On the Vector-Scalar-Scalar Interaction in the Simplest Little Higgs (SLH) Model

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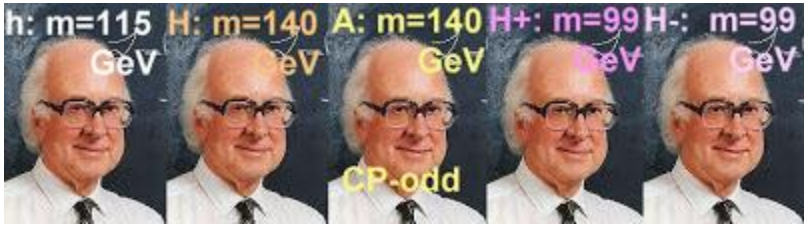
Outline

- Motivation
- General consideration
- Application to the SLH
- Discussion and conclusion

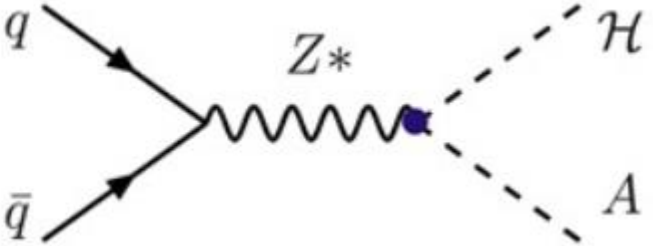
Motivation

- Higgs, as probe of new physics

- Almost all BSMs entail an enlarged Higgs sector.



- We would anticipate new kind of interactions such as Vector-Scalar-Scalar (VSS) interactions which are absent in the SM.



Motivation

- How to derive the VSS vertices in a specific BSM model?
  - Start with the gauge covariant kinetic terms of the scalar fields
  - Expand the fields into vevs and components without vev
  - Extract the 3-point VSS vertex
  - Submit the results to PRD/JHEP/PLB/NPB/EPJC… & wait for the editorial decision

Motivation

- Sometimes the story is not so simple, e.g. for a nonlinearly-realized scalar sector.
- Taking the SLH as an example (Here we focus on the neutral sector), there exists 'unexpected' vector-scalar two-point transitions from a naïve expansion of the gauge kinetic terms. Also, the scalar kinetic terms are not canonically normalized.
- A procedure for the diagonalization of a general vector-scalar system in gauge field theories is needed.

General consideration:

Formulation of the problem

- Consider a gauge field theory with

n_S real scalar fields $G_i, i = 1, 2, ..., n_S$

G_i's need neither be in mass eigenstate nor canonically normalized.

n_M real massive gauge boson fields $Z_p^\mu, p = 1, 2, ..., n_M$.

- Suppose its classical Lagrangian contains

$$\mathcal{L}_{quad} \supset \frac{1}{2} V_{ij} (\partial_\mu G_i)(\partial^\mu G_j) + F_{pi} Z_p^\mu (\partial_\mu G_i) - \frac{1}{2} (\mathbb{M}_G^2)_{ij} G_i G_j + \frac{1}{2} (\mathbb{M}_V^2)_{pq} Z_{p\mu} Z_q^\mu$$

- We also define $\tilde{G}_p = F_{pi} G_i, p = 1, 2, ..., n_M$

- Question: How to diagonalize this system and derive the Vector-Scalar-Scalar vertices?

General consideration:

Gauge-fixing options

- It is natural to consider the tentative gauge-fixing terms

$$\mathcal{L}_{gf} = -\sum_{p=1}^{n_M} \frac{1}{2\xi^p} (\partial_\mu Z_p^\mu - \xi^p \tilde{G}_p)^2$$

- Then the quadratic parts are free of vector-scalar transition

$$\mathcal{L}_{quad} + \mathcal{L}_{gf} \supset$$
$$\frac{1}{2} V_{ij} (\partial_\mu G_i)(\partial^\mu G_j) - \frac{1}{2} \xi^p \tilde{G}_p^2 - \frac{1}{2} (\mathbb{M}_G^2)_{ij} G_i G_j - \frac{1}{2\xi^p} (\partial_\mu Z_p^\mu)^2 + \frac{1}{2} (\mathbb{M}_V^2)_{pq} Z_{p\mu} Z_q^\mu$$

- There is freedom in the gauge-fixing and we will show below there is a theoretically well-motivated choice.

General consideration:

Treatment of scalar kinetic terms

- Suppose we apply a transformation which renders the scalar kinetic terms canonically normalized

$$S_i = U_{ij} G_j \qquad\qquad V = U^T U$$

$$\frac{1}{2} V_{ij} (\partial_\mu G_i)(\partial^\mu G_j) = \frac{1}{2} (\partial_\mu S_i)(\partial^\mu S_i)$$

- The quadratic parts then become

$$\mathcal{L}_{quad} + \mathcal{L}_{gf} \supset \frac{1}{2} (\partial_\mu S_i)(\partial^\mu S_i) - \frac{1}{2} \xi^p \tilde{G}_p^2 - \frac{1}{2} ((U^{-1})^T \mathbb{M}_G^2 U^{-1})_{ij} S_i S_j$$
$$- \frac{1}{2\xi^p} (\partial_\mu Z_p^\mu)^2 + \frac{1}{2} (\mathbb{M}_V^2)_{pq} Z_{p\mu} Z_q^\mu$$

General consideration:

Diagonalization of the original scalar mass terms

- To diagonalize

$$-\tfrac{1}{2}((U^{-1})^T \mathbb{M}_G^2 U^{-1})_{ij} S_i S_j$$

Suppose we use an orthogonal transformation

$$\bar{S}_i = P_{ij} S_j$$

Then the quadratic parts become

$$\mathcal{L}_{quad} + \mathcal{L}_{gf} \supset \frac{1}{2}(\partial_\mu \bar{S}_i)(\partial^\mu \bar{S}_i) - \frac{1}{2}\xi^p \tilde{G}_p^2 - \frac{1}{2}\nu_r^2 \bar{S}_r^2 - \frac{1}{2\xi^p}(\partial_\mu Z_p^\mu)^2 + \frac{1}{2}(\mathbb{M}_V^2)_{pq} Z_{p\mu} Z_q^\mu$$

- Note: $r = n_M + 1, \ldots, n_S$

General consideration:

Highly suggestive scalar mass terms

- Let us take a closer look at the scalar mass terms

$$\mathcal{L}' = -\frac{1}{2}\xi^p \tilde{G}_p^2 - \frac{1}{2}\nu_r^2 \bar{S}_r^2$$

- We want to diagonalize it by an orthogonal transformation

$$\tilde{S}_i = K_{ij}\bar{S}_j \tag{2.12}$$

- Natural/educated guess

$$\tilde{S}_i = \alpha_i H_i, i = 1, 2, ..., n_S \text{ (no summation over i)} \tag{2.13}$$

$$H_i = \begin{cases} \tilde{G}_i, & i = 1, 2, ..., n_M, \\ \bar{S}_i, & i = n_M + 1, ..., n_S. \end{cases} \tag{2.14}$$

Question:

How to determine whether the corresponding K is orthogonal?

General consideration:

Introducing the inner product

- In the real vector space spanned by the G_i's, we introduce an **inner product**, defined through (recall that the S_i's have canonically-normalized kinetic terms)

$$< S_i|S_j >\equiv \delta_{ij}, i, j = 1, 2, ..., n_S$$

- The question of whether the matrix K is orthogonal reduces to determine whether the $\tilde{S}$'s form an orthonormal basis.

- We may always adjust the α_i's so that

$$< \tilde{S}_i|\tilde{S}_i >= 1, \forall i = 1, 2, ..., n_S$$

- Then the crucial question becomes **whether**

$$< \tilde{S}_i|\tilde{S}_j >= 0 \text{ holds when } i \neq j.$$

General consideration:

Physical scalars

- Mass diagonalization is supposed to have already been realized for n_S-n_M physical scalars.
- Eigenvectors which belong to different eigenvalues of a real symmetric matrix must be orthogonal to each other.
- Therefore

$$\tilde{S}_r, r = n_M + 1, ..., n_S$$

must be orthogonal to each other and to the $\tilde{G}_p$'s.

General consideration:

Inner products of Goldstone vectors

- Now the crucial question is whether the $\tilde{G}_p$'s are orthogonal to each other.
- When the scalar fields are canonically normalized in their kinetic part, the vector-scalar two-point transitions in a gauge theory has the form (see Weinberg)

$$i \sum_{nm\alpha} \partial_\mu \phi'_n t^\alpha_{nm} A^\mu_\alpha v_m \tag{2.17}$$

- On the other hand, the elements of the gauge boson mass matrix are

$$\mu^2_{\alpha\beta} = -\sum_{nml} t^\alpha_{nm} t^\beta_{nl} v_m v_l \tag{2.18}$$

- Comparing these two expressions we find

$$< \tilde{G}_p | \tilde{G}_q >= (\mathbb{M}^2_V)_{pq}, \forall p, q = 1, 2, ..., n_M \tag{2.19}$$

General consideration:

A convenient way to eat the Goldstones

$$< \tilde{G}_p|\tilde{G}_q >= (\mathbb{M}_V^2)_{pq}, \forall p, q = 1, 2, ..., n_M \tag{2.19}$$

- Eq. (2.19) suggests that if the gauge bosons are already in their mass eigenstates, then the related Goldstone boson vectors must be orthogonal to each other.
- Physically this implies that **massive gauge bosons eat their corresponding Goldstone bosons along the directions dictated by their mass eigenstates**.
- Therefore it would be desirable we rotate the gauge fields to their mass eigenstates before adding the gauge-fixing terms

$$\mathcal{L}_{gf} = -\sum_{p=1}^{n_M} \frac{1}{2\xi^p}(\partial_\mu Z_p^\mu - \xi^p \tilde{G}_p)^2$$

- The alternative option, although legitimate, could be very inconvenient.

General consideration:

General results of diagonalization

- Now suppose the gauge boson mass matrix can be diagonalized by an orthogonal matrix R as

$$R\mathbb{M}_V^2 R^{-1} = \mathbb{M}_{DV}^2 \equiv \mathrm{diag}\{\mu_1^2, \mu_2^2, ..., \mu_{n_M}^2\}$$

- Let us define

$$G_p^m \equiv \frac{R_{pq}}{\mu_p}\tilde{G}_q = \frac{(RF)_{pi}}{\mu_p}G_i, p = 1, 2, ..., n_M \text{ (no summation over } p)$$

- So that we may easily check

$$< G_p^m | G_q^m >= \frac{1}{\mu_p \mu_q}(R\mathbb{M}_V^2 R^T)_{pq} = \delta_{pq}, \forall p, q = 1, 2, ..., n_M$$

General consideration

General results of diagonalization

- Suppose

$$\bar{S}_r = T_{ri} G_i, r = n_M + 1, ..., n_S$$

- Then we have

$$G_i^m = Q_{ij} G_j, i = 1, 2, ..., n_S \tag{2.24}$$

where the $n_S \times n_S$ matrix Q is defined by (no summation over i)

$$Q_{ij} = \begin{cases} \frac{(RF)_{ij}}{\mu_i}, & i = 1, 2, ..., n_M, \\ T_{ij}, & i = n_M + 1, ..., n_S. \end{cases} \tag{2.25}$$

Application to the SLH:

Review of SLH

- Little Higgs mechanism: Higgs is realized as a PNGB of some global symmetry breaking. And the explicit breaking of the global symmetry is realized collectively. N. Arkani-Hamed et al., JHEP 07(2002)034
- Simplest Little Higgs (SLH): based on $SU(3)_L \times U(1)_X$ electroweak gauge group and the global symmetry breaking pattern

$$[SU(3)_1 \times U(1)_1] \times [SU(3)_2 \times U(1)_2] \to [SU(2)_1 \times U(1)_1] \times [SU(2)_2 \times U(1)_2]$$

realized through two scalar triplets. M. Schmaltz, JHEP 08(2004)056

Application to the SLH:

Parametrization of the scalars

- Double exponential parametrization of the scalar triplets

F. del Aguila et al., JHEP 03(2011)080

$$\Phi_1 = \exp\left(\frac{i\Theta'}{f}\right)\exp\left(\frac{it_\beta\Theta}{f}\right)\begin{pmatrix}0\\0\\fc_\beta\end{pmatrix} \qquad \Phi_2 = \exp\left(\frac{i\Theta'}{f}\right)\exp\left(-\frac{i\Theta}{ft_\beta}\right)\begin{pmatrix}0\\0\\fs_\beta\end{pmatrix}$$

$$\Theta = \frac{\eta}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times 2} & h\\ h^\dagger & 0\end{pmatrix}, \quad \Theta' = \frac{\zeta}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times 2} & k\\ k^\dagger & 0\end{pmatrix}$$

$$h = \begin{pmatrix}h^0\\h^-\end{pmatrix}, \quad h^0 = \frac{1}{\sqrt{2}}(v + H - i\chi)$$

$$k = \begin{pmatrix}k^0\\k^-\end{pmatrix}, \quad k^0 = \frac{1}{\sqrt{2}}(\sigma - i\omega)$$

$$s_\beta \equiv \sin\beta, c_\beta \equiv \cos\beta, t_\beta \equiv \tan\beta$$

Application to the SLH:

Parametrization of the gauge fields

- Gauge kinetic terms for the scalar triplets

$$\mathcal{L}_{gk} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)$$

$$D_\mu = \partial_\mu - igA^a_\mu T^a + ig_x Q_x B^x_\mu, \quad g_x = \frac{g t_W}{\sqrt{1 - t_W^2/3}} \quad Q_x = -\tfrac{1}{3}$$

$$A^a_\mu T^a = \frac{A^3_\mu}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{A^8_\mu}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & W^+_\mu & Y^0_\mu \\ W^-_\mu & 0 & X^-_\mu \\ Y^{0\dagger}_\mu & X^+_\mu & 0 \end{pmatrix}$$

$$Y^0_\mu \equiv \frac{1}{\sqrt{2}}(Y_{R\mu} + iY_{I\mu}), \quad Y^{0\dagger}_\mu \equiv \frac{1}{\sqrt{2}}(Y_{R\mu} - iY_{I\mu})$$

First order gauge boson mixing

$$\begin{pmatrix} A^3 \\ A^8 \\ B_x \end{pmatrix} = \begin{pmatrix} 0 & c_W & -s_W \\ \sqrt{1 - \frac{t_W^2}{3}} & \frac{s_W t_W}{\sqrt{3}} & \frac{s_W}{\sqrt{3}} \\ -\frac{t_W}{\sqrt{3}} & s_W\sqrt{1 - \frac{t_W^2}{3}} & c_W\sqrt{1 - \frac{t_W^2}{3}} \end{pmatrix} \begin{pmatrix} Z' \\ Z \\ A \end{pmatrix}$$

Application to the SLH:

CP-even and CP-odd sectors

- The observed Higgs mass receives contribution from the Coleman-Weinberg potential and the mu term

$$\mathcal{L}_\mu = \mu^2(\Phi_1^\dagger \Phi_2 + \text{h.c.})$$

- CP-even sector: H, σ, Y_R
- CP-odd sector: $\eta, \zeta, \chi, \omega, Z', Z, Y_I$
- Here our goal is to derive the mass eigenstate ZHη vertex. We don't need to show the fermionic sector.

Application to the SLH

Rescaling of η

- The advantage of the double exponential parametrization is that η is only subject to a simple rescaling. Also H & σ does not mix.

- Consider the inner product of two Goldstone fields

$$
\begin{aligned}
< G_i|G_j > &= (U^{-1})_{ik}(U^{-1})_{jl} < S_k|S_l >= (U^{-1})_{ik}(U^{-1})_{jl}\delta_{kl} = (U^{-1})_{ik}(U^{-1})_{jk} \\
&= (V^{-1})_{ij} \qquad (3.14)
\end{aligned}
$$

- Suppose $\eta, \zeta, \chi, \omega$ correspond to indices 1,2,3,4, respectively, then

$$< \eta|\eta >= (V^{-1})_{11}$$

- Therefore we can find the transformation for η

$$\eta = \sqrt{(V^{-1})_{11}}\eta^m$$

Application to the SLH:

Finding the η^m component

- We express the CP-odd sector matrix F as

$$F = \begin{pmatrix} F_{Z\eta} & F_{Z\zeta} & F_{Z\chi} & F_{Z\omega} \\ F_{Z'\eta} & F_{Z'\zeta} & F_{Z'\chi} & F_{Z'\omega} \\ F_{Y\eta} & F_{Y\zeta} & F_{Y\chi} & F_{Y\omega} \end{pmatrix} \qquad \tilde{F} \equiv \begin{pmatrix} F_{Z\zeta} & F_{Z\chi} & F_{Z\omega} \\ F_{Z'\zeta} & F_{Z'\chi} & F_{Z'\omega} \\ F_{Y\zeta} & F_{Y\chi} & F_{Y\omega} \end{pmatrix}$$

- Recall $G_i^m = Q_{ij} G_j, i = 1, 2, ..., n_S$ $\quad Q_{ij} = \begin{cases} \frac{(RF)_{ij}}{\mu_i}, & i = 1, 2, ..., n_M, \\ T_{ij}, & i = n_M + 1, ..., n_S. \end{cases}$

- The application of this result to the SLH leads to

$$\begin{pmatrix} \zeta^m \\ \chi^m \\ \omega^m \end{pmatrix} = \mathbb{M}_{DV}^{-1} R \left[\begin{pmatrix} F_{Z\eta} \\ F_{Z'\eta} \\ F_{Y\eta} \end{pmatrix} \eta + \tilde{F} \begin{pmatrix} \zeta \\ \chi \\ \omega \end{pmatrix} \right] \tag{3.19}$$

Application to the SLH:

Finding the η^m component

- Eq. (3.19) can be inverted to give

$$\begin{pmatrix} \zeta \\ \chi \\ \omega \end{pmatrix} = \tilde{F}^{-1} R^T \mathbb{M}_{DV} \begin{pmatrix} \zeta^m \\ \chi^m \\ \omega^m \end{pmatrix} - \sqrt{(V^{-1})_{11}} \tilde{F}^{-1} \begin{pmatrix} F_{Z\eta} \\ F_{Z'\eta} \\ F_{Y\eta} \end{pmatrix} \eta^m$$

- Define

$$\Upsilon \equiv \begin{pmatrix} \sqrt{(V^{-1})_{11}} \\ -\sqrt{(V^{-1})_{11}} \tilde{F}^{-1} \begin{pmatrix} F_{Z\eta} \\ F_{Z'\eta} \\ F_{Y\eta} \end{pmatrix} \end{pmatrix}$$

$$\mathbb{R}_1 = \begin{pmatrix} R_{11} & R_{12} & R_{13} \end{pmatrix}$$

Application to the SLH:

General formulas for the ZHη vertex

- Also needed are the coefficient matrices

$$\mathbb{C}^{dH} = \begin{pmatrix} C^{dH}_{Z\eta} & C^{dH}_{Z\zeta} & C^{dH}_{Z\chi} & C^{dH}_{Z\omega} \\ C^{dH}_{Z'\eta} & C^{dH}_{Z'\zeta} & C^{dH}_{Z'\chi} & C^{dH}_{Z'\omega} \\ C^{dH}_{Y\eta} & C^{dH}_{Y\zeta} & C^{dH}_{Y\chi} & C^{dH}_{Y\omega} \end{pmatrix}, \quad \mathbb{C}^{Hd} = \begin{pmatrix} C^{Hd}_{Z\eta} & C^{Hd}_{Z\zeta} & C^{Hd}_{Z\chi} & C^{Hd}_{Z\omega} \\ C^{Hd}_{Z'\eta} & C^{Hd}_{Z'\zeta} & C^{Hd}_{Z'\chi} & C^{Hd}_{Z'\omega} \\ C^{Hd}_{Y\eta} & C^{Hd}_{Y\zeta} & C^{Hd}_{Y\chi} & C^{Hd}_{Y\omega} \end{pmatrix}$$

Here $C^{dH}_{Z\eta}$ denotes the coefficient of $Z^\mu \eta \partial_\mu H$, while $C^{Hd}_{Z\eta}$ denotes the coefficient of $Z^\mu H \partial_\mu \eta$.

- Then we have the formulas for the coefficient of the mass eigenstate ZHη vertex (antisymmetric type & symmetric type)

$$c^{as}_{ZH\eta} = \frac{\mathbb{R}_1 \mathbb{C}^{dH} \Upsilon - \mathbb{R}_1 \mathbb{C}^{Hd} \Upsilon}{2} \qquad c^{s}_{ZH\eta} = \frac{\mathbb{R}_1 \mathbb{C}^{dH} \Upsilon + \mathbb{R}_1 \mathbb{C}^{Hd} \Upsilon}{2}$$

Application to the SLH:

Results

- Results for matrix V, F, Y, C^{dH}, C^{Hd} to $\mathcal{O}(\xi^3)$ $\xi \equiv \frac{v}{f}$ (Obtained by Mathematica)

$$V = \begin{pmatrix} 1 & 0 & \frac{\sqrt{2}}{t_{2\beta}}\xi - \frac{7c_{2\beta}+c_{6\beta}}{6\sqrt{2}s_{2\beta}^3}\xi^3 & -\sqrt{2}\xi + \frac{5+3c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 \\ 0 & 1 & -\frac{1}{\sqrt{2}}\xi + \frac{5+3c_{4\beta}}{12\sqrt{2}s_{2\beta}^2}\xi^3 & -\frac{2\sqrt{2}}{3t_{2\beta}}\xi^3 \\ \frac{\sqrt{2}}{t_{2\beta}}\xi - \frac{7c_{2\beta}+c_{6\beta}}{6\sqrt{2}s_{2\beta}^3}\xi^3 & -\frac{1}{\sqrt{2}}\xi + \frac{5+3c_{4\beta}}{12\sqrt{2}s_{2\beta}^2}\xi^3 & 1 - \frac{5+3c_{4\beta}}{12s_{2\beta}^2}\xi^2 & \frac{2}{3t_{2\beta}}\xi^2 \\ -\sqrt{2}\xi + \frac{5+3c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 & -\frac{2\sqrt{2}}{3t_{2\beta}}\xi^3 & \frac{2}{3t_{2\beta}}\xi^2 & 1 \end{pmatrix} + \mathcal{O}(\xi^4) \quad (3.26)$$

$$F = gf \begin{pmatrix} \frac{1}{\sqrt{2}c_W t_{2\beta}}\xi^2 & -\frac{1}{2\sqrt{2}c_W}\xi^2 & \frac{1}{2c_W}\xi - \frac{5+3c_{4\beta}}{24c_W s_{2\beta}^2}\xi^3 & \frac{1}{3c_W t_{2\beta}}\xi^3 \\ \frac{\rho}{t_{2\beta}}\xi^2 & \frac{\sqrt{2}}{\sqrt{3-t_W^2}} - \frac{1+2c_{2W}}{2\sqrt{2}c_W^2\sqrt{3-t_W^2}}\xi^2 & \kappa\xi - \frac{\kappa(5+3c_{4\beta})}{12s_{2\beta}^2}\xi^3 & -\frac{1}{3c_W^2\sqrt{3-t_W^2}t_{2\beta}}\xi^3 \\ -\xi + \frac{5+3c_{4\beta}}{6s_{2\beta}^2}\xi^3 & -\frac{2}{3t_{2\beta}}\xi^3 & \frac{\sqrt{2}}{3t_{2\beta}}\xi^2 & \frac{1}{\sqrt{2}} \end{pmatrix} + \mathcal{O}(\xi^4) \quad (3.27)$$

$$\rho \equiv \sqrt{\frac{1+2c_{2W}}{1+c_{2W}}}, \quad \kappa \equiv \frac{c_{2W}}{2c_W^2\sqrt{3-t_W^2}}$$

Application to the SLH:

Results

$$\Upsilon = \begin{pmatrix} 1+\frac{1}{s_{2\beta}^2}\xi^2+\mathcal{O}(\xi^4) \\ -\frac{1}{t_{2\beta}}\xi^2+\mathcal{O}(\xi^4) \\ -\frac{\sqrt{2}}{t_{2\beta}}\xi-\frac{3-c_{4\beta}}{\sqrt{2}s_{2\beta}^2 t_{2\beta}}\xi^3+\mathcal{O}(\xi^5) \\ \sqrt{2}\xi+\frac{3-c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3+\mathcal{O}(\xi^5) \end{pmatrix} \quad R = \begin{pmatrix} 1+\mathcal{O}(\xi^4) & -\frac{c_{2W}(1+2c_{2W})}{8c_W^5\sqrt{3-t_W^2}}\xi^2+\mathcal{O}(\xi^4) & -\frac{\sqrt{2}}{3c_W t_{2\beta}}\xi^3+\mathcal{O}(\xi^5) \\ \frac{c_{2W}(1+2c_{2W})}{8c_W^5\sqrt{3-t_W^2}}\xi^2+\mathcal{O}(\xi^4) & 1+\mathcal{O}(\xi^4) & -\frac{\sqrt{2}(1+2c_{2W})}{3c_W^2\sqrt{3-t_W^2}t_{2\beta}}\xi^3+\mathcal{O}(\xi^5) \\ \frac{\sqrt{2}}{3c_W t_{2\beta}}\xi^3+\mathcal{O}(\xi^5) & \frac{\sqrt{2}(1+2c_{2W})}{3c_W^2\sqrt{3-t_W^2}t_{2\beta}}\xi^3+\mathcal{O}(\xi^5) & 1+\mathcal{O}(\xi^6) \end{pmatrix}$$

$$\mathbb{C}^{dH} = \begin{pmatrix} 0 & 0 & -\frac{g}{2c_W}+\frac{g(5+3c_{4\beta})}{24c_W s_{2\beta}^2}\xi^2+\mathcal{O}(\xi^4) & 0 \\ 0 & 0 & -\frac{g(1-t_W^2)}{2\sqrt{3-t_W^2}}+\frac{g\kappa(5+3c_{4\beta})}{12s_{2\beta}^2}\xi^2+\mathcal{O}(\xi^4) & 0 \\ 0 & 0 & -\frac{\sqrt{2}g}{3t_{2\beta}}\xi+\frac{g(7c_{2\beta}+c_{6\beta})}{30\sqrt{2}s_{2\beta}^3}\xi^3+\mathcal{O}(\xi^5) & 0 \end{pmatrix}$$

$$\mathbb{C}^{Hd} = \begin{pmatrix} \frac{\sqrt{2}g}{c_W t_{2\beta}}\xi-\frac{g(7c_{2\beta}+c_{6\beta})}{3\sqrt{2}c_W s_{2\beta}^3}\xi^3 & -\frac{g}{\sqrt{2}c_W}\xi+\frac{g(5+3c_{4\beta})}{6\sqrt{2}c_W s_{2\beta}^2}\xi^3 & \frac{g}{2c_W}-\frac{g(5+3c_{4\beta})}{8c_W s_{2\beta}^2}\xi^2 & \frac{g}{c_W t_{2\beta}}\xi^2 \\ \frac{2g\rho}{t_{2\beta}}\xi-\frac{g\rho(7c_{2\beta}+c_{6\beta})}{3s_{2\beta}^3}\xi^3 & -g\rho\xi+\frac{g\rho(5+3c_{4\beta})}{6s_{2\beta}^2}\xi^3 & g\kappa-\frac{g\kappa(5+3c_{4\beta})}{4s_{2\beta}^2}\xi^2 & -\frac{g}{c_W^2\sqrt{3-t_W^2}t_{2\beta}}\xi^2 \\ -g+\frac{g(5+3c_{4\beta})}{2s_{2\beta}^2}\xi^2 & -\frac{2g}{t_{2\beta}}\xi^2 & \frac{2\sqrt{2}g}{3t_{2\beta}}\xi-\frac{\sqrt{2}g(7c_{2\beta}+c_{6\beta})}{15s_{2\beta}^3}\xi^3 & 0 \end{pmatrix} + \mathcal{O}(\xi^4) \qquad (3.31)$$

Application to the SLH:

Results

- The final results, for the coefficient of mass eigenstate antisymmetric ($Z^\mu(\eta\partial_\mu H - H\partial_\mu\eta)$) & symmetric ($Z^\mu(\eta\partial_\mu H + H\partial_\mu\eta)$) ZHη vertex, to $\mathcal{O}(\xi^3)$, are

$$c^{as}_{ZH\eta} = -\frac{g}{4\sqrt{2}c^3_W t_{2\beta}}\xi^3 + \mathcal{O}(\xi^5) \tag{3.33}$$

$$c^{s}_{ZH\eta} = \frac{g}{\sqrt{2}c_W t_{2\beta}}\xi + \frac{g}{24\sqrt{2}c_W s_{2\beta}}\left[\frac{8}{s_{2\beta}t_{2\beta}} + 3c_{2\beta}\left(8 + \frac{6}{c^2_W} - \frac{1}{c^4_W}\right)\right]\xi^3 + \mathcal{O}(\xi^5) \tag{3.34}$$

- From our derivation, the antisymmetric vertex vanishes at O(ξ), which is different from the expression that has existed for a long time.

$$\mathcal{L}_{ZH\eta} = \frac{m_Z}{\sqrt{2}F}N_2 Z_\mu(\eta\partial^\mu H - H\partial^\mu\eta) \qquad N_2 = \frac{F_2^2 - F_1^2}{F_1 F_2}$$

W. Kilian, D. Rainwater & J. Reuter, PRD 71, 015008(2005), PRD 74, 095003(2006).

- The η phenomenology could thus be very different. (On-going study)

Discussion and conclusion

- A general procedure is elucidated to diagonalize a general vector-scalar system in gauge theories. The convenience of rotating to gauge boson mass eigenstate before adding gauge-fixing terms is emphasized, based on the observation that **massive gauge bosons eat their corresponding Goldstone bosons along the directions dictated by their mass eigenstates**.
- The general procedure is then applied to the case of the SLH, which due to its nonlinearly-realized scalar sector entails a general treatment for its non-canonically normalized scalar kinetic terms and 'unexpected' vector-scalar transitions.
- We obtained **$O(\xi^3)$ mass eigenstate antisymmetric and symmetric ZHη vertices** and found them to be different from expressions that has existed in the literature for a long time. The η phenomenology could be quite different.
- The general procedure could also be applied to other models with nonlinearly-realized scalar sector. Finding a convenient parametrization may be important.

Thank you!