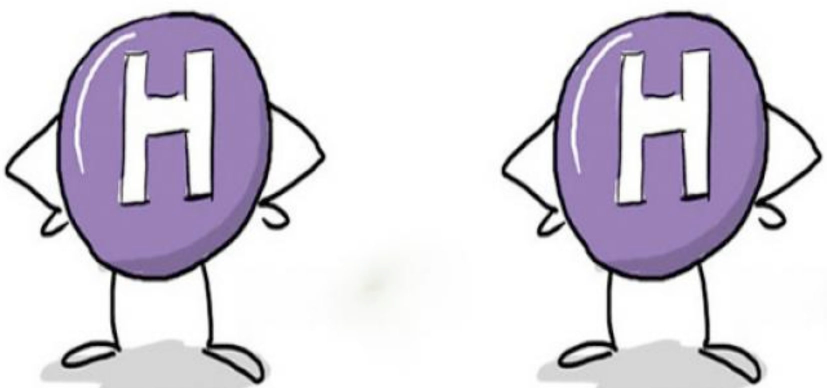
# Alignment in Models of Extended Higgs Sectors


Howard E. Haber


Energy Frontiers in Particle Physics: LHC and Future Colliders

NTU, Taipei, Taiwan

29 September 2017



# Outline

- Motivations for an extended Higgs sector
- Constraints on extended Higgs sectors
- The Higgs field alignment limit—approaching the SM Higgs boson
- The alignment limit of the CP-conserving 2HDM
- Realizing (approximate) alignment via a symmetry principle
- Alignment without decoupling in the MSSM
- Flavor alignment in extended Higgs sectors
- The flavor-aligned 2HDM
- Conclusions

## Why not an extended Higgs sector?

- The fermion and gauge boson sectors of the Standard Model (SM) are not of minimal form ("Who ordered that?"). So, why should the spin-0 (scalar) sector be minimal?

- Extended Higgs sectors can provide a dark matter candidate.

- Extended Higgs sectors can modify the electroweak phase transition.

- Extended Higgs sectors can enhance vacuum stability.

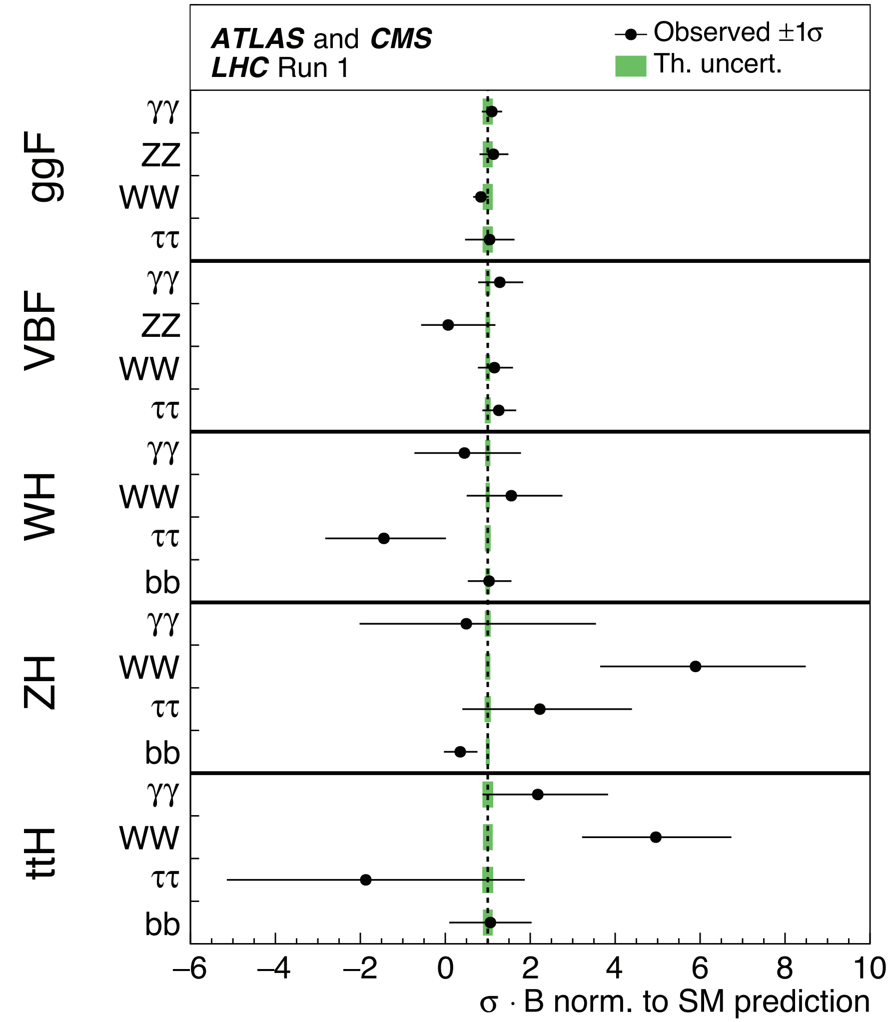
- Models of new physics beyond the SM often require additional scalar Higgs states. E.g., two Higgs doublets are required in the minimal supersymmetric extension of the SM (MSSM).

# Extended Higgs sectors are highly constrained

- The electroweak $\rho$ parameter is very close to 1.
- One neutral Higgs scalar of the extended Higgs sector must be SM-like (and identified with the Higgs boson at 125 GeV).
- Higgs-mediated flavor-changing neutral currents (FCNCs) are suppressed.
- Charged Higgs exchange at tree level (e.g. in $\bar{B} \to D^{(*)}\tau^-\overline{\nu}_\tau$) and at one-loop (e.g. in $b \to s\gamma$) can significantly constrain the charged Higgs masses and the Yukawa couplings.
- At present, only one Higgs scalar has been observed.
- If the scale that governs the non-SM like Higgs bosons is close to the electroweak scale, is the naturalness problem of electroweak symmetry breaking exacerbated?

# A SM-like Higgs boson

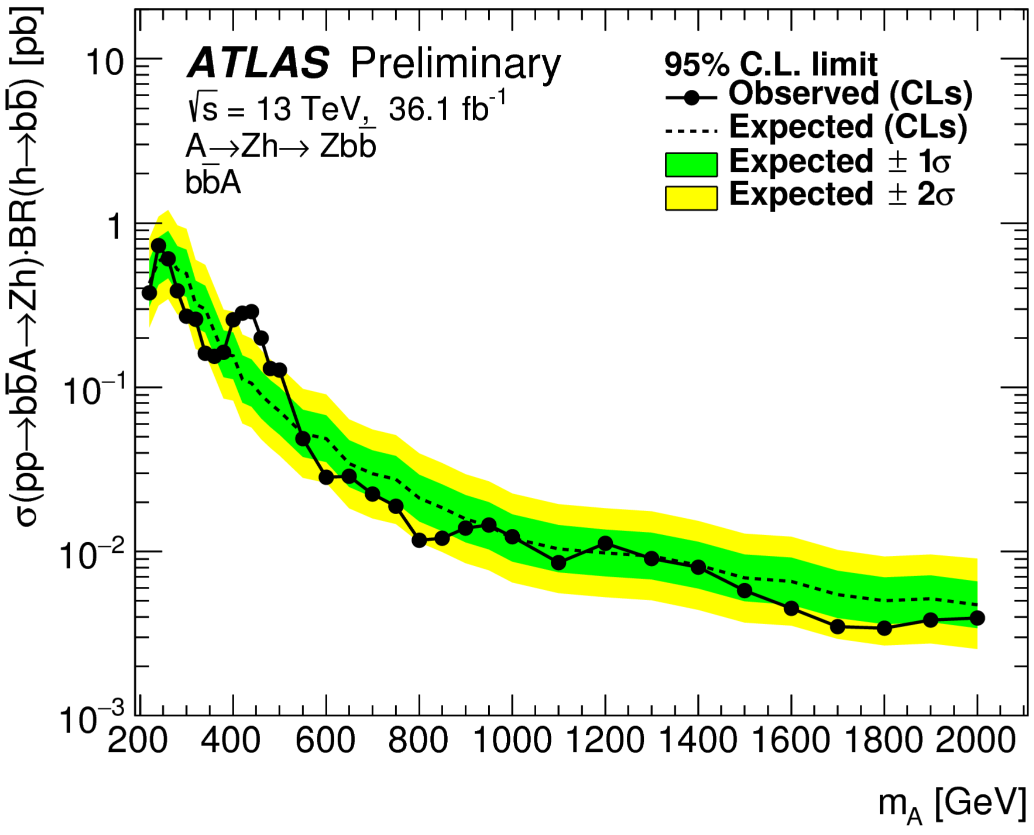
After Run-1 of the LHC, the ATLAS and CMS Collaborations provided a combined analysis of the Higgs data [arXiv:1503.07589].



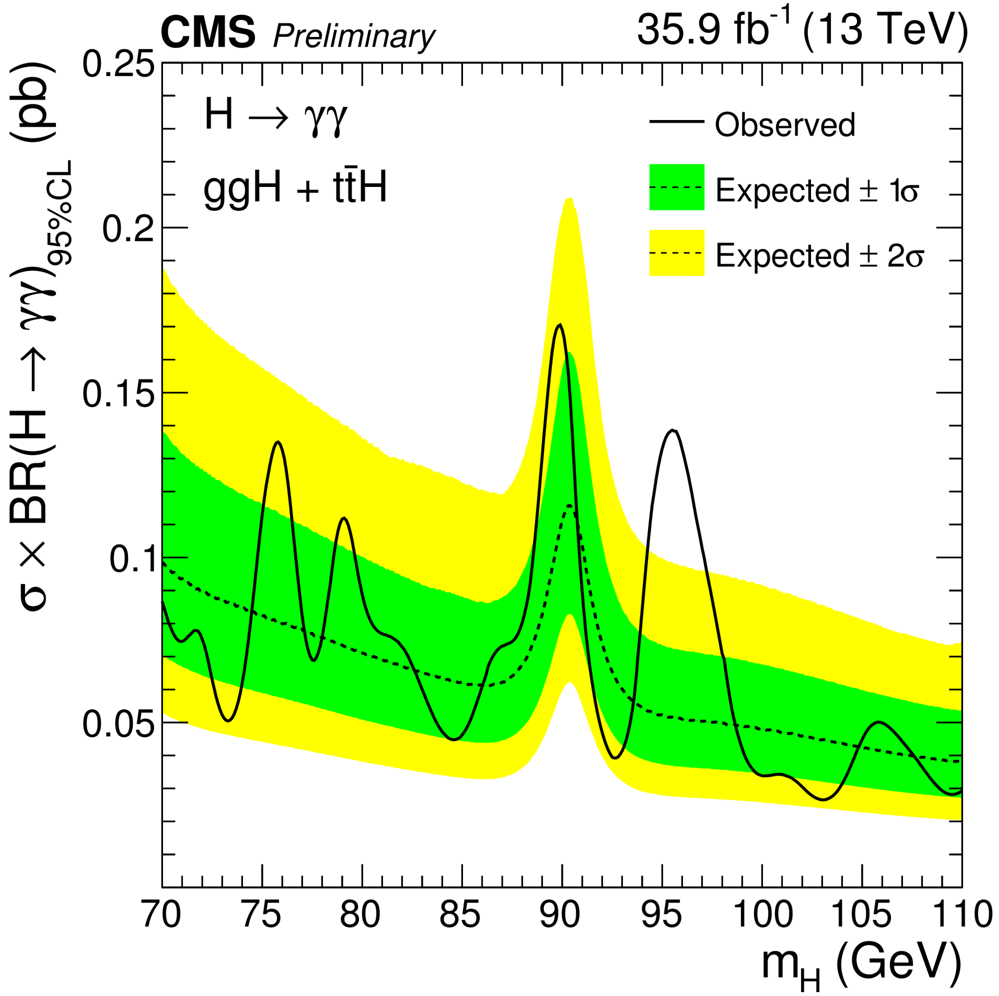

The properties of the Higgs boson are consistent with SM predictions (given the statistical power of the Higgs data).

The Higgs data taken at Run-2 of the LHC have confirmed the Run-1 observations (with potential deviations from SM-like behavior further reduced).

# Hints of additional scalars beyond the SM Higgs boson?




Upper limits at the 95% CL on the product of the production cross-section for pp → A and the branching ratios for A → Z h and h → bb evaluated by combining the 0-lepton and 2-lepton channels. Taken from ATLAS-CONF-2017-055.




Expected and observed exclusion limits (95% CL) on the production cross section times branching ratio into two photons for a second Higgs. Taken from CMS-PAS-HIG-17-013.

# A tale of two alignment mechanisms

## 1. Higgs field alignment

In the limit in which one of the Higgs mass eigenstate fields is approximately aligned with the direction of the scalar doublet vacuum expectation value (vev) in field space, the tree-level properties of corresponding scalar mass eigenstate approximate those of the SM Higgs boson.

## 2. Flavor alignment

The quark mass matrices arise from the Higgs-fermion Yukawa couplings when the neutral Higgs fields acquire vevs. If flavor alignment is realized, then the diagonalization of the quark mass matrices simultaneously diagonalize the neutral Higgs quark interactions, which implies the absence of tree-level Higgs-mediated FCNCs in hadron physics.

# The Higgs field alignment limit: approaching the SM Higgs boson

Consider an extended Higgs sector with $n$ hypercharge-one Higgs doublets $\Phi_i$ and $m$ additional singlet Higgs fields $\phi_i$ (which naturally yields a tree-level $\rho$-parameter equal to one).

After minimizing the scalar potential, we assume that only the neutral Higgs fields acquire vacuum expectation values (in order to preserve $\mathrm{U}(1)_{\rm EM}$),

$$\langle\Phi_i^0\rangle = v_i/\sqrt{2}\,, \qquad\qquad \langle\phi_j^0\rangle = x_j\,.$$

Note that $v^2 \equiv \sum_i |v_i|^2 = 4m_W^2/g^2 = (246~\mathrm{GeV})^2$.

## The Higgs basis

Define new linear combinations of the hypercharge-one doublet Higgs fields (the so-called *Higgs basis*). In particular,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} = \frac{1}{v}\sum_i v_i^* \Phi_i \,, \qquad\qquad \langle H_1^0 \rangle = v/\sqrt{2}\,,$$

and $H_2, H_3, \ldots, H_n$ are the other linear combinations of doublet scalar fields such that $\langle H_i^0 \rangle = 0$ (for $i = 2, 3, \ldots, n$).

That is $H_1^0$ is aligned in field space with the direction of the Higgs vacuum expectation value (vev). Thus, if $\sqrt{2}\,\mathrm{Re}(H_1^0) - v$ is a mass-eigenstate, then the tree-level couplings of this scalar to itself, to gauge bosons and to fermions are precisely those of the SM Higgs boson, $h^0$. This is the exact alignment limit.

## A SM-like Higgs boson

In general, $\sqrt{2}\,\mathrm{Re}(H_1^0)-v$ is not a mass-eigenstate due to mixing with other neutral scalars. Nevertheless, a SM-like Higgs boson exists if either:

- the diagonal squared masses of the other Higgs basis scalar fields are all large compared to the mass of the observed Higgs boson (the so-called *decoupling limit*).

and/or

- the elements of the neutral scalar squared-mass matrix that govern the mixing of $\sqrt{2}\,\mathrm{Re}(H_1^0)-v$ with other neutral scalars are suppressed.

## Higgs field alignment with or without decoupling

### 1. The decoupling limit

Approximate Higgs field alignment is most naturally achieved in the decoupling limit, where there is a new mass parameter, $M \gg v$, such that all physical Higgs masses with one exception are of $\mathcal{O}(M)$. The Higgs boson, with $m_h \sim \mathcal{O}(v)$, is SM-like, due to approximate alignment.

### 2. Higgs field alignment without decoupling[1]

In models of alignment without decoupling (due to suppressed scalar mixing), the masses of all Higgs scalars (both SM-like and non-SM-like) can be of $\mathcal{O}(v)$. Hence, the non-SM Higgs scalars may be more easily accessible at the LHC. In some theories, this can be achieved by a symmetry (e.g., the inert doublet model). In most cases, approximate alignment is an accidental (fine-tuned?) region of the model parameter space.

[1] J.F. Gunion and H.E. Haber, hep-ph/0207010; N. Craig, J. Galloway and S. Thomas, arXiv:1305.2424.

## Example: the alignment limit of the CP-conserving 2HDM

The most relevant terms of the scalar potential, written in terms of Higgs basis scalar fields, are:

$$\mathcal{V} \ni \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 + \left\{\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + Z_6(H_1^\dagger H_1)(H_1^\dagger H_2) + \text{h.c.}\right\} .$$

For simplicity, the scalar potential is taken to be CP-conserving, in which case it is possible to rephase the Higgs basis field $H_2$ such that $Z_5$ and $Z_6$ are real.

We identify the CP-odd Higgs boson as $A = \sqrt{2}\,\text{Im}\,H_2^0$ with mass $m_A$. The CP-even Higgs squared-masses are obtained by diagonalizing the corresponding $2\times 2$ squared-mass matrix, $\mathcal{M}_H^2$.

With respect to Higgs basis states, $\{\sqrt{2}\,\mathrm{Re}\,H_1^0 - v\,,\,\sqrt{2}\,\mathrm{Re}\,H_2^0\}$,

$$\mathcal{M}_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix}.$$

The CP-even Higgs bosons are $h$ and $H$ with $m_h \leq m_H$. The couplings of $\sqrt{2}\,\mathrm{Re}\,H_1^0 - v$ coincide with those of the SM Higgs boson. Alignment arises two limiting cases:

1. $m_A^2 \gg (Z_1 - Z_5)v^2$. This is the *decoupling limit*, where $h$ is SM-like and $m_A^2 \sim m_H^2 \sim m_{H^\pm}^2 \gg m_h^2 \simeq Z_1 v^2$.

2. $|Z_6| \ll 1$. Then, $h$ is SM-like if $m_A^2 + (Z_5 - Z_1)v^2 > 0$. Otherwise, $H$ is SM-like.

In particular, the CP-even mass eigenstates are:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2}\ \mathrm{Re}\ H_1^0 - v \\ \sqrt{2}\,\mathrm{Re}\ H_2^0 \end{pmatrix},$$

where $c_{\beta-\alpha} \equiv \cos(\beta-\alpha)$ and $s_{\beta-\alpha} \equiv \sin(\beta-\alpha)$ are defined in terms of the mixing angle $\alpha$ that diagonalizes the CP-even Higgs squared-mass matrix when expressed in the $\Phi_1$–$\Phi_2$ basis of scalar fields, $\{\sqrt{2}\,\mathrm{Re}\ \Phi_1^0 - v_1\,,\ \sqrt{2}\,\mathrm{Re}\ \Phi_2^0 - v_2\}$, and $\tan\beta \equiv v_2/v_1$.

Remark: The generalization of this analysis to the most general (CP-violating) 2HDM again yields the condition $|Z_6| \ll 1$ in the limit of alignment due to suppressed scalar mixing.

Since the SM-like Higgs boson must be approximately $\sqrt{2}\,\mathrm{Re}\;H_1^0 - v$, it follows that

- $h$ is SM-like if $|c_{\beta-\alpha}| \ll 1$ (alignment with or without decoupling, depending on the magnitude of $m_A$),

- $H$ is SM-like if $|s_{\beta-\alpha}| \ll 1$ (alignment without decoupling).

Remark: Although the tree-level couplings of $\sqrt{2}\,\mathrm{Re}\;H_1^0 - v$ coincide with those of the SM Higgs boson, the one-loop couplings can differ due to the exchange of non-minimal Higgs states (if not too heavy). For example, the $H^\pm$ loop contributes to the decays of the SM-like Higgs boson to $\gamma\gamma$ and $\gamma Z$.

## The alignment limit in equations

The CP-even Higgs squared-mass matrix yields,

$$Z_1 v^2 = m_h^2 s_{\beta-\alpha}^2 + m_H^2 c_{\beta-\alpha}^2 \,,$$

$$Z_6 v^2 = (m_h^2 - m_H^2) s_{\beta-\alpha} c_{\beta-\alpha} \,,$$

$$Z_5 v^2 = m_H^2 s_{\beta-\alpha}^2 + m_h^2 c_{\beta-\alpha}^2 - m_A^2 \,.$$

If $h$ is SM-like, then $m_h^2 \simeq Z_1 v^2$ (i.e., $Z_1 \simeq 0.26$) and

$$|c_{\beta-\alpha}| = \frac{|Z_6| v^2}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}} \simeq \frac{|Z_6| v^2}{m_H^2 - m_h^2} \ll 1 \,,$$

If $H$ is SM-like, then $m_H^2 \simeq Z_1 v^2$ (i.e., $Z_1 \simeq 0.26$) and

$$|s_{\beta-\alpha}| = \frac{|Z_6| v^2}{\sqrt{(m_H^2 - m_h^2)(Z_1 v^2 - m_h^2)}} \simeq \frac{|Z_6| v^2}{m_H^2 - m_h^2} \ll 1 \,.$$

| Higgs interaction | 2HDM coupling | approach to the alignment limit |
|---|---|---|
| $hVV$ | $s_{\beta-\alpha}$ | $1-\frac{1}{2}c^2_{\beta-\alpha}$ |
| $hhh$ | * | $1+2(Z_6/Z_1)c_{\beta-\alpha}$ |
| $hhhh$ | * | $1+3(Z_6/Z_1)c_{\beta-\alpha}$ |
| $h\overline{D}D$ | $s_{\beta-\alpha}\mathbb{1}+c_{\beta-\alpha}\rho^D_R$ | $\mathbb{1}+c_{\beta-\alpha}\rho^D_R$ |
| $h\overline{U}U$ | $s_{\beta-\alpha}\mathbb{1}+c_{\beta-\alpha}\rho^U_R$ | $\mathbb{1}+c_{\beta-\alpha}\rho^U_R$ |

Couplings of the SM-like Higgs boson $h$ in the CP-conserving 2HDM normalized to those of the SM Higgs boson, in the alignment limit. For the fermion couplings, $D$ is a column vector of three down-type fermion fields (either down-type quarks or charged leptons) and $U$ is a column vector of three up-type quark fields. In the third column, the first non-trivial correction to alignment is exhibited. Finally, complete expressions for the entries marked with a * can be found in H.E. Haber and D. O'Neil, arXiv: hep-ph/0602242.

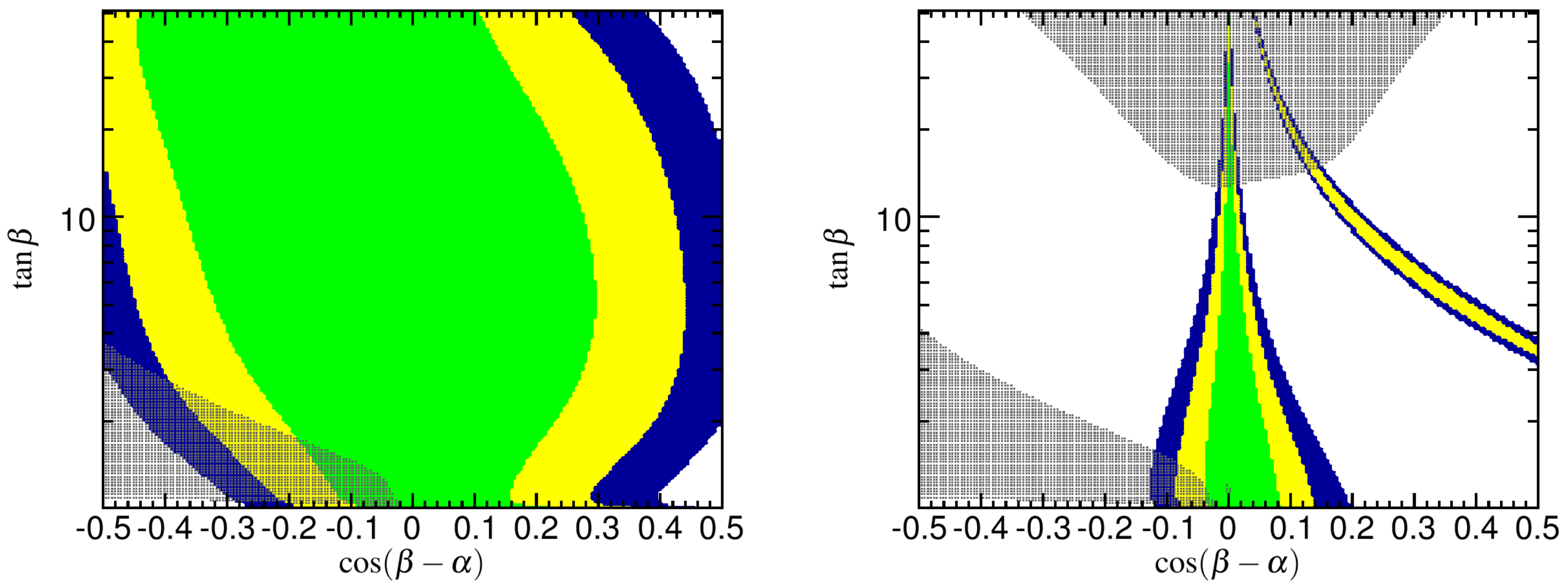
In the case of a flavor aligned Higgs sector, the coupling matrices $\rho^D_R$ and $\rho^U_R$ are separately diagonal. In particular, in the so-called Type-I and Type-II 2HDMs, these matrices are proportional to the identity matrix.

$$\text{Type I}: \qquad \rho^D_R=\rho^U_R=\mathbb{1}\cot\beta\,,$$

$$\text{Type II}: \qquad \rho^D_R=-\mathbb{1}\tan\beta\,, \qquad \rho^U_R=\mathbb{1}\cot\beta\,.$$

Note the possibility of "delayed alignment" in the $h\overline{D}D$ coupling if $\tan\beta\gg 1$.

# Constraints on Type-I and II 2HDMs from LHC Higgs data




Direct constraints from LHC Higgs searches for Type-I (left) and Type-II (right) 2HDM with $m_H = 300$ GeV with $m_h = 125$ GeV, $Z_4 = Z_5 = -2$ and $Z_7 = 0$. Colors indicate compatibility with the observed Higgs signal at $1\,\sigma$ (green), $2\,\sigma$ (yellow) and $3\,\sigma$ (blue). Exclusion bounds at 95% C.L. from the non-observation of the additional Higgs states overlaid in gray. From H.E. Haber and O. Stål, arXiv:1507.04281.

# Alignment as a consequence of a symmetry?

Typically, alignment without decoupling arises due to a (fine-tuned) choice of Higgs sector parameters. In such cases, the exact alignment limit is not a consequence of a symmetry.

In the Higgs basis, the 2HDM scalar potential has the form

$$\mathcal{V} = Y_1 H_1^\dagger H_1 + \big(Y_3 H_1^\dagger H_2 + \text{h.c.}\big) + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 + H_1^\dagger H_1 \big(Z_6 H_1^\dagger H_2 + \text{h.c.}\big) + \cdots$$

Since $\langle H_1 \rangle = v$ and $\langle H_2 \rangle = 0$, the minimum conditions $\partial\mathcal{V}/\partial H_k = 0$ yield $Y_1 = -\frac{1}{2} Z_1 v^2$ and $Y_3 = -\frac{1}{2} Z_6 v^2$.

Exact alignment implies that $Z_6 = 0$, which via the minimum condition yields $Y_3 = 0$.

Exact alignment can therefore be achieved by imposing a discrete $\mathbb{Z}_2$ symmetry in the Higgs basis,

$$H_1 \to +H_1\,, \qquad\qquad H_2 \to -H_2,.$$

Assuming that all SM fermions and gauge bosons are even under the $\mathbb{Z}_2$ symmetry, the end result is the inert doublet model (IDM) in which $h=\sqrt{2}\,\mathrm{Re}(H_1^0)-v$ is identified as the SM Higgs boson.

The lightest $\mathbb{Z}_2$–odd particle (LOP) is stable. Parameter regimes exist where the LOP is a neutral scalar, which can be a viable candidate for dark matter.[2]

---

[2] In A. Goudelis, B. Herrmann and O. Stål, arXiv:1303.3010, a cosmologically relevant LOP is ruled out for all LOP masses below 500 GeV except for a narrow window around $\frac{1}{2}m_h$.

# Natural alignment without decoupling

The scalar potential parameters of the CP-conserving 2HDM in the $\Phi_1$–$\Phi_2$ basis, $m_{ij}^2$ and $\lambda_{1,\ldots,7}$, are related to the corresponding Higgs basis parameters,

$$Y_3 = \tfrac{1}{2}(m_{11}^2 - m_{22}^2)s_{2\beta} + m_{12}^2 c_{2\beta}\,,$$

$$Z_6 = -\tfrac{1}{2}\big[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345}c_{2\beta}\big]s_{2\beta} + \lambda_6 c_\beta c_{3\beta} + \lambda_7 s_\beta s_{3\beta}\,,$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$. If the alignment condition $Z_6 = 0$ holds independently of $\tan\beta$, then it follows that

$$\lambda_1 = \lambda_2 = \lambda_{345}\,, \qquad \lambda_6 = \lambda_7 = 0\,,$$

which is called the *natural alignment condition*.[3]

In order to associate natural alignment with a symmetry, we employ the alignment condition $Y_3 = 0$. If this is satisfied independently of $\tan\beta$, then

$$m_{11}^2 = m_{22}^2\,, \qquad m_{12}^2 = 0\,.$$

Only one squared-mass parameter requires fine-tuning as in the SM.

[3] See P.S. Bhupal Dev and A. Pilaftsis, arXiv:1408.3405.

## Exceptional region of the parameter space (ERPS)

An exceptional region of the 2HDM parameter space consists of:

$$\mathrm{ERPS}: \qquad m_{22}^2 = m_{11}^2\,, \qquad m_{12}^2 = 0\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_7 = -\lambda_6$$

The corresponding conditions in the Higgs basis are,

$$Y_2 = Y_1\,, \qquad Y_3 = Z_6 = Z_7 = 0\,, \qquad Z_1 = Z_2\,.$$

This leads to three possible symmetry choices:

| symmetry | $m_{22}^2$ | $m_{12}^2$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ | $\lambda_7$ |
|---|---|---|---|---|---|---|---|---|
| CP2 | $m_{11}^2$ | 0 | $\lambda_1$ | | | | | $-\lambda_6$ |
| CP3 | $m_{11}^2$ | 0 | $\lambda_1$ | | | $\lambda_1 - \lambda_3 - \lambda_4$ (real) | 0 | 0 |
| SO(3) | $m_{11}^2$ | 0 | $\lambda_1$ | | $\lambda_1 - \lambda_3$ | 0 | 0 | 0 |

The latter two symmetries are consistent with "natural alignment." However, none of the ERPS models can be extended to the Yukawa interactions without generating some phenomenologically untenable feature in the flavor sector.[4]

[4] See, P.M. Ferreira and J.P. Silva, arXiv:1001.0574 [hep-ph].

## Higgs family symmetries

$\mathbb{Z}_2$ : $\Phi_1 \to \Phi_1, \qquad \Phi_2 \to -\Phi_2$

$\Pi_2$ : $\Phi_1 \longleftrightarrow \Phi_2$

U(1)$_{\rm PQ}$ [Peccei-Quinn]: $\Phi_1 \to e^{-i\theta}\Phi_1, \qquad \Phi_2 \to e^{i\theta}\Phi_2$

SO(3): $\Phi_a \to U_{ab}\Phi_b\,, \qquad U \in {\rm U}(2)/{\rm U}(1)_Y$

## Generalized CP (GCP) transformations

$$\text{CP1}: \quad \Phi_1 \to \Phi_1^*, \qquad \Phi_2 \to \Phi_2^*$$

$$\text{CP2}: \quad \Phi_1 \to \Phi_2^*, \qquad \Phi_2 \to -\Phi_1^*$$

$$\text{CP3}: \quad \Phi_1 \to \Phi_1^* c_\theta + \Phi_2^* s_\theta, \qquad \Phi_2 \to -\Phi_1^* s_\theta + \Phi_2^* c_\theta, \qquad \text{for } 0 < \theta < \tfrac{1}{2}\pi$$

where $c_\theta \equiv \cos\theta$ and $s_\theta \equiv \sin\theta$. Some observations of note:

1. $\Pi_2$ symmetry is equivalent to $\mathbb{Z}_2$ symmetry in a different basis.
2. Applying $\mathbb{Z}_2$ and $\Pi_2$ simultaneously $\Longleftrightarrow$ CP2 in a different basis.
3. Applying U(1)$_{\rm PQ}$ and $\Pi_2$ simultaneously $\Longleftrightarrow$ CP3 in a different basis.

## A strategy for achieving approximate alignment, $|Z_6| \ll 1$

- Extend the Yukawa sector by introducing new degrees of freedom, such that the symmetries of the ERPS models are broken at most softly.

- Integrating out the new degrees of freedom can yield approximate alignment without re-introducing a new fine-tuning problem associated with additional scalar squared-mass parameters.

Example: The CP2-symmetric 2HDM with mirror fermions.[5]

- Introduce mirror fermion partners of the top and bottom quark. Then a vectorlike mass term for the mirror fermion softly breaks the CP2 symmetry.

- Integrating out the mirror fermions, a mass splitting $\Delta m^2 \equiv m_{22}^2 - m_{11}^2 \neq 0$ is generated.

- Depending on the parameters of the scalar potential, the end result is either the IDM or a model in which $\cos(\beta - \alpha) \propto \Delta m^2/v^2$.

[5] P. Draper, H.E. Haber and J.T. Ruderman, arXiv:1605.03237 [hep-ph]. An extension of this model in the case of a CP3-symmetric scalar potential is currently under investigation.

# Alignment without decoupling in the MSSM?

The MSSM Higgs sector is a CP-conserving 2HDM. At tree level, the Higgs basis parameters of interest are fixed by SUSY:

$$Z_1 v^2 = m_Z^2 c_{2\beta}^2\,, \qquad Z_5 v^2 = m_Z^2 s_{2\beta}^2\,, \qquad Z_6 v^2 = -m_Z^2 s_{2\beta} c_{2\beta}\,.$$

It follows that,

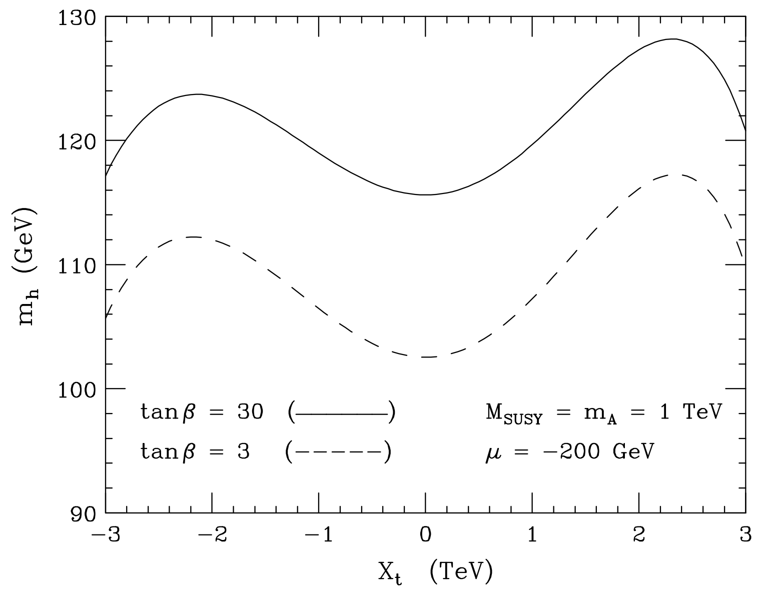
$$\cos^2(\beta-\alpha) = \frac{m_Z^4\, s_{2\beta}^2 c_{2\beta}^2}{(m_H^2 - m_h^2)(m_H^2 - m_Z^2 c_{2\beta}^2)}\,.$$

The decoupling limit is achieved when $m_H \gg m_h$ as expected. Alignment without decoupling is (naively) possible at tree-level when $Z_6 = 0$, which yields $\sin 4\beta \simeq 0$. However, this limit is not phenomenologically viable. In any case, radiative corrections are required to obtain the observed Higgs mass of 125 GeV.

# The radiatively corrected MSSM Higgs Sector

The leading effects due to radiative corrections can be illustrated in the limit where $m_h$, $m_A$, $m_H$, $m_{H^\pm} \ll M_S$, where $M_S$ is the SUSY-breaking scale. In this case, we can formally integrate out the squarks and generate a low-energy effective 2HDM Lagrangian (which is no longer of the tree-level MSSM form).

Large radiative corrections can easily accommodate the observed Higgs mass of 125 GeV (in some regions of the MSSM parameter space).



The dominant one-loop corrected expressions for $Z_1$ and $Z_6$ are given by[6]

$$Z_1 v^2 = m_Z^2 c_{2\beta}^2 + \frac{3v^2 s_\beta^4 h_t^4}{8\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

$$Z_6 v^2 = -s_{2\beta}\left\{m_Z^2 c_{2\beta} - \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t(X_t+Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4}\right]\right\},$$

where $M_S^2 \equiv m_{\tilde{t}_1} m_{\tilde{t}_2}$, $X_t \equiv A_t - \mu\cot\beta$ and $Y_t = A_t + \mu\tan\beta$.

Note that $m_h^2 \simeq Z_1 v^2$ is consistent with $m_h \simeq 125$ GeV for suitable choices for $M_S$ (as a function of $\tan\beta$ and $X_t$). Exact alignment (i.e., $Z_6 = 0$) can now be achieved due to an accidental cancellation between tree-level and loop contributions.

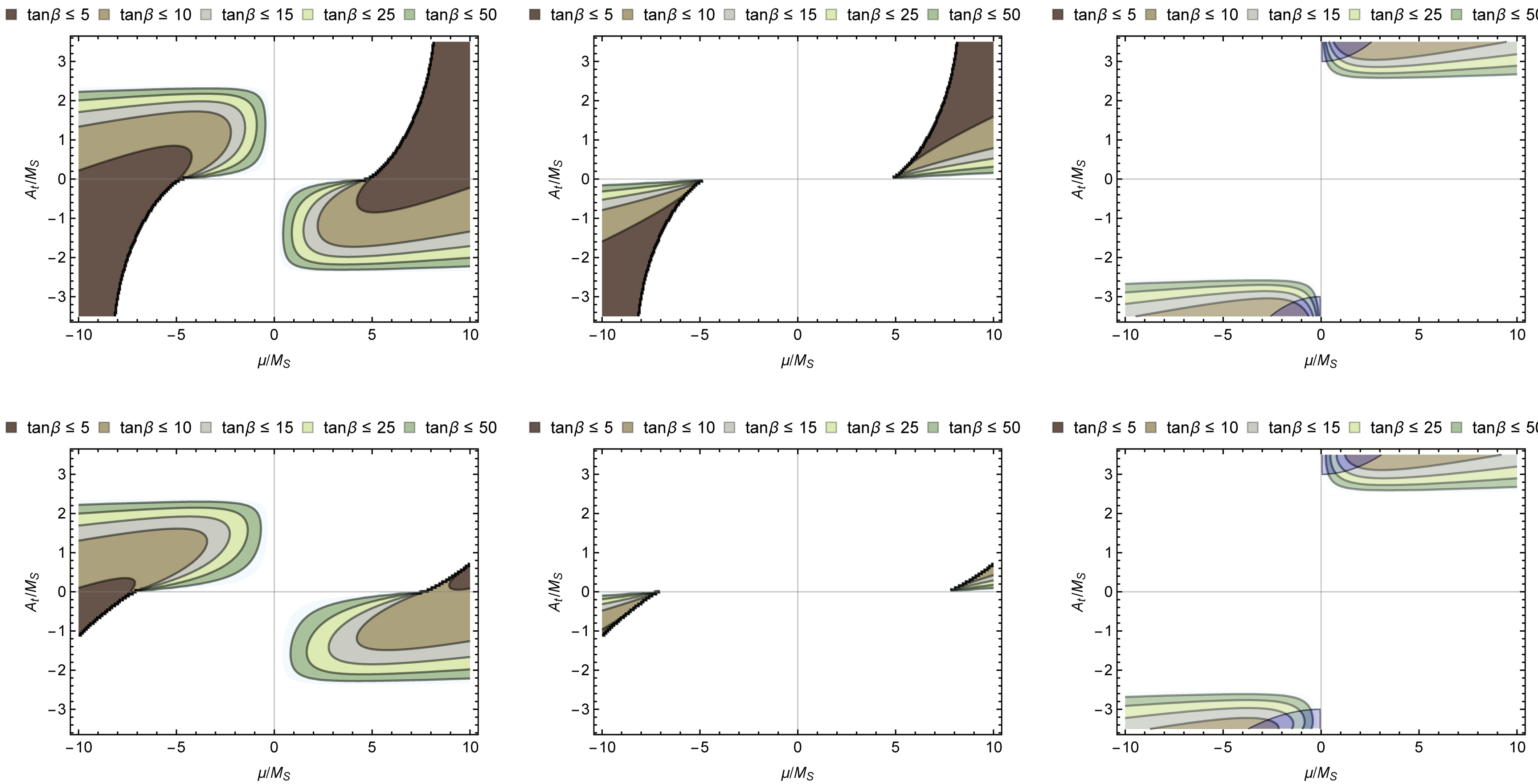
[6] CP-violating phases that could appear in the MSSM parameters such as $\mu$ and $A_t$ are neglected. The expression for $Z_6$ exhibited above first appears in M. Carena, H.E. Haber, I. Low, N.R. Shah and C.E.M. Wagner, arXiv:1410.4969.

Setting $Z_6 = 0$, one obtains a 7th order polynomial equation for $t_\beta \equiv \tan\beta$ as a function of $\widehat{A}_t \equiv A_t/M_S$ and $\widehat{\mu} \equiv \mu/M_S$,

$$m_Z^2 t_\beta^4(1-t_\beta^2) - Z_1 v^2 t_\beta^4(1+t_\beta^2) + \frac{3m_t^4\widehat{\mu}(\widehat{A}_t t_\beta - \widehat{\mu})(1+t_\beta^2)^2}{4\pi^2 v^2}\left[\tfrac{1}{6}(\widehat{A}_t t_\beta - \widehat{\mu})^2 - t_\beta^2\right] = 0\,.$$

which can be solved numerically for real positive solutions.

Typically, we identify $h$ as the SM-like Higgs boson. However, in the alignment limit there exist parameter regimes, corresponding to the case of $m_A^2 + (Z_5 - Z_1)v^2 < 0$ (where the radiatively corrected $Z_1$ and $Z_5$ are employed), in which $H$ is the SM-like Higgs boson. In either case, $Z_1 v^2$ is the (approximate) squared mass of the SM-like Higgs boson.




Contours of $\tan\beta$ corresponding to exact alignment, $Z_6 = 0$, in the $(\mu/M_S, A_t/M_S)$ plane. $Z_1$ is adjusted to give the correct Higgs mass, taken from P. Bechtle, H.E. Haber, S. Heinemeyer, O. Stål,, T. Stefaniak, G. Weiglein and L. Zeune, arXiv:1608.00638 [hep-ph]].

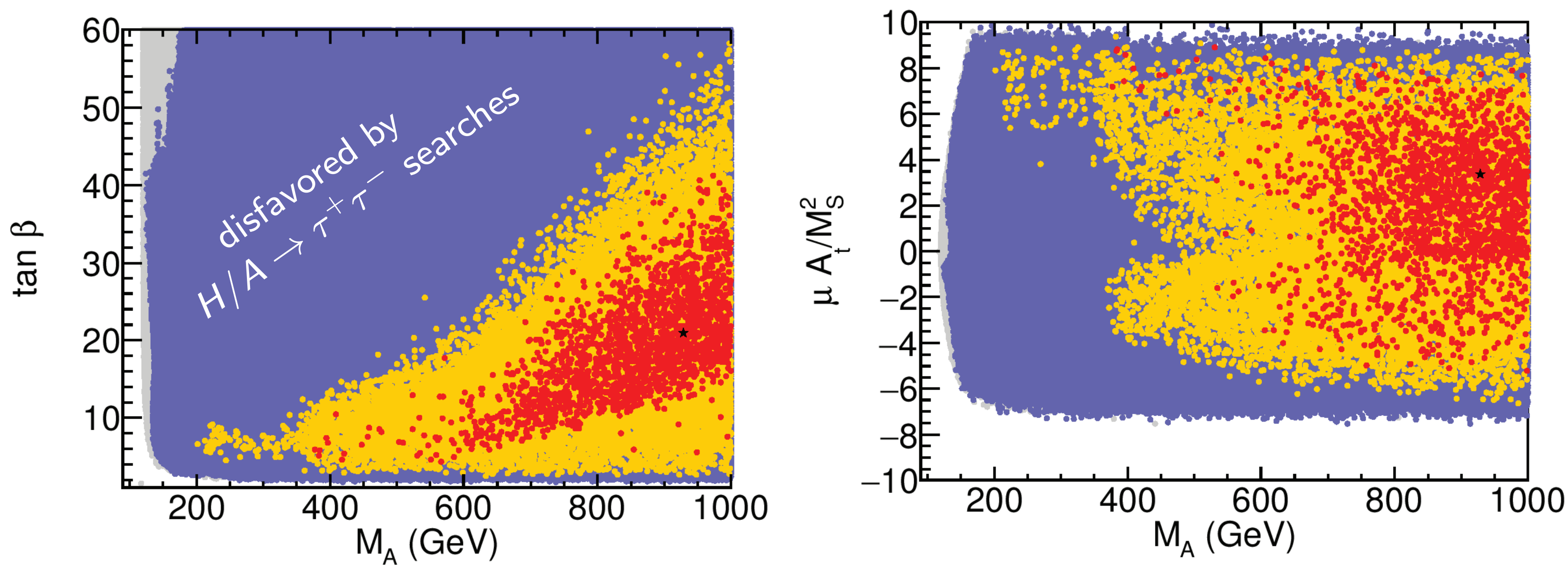
Top panels: Approximate one-loop result.

Bottom panels: Two-loop improved result, which incorporates the leading $\mathcal{O}(\alpha_s h_t^2)$ corrections, using the results of H.E. Haber, S. Heinemeyer and T. Stefaniak, arXiv:1708.04416 [hep-ph]..

Taking the top (bottom) three panels together yields the regions of zero, one, two and three values of $\tan\beta$ in which exact alignment is realized. In the overlaid blue regions we have (unstable) values of $|X_t/M_S| \geq 3$.

# Preferred parameter regions in a pMSSM-8 scan

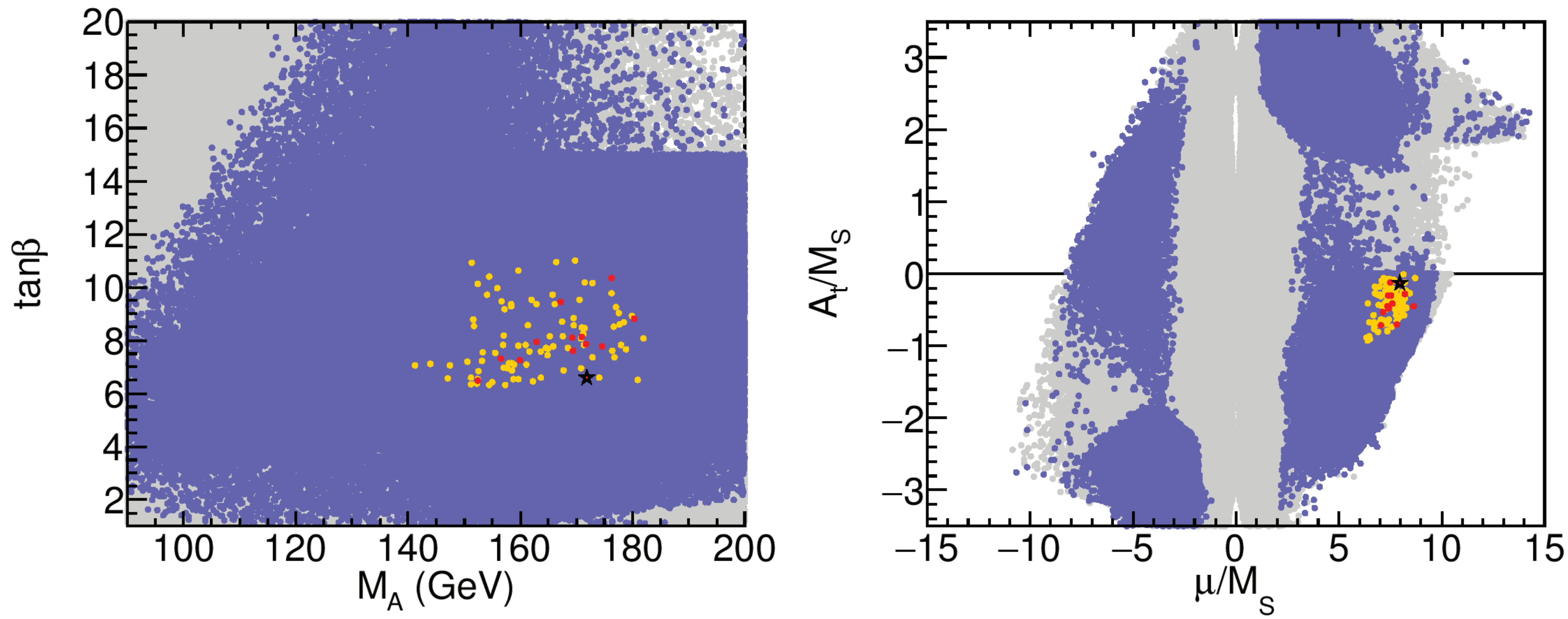
## Case 1: $h$ is SM-like




Points that do not pass the direct constraints from Higgs searches from `HiggsBounds` and from LHC SUSY particle searches from `CheckMATE` are shown in gray. Applying a global likelihood analysis to the points that pass the direct constraints, the color code employed is red for $\Delta\chi^2_h < 2.3$, yellow for $\Delta\chi^2_h < 5.99$ and blue otherwise. The best fit point is indicated by a black star. Taken from P. Bechtle et al., arXiv:1608.00638 [hep-ph]].

Bottom line: $m_A$ values as low as 200 GeV are still allowed in the MSSM.

## Case 2: $H$ is SM-like

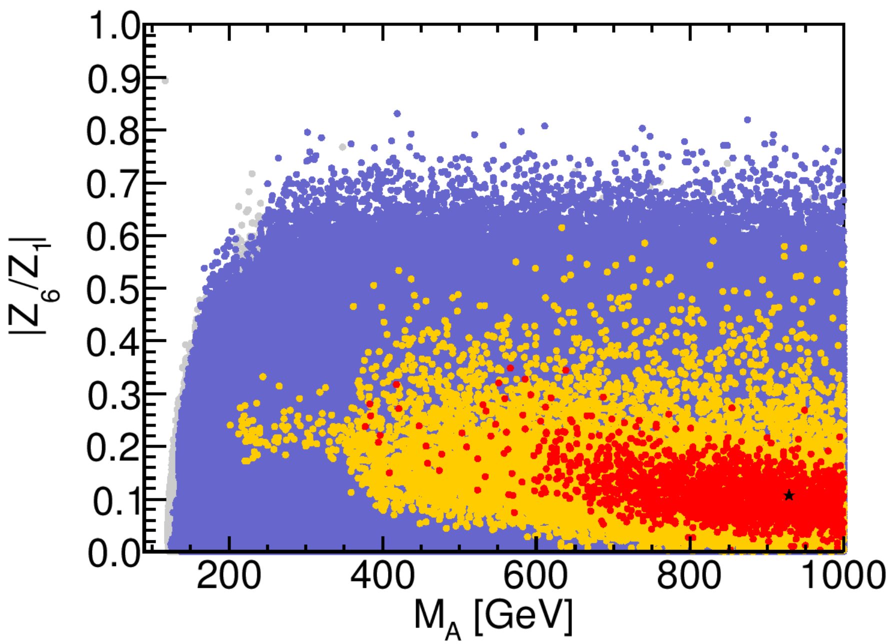



Note: In the preferred region of the pMSSM-8 parameter space with a SM-like $H$, $X_t \sim -1.5M_S$ with $150~\mathrm{GeV} \lesssim m_{H^\pm} \lesssim 200~\mathrm{GeV}$ and $m_h \lesssim 100~\mathrm{GeV}$.

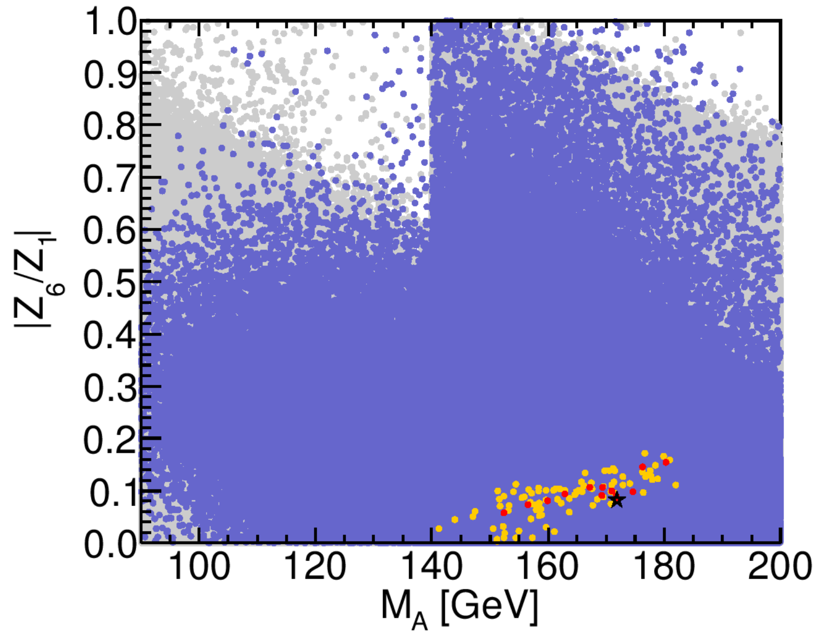
However, this parameter regime requires values of $\mu/M_S \gtrsim 6$ which can lead to the existence of color and charge breaking minima (and a destabilization of the electroweak vacuum).

Bottom line: The possibility that the heavier of two CP-even Higgs bosons of the MSSM is the observed 125 GeV Higgs boson is not yet totally excluded.

# How fine-tuned is the alignment without decoupling region of the MSSM?




SM-like $h$




SM-like $H$

Near the alignment limit, $m_h = 125$ GeV corresponding to $Z_1 \simeq 0.26$. Parameter regions with $Z_6 \sim 0.05$ are compatible with approximate alignment without decoupling (to be compared with $Z_6 = 0$ at exact alignment).

# Flavor alignment in extended Higgs sectors

In the SM (with a single Higgs doublet), the diagonalization of the fermion mass matrices automatically yields diagonal neutral Higgs-fermion couplings. In models with multiple Higgs doublets, these couplings are generically non-diagonal in the fermion mass basis.

The Glashow-Weinberg and Paschos (GWP) condition for natural flavor conservation (1977) imposes a symmetry so that all right-handed fermions with a given electric charge $q$ couple to exactly one Higgs doublet.

E.g., for the Higgs-quark interactions of the 2HDM:

Type-I: All right-handed quarks couple to the same Higgs doublet.
Type-II: Right-handed quarks with $q = 2/3$ and $q = -1/3$ couple to different Higgs doublets.

## Higgs flavor alignment more generally

- One can impose by fiat that the diagonalization of the fermion mass matrix simultaneously diagonalizes the neutral Higgs-fermion Yukawa couplings. In absence of a symmetry, this is unstable with respect to RG-running.

- One can assert that flavor alignment is imposed at a very high energy scale (by new dynamics not specified). In this case, RG-running yields small violations of Higgs flavor alignment at the electroweak scale that can be consistent with present data.

Example: the Yukawa couplings of the flavor aligned 2HDM are governed by three alignment parameters $a_U$, $a_D$ and $a_E = \tan\beta$, which relate the two independent Yukawa coupling matrices of the up-type and down-type quarks and charged leptons, respectively.

# High-scale flavor alignment in the 2HDM

The Yukawa Lagrangian in the Higgs basis of the 2HDM is

$$-\mathscr{L}_{\rm Y} = \overline{U}_L \left(\kappa^U H_1^{0\,\dagger} + \rho^U H_2^{0\,\dagger}\right) U_R - \overline{D}_L K^\dagger \left(\kappa^U H_1^- + \rho^U H_2^-\right) U_R$$
$$+\overline{U}_L K \left(\kappa^D H_1^+ + \rho^{D\,\dagger} H_2^+\right) D_R + \overline{D}_L \left(\kappa^D H_1^0 + \rho^{D\,\dagger} H_2^0\right) D_R\,,$$

where $\kappa^Q \equiv \sqrt{2} M_Q/v$ (for $Q = U,\ D$) and $M_U$, $M_D$ are the diagonal quark mass matrices. In the most general 2HDM, $\rho^U$ and $\rho^D$ are arbitrary complex $3\times 3$ matrices, which yield neutral Higgs-mediated CP-violating and flavor-changing interactions. The Higgs couplings to leptons is similarly treated.

In the flavor-aligned 2HDM, $\rho^F = a^F \kappa^F$ for $F = U, D, E$, where $a^F$ is called the alignment parameter.[7]

[7] A. Pich and P. Tuzon, arXiv:0908.1554 [hep-ph].

We impose $\rho^F = a^F \kappa^F$ at the Planck scale $M_{\rm P}$, and then generate flavor non-diagonal Higgs-fermion couplings via RG-running. We also work in the decoupling limit where the mass scale of the non-SM-like Higgs bosons is $\Lambda_H \gg m_h$. We then compare our numerical results to a one-loop leading log approximation,[8]

$$\rho^U(\Lambda_H)_{ij} \simeq a^U \delta_{ij} \frac{\sqrt{2}(M_U)_{jj}}{v} + \frac{(M_U)_{jj}}{4\sqrt{2}\pi^2 v^3} \log\left(\frac{\Lambda_H}{\Lambda_{\rm P}}\right) \Big\{ (a^E - a^U)\left[1 + a^U (a^E)^*\right] \delta_{ij} \sum_k (M_E^2)_{kk} \\ + (a^D - a^U)\left[1 + a^U (a^D)^*\right] \sum_k \left[3\delta_{ij}(M_D^2)_{kk} - 2(M_D^2)_{kk} K_{ik} K^*_{jk}\right] \Big\},$$

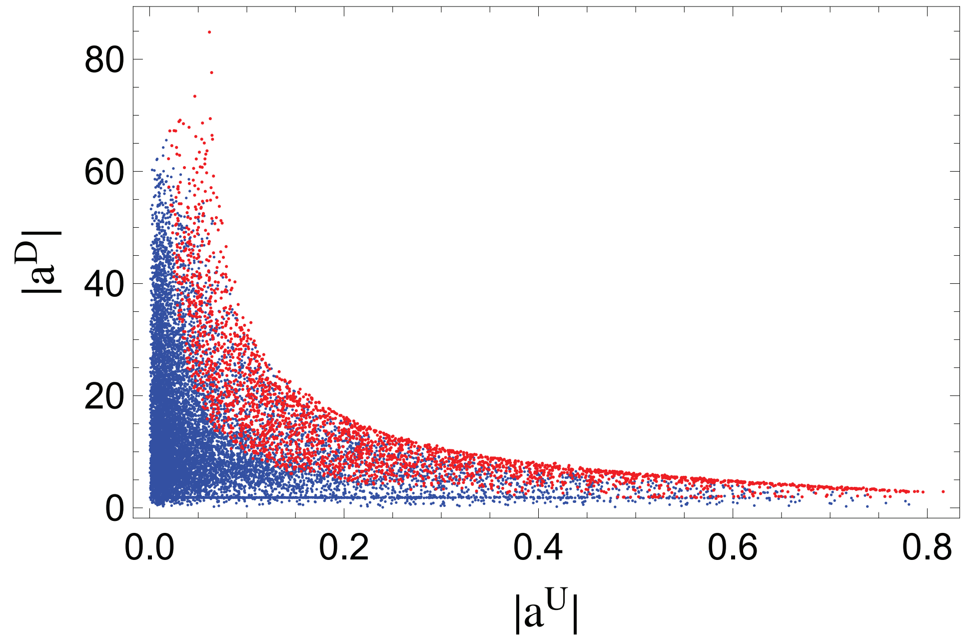
$$\rho^D(\Lambda_H)_{ij} \simeq a^D \delta_{ij} \frac{\sqrt{2}(M_D)_{ii}}{v} + \frac{(M_D)_{ii}}{4\sqrt{2}\pi^2 v^3} \log\left(\frac{\Lambda_H}{\Lambda_{\rm P}}\right) \Big\{ (a^E - a^D)\left[1 + a^D (a^E)^*\right] \delta_{ij} \sum_k (M_E^2)_{kk} \\ + (a^U - a^D)\left[1 + a^D (a^U)^*\right] \sum_k \left[3\delta_{ij}(M_U^2)_{kk} - 2(M_U^2)_{kk} K^*_{ki} K_{kj}\right] \Big\}.$$

where $K$ is the CKM mixing matrix.

---

[8]S. Gori, H.E. Haber and E. Santos, arXiv:1703.05873 [hep-ph]. See also, C.B. Braeuninger, A. Ibarra and C. Simonetto, arXiv:1005.5706 [hep-ph]].

The validity of the one-loop leading log approximation breaks down for large values of the alignment parameters.




Blue: region of the A2HDM parameter space where the prediction for all the off-diagonal terms of the $\rho^Q$ matrices lies within a factor of 3 from the results obtained with the full running. Red: region where the one-loop leading log approximation differs significantly from the the results obtained by numerically solving the RGEs.

Remark: In our numerical analysis, we require that no Landau poles in the Yukawa couplings $\kappa^Q$ and $\rho^Q$ appear below $\Lambda = M_{\mathrm{P}}$. This constraint is reflected in the upper boundary of the red curve shown above.

## The significance of the parameter $\tan\beta$ in the A2HDM

Since $\tan\beta$ is a basis-dependent quantity, it has no significance in the A2HDM. In the CP-conserving case, only $\beta-\alpha$ (which is basis independent) has significance. Indeed, $\tan\beta$ does not appear in the Yukawa couplings of the A2HDM.

In our analysis, we have neglected neutrino masses, so that alignment in the leptonic sector is preserved by RG running. Thus, it is convenient to define $\tan\beta$ via

$$a^E \equiv \tan\beta\,,$$

which is a real number of either sign. The significance of $\tan\beta$ is that in the $\Phi_1$–$\Phi_2$ basis, the couplings of the right handed charged leptons to $\Phi_2$ vanish, although this is not enforced by a discrete symmetry. The Yukawa couplings to leptons then resemble those of a Type II or Type X 2HDM.

# Phenomenological consequences

1. Flavor-changing top decays are too small to be seen at the LHC or at future colliders under consideration.

2. Higgs mediated contributions to neutral meson mixing ($B_{d,s}$–$\overline{B}_{d,s}$, $K$–$\overline{K}$ and $D$–$\overline{D}$ mixing) arise in our model.

3. The most stringent constraints of the alignment parameters are due to $B_{s,d} \to \mu^+\mu^-$. At present the SM predicted rates,

$$\mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{SM}} = (3.65 \pm 0.23) \times 10^{-9},$$

$$\mathrm{BR}(B_d \to \mu^+\mu^-)_{\mathrm{SM}} = (1.06 \pm 0.09) \times 10^{-10}.$$

are in good agreement with the combination of the LHCb and the CMS measurements at Run I for the $B_s$ decay,

$$\mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{exp}} = (2.8^{+0.7}_{-0.6}) \times 10^{-9},$$

$$\mathrm{BR}(B_d \to \mu^+\mu^-)_{\mathrm{exp}} = (3.9^{+1.6}_{-1.4}) \times 10^{-10}\,.$$

4. Interesting constraints also arise in $B \to \tau\nu$. The present data yields

$$\mathrm{BR}(B \to \tau\nu)_{\mathrm{exp}} = (1.06 \pm 0.19) \times 10^{-4},$$

which is in a relatively good agreement with the SM prediction,

$$\mathrm{BR}(B \to \tau\nu)_{\mathrm{SM}} = (0.848^{+0.036}_{-0.055}) \times 10^{-4}.$$

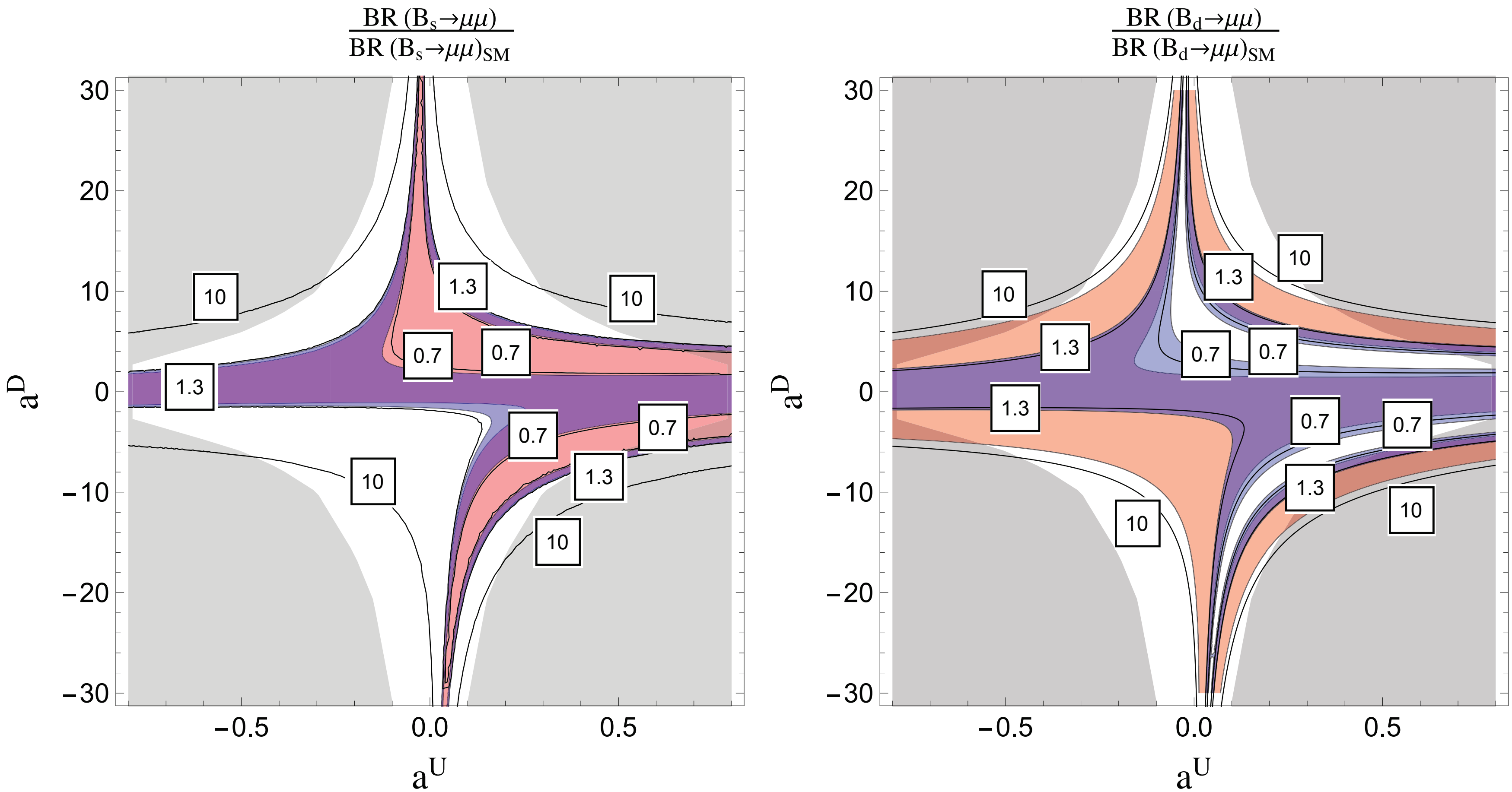
The branching ratio in the 2HDM relative to that of the SM is given by

$$\frac{\mathrm{BR}(B \to \tau\nu)}{\mathrm{BR}(B \to \tau\nu)_{\mathrm{SM}}} = \left| 1 - \frac{m_B^2}{m_b} \frac{v \tan\beta}{\sqrt{2} K_{ub} m_{H^\pm}^2} \sum_i \left[ K_{ui} \rho_{3i}^{D*} + K_{ib}^* \rho_{i1}^{U*} \right] \right|^2 .$$

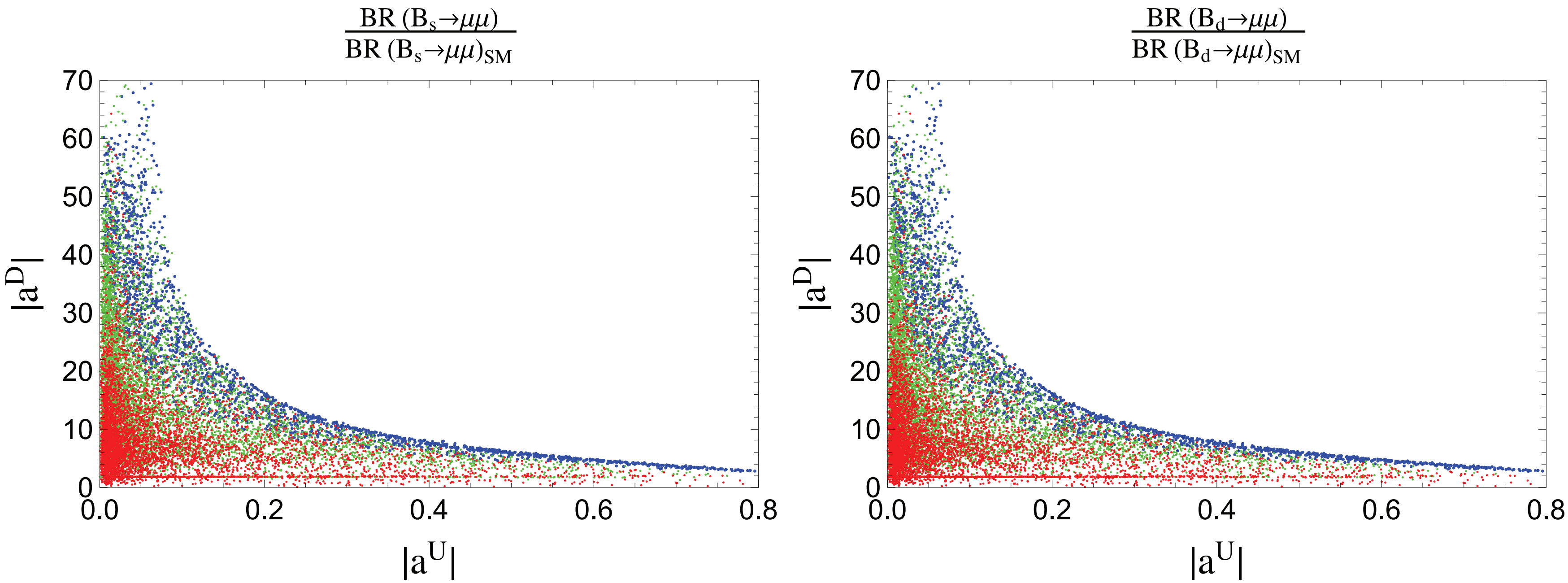
5. If a heavy CP-even Higgs boson $H$ is discovered, then its branching ratios provide critical tests of the A2HDM approach.

- Possible flavor non-diagonal decays, e.g. $H \to b\bar{s}$, $\bar{b}s$
- Non-standard ratios of BRs, e.g.

$$\frac{\mathrm{BR}(H \to \bar{b}b)}{\mathrm{BR}(H \to \tau^+\tau^-)} \neq \frac{3m_b^2}{m_\tau^2} .$$



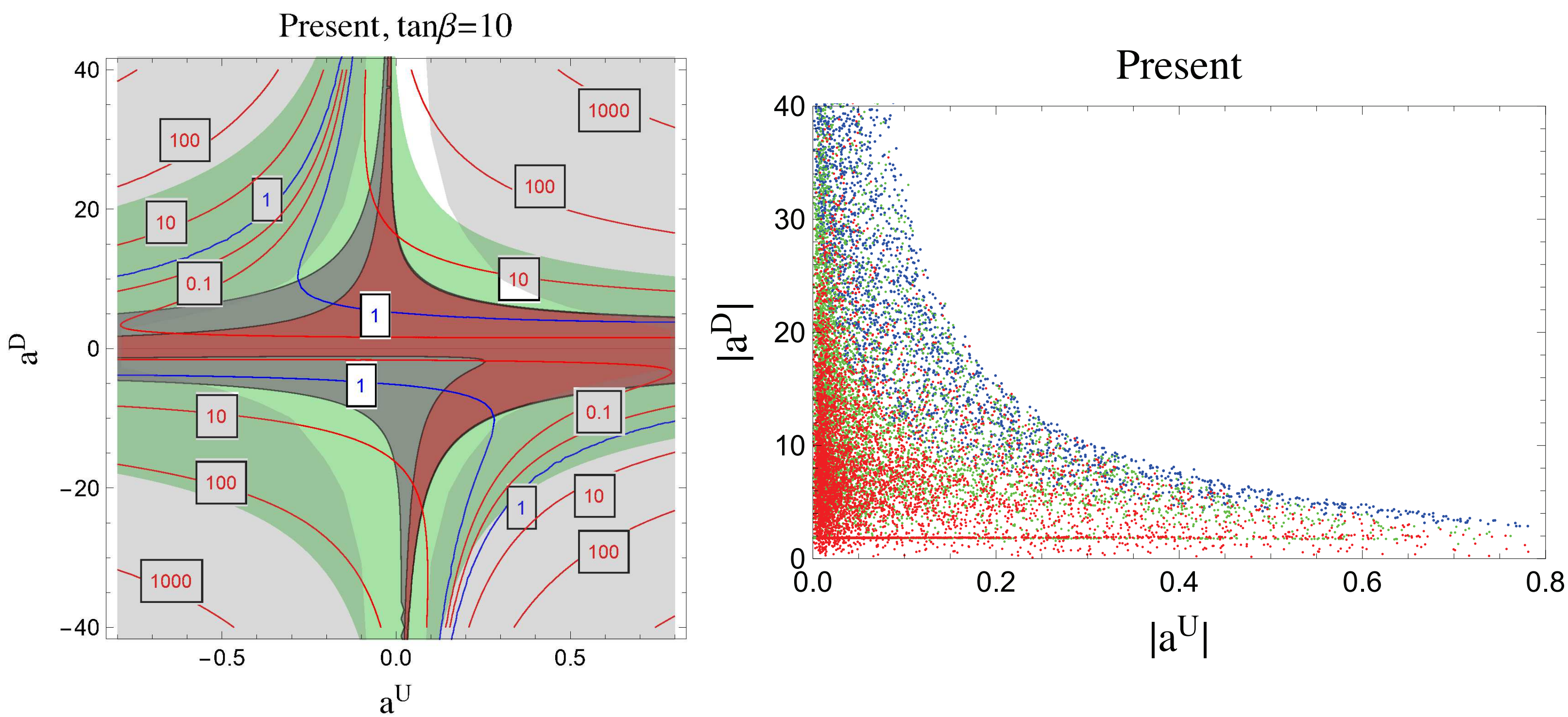

Leading log prediction for the branching ratios for $B_s \to \mu^+\mu^-$ (left panel) and $B_d \to \mu^+\mu^-$ (right panel) relative the the SM, as a function of $a^U$ and $a^D$, with fixed $\tan\beta = 10$, $\cos(\beta - \alpha) = 0$, and $m_A = m_H = 400$ GeV. The regions in pink are allowed at the $2\sigma$ level by the present measurements. The purple shaded regions are anticipated by the more precise HL-LHC measurements, assuming a measured central value equal to the SM prediction. The gray shaded regions produce Landau poles in the Yukawa couplings below $M_{\mathrm{P}}$.




The branching ratio for $B_s \to \mu^+\mu^-$ (left panel) and for $B_d \to \mu^+\mu^-$ (right panel) relative to the SM, obtained via scanning the parameter space and using the full RG running, with fixed $\tan\beta = 10$, $\cos(\beta-\alpha) = 0$, and $m_A = m_H = 400$ GeV. The yellow, red, green and blue points corresponds to branching ratios normalized to the SM prediction $< \mathbf{0.4}, [\mathbf{0.4}, \mathbf{1.1}], [\mathbf{1.1}, 10], > 10$. In boldface we denote the range preferred by the LHCb and ATLAS measurements of $B_s \to \mu^+\mu^-$.

The red points shown in the left plot above correspond roughly to the regions allowed by the experimental measurements at the $2\sigma$ level.

# Summary Plots




Summary of the present day constraints and predictions for the heavy Higgs phenomenology, with $\cos(\beta - \alpha) = 0$, $\tan\beta = 10$ and $m_A = m_H = m_{H^\pm} = 400$ GeV. Left panel: Predictions of the leading log approximation. The contours represent the ratio $\mathrm{BR}(H \to b\bar{b}) m_\tau^2 / [\mathrm{BR}(H \to \tau^+\tau^-) 3 m_b^2]$. The reddish-brown regions are favored by all flavor constraints. Green, blue-gray and tan regions are favored by the measurement of $B \to \tau\nu$, $B_s$ mixing and $B_s \to \mu^+\mu^-$, respectively. The gray shaded regions produce Landau poles in the Yukawa couplings below $M_\mathrm{P}$. Right panel: Result of the parameter scan using full RG running. Blue points correspond to points allowed by the measurement of $B \to \tau\nu$, but not by the measurement of $B_s$ mixing or $B_s \to \mu^+\mu^-$. Green points are allowed by the measurements of $B \to \tau\nu$ and of meson mixing but not by $B_s \to \mu^+\mu^-$. Red points are allowed by all constraints. In the solid white region, Landau poles in the Yukawa couplings are produced below $M_\mathrm{P}$.

# Conclusions

- Since the structure of the gauge bosons and fermions of the SM is non-minimal, one should entertain the possibility of a non-minimal Higgs sector.

- Phenomenological considerations already significantly constrain an extended Higgs sector.

  - The Higgs data strongly suggests that the observed Higgs boson is SM-like. Thus, any non-minimal Higgs sector must contain a SM-like Higgs boson.

  - Higgs-mediated FCNCs are strongly suppressed.

- In the Higgs field alignment limit, the mass eigenstate corresponding to the observed Higgs boson is aligned with direction (in field space) of the scalar doublet vacuum expectation value. Departures from the alignment limit encode critical information that will provide important clues for the structure of the non-minimal Higgs sector.

- If an extended Higgs sector is revealed, it will be important to determine the flavor alignment mechanism that suppresses Higgs-mediated FCNCs.

# Backup slides

# The alignment limit in the general 2HDM

In the general 2HDM, the scalar potential is generically CP-violating. In this case, the neutral Higgs mass-eigenstates are linear combinations of $\{\sqrt{2}\,\mathrm{Re}\,H_1^0 - v\,,\ \mathrm{Re}\,H_2^0\,,\ \mathrm{Im}\,H_2^0\}$, which are determined by diagonalizing the $3\times 3$ real symmetric squared-mass matrix

$$\mathcal{M}^2 = v^2 \begin{pmatrix} Z_1 & \mathrm{Re}(Z_6) & -\mathrm{Im}(Z_6) \\ \mathrm{Re}(Z_6) & \frac{1}{2}Z_{345} + Y_2/v^2 & -\frac{1}{2}\mathrm{Im}(Z_5) \\ -\mathrm{Im}(Z_6) & -\frac{1}{2}\mathrm{Im}(Z_5) & \frac{1}{2}Z_{345} - \mathrm{Re}(Z_5) + Y_2/v^2 \end{pmatrix},$$

where $Z_{345} \equiv Z_3 + Z_4 + \mathrm{Re}(Z_5)$. The diagonalizing matrix is a $3\times 3$ real orthogonal matrix that depends on three angles: $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$, such that $\theta_{12}$ and $\theta_{13}$ are invariant whereas $\theta_{23} \to \theta_{23} - \chi$ under the rephasing of $H_2$.[9]

The alignment limit again corresponds to two cases:

1. $Y_2 \gg v^2$, corresponding to the decoupling limit.
2. $|Z_6| \ll 1$, corresponding to alignment with or without decoupling.

[9] See H.E. Haber and D. O'Neil, arXiv: hep-ph/0602242.

## The alignment limit of the general 2HDM in equations

To obtain the conditions in which $h_1$ is the SM-like Higgs boson, noting that:

$$\frac{g_{h_1VV}}{g_{h_{\rm SM}VV}} = c_{12}c_{13}\,, \qquad \text{where } V=W \text{ or } Z\,,$$

where $h_{\rm SM}$ is the SM Higgs boson, we demand that

$$s_{12}\,,\; s_{13} \ll 1\,.$$

Here, $s_{12} \equiv \sin\theta_{12}$, $c_{12} \equiv \cos\theta_{12}$, etc. We denote the masses of the neutral Higgs mass eigenstates by $m_1$, $m_2$ and $m_3$. It follows that:

$$Z_1 v^2 = m_1^2 c_{12}^2 c_{13}^2 + m_2^2 s_{12}^2 c_{13}^2 + m_3^2 s_{13}^2\,,$$

$$\mathrm{Re}(Z_6\, e^{-i\theta_{23}})\, v^2 = c_{13}s_{12}c_{12}(m_2^2 - m_1^2)\,,$$

$$\mathrm{Im}(Z_6\, e^{-i\theta_{23}})\, v^2 = s_{13}c_{13}(c_{12}^2 m_1^2 + s_{12}^2 m_2^2 - m_3^2)\,,$$

$$\mathrm{Re}(Z_5\, e^{-2i\theta_{23}})\, v^2 = m_1^2(s_{12}^2 - c_{12}^2 s_{13}^2) + m_2^2(c_{12}^2 - s_{12}^2 s_{13}^2) - m_3^2 c_{13}^2\,,$$

$$\mathrm{Im}(Z_5\, e^{-2i\theta_{23}})\, v^2 = 2s_{12}c_{12}s_{13}(m_2^2 - m_1^2)\,.$$

Assuming no mass degeneracies in the neutral scalar sector, it then follows that in the alignment limit,

$$\boxed{\begin{aligned} s_{12} &\equiv \sin\theta_{12} \simeq \frac{\mathrm{Re}(Z_6 e^{-i\theta_{23}})v^2}{m_2^2 - m_1^2} \ll 1\,, \\ s_{13} &\equiv \sin\theta_{13} \simeq -\frac{\mathrm{Im}(Z_6 e^{-i\theta_{23}})v^2}{m_3^2 - m_1^2} \ll 1\,, \end{aligned}}$$

One additional small quantity characterizes the alignment limit,

$$\mathrm{Im}(Z_5 e^{-2i\theta_{23}}) \simeq \frac{(m_2^2 - m_1^2)s_{12}s_{13}}{v^2} \simeq -\frac{2\,\mathrm{Im}(Z_6^2 e^{-2i\theta_{23}})v^2}{m_3^2 - m_1^2} \ll 1\,.$$

Finally, the following mass relations in the alignment limit are noteworthy,

$$\begin{aligned} m_1^2 &\simeq Z_1 v^2\,, \\ m_2^2 - m_3^2 &\simeq \mathrm{Re}(Z_5 e^{-2i\theta_{23}})v^2\,. \end{aligned}$$