Supersymmetry, old and new

Jean-Pierre Derendinger

Albert Einstein Center for Fundamental Physics, ITP, University of Bern

“ Frontier Topics in Physics "

20th Anniversary, National Center for Theoretical Sciences, NCTS,
National Tsing Hua University, Hsinchu, August 3, 2017

UNIVERSITÄT BERN

AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

Supersymmetry

Available LHC data do not show a sign of supersymmetry, as expected from the most plausible and elegant supersymmetric extensions of the Standard Model.

More data needed, but already some options in the theorists' community:

- Think of something else (try to. . .)
  (Exhausted MSSM phenomenologists)
- A new (or renewed) interest in ideas which appeared earlier in the literature, reformulated, extended or completed with recent inputs and ideas.
- A problem of principle however: hidden, invisible supersymmetry ?
  Predictions, numbers are needed.

Supersymmetry

1. Has been developed in the 70's without strong phenomenological implications or requirements (in the wake of the success of quantum field theories and symmetries)
2. Considered a plausible proposal (MSSM) to approach the (technical) hierarchy problem and the stability or generation of the weak scale
3. Has been promoted at the turn of the 90's as a experimental challenge for the LHC and also, as a consequence of the slow decrease of $\sin^2\theta_W$ (coupling unification with supersymmetry instead of without in the 70's).

Point 3 in apparent trouble, point 2 exhausted after 25 years of increasingly sophisticated and often marginal studies.

The absence of any sign of supersymmetry at LHC is not a good argument for investing in much higher energies.

But susy remains a tool of primary importance in studies of quantum field theories and is an ingredient of superstring theories.

Supersymmetry

As usual, the crucial issues are: if supersymmetry relevant to Nature, how does it break ? How could we see it ?

- Global, perturbative, spontaneous: massless Goldstino
- Local, spontaneous: requires supergravity, classical, exp. elusive
  Or superstrings, even more exp. elusive
  At the origin of 35 years of susy SM pushed to ultimate sophistication ...
- Dynamical, nonperturbative: mediation, *a model please*
- *Accept susy partners very high in energy, use then effective techniques for low-energy (LHC or next generation) descriptions. Relevant then is the effective description of the Goldstino modes (of susy breaking, of the gravitino/gravitini)*

$\Longrightarrow$ Last point: nonlinear realizations of supersymmetry again on the market
Recently promoted by Komargodski, Seiberg (09), and now many others, an old idea

$\Longrightarrow$ *An interesting link with partial breaking, or several-scale breaking (superstring-suggested)*

Nonlinear realizations

- Nonlinear symmetries play a considerable role in effective lagrangian descriptions (chiral lagrangians, G/H sigma models, ...)
- Nonlinear symmetries in conflict with quantum field theory, however. Generically lead to non-renormalizable models (physical cutoff then)
- Nonlinear supersymmetry already in 1973 ... (before linear susy)
- Spinors only, no superpartner. By construction, or by constraints
- Ambitious attemps at a nonlinear supersymmetric Standard Model in 83–84
  Tremendous complexity
  Mild to cold reception, facing the simplicity and the many new particles of the linear option (MSSM, since 1981)

A way to eliminate the missing susy partners ... Any testable prediction remaining ?

Nonlinear susy, constrained superfields, old

Volkov and Akulov (1973) describe a massless neutrino as a Goldstino:

Volume 46B, number 1 PHYSICS LETTERS 3 September 1973

IS THE NEUTRINO A GOLDSTONE PARTICLE?

D.V. VOLKOV and V.P. AKULOV
Physico-Technical Institute, Academy of Sciences of the Ukrainian SSR, Kharkov 108, USSR

Received 5 March 1973

Using the hypotheses, that the neutrino is a goldstone particle, a phenomenological Lagrangian is constructed, which describes an interaction of the neutrino with itself and with other particles.

From nonlinear variation: $\boldsymbol{\delta\lambda_\alpha = \epsilon_\alpha}$ $\quad$ $\boldsymbol{\delta x^\mu = \frac{ia}{2}(\lambda\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\lambda})}$

introduce: $\boldsymbol{\omega^\mu = dx^\mu - \frac{ia}{2}(\psi\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\lambda})}$

and invariant action: $\boldsymbol{S = a^{-1}\int d^4\omega}$

Nonlinear susy, constrained superfields, old

Then Ivanov, Kapustnikov, . . . and

Martin Roček rederives the VA action from a constrained chiral superfield $\mathbf{\Phi}$

Phys. Rev. Lett. 41 (1978) 451

Linearizing the Volkov-Akulov Model

Martin Roček
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 9 February 1978)

The nonlinear realization of supersymmetry of Volkov and Akulov is related to a constrained linear realization in two and four dimensions.

- Imposes: $\mathbf{\Phi^2 = 0}$ $\quad$ $\mathbf{\Phi\overline{DD\Phi} = -4\Phi}$
- The second condition indeed follows from the field equation of

$$\int d^2\theta d^2\overline{\theta}\,\overline{\Phi}\Phi + \int d^2\theta\,\Phi + \int d^2\overline{\theta}\,\overline{\Phi}$$

The lagrangian compatible with $\mathbf{\Phi^2 = 0}$

Towards matter/gravity couplings with nonlinear susy

In three papers Stuart Samuel and Julius Wess work out a formulation of global and local nonlinear supersymmetry (83–84)

They attempt to construct realistic models, and to find out how to confront them to experiment

Nuclear Physics B221 (1983) 153–177

A SUPERFIELD FORMULATION OF THE NON-LINEAR REALIZATION OF SUPERSYMMETRY AND ITS COUPLING TO SUPERGRAVITY*

Stuart SAMUEL

Department of Physics, Columbia University, New York, New York 10027, USA

Julius WESS

Institut für Theoretische Physik, Universität Karlsruhe, D-7500 Karlsruhe 1, Federal Republic of Germany

Received 23 December 1982

A thorough investigation of the non-linear realization of supersymmetry is carried out both in flat space and in curved space (supergravity). A manageable superfield formulation is developed which allows one to evaluate the physical effects of the non-linear field when it is coupled to other multiplets. We present several interesting applications (mostly in the context of supergravity) useful in model building.

Towards matter/gravity couplings with nonlinear susy

Nuclear Physics B226 (1983) 289–298

REALISTIC MODEL BUILDING WITH THE AKULOV–VOLKOV SUPERFIELD AND SUPERGRAVITY*

Stuart SAMUEL

Department of Physics, Columbia University, New York, New York 10027, USA

Julius WESS

Institut für Theoretische Physik, Universität Karlsruhe, Kaiserstrasse 12, 7500 Karlsruhe, West Germany

Received 22 February 1983
(Revised 25 April 1983)

The flat-space limit of a supergravity theory involving the Akulov–Volkov field as well as other matter multiplets is found to be an ordinary renormalizable globally supersymmetric theory with explicit soft-breaking mass terms. This allows one to easily construct realistic models of nature which are analyzable at the tree and one-loop level. This is carried out for QED-like and Weinberg–Salam-like models.

Towards matter/gravity couplings with nonlinear susy

Nuclear Physics B233 (1984) 488–510

SECRET SUPERSYMMETRY*

Stuart SAMUEL

Department of Physics, Columbia University, New York, New York 10027, USA

Julius WESS

Institut fur Theoretische Physik, Universitat Karlsruhe, Kaiserstrasse 12, 7500 Karlsruhe, Fed. Rep. Germany

Received 26 July 1983

We present new non-linear realizations of the $N=1$ supergravity algebra. They allow us to build interesting realistic models of the four forces of nature. These models are different from all previous ones in that particles do not appear in (broken) supersymmetric multiplets.

These new non-linear realizations also permit us to construct the effective low-energy lagrangian of an arbitrary supergravity theory in which supersymmetry is spontaneously broken. We are thus able to analyze what are the model-independent low-energy effects of supergravity. We find that the number of Higgs fields and the way they couple to quark and lepton matter is a feature which distinguishes supersymmetric theories from non-supersymmetric ones.

Towards matter/gravity couplings with nonlinear susy

In “Secret supersymmetry":

> "Since there are no supersymmetric particle states, are there any indications that the underlying theory came from a supersymmetric one? In simple models like QED, we find none. The situation is slightly different for the Weinberg-Salam model. There are no signs of supersymmetry except in the Higgs sector . . . "

The sign for supersymmetry is actually the need for (at least) two Higgs doublets, as in the standard MSSM case . . .

Hence, a second Higgs doublet could be either a hint of nonlinearly-realized supersymmetry, or simply a multi-Higgs ordinary Standard Model. The absence of further partner states does not remove the ambiguity. Not the most healthy situation, and maybe not a strong enough argument to justify a 88 TeV machine

The analysis of Samuel and Wess deserves a critical examination, in view of its complexity

The $\Phi^2 = 0$ condition

- Simplest case, chiral $\mathcal{N} = 1$ supermultiplet, (off-shell) fields z, ψ_α, f
- Deform the supersymmetry variation

$$\delta z = \sqrt{2}\,\epsilon\psi \qquad \delta\psi_\alpha = -\sqrt{2}M^2\epsilon_\alpha - \sqrt{2}f\epsilon_\alpha - \sqrt{2}i(\sigma^\mu\overline{\epsilon})_\alpha\partial_\mu z$$

$$\delta f = -\sqrt{2}i\,\partial_\mu\,\psi\sigma^\mu\overline{\epsilon}$$

Nonlinear deformation, M is a scale, ψ_α transforms like a goldstino

- The algebra is not deformed:

$$[\delta_1, \delta_2](z, \psi, f) = -2i(\epsilon_2\sigma^\mu\overline{\epsilon}_1 - \epsilon_1\sigma^\mu\overline{\epsilon}_2)\,\partial_\mu(z, \psi, f)$$

- Nothing particular at this point, M^2 equivalent to $\langle f\rangle$, which would spontaneously break supersymmetry
- Would be induced by linear superpotential $W = M\Phi$ in the canonical theory (but susy does not break)
- Deformation more significant in $\mathcal{N} = 2$ theories. *(see later)*

The $\Phi^2 = 0$ condition

- Now: eliminate the scalar z, using a supersymmetric nonlinear constraint
- Deformed superfield: $\Phi = z + \sqrt{2}\,\theta\psi - \theta\theta\,(M^2 + f)$,

$$\Phi^2 = z^2 + \sqrt{2}\theta\psi z - \theta\theta\Big[z(M^2 + f) + \frac{1}{2}\psi\psi\Big)\Big]$$

$\Phi^2 = 0$ solved by $z = -\frac{1}{2}\frac{\psi\psi}{M^2 + f}$ $\Longrightarrow$ $z\psi = z^2 = 0$

Only makes sense if $M^2 + f \neq 0$, hence the deformation parameter M^2

Or: viewed as an expansion of the theory around the point $f = M^2$

- Since $z\psi = z^2 = 0$, solves also condition $\Phi^n = 0, n > 2$

The $\Phi^2 = 0$ condition

- Since $\Phi^n = 0$, $n \geq 2$, the lagrangian reduces to

$$\int d^2\theta d^2\overline{\theta}\ \overline{\Phi}\Phi + \lambda^2 \int d^2\theta\,\Phi + \mathrm{h.c.}$$

  Would be free (susy unbroken) without the constraint

- Dynamical equation: $\overline{DD}\Phi = 4\lambda^2\Phi$ (Roček: a constraint)

- With the constraint, $z = -\frac{1}{2}\frac{\psi\psi}{M^2+f}$ and the field equation for f is nontrivial:

$$f = \overline{\lambda}^2 - \frac{1}{4}\frac{\overline{\psi\psi}}{(M^2+f)^2}\Box\frac{\psi\psi}{M^2+f}$$

  Can be solved in powers of $\psi\psi$, $\overline{\psi\psi}$, $\psi\psi\overline{\psi\psi}$

  See Komargodski, Seiberg (0907.2441)

- The result is the Volkov-Akulov theory with some higher-order corrections

An excursion to $\mathcal{N} = 2$ and partial susy breaking

- Using simple deformations of the supersymmetry representation to generate nonlinear susy, or to formulate the theory “around" a susy-breaking state witlh well-identified Goldstino, leads naturally to partial supersymmetry breaking $\mathcal{N} = 2 \to \mathcal{N} = 1$
- Needed for low-energy descriptions of compactified superstrings (gauge fields on branes, 1/2 susy, DBI lagrangians)
- First field theory example: Maxwell $\mathcal{N} = 2$ (Antoniadis, Partouche, Taylor, 1995) *Strictly speaking not a spontaneous breaking . . .*
- First example in supergravity: Ferrara, Girardello, Porrati (1995)
- Recently: partial breaking in a $\mathcal{N} = 2$ hypermultiplet with a (translational) isometry (Antoniadis, Markou, JPD, 2017)
- The occurence of partial breaking has been a source of confusion and controversy, due to “common knowledge" and a wrong no-go theorem

Partial susy breaking

- Pseudo no-go:
  If susy 1 is broken, $Q_1|0\rangle \neq 0$: $\langle 0|Q_1^\dagger Q_1|0\rangle \sim \langle H\rangle > 0$
  But $\langle H\rangle \sim \langle 0|Q_A^\dagger Q_A|0\rangle > 0$ and all susys are broken
- Broken susy: $\langle V\rangle = \langle T_{00}\rangle > 0 \quad \Rightarrow \quad \langle P_0\rangle = \langle H\rangle = \int d^3x\, \langle T_{00}\rangle$
  and Noether charges are not defined
- And anyway $\langle H\rangle$ does not have physical significance in global susy
- Needs to consider the current algebra:

$$\frac{\partial}{\partial x_\mu} T S^A_{\mu\alpha}(x) \overline{S}^B_{\nu\dot\beta}(y) = 2(\sigma^\rho)_{\alpha\dot\beta}\Big[T_{\rho\nu}\delta_{AB} + \eta_{\rho\nu} C_{AB}\Big]\delta^4(x-y)$$

C_{AB} are finite constant, $\neq 0$ when partial breaking occurs

[Hughes, Polchinski, Nucl. Phys. B278 (1986) 147]

Partial breaking, $\mathcal{N} = 2$ Maxwell

In the case of $\Phi^2 = 0$, there are two deformation parameters (the complex number M^2) which can be absorbed in the auxiliary field f

In $N = 2$ cases, the deformation parameters cannot in general be absorbed in auxiliary-field vev's.
This is the source of partial breaking (which is not, strictly speaking) a spontaneous breaking.

An example is the $N = 2$ Maxwell system:

$\mathcal{N} = 1$ superfields: W_α (gauge field, gaugino, real auxiliary field) and X (2nd gaugino, complex scalar and complex auxiliary field).

This is an off-shell linear representation which can be assembled in a chiral (constrained) $\mathcal{N} = 2$ superfield

Partial breaking, $\mathcal{N} = 2$ Maxwell

$$\mathcal{W} = X + \sqrt{2}i\,\widetilde{\theta}^{\alpha}W_{\alpha} - \widetilde{\theta}\widetilde{\theta}\,\frac{1}{4}\overline{DDX}$$

Both gauginos are in the θ and $\widetilde{\theta}$ components and deformation parameters can be introduced at the two-theta level:

$$\mathcal{W} = \ldots + \sqrt{2}\,\theta^{i}\lambda_{i} + \theta\theta(A^{2} - f) + \widetilde{\theta}\widetilde{\theta}(B^{2} - \overline{f}) + \sqrt{2}i\,\widetilde{\theta}\theta(2\Gamma + D) + \ldots$$

f, D: three (“electric") auxiliary fields (real $SU(2)_R$ triplet)

A^2, B^2, Γ: 6 deformation parameters (complex $SU(2)_R$ triplet)

If $\Gamma = \pm AB$ the deformations arrange into $(A\theta \pm B\widetilde{\theta})^2$, the theory has one Goldstino, partial breaking $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$.

This cannot be achieved by vev’s of auxiliary fields: the condition would be

$$iD = \sqrt{2f\overline{f}}$$

6 deformation parameters: “electric" and “magnetic"

Partial breaking, $\mathcal{N} = 2$ Maxwell

Example, with $B^2 = -iM^2$ deformation: APT model

$$\mathcal{L}_{Max.} = \frac{1}{2}\int d^2\theta \int d^2\widetilde{\theta}\, \mathcal{F}(\mathcal{W})$$

$$= \int d^2\theta \left[\frac{1}{4}\mathcal{F}_{XX}WW - \frac{1}{8}\mathcal{F}_X\overline{DD}\,\overline{X} + \frac{m^2}{2}X - i\frac{M^2}{2}\mathcal{F}_X\right] + \text{h.c.} + \mathcal{L}_{FI}$$

- Has a supersymmetric $\mathcal{N} = 1$ ground state $\langle f\rangle, \langle D\rangle = 0$
- Mass of X controlled by $\langle\mathcal{F}_{XXX}\rangle$. W_α of course massless [exact U(1)]
- The infinite mass limit leads to a nonlinear constraint: $|\langle\mathcal{F}_{XXX}\rangle| \rightarrow \infty$, $\langle \mathrm{Re}\,\mathcal{F}_{XX}\rangle$ kept fixed:

$$WW - \frac{1}{2}X\overline{DD}\,\overline{X} = M^2 X$$

Eliminates X, $\mathcal{N} = 2$ theory with an abelian gauge field and one spinor Goldstino/gaugino

[Bagger, Galperin (1996), Roček, Tseytlin (1998)]

Partial breaking, $\mathcal{N} = 2$ Maxwell

Solved by Bagger and Galperin:

$$X = -\frac{W^2}{2B^2}\left[1 - \overline{D}^2\left(\frac{\overline{W}^2}{4B^4 + a + 4B^4\sqrt{1 + \frac{a}{2B^4} + \frac{b^2}{16B^8}}}\right)\right]$$

$$a = \frac{1}{2}(D^2W^2 + \overline{D}^2\overline{W}^2) \qquad\qquad b = \frac{1}{2}(D^2W^2 - \overline{D}^2\overline{W}^2)$$

Bosonic part of lagrangian $m^2\int d^2\theta\, X + \text{h.c.}$

$$\begin{aligned}\mathcal{L}|_{bos} &= 8m^2B^2\left(1 - \sqrt{1 + \tfrac{1}{B^4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4B^8}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2}\right) \\ &= 8m^2B^2\left(1 - \sqrt{-\det\left(\eta_{\mu\nu} - \tfrac{\sqrt{2}}{B^2}F_{\mu\nu}\right)}\right) \qquad \textit{Born-Infeld}\end{aligned}$$

Hence, relevant to compactified branes. Coupling to the dilaton hypermultiplet: Ambrosetti, Antoniadis, Tziveloglou, JPD (2010)

Constraint superfields

- Very recent literature presents studies of many different constraints applied to various sets of supermultiplets
- Either as Minkowski theories with global supersymmetry or coupled to supergravity (compatible in principle with de Sitter)
- In general: describes the Goldstino coupled to matter and gauge multiplets, with missing superpartners
- Useful scheme for realistic theories ?
- Quantum aspects ?
- Experimentally testable ? (even in principle)