

# ELECTROWEAK BARYOGENESIS AND STOCHASTIC GRAVITATIONAL WAVES

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COSMOLOGY FRONTIER IN PARTICLE PHYSICS: ASTRO-PARTICLE PHYSICS AND EARLY UNIVERSE

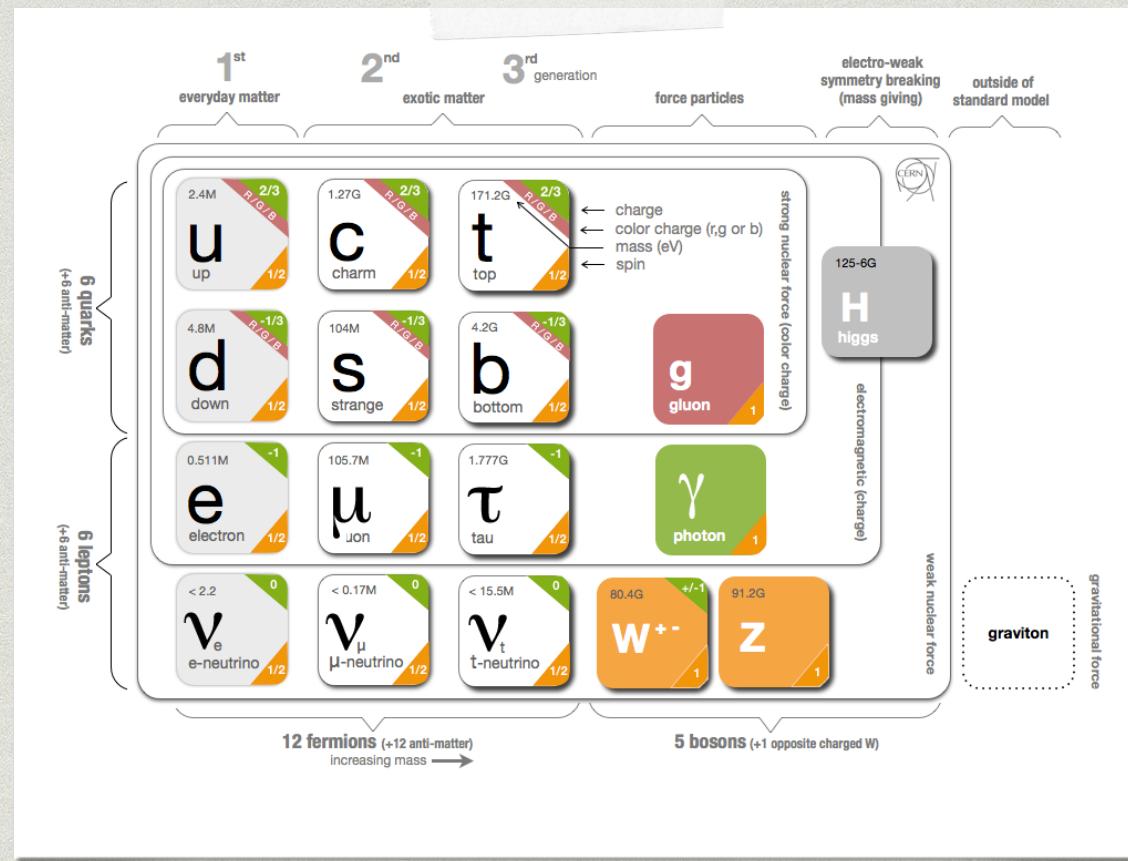
# Outline&preview

- \* EWBG from spontaneous CPV at the finite T and a two-step phase transition
- \* Stochastic gravitational wave signals from two-step PT

## Preview

- ◆ *Show you how to avoid constraints on the electroweak baryogenesis from the energy and intensity frontiers.*
- ◆ *Show you the space-based interferometer is an efficient facility for the indirect detection of the PT with the stochastic gravitational wave signal.*

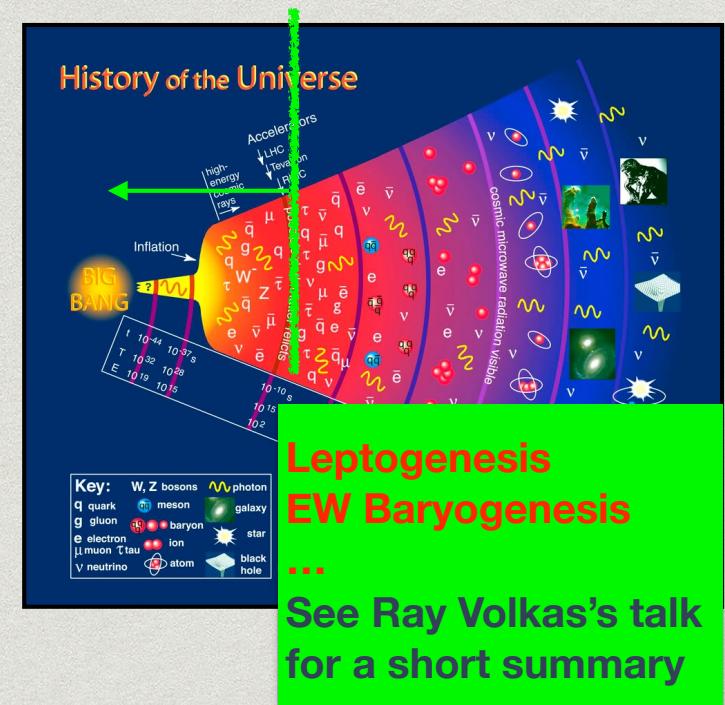
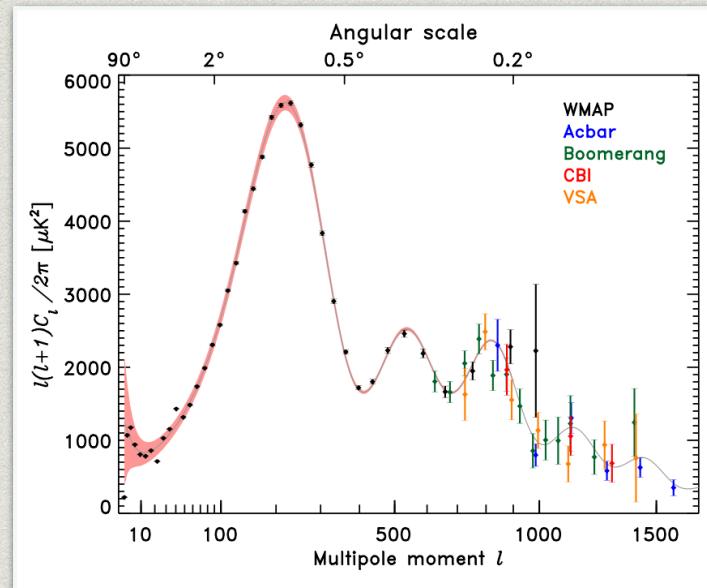
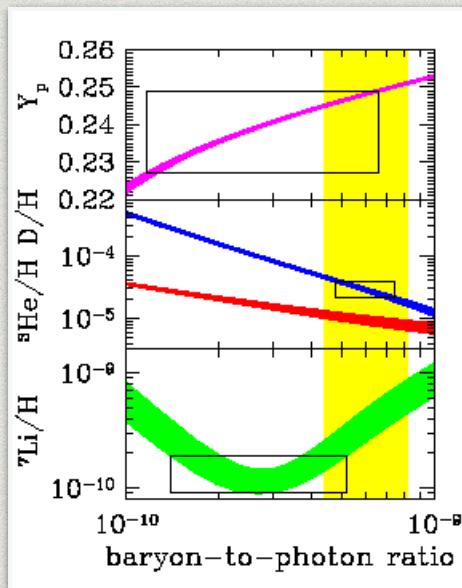
# Particle Zoo



- \* Neutrino masses
- \* Dark matter
- \* Baryon asymmetry

# Baryon asymmetry

- \* No anti-galaxy was observed
- \* The abundance of the primordial elements and the height of the CMB power spectrum depend on the ratio of baryon to photons



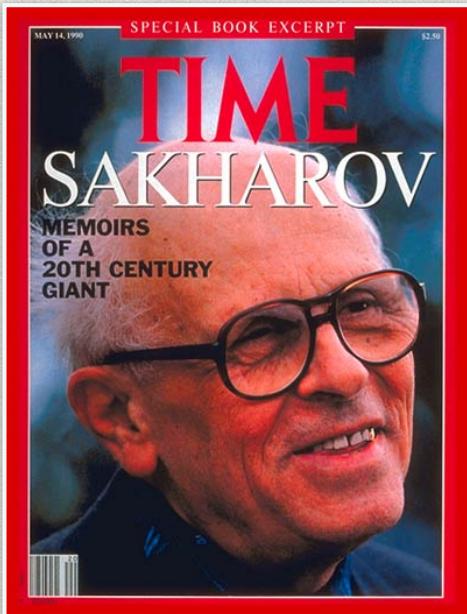
**Baryon asymmetry:**

$$Y_B = \frac{\rho_B}{s} = (8.59 \pm 0.11) \times 10^{-11}$$

(Planck 2015)

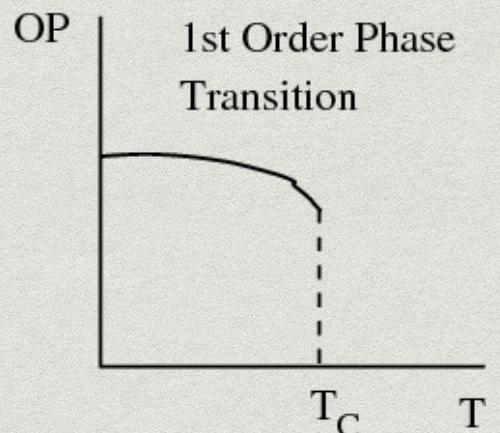
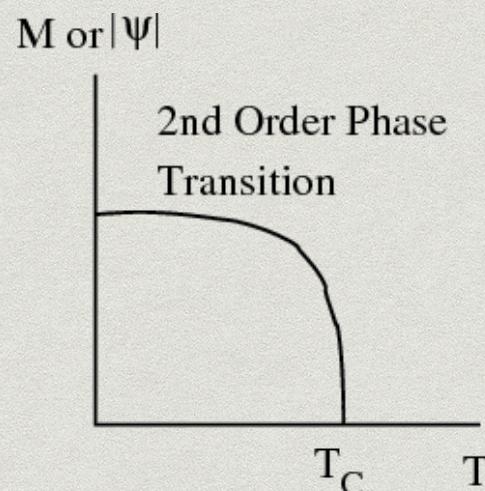
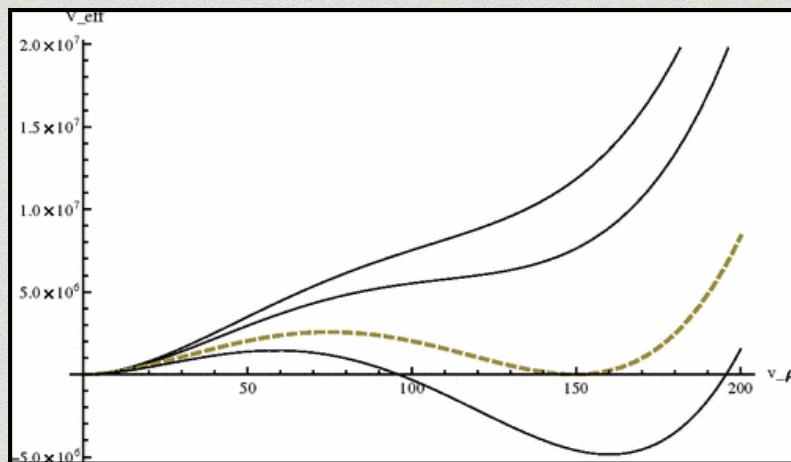
# BAU from First order EWPT

BAU might be generated during the electroweak phase transition



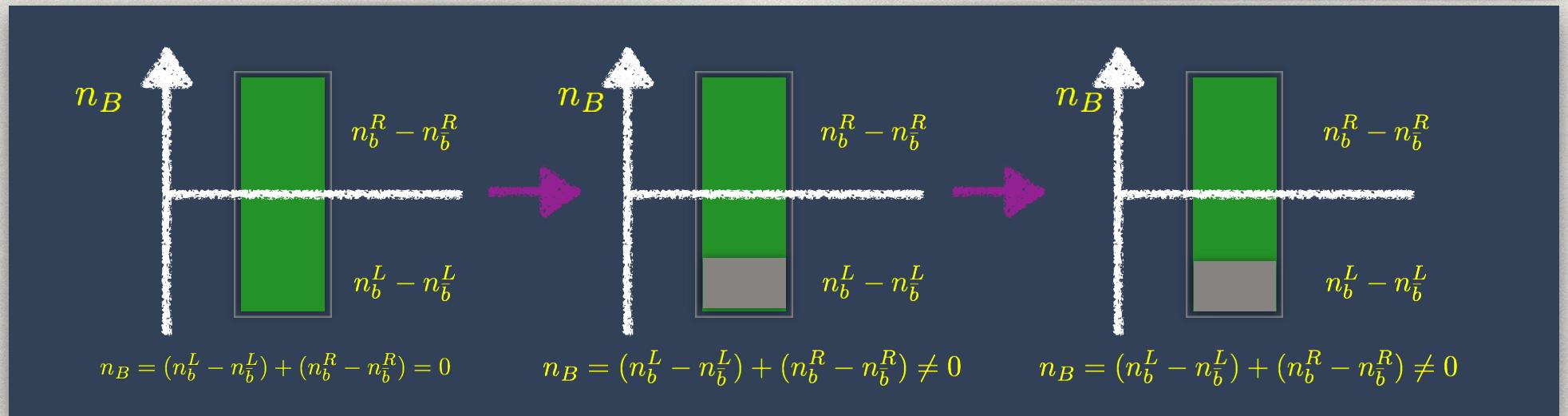
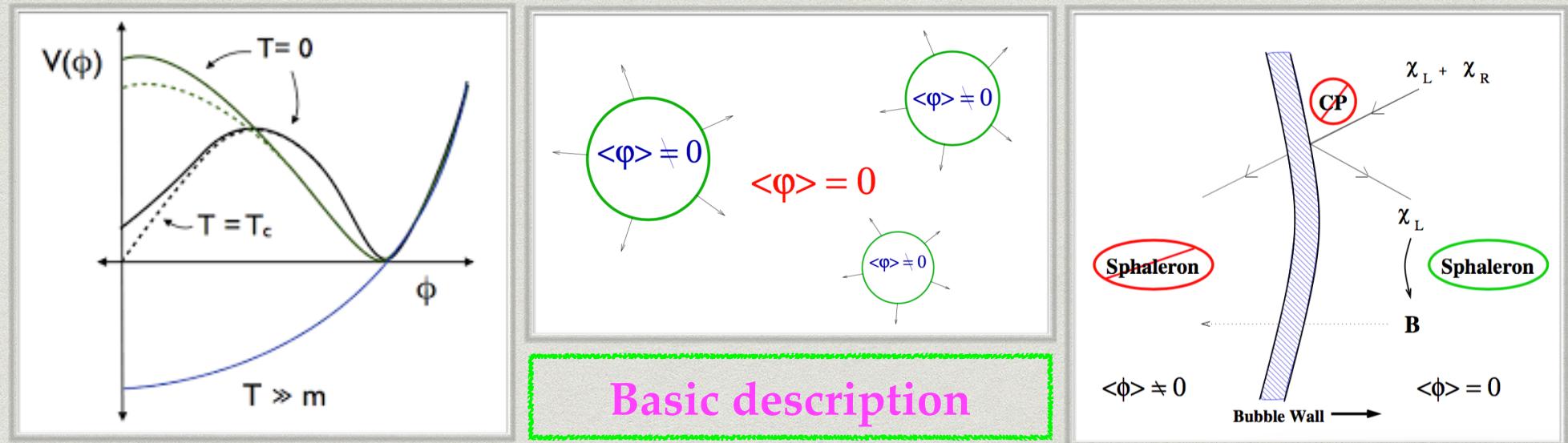
- ★ Baryon number violation
- ★ C&CP violation
- ★ Departure from thermal equilibrium

First order electroweak phase transition if baryon asymmetry is generated during the EWPT without CPT violation.



# Electroweak Baryogenesis

- \* Generate BAU during the electroweak phase transition



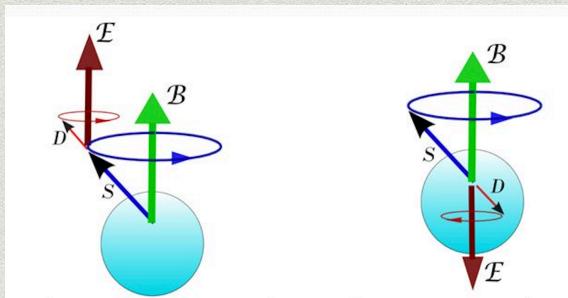
# Fate of the EWBG

## Three Detectives

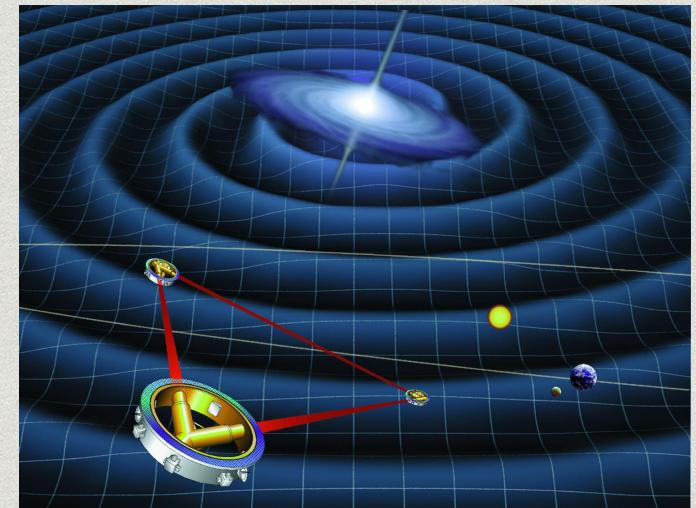
LHC



EDM



GW



Conventional EWBG mechanism might be found or excluded in the near future when these three detection methods are combined.

A typical example: Wino-catalyzed EWBG is excluded by the ACME result(intensity frontier) and the Higgs search results at the LHC(energy frontier).

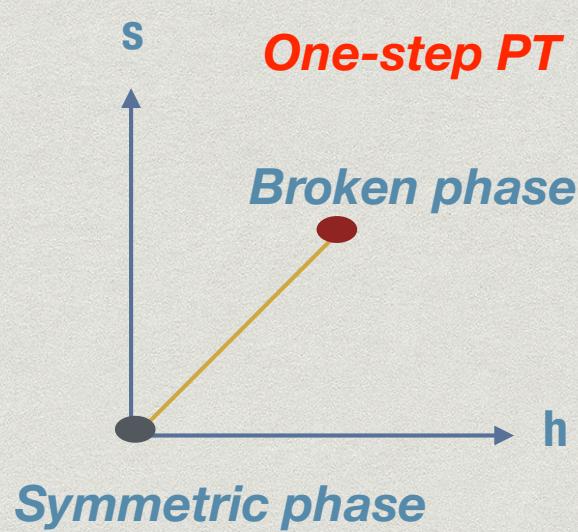
Questions: Is there a mechanism of electroweak baryogenesis that can escape from these hunters?

# Our little aim: a EWBG with less signature

Exploring the scenario of electroweak baryogenesis that may escape from the combined detection of the cosmic, energy and intensity frontiers.

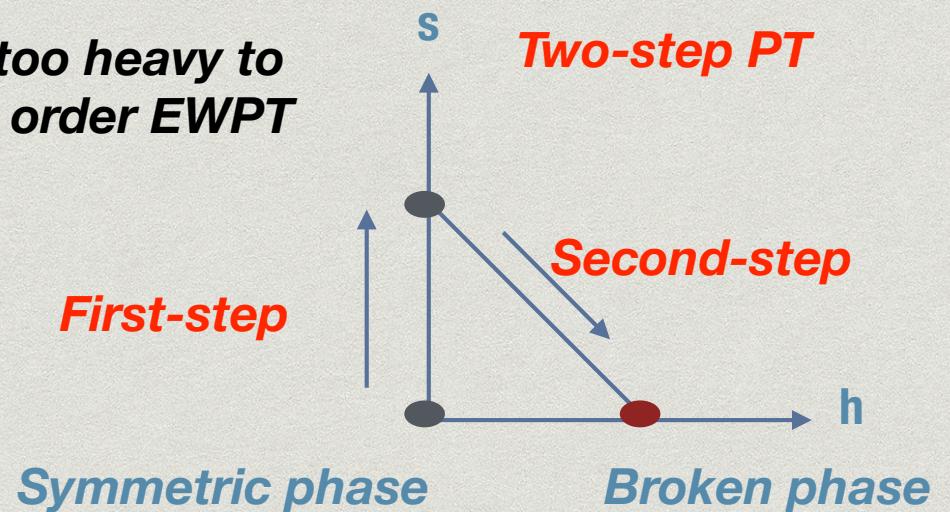
One observation:

A two-step phase transition may avoid constraint arising from Higgs searches at the LHC



The barrier between the symmetric and the broken phase usually comes from radiative corrections

$$V_{\text{eff}}(\phi, T) = \mathcal{A}(T)\phi^2 + \mathcal{B}(T)\phi^3 + \mathcal{C}(T)\phi^4 + \dots$$



The barrier exists at the tree-level  
Merits:  
1. No mixing with the SM Higgs  
2. Correlated with the dark matter

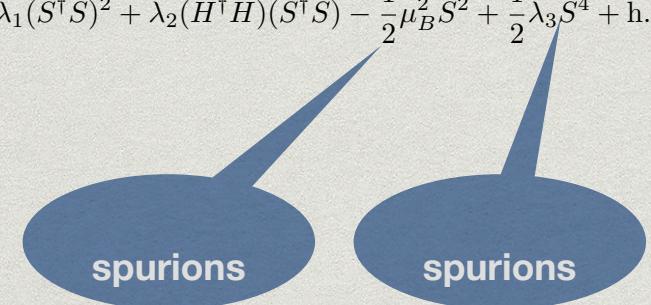
# Our little aim: a EWBG with less signature

Another observation:

There exists spontaneous CP phase in the scalar singlet sector

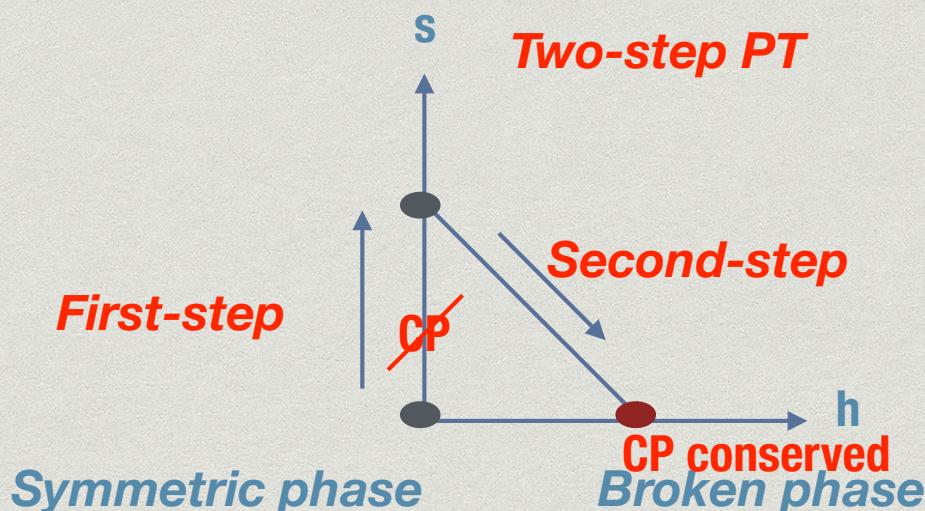
**Lemma:** Haber, Surujon, 2012  
*spontaneous CP violation in the theory of one complex scalar field may occur only when the related U(1) is explicitly broken by at least two spurions whose U(1) charges are different in magnitude*

$$V = -\mu^2(H^\dagger H) + \lambda(H^\dagger H)^2 - \mu_A^2(S^\dagger S) + \lambda_1(S^\dagger S)^2 + \lambda_2(H^\dagger H)(S^\dagger S) - \frac{1}{2}\mu_B^2 S^2 + \frac{1}{2}\lambda_3 S^4 + \text{h.c.}$$



A possible strategy:

There might be spontaneous CPV phase only at finite T!



$$\varphi = \pm \frac{1}{2} \arccos \left[ \frac{\lambda_1 - \lambda_3}{2\lambda_3} \frac{m_\beta^2 - m_\alpha^2}{\lambda_2 v^2 - m_\alpha^2 - m_\beta^2 + 2\Pi_\alpha} \right]$$

NO constraint of EDM and Higgs search!

# Sketch of the mechanism

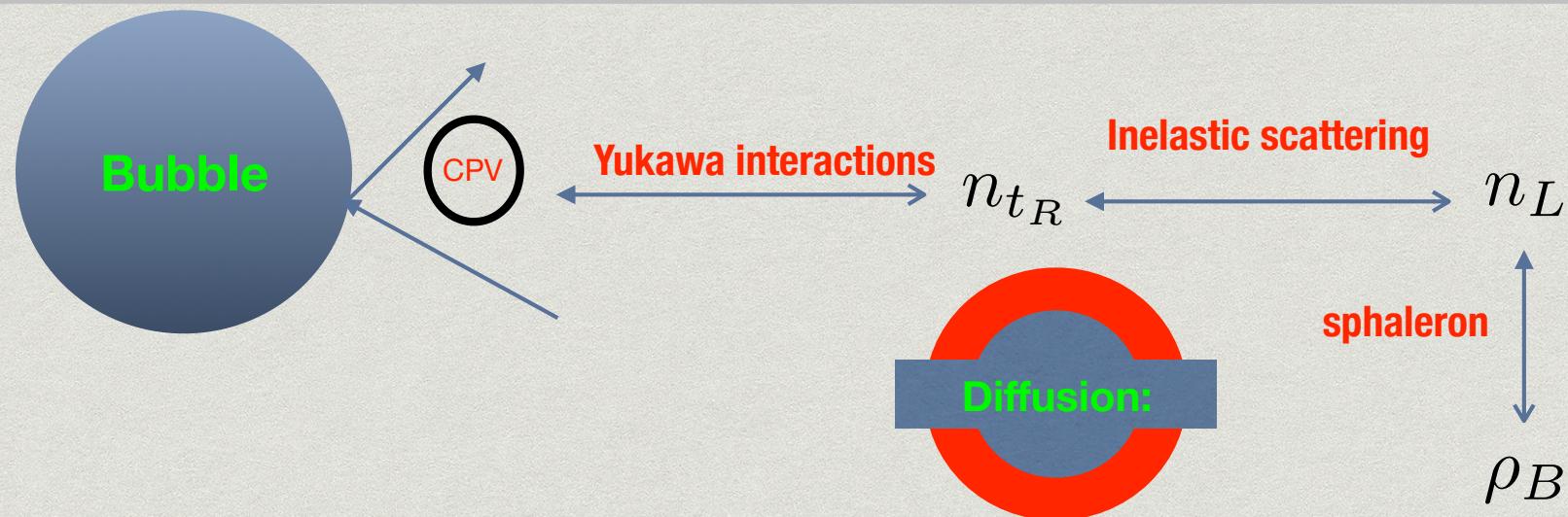
## Basic description

Sakharov:

{

- \*  $\mathcal{B}$
- \*  $\mathcal{C} \& \mathcal{CP}$
- \* First order EWPT

Sphaleron
Spontaneous CPV
Two-step PT



Transport equation:

$$\partial_t \rho_B(x) - D \nabla^2 \rho_B(x) = -\Gamma_{ws} F_{ws}(x)[n_L(x) - R\rho_B(x)]$$

# The model:

**SM+ complex scalar singlets**

**Potential:**

$$V = -\mu^2(H^\dagger H) + \lambda(H^\dagger H)^2 - \mu_A^2(S^\dagger S) + \lambda_1(S^\dagger S)^2 + \lambda_2(H^\dagger H)(S^\dagger S) - \frac{1}{2}\mu_B^2 S^2 + \frac{1}{2}\lambda_3 S^4 + \text{h.c.}$$

**Yukawa:**

$$-\mathcal{L} \sim \frac{1}{\Lambda} \overline{Q}_L \tilde{H} S t_R + \text{h.c.}$$

$$-\mathcal{L} \sim \eta \overline{T}_L S t_R + M \overline{T}_L T_R + \text{h.c.}$$

**$T_{L,R}$ : vector-like top quark**

$$J_{B(F)}(x) = \int_0^\infty dt t^2 \ln \left( 1 \mp \exp \{-\sqrt{t^2 + x}\} \right)$$

$$V_T = \frac{T^4}{2\pi^2} \left\{ \sum_{i \in B} n_i J_B \left[ \frac{m_i^2(h, s, \xi)}{T^2} \right] - \sum_{j \in F} n_j J_F \left[ \frac{m_j^2(h)}{T^2} \right] - \sum_{k \in G} n_k J_B \left[ \frac{m_k^2(h, s, \xi)}{T^2} \right] \right\}$$

\*  **$V_0$ :** The tree-level potential

$$V_{\text{eff}} = V_0 + V_{\text{CW}} + V_T + V_{\text{Daisy}}$$

\*  **$V_{\text{cw}}$ :** Coleman-Weinberg term

$$V_{\text{CW}} = \frac{1}{64\pi^2} \sum_i (-1)^{2s_i} n_i m_i^4(h, s, \xi) \left[ \log \frac{m_i^2(h, s, \xi)}{\mu^2} - C_i \right]$$

\*  **$V_T$ :** Finite temperature contribution

\*  **$V_{\text{ring}}$ :** The ring contribution

$$V_T^{\text{ring}} = \frac{T}{12\pi} \sum_i n_i \left\{ (m_i^2(h, s))^{3/2} - (M_i^2(h, s, T))^{3/2} \right\}$$

# Bubble dynamics

## 1. Bubble nucleation

**Euclidean equation of motion**

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} - V''(\phi) = 0$$

**Euclidean action for the solution of EoM**

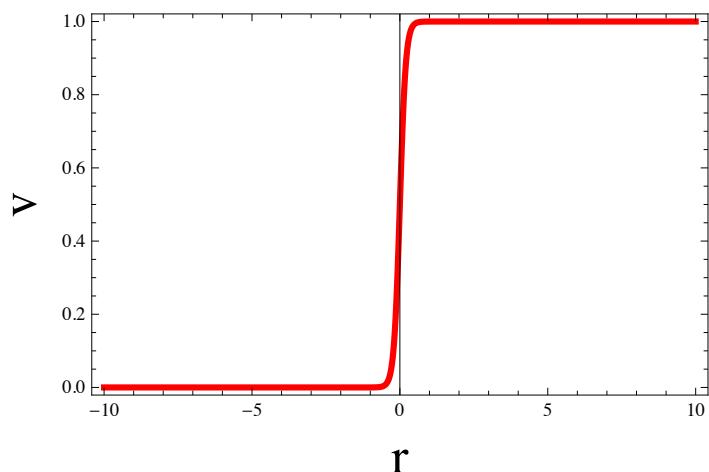
$$S_3 = 4\pi \int r^2 dr \left[ \frac{1}{2} \left( \frac{d\phi}{dr} \right)^2 + V(\phi) \right]$$

**Bubble nucleation rate per unit time per unit volume**

$$\Gamma_n(T) \approx T^4 \left( \frac{S_3(T)}{2\pi T} \right)^{3/2} \exp \left[ -\frac{S_3(T)}{T} \right]$$

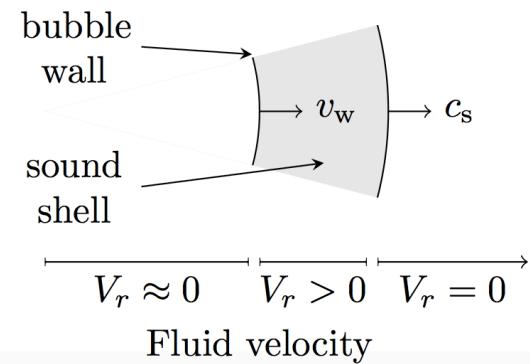
**Bounce solution to the background field**

$$V(z) = \frac{1}{2} v(T) \left[ 1 + \tanh \left( 3 \frac{z}{L_w} \right) \right]$$



Vacuum expectation value

$$\overbrace{\qquad\qquad\qquad}^{\langle \phi \rangle \neq 0} \qquad \overbrace{\qquad\qquad\qquad}^{\langle \phi \rangle = 0}$$



# Bubble dynamics

## 2. Typical temperatures

**Critical temperature  $T_c$ :**

**Bubble nucleation Temperature  $T_n$ :**

**PT completed Temperature  $T_d$ :**

★**Relationships**

$$T_c > T_n > T_d$$

$$V_{\text{eff}}(\phi_{\text{symmetric}}, T)|_{T_C} = V_{\text{eff}}(\phi_{\text{broken}}, T)|_{T_C}$$

$$\int_0^{t_n} \Gamma V_H(t) dt = \int_{T_n}^{\infty} \frac{dT}{T} \left( \frac{2\zeta M_{\text{pl}}}{T} \right)^4 e^{-S_3/T} = \mathcal{O}(1),$$

$$\Gamma$$

$$V_H(t)$$

**Bubble nucleation rate**

**One-horizon volume**

$$f(T_d) = \frac{4\pi}{3} \int_{T_d}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} v_w^3 \left(1 - \frac{T_d}{T}\right)^3 \equiv 1$$

$$H(T)$$

$$v_w$$

**Hubble constant**

**Bubble wall velocity**

$$f(T)$$

**Friction of the universe covered by the broken phase**

# Bubble dynamics

## 3. Physical parameters relating to PT

$v_w$	<i>Bubble wall velocity</i>	<i>calculated numerically</i>
$l_w$	<i>Bubble wall width</i>	<i>calculated numerically</i>
$\alpha$	<i>Released energy to radiation energy</i>	$\alpha = \Lambda / \rho_{\text{rad}}$
$\kappa$	<i>The efficiency factor</i>	$\kappa = \frac{3}{\varepsilon v_w^3} \int w(\xi) v^2 \gamma^2 \xi^2 d\xi$
$\Lambda$	<i>Latent heat</i>	$\Lambda = \Delta \left( V - \frac{dV}{dt} T \right)$

$v_w$
$l_w$

Relevant to the calculation of baryon number density generated during the EWPT

$\alpha$
$\kappa$
$\Lambda$

Relevant to the calculation of stochastic gravitational wave spectrum emitted during the EWPT

# CP phase and bubble wall width

**EoM for three background fields:**

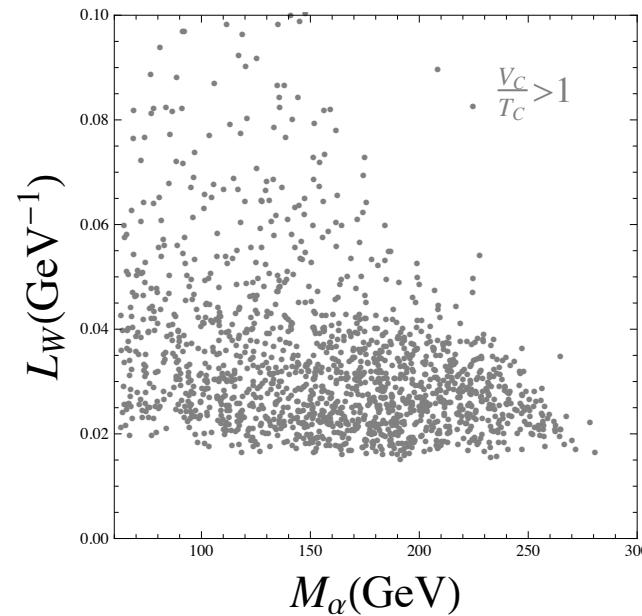
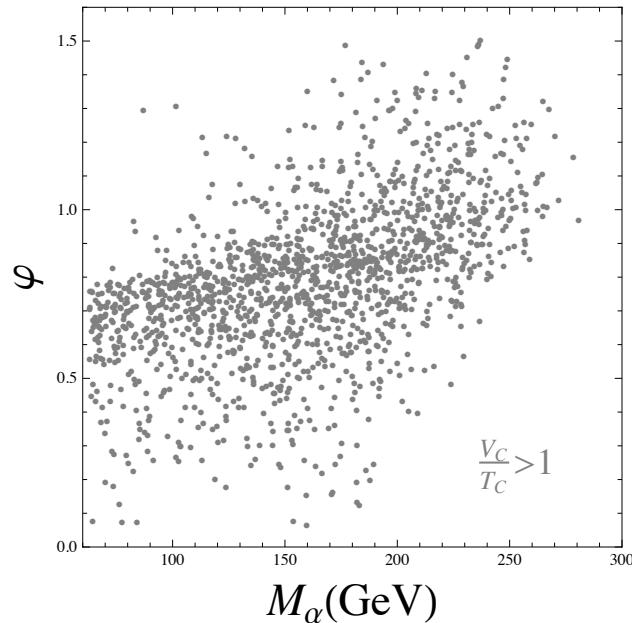
$$\frac{d^2\phi_i}{dr^2} + \frac{2}{r} \frac{d\phi_i}{dr} = \bar{V}'(\vec{\phi})$$

**Bubble wall width:**

$$L_w^2 \approx 1.35 \frac{\lambda + \sqrt{\lambda\lambda_\varrho}}{(\lambda_2 - 2\sqrt{\lambda\lambda_\varrho})[\lambda v_0^2 - \Pi_h(T_C^2)]} \times \left(1 + \sqrt{\frac{\lambda_2^2}{4\lambda\lambda_\varrho}}\right)$$

**Spontaneous CP phase:**

$$\varphi = \pm \frac{1}{2} \arccos \left[ \frac{\lambda_1 - \lambda_3}{2\lambda_3} \frac{m_\beta^2 - m_\alpha^2}{\lambda_2 v^2 - m_\alpha^2 - m_\beta^2 + 2\Pi_\alpha} \right] .$$



# Transport equations

EWBG

*Transport equation*

$$\frac{\partial n}{\partial t} + \nabla \cdot j(x) = - \int d^3z \int_{-\infty}^{x_0} dz^0 \text{Tr}[\Sigma^>(x, z)S^<(z, x) - S^>(x, z)\Sigma^<(z, x) \\ + S^<(x, z)\Sigma^>(z, x) - \Sigma^<(x, z)S^>(z, x)]$$

**Source term:**

$$S_{\text{top}}^{\text{CPV}} = -2\zeta^2 v_s^2 \dot{\varphi} \int \frac{k^2 dk}{\pi^2 \omega_L \omega_R} \text{Im} \left\{ (\varepsilon_L \varepsilon_R^* - k^2) \frac{n(\varepsilon_L) - n(\varepsilon_R^*)}{(\varepsilon_L - \varepsilon_R^*)^2} + (\varepsilon_L \varepsilon_R + k^2) \frac{n(\varepsilon_L) + n(\varepsilon_R)}{(\varepsilon_L + \varepsilon_R)^2} \right\}$$



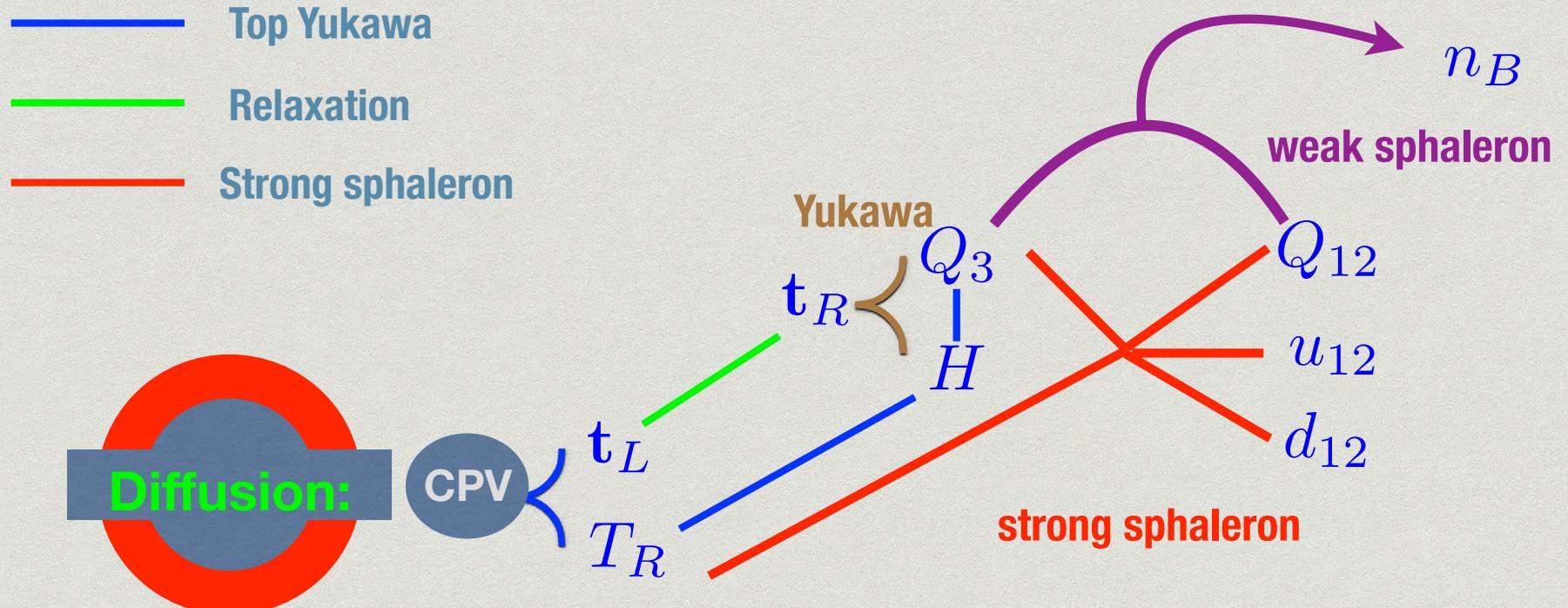
$$\zeta \overline{\mathfrak{t}_L} St_R + (M_{\mathfrak{t}}) \overline{\mathfrak{t}_L} \mathfrak{t}_R + \text{h.c.} \longrightarrow \frac{y_t \zeta}{\Lambda} \overline{Q_3} \tilde{H} St_R$$

**All equations**

$$\begin{aligned} \partial^\mu Q_\mu &= +\Gamma_{m_t} \mathcal{R}_T^- + \Gamma_{Y_t} \delta_t + \Gamma_{y'} \delta_{\mathfrak{t}'} + 2\Gamma_s \delta_s \\ \partial^\mu T_\mu &= -\Gamma_{m_t} \mathcal{R}_T^- - \Gamma_{Y_t} \delta_t - \Gamma_s \delta_s - \Gamma_\zeta \delta_{\mathfrak{t}} \\ &\quad + \Gamma_{\mathfrak{t}}^+ \mathcal{R}_{\mathfrak{t}}^+ + \Gamma_{\mathfrak{t}}^- \mathcal{R}_{\mathfrak{t}}^- + S_{\text{top}}^{\text{CPV}} \\ \partial^\mu \mathfrak{t}_\mu &= +\Gamma_{m_t} \mathcal{R}_\Lambda^- - \Gamma_{\mathfrak{t}}^+ \mathcal{R}_{\mathfrak{t}}^+ - \Gamma_{\mathfrak{t}}^- \mathcal{R}_{\mathfrak{t}}^- + \Gamma_\zeta \delta_{\mathfrak{t}} - S_{\text{top}}^{\text{CPV}} \\ \partial^\mu \mathfrak{t}'_\mu &= -\Gamma_{m_t} \mathcal{R}_\Lambda^- - \Gamma_{y'} \delta_{\mathfrak{t}'} \\ \partial^\mu S_\mu &= -\Gamma_\zeta \delta_{\mathfrak{t}} \\ \partial^\mu H_\mu &= -\Gamma_{Y_t} \delta_t - \Gamma_{y'} \delta_{\mathfrak{t}'} \end{aligned} \tag{13}$$

# Diffusions

EWBG



**Baryon number density:**

$$\hat{n}_B = -\frac{3\Gamma_{ws}}{2D_Q\lambda_+} \int_{-\infty}^{-L_w/2} dz n_L(z) e^{-\lambda_- z}$$

# Damping of the domain wall

EWBG

*Problems*

$$+\varphi + -\varphi = 0$$

*No BAU left*

*Solution: Adding a  $Z_2$  breaking term to the Higgs potential:  $\Delta s + h.c.$*

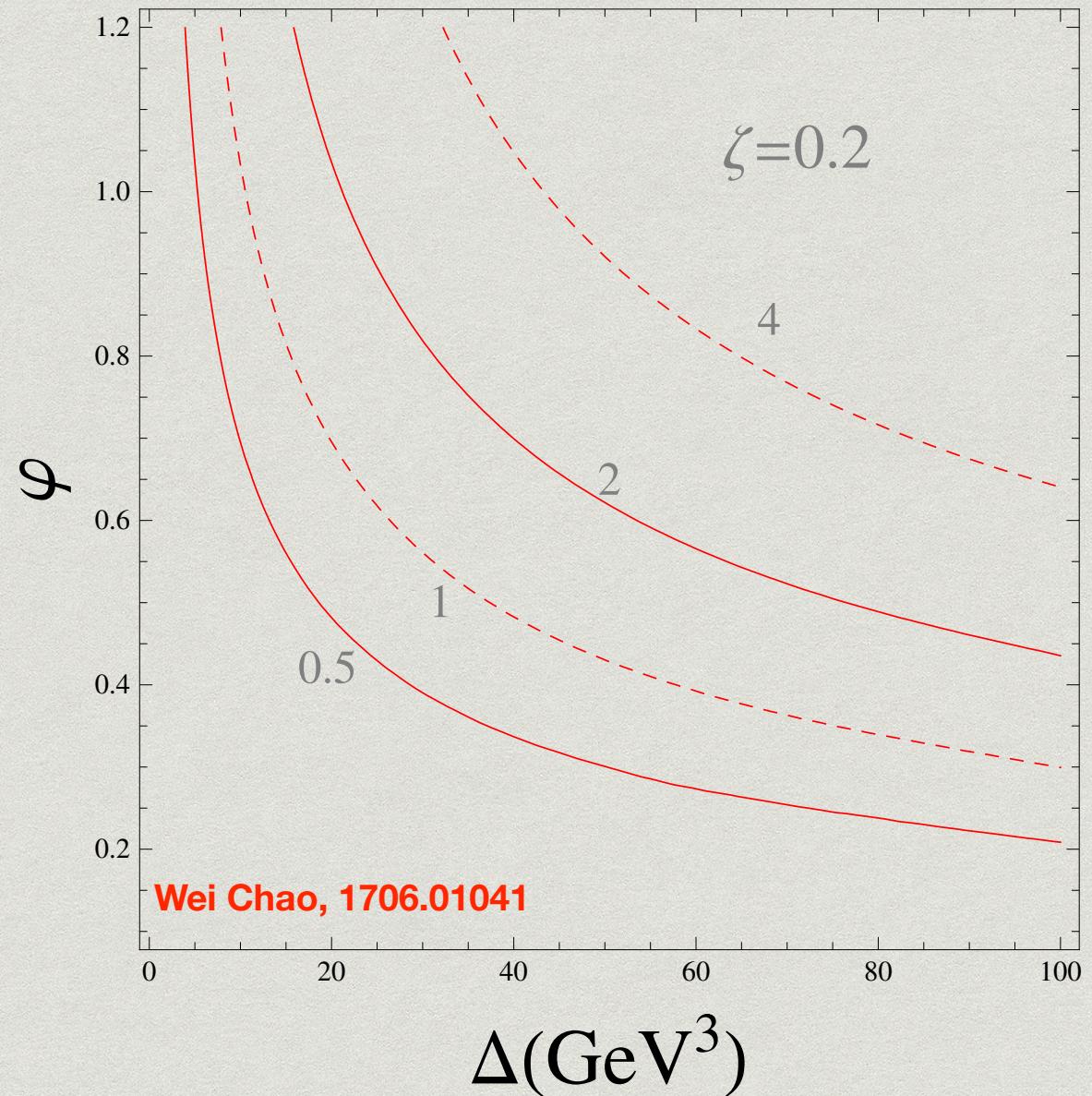
*Ratio of bubbles*

$$\frac{N_+}{N_-} = \exp\left(\frac{\Delta F}{T}\right)$$

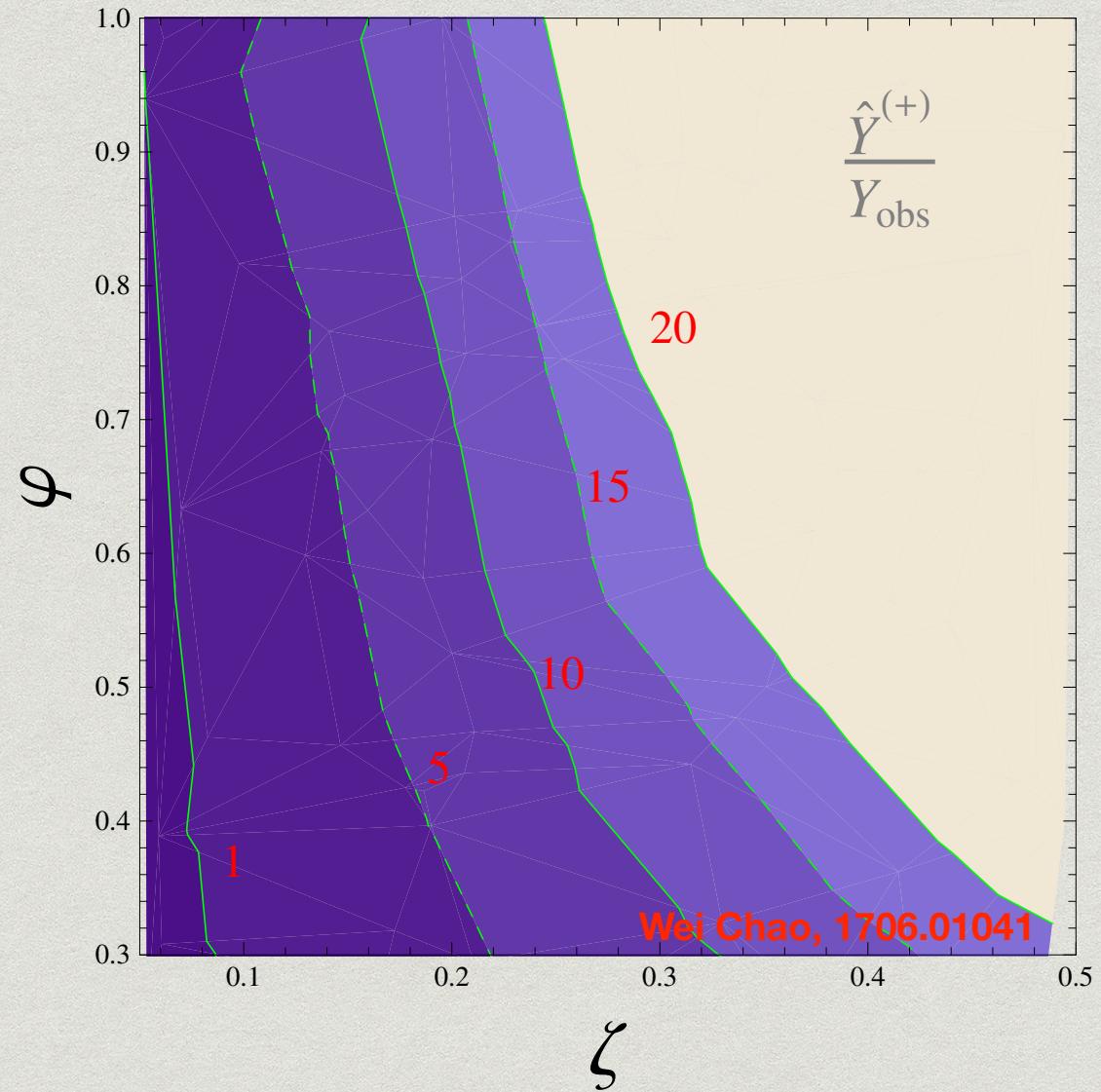
*Final BAU*

$$n_B = \hat{n}_B^{(+)} \frac{N_+ - N_-}{N_+ + N_-}$$

# Numerical results



# Numerical results



# Outline

- \* EWBG with two-step PT and spontaneous CPV
- \* **Stochastic gravitational wave from two-step PT**

# GW from the PT

## Basics of gravitational wave from EWPT

*Gravitational waves are described by a transverse-traceless gauge invariant perturbation,  $h_{ij}$ , in a FRW metric,*

*Einstein eq for transverse-traceless part*

*Gravitational wave energy density*

*Energy spectrum*

$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

$$h''_{ij} - \Delta h_{ij} = 16\pi G[(e + p)\gamma^2 v_i v_j + \partial_i \phi \partial_j \phi]$$

$$\rho_{gw}(t) = \frac{\langle \dot{h}_{ij}(t, \vec{x}) \dot{h}_{ij}(t, \vec{x}) \rangle}{8\pi G}$$

$$h^2 \Omega_{\text{GW}}(f) = \frac{h^2}{\rho_c} \frac{d\rho_{\text{gw}}}{d \log f}$$

# GW from the PT

Sources of GW from  
EWPT:

- ◆ *Collisions of bubble wall and shocks in the plasma*
- ◆ *Sound wave after the collision but before the expansion has dissipated the kinetic energy.*
- ◆ *Magnetohydrodynamic turbulence : percolation can also induce MHD turbulence since the plasma is fully ionized.*

Fitted results of GW spectrum

**Bubble  
collision**

$$h^2\Omega_{\text{coll}}(f) = 1.67 \times 10^{-5} \left(\frac{H_n}{\beta}\right)^2 \left(\frac{\kappa\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \times \left(\frac{0.11v_w^3}{0.42 + v_w^2}\right) \left[\frac{3.8(f/f_{\text{coll}})^{2.8}}{1 + 2.8(f/f_{\text{coll}})^{3.8}}\right],$$

**Sound wave**

$$h^2\Omega_{\text{sw}}(f) = 2.65 \times 10^{-6} \left(\frac{H_n}{\beta}\right) \left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \times v_w \left(\frac{f}{f_{\text{sw}}}\right)^3 \left[\frac{7}{4 + 3(f/f_{\text{sw}})^2}\right]^{7/2}$$

**MHD  
turbulence**

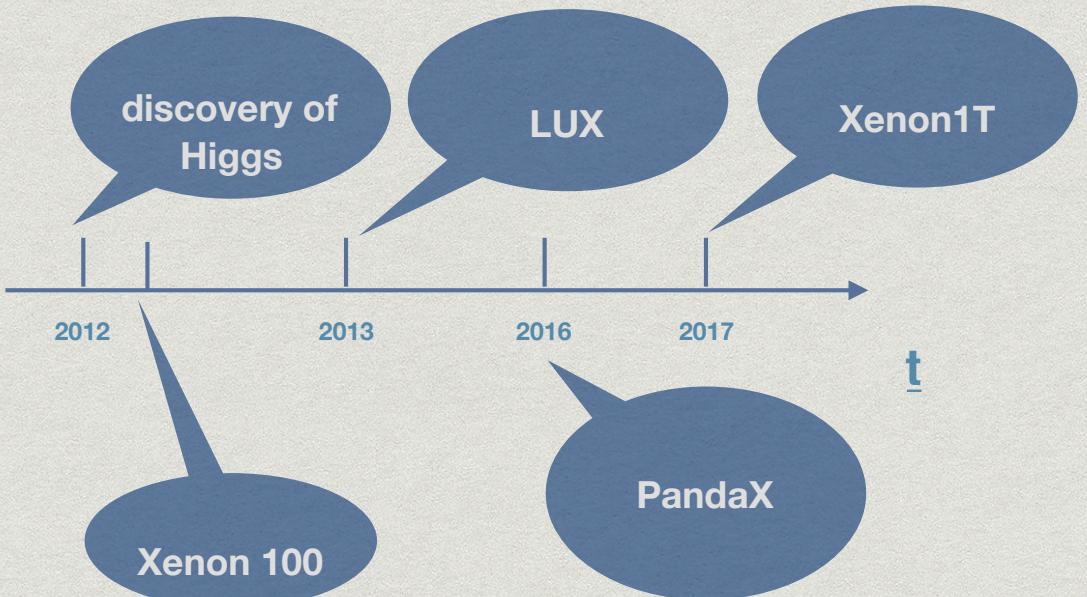
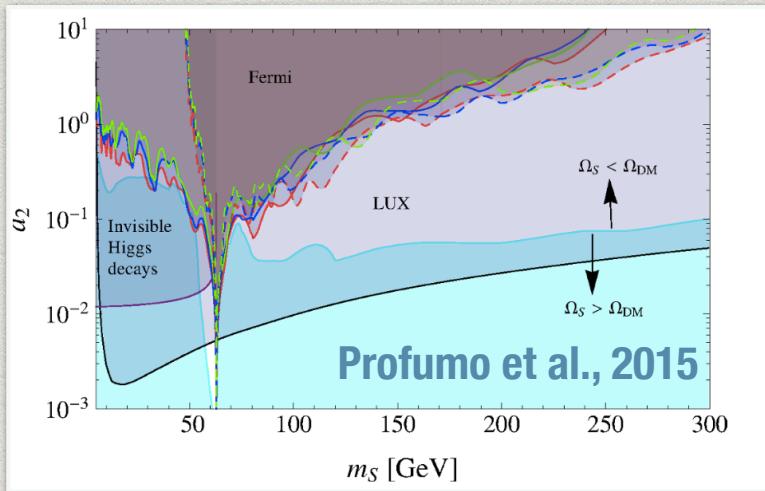
$$h^2\Omega_{\text{turb}}(f) = 3.35 \times 10^{-4} \left(\frac{H_n}{\beta}\right) \left(\frac{\kappa_{\text{tu}}\alpha}{1+\alpha}\right)^{3/2} \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \times v_w \frac{(f/f_{\text{tu}})^3}{(1 + f/f_{\text{tu}})^{11/3}(1 + 8\pi f/h_n)}$$

**Total energy spectrum:**

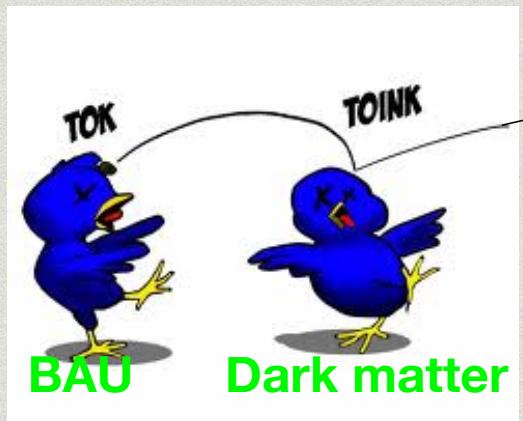
$$h^2\Omega_{\text{GW}} \approx h^2\Omega_{\text{coll}} + h^2\Omega_{\text{sw}} + h^2\Omega_{\text{turb}}$$

# Dark matter in a two-step PT

GW from EWPT, a typical model:



Ordinary Higgs portal is dead  
since 2013!



*Non-minimal  
Higgs portal*

# GW from the PT

The model: SM + two real scalar singlets

$$V_0 = -\frac{1}{2}\mu_\Phi^2\Phi^2 + \frac{1}{4}\lambda_\Phi\Phi^4 - \frac{1}{2}\mu_S^2S^2 + \frac{1}{4}\lambda_SS^4 - \mu^2H^\dagger H + \lambda(H^\dagger H)^2 + \lambda_1S^2H^\dagger H + \lambda_2\Phi^2H^\dagger H + \lambda_3S^2\Phi^2,$$



Real singlet



Dark matter



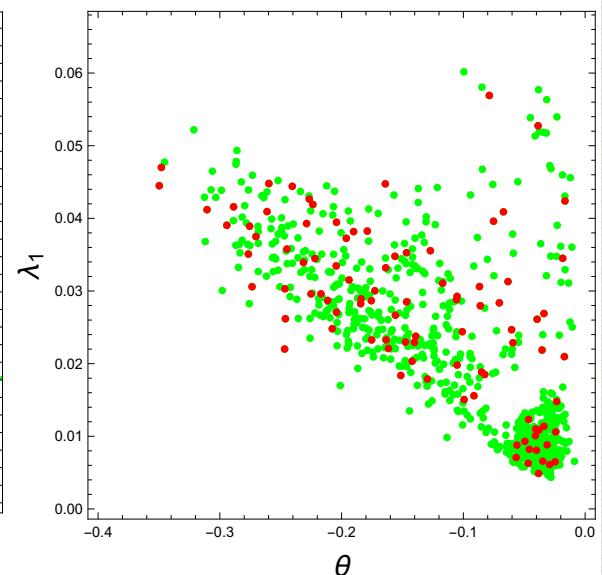
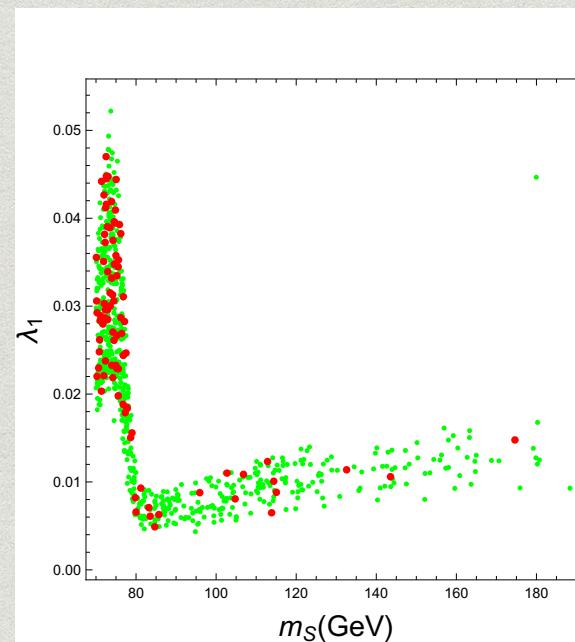
SM Higgs

Physical parameters:

$v_\Phi, m_\Phi, m_S, \lambda_S, \theta, \lambda_1, \lambda_3$

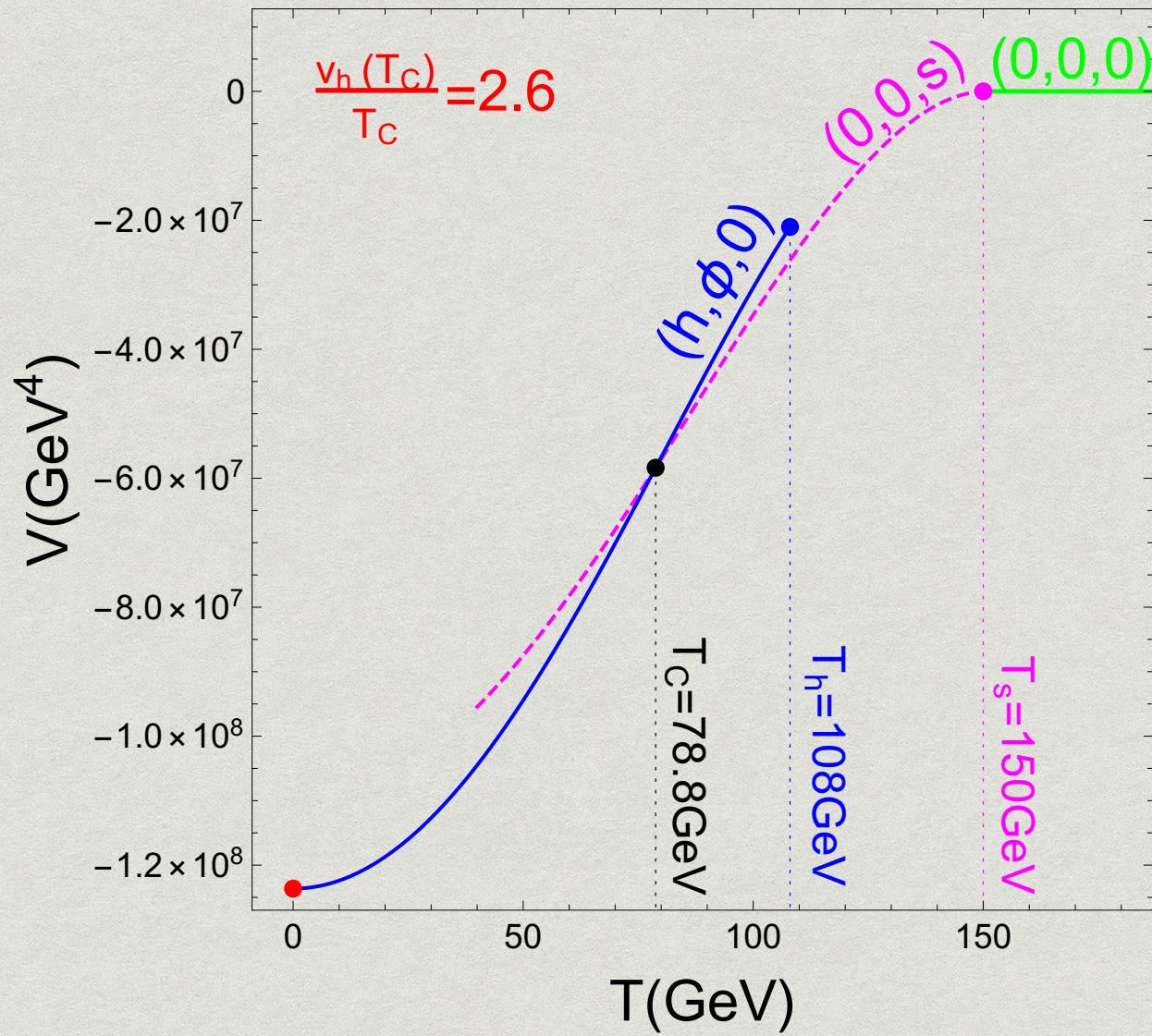
Conditions for a negligible DD cross section:

$$\lambda_3 = \frac{v_{EW}\lambda_1(m_{\hat{h}}^2 \tan \theta + m_{\hat{\phi}}^2 \cot \theta)}{2v_\Phi(m_{\hat{h}}^2 - m_{\hat{\phi}}^2)}$$



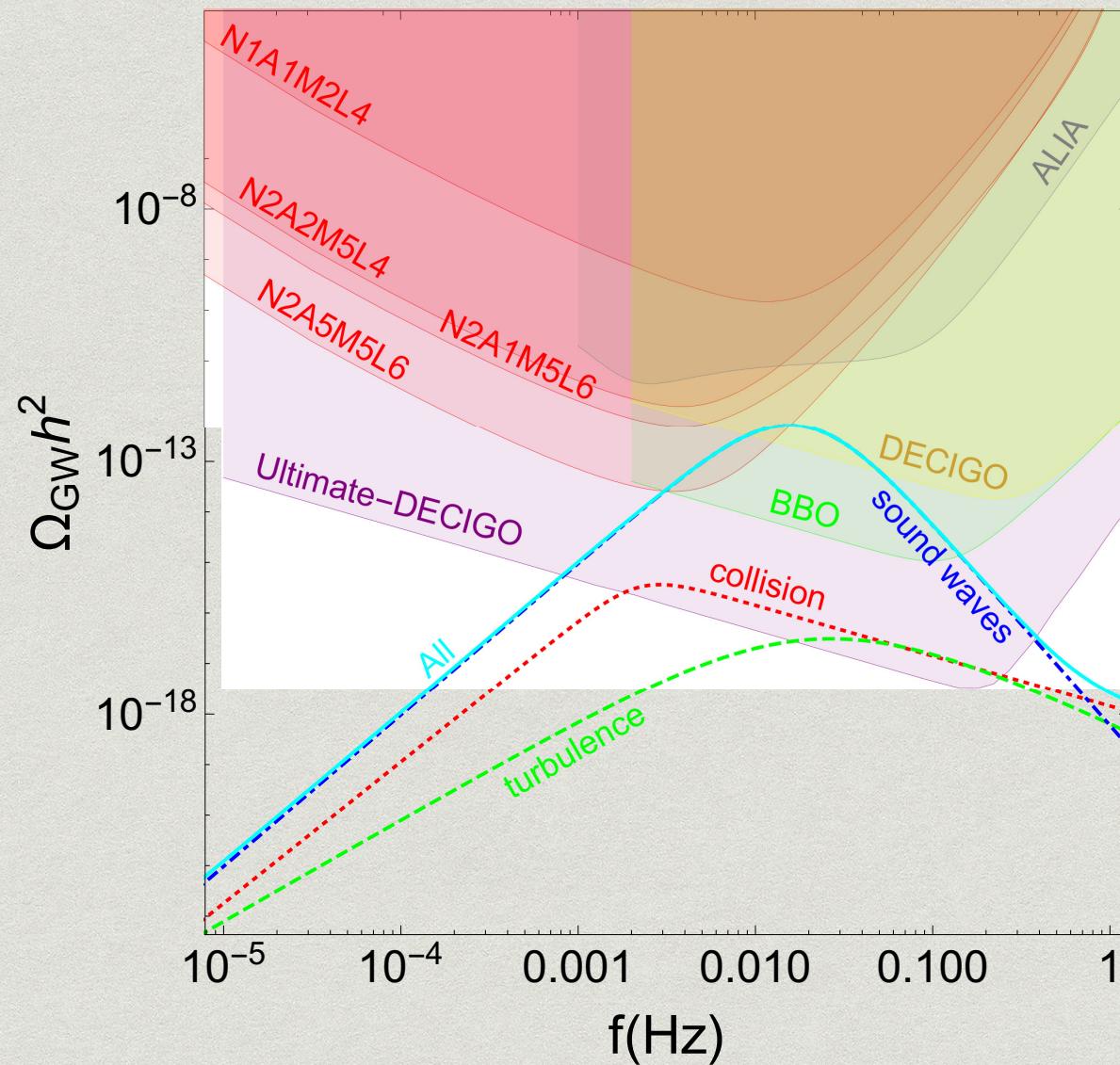
# GW from the PT

PT details:



# GW from the PT

GW spectrum: ( Wei Chao, H. Guo, J. Shu, JCAP1709,009)



# Summary

- ◆ Physics relevant to electroweak phase transition are briefly reviewed.
- ◆ The baryon asymmetry of the universe generated during the first order EWPT is discussed, especially I showed how to generate sufficient BAU with the spontaneous CP phase and a two-step EWPT.
- ◆ Show you stochastic gravitational wave can be taken as an indirect signal of the EWBG.

Thank you