

- Jet mass $\mathbf{M_{jet}}$ **can be measured in polarized** $\mathbf{e^+ + e^-}$ **:**



  - Needs **LT asymmetry** in semi-inclusive **Lambda** production

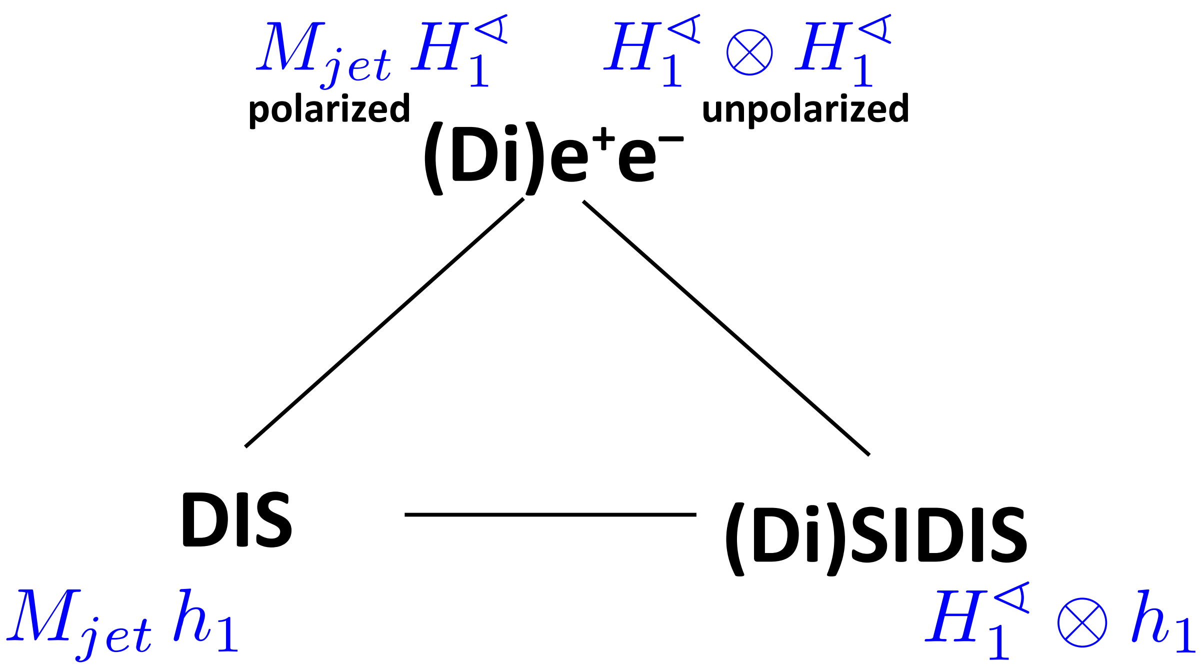
$$\frac{d\sigma^L(e^+e^- \to \text{jet}\, h\, X)}{d\Omega dz} = \frac{3\alpha^2}{Q^2}\lambda_e \sum_a e_a^2 \left\{ \frac{C(y)}{2}\lambda_h G_1 + D(y)|\boldsymbol{S}_T| \cos(\phi_S)\frac{2M_h}{Q}\left(\frac{G_T}{z} + \frac{M_q - m_q}{M_h}H_1\right)\right\}$$

  - Similarly a **LU asymmetry** in unpolarized **dihadron** production

A new “universal” fits

- **Chiral even case**
  - Discussed before
  - Unpolarized PDFs, long. polarized PDFs, FFs simultaneously

- **Chiral-odd collinear case:**



Where can we measure jet correlators?

- Can we get a (polarized) **e+ e- collider at JLab / BNL?**
  - At JLab12 ? EIC + positron beam ?
- Are **existing facilities** enough?

	BEPC	super KEKB	ILC	JLab/BNL
E beam [GeV]	1.9	4 (e^{-}) 7 (e^{-})	250	?
√s [GeV]	3 – 5	10	500	?
polarization	?	maybe	80% e^{-} 60% e^{+}	YES!

- What else is interesting to study?
  - **Factorization tests for FFs** (low s, unpol), …

Ideas?

Final thoughts

A partial summary

- **Entering a new FF era:**
  - LHC rich set of data:
    - Tensions with SIDIS, RHIC
    - Novel possibilities: in-jet fragmentation (not discussed here… see reviews)
  - Future EIC: precision SIDIS, collinear and TMD; jets in e+p / e+A
  - Universal fits needed: control FF vs. PDF interplay in global fits
- **Near future:**
  - Low-energy e+e- , SIDIS at Jlab and COMPASS
  - Need Hadron Mass Corrections (even more for TMD program!)
  - Need large-z resummation, QCD factorization tests

    (not discussed here, either… see AA, Anderle Ringer, 2014 + recent COMPASS results)
- **“Inclusive” fragmentation and jet correlators:**
  - Novel FF sum rules
  - New phenomenology → chiral cond., tensor charge in DIS, SIA !!
    - **Need polarized e+e- colliders** for best results

Time for universal fits!

- **Universal fits:**
  - Simultaneous analysis of:
    - PDFs and FFs
    - Inclusive and semi-inclusive processes
  - Controls correlations
  - Propagates exp. uncertainties across "sectors"

- **Theoretical advantages**
  - Lessen the need for theoretical simplifications
  - Sophisticated statistical analysis
  - **Maximize access to known and novel correlators!**

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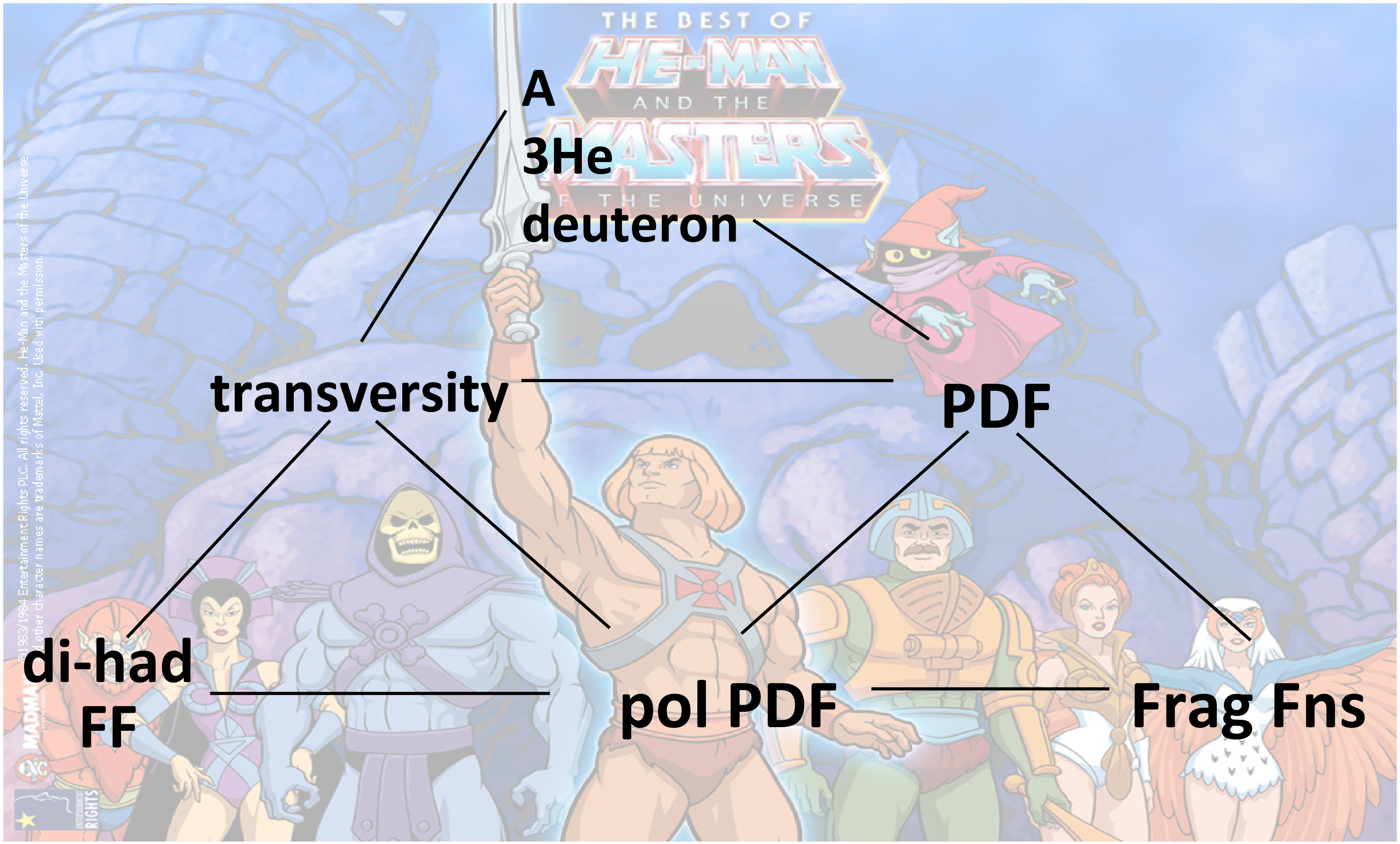

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