Global fits of PDF

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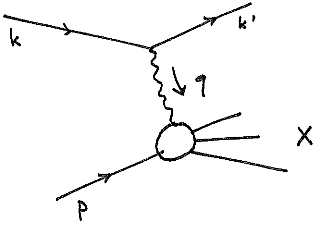
understanding the structure of the proton

- definition of PDFs - theoretical issues, comparison with NP (lattice) formulations
- current fits - methodologies & results
- impact on phenomenology - future targets
- future results: new data, theoretical progress

PDF from DIS



$$k^\mu = k'^\mu + q^\mu$$
$$Q^2 = -q^2$$
$$\nu \;= p \cdot q, \quad x = Q^2/(2\nu)$$

$$\boxed{d\sigma = d\Phi\, L^{\mu\nu}(k,k') W_{\mu\nu}(p,q)}$$

$$\begin{aligned} W_{\mu\nu}(p,q) &= \frac{1}{4\pi} \sum_X \langle p | j_\mu(0)^\dagger | X \rangle \langle X | j_\nu(0) | p \rangle (2\pi)^4 \delta\left(p_X - p - q\right) \\ &= \frac{1}{4\pi} \int d^4y\, e^{iq\cdot y}\, \langle p | j_\mu(y)^\dagger j_\nu(0) | p \rangle \end{aligned}$$

structure functions and PDFs

$$W_{\mu\nu}(p,q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(Q^2,\nu) + \\ + \left(p_\mu - q_\mu \frac{\nu}{q^2}\right)\left(p_\nu - q_\nu \frac{\nu}{q^2}\right) F_2(Q^2,\nu)/\nu$$

Factorization:

$$\boxed{\begin{aligned} F_2(x,Q^2) &= x \sum_i \int \frac{dz}{z}\, C_i\left(z,\mu^2\right)\, f_i\left(\frac{x}{z},\mu^2\right) \\ &= x \sum_{\mathbf{i}} C_i(x,\mu^2) \otimes f_i(x,\mu^2) \end{aligned}}$$

DGLAP evolution

- dependence on the factorization scale

$$\mu^2 \frac{d}{d\mu^2} f_i(x, \mu^2) = \sum_j P_{ij}\left(x, \alpha_s(\mu^2)\right) \otimes f_j(x, \mu^2)$$

- $P_{ij}\left(x, \alpha_s(\mu^2)\right)$: perturbative Altarelli-Parisi splitting functions

$\hookrightarrow$ LO, NLO, NNLO PDFs

- Solution of the evolution equation:

$$\boxed{f_i(x, \mu^2) = \sum_j \Gamma_{ij}(x, \alpha_s, \alpha_s^0) \otimes f_j(x, \mu_0^2)}$$

the non-singlet PDF

Simple example:

$$
\begin{aligned}
F_2^{\mathrm{NS}}(x,Q^2) &= F_2^{\mathrm{p}}(x,Q^2) - F_2^{\mathrm{d}}(x,Q^2) \\
&= C_{\mathrm{NS}}(x,Q^2) \otimes f_{\mathrm{NS}}\left(x,Q^2\right)
\end{aligned}
$$

where

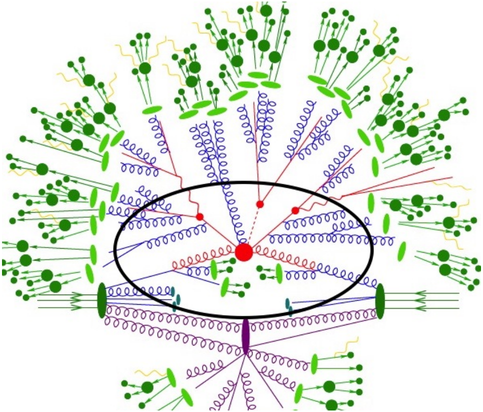
$$
f_{\mathrm{NS}}(x,Q^2) = \left[\left(u(x,Q^2) + \bar{u}(x,Q^2)\right) - \left(d(x,Q^2) + \bar{d}(x,Q^2)\right)\right]
$$

combining the evolution and the coefficient function

$$
F_2^{\mathrm{NS}}(x,Q^2) = \int_x^1 \frac{dy}{y}\, K_{\mathrm{NS}}\left(y, \alpha_s\left(Q^2\right), \alpha_s\left(Q_0^2\right)\right)\, f_{\mathrm{NS}}\left(\frac{x}{y}, Q_0^2\right)
$$

collider processes

$$\boxed{\sigma(H_1H_2 \to X) = \sum_{i,j} \int dx_1 dx_2\ f_{i/H_1}(x_1,\mu^2) f_{j/H_2}(x_2,\mu^2) \times \\ \times\ \hat{\sigma}_{ij\to X}(x_1 x_2 s, \mu^2, \mu_R^2)}$$



lattice data

a lot of activity on proton structure recently (cfr Kostas talk)

quasi-PDF [Ji et al]

$$\boxed{\mathcal{M}_i(\zeta, P) = \langle P|\bar{\psi}(\zeta)\Gamma_i \,\mathrm{P}\exp\left(-ig\int_0^\zeta d\eta\, A(\eta)\right)\psi(0)|P\rangle}$$

$$q(x,\mu,M_N,P_z) = \int \frac{dz}{4\pi}\, e^{-i(xP_z)z}\,\mathcal{M}_z(z,P_z)$$

$$q(x,\mu,M_N,P_z) = C_Q\left(x,\frac{P_z}{\mu}\right)\otimes f(x,\mu) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M_N^2}{P_z^2}\right)$$

fit methodology

χ^2 minimization

$$\chi^2[f] = \sum_{i,j=1}^{N_{\rm dat}} \Big(D_i - T_i[f]\Big) \left(\mathrm{cov}^{-1}\right)_{ij} \Big(D_j - T_j[f]\Big)$$

for each PDF

$$xf(x, Q_0^2) = Ax^\alpha (1-x)^\beta I(x)$$

$I(x)$: polynomial function or neural network

NN allow ≈ 40 parameters for each PDF
$\hookrightarrow$ redundant parametrization, small bias

Hessian method

$$H_{ij} = \frac{1}{2} \left. \frac{\partial^2 \chi^2}{\partial a_i \partial a_j} \right|_0$$

error on quantity $\mathcal{F}$

$$\Delta \mathcal{F} = \frac{1}{2} \sqrt{\sum_i \left[\mathcal{F}(S_i^+) - \mathcal{F}(S_i^-) \right]^2}$$

correlations

$$\rho(\mathcal{F}, \mathcal{G}) = \frac{1}{4 \Delta \mathcal{F} \Delta \mathcal{G}} \sum_i \left[\mathcal{F}(S_i^+) - \mathcal{F}(S_i^-) \right] \left[\mathcal{G}(S_i^+) - \mathcal{G}(S_i^-) \right]$$

MC method generate *replicas* that reproduce the data distribution

$$\langle \mathcal{F} \rangle = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \mathcal{F}[q^{(k)}]$$

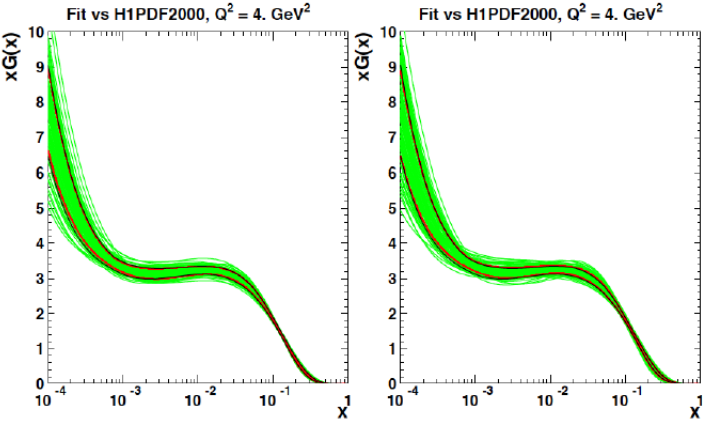
error on quantity $\mathcal{F}$

$$\Delta \mathcal{F} = \sqrt{\frac{1}{N_{\rm rep}-1} \sum_{k=1}^{N_{\rm rep}} \left(\mathcal{F}[q^{(k)}] - \langle \mathcal{F} \rangle \right)^2}$$

correlations

$$\rho(\mathcal{F}, \mathcal{G}) = \frac{1}{\Delta \mathcal{F} \Delta \mathcal{G}} \Big[\langle \mathcal{F} \mathcal{G} \rangle - \langle \mathcal{F} \rangle \langle \mathcal{G} \rangle \Big]$$

comparing errors



bayesian reweighting

- in the MC approach **all** replicas are equivalent
- new data can be incorporated by a new fit **or**
- by changing the weight of the existing replicas

$$\boxed{w_k \propto \left(\chi^2_k\right)^{(n-1)/2} e^{-\chi^2_k/2}}$$

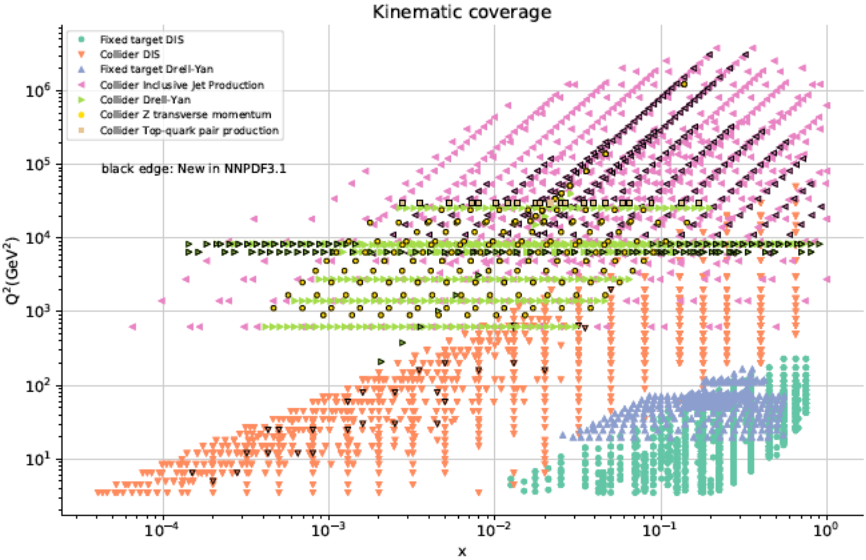
current fits

- focus on latest NNPDF3.1 set – fitted charm + LHC data
- discuss new data and their impact
- current precision on PDFs
- can we identify areas for synergy with lattice results?

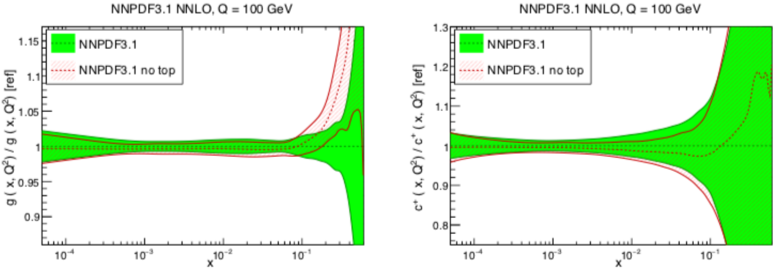
datasets

- fixed-target & collider DIS (NMC, BCDMS, SLAC, HERA final)
- charm production, F_2^b (HERA)
- neutrino DIS (CHORUS, NuTeV)
- inclusive jets, Z rapidity distribution, lepton asymmetries, fixed target DY (Tevatron)
- Z boson double differential, inclusive W/Z rapidity distribution, lepton asymmetries, W+c, Drell-Yan (LHC)
- $t\bar{t}$ distributions, total $t\bar{t}$ cross-section (LHC)
- inclusive jet data (LHC)

kinematic coverage

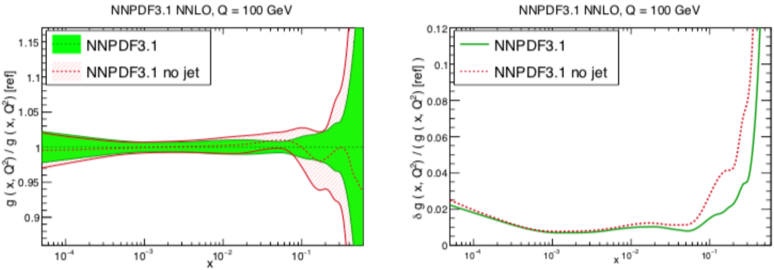


ttbar production



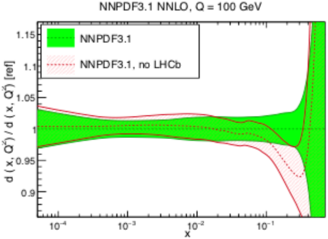
[NNPDF3.1]

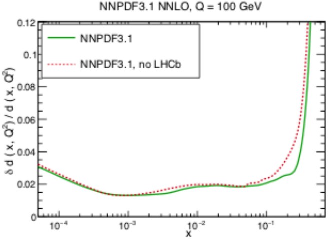
inclusive jet



[NNPDF3.1]

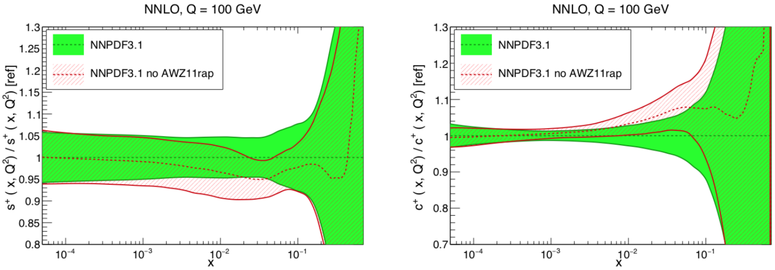
forward EW boson production





[NNPDF3.1]

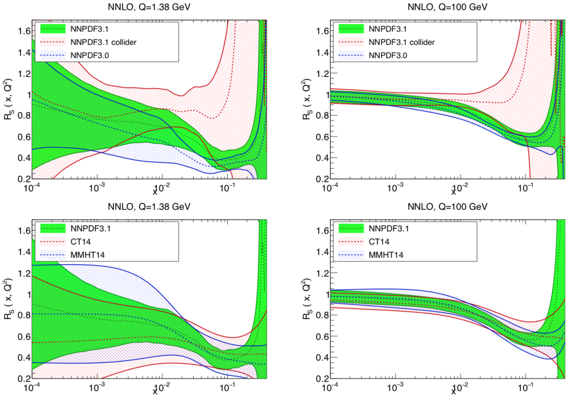
ATLAZ W Z production



[NNPDF3.1]

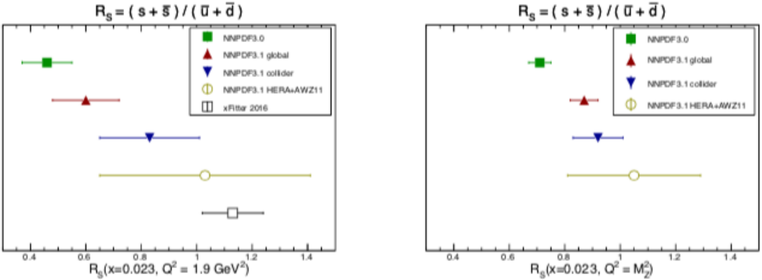
strangeness of the proton

$$R_s(x, Q^2) = \frac{s(x, Q^2) + \bar{s}(x, Q^2)}{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}$$



[NNPDF3.1]

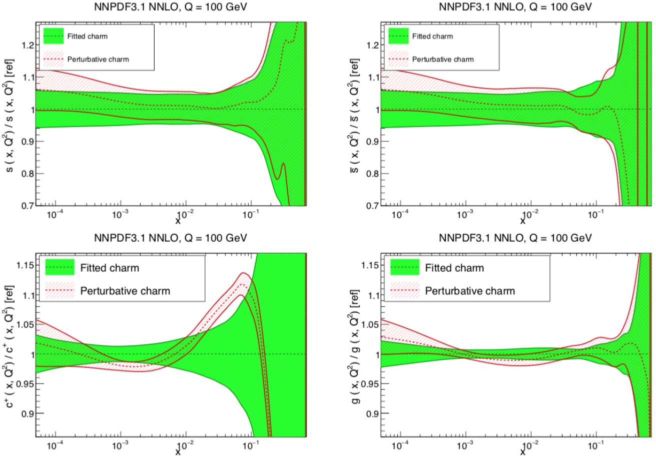
strangeness of the proton



[NNPDF3.1]

charm parametrization

same data, fitted charm vs. perturbative charm

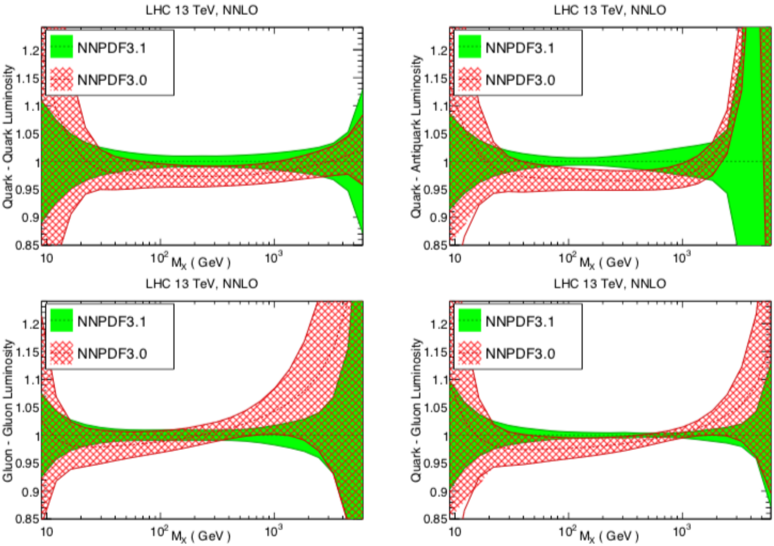


[NNPDF3.1]

phenomenology

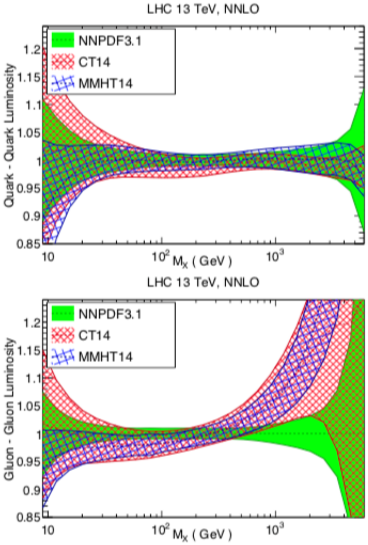
- prediction of LHC observables
- current PDF errors on LHC observables
- future PDF determinations – 'Ultimate' PDFs

parton lumi for LHC 13TeV



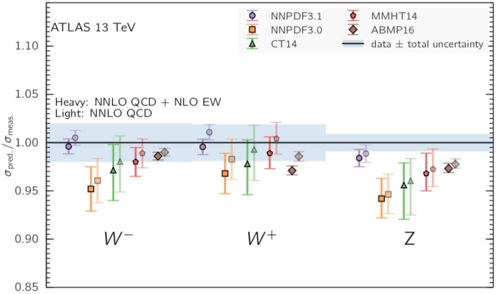
[NNPDF3.1]

parton lumi comparison



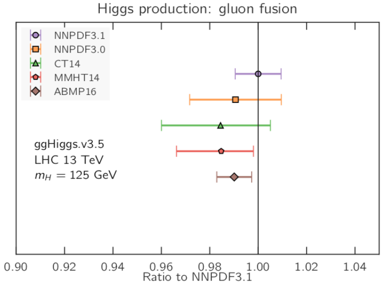
[NNPDF3.1]

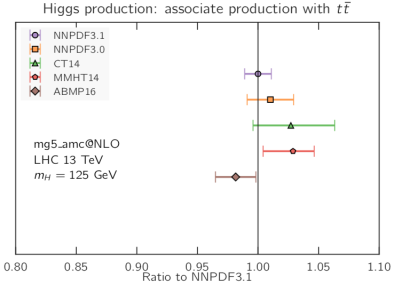
W & Z production

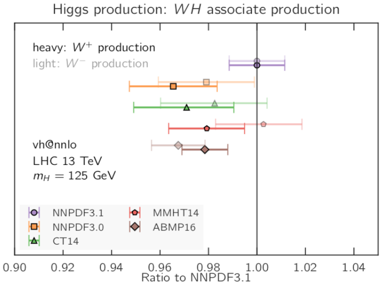


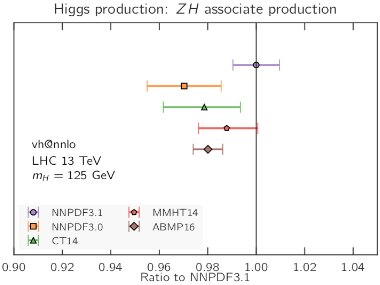
[NNPDF3.1]

Higgs production

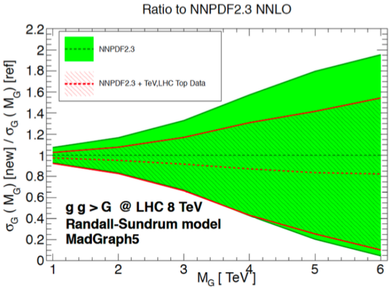


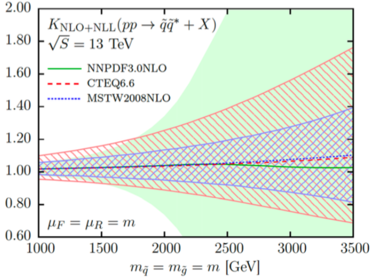






heavy new states





[Gao et al 17]

'ultimate' PDF

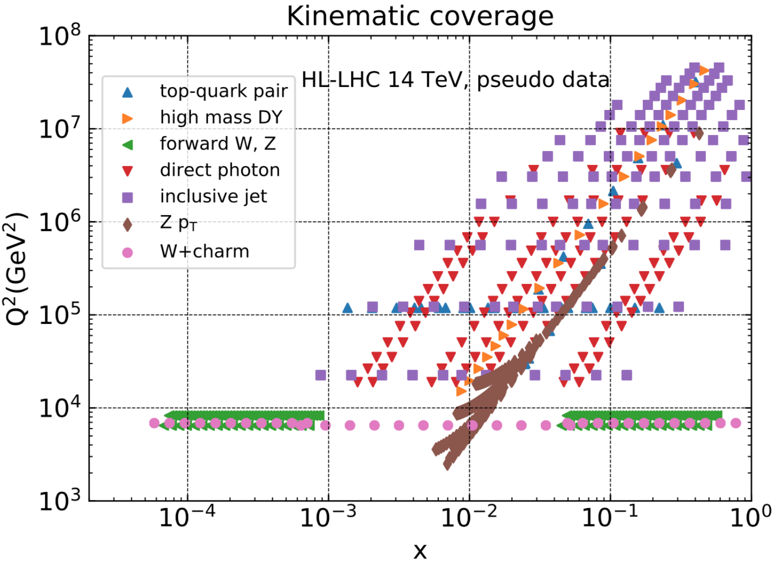
Generation of artificial HL-LHC data

- generate σ_i^{th} using NLO theory and PDF4LHC15
- compute the total uncertainty

$$\left(\delta^{\mathrm{exp}}_{\mathrm{tot},i}\right)^2 = \left(\delta^{\mathrm{exp}}_{\mathrm{stat},i}\right)^2 + \left(f_{\mathrm{corr}} \times f_{\mathrm{red}} \times \delta^{\mathrm{exp}}_{\mathrm{sys},i}\right)^2$$

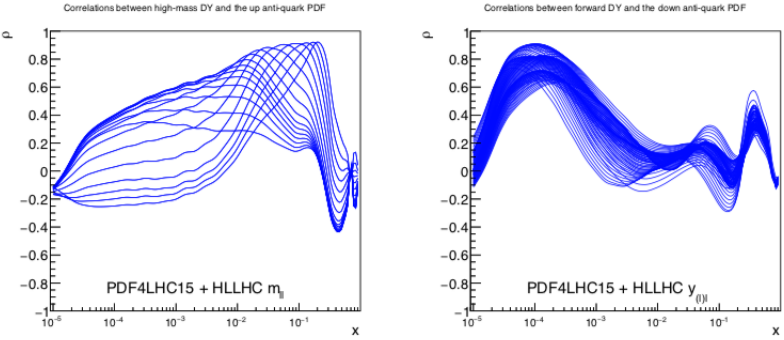
$$\left(\delta^{\mathrm{exp}}_{\mathrm{stat},i}\right)^2 = f_{\mathrm{acc}} \times N_{\mathrm{ev},i}$$

- fit/reweight PDFs to this set of artificial data



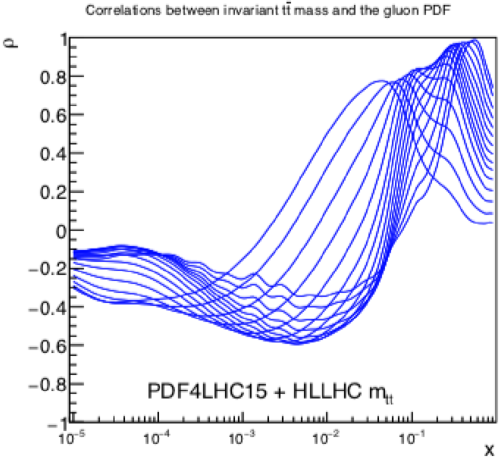
[Abdul Khalek et al 18]

DY - sea quark correlation



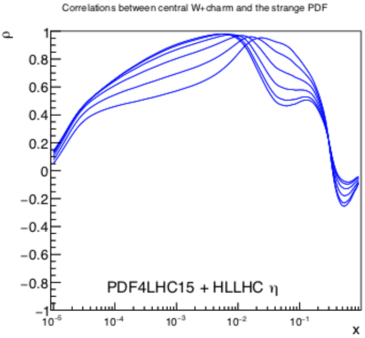
[Abdul Khalek et al 18]

ttbar - gluon correlation



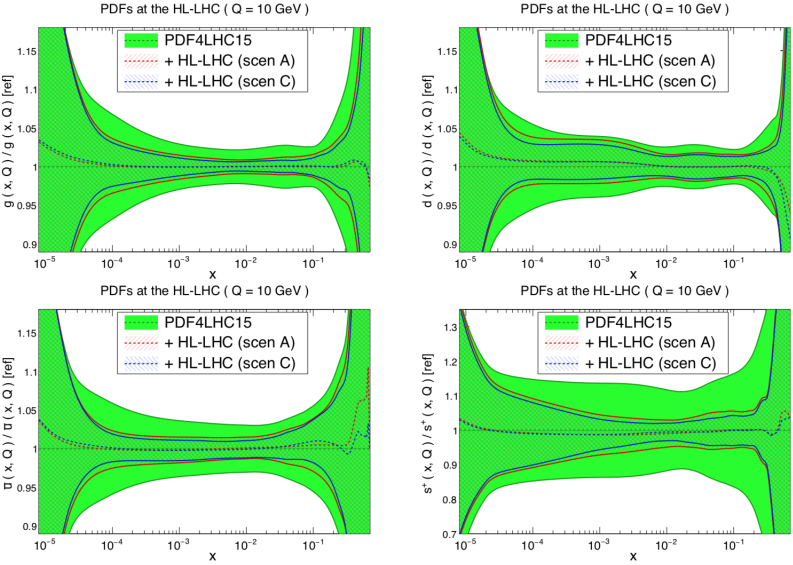
[Abdul Khalek et al 18]

W+c - strange correlation



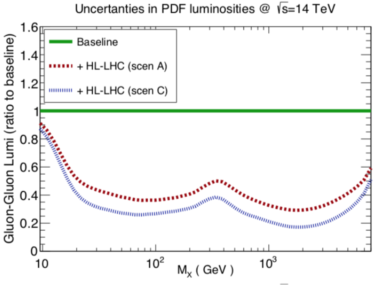
[Abdul Khalek et al 18]

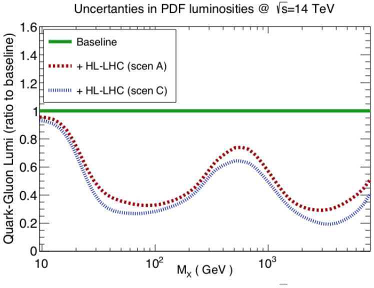
PDF uncertainty reduction



[Abdul Khalek et al 18]

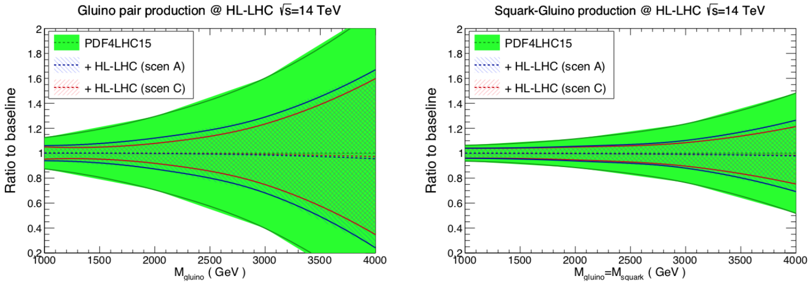
lumi uncertainty reduction





[Abdul Khalek et al 18]

lumi uncertainty reduction



[Abdul Khalek et al 18]

conclusions

- current LHC data have allowed flexible, unbiased parametrizations of PDFs
- broad agreement between different fits, robustness of the results
- good statistical accuracy in the kinematic region covered by data
- improved accuracy at HL-LHC
- at this level of accuracy it is mandatory to understand systematics better – theory errors, bias in the fit, extrapolation region
- incorporate lattice data in fits, targeting kinematic regions that are not covered by experiments