HOW TO BREAK CP SYMMETRY IN A 2HDM

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Prediction on CP phases in Standard Model

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In standard model:

$$-\mathcal{L}_Y = Y^{(d)}\bar{Q}_L\phi d_R + Y^{(u)}\bar{Q}_L\epsilon\phi^* u_R + h.c., \qquad (1)$$

After Spontaneous symmetry Breaking

$$\langle\phi^0\rangle = v/\sqrt{2}$$

$$-\mathcal{L}_{mass} = \bar{u}_L M^{(u)} u_R + \bar{d}_L M^{(d)} d_R + h.c., \qquad (2)$$

where $M^{(f)} = Y^{(f)}\, v/\sqrt{2}$

There must be a unitary transformation *U* the charged-current $W^{\pm}$ interactions

$$\frac{-g}{\sqrt{2}}(\bar{u}_L,\ \bar{c}_L,\ \bar{t}_L)\gamma^{\mu}W_{\mu}^{+}V_{CKM}\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c., \qquad (3)$$

$$V_{CKM} = U_L^{(u)}U_L^{(d)\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

In the most general case, a 3×3 mass matrix can be given

$$M = \begin{pmatrix} A_1 + iD_1 & B_1 + iC_1 & B_2 + iC_2 \\ B_4 + iC_4 & A_2 + iD_2 & B_3 + iC_3 \\ B_5 + iC_5 & B_6 + iC_6 & A_3 + iD_3 \end{pmatrix}, \qquad (4)$$

If the Lagrangian were Hermitian:

$$M = \begin{pmatrix} A_1 & B_1 + iC_1 & B_2 + iC_2 \\ B_1 - iC_1 & A_2 & B_3 + iC_3 \\ B_2 - iC_2 & B_3 - iC_3 & A_3 \end{pmatrix}, \qquad (5)$$

with $B_4 = B_1$, $B_5 = B_2$, $B_6 = B_3$, $C_4 = -C_1$, $C_5 = -C_2$ and $C_6 = -C_3$.

$$M_R = \begin{pmatrix} A_1 & B_1 & B_2 \\ B_1 & A_2 & B_3 \\ B_2 & B_3 & A_3 \end{pmatrix}, \qquad M_I = i \begin{pmatrix} 0 & C_1 & C_2 \\ -C_1 & 0 & C_3 \\ -C_2 & -C_3 & 0 \end{pmatrix}, \quad (6)$$

since $U M_{(R,I)} U^\dagger = U M^\dagger_{(R,I)} U^\dagger = M^{diagonal}_{(R,I)}$.

$$U\,(\,M_R M_I^\dagger - M_I M_R^\dagger = 0\,)\,U^+ \qquad (7)$$

$$(U\,M_R\,U^+)(U\,M_I^{+}\,U^+)\text{-}(U\,M_I\,U^+)(U\,M_R^{+}\,U^+)=0$$

$$M_I M_R^\dagger = i\begin{pmatrix} B_1C_1 + B_2C_2 & A_2C_1 + B_3C_2 & B_3C_1 + A_3C_2 \\ B_2C_3 - A_1C_1 & B_3C_3 - B_1C_1 & A_3C_3 - B_2C_1 \\ -A_1C_2 - B_1C_3 & -B_1C_2 - A_2C_3 & -B_2C_2 - B_3C_3 \end{pmatrix},$$

$$M_R M_I^\dagger = i\begin{pmatrix} -B_1C_1 - B_2C_2 & A_1C_1 - B_2C_3 & A_1C_2 + B_1C_3 \\ -A_2C_1 - B_3C_2 & B_1C_1 - B_3C_3 & B_1C_2 + A_2C_3 \\ -B_3C_1 - A_3C_2 & B_2C_1 - A_3C_3 & B_2C_2 + B_3C_3 \end{pmatrix}. \quad (8)$$

The diagonal elements give us following conditions:

$$B_1C_1 = -B_2C_2 = B_3C_3 \quad (9)$$

and the off-diagonal ones give us three others:

$$(A_1 - A_2) = \quad (B_3C_2 + B_2C_3)/C_1, \quad (10)$$
$$(A_3 - A_1) = \quad (B_1C_3 - B_3C_1)/C_2, \quad (11)$$
$$(A_2 - A_3) = -(B_2C_1 + B_1C_2)/C_3. \quad (12)$$

$$A_1 = A_3 + B_2(B_1^2 - B_3^2)/B_1 B_3,$$
$$A_2 = A_3 + B_3(B_1^2 - B_2^2)/B_1 B_2,$$
$$C_1 = B_3 C_3/B_1, \qquad C_2 = -B_3 C_3/B_2. \tag{13}$$

⇓

$$M = \begin{pmatrix} A + xB(y - \frac{1}{y}) & yB + \frac{iC}{y} & xB - \frac{iC}{x} \\ yB - \frac{iC}{y} & A + B(\frac{y}{x} - \frac{x}{y}) & B + iC \\ xB + \frac{iC}{x} & B - iC & A \end{pmatrix}, \tag{14}$$

if one lets $A \equiv A_3$, $B \equiv B_3$, $C \equiv C_3$ and $x \equiv B_2/B_3$, $y \equiv B_1/B_3$.

Mass eigenvalues

$$M^{\text{diag.}} = \begin{pmatrix} A - \frac{x}{y}B - \frac{\sqrt{x^2+y^2+x^2y^2}}{xy}C & 0 & 0 \\ 0 & A - \frac{x}{y}B + \frac{\sqrt{x^2+y^2+x^2y^2}}{xy}C & 0 \\ 0 & 0 & A + \frac{(x^2+1)y}{x}B \end{pmatrix}.$$

Unitary transformation matrix:

$$U^{(u)} = \begin{pmatrix} \frac{-\sqrt{x^2+y^2}}{\sqrt{2(x^2+y^2+x^2y^2)}} & \frac{x(y^2-i\sqrt{x^2+y^2+x^2y^2})}{\sqrt{2}\sqrt{x^2+y^2}\sqrt{x^2+y^2+x^2y^2}} & \frac{y(x^2+i\sqrt{x^2+y^2+x^2y^2})}{\sqrt{2}\sqrt{x^2+y^2}\sqrt{x^2+y^2+x^2y^2}} \\ \frac{-\sqrt{x^2+y^2}}{\sqrt{2(x^2+y^2+x^2y^2)}} & \frac{x(y^2+i\sqrt{x^2+y^2+x^2y^2})}{\sqrt{2}\sqrt{x^2+y^2}\sqrt{x^2+y^2+x^2y^2}} & \frac{y(x^2-i\sqrt{x^2+y^2+x^2y^2})}{\sqrt{2}\sqrt{x^2+y^2}\sqrt{x^2+y^2+x^2y^2}} \\ \frac{xy}{\sqrt{x^2+y^2+x^2y^2}} & \frac{y}{\sqrt{x^2+y^2+x^2y^2}} & \frac{x}{\sqrt{x^2+y^2+x^2y^2}} \end{pmatrix} \cdot (16)$$

A, B and C independent !

Thus,

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} ce^{-i\delta_1} & de^{i\delta_2} & ae^{-i\delta_3} \\ de^{-i\delta_2} & ce^{i\delta_1} & ae^{i\delta_3} \\ ae^{i\delta_4} & ae^{-i\delta_4} & b \end{pmatrix}, \quad (17)$$

The magnitudes of V_{td}, V_{ts}, V_{cb} and V_{ub} are the same "a"

$$V_{ud} = V_{cs}^{*} = c\, e^{-i\delta_1} = \frac{(x^2+y^2)(x'^2+y'^2) + (xx'+yy')(xyx'y' + \sqrt{x^2+y^2+x^2y^2}\,\sqrt{x'^2+y'^2+x'^2y'^2})}{2\sqrt{x^2+y^2}\,\sqrt{x'^2+y'^2}\,\sqrt{x^2+y^2+x^2y^2}\,\sqrt{x'^2+y'^2+x'^2y'^2}}$$

$$+\, i\, \frac{(xy'-x'y)(x'y'\sqrt{x^2+y^2+x^2y^2} + xy\sqrt{x'^2+y'^2+x'^2y'^2})}{2\sqrt{x^2+y^2}\,\sqrt{x'^2+y'^2}\,\sqrt{x^2+y^2+x^2y^2}\,\sqrt{x'^2+y'^2+x'^2y'^2}},$$

$$V_{us} = V_{cd}^{*} = d\, e^{i\delta_2} = \frac{(x^2+y^2)(x'^2+y'^2) + (xx'+yy')(xyx'y' - \sqrt{x^2+y^2+x^2y^2}\,\sqrt{x'^2+y'^2+x'^2y'^2})}{2\sqrt{x^2+y^2}\,\sqrt{x'^2+y'^2}\,\sqrt{x^2+y^2+x^2y^2}\,\sqrt{x'^2+y'^2+x'^2y'^2}}$$

$$+\, i\, \frac{(xy'-x'y)(x'y'\sqrt{x^2+y^2+x^2y^2} - xy\sqrt{x'^2+y'^2+x'^2y'^2})}{2\sqrt{x^2+y^2}\,\sqrt{x'^2+y'^2}\,\sqrt{x^2+y^2+x^2y^2}\,\sqrt{x'^2+y'^2+x'^2y'^2}},$$

$$V_{ub} = V_{cb}^{*} = a\, e^{-i\delta_3} = \frac{[y'y^2(x-x') + x'x^2(y-y')] + i(xy'-x'y)\sqrt{x^2+y^2+x^2y^2}}{2\sqrt{x^2+y^2}\,\sqrt{x^2+y^2+x^2y^2}\,\sqrt{x'^2+y'^2+x'^2y'^2}},$$

$$V_{td} = V_{ts}^{*} = a\, e^{i\delta_4} = \frac{[yy'^2(x'-x) + xx'^2(y'-y)] + i(xy'-x'y)\sqrt{x'^2+y'^2+x'^2y'^2}}{2\sqrt{x^2+y^2+x^2y^2}\,\sqrt{x'^2+y'^2}\,\sqrt{x'^2+y'^2+x'^2y'^2}},$$

$$V_{tb} = b = \frac{xx'+yy'+xyx'y'}{\sqrt{x^2+y^2+x^2y^2}\,\sqrt{x'^2+y'^2+x'^2y'^2}}.$$

From PDG (2016)

$$V_{CKM}^{empirical} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97434^{+0.00011}_{-0.00012} & 0.22506\pm0.00050 & 0.00357\pm0.00015 \\ 0.22492\pm0.00050 & 0.97351\pm0.00013 & 0.0411\pm0.0013 \\ 0.00875^{+0.00032}_{-0.00033} & 0.0403\pm0.0013 & 0.99915\pm0.00005 \end{pmatrix}, \quad (23)$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} ce^{-i\delta_1} & de^{i\delta_2} & ae^{-i\delta_3} \\ de^{-i\delta_2} & ce^{i\delta_1} & ae^{i\delta_3} \\ ae^{i\delta_4} & ae^{-i\delta_4} & b \end{pmatrix}, (17)$$

The magnitudes of V_{td}, V_{ts}, V_{cb} and V_{ub} are the same "a"

Employing Fine-tuning parameters

$$|V_{CKM}| = \begin{pmatrix} c\sqrt{1+\gamma} & d\sqrt{1+\beta} & a\sqrt{1-\alpha} \\ d\sqrt{1-\beta} & c\sqrt{1-\gamma} & a\sqrt{1+\alpha} \\ a\sqrt{1-\alpha'} & a\sqrt{1+\alpha'} & b \end{pmatrix}. \qquad (24)$$

b=0.99915

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = a^2(1-\alpha) + a^2(1+\alpha) + b^2 = 2a^2 + b^2 = 1$$

$a = 0.291486$

Consequently, $\alpha = 0.986573$ ………

$b = 0.99915$

$a = 0.291486$ with the unitarity condition

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = a^2(1-\alpha) + a^2(1+\alpha\ ) + b^2 = 2a^2 + b^2 = 1$$

$\alpha = 0.986573$ is derived from the relation $|V_{cb}|^2 - |V_{ub}|^2 = 2a^2\alpha$

$\alpha' = 0.910695$ from $|V_{ts}|^2 - |V_{td}|^2 = 2a^2\alpha'$

$|V_{ud}|^2 + |V_{cs}|^2 = 2c^2$ gives $c = 0.973925$

$|V_{us}|^2 + |V_{cd}|^2 = 2d^2$ gives $d = 0.22499$

and $\gamma = 0.000852$

$\beta = 0.000622$

Substituting these parameters into Eq.(24), we will receive

$$|V_{CKM}| = \begin{pmatrix} 0.97434 & 0.22506 & 0.003378 \\ 0.22492 & 0.97351 & 0.041084 \\ 0.008711 & 0.040291 & 0.99915 \end{pmatrix}, \qquad (25)$$

which coincide the empirival values to $\mathbf{O}(10^{-4})$.

PDG (2016)

$$V_{CKM}^{empirical} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97434^{+0.00011}_{-0.00012} & 0.22506\pm0.00050 & 0.00357\pm0.00015 \\ 0.22492\pm0.00050 & 0.97351\pm0.00013 & 0.0411\pm0.0013 \\ 0.00875^{+0.00032}_{-0.00033} & 0.0403\pm0.0013 & 0.99915\pm0.00005 \end{pmatrix}, \quad (23)$$

Numerically,

$$x \sim 0.437, \quad y \sim 2.994, \quad x' \sim 0.396 \quad \text{and} \quad y' \sim 0.545.$$

$$|V_{tb}|^2 = \frac{(xx'+yy'+xyx'y')^2}{(x^2+y^2+x^2y^2)(x'^2+y'^2+x'^2y'^2)} = 0.99915^2$$

Accordingly, the phases can be predicted as:

$$\delta_1 \sim 9.91553^o,$$

$$\delta_2 \sim 49.9335^o,$$

$$\delta_3 \sim 82.9455^o$$

and $\delta_4 \sim 73.03^o$.