Measuring properties of a Heavy Higgs boson in the $H \to ZZ \to 4l$ decay

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Motivations

- The SM with one Higgs doublet is not natural. Another heavy scalar boson can appear soon.
- LHC searched for H -> ZZ -> 4l and there are some 2-3 sigma here and there.
- The decay H ->ZZ->4l involves a number of angles that one can investigate the CP properties of the boson.

Interactions of HZZ

$$
\begin{aligned}
i\mathcal{M}^{H\to ZZ} &\equiv i\frac{gM_W}{c_W^2}\,\Gamma^{ZZ}_{\mu\nu}\epsilon_1^{*\mu}\epsilon_2^{*\nu} \\
&= i\frac{gM_W}{c_W^2}\Bigg\{g_{_{HZZ}}\,\epsilon_1^*\cdot\epsilon_2^* + S_H^{ZZ}(s)\left[\frac{-2k_1\cdot k_2}{s}\,\epsilon_1^*\cdot\epsilon_2^* \;+\; \frac{2}{s}\,k_1\cdot\epsilon_2^*\,k_2\cdot\epsilon_1^*\right] \\
&\qquad\qquad + P_H^{ZZ}(s)\,\frac{2}{s}\,\langle\epsilon_1^*\epsilon_2^*k_1k_2\rangle\Bigg\}
\end{aligned}
$$

The first term comes from $\mathcal{L} = \frac{gM_W}{2c_W^2}\,g_{_{HZZ}}\,Z_\mu Z^\mu H$

The second and third term come from higher-order or from genuine dim-6 operators. They can be complex if developed non-vanishing absorptive part.

Helicity Amplitude

$$H \quad \to \; Z(k_1, \epsilon_1) Z(k_2, \epsilon_2) \; \to f_1(p_1, \sigma_1) \bar{f}_1(\bar{p}_1, \bar{\sigma}_1) \;\; f_2(p_2, \sigma_2) \bar{f}_2(\bar{p}_2, \bar{\sigma}_2).$$

$$\begin{aligned}
i\mathcal{M}_{\sigma_1 \bar{\sigma}_1 : \sigma_2 \bar{\sigma}_2} &= \left(i \frac{g M_W}{c_W^2} \Gamma^{ZZ}_{\mu\nu} \right) \; \frac{-i \left(g^{\mu\rho} - \frac{k_1^\mu k_1^\rho}{M_Z^2} \right)}{k_1^2 - M_Z^2 + i M_Z \Gamma_Z} \; \frac{-i \left(g^{\nu\sigma} - \frac{k_2^\nu k_2^\sigma}{M_Z^2} \right)}{k_2^2 - M_Z^2 + i M_Z \Gamma_Z} \\
&\times \left[-i \frac{g}{c_W} \sum_{A=L,R} \bar{u}(p_1, \sigma_1) \gamma_\rho (v_{f_1} - A a_{f_1}) P_A v(\bar{p}_1, \bar{\sigma}_1) \right] \\
&\times \left[-i \frac{g}{c_W} \sum_{B=L,R} \bar{u}(p_2, \sigma_2) \gamma_\sigma (v_{f_2} - B a_{f_2}) P_B v(\bar{p}_2, \bar{\sigma}_2) \right] \\
&= i \sum_{\lambda_1, \lambda_2} \mathcal{M}^{H \to ZZ}_{\lambda_1 \lambda_2} \; \frac{1}{k_1^2 - M_Z^2 + i M_Z \Gamma_Z} \; \frac{1}{k_2^2 - M_Z^2 + i M_Z \Gamma_Z} \; \mathcal{M}^{Z \to f_1 \bar{f}_1}_{\lambda_1 : \sigma_1 \bar{\sigma}_1} \mathcal{M}^{Z \to f_2 \bar{f}_2}_{\lambda_2 : \sigma_2 \bar{\sigma}_2}
\end{aligned}$$

The helicity amplitude for the decay $H \to Z(k_1, \epsilon_1)Z(k_2, \epsilon_2)$ in the rest frame of H

by

$$\mathcal{M}^{H\to ZZ}_{\lambda_1\lambda_2} = \frac{gM_W}{c_W^2} \langle\lambda_1\rangle \, \delta_{\lambda_1\lambda_2}$$

with the reduced amplitudes $\langle\lambda_1\rangle$ defined by

$$\langle+\rangle \equiv g_{HZZ} + (1-\alpha_1-\alpha_2)\, S_H^{ZZ} \; - i\lambda^{1/2}(1,\alpha_1,\alpha_2)\, P_H^{ZZ} \,,$$

$$\langle-\rangle \equiv g_{HZZ} + (1-\alpha_1-\alpha_2)\, S_H^{ZZ} \; + i\lambda^{1/2}(1,\alpha_1,\alpha_2)\, P_H^{ZZ} \,,$$

$$\langle 0\rangle \equiv g_{HZZ} \left(\frac{1-\alpha_1-\alpha_2}{2\sqrt{\alpha_1\alpha_2}}\right) - 2\sqrt{\alpha_1\alpha_2}\, S_H^{ZZ} \,,$$

$$\alpha_i = k_i^2/M_H^2.$$

The longitudinal amplitude <0> is enhanced by a factor $M_H^2/2\,M_Z^2$ in large M_H limit.

helicity amplitude for the decay $Z(k,\epsilon(k,\lambda)) \to f(p,\sigma)\bar{f}(\bar{p},\bar{\sigma})$ is

$$\mathcal{M}^{Z\to f\bar{f}}_{\lambda:\sigma\bar{\sigma}} = \begin{cases} -\frac{g}{c_W}\Big[\sqrt{2}m_f v_f\ \lambda\sigma e^{-i(\sigma-\lambda)\phi}\ s_\theta\ \delta_{\sigma\bar{\sigma}} \\ \qquad +\frac{\sqrt{k^2}}{\sqrt{2}}(v_f-\sigma\beta_f a_f)(\lambda c_\theta+\sigma)\ e^{i\lambda\phi}\ \delta_{\sigma-\bar{\sigma}}\Big] & \text{for } \lambda=\pm \\ -\frac{g}{c_W}\Big[2m_f v_f\ e^{-i\sigma\phi}\ (-\sigma c_\theta)\ \delta_{\sigma\bar{\sigma}}+\sqrt{k^2}(v_f-\sigma\beta_f a_f)s_\theta\ \delta_{\sigma-\bar{\sigma}}\Big] & \text{for } \lambda=0 \end{cases}$$

Combining all sub-amplitudes

$$\begin{aligned}\mathcal{M}_{\sigma_1\bar{\sigma}_1:\sigma_2\bar{\sigma}_2} &= \frac{gM_W}{2c_W^2}\left(\frac{g}{c_W}\right)^2 \frac{\sqrt{k_1^2}}{k_1^2-M_Z^2+iM_Z\Gamma_Z}\ \frac{\sqrt{k_2^2}}{k_2^2-M_Z^2+iM_Z\Gamma_Z} \\ &\times (v_{f_1}-\sigma_1 a_{f_1})(v_{f_2}-\sigma_2 a_{f_2}) \\ &\times \Big[\langle+\rangle(c_{\theta_1}+\sigma_1)(c_{\theta_2}+\sigma_2)e^{i(\phi_1+\phi_2)}+\langle-\rangle(-c_{\theta_1}+\sigma_1)(-c_{\theta_2}+\sigma_2)e^{-i(\phi_1+\phi_2)} \\ &\quad +2\langle 0\rangle s_{\theta_1}s_{\theta_2}\Big]\,\delta_{\sigma_1-\bar{\sigma}_1}\delta_{\sigma_2-\bar{\sigma}_2}\,.\end{aligned}$$

The amplitude squared can be written as a sum of independent combinations of angular variables:

$$\sum_{\sigma_1,\bar{\sigma}_1,\sigma_2,\bar{\sigma}_2} |\mathcal{M}_{\sigma_1\bar{\sigma}_1:\sigma_2\bar{\sigma}_2}|^2 = \left(\frac{gM_W}{c_W^2}\right)^2 \left(\frac{g}{c_W}\right)^4 \frac{k_1^2}{(k_1^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2} \frac{k_2^2}{(k_2^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$

$$\times\, (v_{f_1}^2 + a_{f_1}^2)(v_{f_2}^2 + a_{f_2}^2) \frac{128\pi}{9} \sum_{i=1}^{9} C_i\ f_i(\theta_1, \theta_2, \Phi) \qquad (10)$$

$$f_1(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[(1+c_{\theta_1}^2)(1+c_{\theta_2}^2) + 4\eta_1\eta_2 c_{\theta_1}c_{\theta_2}\right],$$

$$f_2(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left\{-2\left[\eta_1 c_{\theta_1}(1+c_{\theta_2}^2) + \eta_2 c_{\theta_2}(1+c_{\theta_1}^2)\right]\right\},$$

$$f_3(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[4s_{\theta_1}^2 s_{\theta_2}^2\right],$$

$$f_4(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[4(c_{\theta_1}c_{\theta_2} + \eta_1\eta_2)s_{\theta_1}s_{\theta_2}c_\Phi\right],$$

$$f_5(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[-4(c_{\theta_1}c_{\theta_2} + \eta_1\eta_2)s_{\theta_1}s_{\theta_2}s_\Phi\right],$$

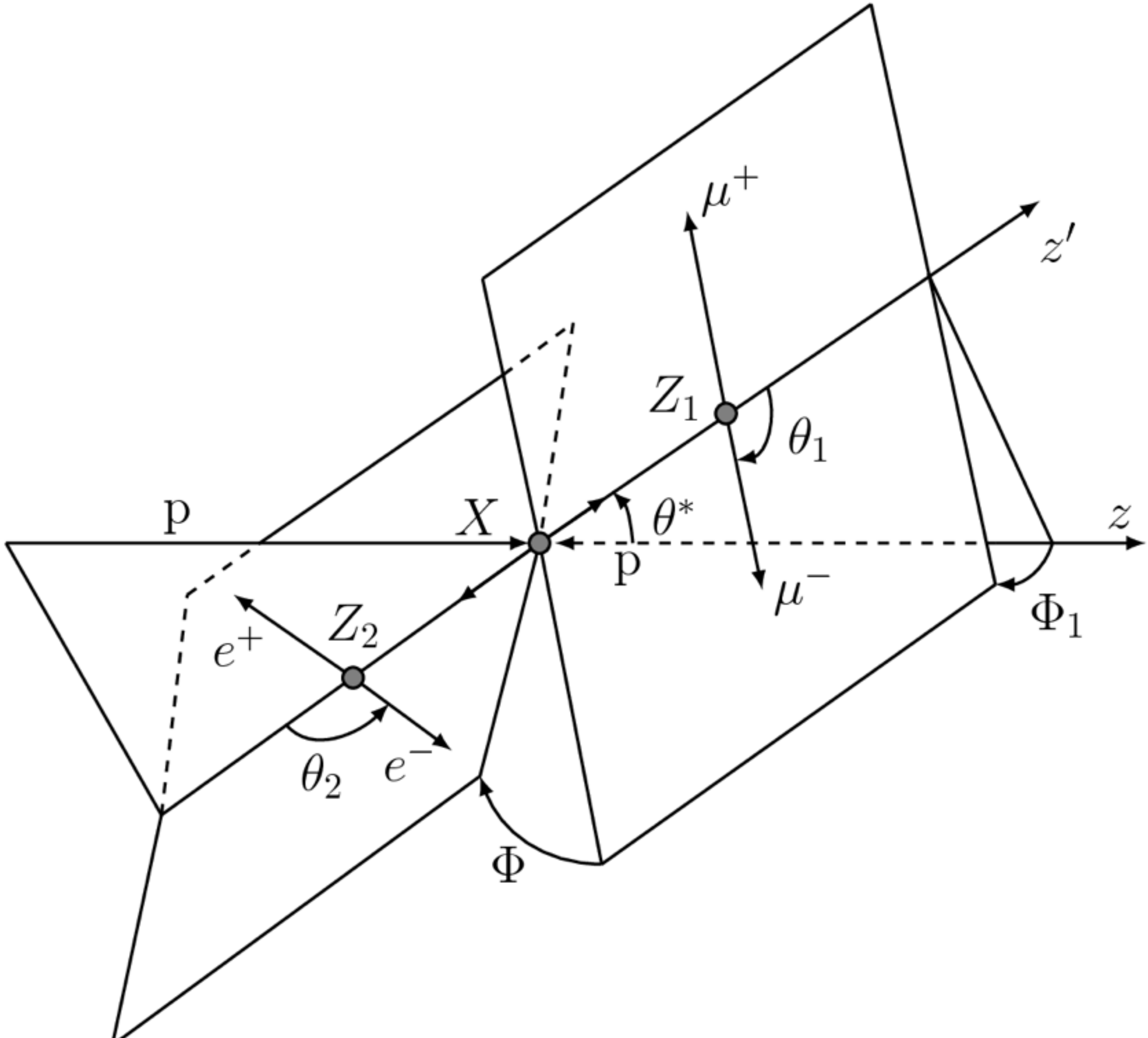
$$f_6(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[-4(\eta_1 c_{\theta_2} + \eta_2 c_{\theta_1})s_{\theta_1}s_{\theta_2}c_\Phi\right],$$

$$f_7(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[4(\eta_1 c_{\theta_2} + \eta_2 c_{\theta_1})s_{\theta_1}s_{\theta_2}s_\Phi\right],$$

$$f_8(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[s_{\theta_1}^2 s_{\theta_2}^2 c_{2\Phi}\right],$$

$$f_9(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[-s_{\theta_1}^2 s_{\theta_2}^2 s_{2\Phi}\right].$$

with $\Phi = \phi_1 + \phi_2$ and $\eta_i = 2v_{f_i}a_{f_i}/(v_{f_i}^2 + a_{f_i}^2)$.



μ^+
z'
Z_1
θ_1
p
X
θ^*
z
p
μ^-
Φ_1
Z_2
e^+
θ_2
e^-
Φ

Also, the 9 angular coefficients C_{1-9}, which are combinations of the reduced helicity amplitudes $\langle + \rangle$, $\langle - \rangle$, and $\langle 0 \rangle$, are defined as

$$
\begin{aligned}
&C_1 \equiv |\langle + \rangle|^2 + |\langle - \rangle|^2 \,, \quad C_2 \equiv |\langle + \rangle|^2 - |\langle - \rangle|^2 \,, \quad C_3 \equiv |\langle 0 \rangle|^2 \,, \\
&C_4 \equiv \Re\mathrm{e}\,[\langle + \rangle\langle 0 \rangle^* + \langle - \rangle\langle 0 \rangle^*] \,, \quad C_5 \equiv \Im\mathrm{m}\,[\langle + \rangle\langle 0 \rangle^* - \langle - \rangle\langle 0 \rangle^*] \,, \\
&C_6 \equiv \Re\mathrm{e}\,[\langle + \rangle\langle 0 \rangle^* - \langle - \rangle\langle 0 \rangle^*] \,, \quad C_7 \equiv \Im\mathrm{m}\,[\langle + \rangle\langle 0 \rangle^* + \langle - \rangle\langle 0 \rangle^*] \,, \\
&C_8 \equiv 2\Re\mathrm{e}\,[\langle + \rangle\langle - \rangle^*] \,, \quad C_9 \equiv 2\Im\mathrm{m}\,[\langle + \rangle\langle - \rangle^*] \,.
\end{aligned}
\tag{12}
$$

Under CP and CPT~ the reduced amplitudes transform like

$$
\langle \lambda \rangle \stackrel{\mathrm{CP}}{\leftrightarrow} \langle -\lambda \rangle \,, \qquad \langle \lambda \rangle \stackrel{\mathrm{CP\widetilde{T}}}{\leftrightarrow} \langle -\lambda \rangle^* \,.
$$

- C_2, C_5, C_6, C_9 are CP-odd and nonzero when g_{HZZ}/S^{ZZ}_H and P^{ZZ}_H exist.

- C_2, C_6, C_7 are CPT~ odd and nonzero when induced by the absorptive parts of S^{ZZ}_H and/or P^{ZZ}_H

The partial decay width of the process $H \to ZZ \to 2\ell_1 2\ell_2$ is given by

$$
\begin{aligned}
d\Gamma &= \frac{1}{2M_H} \left(\sum_{\sigma_1,\bar\sigma_1,\sigma_2,\bar\sigma_2} \left|\mathcal{M}_{\sigma_1\bar\sigma_1:\sigma_2\bar\sigma_2}\right|^2 \right) d\Phi_4 \\
&= \frac{1}{2^{13}\,\pi^6\,M_H}\,\lambda^{1/2}(1, k_1^2/M_H^2, k_2^2/M_H^2)\sqrt{k_1^2}\sqrt{k_2^2} \\
&\qquad \times \left(\sum_{\sigma_1,\bar\sigma_1,\sigma_2,\bar\sigma_2} \left|\mathcal{M}_{\sigma_1\bar\sigma_1:\sigma_2\bar\sigma_2}\right|^2 \right)\, d\sqrt{k_1^2}\, d\sqrt{k_2^2}\, dc_{\theta_1}\, dc_{\theta_2}\, d\Phi\,.
\end{aligned}
$$

After integrating over $\sqrt{k_1^2}$ and $\sqrt{k_2^2}$, we obtain

$$
\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}c_{\theta_1}\mathrm{d}c_{\theta_2}\mathrm{d}\Phi} = \sum_{i=1}^{9} \overline{R}_i f_i(\theta_1, \theta_2, \Phi)
$$

with the 9 angular observables defined by

$$
\overline{R}_i \equiv \frac{w_i\overline{C}_i}{w_1\overline{C}_1 + w_3\overline{C}_3}.
$$

$$
\overline{C}_i = C_i(k_1^2 = M_Z^2, k_2^2 = M_Z^2)
$$

All $w_i = 1$ for on shell Z's. We shall take the NWA.

We can integrate any 2 of the angles θ_1, θ_2, and Φ to obtain 1-dim angular distributions

$$\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}c_{\theta_{1,2}}} = \frac{3}{8}\overline{R}_1\left(1+c^2_{\theta_{1,2}}\right) - \frac{3\eta_{1,2}}{4}\overline{R}_2\ c_{\theta_{1,2}} + \frac{3}{4}\overline{R}_3\left(1-c^2_{\theta_{1,2}}\right)\ ,$$

$$\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi} = \frac{1}{2\pi} + \frac{9\pi\eta_1\eta_2}{128}\left(\overline{R}_4\ c_\Phi - \overline{R}_5\ s_\Phi\right) + \frac{1}{8\pi}\left(\overline{R}_8\ c_{2\Phi} - \overline{R}_9\ s_{2\Phi}\right)$$

$$\Gamma = \frac{1}{2^6 3^2 \pi^5 M_H}\left(\frac{gM_W}{c_W^2}\right)^2\left(\frac{g}{c_W}\right)^4\left(v^2_{f_1}+a^2_{f_1}\right)\ \left(v^2_{f_2}+a^2_{f_2}\right)\ \left(w_1\overline{C}_1+w_3\overline{C}_3\right)\ \mathcal{F}$$

* Only $C_{1,2,3}$ contribute to cos-theta$_{1,2}$ distributions. When S^{ZZ}_H and P^{ZZ}_H are real, C_2=0.
* $R_{6,7}$ never appear in 1-dim distributions. We need 2-dim distributions, e.g., c_{θ_1}-Φ and c_{θ_2}-Φ distributions.

* The angular observables $R_{1,2,3}$ can be obtained from fitting to cos-theta$_{1,2}$ distributions.

* $R_{4,5,8,9}$ can be obtained from Fourier analysis or fitting to Ø distribution.

* A non-vanishing R_2 signals may imply new particles of mass < $M_H/2$, such that develops absorptive part for $S^{ZZ}{}_H$ and $P^{ZZ}{}_H$.
* Measurements of R's cannot determine the absolute size of $S^{ZZ}{}_H$, $P^{ZZ}{}_H$ and g_{HZZ}.

* We need to measure $C_1 + C_3$ in the partial width

$$\Gamma = 2.78 \times 10^{-4}\ (w_1 \overline{C}_1 + w_3 \overline{C}_3)\ \mathrm{GeV}$$

$$= \Gamma^H_{\rm tot}\ B(H \to ZZ \to 2\ell_1 2\ell_2) \simeq \Gamma^H_{\rm tot}\ B(H \to ZZ)\ \ [B(Z \to \ell\ell)]^2$$

Numerical Analysis

- We used M_H=260 GeV
- B(H->ZZ, Zh, hh) are comparable, B(H->tt) = 0.
- S^{ZZ}_H and P^{ZZ}_H are real. Only 3 input parameters of couplings.

	g_{HZZ}	S_H^{ZZ}	P_H^{ZZ}	$\overline{C}_1[++]$	$\overline{C}_2[--]$	$\overline{C}_3[++]$	$\overline{C}_4[++]$	$\overline{C}_5[-+]$	$\overline{C}_6[--]$	$\overline{C}_7[+-]$	$\overline{C}_8[++]$	$\overline{C}_9[-+]$
S1	1	0	0	2.00	0.00	9.39	-6.13	0.00	0.00	0.00	2.00	0.00
S2	0	1	0	1.14	0.00	0.0605	-0.371	0.00	0.00	0.00	1.14	0.00
S3	0	0	1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	-1.02	0.00
S4	0	1	1	2.15	0.00	0.0605	-0.371	0.351	0.00	0.00	0.121	-2.15
S5	0	1	-1	2.15	0.00	0.0605	-0.371	-0.351	0.00	0.00	0.121	2.15
S6	0.32	1	1	1.39	0.00	0.540	0.638	-1.05	0.00	0.00	-0.639	-1.24

* $S_{1,2,3}$ are CP conserving because one coupling is nonzero.
* $S_{4,5}$ are CP violating.
* S_6 : parameters are chosen s.t. they contribute equally to amplitude squared. $\langle O \rangle$ is enhanced for large M_H.

Non-vanishing R's

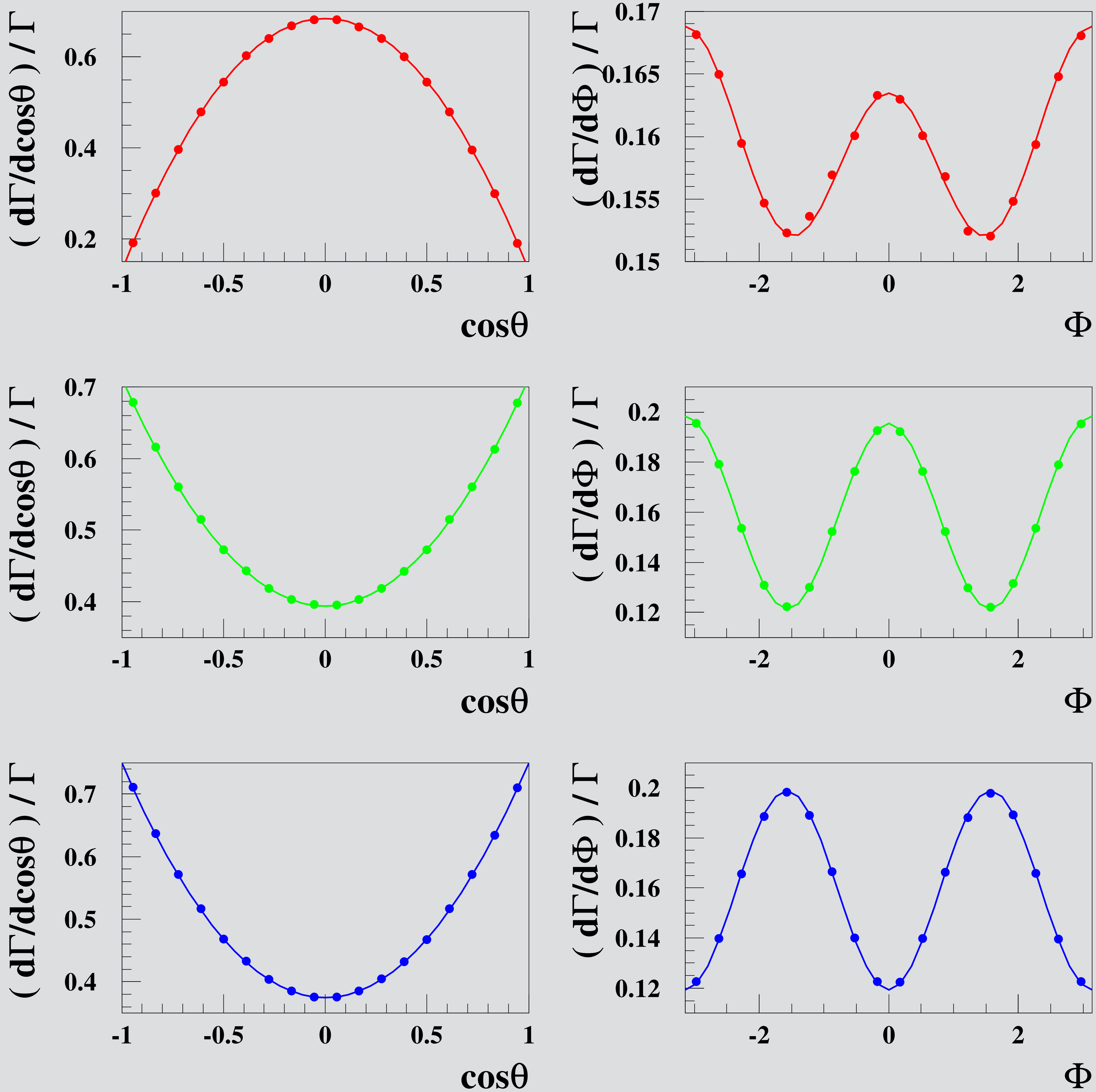
	g_{HZZ}	S_H^{ZZ}	P_H^{ZZ}	$\overline{R}_1[++]$	$\overline{R}_3[++]$	$\overline{R}_4[++]$	$\overline{R}_5[-+]$	$\overline{R}_8[++]$	$\overline{R}_9[-+]$	$(\overline{C}_1+\overline{C}_3)[++]$
S1	1	0	0	0.176	0.824	-0.538	0.00	0.176	0.00	11.4
S2	0	1	0	0.950	0.0505	-0.310	0.00	0.945	0.00	1.20
S3	0	0	1	1.00	0.00	0.00	0.00	-1.00	0.00	1.02
S4	0	1	1	0.973	0.0273	-0.168	0.158	0.0547	-0.971	2.21
S5	0	1	-1	0.973	0.0273	-0.168	-0.158	0.0547	0.971	2.21
S6	0.32	1	1	0.721	0.280	0.330	-0.542	-0.331	-0.640	1.93

CP Conserving

S_1 (1,0,0)

S_2 (0,1,0)

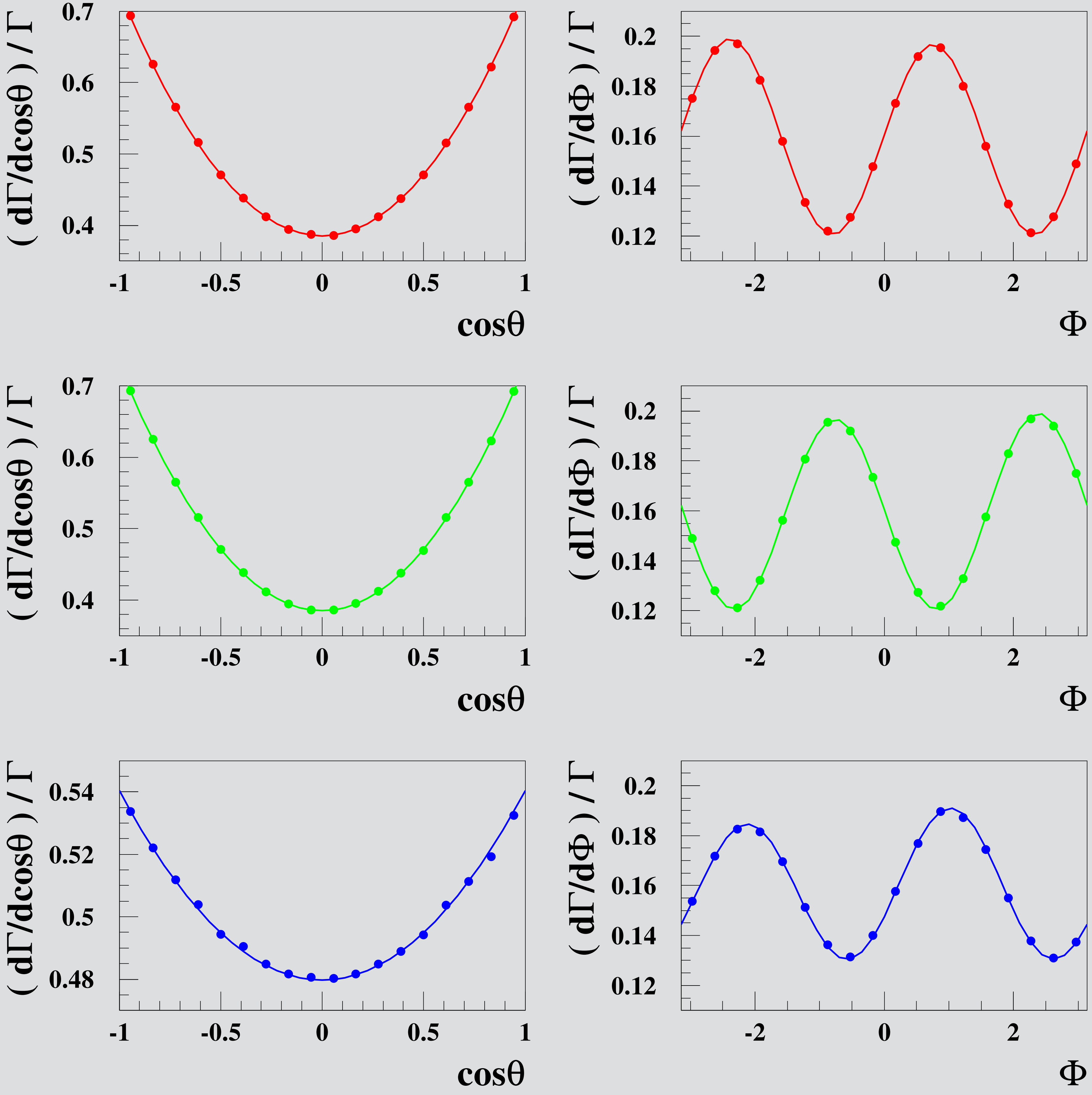
S_3 (0,0,1)



S$_4$ (0,1,1)

S$_5$ (0,1,-1)

S$_6$ (0.32,1,1)



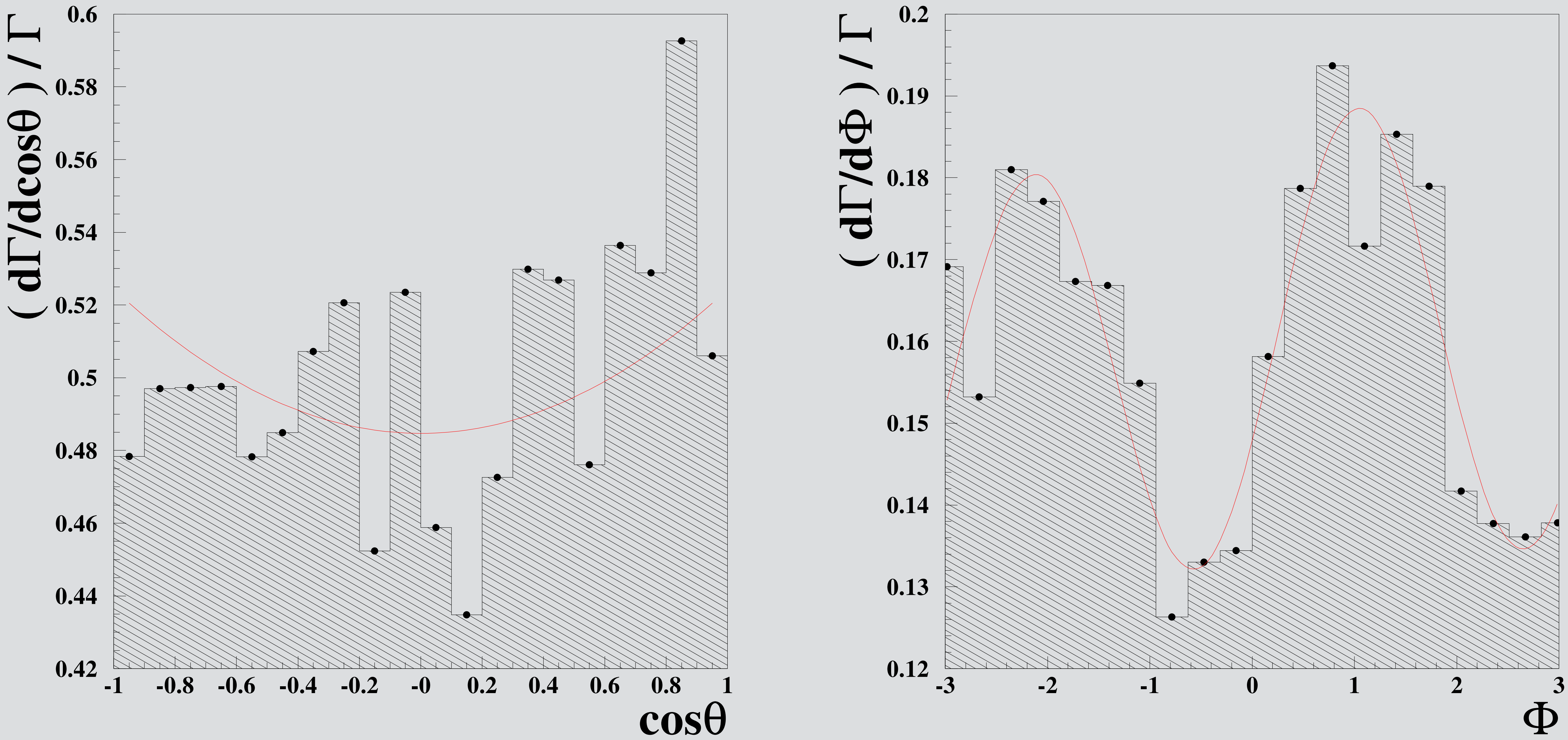
Illustrate the measurement in S_6

S6 with $\left(g_{HZZ}, S_H^{ZZ}, P_H^{ZZ}\right) = (0.32, 1, 1)$

Current upper limit: $\sigma(gg \to H) \cdot B(H \to ZZ) \simeq 1$ pb for a 260 GeV Higgs boson

$$\sigma(gg \to H) \cdot B(H \to ZZ) \cdot 4[B(Z \to \ell\ell)]^2 \cdot \epsilon_{4\ell} \cdot \mathcal{L} \simeq 10^4$$

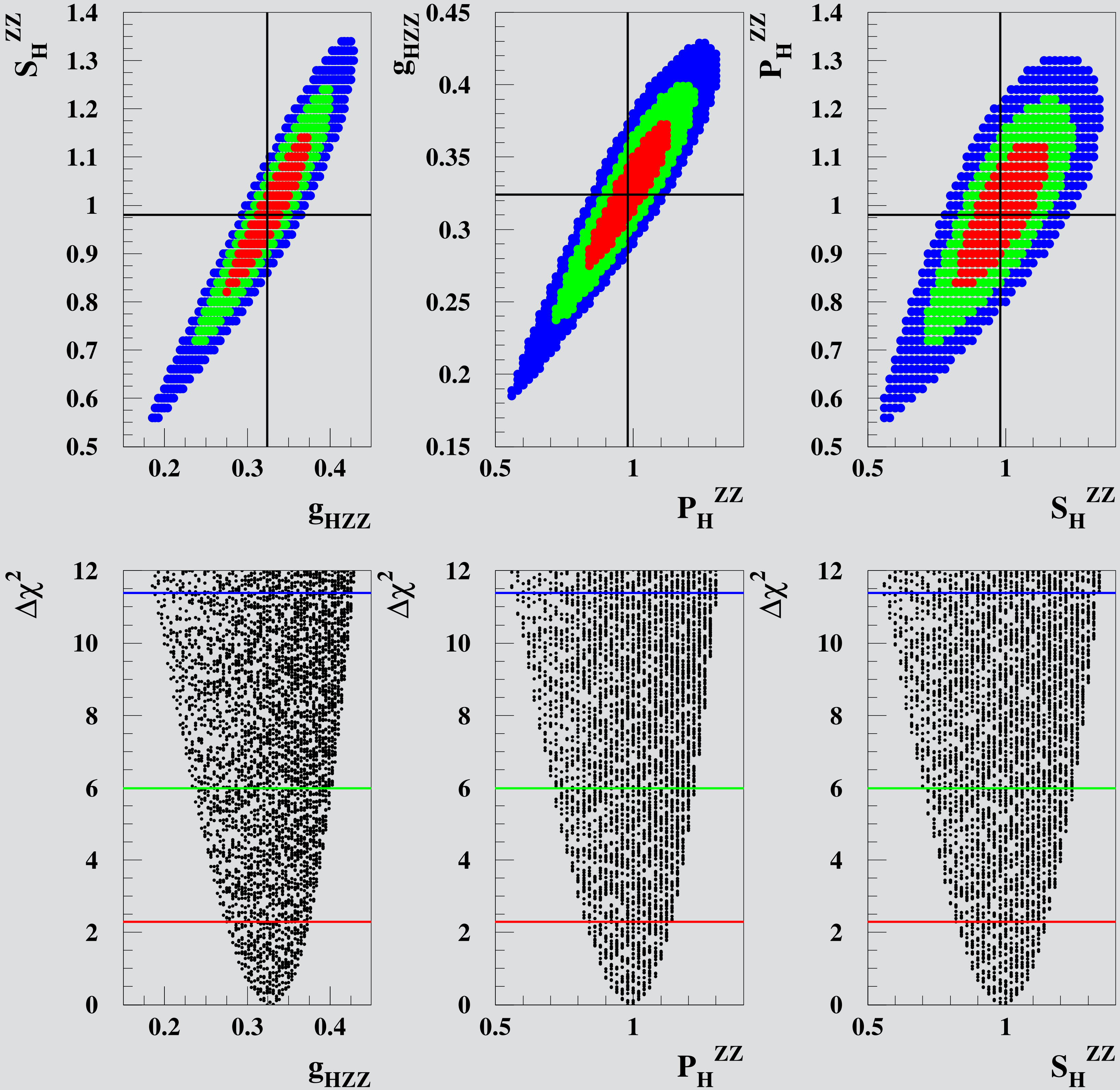
We take 10^4events in the following, and angular resolution of cos-theta = 0.1, Phi=0.1π.



S6	$\overline{R}_1[++]$	$\overline{R}_3[++]$	$\overline{R}_4[++]$	$\overline{R}_5[-+]$	$\overline{R}_8[++]$	$\overline{R}_9[-+]$	$(\overline{C}_1+\overline{C}_3)[++]$
Input	0.721	0.280	0.330	-0.542	-0.331	-0.640	1.93
Output (center value)	0.699	0.297	0.196	-0.832	-0.305	-0.564	1.93
Output (parabolic error)	±0.018	±0.011	±0.443	±0.461	±0.057	±0.056	±0.386

$\overline{R}_1$ *and* $\overline{R}_3$ *is* $\rho = -0.805,$

Fit to the original inputs g_{HZZ}, $S^{ZZ}{}_H$, $P^{ZZ}{}_H$



Best Fit

g_{HZZ}=0.324±0.031

$S^{ZZ}{}_H$ =0.980±0.11

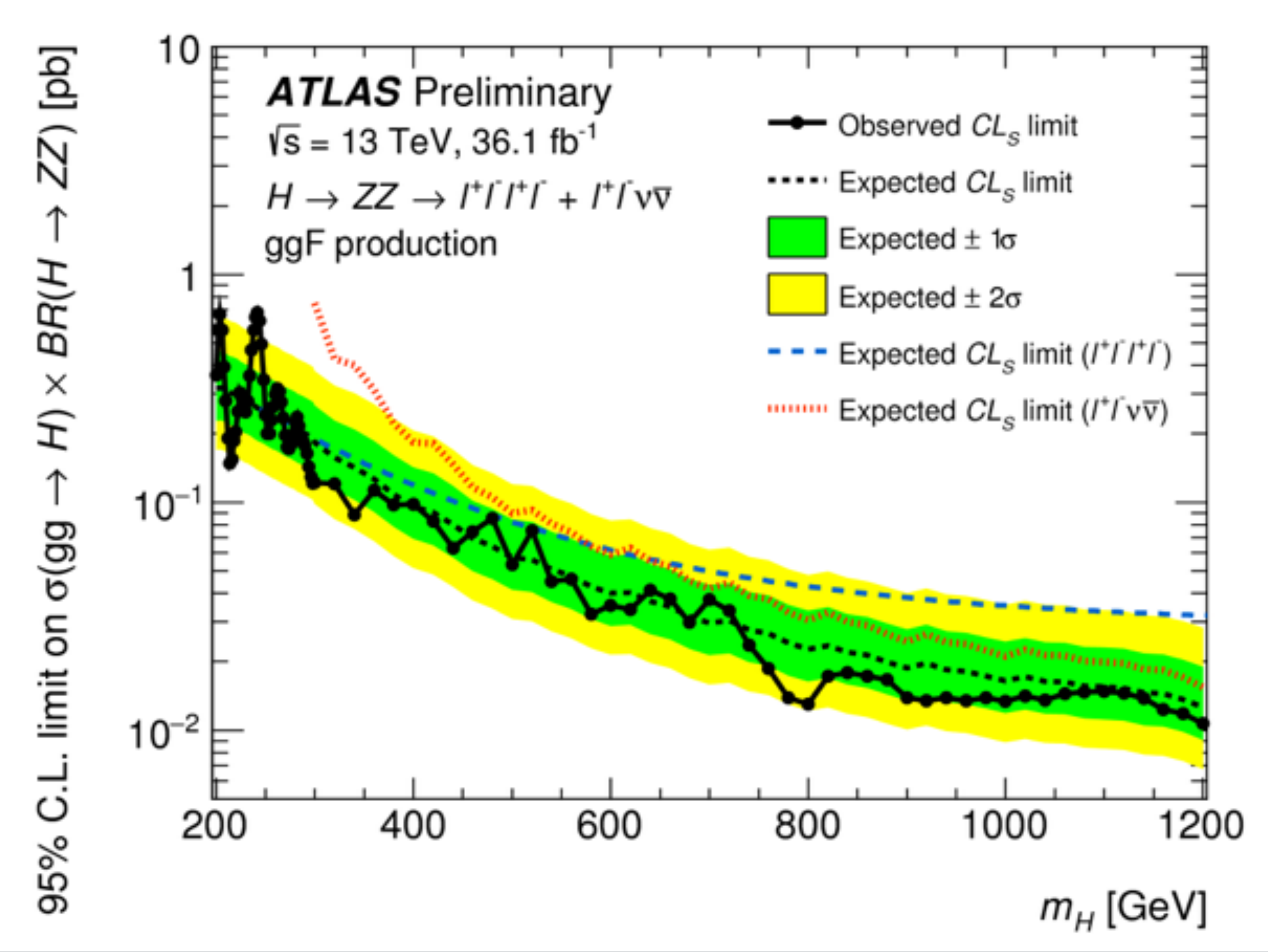
$P^{ZZ}{}_H$ =0.980±0.10

10% error for
10^4 events

ATLAS-CONF-2017-058

The upper limit around 240-270 GeV is 0.5 pb, corresponding to about 3000 events at HL-LHC.

The uncertainty will increases to 20%.



Summary

- The angular distributions of $theta_{1,2}$ and Phi can be analysed and fitted to the observables R_i's and then to Higgs couplings.

- With 10^4 H -> ZZ -> 4l events one can determine the couplings g_{HZZ}, $S^{ZZ}{}_H$ and $P^{ZZ}{}_H$ with 10% uncertainty. With 3000 events uncertainty goes down to 20%.

- Appearance of C_2 signals new particles with mass < $M_H/2$ running in the loop.

- One can also extend to 2-dim distributions to analyse other observables $R_{6,7}$.