Probing Dark Energy Directly *via* Gravitational Lensing

Hong-Jian He

Based on arXiv:1701.03418 [astro-ph.CO] with Zhen Zhang
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New Physics $\stackrel{?}{=}$ New Particles

New Physics = New Phenomena !

New Physics = New Principles !!

What is Dark Energy ?

Dark Energy (DE)

- **Einstein-Hilbert Action:**

$$S = \int d^4x \sqrt{-g}\, [\kappa^{-2}(R + \Lambda) + \mathcal{L}_m]$$

- **Einstein Equation with Cosmological Constant:**

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

- **Indirect Probe from CMB Measurements on scale factor a(t) at Cosmological Scales > 100-200Mpc , in FRW metric.**

- **Friedmann Eq:**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}$$

Dark Energy: Planck 2018

arXiv:1807.06209

- ΛCDM Fit (Planck+TT,TE,EE+lowE+lensing+BAO):

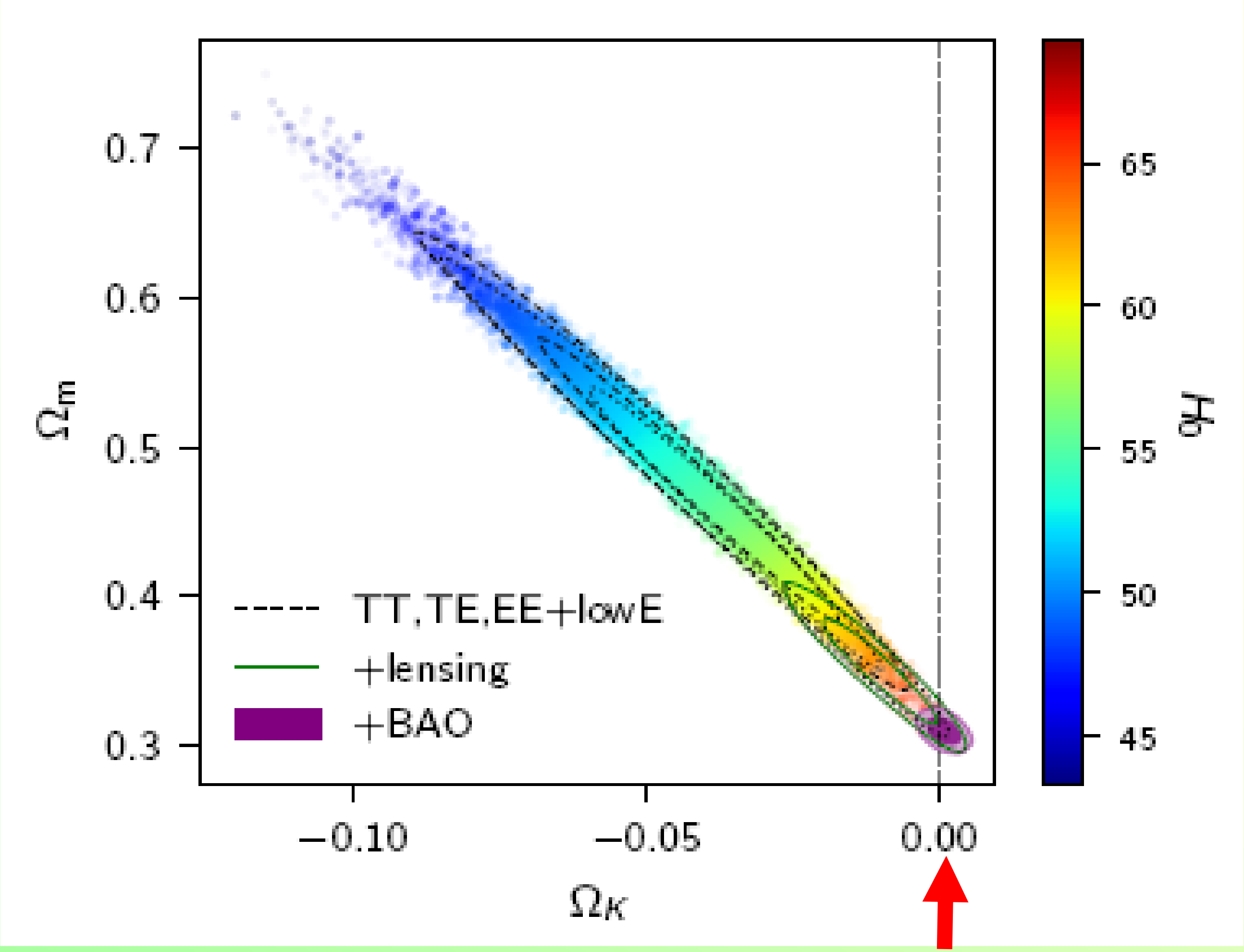
$$\Omega_\Lambda = 0.6889 +/- 0.0056$$
$$\Omega_m = 0.3111 +/- 0.0056$$
$$\Omega_K = 0.001 +/- 0.002$$

where $\Omega_m = \Omega_B + \Omega_{DM} = 0.04897 + 0.2607.$

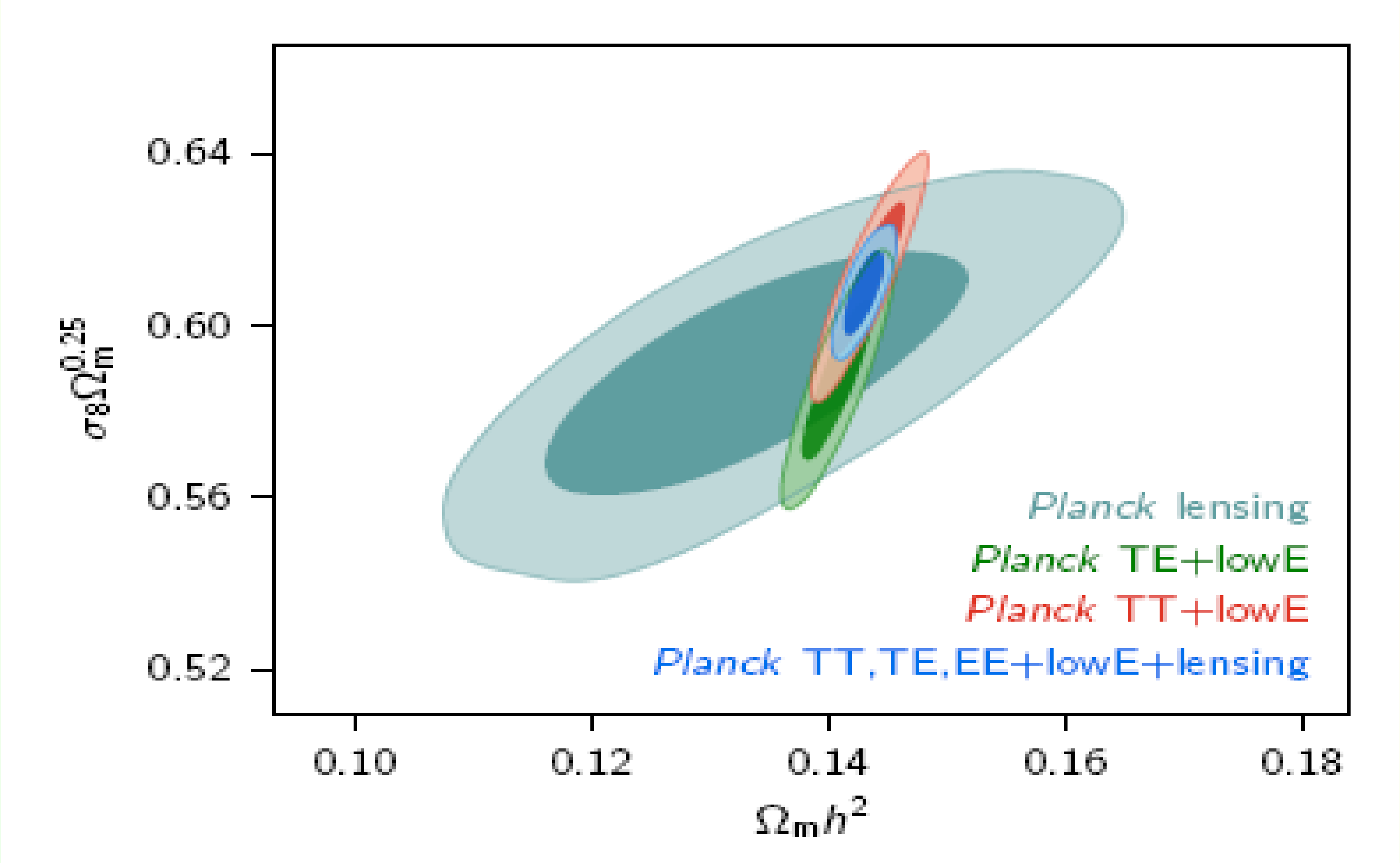
Hubble constant: $H_0 = 67.66 +/- 0.42$ km s^{-1} Mpc^{-1} .

Nearly Flat Universe

- **Constraint: the Universe is fairly flat:** $\Omega_K = 0.001 +/- 0.002$
- **Planck 2018:** [arXiv:1807.06209]



Planck Fit 2018

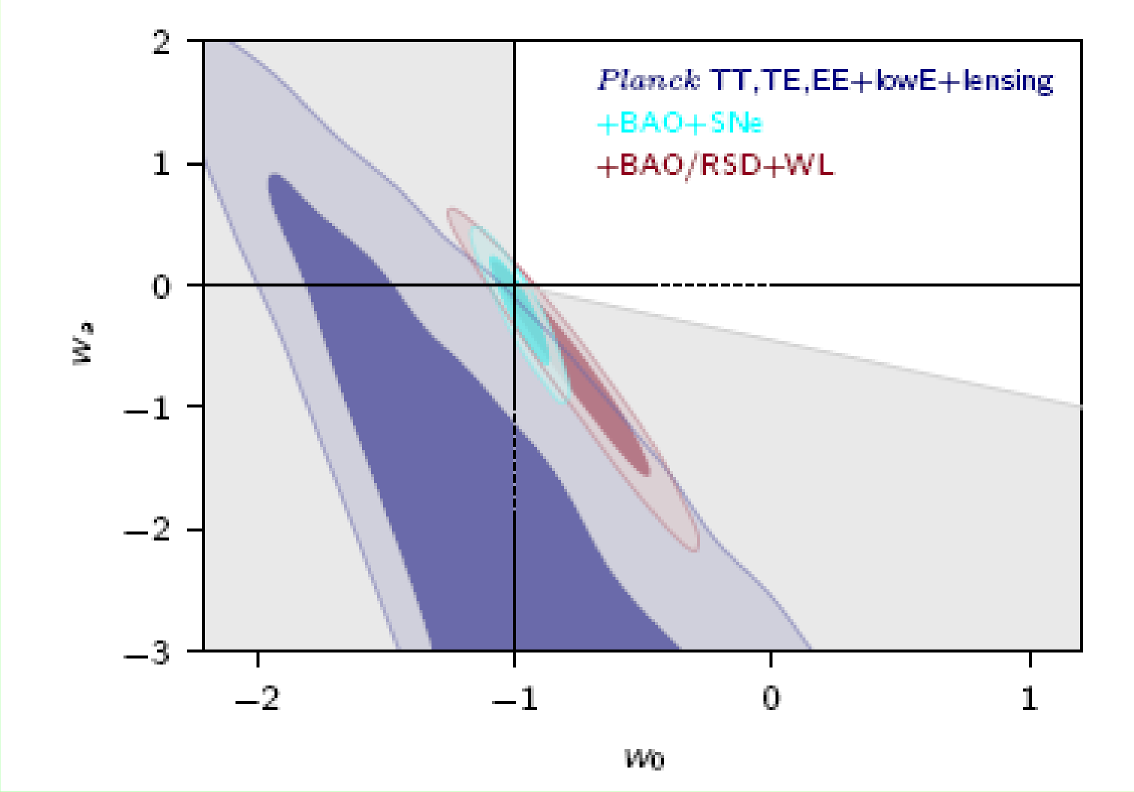


arXiv:1807.06209

Dark Energy: Planck 2018

- **Parametrization of w = p/ρ :** $w(a) = w_0 + (1 - a)w_a$

Parameter	*Planck*+SNe+BAO	*Planck*+BAO/RSD+WL
w_0	-0.961 ± 0.077	-0.76 ± 0.20
w_a	$-0.28^{+0.31}_{-0.27}$	$-0.72^{+0.62}_{-0.54}$



arXiv:1807.06209

Dark Energy: 2 Big Challenges

- **Does Dark Energy Really exist ?**
  --- How to find Direct Evidence of DE ?

- **What is the Identity of Dark Energy ??**
  --- Cosmological Constant?
  --- Many other models:
  Phamtom, Quintessence, Quintom……?

- **We study Model-Independent, Direct Probe.**

Direct Probe of Dark Energy

- How to find **Direct Evidence** of DE ?

- We propose to **Directly Probe** DE at **Astrophysical Scales**, such as galaxies / galaxy clusters, around 1-20Mpc scales.

- Probe DE by **Gravitational Lensing**:
  This is **Model-Independent Probe** of DE.

Many other Specific Models of DE

- **Quintessence: - 1/3 > w > -1** (Wetterich 1988)

$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-V(\varphi)\right]$$

$$w=\frac{\frac{1}{2}\dot{\varphi}^2-V(\varphi)}{\frac{1}{2}\dot{\varphi}^2+V(\varphi)}$$

- **Phantom: w < -1** (Caldwell 2002)

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-V(\varphi)\right]$$

$$w=\frac{\frac{1}{2}\dot{\varphi}^2+V(\varphi)}{\frac{1}{2}\dot{\varphi}^2-V(\varphi)}$$

- **Quintom (Quintessence+Phantom): w cross -1** (Feng,Wang,Zhang 2005)

$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi+\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma-V(\varphi,\sigma)\right]$$

$$w=\frac{\frac{1}{2}\dot{\varphi}^2-\frac{1}{2}\dot{\sigma}^2-V}{\frac{1}{2}\dot{\varphi}^2-\frac{1}{2}\dot{\sigma}^2+V}$$

- **and many other models …….**

Taking Phantom as Example

- **Energy Density and Pressure:**

$$\rho_p = -\frac{\dot{\phi}^2}{2} + V(\phi)$$
$$p_p = -\frac{\dot{\phi}^2}{2} - V(\phi)$$

Obey: $w_p = \frac{p_p}{\rho_p} < -1$

- **Energy-Momentum Tensor:**

$$T_t^{\ t} = \rho(r)\,,$$
$$T_i^{\ j} = 3\,w\,\rho(r)\left[-(1+3\,\beta)\frac{r_i\,r^j}{r_n r^n} + \beta\,\delta_i^{\ j}\right]$$

- **Equation of State Parameter:**

$$\langle T_i^{\ j}\rangle = -\rho(r)\,w\,\delta_i^{\ j} = -p(r)\,\delta_i^{\ j}$$

Kiselev 2002

Astrophysical Scale: Galaxy & Galaxy Cluster

- **Galaxies & Galaxy Clusters (with mass $10^{12\text{-}16}\,M_{sun}$) exist at Scales ≤ O(1-20) Mpc << CMB Scale (above 100-200 Mpc).**

- **Astrophysical system is fairly Static**
  $|\Delta a(t)|/a \sim 10^{-3}$—$10^{-4}$

- **This corresponds to a Schwartzschild-de-Sitter Spacetime with generic state parameter w .**

Schwartzschild-de-Sitter-w metric (SdSw)

- **General Static & Spherically Symmetric Spacetime:**

$$\mathrm{d}S^2 = e^{\mathbb{A}}\mathrm{d}t^2 - e^{\mathbb{B}}\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$

- **Energy-Momentum Tensor:**

$$8\pi\, T_t{}^t = -e^{-\mathbb{B}}\left(\frac{1}{r^2} - \frac{\mathbb{B}'}{r}\right) + \frac{1}{r^2}\,,$$

$$8\pi\, T_r{}^r = -e^{-\mathbb{B}}\left(\frac{1}{r^2} + \frac{\mathbb{A}'}{r}\right) + \frac{1}{r^2}\,,$$

$$8\pi\, T_\theta{}^\theta = 8\pi\, T_\phi{}^\phi = -\frac{e^{-\mathbb{B}}}{2}\left(\mathbb{A}'' + \frac{\mathbb{A}'^2}{2} + \frac{\mathbb{A}' - \mathbb{B}'}{r} - \frac{\mathbb{A}'\mathbb{B}'}{2}\right)$$

where $\mathbb{A}' = \mathrm{d}\mathbb{A}(r)/\mathrm{d}r$ and $\mathbb{B}' = \mathrm{d}\mathbb{B}(r)/\mathrm{d}r$

Solving Einstein Equation

- **Linearity and Additivity:**

Gauge Fixing

$$T_t{}^t = T_r{}^r$$

$$\beta = -(3w+1)/(6w)$$

$$T_t{}^t = T_r{}^r = \rho$$

$$T_\theta{}^\theta = T_\phi{}^\phi = -\frac{\rho}{2}(3w+1)$$

$$\mathbb{A} + \mathbb{B} = 0$$

$$\mathbb{B} = -\ln(1+F)$$

$$4\pi T_t{}^t = 4\pi T_r{}^r = -\frac{1}{2r^2}(F + rF'),$$

$$4\pi T_\theta{}^\theta = 4\pi T_\phi{}^\phi = -\frac{1}{4r}(2F' + rF'')$$

$$F_{\rm N} = -2\frac{r_g}{r}$$

$$F_{\rm DE} = -2\left(\frac{r_o}{r}\right)^{3w+1}$$

Potential

$$\Phi_{\rm N} = -\frac{M}{r}$$

$$\Phi_{\rm DE} = -\left(\frac{r_o}{r}\right)^{3w+1}$$

(similar to Kiselev 2002)

Schwartzschild-de-Sitter-w metric (SdSw)

$$dS^2 = \left[1 - 2\frac{M}{r} - 2\left(\frac{r_o}{r}\right)^{3w+1}\right] dt^2 - \frac{dr^2}{\left[1 - 2\frac{M}{r} - 2\left(\frac{r_o}{r}\right)^{3w+1}\right]} - r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

Matter Potential

DE Potential

★ $$\Phi = -\frac{M}{r} - \left(\frac{r_o}{r}\right)^{3w+1}$$

$$\Phi_\Lambda = -\left(\frac{r}{r_o}\right)^2 = -\frac{\Lambda}{6} r^2$$

For some specific models:

$$r_o = \sqrt{6/\Lambda}$$

$$\simeq 0.72 \times 10^4 \, \mathrm{Mpc}$$

- **Cosmological Constant， $w = -1$;**
- **Quntenssence， $-1 < w < -1/3$;**
- **Phantom， $w < -1$.**

Light Orbital Equation under SdSw

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\phi^2} = -u + 3Mu^2 + 3(w+1)\, r_o^{3w+1} u^{3w+2}$$

- **DE Correction term: -- it is w and r dependent !**

$$\left(\frac{\mathrm{d}u}{\mathrm{d}\phi}\right)^2 = \frac{1}{b^2} + \frac{2}{r_o^2} - u^2 + 2Mu^3 + \frac{2}{r_o^2}\left[(r_o u)^{3(w+1)} - 1\right]$$

DE Correction

(u=1/r)

He & Zhang, arXiv:1701.03418

Critical Radius

$$g_{00} = 1 + 2\Phi$$

- **Gravity Potential:** $$\Phi = -\frac{M}{r} - \left(\frac{r_o}{r}\right)^{3w+1}$$
- **Dark Force:** $$\vec{F} = -\nabla\Phi = \left(-\frac{M}{r^2} + \frac{|3w+1|}{r_o}\left(\frac{r}{r_o}\right)^{|3w+2|}\right)\hat{\boldsymbol{r}}$$
- **Critical Radius:**

F=0

$$r_{\rm cri} = r_o\left(\frac{M}{|3w+1|\,r_o}\right)^{-\frac{1}{3w}}$$

- **Important:**

r_{cri} **defines Lensing System as an Isolated System !**

He & Zhang, arXiv:1701.03418

Critical Radius (w = −1)

For cosmological constant (w = −1):

- **Dark force:**

$$\vec{g} = -\vec{\nabla}\Phi = \left(-\frac{GM}{r^2} + \frac{\Lambda}{3} r \right) \hat{r}$$

- **Critical Radius:**

$$r_c = \left(\frac{3GM}{\Lambda} \right)^{1/3}$$

- **For $M = 10^{12\text{-}16} M_{sun}$ → $r_c = (1.1—23)\text{Mpc}$.**

Critical Radius: Matching Condition

Matching at boundary r_{eff} of the Lensing System:

$$\frac{M}{\frac{4\pi}{3} r_{\text{eff}}^3} = \rho_M \,,$$

$$r_{\text{eff}} = \left(\frac{3M}{4\pi\rho_M}\right)^{\frac{1}{3}} = r_{\text{cri}} \left(\frac{3|3w+1|}{4\pi r_o^2 \, \rho_M}\right)^{\frac{1}{3}} \left(\frac{r_{\text{cri}}}{r_o}\right)^{|w|-1} .$$

$$r_{\text{eff}} = r_{\text{cri}} \left(\frac{2}{\Omega_{\text{M}}/\Omega_{\Lambda}}\right)^{\frac{1}{3}} \simeq 1.7\, r_{\text{cri}}$$

(w = -1)

Post Newtonian Approximation

Under Newtonian gauge:

(1) Weak field approx： $\frac{M}{r} \ll 1$ $\left(\frac{r_0}{r}\right)^{3w+1} \ll 1$

(2) Poisson equation： $-\nabla^2\Phi = 4\pi(\rho_{\mathrm{M}} - \rho_w)$

(3) Self-Bounded System： $r \leq r_{cri}$

$\rightarrow$ $$\mathrm{d}S^2 = (1+2\Phi)\mathrm{d}t^2 - (1-2\Phi)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$$

Dark Energy Lensing

- **For isolated astrophysical system in post-Newtonian approximation:**

$$\mathrm{d}S^2 = (1+2\Phi)\mathrm{d}t^2 - (1-2\Phi)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$$

- **Weak field condition:**

$$\Phi = \Phi_{\mathrm{N}} + \Phi_{\mathrm{DE}} \quad \begin{cases} \Phi_{\mathrm{N}} \ll 1 \\ \Phi_{\mathrm{DE}} \ll 1 \end{cases}$$

- **Refractive Index:**

$$n = 1 - 2\Phi_{\mathrm{N}} - 2\Phi_{\mathrm{DE}}$$

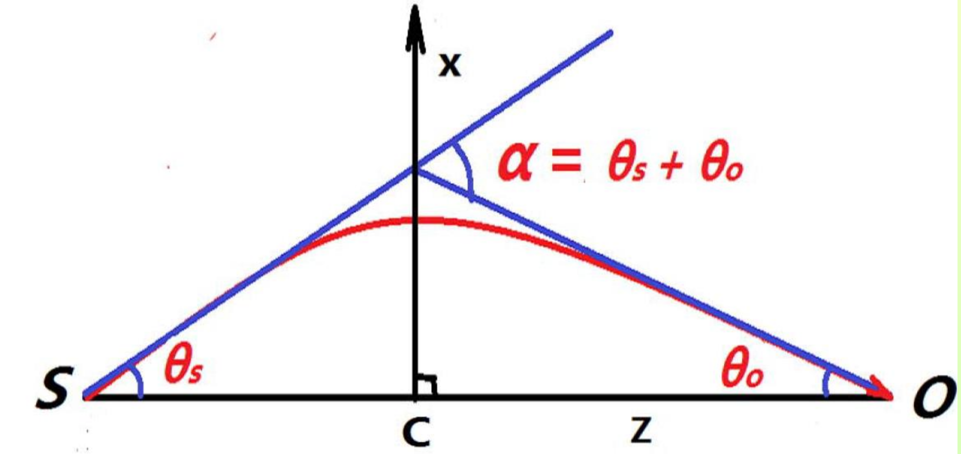
Newtonian term: n_{M}

DE term: Δn_{DE}

Light Deflection

- **Fermat Principle:** $$\left(n\frac{\mathrm{d}x}{\mathrm{d}\ell}\right)_{\mathcal{O}} - \left(n\frac{\mathrm{d}x}{\mathrm{d}\ell}\right)_{\mathcal{S}} = \int_{\mathcal{S}}^{\mathcal{O}} \mathrm{d}\ell\, \frac{\partial n}{\partial x}$$

- **Incident & Outgoing angles:**



$$n_{\mathcal{S}} \simeq 1 \text{ and } n_{\mathcal{O}} \simeq 1.$$

$$\theta_{\mathcal{S}} \simeq n_{\mathcal{S}} \left(\frac{\mathrm{d}x}{\mathrm{d}\ell}\right)_{\mathcal{S}}, \qquad \theta_{\mathcal{O}} \simeq -n_{\mathcal{O}} \left(\frac{\mathrm{d}x}{\mathrm{d}\ell}\right)_{\mathcal{O}}$$

- **Deflection angle:**

$$\vec{\alpha} = \vec{\theta}_{\mathcal{O}} + \vec{\theta}_{\mathcal{S}} = -2\int_{\mathcal{S}}^{\mathcal{O}} \mathrm{d}\ell\, \boldsymbol{\nabla}_{\perp}\Phi_{\mathrm{N}} - 2\int_{\mathcal{S}}^{\mathcal{O}} \mathrm{d}\ell\, \boldsymbol{\nabla}_{\perp}\Phi_{\mathrm{DE}}$$

Dark Energy: Concave Lensing

- **Convex Lens（凸透镜）**

$$\vec{\alpha}_{\rm M} = -2\int_{\mathcal{S}}^{\mathcal{O}} {\rm d}\ell\, \boldsymbol{\nabla}_{\perp}\Phi_{\rm N}$$

$$\simeq \boxed{-2M}\int_{\mathcal{S}}^{\mathcal{O}} {\rm d}\ell\, \frac{(\mathbf{x}-\mathbf{x}_c)_{\perp}}{|\mathbf{x}-\mathbf{x}_c|^3}$$

- **Concave Lens（凹透镜）**

$$\Delta\vec{\alpha}_{\rm DE} = -2\int_{\mathcal{S}}^{\mathcal{O}} {\rm d}\ell\, \boldsymbol{\nabla}_{\perp}\Phi_{\rm DE}$$

$$= \boxed{2(-3w-1)}\, r_o^{3w+1}\int_{\mathcal{S}}^{\mathcal{O}} {\rm d}\ell\, \frac{(\mathbf{x}-\mathbf{x}_c)_{\perp}}{|\mathbf{x}-\mathbf{x}_c|^{3w+3}}$$

Gravitational Lens = Convex + Concave Lens

$$\vec{\alpha} = \vec{\alpha}_{\rm M} + \Delta\vec{\alpha}_{\rm DE}$$

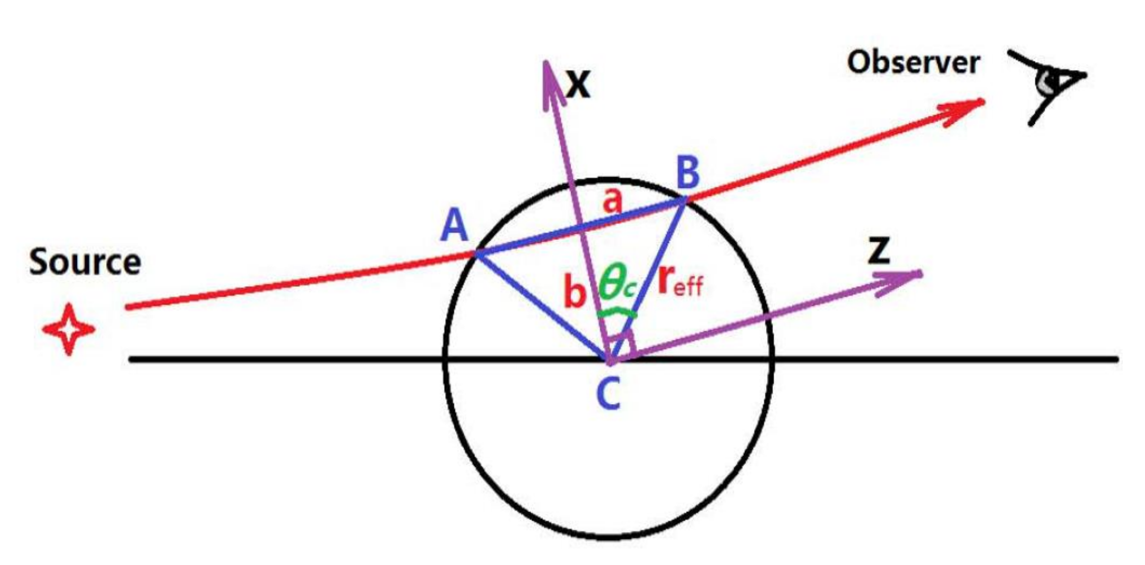
$$\mathrm{sign}(\Delta\alpha^x_{\rm DE}) = -\mathrm{sign}(\alpha^x_{\rm M})$$

He & Zhang, arXiv:1701.03418

Dark Energy Induced Deflection

- **DE Deflection:**

$$\Delta\vec{\alpha}_{\rm DE} = -2\int_{\mathcal{S}}^{\mathcal{O}} {\rm d}\ell\, \nabla_{\perp}\Phi_{\rm DE}$$

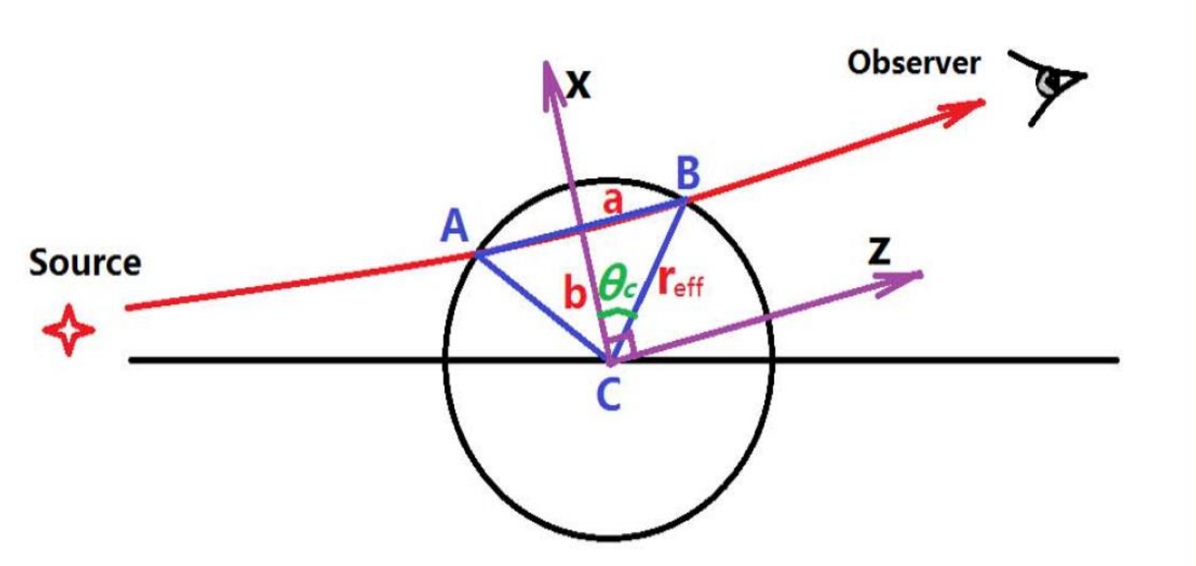


- **For Cosmological Constant (w = -1):**

$$\Delta\alpha_{\rm DE} = 4\left(\frac{r_{\rm eff}}{r_o}\right)^2 \sin 2\theta_c = 2n_{\rm eff}^2 \sin 2\theta_c \left(\frac{\Lambda M^2}{3}\right)^{\frac{1}{3}},$$

- **DE Lensing Effect: For Galalaxy & Galaxy Cluster, Deflection Angle can reach:** $\Delta\alpha_{\rm DE}^{\rm max} \simeq n_{\rm eff}^2 \times (0.018'' - 8.5'')$

Dark Energy Induced Deflection



- **For Cosmological Constant (w = -1):**

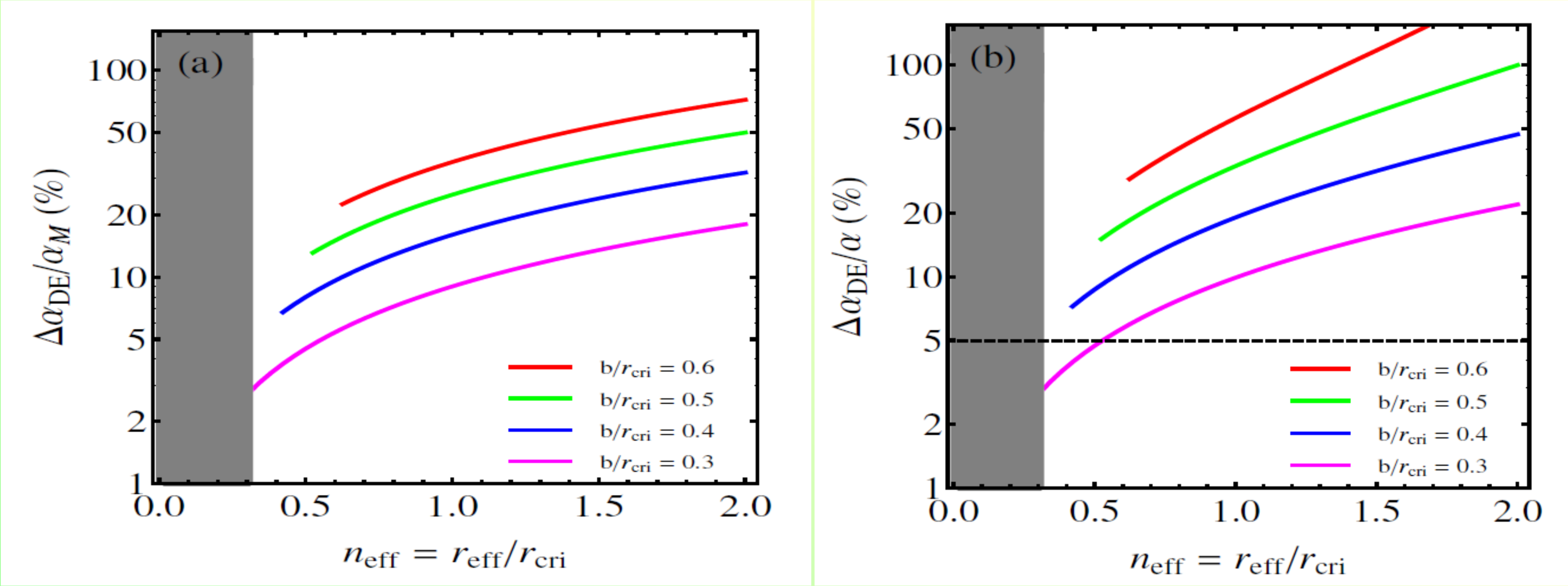
$$\alpha_M = \frac{4M}{r_{\rm eff}} \tan\theta_c \qquad \frac{\Delta\alpha_{\rm DE}}{\alpha_M} = n_{\rm eff}^3 \cos^2\theta_c = n_{\rm eff}\frac{b^2}{r_{\rm cri}^2}$$

- **For visible DE effect, we need:**

$b = O(0.3—1)\ r_{cri}$

Dark Energy Induced Deflection

- **At Effective Radius $r_{eff} = n_{eff}\, r_{cri}$,** $$\frac{\Delta\alpha_{DE}}{\alpha_M} = n_{eff}^3 \cos^2\theta_c = n_{eff}\frac{b^2}{r_{cri}^2}$$



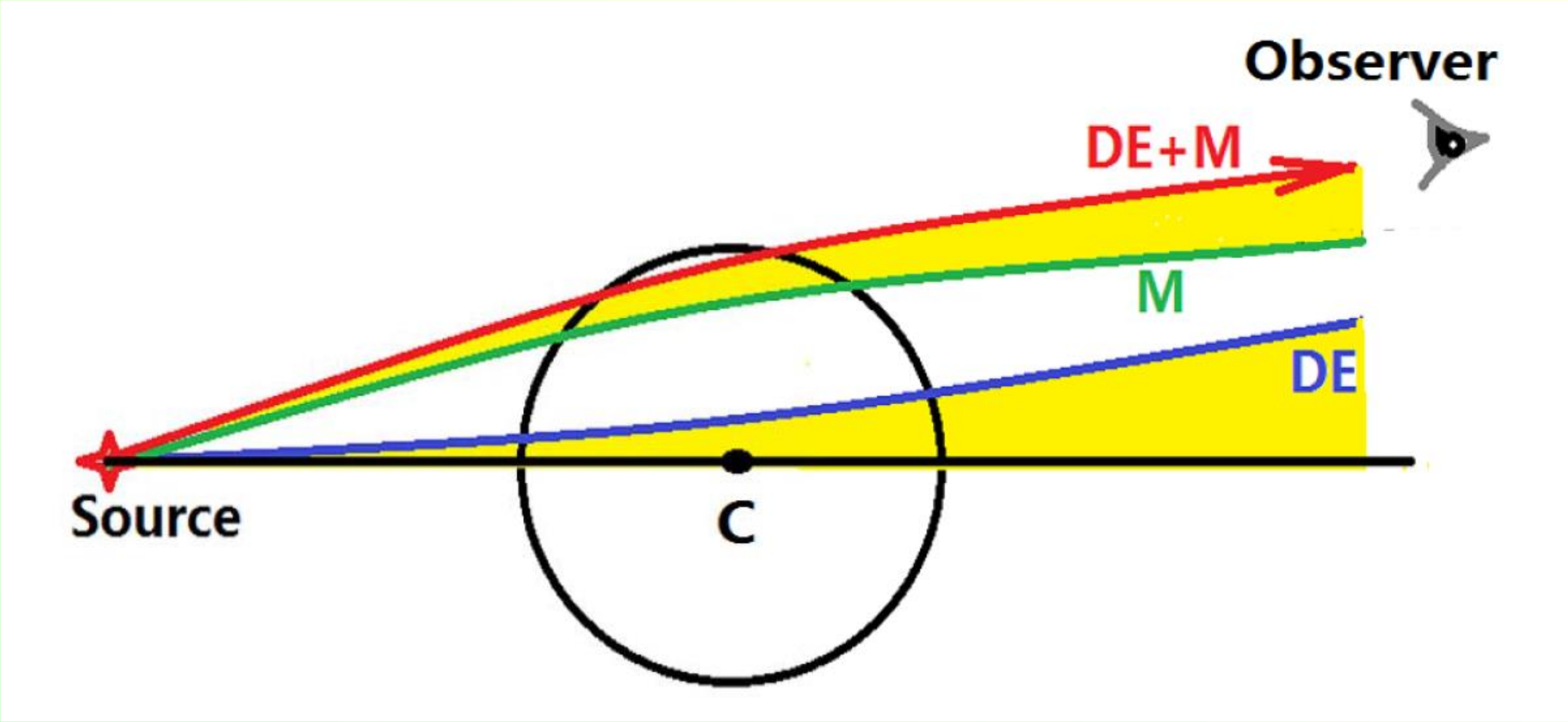
He & Zhang, arXiv:1701.03418

Gravitational Lensing

Gravitational Lens = Convex + Concave Lens

Lensing Equation:

$$\vec{\theta} = \vec{\beta} + (\vec{\alpha}_{\mathrm{M}} + \Delta\vec{\alpha}_{\mathrm{DE}})\frac{D_{\mathcal{SC}}}{D_{\mathcal{SO}}}$$



Outside Lensing System

- **For** $r_{\rm cri}^{3|w|}/r^{3|w|} \ll 1$, **SdSw → dSw.**

  we have Conformally flat metric: de Sitter-w (dSw):

$$\mathrm{d}S^2 \simeq \mathrm{d}S^2_{\rm dSw} = \left[1 - 2\left(\frac{r_o}{r}\right)^{3w+1}\right]\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{\left[1 - 2\left(\frac{r_o}{r}\right)^{3w+1}\right]} - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$

- **Under conformally flat dSw spacetime, the light orbit is not affected, so the Deflection Angle is unchanged.**

Prediction of New Phenomenon

- **For galaxy (cluster), Dark Energy contributes a Replusive Power-Law term to Gravitational Potential, taking a Model-Independent form.**

- **Dark Energy Lensing: Concave Lensing Effect**
  Matter: Convex lensing

- **Distinguish DE Models: $w \neq -1$ vs $w = -1$**
  $w \neq -1$ causes deformation of shear image

- **Much more can be pursued along this direction…**


Thank You !

Backup Slides

Dark Energy (DE)

- **Einstein Equation: CC**

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- **Indirect Probe from CMB Measurements on scale factor a(t) at Cosmological Scales > 100-200Mpc , in FLRW metric.**