Cosmological adiabatic conversion between the QCD axion and ALP dark matter

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Outline

- Axions : The QCD axion & Axion-like particles (ALPs)
- Level crossing & Adiabatic conversion
- Cosmological abundances
- Axion-photon coupling
- Summary

Axions : The QCD axion (a)

- The strong CP problem

$$\mathcal{L}_\theta = \overset{\text{Strong CP phase}}{\theta} \frac{g_s^2}{32\pi^2} G^b_{\mu\nu}\widetilde{G}^{b\mu\nu}$$

- ✓ Exp. bound from neutron electron dipole moment : $|\theta| < 10^{-10}$
- ✓ The problem is more puzzling

$$\theta = \underbrace{\theta_0}_{\text{Theta vacuum}} + \underbrace{\arg\det\left(M_u M_d\right)}_{\text{Chiral transformation}}$$

Why θ is so small is the strong CP problem

Axions : The QCD axion (a)

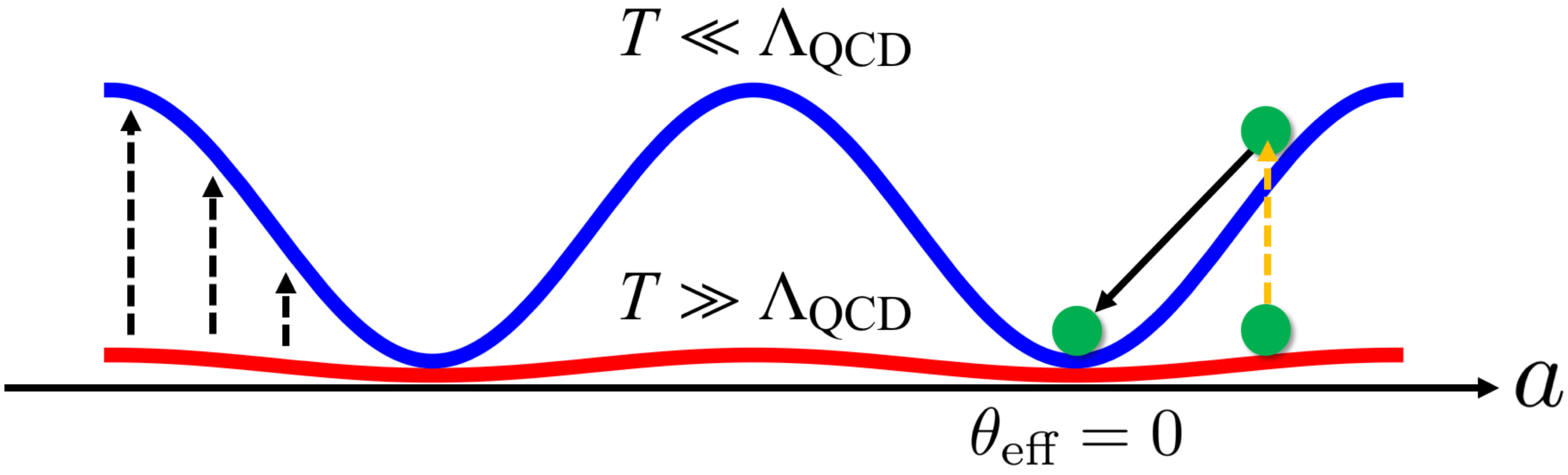
- Peccei-Quinn mechanism : Strong CP phase is a dynamical var.

$$\mathcal{L}_\theta = \underbrace{\left(\theta + \frac{a}{f_a}\right)}_{\theta_{\text{eff}}} \frac{g_s^2}{32\pi^2} G^b_{\mu\nu}\widetilde{G}^{b\mu\nu}$$

Shift symmetry

$$a \to a + \epsilon \; \underline{f_a}$$

Decay constant



Axions : The QCD axion (a)

- **The QCD axion gets mass through non-perturbative QCD effects.**

Sz. Borsanyi *et al.* `16

$$m_a(T) \simeq \begin{cases} \dfrac{\sqrt{\chi_0}}{f_a}\left(\dfrac{T_{\rm QCD}}{T}\right)^{4.08} & T > T_{\rm QCD} \\ m_a \equiv m_a(T \to 0) & T < T_{\rm QCD} \end{cases}$$

✓ **Zero temperature mass**

$$m_a \simeq 5.7 \times 10^{-6}\,{\rm eV}\left(\frac{f_a}{10^{12}\,{\rm GeV}}\right)^{-1}$$

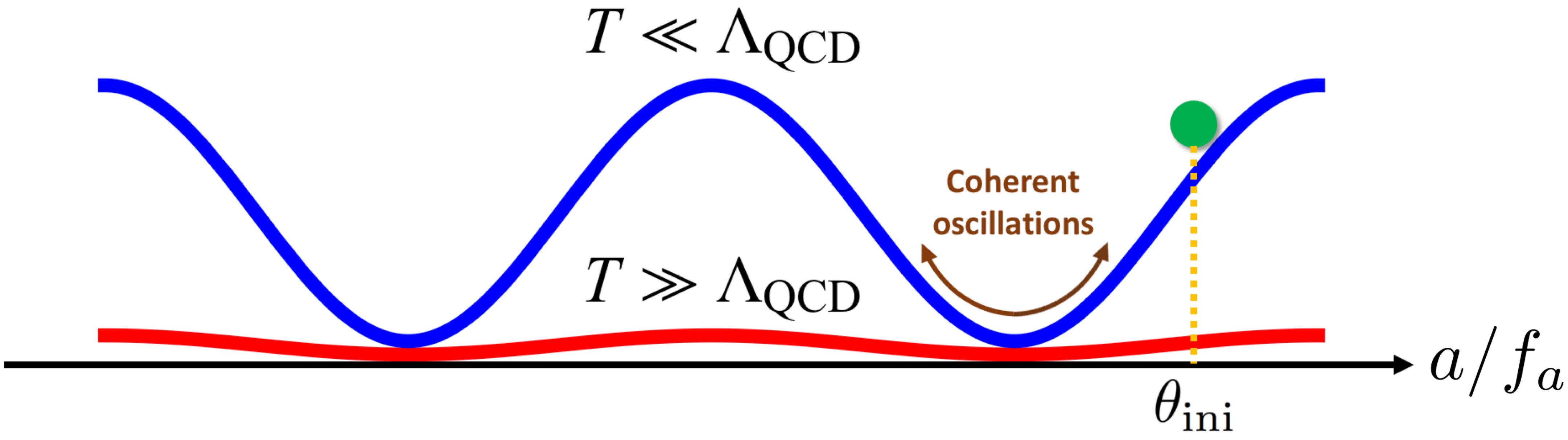
G. Cortona *et al.* `16

Axion-like particles (ALPs) do not satisfy the above relation.

Axions : The QCD axion (a)

- The QCD axion dark matter (DM) can be produced through the **realignment mechanism**.

$$\Omega_a h^2 \simeq 0.11\,\theta_{\rm ini}^2\,\underset{\text{Anharmonic effect}}{\underline{\mathcal{F}(\theta_{\rm ini})}}\left(\frac{f_a}{5\times 10^{11}\,{\rm GeV}}\right)^{1.184}$$



Axions : The QCD axion (a)

- The QCD axion coupling to photon

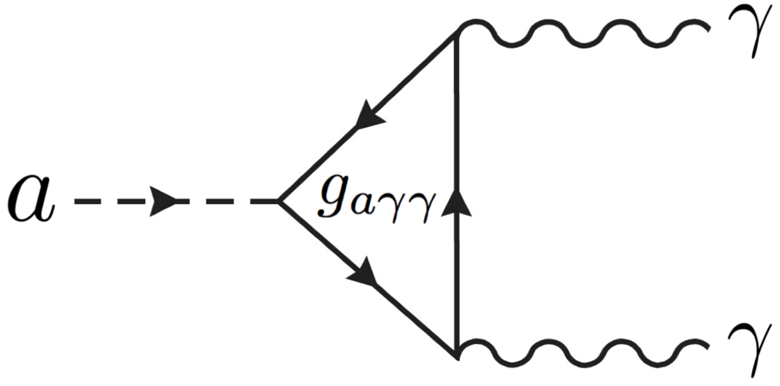
$$\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu}\widetilde{F}^{\mu\nu} , \quad \underline{g_{a\gamma\gamma}} = \frac{\alpha}{2\pi f_a}\left(\underline{-1.92} + \frac{\mathcal{E}}{\mathcal{N}}\right)$$

Axion-photon coupling

Axion-meson mixing

- ✓ $\mathcal{E}$: electromagnetic anomaly $\mathcal{N}$: color anomaly
- ✓ **KSVZ model :** $\mathcal{E}/\mathcal{N} = 0$
- ✓ **DFSZ model :** $\mathcal{E}/\mathcal{N} = 8/3$
- ✓ **The QCD axion DM :**

$$g_{a\gamma\gamma} \simeq 10^{-15}\,\mathrm{GeV}^{-1}$$ **(Stability)**



Axions : ALPs (axion-like particles) (φ)

- ALPs are motivated by string theory (Axiverse)
- The properties of ALPs :
  - ✓ They do not directly interact with QCD gluons
  - ✓ They can couple to photons

$$\mathcal{L}_{\varphi\gamma\gamma} = -\frac{g_{\varphi\gamma\gamma}}{4}\varphi F_{\mu\nu}\widetilde{F}^{\mu\nu} \ , \quad g_{\varphi\gamma\gamma} = \frac{\alpha}{2\pi f_\varphi} C_{\varphi\gamma}$$

  - ✓ They do not acquire a mass from the QCD effects
  - ✓ They can also be produced by the realignment mechanism (ALP DM)

$$\Omega_\varphi h^2 \simeq 0.3\,\theta_{\varphi,0}^2 \left(\frac{m_\varphi}{1\,\mathrm{eV}}\right)^{1/2} \left(\frac{f_\varphi}{10^{12}\,\mathrm{GeV}}\right)^2$$

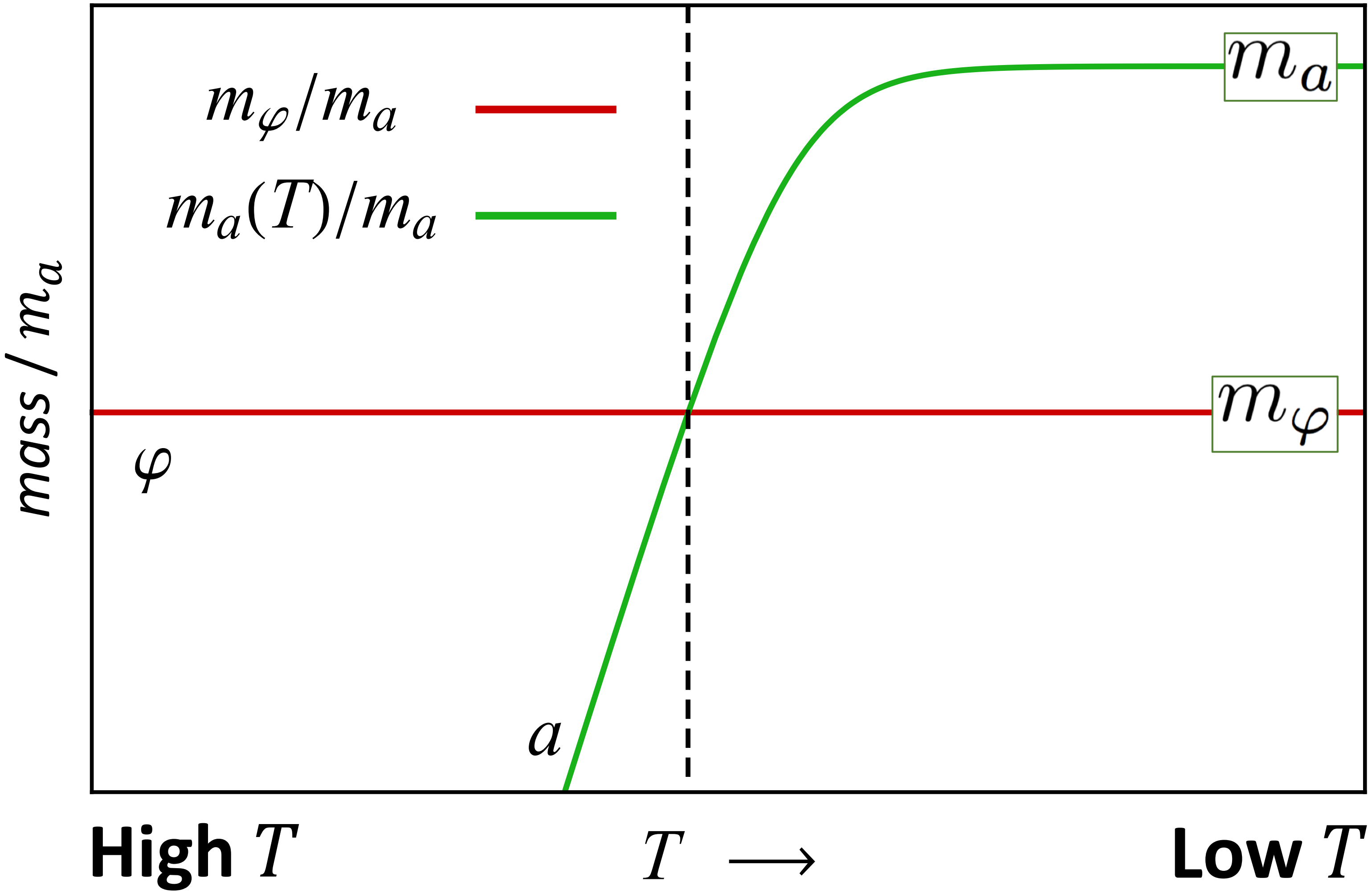
P. Arias, et al., `12

What if QCD axion a and ALP φ co-exist in nature?

What is the consequence if they have a mass mixing?

Level crossing

Without mass mixing

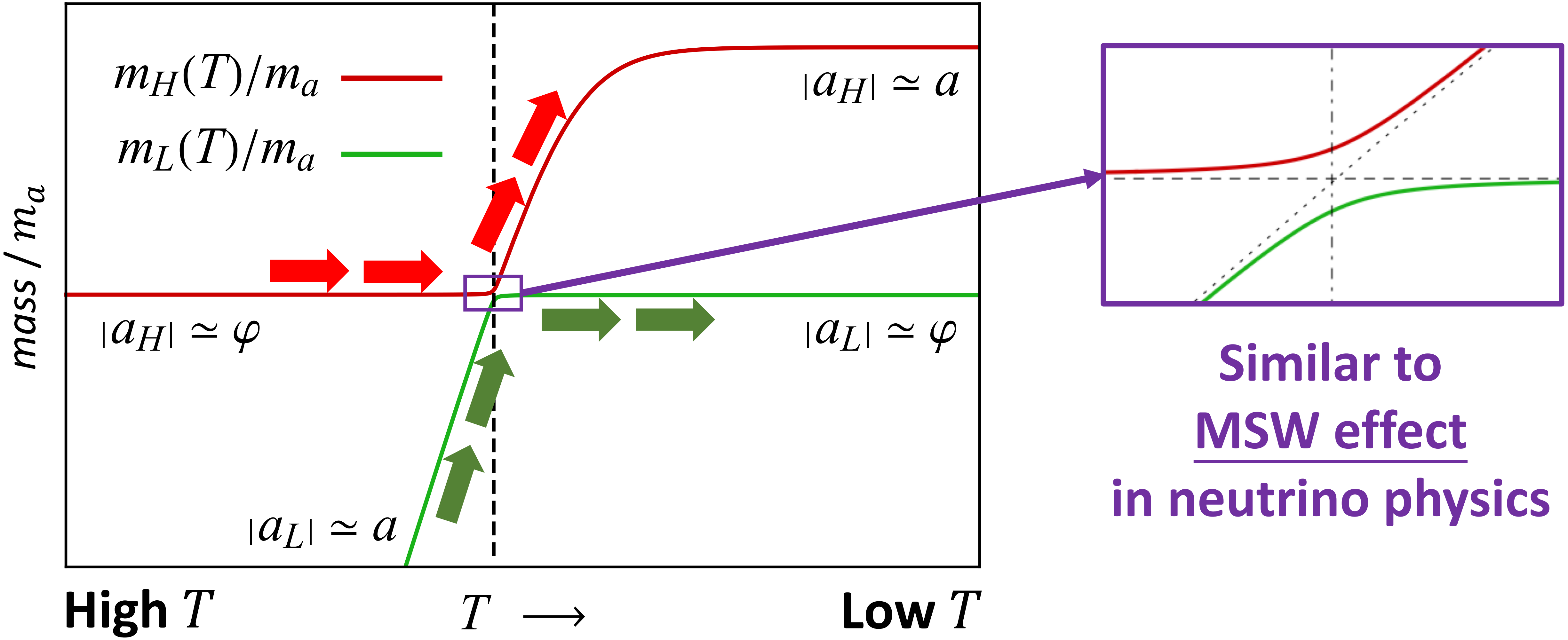


$$\begin{cases} m_a(T) \to 0 & T \gg \Lambda_{\rm QCD} \\ m_a(T) \to m_a & T \ll \Lambda_{\rm QCD} \end{cases}$$

ALPs do not acquire a mass from the effects of QCD.

Level crossing

With mass mixing



Similar to
MSW effect
in neutrino physics

What if QCD axion a and ALP φ co-exist in nature?

Adiabatic conversion can take place!

$$a \longleftrightarrow \varphi$$

Then, the axion abundance is suppressed by the mass ratio.

C. T. Hill and G. G. Ross (1988)
N. Kitajima & F. Takahashi (2014)

Mass mixing of the QCD axion and ALP

- **The Model**

$$V_{\mathrm{QCD}}(a) = \underbrace{m_a^2(T)}_{\text{QCD axion mass}} f_a^2 \left[1-\cos\left(\frac{a}{\underbrace{f_a}_{\text{QCD axion decay constant}}}\right)\right], \quad V_{\mathrm{mix}}(a,\varphi) = \underbrace{m_\varphi^2}_{\text{ALP mass}} f_\varphi^2 \left[1-\cos\left(\overbrace{\frac{a}{f_a}+\frac{\varphi}{\underbrace{f_\varphi}_{\text{ALP decay constant}}}}^{\text{Mass mixing}}\right)\right]$$

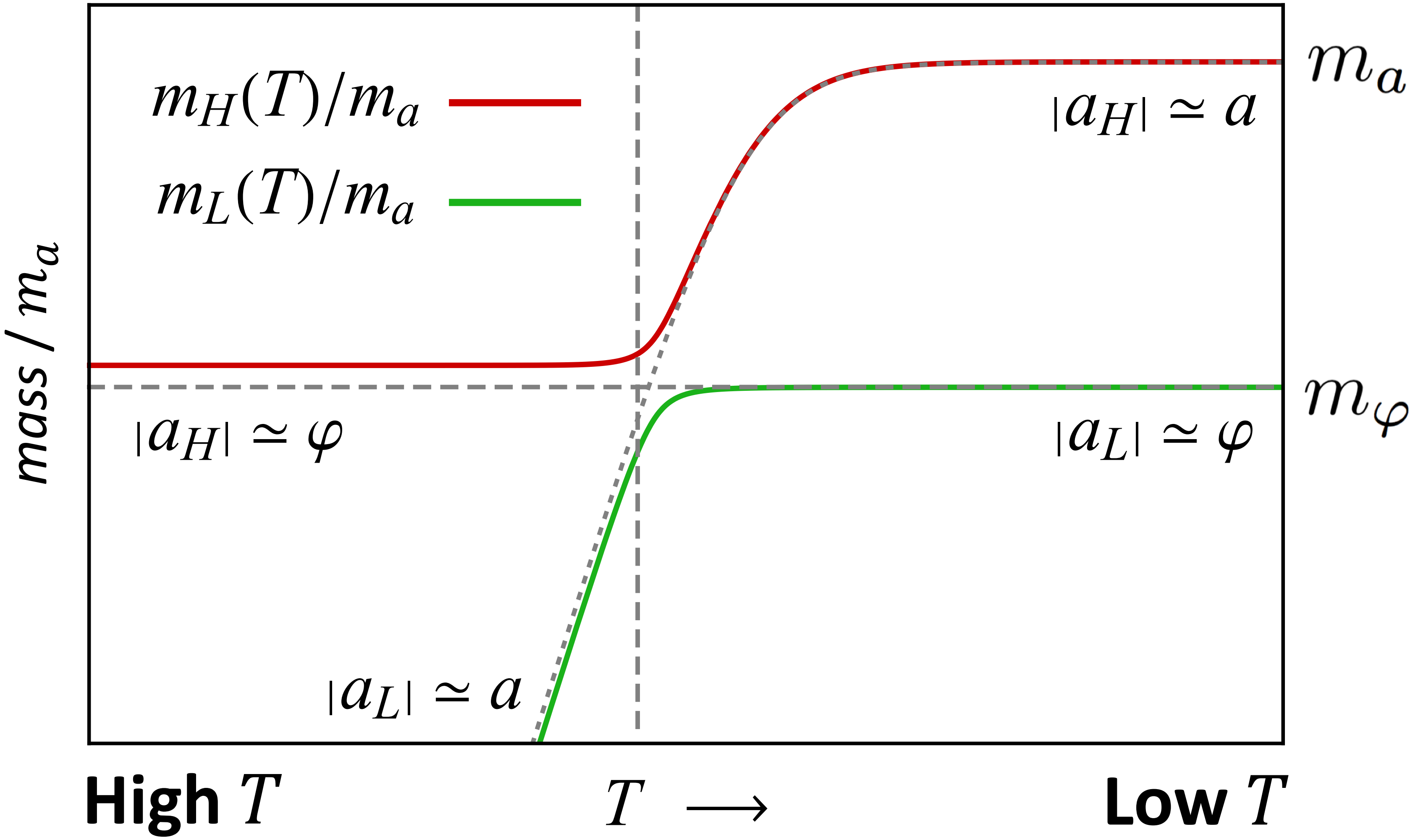
- **Mass eigenstate and mixing angle ξ :**

$$\begin{pmatrix} a_H \\ a_L \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} \varphi \\ a \end{pmatrix}$$

$$m_{H,L}^2(T) = \frac{1}{2} m_a^2(T) \left\{ 1 + \frac{m_\varphi^2}{m_a^2(T)} \left[1 + \frac{f_\varphi^2}{f_a^2} \pm \sqrt{\left(1 - \frac{f_\varphi^2}{f_a^2} - \frac{m_a^2(T)}{m_\varphi^2}\right)^2 + 4\frac{f_\varphi^2}{f_a^2}} \right] \right\}$$

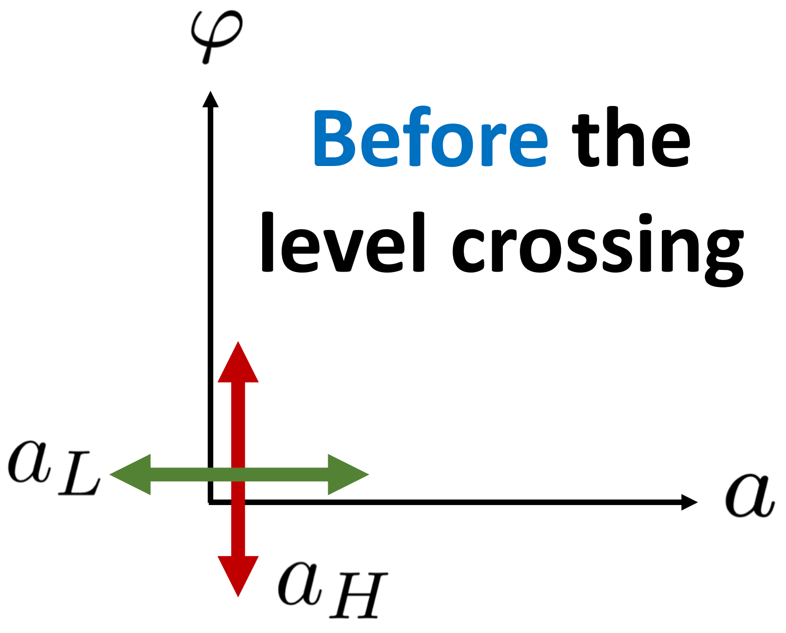
Level crossing between the QCD axion and ALP

- **Mass eigenvalues**



$$f_\varphi < f_a$$

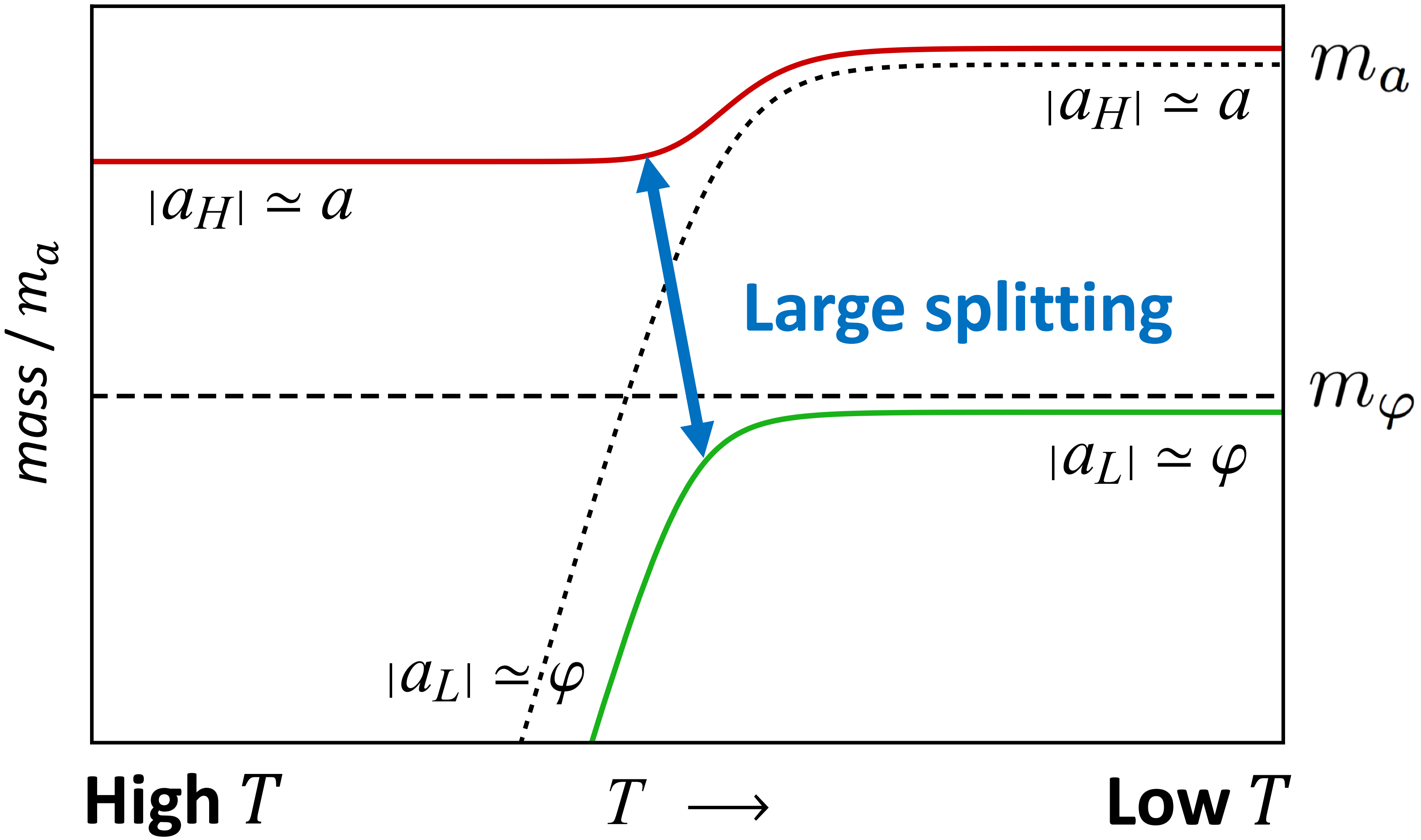
$$m_\varphi < m_a$$



Before the level crossing

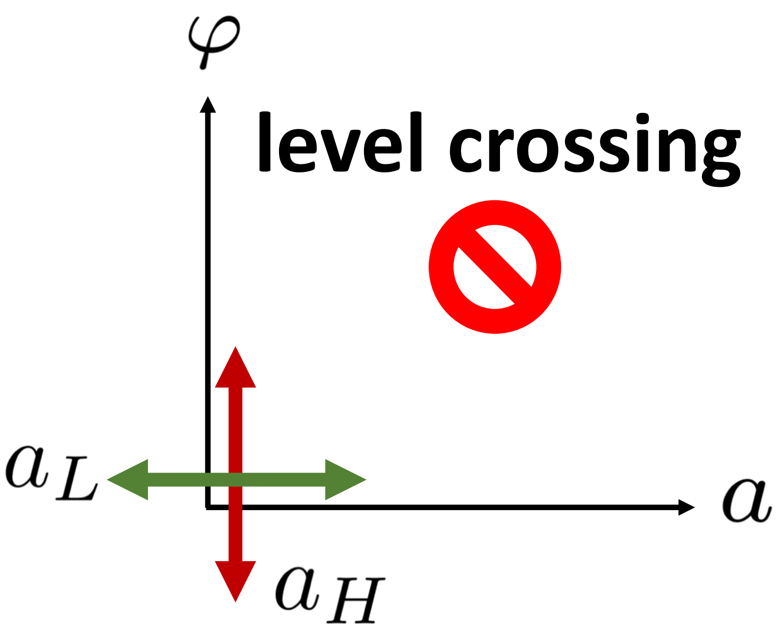
Level crossing between the QCD axion and ALP

- **Mass eigenvalues**



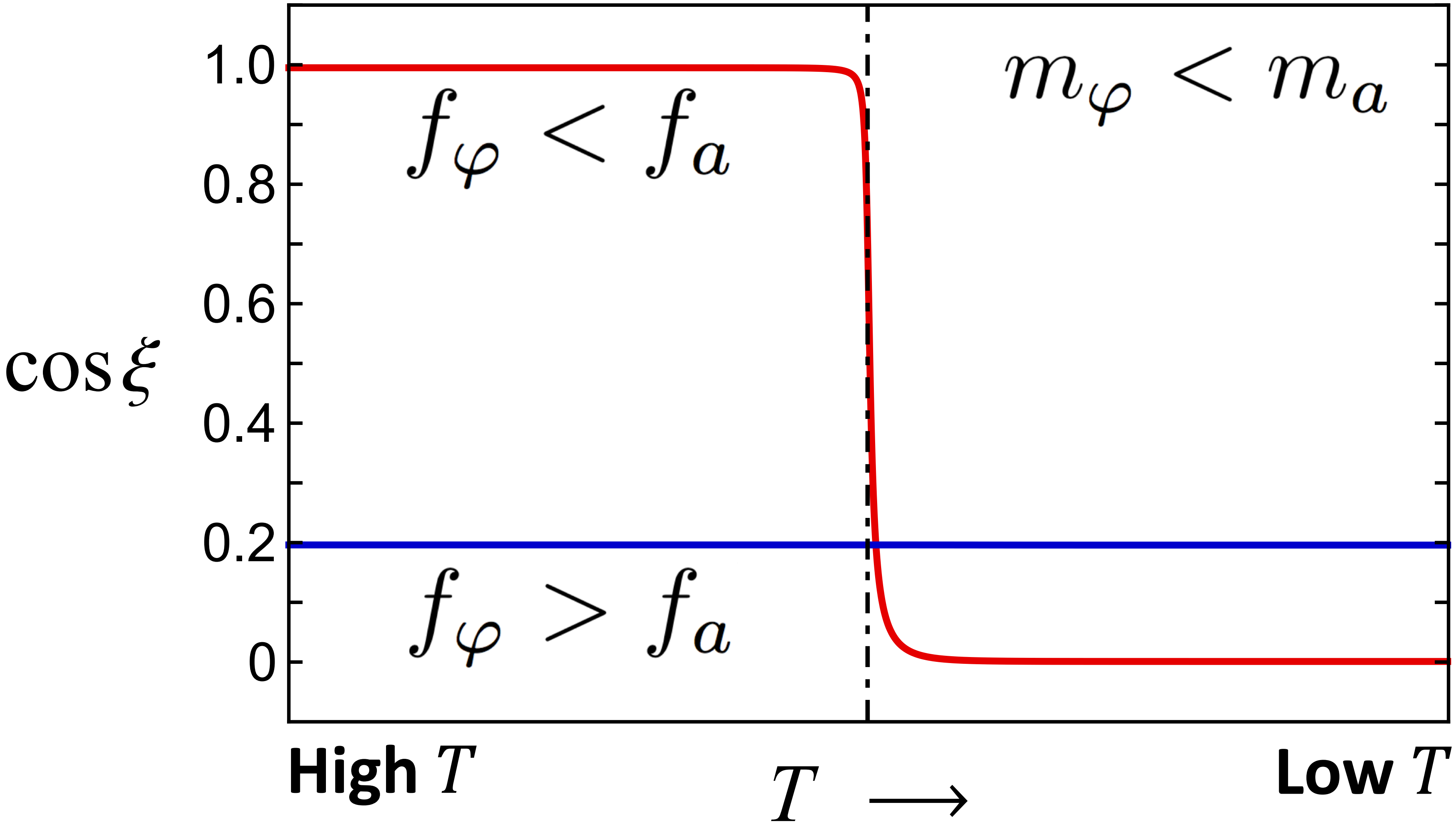
$$f_\varphi > f_a$$

$$m_\varphi < m_a$$



Level crossing between the QCD axion and ALP

- **Mixing angle**

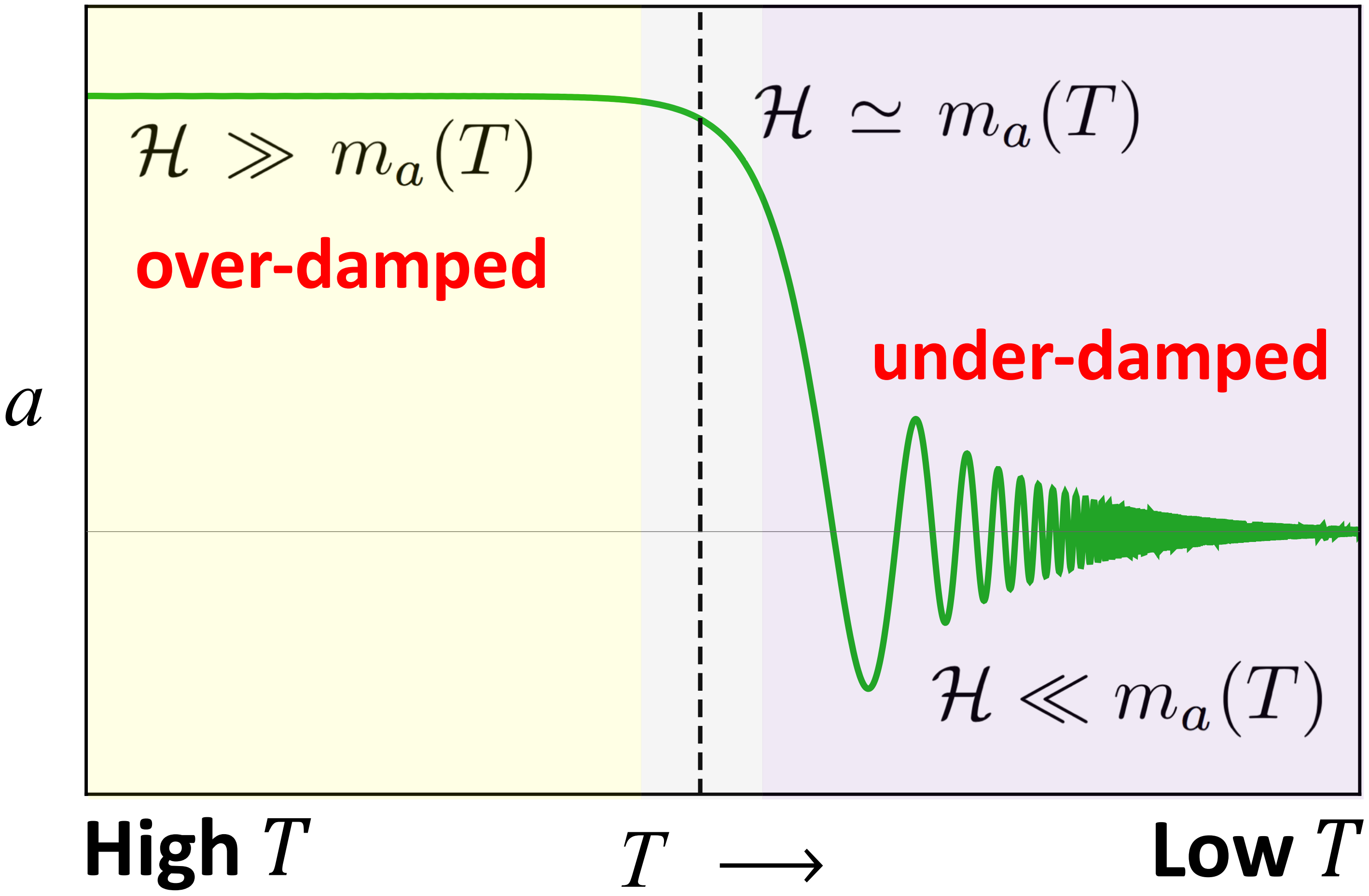


Adiabatic invariant of the QCD axion

- **E.O.M of the QCD axion :** $\ddot{a} + 3\mathcal{H}\dot{a} + m_a^2(T)\, a = 0$

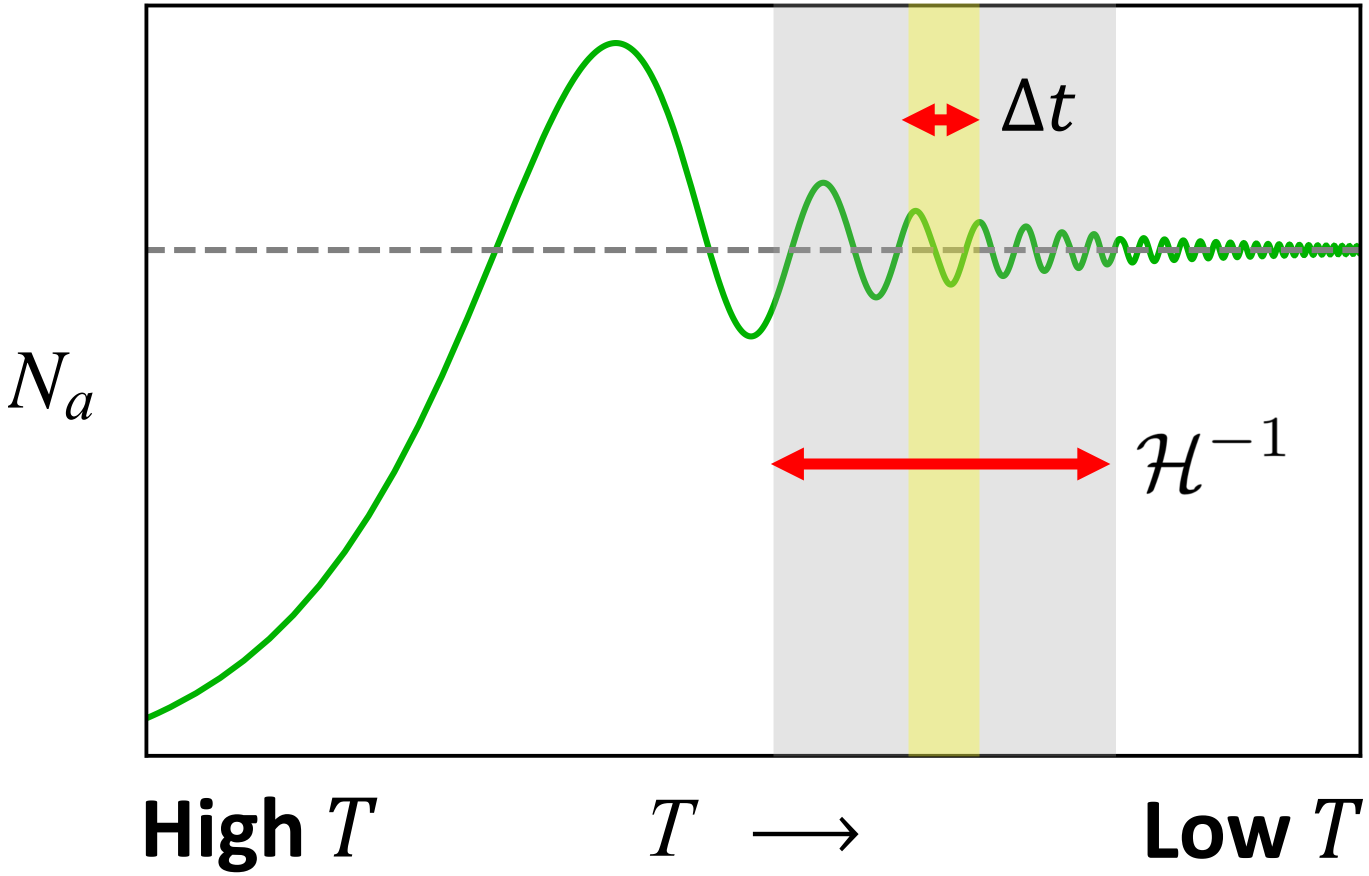
$T_{osc} \sim$ 1 GeV

$\mathcal{H} = \mathcal{H}(T)$: Hubble parameter



Adiabatic invariant of the QCD axion

- **Comoving axion number :** $N_a(T) = \dfrac{\rho_a(T)}{m_a(T)s(T)}$



$$N_a(T < T_{\rm osc}) = \text{ const.}$$

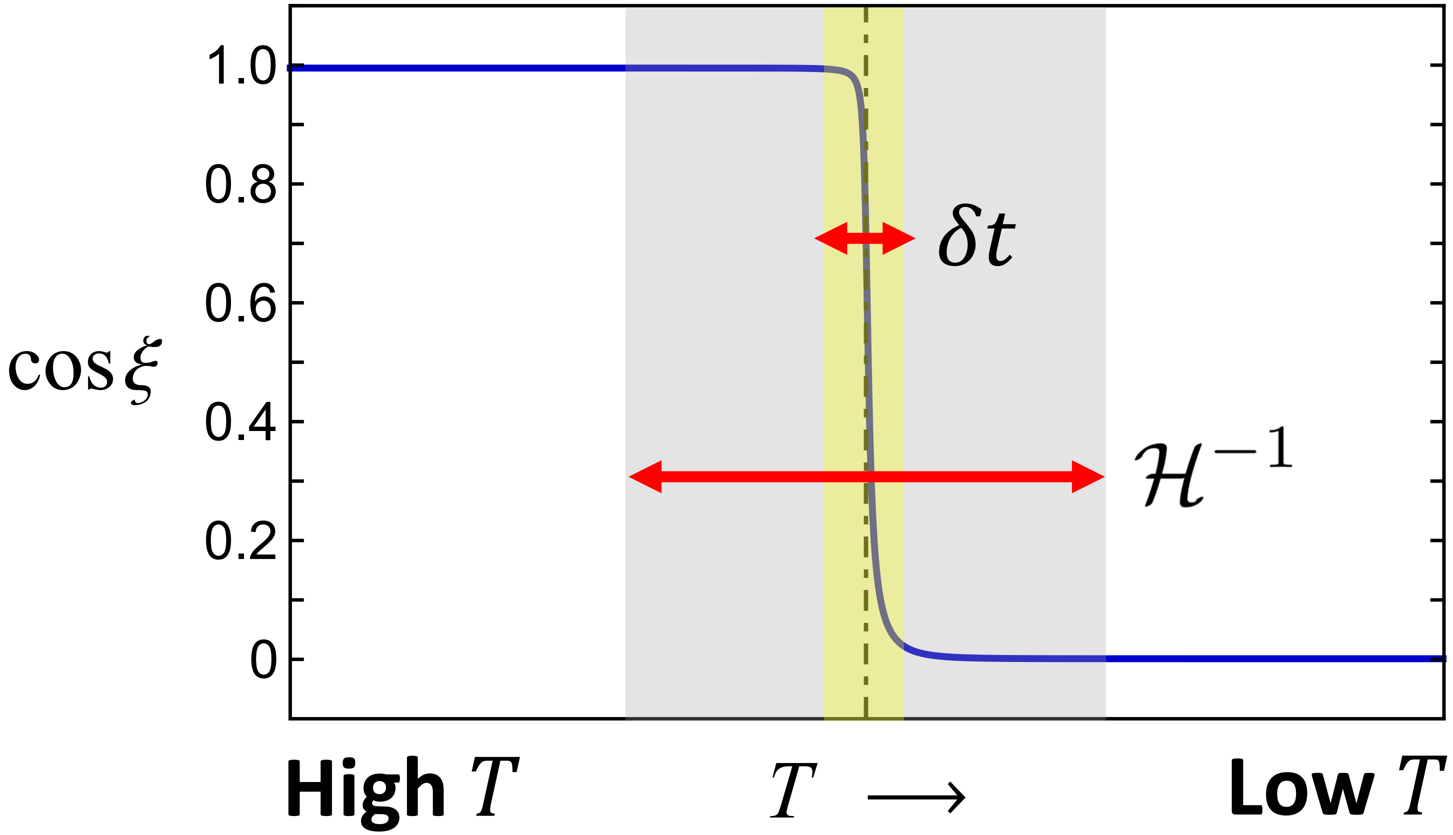
Adiabatic process

Ext. time scale >> Int. time scale

$$\mathcal{H}^{-1} \gg \Delta t = \frac{2\pi}{m_a(T)}$$

Adiabatic invariant in the case of two axions

- **What is the external time scale if there is a mass mixing between the QCD axion and ALP?**



✓ **Hubble time : $\mathcal{H}^{-1}$**

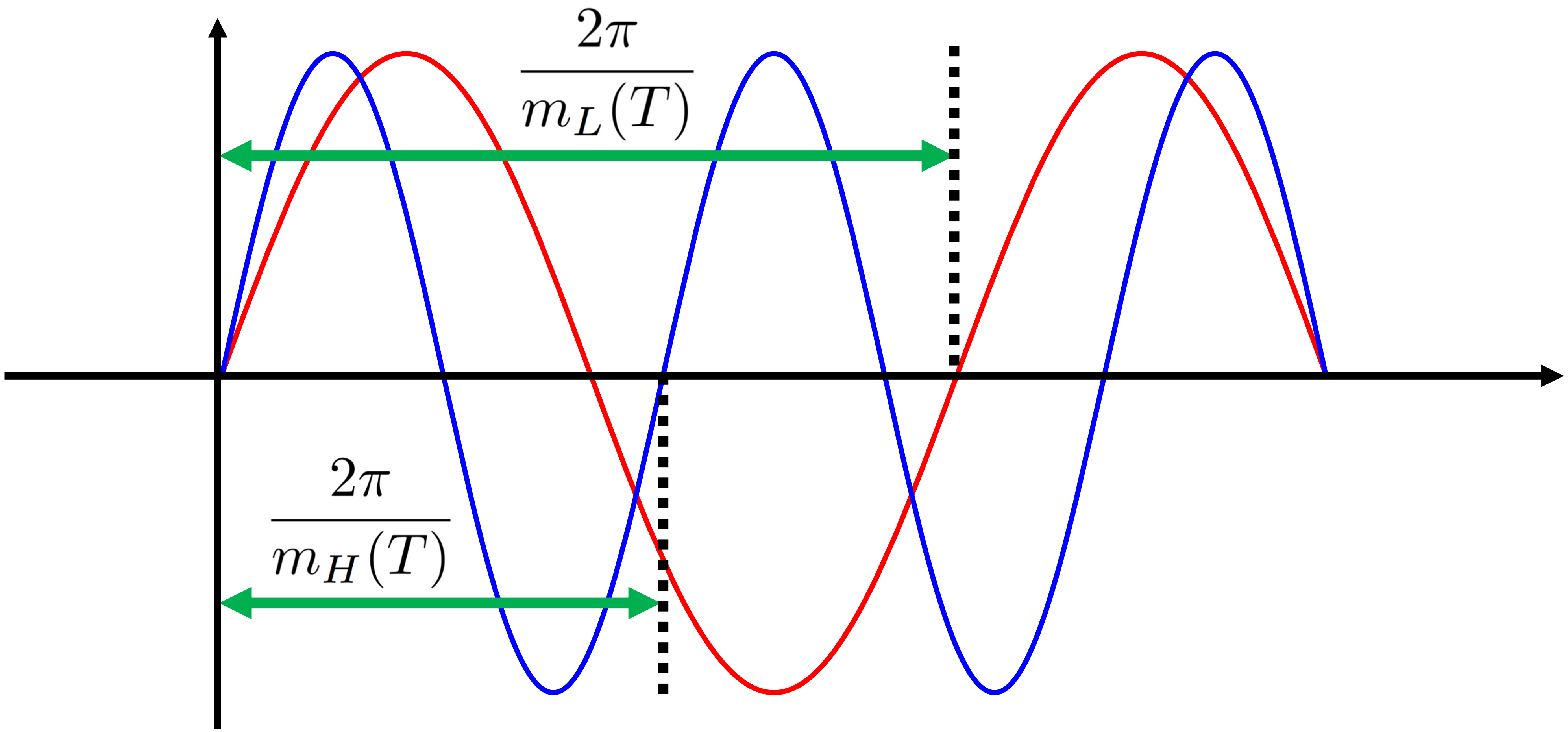
✓ **Transition time at the level crossing :**

$$\delta t \sim \left| \frac{d \ln \cos\xi(T)}{dt} \right|^{-1}$$

$$(\mathcal{H}^{-1} \gg dt)$$

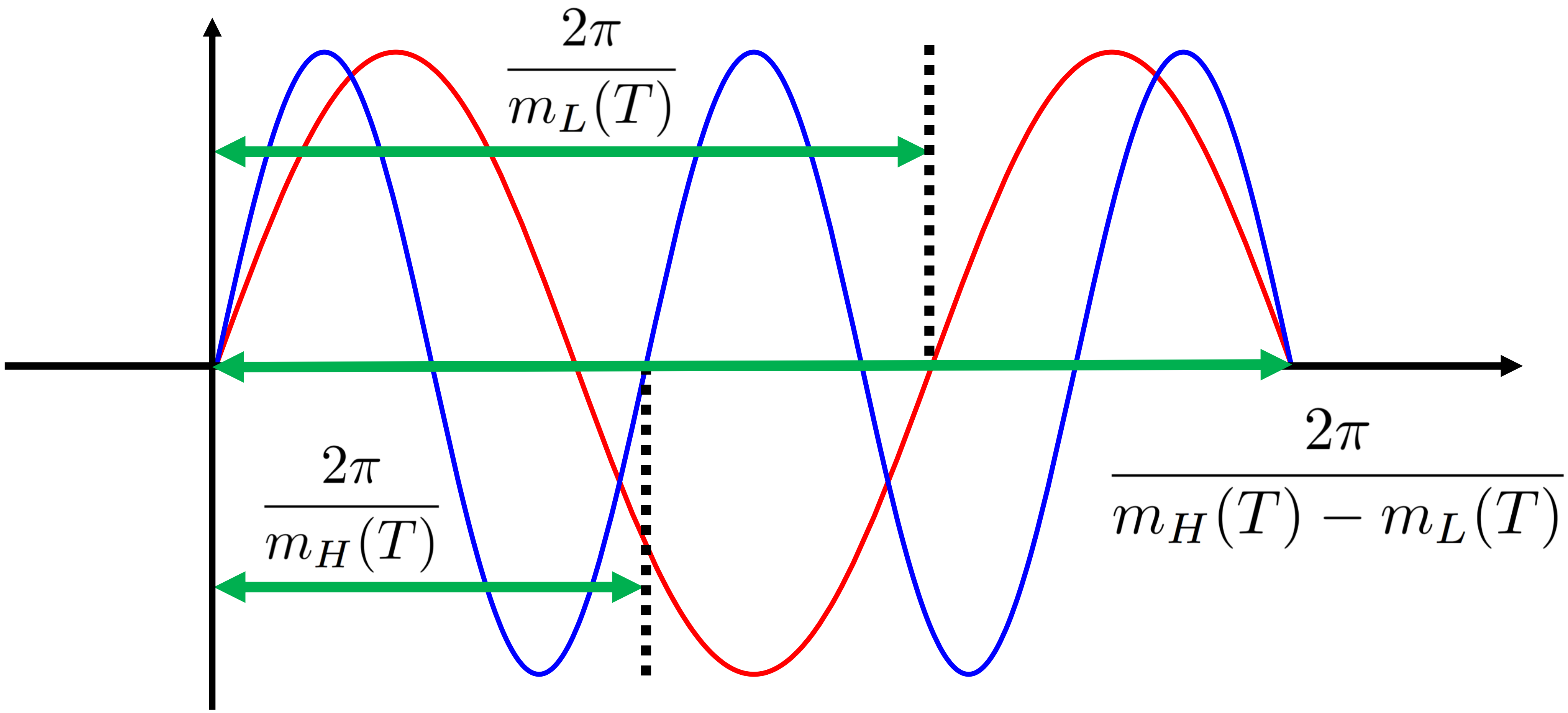
Adiabatic invariant in the case of two axions

- What is the internal time scale if there is a mass mixing between the QCD axion and ALP?



Adiabatic invariant in the case of two axions

- **What is the internal time scale if there is a mass mixing between the QCD axion and ALP?**



Adiabatic invariant in the case of two axions

- **Adiabatic condition : External time scale >> Internal time scale**

$$\delta t \sim \left|\frac{d\ln\cos\xi(T)}{dt}\right|^{-1} \gg \mathrm{Max}\left[\frac{2\pi}{m_H(T)}, \frac{2\pi}{m_L(T)}, \frac{2\pi}{m_H(T)-m_L(T)}\right]$$

Beat frequency

- **At the level crossing with** $f_\varphi \ll f_a$

$$\boxed{\gamma \equiv \beta\,\frac{f_\varphi}{f_a}\sqrt{\frac{m_\varphi}{\mathcal{H}(T_{\rm lc})}} \gg 1}$$

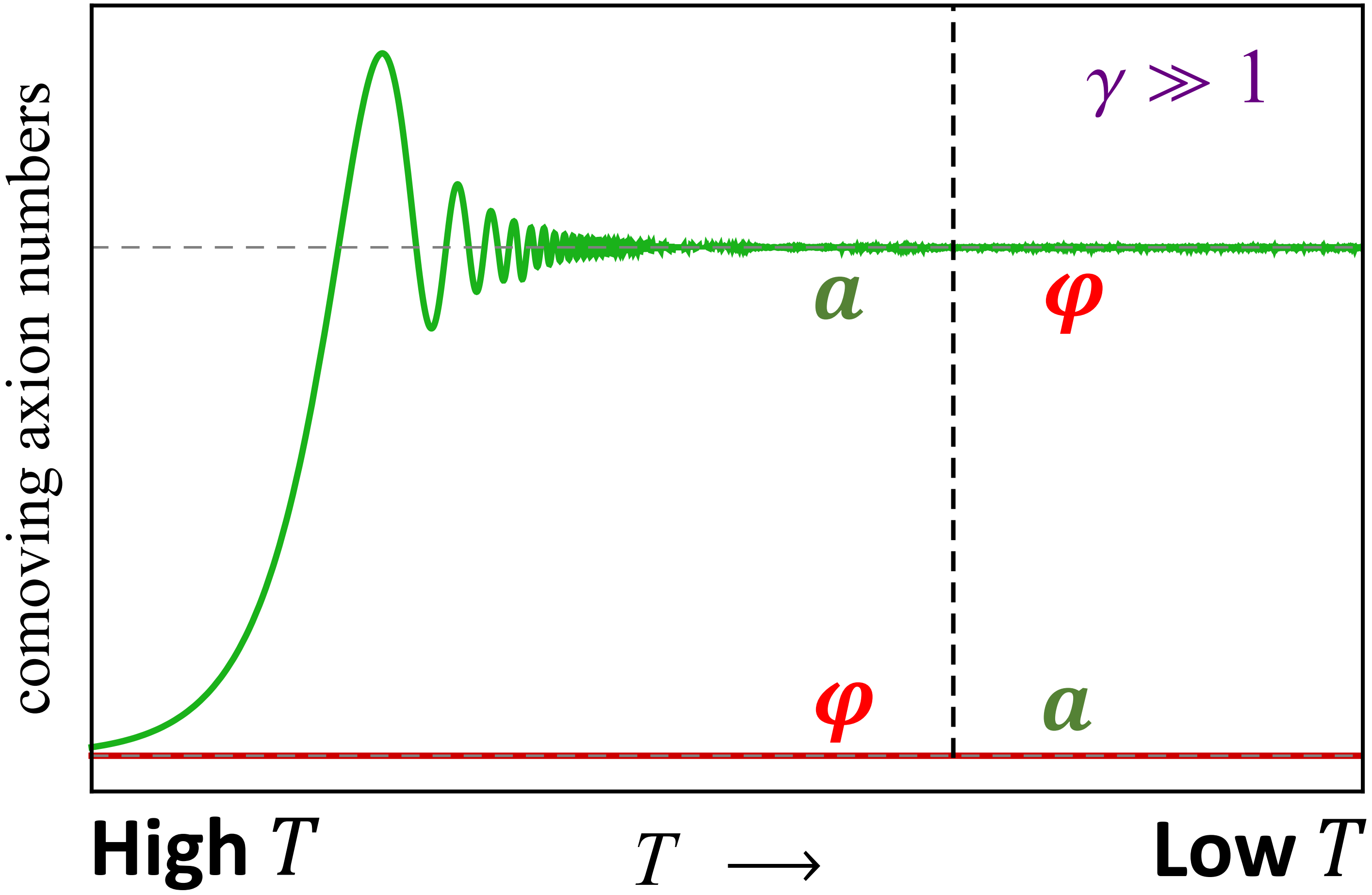
$$\beta \sim 0.3$$

(N_L and N_H are separately conserved)

Adiabatic conversion btw. the QCD axion and ALP

- **Evolution of $N_{L,H}(T)$** $N_L(T)$ 🟩 $N_H(T)$ 🟥

$f_\varphi/f_a = 0.05,\, m_\varphi/m_a = 0.5$



$$\gamma = \beta \frac{f_\varphi}{f_a} \sqrt{\frac{m_\varphi}{\mathcal{H}(T_{\rm lc})}}$$

Adiabatic conversion btw. the QCD axion and ALP

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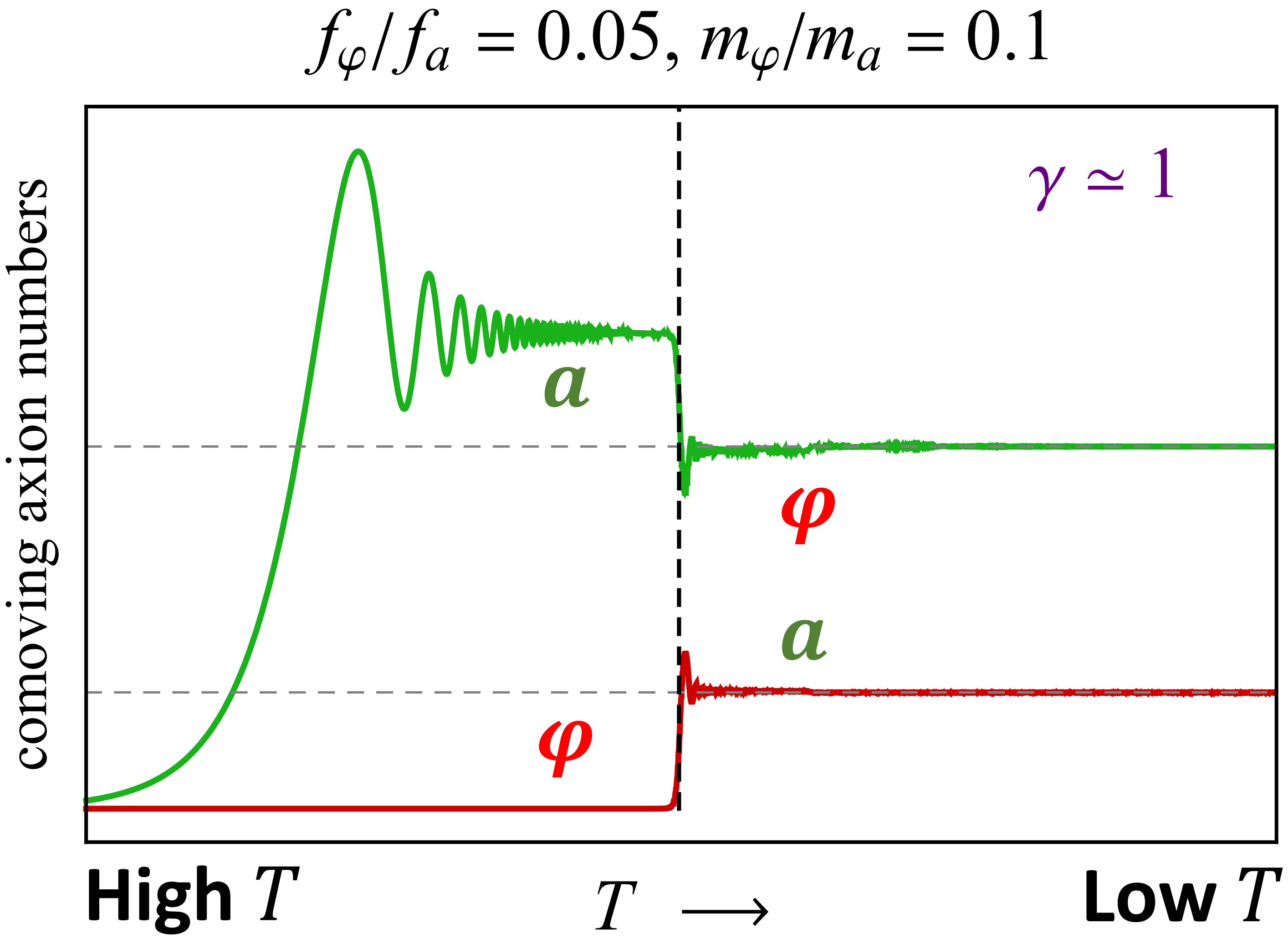
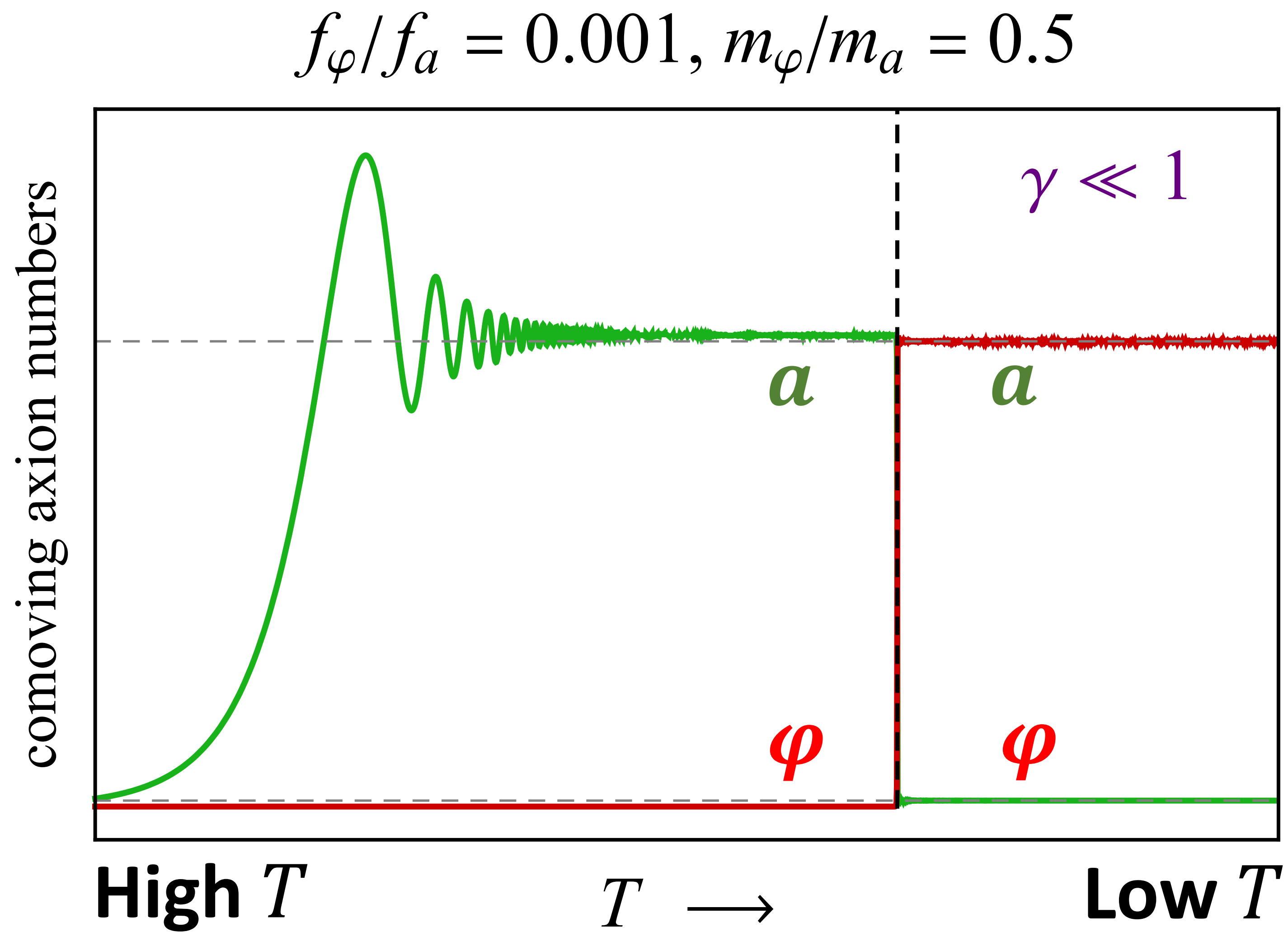


$$\gamma = \beta \frac{f_\varphi}{f_a} \sqrt{\frac{m_\varphi}{\mathcal{H}(T_{\rm lc})}}$$

Adiabatic conversion btw. the QCD axion and ALP

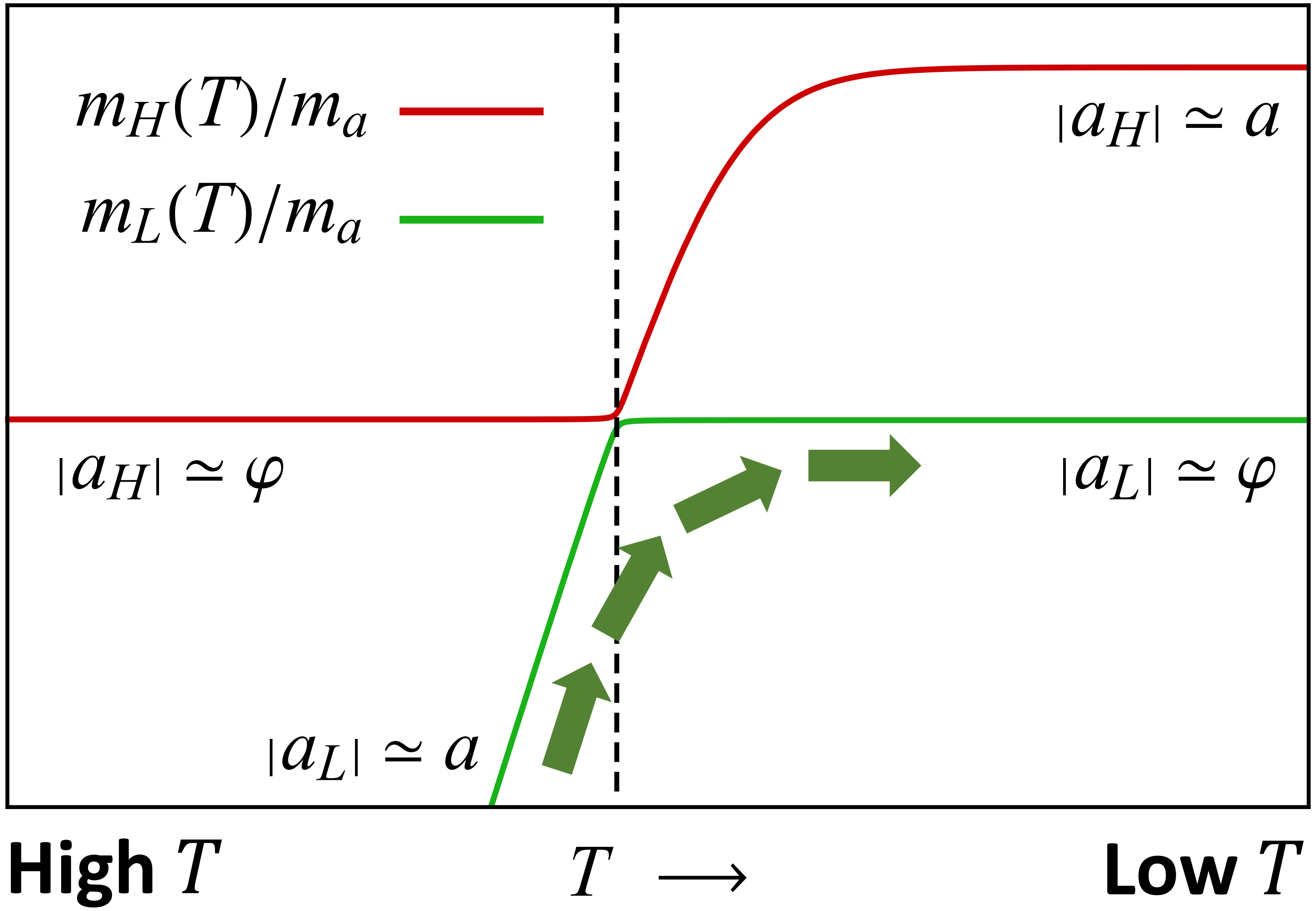
- **Evolution of $N_{L,H}(T)$** $N_L(T)$ 🟩 $N_H(T)$ 🟥



$$\gamma = \beta \frac{f_\varphi}{f_a} \sqrt{\frac{m_\varphi}{\mathcal{H}(T_{\rm lc})}}$$

Cosmological abundance

- **If adiabatic conversion occurs, then the QCD axion becomes ALP**



$$\Omega_a \propto m_a N_a$$

(w/o adiabatic conversion)

$$\Omega_L \propto m_\varphi N_L$$

(w/ adiabatic conversion)

The abundance is suppressed by the mass ratio m_φ/m_a

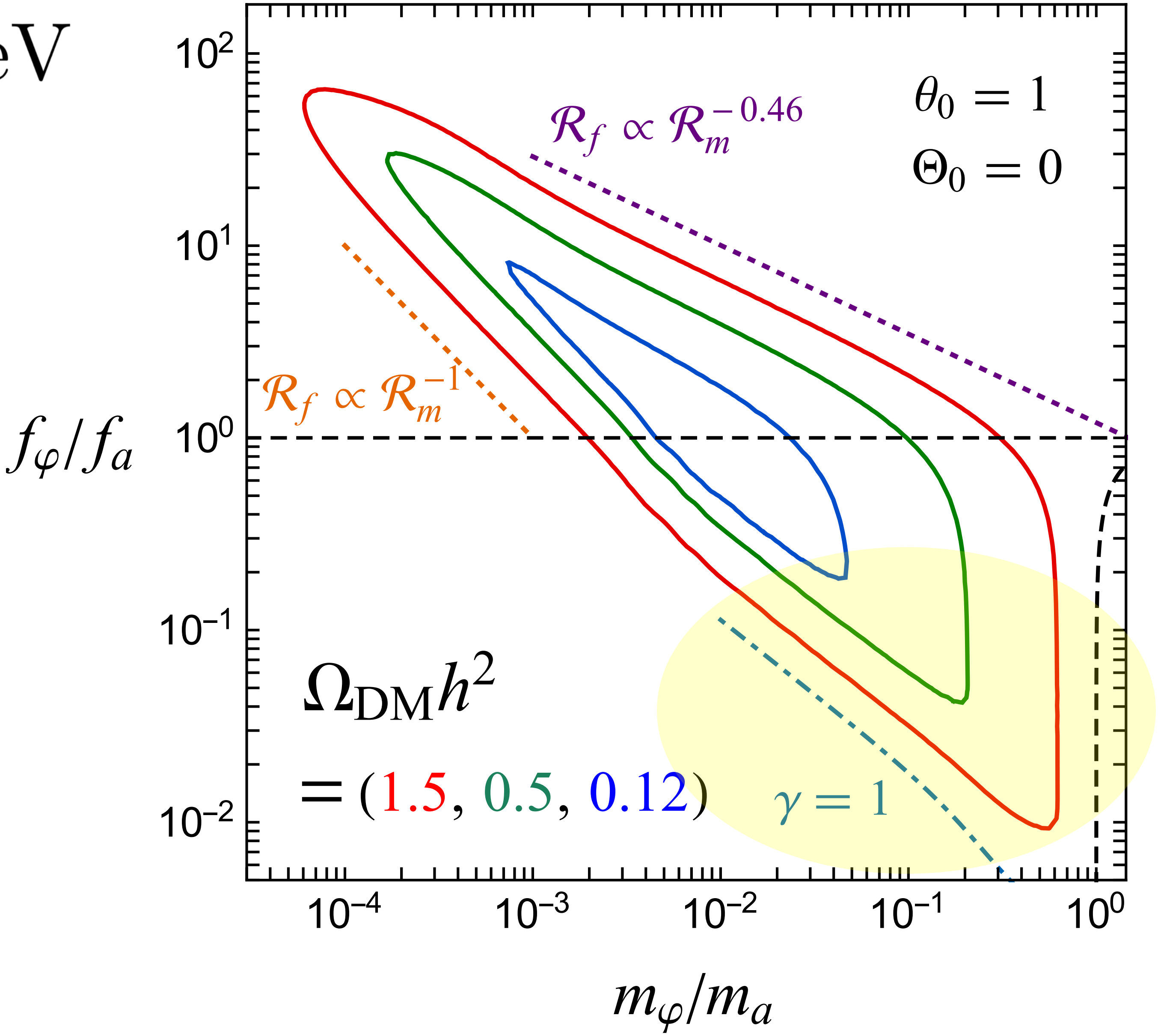
Cosmological abundance

- **Contours of** $f_a = 10^{13}\,\mathrm{GeV}$

$$\Omega_{\mathrm{DM}}h^2 = \Omega_H h^2 + \Omega_L h^2$$

$$\Omega_H h^2 = \frac{m_H s_0}{\rho_{c,0}} N_H$$

$$\Omega_L h^2 = \frac{m_L s_0}{\rho_{c,0}} N_L$$



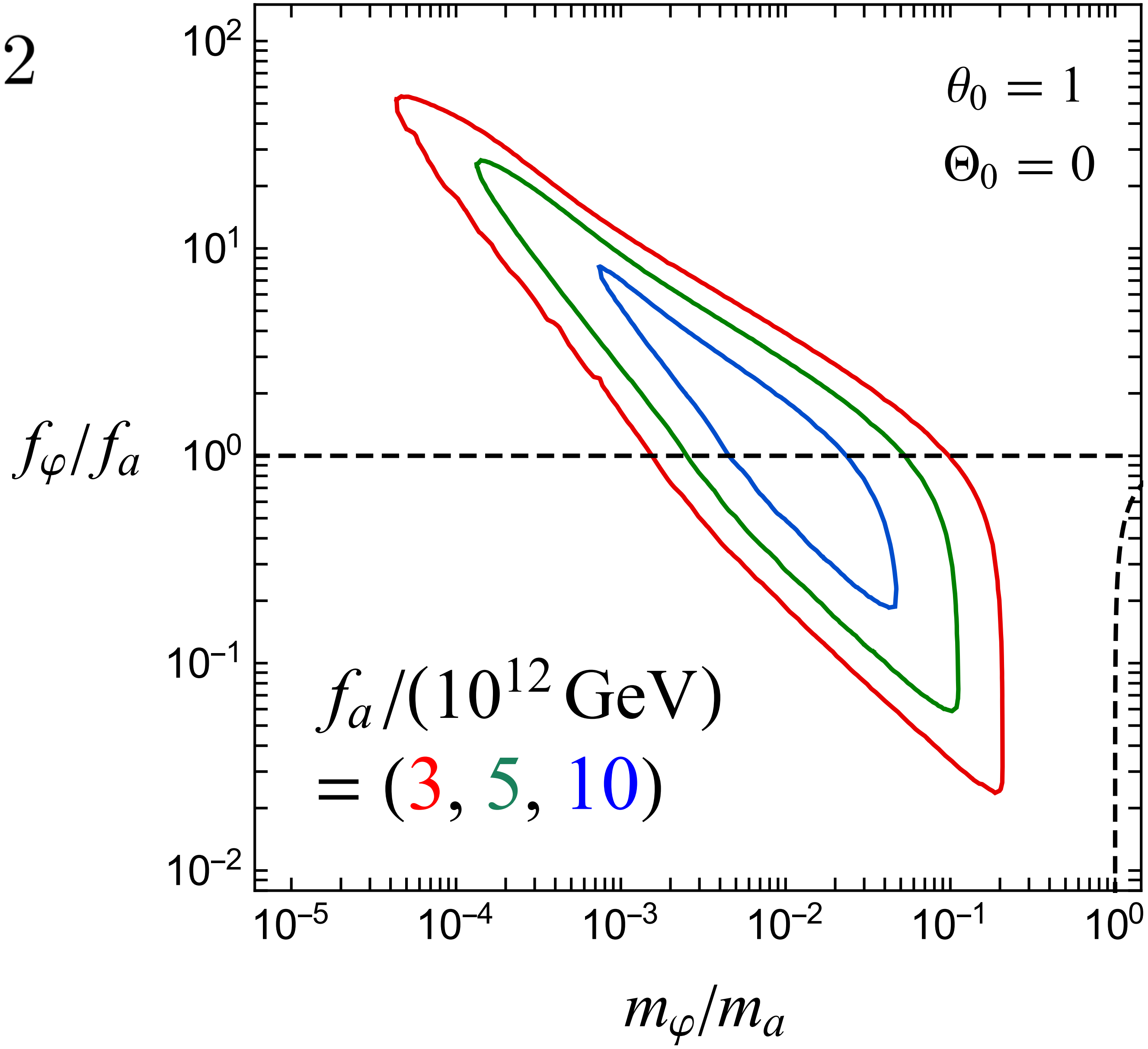
Cosmological abundance

- **Contours of** $\Omega_{\mathrm{DM}} h^2 = 0.12$

$$\Omega_{\mathrm{DM}} h^2 = \Omega_H h^2 + \Omega_L h^2$$

$$\Omega_H h^2 = \frac{m_H s_0}{\rho_{c,0}} N_H$$

$$\Omega_L h^2 = \frac{m_L s_0}{\rho_{c,0}} N_L$$



Implications for the axion search experiments

- **Axion-photon couplings**

$$\begin{aligned}\mathcal{L}_{\text{axion-}\gamma\text{-}\gamma} &= -\frac{\alpha}{8\pi}\left(C_{a\gamma}\frac{a}{f_a} + C_{\varphi\gamma}\frac{\varphi}{f_\varphi}\right)F_{\mu\nu}\widetilde{F}^{\mu\nu} \\ &= -\frac{1}{4}\left(\underline{g_{L\gamma\gamma}}\,a_L + \underline{g_{H\gamma\gamma}}\,a_H\right)F_{\mu\nu}\widetilde{F}^{\mu\nu}\end{aligned}$$

Couplings of the light (heavy) axion mode to photons

where

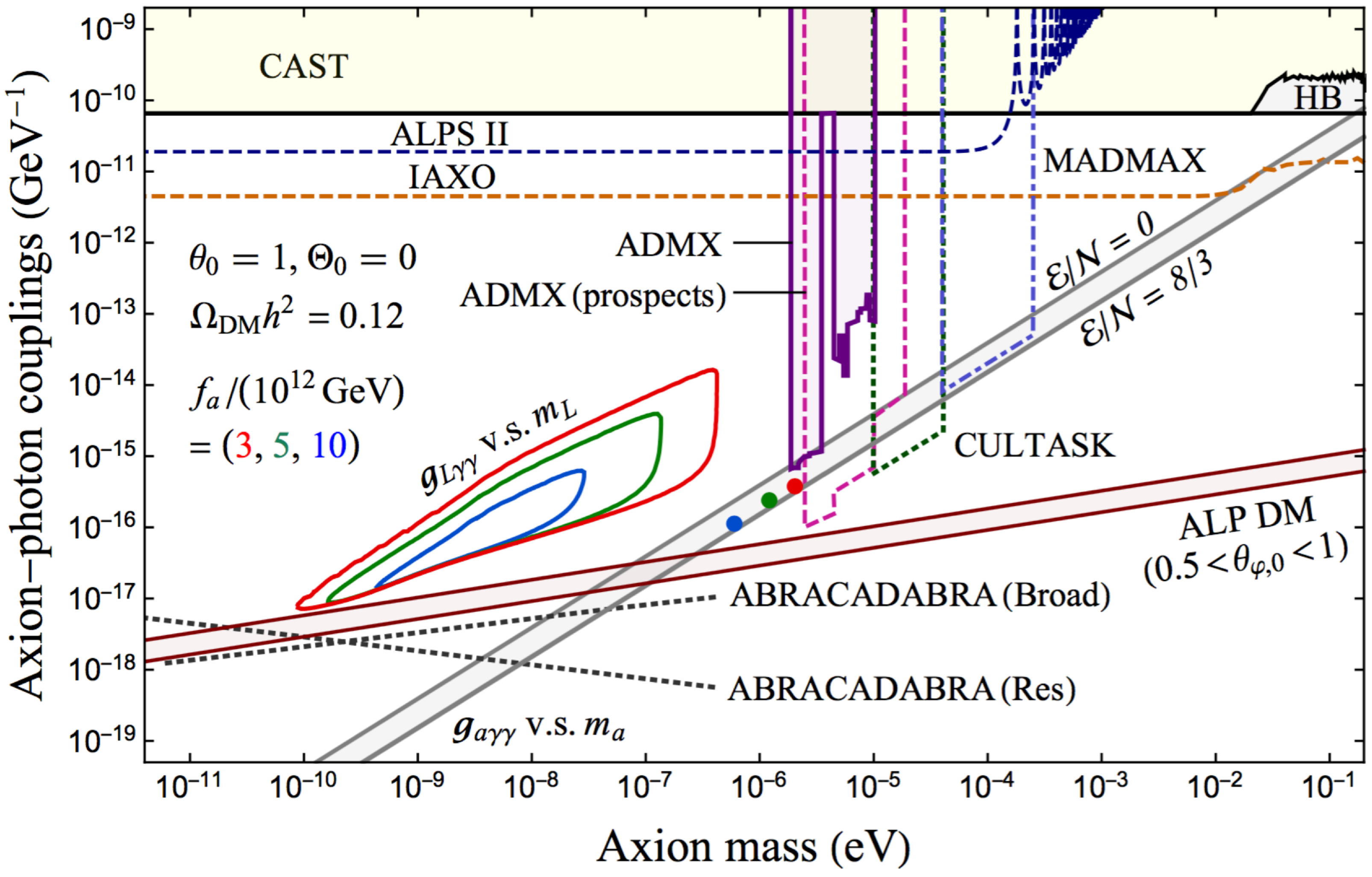
$$g_{L\gamma\gamma} = \frac{\alpha}{2\pi f_a}\left(C_{a\gamma}\cos\xi_0 - C_{\varphi\gamma}\frac{\sin\xi_0}{\mathcal{R}_f}\right), \quad g_{H\gamma\gamma} = \frac{\alpha}{2\pi f_a}\left(C_{a\gamma}\sin\xi_0 + C_{\varphi\gamma}\frac{\cos\xi_0}{\mathcal{R}_f}\right)$$

$$\xi_0 \equiv \xi(T \to 0)$$

Fiducial values : $C_{a\gamma} = C_{\varphi\gamma} = 1$

Implications for the axion search experiments

- **Our result : ALP-photon coupling is enhanced by a factor of 10-1000 compared with the ALP DM without mass mixing.**



Summary

- We studied the scenario where the QCD axion and ALP have a nonzero mass mixing.
- We clarified when the adiabatic conversion takes place.
- We showed that the ALP produced by the adiabatic conversion of the QCD axion explains the observed DM abundance.
- In this scenario, the ALP-photon coupling is enhanced by a few orders of magnitude, which is advantageous for the future axion-search experiments using the axion-photon coupling.

Thank you for your attention!!