ICTP-AP
International Center
for Theoretical Physics Asia-Pacific
国际理论物理中心-亚太地区

Taiji Program in Space
空间太极计划
Unified Field Theory in Hyper-spacetime
超时空统一场论

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Taiji Program in Space
Gravitational wave physics
空间太极计划

1. Introduction

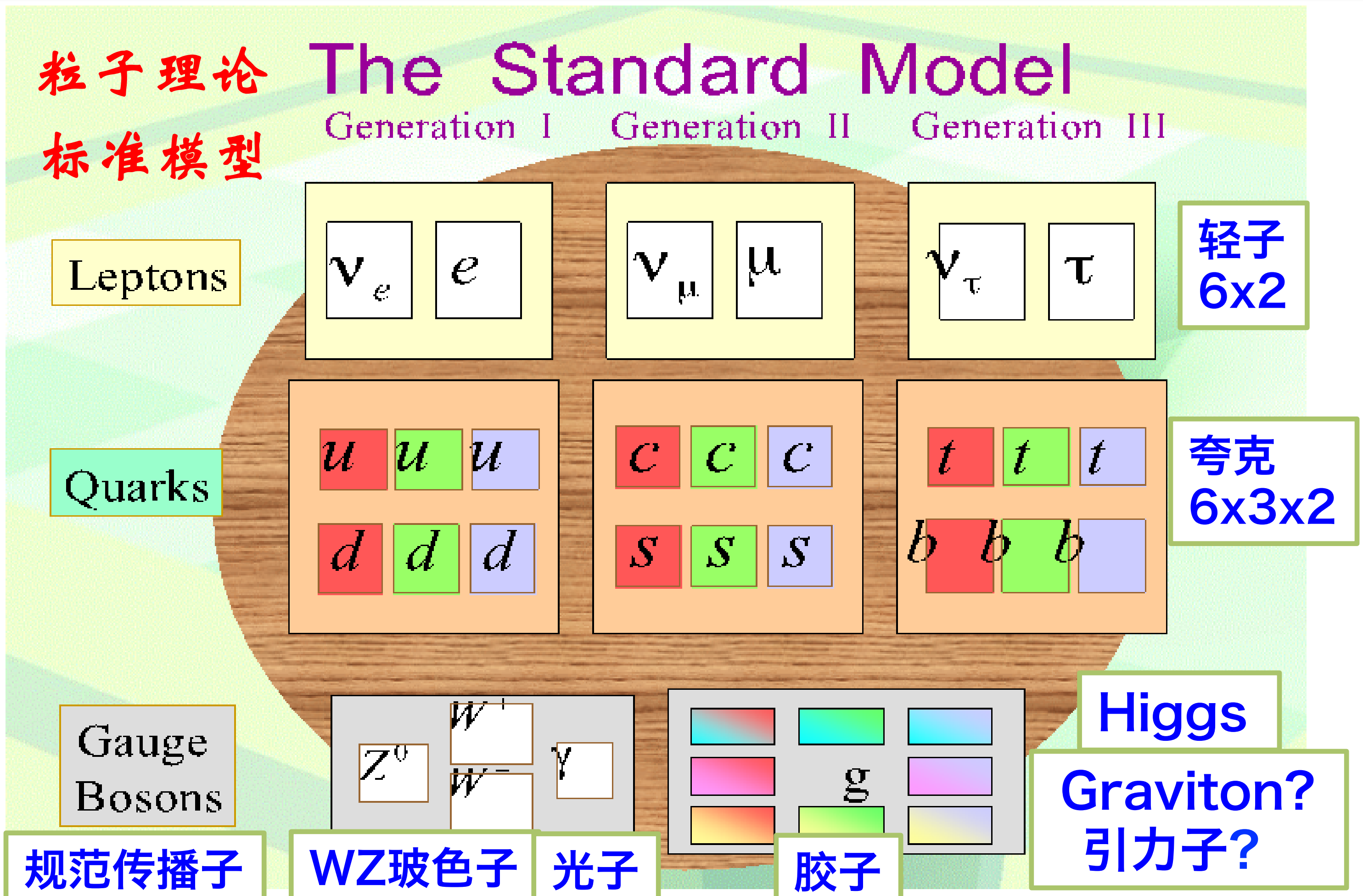
2. Taiji Program in Space
   for Gravitational Physics

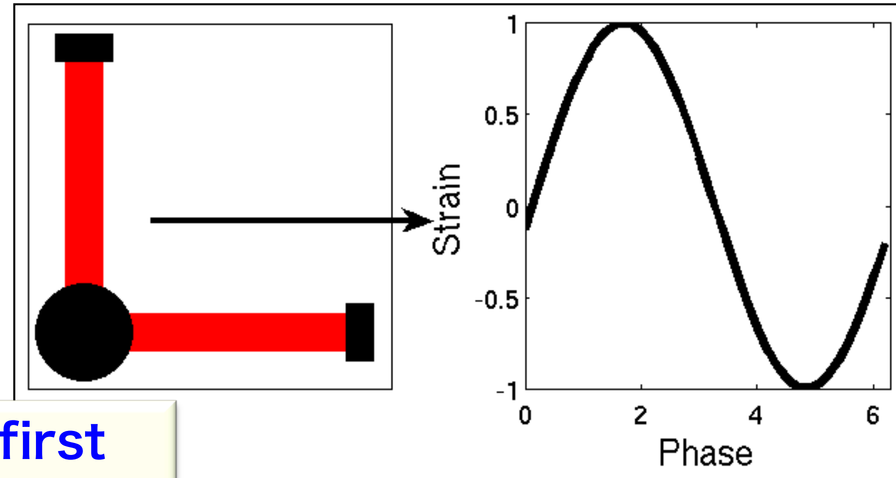
3. Unified Field Theory of All Basic
   Forces & Elementary Particles
   in Hyper-spacetime

Understanding on Elementary Particles

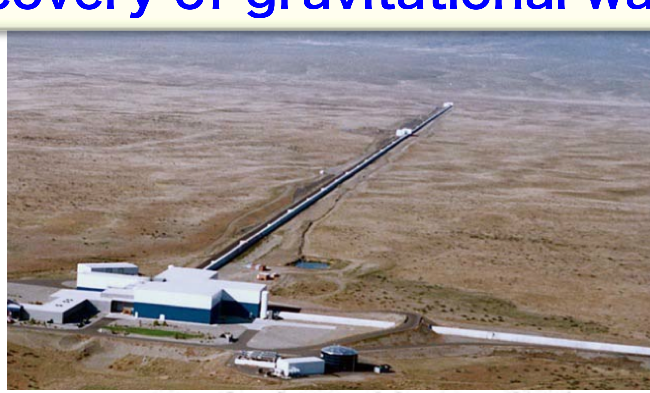
After the discovery of Higgs Boson, all elementary particles(61) in standard model have been observed

- The only undiscovered particle in SM is graviton





Feb. 11, 2016, LIGO announced the first direct discovery of gravitational wave



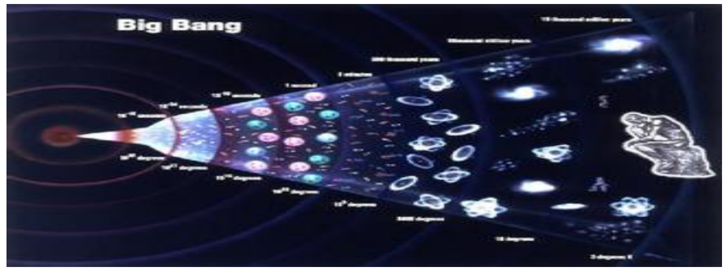
Hanford, Washington (H1)

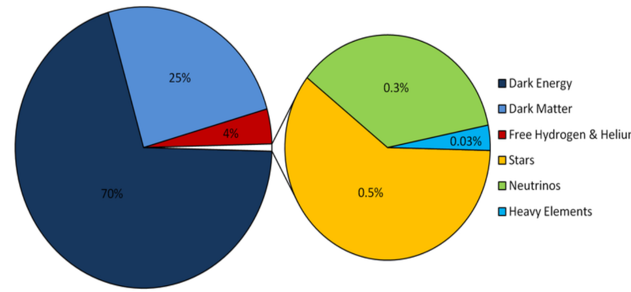
Livingston, Louisiana (L1)

Though a single graviton cannot be observed yet, while the gravitational wave composed of gravitons has been confirmed, which provides a direct test on the prediction made by Albert Einstein a century ago based on his general theory of relativity

Understanding on the Universe

Picture of Universe: Inflation⊕DE⊕DM⊕Atomic Matter





Cosmic Component in Universe

- **Dark Energy** 暗能量～70%
- **Matter** 物质～30%
  - **Dark Matter** 暗物质 85%
  - **Quark Matter** 夸克物质 15%
    - H (77%)
    - He (23%)

Our understanding on the nature of such a universe is very limited!

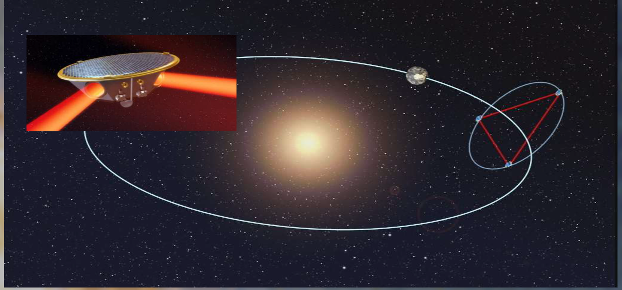
The early universe is filled with elementary particles at a very high energy and temperature, so that quantum gravity plays a role!

Exploring the intrinsic correlation between the very small elementary particles and the extreme large universe has been regarded as the frontier of particle physics and cosmology in basic sciences

Taiji Program in Space

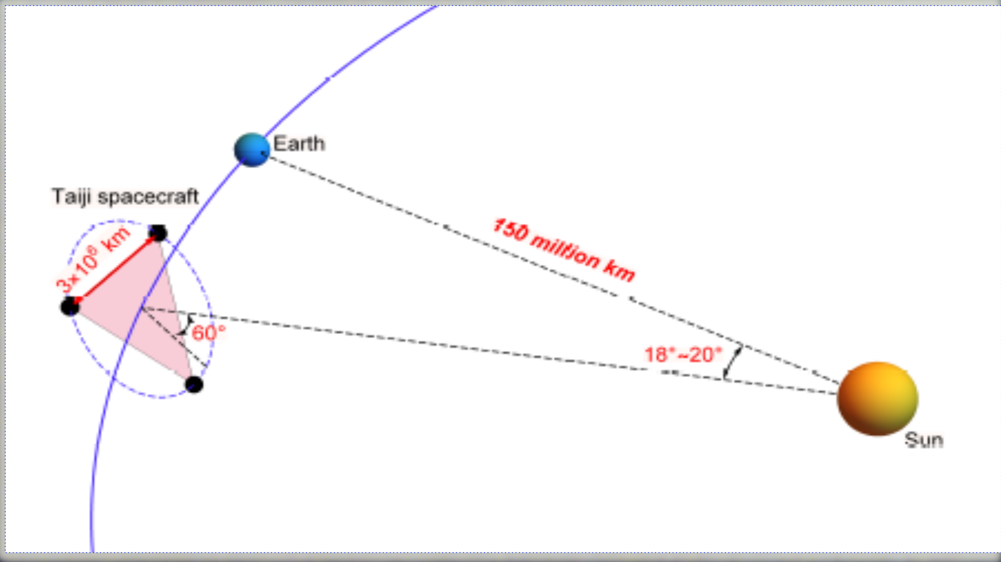
Gravitational wave physics

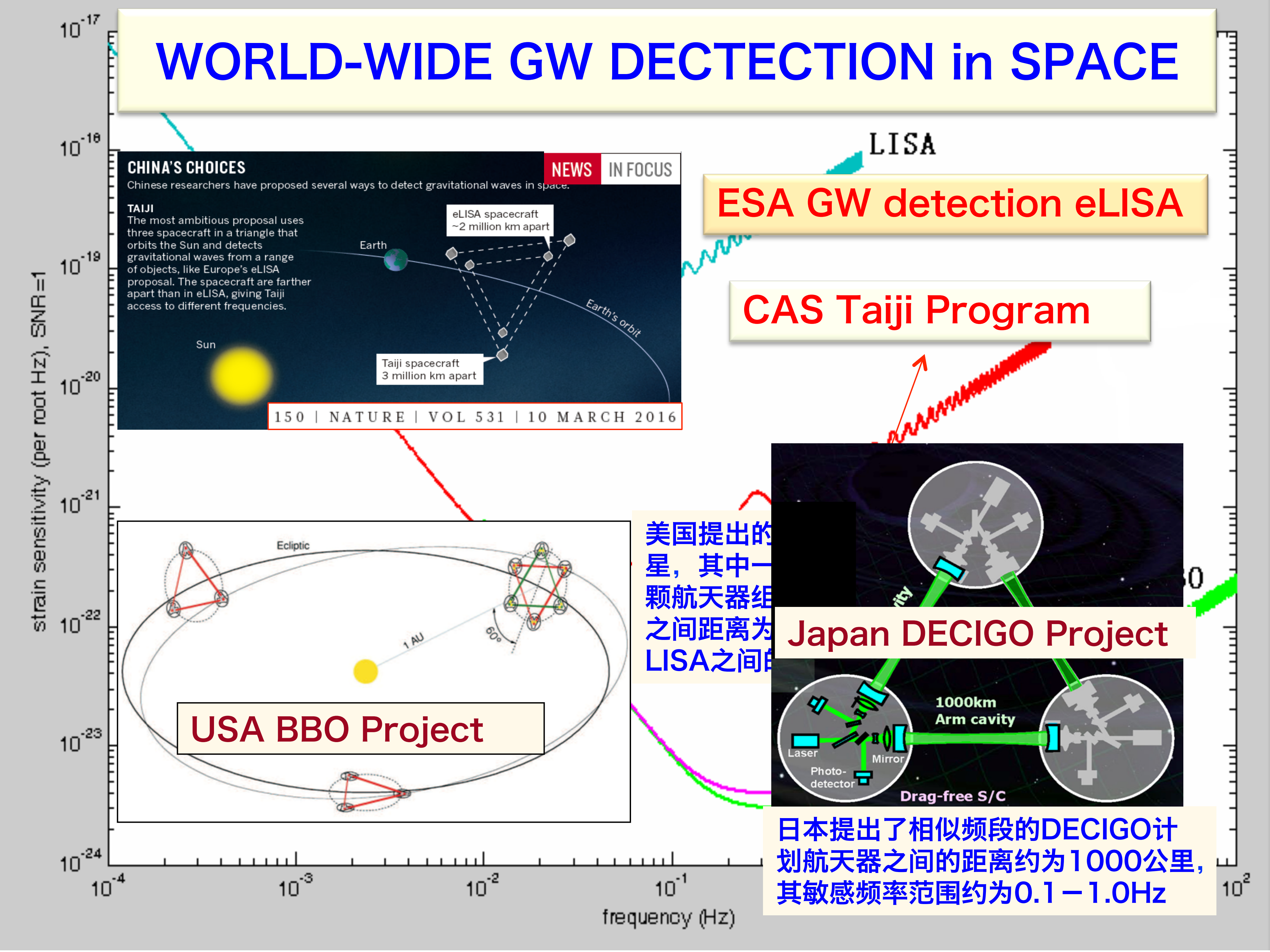
空间太极计划



Space-Based Gravitational Wave Detection

空间太极计划 Taiji Program In Space



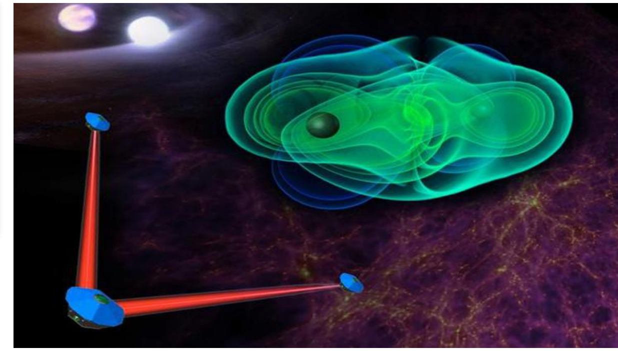


WORLD-WIDE GW DECTECTION in SPACE
CHINA'S CHOICES
NEWS IN FOCUS
Chinese researchers have proposed several ways to detect gravitational waves in space.
TAIJI
The most ambitious proposal uses three spacecraft in a triangle that orbits the Sun and detects gravitational waves from a range of objects, like Europe's eLISA proposal. The spacecraft are farther apart than in eLISA, giving Taiji access to different frequencies.
eLISA spacecraft ~2 million km apart
Earth
Earth's orbit
Sun
Taiji spacecraft 3 million km apart
150 | NATURE | VOL 531 | 10 MARCH 2016
LISA
ESA GW detection eLISA
CAS Taiji Program
Ecliptic
1 AU
60°
USA BBO Project
美国提出的
星，其中一
颗航天器组
之间距离为
LISA之间的
Japan DECIGO Project
1000km
Arm cavity
Laser
Mirror
Photo-detector
Drag-free S/C
日本提出了相似频段的DECIGO计划航天器之间的距离约为1000公里，其敏感频率范围约为0.1－1.0Hz
strain sensitivity (per root Hz), SNR=1
frequency (Hz)
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{2}

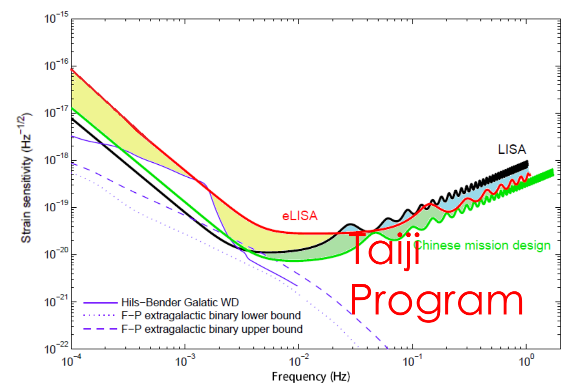
Taiji Program in Space
Gravitational wave physics
空间太极计划

Taiji Program in Space

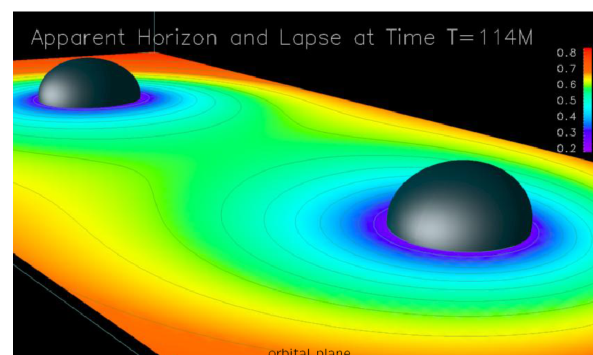
- Taiji Program is proposed to detect GW with frequency covering over the range of eLISA (0.1mHz-1.0Hz).



- Focus on the intermediate BH binaries (10^2~10^4 M_sun)
- With more sensitivity around (0.01-1Hz) (in comparison eLISA)

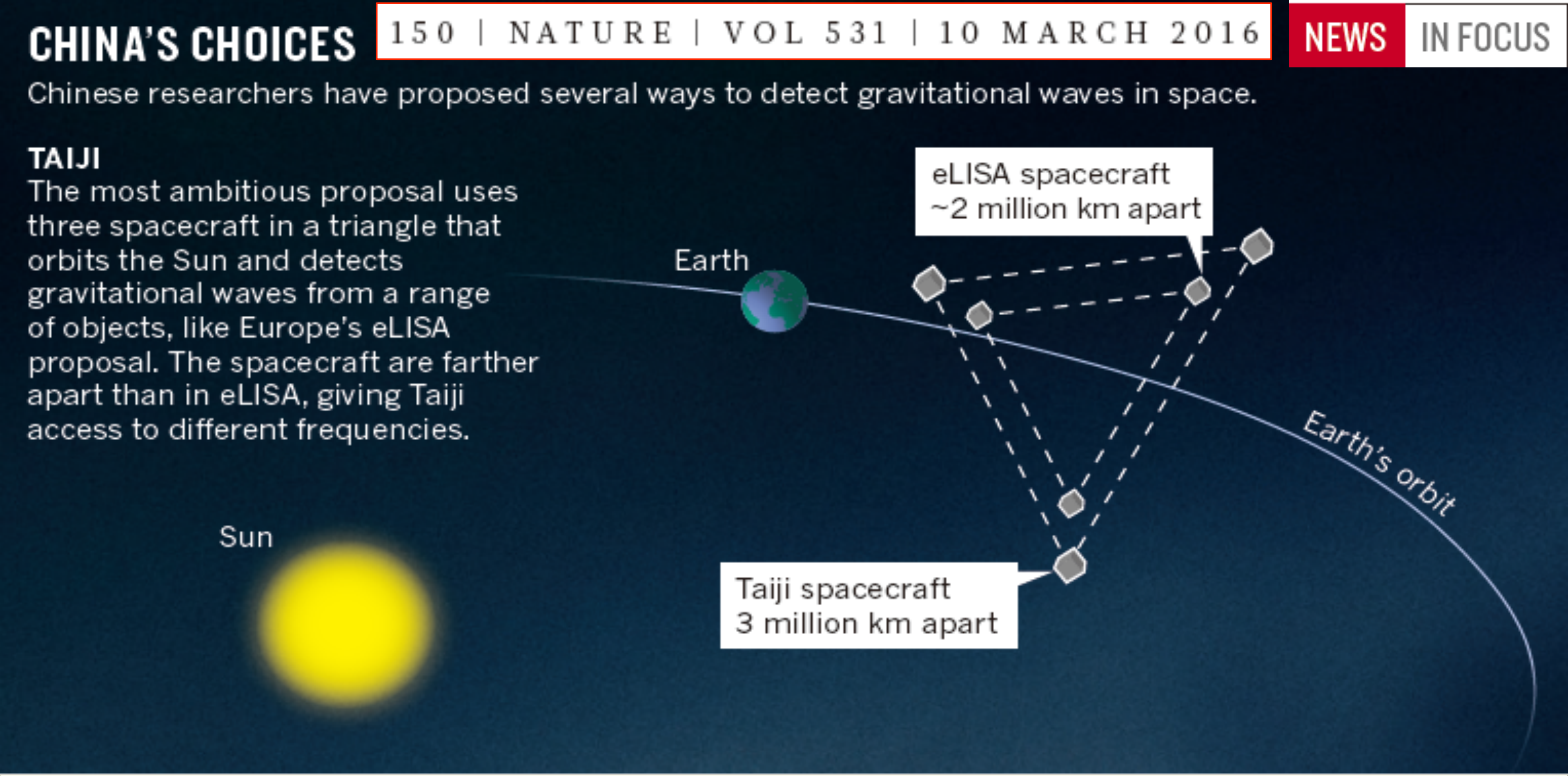


- How did the intermediate mass seed BH formed in early universe
- Whether DM could form BH
- How the seed BH grows into large or extreme-large BH
- Probe the polarization of GW and understand the nature of gravity



Taiji Program Baseline Design Parameters (preliminary mission proposal)

	Taiji Program preliminary mission proposal	LISA	eLISA
Arm length	$3\times10^9 m$	$5\times10^9 m$	$1\times10^9 m$
1-way position noise budget	$5\sim10\ \mathrm{pm\ Hz}^{-\frac{1}{2}}$	$18\ \mathrm{pm\ Hz}^{-\frac{1}{2}}$	$11\ \mathrm{pm\ Hz}^{-\frac{1}{2}}$
Laser power	2W	2W	2W
Telescope diameter	~50cm	40cm	20cm
Acceleration noise budget	$3\times10^{-15}\mathrm{ms^{-2}Hz^{-1/2}}$	$3\times10^{-15}\mathrm{ms^{-2}Hz^{-1/2}}$	$3\times10^{-15}\mathrm{ms^{-2}Hz^{-1/2}}$

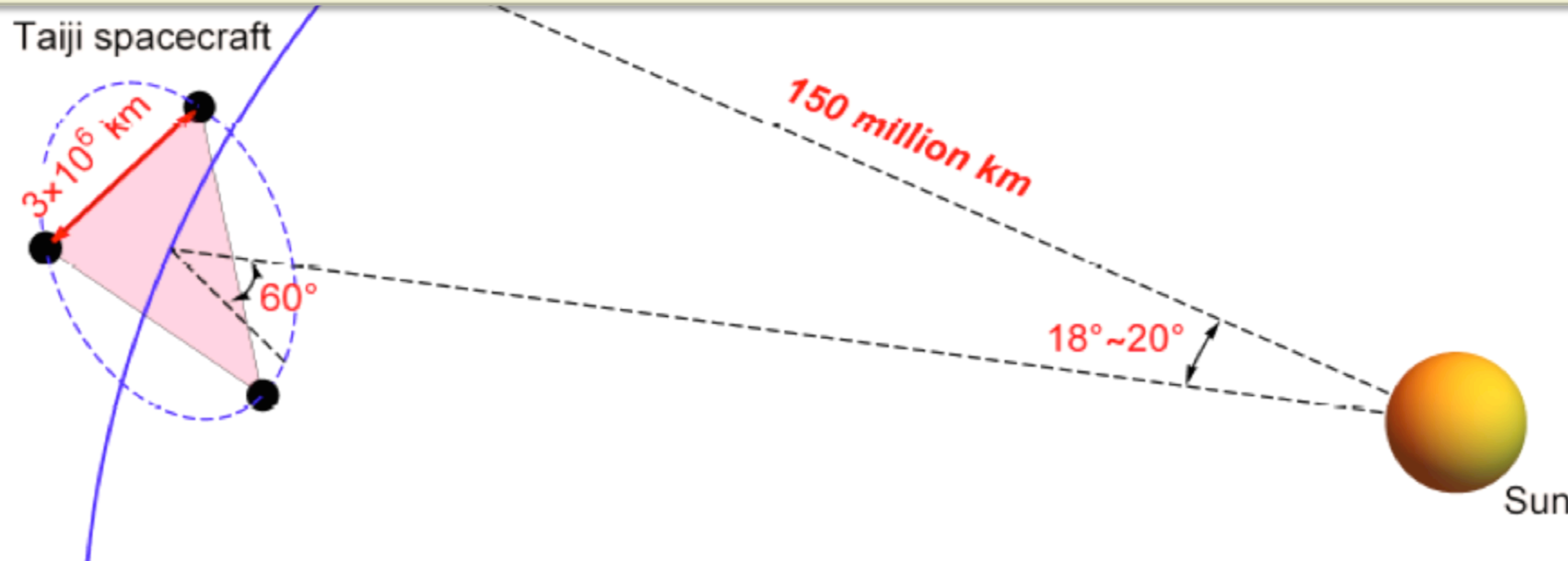


Compared with LIGO & other earth-based detectors,

- ✓ The orbit around the Sun can effectively avoid noises and signal pollutions in the earth
- ✓ Meet the high thermomechanical stability
- ✓ Obtain more accurate gravitational wave with wider low and medium frequencies.

Taiji program with 6 laser beams which are sent both ways between each pair of spacecraft. The differences in the phase changes between the transmitted and received laser beams at each spacecraft are measured.

Namely, signals between any two of the three satellites can be used to detect gravitational wave, it can provide a cross check for GW signals. Meanwhile, it allows us to measure the polarization of GWs and probe the nature of dark energy.



Taiji program is proposed to be launched around 2033.

Road Map of Taiji Program in Space

Phase I: 2016-2020 Pre-study on EP. & TH.

Concept & Design

Taiji concept
- ✓ Design concept
- ✓ Measurement scheme
- ✓ Data Analysis

Mission Design
- ✓ Spacecraft design
- ✓ Mission Analysis

Scientific Research

- ✓ Sources of GWs
- ✓ Astrophysics BH
- ✓ Ultra-Compact Binaries
- ✓ Extreme mass ratio inspirals
- ✓ Astrophysics of dense stellar sys.
- ✓ Test GR & Nature of gravity
- ✓ Fundamental laws
- ✓ Cosmology

Key technologies & Payload

- ✓ Laser system
- ✓ Optical sys.
- ✓ Phase lock & measurement
- ✓ Telescope
- ✓ Micropropulsion design

Road Map of Taiji Program in Space

Phase II: 2020-2025 Taiji Pathfinder mission

Launch Taiji-pathfinder
with two satellites
Test crucial technologies

Study on EP.
Prototype of GW

Phase III: 2025-2033 Taiji GW detection

2025-2029:
Develop
Engineering
prototype

2029-2033:
Flight load
development

~2033:
Launch GW
spacecraft

More than 10 institutes in China, most of them from the Chinese Academy of Sciences (CAS) have jointly studied the space-based GWD, it dates back to 2008 pioneered by Hu et.al.

The institutions of working groups include:

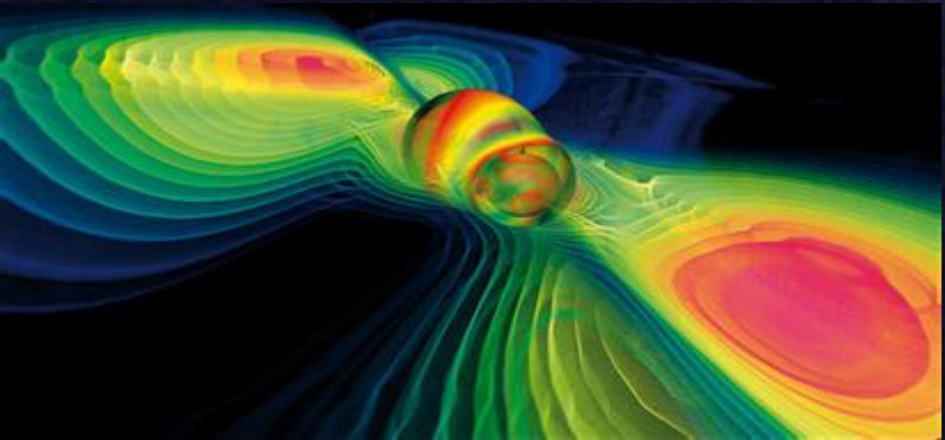
- ✧ Institute of Mechanics,
- ✧ University of Chinese Academy of Sciences (UCAS),
- ✧ Institute of Theoretical Physics,
- ✧ Academy of Mathematics and Systems Science,
- ✧ Institute of Physics,
- ✧ Institute of high Energy Physics,
- ✧ Nanjing Institute of Astronomy and Optics,
- ✧ National Astronomical Observatory,
- ✧ Wuhan Institute of Physics and Mathematics
- ✧ Changchun Institute of Optics,
- ✧ Fine Mechanics and Physics, CAS,
- ✧ Institute of Geodesy and Geophysics, Wuhan,
- ✧ Huazhong University of Science and Technology in Wuhan

We are organizing the Taiji Alliance to make both national & international efforts on GW studies.

GWD covers physics, photonics, astronomy, cosmology, precision measurement, navigation technology and space engineering, etc. We shall work all together with our international colleagues to explore the unknown aspects.

Nature of Gravity

Gravitational Gauge Field Theory & Gravitational Quantum Field Theory (GGFT&GQFT)



Elementary Particle Physics & Quantum Cosmophysics

Special Relativity 相对论 + Quantum Mechanics 量子力学 = Framework of Quantum Field Theory 量子场论

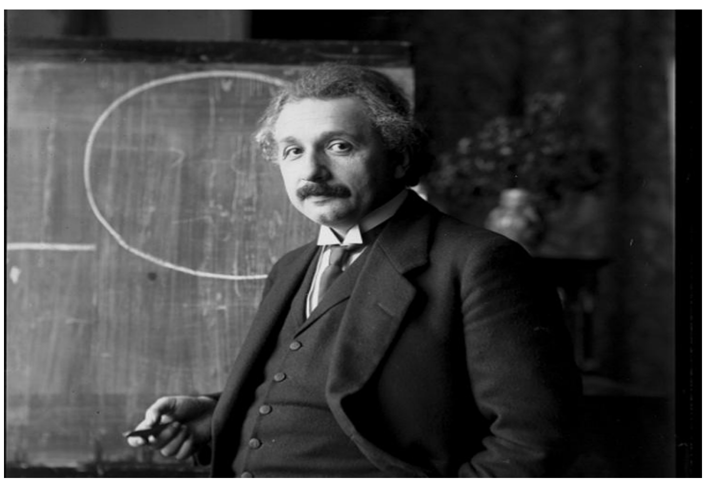
Symmetry dictates Interactions U(1)xSU(2)xSU(3) SO(1,3); Symmetry breaking generates our world

\+

Elementary Particle Physics & Quantum Cosmophysics 基本粒子物理 & 量子宇宙物理 = Symmetry Principle 对称原理

Foundation on General Theory of Relativity (1916)

But if $-g$ is always finite and positive, it is natural to settle the choice of co-ordinates *a posteriori* in such a way that this quantity is always equal to unity. We shall see later that by such a restriction of the choice of co-ordinates it is possible to achieve an important simplification of the laws of nature.



$$\left.\begin{aligned} \frac{\partial \Gamma^{\alpha}_{\mu\nu}}{\partial x_{\alpha}} + \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} &= 0 \\ \sqrt{-g} &= 1 \end{aligned}\right\}$$

Foundation on General Theory of Relativity (1916)

In place of (18), we then have simply $d\tau' = d\tau$, from which, in view of Jacobi's theorem, it follows that

$$\left| \frac{\partial x'_\sigma}{\partial x_\mu} \right| = 1 \tag{19}$$

Thus, with this choice of co-ordinates, only substitutions for which the determinant is unity are permissible.

But it would be erroneous to believe that this step indicates a partial abandonment of the general postulate of relativity. We do not ask "What are the laws of nature which are co-variant in face of all substitutions for which the determinant is unity?" but our question is "What are the generally co-variant laws of nature?" It is not until we have formulated these that we simplify their expression by a particular choice of the system of reference.

Limitation of Einstein General Relativity & Inspiration from Dirac Equation for Relativistic QM

Understanding on physical phenomena & experiments is often limited from the experimental means and the known knowledge

In 1915
- Quantum mechanics & QFT have not yet been established
- Basic constituents of matter and their structure are unknown

Einstein: SR→GR: Minkowski spcetime→Riemannian spacetime

$$\boxed{G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}}$$

Geometry ⟺ Matter

1928: SR+QM→RQM: Schrodinger Equation→Dirac equation

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{r},t) = \hat{H}\psi(\vec{r},t)$$

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})$$

→

$$\gamma^\mu i\partial_\mu \Psi - m\Psi = 0$$

ψ_i (i=1,2,3,4), SO(1,3)

Spacetime Dimension ⟺ Degrees of Freedom of Matter Field ⟺ Conserv. of total Angular Mom.

RQM → RQFT (QED) → SM of Particle Physics

Key Progresses after General Theory of Relativity

QED

- S. Tomonaga, Prog. Theor. Phys. 1, 27 (1946).
- J. Schwinger, Phys. Rev. 73, 416 (1948); 74, 1439 (1948).
- R. P. Feynman, Phys. Rev. 76, 769 (1949); 76, 749 (1949); 80, 440 (1950)
- F. Dyson, Phys. Rev. 75, 486502 (1949); 75, 1736 (1949).

Gauge Theory and Parity nonconservation

- C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).
- T.D. Lee and C.N. Yang, Phys. Rev., 104, 254 (1956).

Model for Electroweak Interaction

- S. L. Glashow, Nucl. Phys. 22, 579 (1961).
- S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
- A. Salam, in Proceedings of the Eight Nobel Symposium,on Elementary Particle Theory, Relativistic Groups, and Analyticit, Stochholm, Sweden, 1968, edited by

Key Progresses after General Theory of Relativity

QCD –asymptotic freedom & SU(3) color

- D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973).
- H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).
- H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973).

Inflation of Universe

- A. A. Starobinsky, Phys. Lett. 91B, 99 (1980).
- A. H. Guth, Phys. Rev. D 23, 347 (1981).
- A. D. Linde, Phys. Lett. 108B, 389 (1982).
- For a recent review, see, e.g., A. D. Linde, arXiv:1402.0526, and references therein.

Constituents of Matter & Universe:

Quarks、 Leptons、 Dark Matter 、 Dark Energy

QUESTION & CHALLENGE

Can we describe the gravitational interaction within the framework of QFTs ?

Can we establish the theory of gravity not based on Einstein's postulate/principle of the general coordinate transformation ?

Can we find out the laws of nature which are covariant with the transformations for which the determinant is unity

It is necessary to go beyond Einstein's Theory of General Relativity !

Gauge Theory of Gravity

Gravity gauge theories: based on Riemannian or non-Riemannian geometry on curved space-time manifolds

* R.Utiyama, Phys. Rev. 101, 1597 (1956).
* T. W. B. Kibble, J. Math. Phys. 2, 212 (1961).

It has been motivated 60 years ago

* D.W. Sciama, ”On the analogy between charge and spin in general relativity, ” in Recent Developments in General Relativity (Pergamon+ PWN, Oxford), p. 415, 1962;
* D.W. Sciama. ”The physical structure of general relativity”, Rev. Mod. Phys. 36, 463-469 (1964).
* H.Y. Guo, Y.S. Wu, Y.Z. Zhang, Science Bulletin, 2, 72 (1973) (in Chinese).
* C. N. Yang, Phys. Rev. Lett. 33, 445 (1974).
* F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, ”General relativity with spin and torsion: Foundations and prospects.” Rev. Mod. Phys. 48, 393416 (1976)
* D. Ivanenko and G. Sardanashvily, ”The gauge treatment of gravity”, Phys. Rep. 94, 1 (1983)
* F.Hehl, J.McCrea, E.Mielke and Y.Neeman, ”Metric-affine gauge theory of gravity: field equations, Noether identities, world spinors, and breaking of dilaton invariance”, Phys. Rep. 258, 1 (1995).

Open questions for curved spacetime:

- Definitions of space and time;
- Quantization of gravity gauge theories;
- Basic structure of Lagrangian;
- Dynamic properties of gravity gauge

Quantum Field Theory of Gravity with Spin and Scaling Gauge Invariance and Spacetime Dynamics with Quantum Inflation

PHYSICAL REVIEW D **93**, 024012 (2016)

Quantum field theory of gravity with spin and scaling gauge invariance and spacetime dynamics with quantum inflation

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Principle & Postulate of GGFT/GQFT Beyond Einstein

- ✓ Gauge theory of gravity is constructed within the framework of QFT in a flat Minkowski space-time
- ✓ Treat the gravitational force on the same footing as other three basic forces

- ➢ The basic constituents of matter are assumed to be the quantum fields of Dirac fermions
- ➢ Kinematics of all quantum fields obeys the principles of special relativity and quantum mechanics;
- ➢ Dynamics of all quantum fields is characterized by basic interactions governed via gauge symmetries;

Principle & Postulate of GGFT/GQFT Beyond Einstein

- Gauge symmetry group is determined through the intrinsic degrees of freedom or quantum numbers of the basic constituents of fermionic quantum fields
- Spin quantum numbers correspond to the spin gauge symmetry SP(1,3) that governs gravitational interaction

- The action including gravitational interaction is constructed in the Gravifield space-time to be coordinate and scaling independent, it is based solely on gauge symmetries (spin, scaling, internal).
- The action of theory is invariant under the local spin and scaling as well as internal gauge transformations for the fields in the locally flat gravifield spacetime , and also invariant under the global Lorentz and scaling as well as translational transformations for the coordinates in the globally flat Minkowski space-time

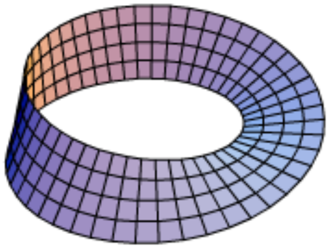
Global & local gauge symmetries of GGFT/GQFT

P(1,3)x S(1)✖U(1)xSU(2)xSU(3)xSP(1,3)xSG(1)

Principle & Postulate of GGFT&GQFT Beyond Einstein

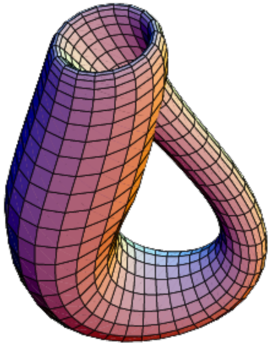
- **A biframe spacetime is proposed to describe GGFT&GQFT**

One frame space-time is a globally flat coordinate Minkowski space-time, which acts as an inertial reference frame for characterizing the motions of fields. As a base spacetime V^4



Möbius strip is a bundle of the line segment over the circle

The other is a locally flat non-coordinate Gravifield spacetime, which functions as an interaction representation frame for characterizing the degrees of freedom of fields. As a fiber G^4, such a spacetime behaves as a noncommutative geometry



Klein bottle twisted circle bundle over another circle

The total spcetime is in general a Fiber Bundle: $E \sim V^4 \times G^4$

Basic gravitational field is no longer the metric field, but a bi-covariant vector field defined in the globally flat Minkowski spacetime and valued in the locally flat gravifield spacetime

$$g_{\mu\nu} \rightarrow \chi_{\mu}^{\ a}$$

Gauge field in SP(1,4)/SP(1,3)

Gravifield Space-time and Gauge Theory of Gravity

Principle of Gauge-invariance and coordinate-independence

action for gauge theory of gravity in 4D Gravifield space-time

$$
\begin{aligned}
S_\chi \, = \, \int \{ \, & \frac{1}{2} [i\bar{\Psi} \ast \chi \wedge \mathcal{D}\, \Psi \, + i\bar{\psi} \ast \chi \wedge \bigtriangledown \psi \, + H.c.] + y_s Tr\, (\chi \wedge \chi) \wedge \ast (\chi \wedge \chi)\, \bar{\psi}\phi\psi \\
- \, & \frac{1}{g_A^2} Tr\, \mathcal{F} \wedge \ast \mathcal{F} - \frac{1}{g_s^2} Tr\, \mathcal{R} \wedge \ast \mathcal{R} - \frac{1}{2} \mathcal{W} \wedge \ast \mathcal{W} + \frac{1}{2}\, \alpha_G\, \phi^2 Tr\, \mathcal{G} \wedge \ast \mathcal{G} \\
- \, & \frac{1}{2} d\phi \wedge \ast d\phi - \alpha_E Tr\, \mathcal{R} \wedge \ast (\chi \wedge \chi)\, \phi^2 + \lambda_S Tr\, (\chi \wedge \chi) \wedge \ast (\chi \wedge \chi)\, \phi^4 \, + \mathcal{L}' \} \, ,
\end{aligned}
$$

Covariant derivative in locally flat Gravifield spacetime & Hodge star

$$\mathcal{D} = \chi^a \mathcal{D}_a$$

$$\mathcal{D}_a = \hat{\chi}_a - i\mathcal{A}_a - i\Omega_a = \hat{\chi}_a{}^\mu \mathcal{D}_\mu = \hat{\chi}_a{}^\mu (\, \partial_\mu - i\mathcal{A}_\mu - i\Omega_\mu \,)$$

$$\ast \mathcal{R} = \frac{1}{4i} \epsilon^{ab}{}_{cd}\, \mathcal{R}_{ab}\, \chi^c \wedge \chi^d$$

$$\ast \chi^a = \frac{1}{3!} \epsilon^a{}_{bcd}\, \chi^b \wedge \chi^d \wedge \chi^c$$

$$(\chi \wedge \chi) = \chi^a \wedge \chi^b \, \frac{1}{2i} \Sigma_{ab}$$

$$\ast (\chi \wedge \chi) = \frac{1}{2} \epsilon^{ab}{}_{cd}\, \chi^c \wedge \chi^d \, \frac{1}{2i} \Sigma_{ab}$$

$$d\phi = (d_\chi - i g_w W)\phi$$

$$\ast d\phi = \frac{1}{3!} \epsilon^a{}_{bcd}\, \chi^b \wedge \chi^d \wedge \chi^c \, (\hat{\chi}_a - g_w W_a)\phi$$

Gravitational Gauge Field Theory & GQFT

Action of gauge theory of gravity in Gravifield space-time → Action expressed in the globally flat Minkowski space-time

→ Convert Gravifield basis into coordinate basis via dual Gravifield

$$\begin{aligned} S_\chi \;=\; \int d^4x\, \chi\, \frac{1}{2}\{ [\, \hat{\chi}^{\mu\nu}(\bar{\Psi}\chi_\mu i\mathcal{D}_\nu \Psi + \bar{\psi}\chi_\mu i\bigtriangledown_\nu \psi) + H.c.\,] - y_s \bar{\psi}\phi\psi \\ - \frac{1}{4}\hat{\chi}^{\mu\mu'}\hat{\chi}^{\nu\nu'}[\,\mathcal{F}^I_{\mu\nu}\mathcal{F}^I_{\mu'\nu'} + \mathcal{R}^{ab}_{\mu\nu}\mathcal{R}_{\mu'\nu' ab} + \mathcal{W}_{\mu\nu}\mathcal{W}_{\mu'\nu'} - \alpha_G\,\phi^2\,\mathcal{G}^a_{\mu\nu}\mathcal{G}_{\mu'\nu' a}\,] \\ + \frac{1}{2}\hat{\chi}^{\mu\nu}d_\mu\phi d_\nu\phi - \alpha_E g_s \phi^2 \hat{\chi}_a{}^\mu \hat{\chi}_b{}^\nu \mathcal{R}^{ab}_{\mu\nu} - \lambda_S\phi^4 + \mathcal{L}'(x)\}\,, \end{aligned}$$

$$\hat{\chi}^{\mu\nu}(x) = \hat{\chi}_a{}^\mu(x)\hat{\chi}_b{}^\nu(x)\eta^{ab}$$

✓ **Gauge-type field:** $\boxed{\chi_\mu{}^a(x)}$ **couples inversely to all kinematic terms and also interaction terms of quantum fields → Basic gravitational field**

✓ **Gauge-type interaction:** $\boxed{\mathcal{G}^a_{\mu\nu}(x)}$ **governs dynamics of Gravifield → characterizes the gravitational force** $\quad \mathrm{SP(1,3)}$

$$T_G^{1,3} = \mathrm{PG(1,3)/SP(1,3)} \qquad \mathrm{SP(1,3)} \times \mathrm{P(1,3)} \quad \neq \quad \mathrm{GL(4,R)}$$

Energy Momentum Conservation in GQFT

Translational invariance:

$$\boxed{x^\mu \to x'^\mu = x^\mu + \alpha^\mu}$$

Total Energy-momentum conservation

$$\boxed{\delta S_\chi = \int d^4x\, \partial_\mu(\mathcal{T}_\nu{}^\mu)a^\nu = 0} \longrightarrow \boxed{\partial_\mu \mathcal{T}_\nu{}^\mu = 0}$$

Gauge Invariant Energy-momentum Tensor

$$\boxed{\begin{aligned} \mathcal{T}_\nu{}^\mu &= -\eta^\mu_\nu \chi \mathcal{L} + \frac{1}{2}\chi \hat{\chi}_a{}^\mu [\, i\bar{\Psi}\gamma^a \mathcal{D}_\nu \Psi + i\bar{\psi}\gamma^a \bigtriangledown_\nu \psi + H.c.\,] \\ &- \chi \hat{\chi}^{\mu\mu'} \hat{\chi}^{\rho\sigma} [\, \mathcal{F}^I_{\mu'\rho} \mathcal{F}^I_{\nu\sigma} + \mathcal{R}^{ab}_{\mu'\rho} \mathcal{R}_{\nu\sigma\, ab} + \mathcal{W}_{\mu'\rho} \mathcal{W}_{\nu\sigma} - \alpha_W \phi^2 \mathcal{G}^a_{\mu'\rho} \mathcal{G}_{\nu\sigma a}\,] \\ &+ \chi \hat{\chi}^{\mu\mu'} d_{\mu'}\phi d_\nu \phi - 2\alpha_E g_s \chi \phi^2 \hat{\chi}_a{}^\mu \mathcal{R}^{ab}_{\nu\rho} \hat{\chi}_b{}^\rho \end{aligned}}$$

$$\boxed{\mathcal{T}_{\mu\nu} \neq \mathcal{T}_{\nu\mu}}$$

Equation of Motion for Gravifield & Conservation Law of Gravifield Tensor Current

Relation between the energy-momentum tensor and bi-covariant vector current for the Gravifield

$$\mathcal{T}_{\nu}{}^{\mu} = \chi_{\nu}{}^{a} J_{a}^{\mu}$$

Equation of motion for Gravifield

$$\partial_{\rho}\mathcal{G}_{\nu}{}^{\mu\rho} - \mathcal{G}_{\nu}{}^{\mu} = \mathcal{T}_{\nu}{}^{\mu}$$

Gravity Equation Beyond & Extension to Einstein equation of GR

Gravifield Tensor

$$\mathcal{G}_{\nu}^{\mu\rho} \equiv \alpha_G \phi^2 \chi \hat{\chi}^{\mu\mu'} \hat{\chi}^{\rho\nu'} \mathcal{G}_{\mu'\nu' a} \chi_{\nu}{}^{a} = -\mathcal{G}_{\nu}^{\rho\mu}$$

Gravifield Tensor Current

$$\mathcal{G}_{\nu}{}^{\mu} \equiv \alpha_G \phi^2 \chi \hat{\chi}^{\mu\mu'} \hat{\chi}^{\rho\nu'} \mathcal{G}_{\mu'\nu' a} \nabla_{\rho} \chi_{\nu}{}^{a} = \left(\hat{\chi}_{a}^{\sigma} \nabla_{\rho} \chi_{\nu}{}^{a}\right) \mathcal{G}_{\sigma}^{\mu\rho}$$

Conservation law for the Gravifield Tensor Current

based on the energy momentum conservation

$$\partial_{\mu}\mathcal{T}_{\rho}{}^{\mu} = \partial_{\mu}(J_{a}^{\mu}\chi_{\rho}{}^{a}) = 0$$

$$\partial_{\mu}\mathcal{G}_{\nu}{}^{\mu} = \partial_{\mu}(\hat{\chi}_{a}^{\sigma}\nabla_{\rho}\chi_{\nu}{}^{a}\mathcal{G}_{\sigma}^{\mu\rho}) = \partial_{\mu}\left(\phi^2\chi\hat{\chi}^{\mu\mu'}\hat{\chi}^{\rho\nu'}\mathcal{G}_{\mu'\nu' a}\nabla_{\rho}\chi_{\nu}{}^{a}\right) = 0$$

Nature of Gravity
&
Unification Theory

Unified Field Theory of all Basic Forces & Elementary Particles with Gravitational Origin of Gauge Symmetry in Hyper-spacetime

Structure of Spacetime & Matter

Inspired from Relativistic Quantum Mechanics of Dirac equation

$$\boxed{(\gamma^\mu i\partial_\mu - m)\,\psi = 0}$$

$$\psi^T = (\psi_1\,, \psi_2\,, \psi_3\,, \psi_4)$$

$$\{\gamma_\mu\ \gamma_\nu\} = \eta_{\mu\nu}$$

$$\boxed{(\eta^{\mu\nu}\partial_\mu\partial_\nu + m^2)\psi = 0}$$

$$\eta_{\mu\nu} = diag.(1, -1, -1, -1)$$

The 4-dimensional spacetime of coordinates with movement and rotation corresponds to the 4-component entity of Dirac spinor that reflects the boost spin and helicity spin.

Dirac equation reveals a correlation between the quantum numbers and the geometry of spacetime at a profound level

The dimensions of spacetime are coherently related to the degrees of freedom of Dirac spinor

The Dirac equation as a unity of quantum mechanics and special relativity has led to a successful development of relativistic quantum mechanics (QM) and quantum field theory (QFT)

Maximal symmetry and mass generation of Dirac fermion and gravitational gauge field theory in six-dimensional spacetime

Yue-Liang Wu*

Massless Dirac Spinor → Chiral Symmetry

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International Centre for Theoretical Physics Asia-Pacific (ICTP-AP)

$$\mathrm{P}(1,5) = \mathrm{SO}(1,5) \ltimes P^{1,5}$$

$$G_S = SU(1,3) \times SU(2) \times SG(1)$$

The relativistic Dirac equation in four-dimensional spacetime reveals a coherent relation be-tween the dimensions of spacetime and the degrees of freedom of fermionic spinor. A massless

Gravitational Relativistic Dirac Equation in 6D-spacetime

fermion can be treated a ... in a six-dimensional spacetime that reflects the intrinsic ... neralized Dirac equation is ob-tained in the six-dimensi... y. Based on the framework of gravitational quantum field theory proposed in Ref [1] with the postulate of gauge invariance and coo... for the ... the loca... as well ... bly elec... to maintain both global and local conformal scaling symmetries. A generalized gravitational Dirac

$$\Gamma^{\hat{a}} \hat{\chi}_{\hat{a}}{}^{\hat{\mu}} i(\mathcal{D}_{\hat{\mu}} + \mathsf{V}_{\hat{\mu}})\Psi_- = 0$$

$$\hat{\chi}^{\hat{\mu}\hat{\nu}}(\nabla_{\hat{\mu}} + \mathsf{V}_{\hat{\mu}})(\mathcal{D}_{\hat{\nu}} + \mathsf{V}_{\hat{\nu}})\Psi_- = \Sigma^{\hat{a}\hat{b}} \hat{\chi}_{\hat{a}}{}^{\hat{\mu}} \hat{\chi}_{\hat{b}}{}^{\hat{\nu}} [\, \mathcal{F}_{\hat{\mu}\hat{\nu}} + F_{\hat{\mu}\hat{\nu}} + i\mathcal{V}_{\hat{\mu}\hat{\nu}} - \mathcal{G}^{\hat{c}}_{\hat{\mu}\hat{\nu}} \hat{\chi}_{\hat{c}}{}^{\hat{\rho}} i(\mathcal{D}_{\hat{\rho}} + \mathsf{V}_{\hat{\rho}})\,]\Psi_-$$

Generalized Gravitational Equation in 6D-spacetime

fects. The dynamics of gauge-type gravifield as a Goldstone-like boson is shown to be governed by a co... ...etric part provi... ...uation of gr... ...breaking mec... ...ion of Dira...

$$\partial_{\hat{\nu}} \mathcal{G}_{\hat{\rho}}{}^{\hat{\mu}\hat{\nu}} - \mathcal{G}_{\hat{\rho}}{}^{\hat{\mu}} = \mathcal{T}_{\hat{\rho}}{}^{\hat{\mu}}$$

$$\partial_{\hat{\mu}} \mathcal{T}_{\hat{\nu}}{}^{\hat{\mu}} = 0$$

PAC...

Unified field theory of basic forces and elementary particles with gravitational origin of gauge symmetry in hyper-spacetime

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A unified field theory of all known basic forces and elementary particles is built based on a postulate of gauge invariance and coordinate independence along with a conformal scaling gauge symmetry. The hyper-spin charge of a unified hyper-spinor field is conjectured to correlate to the dimension of a hyper-spacetime with $D_h = 19$ via a maximal symmetry. A unified fundamental interaction i … The gravitational origin … es us to study gauge gravit … duality.

Unified Field Theory in Hyper-spacetime

超时空统一场论

(To appear in Science Bulletin)

The theory of general … etry of spacetime and the energy-momentum tens … odels of electromagnetism and gravity at a classic … the search for a unified field theory. A scaling g … nagnetism. Kaluza[3] and Klein[4] extended the G … gnetism. The relativistic Dirac spinor theory[5] l … etime and the degrees of freedom of Dirac spinor … el. A modern unified field theory is an attempt to … ty, electromagnetic, weak and strong. Three of the … gauge fields[6], which lays the foundation of the st … ry (QFT). The discovery of asymptotic freedom[… ries with enlarged gauge symmetries initiating from unifying quarks and leptons[12] led to the construction of grand unified theories (GUTs) SU(5)[13] and SO(10)[14, 15] for the electroweak and strong interactions. An enlarged SO(1,13) gauge model[16] was proposed to unify SO(1,3) and SO(10).

Inspiring from the relativistic Dirac spinor theory and Einstein theory of GR as well as GUTs, we are going to build a unified field theory for all known basic forces and elementary particles. The construction of the theory is based on the postulate of gauge invariance and coordinate independence described in [17] for a gravitational gauge field theory within the framework of QFT.

By treating all spin-like charges of elementary particles on the same footing as a *hyper-spin charge* and expressing the degrees of freedom of all elementary particles into a single column vector in a spinor representation of a high-dimensional *hyper-spacetime*, we shall be able to establish a coherent relation between the spinor structure of elementary particles and the dimension of a *hyper-spacetime*. A minimal unified spinor field is found to be a Majorana-type *hyper-spinor field* $\Psi(\hat{x})$ in an irreducible spinor representation of a *hyper-spin group* SP(1,18)≅SO(1,18), which results in a Minkowski *hyper-spacetime* with dimension $D_h = 19$.

Unification of All Elementary Particles into a Single Hyper-spinor Field

Treat all spin-like charges of elementary particles on the same footing as a hyper-spin charge (boost spin, helicity spin, chirality spin, isometric spin, color spin, family spin)

Express the degrees of freedom of all elementary particles into a single column vector in a spinor representation of a high-dimensional hyper-spacetime

Establish coherent relation between the spinor structure of elementary particles and the dimension of a hyper-spacetime

A minimal unified spinor field is found to be a Majorana-type hyper-spinor field in an irreducible spinor representation of a hyper-spin group SP(1,18) =SO(1,18)

A Minkowski hyper-spacetime with dimension D_h =19

Freely moving massless hyper-spinor field in a flat Minkowski hyper-spacetime →self-hermitian action via maximal symmetry

$$I_H = \int [d\hat{x}]\, \frac{1}{2} \bar{\Psi}(\hat{x}) \Gamma^{\mathbf{A}}\, \delta_{\mathbf{A}}{}^{\mathbf{M}} i\partial_{\mathbf{M}} \Psi(\hat{x})$$

$$\hat{x} \equiv x^{\mathbf{M}}$$

$$\mathbf{A}, \mathbf{M} = 0, 1, 2, 3, 5, \cdots, D_h$$

$$D_h = 19$$

$$\Psi(\hat{x}) = \begin{pmatrix} \Psi(\hat{x}) \\ \Psi^c(\hat{x}) \\ \Psi'(\hat{x}) \\ -\Psi'^c(\hat{x}) \end{pmatrix}$$

$$\Psi_{W,E} = \tfrac{1}{2}(1 \mp \gamma_{15})\Psi$$

$$\Psi_i \equiv \Psi_{Wi} + \Psi_{Ei}$$

$$\Gamma^{\mathbf{A}} \equiv (\Gamma^a, \Gamma^A, \Gamma^m)$$

$$\begin{aligned} \Gamma^a &= \ \sigma_0 \otimes \sigma_0 \otimes I_{32} \otimes \gamma^a \\ \Gamma^A &= i\sigma_0 \otimes \sigma_0 \otimes \Gamma^A \otimes \gamma_5 \\ \Gamma^{15} &= i\sigma_2 \otimes \sigma_3 \otimes \gamma^{11} \otimes \gamma_5 \\ \Gamma^{16} &= i\sigma_1 \otimes \sigma_0 \otimes \gamma^{11} \otimes \gamma_5 \\ \Gamma^{17} &= i\sigma_2 \otimes \sigma_1 \otimes \gamma^{11} \otimes \gamma_5 \\ \Gamma^{18} &= i\sigma_2 \otimes \sigma_2 \otimes \gamma^{11} \otimes \gamma_5 \\ \Gamma^{19} &= \ \sigma_0 \otimes \sigma_0 \otimes I_{32} \otimes I_4 \end{aligned}$$

$$\Psi \equiv \Psi_W + \Psi_E \equiv \Psi_1 + i\Psi_2$$

$$\begin{aligned} \Psi_{Wi}^T = [& (U_i^r, U_i^b, U_i^g, U_i^w, D_{ic}^r, D_{ic}^b, D_{ic}^g, D_{ic}^w, \\ & D_i^r, D_i^b, D_i^g, D_i^w, -U_{ic}^r, -U_{ic}^b, -U_{ic}^g, -U_{ic}^w)_L, \\ & (U_i^r, U_i^b, U_i^g, U_i^w, D_{ic}^r, D_{ic}^b, D_{ic}^g, D_{ic}^w, \\ & D_i^r, D_i^b, D_i^g, D_i^w, -U_{ic}^r, -U_{ic}^b, -U_{ic}^g, -U_{ic}^w)_R]^T \end{aligned}$$

$$a = 0, 1, 2, 3$$

$$A = 5, \cdots, 14$$

$$m = 15, \cdots, D_h$$

$$D_h = 19$$

Global Symmetries of freely moving hyper-spinor field

Maximal hyper-spin symmetry

$$\boxed{SP(1, D_h - 1) \cong SO(1, D_h - 1)}$$

$\boxed{D_h = 19}$

SO(1,3)xSO(10)xSO(5)

$$\Sigma^{\mathbf{AB}} = -\Sigma^{\mathbf{BA}} = i[\Gamma^{\mathbf{A}}, \Gamma^{\mathbf{B}}]/4 \qquad \mathbf{A}, \mathbf{B} = 0, 1, 2, 3, 5, \cdots, 18$$

$$\Sigma^{\mathbf{A}19} = -\Sigma^{19\mathbf{A}} = i\Gamma^{\mathbf{A}}/2$$

Invariance under discrete symmetries: charge-conjugation C parity-inversion P & time-reversal T in the hyper-spacetime

$$\begin{aligned}
\mathcal{C}\Psi(\hat{x})\mathcal{C}^{-1} &= C_{19}\bar{\Psi}^T(\hat{x}) = \Psi(\hat{x}) \,, \; C_{19}^{-1}\Gamma^{\mathbf{A}}C_{19} = (\Gamma^{\mathbf{A}})^T \\
\mathcal{P}\Psi(\hat{x})\mathcal{P}^{-1} &= P_{19}\Psi(\tilde{x}) \,, \quad P_{19}^{-1}\Gamma^{\mathbf{A}}P_{19} = (\Gamma^{\mathbf{A}})^{\dagger}, \\
\mathcal{T}\Psi(\hat{x})\mathcal{T}^{-1} &= T_{19}\Psi(-\tilde{x}) \,, \quad T_{19}^{-1}\Gamma^{\mathbf{A}}T_{19} = (\Gamma^{\mathbf{A}})^T \,,
\end{aligned}$$

$$C_{19} = \Gamma_2\Gamma_0\Gamma_6\Gamma_8\Gamma_{10}\Gamma_{12}\Gamma_{14}\Gamma_{16}\Gamma_{18} = i\sigma_3 \otimes \sigma_2 \otimes C_{14}$$

$$P_{19} = \Gamma_0 \qquad \tilde{x} \equiv (x^0, -x^1, \cdots, -x^{18}, x^{19})$$

$$T_{19} = i\Gamma_1\Gamma_3\Gamma_5\Gamma_7\Gamma_9\Gamma_{11}\Gamma_{13}\Gamma_{15}\Gamma_{17}\gamma_{19}$$

Generalized Dirac equation for a unified hyper-spinor field

$$\Gamma^{\mathrm{A}}\,\delta_{\mathrm{A}}{}^{\mathrm{M}} i\partial_{\mathrm{M}}\Psi(\hat{x}) = 0$$

$$\eta^{\mathrm{MN}}\partial_{\mathrm{M}}\partial_{\mathrm{N}}\Psi(\hat{x}) = 0$$

$$\eta^{\mathrm{MN}} = \mathrm{diag}(1,-1,\cdots,-1)$$

$$SP(1, D_h - 1) \cong SO(1, D_h - 1)$$

A fundamental interaction is postulated to be governed by taking SP(1, D_h-1) as a local hyper-spin gauge symmetry

$$\mathcal{A}_{\mathrm{M}}(\hat{x}) = \mathcal{A}_{\mathrm{M}}^{\mathrm{BC}}(\hat{x})\,\frac{1}{2}\Sigma_{\mathrm{BC}}$$

$$\hat{\chi}_{\mathrm{A}}{}^{\mathrm{M}}(\hat{x})$$

Hyper-spin gauge field & Bicovariant vector field

$$I_H = \int [d\hat{x}]\,\phi^{D_h-4}\chi(\hat{x})\frac{1}{2}\bar{\Psi}(\hat{x})\Gamma^{\mathrm{A}}\,\hat{\chi}_{\mathrm{A}}{}^{\mathrm{M}}(\hat{x})i\mathcal{D}_{\mathrm{M}}\Psi(\hat{x})$$

ϕ–scaling scalar field

$$x'^{\mathrm{M}} = \lambda^{-1}x^{\mathrm{M}}\,; \quad \mathcal{A}'_{\mathrm{M}}(\hat{x}') = \lambda\mathcal{A}_{\mathrm{M}}(\hat{x})$$
$$\Psi'(\hat{x}') = \lambda^{3/2}\Psi(\hat{x})\,;\; \phi'(\hat{x}') = \lambda\phi(\hat{x})\,.$$

Global conformal scaling symmetry

Local conformal scaling gauge symmetry

$$\hat{\chi}'_{\mathrm{A}}{}^{\mathrm{M}}(\hat{x}) = \xi(\hat{x})\hat{\chi}_{\mathrm{A}}{}^{\mathrm{M}}(\hat{x})\,;\; \chi'(\hat{x}) = \xi^{-D_h}(\hat{x})\chi(\hat{x})$$
$$\Psi'(\hat{x}) = \xi^{3/2}(\hat{x})\Psi(\hat{x})\,;\; \phi'(\hat{x}) = \xi(\hat{x})\phi(\hat{x})\,.$$

Maximal global and local symmetries of UFT

$$G = P(1, D_h\text{-}1) \times S(1) \times SP(1, D_h\text{-}1) \times SG(1)$$

$$\mathrm{P}(1,D_h\text{-}1) = \mathrm{SO}(1,D_h\text{-}1) \ltimes P^{1,D_h\text{-}1}$$

$$\Gamma^{\mathbf{A}} \delta_{\mathbf{A}}{}^{\mathrm{M}} i\partial_{\mathrm{M}} \to \Gamma^{\mathbf{A}} \hat{\chi}_{\mathbf{A}}{}^{\mathrm{M}}(\hat{x}) i\mathcal{D}_{\mathrm{M}}$$

Gravitational relativistic quantum equation for a unified hyper-spinor field

$$\varGamma^{\mathbf{A}} \hat{\chi}_{\mathbf{A}}{}^{\mathbf{M}}(\hat{x}) i(\mathcal{D}_{\mathbf{M}} + \mathsf{V}_{\mathbf{M}}(\hat{x}))\varPsi(\hat{x}) = 0$$

$$\mathsf{V}_{\mathrm{M}}(\hat{x}) = \frac{1}{2}\partial_{\mathrm{M}} \ln(\chi\phi^{D_h-3}) - \frac{1}{2}\hat{\chi}_{\mathrm{B}}{}^{\mathrm{N}} \mathcal{D}_{\mathrm{N}} \chi_{\mathrm{M}}{}^{\mathrm{B}}$$

Induced-gauge field→invar.CSG

$$\mathcal{D}_{\mathrm{M}} \chi_{\mathrm{N}}{}^{\mathbf{A}} = (\partial_{\mathrm{M}} + \partial_{\mathrm{M}} \ln\phi)\chi_{\mathrm{N}}{}^{\mathbf{A}} + \mathcal{A}_{\mathrm{M}\,\mathbf{B}}^{\mathbf{A}} \chi_{\mathrm{N}}{}^{\mathbf{B}}$$

Dual bicovariant vector field

$$\chi_{\mathrm{M}}{}^{\mathbf{A}}(\hat{x})\, \hat{\chi}_{\mathbf{B}}{}^{\mathrm{M}}(\hat{x}) = \eta_{\mathbf{B}}{}^{\mathbf{A}}$$

$$\chi_{\mathrm{M}}{}^{\mathbf{A}}(\hat{x}) \hat{\chi}_{\mathbf{A}}{}^{\mathrm{N}}(\hat{x}) = \eta_{\mathrm{M}}{}^{\mathrm{N}}$$

$\chi_{\mathrm{M}}{}^{\mathbf{A}}(\hat{x})$ **Gravitational interaction → hyper-gravifield**

Gauge invariant quadratic form for the equation of motion of a hyper-spinor field

$$\hat{\chi}^{\mathrm{MN}}(\nabla_{\mathrm{M}} + \mathrm{V}_{\mathrm{M}})(\mathcal{D}_{\mathrm{N}} + \mathrm{V}_{\mathrm{N}})\Psi = \Sigma^{\mathrm{AB}}\hat{\chi}_{\mathrm{A}}{}^{\mathrm{M}}\hat{\chi}_{\mathrm{B}}{}^{\mathrm{N}} \cdot [\,\mathcal{F}_{\mathrm{MN}} + i\mathcal{V}_{\mathrm{MN}} - \mathcal{G}^{\mathrm{C}}_{\mathrm{MN}}\hat{\chi}_{\mathrm{C}}{}^{\mathrm{P}} i(\mathcal{D}_{\mathrm{P}} + \mathrm{V}_{\mathrm{P}})\,]\Psi\,,$$

$$\nabla_{\mathrm{M}}(\mathcal{D}_{\mathrm{N}} + \mathrm{V}_{\mathrm{N}}) \equiv \mathcal{D}_{\mathrm{M}}(\mathcal{D}_{\mathrm{N}} + \mathrm{V}_{\mathrm{N}}) + \Gamma^{\mathrm{P}}_{\mathrm{MN}}(\mathcal{D}_{\mathrm{P}} + \mathrm{V}_{\mathrm{P}})$$

$$\hat{\chi}^{\mathrm{MN}} = \hat{\chi}_{\mathrm{A}}{}^{\mathrm{M}}\hat{\chi}_{\mathrm{B}}{}^{\mathrm{N}}\eta^{\mathrm{AB}} \qquad \Gamma^{\mathrm{P}}_{\mathrm{MN}} = \hat{\chi}_{\mathrm{A}}{}^{\mathrm{P}}\mathcal{D}_{\mathrm{M}}\chi_{\mathrm{N}}{}^{\mathrm{A}}$$

Field strengths of gauge field

$$\mathcal{F}_{\mathrm{MN}} = \partial_{\mathrm{M}}\mathcal{A}_{\mathrm{N}} - \partial_{\mathrm{N}}\mathcal{A}_{\mathrm{M}} - i[\mathcal{A}_{\mathrm{M}}, \mathcal{A}_{\mathrm{N}}] \equiv \mathcal{F}^{\mathrm{AB}}_{\mathrm{MN}}\frac{1}{2}\Sigma_{\mathrm{AB}}$$

$$\mathcal{V}_{\mathrm{MN}} = (\partial_{\mathrm{M}}\mathrm{V}_{\mathrm{N}} - \partial_{\mathrm{N}}\mathrm{V}_{\mathrm{M}}) \qquad \mathcal{G}^{\mathrm{A}}_{\mathrm{MN}} = \mathcal{D}_{\mathrm{M}}\chi_{\mathrm{N}}{}^{\mathrm{A}} - \mathcal{D}_{\mathrm{N}}\chi_{\mathrm{M}}{}^{\mathrm{A}}$$

$$\digamma_{\mathrm{M}}(\hat{x}) = \chi^{\mathrm{A}}_{\mathrm{M}}(\hat{x})\frac{1}{2}\Gamma_{\mathrm{A}}$$

$\chi_{\mathrm{M}}{}^{\mathrm{A}}$ Gauge-type hyper-gravifield

Hyper-gravifield fiber bundle structure of spacetime

Dual hyper-spacetime of coordinates

Dual coordinate basis

$$\{\partial_{\mathrm{M}}\} \equiv \{\partial/\partial x^{\mathrm{M}}\}$$

$$\{dx^{\mathrm{M}}\}$$

$$< dx^{\mathrm{M}},\, \partial/\partial x^{\mathrm{N}} >= \frac{\partial x^{\mathrm{M}}}{\partial x^{\mathrm{N}}} = \eta_{\mathrm{N}}^{\mathrm{M}}$$

Dual hyper-gravifield spacetime of non-coordinate system

$$\eth_{\mathbf{A}} \equiv \hat{\chi}_{\mathbf{A}}(\hat{x}) = \hat{\chi}_{\mathbf{A}}{}^{\mathrm{M}}(\hat{x})\partial_{\mathrm{M}}$$

$$\delta\chi^{\mathbf{A}} \equiv \chi^{\mathbf{A}}(\hat{x}) = \chi_{\mathrm{M}}{}^{\mathbf{A}}(\hat{x})dx^{\mathrm{M}}$$

Dual hyper-gravifield basis

$$\{\eth_{\mathbf{A}}\}$$

$$\{\delta\chi_{\mathbf{A}}\}$$

$$\langle\delta\chi^{\mathbf{A}}, \eth_{\mathbf{B}}\rangle = \chi_{\mathrm{M}}{}^{\mathbf{A}}(\hat{x})\hat{\chi}_{\mathbf{B}}{}^{\mathrm{N}}(\hat{x})\langle dx^{\mathrm{M}}, \partial_{\mathrm{N}}\rangle = \eta_{\mathbf{B}}{}^{\mathbf{A}}$$

Noncommutative geometry via non-commutation relation

$$[\eth_{\mathbf{A}},\, \eth_{\mathbf{B}}] = f_{\mathbf{AB}}^{\mathbf{C}}\,\eth_{\mathbf{C}}$$

$$f_{\mathbf{AB}}^{\mathbf{C}} \equiv -\hat{\chi}_{\mathbf{A}}{}^{\mathrm{M}}\hat{\chi}_{\mathbf{B}}{}^{\mathrm{N}}\mathsf{G}_{\mathrm{MN}}^{\mathbf{C}}$$

$$\mathsf{G}_{\mathrm{MN}}^{\mathbf{C}}(\hat{x}) = \partial_{\mathrm{M}}\chi_{\mathrm{N}}{}^{\mathbf{C}}(\hat{x}) - \partial_{\mathrm{N}}\chi_{\mathrm{M}}{}^{\mathbf{C}}(\hat{x})$$

Gravitational field strength

Non-coordinate exterior differential operator in the hyper-gravifield spacetime

$$d_\chi = \chi^{\mathbf{A}}(\hat{x}) \wedge \hat{\chi}_{\mathbf{A}}(\hat{x}) \equiv \delta\chi^{\mathbf{A}} \wedge \eth_{\mathbf{A}}$$

Gauge fields & field strengths as an one-form & two-form in the hyper-gravifield spacetime

$$\mathcal{A} = -i\mathcal{A}_{\mathbf{A}}\,\delta\chi^{\mathbf{A}}$$

$$\mathcal{F} = d_\chi\,\mathcal{A} + \mathcal{A}\wedge\mathcal{A} = \frac{1}{2i}\mathcal{F}_{\mathbf{AB}}\,\delta\chi^{\mathbf{A}}\wedge\delta\chi^{\mathbf{B}}$$

$$\mathsf{F} = -i\mathsf{F}_{\mathbf{A}}\,\delta\chi^{\mathbf{A}}$$

$$\mathcal{G} = d_\chi\,\mathsf{F} + \mathcal{A}\wedge\mathsf{F} + \mathsf{W}\wedge\mathsf{F} = \frac{1}{2i}\mathcal{G}_{\mathbf{AB}}\,\delta\chi^{\mathbf{A}}\wedge\delta\chi^{\mathbf{B}}$$

$$\mathsf{W} = -i\mathsf{W}_{\mathbf{A}}\,\delta\chi^{\mathbf{A}}$$

$$\mathcal{W} = d_\chi\,\mathsf{W} = \frac{1}{2i}\mathcal{W}_{\mathbf{AB}}\,\delta\chi^{\mathbf{A}}\wedge\delta\chi^{\mathbf{B}}$$

Covariant derivative as one-form & Hodge star *

$$\mathcal{D} \equiv \chi^{\mathbf{A}}(\hat{x})\mathcal{D}_{\mathbf{A}} = \delta\chi^{\mathbf{A}}\,(\eth_{\mathbf{A}} - i\mathcal{A}_{\mathbf{A}})$$

$$\begin{aligned} *\mathcal{F} \;=\; & \frac{1}{2!(D_h-2)!2i}\varepsilon_{\mathbf{A}_1\mathbf{A}_2\mathbf{A}_3\cdots\mathbf{A}_{D_h}} \\ & \eta^{\mathbf{A}_1\mathbf{A}'_1}\eta^{\mathbf{A}_2\mathbf{A}'_2}\mathcal{F}_{\mathbf{A}'_1\mathbf{A}'_2}\,\delta\chi^{\mathbf{A}_3}\wedge\cdots\wedge\delta\chi^{\mathbf{A}_{D_h}} \end{aligned}$$

Gauge invariant & coordinate independent unified field theory in the hyper-gravifield spacetime

$$
\begin{aligned}
I_H = \int \{ \phi^{D_h-4} [& i\bar{\Psi}\, F \wedge *\mathcal{D}\Psi - \frac{1}{2g_w^2} \mathcal{W} \wedge *\mathcal{W} \\
& - \frac{4}{D_h} \sum_{k=0}^{D_h-3} \alpha_k \,\mathrm{Tr}(\mathcal{F} \wedge F^k) \wedge *(\mathcal{F} \wedge F^k) \\
& + \frac{2}{D_h} \phi^2 (\sum_{k=0}^{D_h-3} \beta_k \,\mathrm{Tr}(\mathcal{G} \wedge F^k) \wedge *(\mathcal{G} \wedge F^k) \\
& - 2\alpha_E\, Tr\, \mathcal{F} \wedge *(F \wedge F)) - \frac{1}{2} d\phi \wedge *d\phi \\
& + \frac{4}{D_h} \beta_E \phi^4\, Tr\, (F \wedge F) \wedge *(F \wedge F)] \} .
\end{aligned}
$$

W_M Weyl gauge field introduced to characterize a conformal scaling gauge invariant dynamics of the scaling scalar field

Gauge invariant & coordinate independent unified field theory in the hyper-gravifield spacetime

$$
\begin{aligned}
I_H \;\equiv\; & \int [\delta\chi]\, \mathfrak{L} = \int [\delta\chi]\, \phi^{D_h-4}\{ \frac{1}{2}\bar{\Psi}\Gamma^{\mathbf{C}} i\mathcal{D}_{\mathbf{C}}\Psi \\
- \;& \frac{1}{4}\,[\, g_h^{-2}\tilde{\eta}^{\mathbf{CDC'D'}}_{\mathbf{ABA'B'}} \mathcal{F}^{\mathbf{AB}}_{\mathbf{CD}} \mathcal{F}^{\mathbf{A'B'}}_{\mathbf{C'D'}} + \mathcal{W}_{\mathbf{CD}}\mathcal{W}^{\mathbf{CD}}\,] \\
+ \;& \alpha_E \phi^2 [\frac{1}{4}\,\tilde{\eta}^{\mathbf{CDC'D'}}_{\mathbf{AA'}} \mathcal{G}^{\mathbf{A}}_{\mathbf{CD}} \mathcal{G}^{\mathbf{A'}}_{\mathbf{C'D'}} - \eta^{\mathbf{C}}_{\mathbf{A}}\eta^{\mathbf{D}}_{\mathbf{B}} \mathcal{F}^{\mathbf{AB}}_{\mathbf{CD}}] \\
+ \;& \frac{1}{2}\eta^{\mathbf{CD}} \bar{\eth}_{\mathbf{C}}\phi \bar{\eth}_{\mathbf{D}}\phi - \beta_E \phi^4 \}\,,
\end{aligned}
$$

$$i\mathcal{D}_{\mathbf{C}} \equiv i\eth_{\mathbf{C}} + \mathcal{A}_{\mathbf{C}}$$

$$\bar{\eth}_{\mathbf{C}}\phi = (\eth_{\mathbf{C}} - \mathsf{W}_{\mathbf{C}})\phi$$

$$\mathcal{F}^{\mathbf{AB}}_{\mathbf{CD}} = \tilde{\mathcal{D}}_{\mathbf{C}}\mathcal{A}^{\mathbf{AB}}_{\mathbf{D}} - \tilde{\mathcal{D}}_{\mathbf{D}}\mathcal{A}^{\mathbf{AB}}_{\mathbf{C}} + (\mathcal{A}^{\mathbf{A}}_{\mathbf{CE}}\mathcal{A}^{\mathbf{EB}}_{\mathbf{D}} - \mathcal{A}^{\mathbf{A}}_{\mathbf{DE}}\mathcal{A}^{\mathbf{EB}}_{\mathbf{C}})$$

$$\tilde{\mathcal{D}}_{\mathbf{C}}\mathcal{A}^{\mathbf{AB}}_{\mathbf{D}} = \eth_{\mathbf{C}}\mathcal{A}^{\mathbf{AB}}_{\mathbf{D}} - \Omega^{\mathbf{E}}_{\mathbf{CD}}\mathcal{A}^{\mathbf{AB}}_{\mathbf{E}}$$

$$\mathcal{W}_{\mathbf{CD}} = \tilde{\mathcal{D}}_{\mathbf{C}}\mathsf{W}_{\mathbf{D}} - \tilde{\mathcal{D}}_{\mathbf{D}}\mathsf{W}_{\mathbf{C}}$$

$$\tilde{\mathcal{D}}_{\mathbf{C}}\mathsf{W}_{\mathbf{D}} = \eth_{\mathbf{C}}\mathsf{W}_{\mathbf{D}} - \Omega^{\mathbf{E}}_{\mathbf{CD}}\mathsf{W}_{\mathbf{E}}$$

Tensor structure to achieve general conformal scaling gauge invariance

$$
\begin{aligned}
\tilde{\eta}^{\mathbf{CDC'D'}}_{\mathbf{ABA'B'}} &\equiv \frac{1}{4}\{ [\eta^{\mathbf{CC'}}\eta_{\mathbf{AA'}}(\eta^{\mathbf{DD'}}\eta_{\mathbf{BB'}} - 2\eta^{\mathbf{D}}_{\mathbf{B'}}\eta^{\mathbf{D'}}_{\mathbf{B}}) \\
&+ \eta^{(\mathbf{C},\mathbf{C'}\leftrightarrow \mathbf{D},\mathbf{D'})}] + \eta_{(\mathbf{A},\mathbf{A'}\leftrightarrow \mathbf{B},\mathbf{B'})} \} \\
&+ \frac{1}{4}\alpha_W \{ [(\eta^{\mathbf{C}}_{\mathbf{A'}}\eta^{\mathbf{C'}}_{\mathbf{A}} - 2\eta^{\mathbf{CC'}}\eta_{\mathbf{AA'}})\eta^{\mathbf{D}}_{\mathbf{B'}}\eta^{\mathbf{D'}}_{\mathbf{B}} \\
&+ \eta^{(\mathbf{C},\mathbf{C'}\leftrightarrow \mathbf{D},\mathbf{D'})}] + \eta_{(\mathbf{A},\mathbf{A'}\leftrightarrow \mathbf{B},\mathbf{B'})} \} \\
&+ \frac{1}{2}\beta_W \{ [(\eta_{\mathbf{AA'}}\eta^{\mathbf{CC'}} - \eta^{\mathbf{C'}}_{\mathbf{A}}\eta^{\mathbf{C}}_{\mathbf{A'}})\eta^{\mathbf{D}}_{\mathbf{B}}\eta^{\mathbf{D'}}_{\mathbf{B'}} \\
&+ \eta^{(\mathbf{C},\mathbf{C'}\leftrightarrow \mathbf{D},\mathbf{D'})}] + \eta_{(\mathbf{A},\mathbf{A'}\leftrightarrow \mathbf{B},\mathbf{B'})} \} ,
\end{aligned}
$$

Tensor structure to arrive at a general hyper-spin gauge symmetry

$$
\begin{aligned}
\tilde{\eta}^{\mathbf{CDC'D'}}_{\mathbf{AA'}} &\equiv \eta^{\mathbf{CC'}}\eta^{\mathbf{DD'}}\eta_{\mathbf{AA'}} + \eta^{\mathbf{CC'}}(\eta^{\mathbf{D}}_{\mathbf{A'}}\eta^{\mathbf{D'}}_{\mathbf{A}} - 2\eta^{\mathbf{D}}_{\mathbf{A}}\eta^{\mathbf{D'}}_{\mathbf{A'}}) \\
&+ \eta^{\mathbf{DD'}}(\eta^{\mathbf{C}}_{\mathbf{A'}}\eta^{\mathbf{C'}}_{\mathbf{A}} - 2\eta^{\mathbf{C}}_{\mathbf{A}}\eta^{\mathbf{C'}}_{\mathbf{A'}}) .
\end{aligned}
$$

Gauge Invariant Unified Field Theory for All Basic Forces in Hyper-spacetime

With the projection $\chi_M{}^A$

$\mathcal{A}_{\mathbf{C}}^{\mathbf{AB}} = \hat{\chi}_{\mathbf{C}}{}^{\mathbf{M}} \mathcal{A}_{\mathbf{M}}^{\mathbf{AB}}$

$W_C = \hat{\chi}_C{}^M W_M$

$\mathcal{F}_{\mathbf{CD}}^{\mathbf{AB}} = \mathcal{F}_{\mathbf{MN}}^{\mathbf{AB}} \hat{\chi}_{\mathbf{C}}{}^{\mathbf{M}} \hat{\chi}_{\mathbf{D}}{}^{\mathbf{N}}$

$\mathcal{G}_{\mathbf{CD}}^{\mathbf{A}} \equiv \mathcal{G}_{\mathbf{MN}}^{\mathbf{A}} \hat{\chi}_{\mathbf{C}}{}^{\mathbf{M}} \hat{\chi}_{\mathbf{D}}{}^{\mathbf{N}}$

$\mathcal{W}_{\mathbf{CD}} \equiv \mathcal{W}_{\mathbf{MN}} \hat{\chi}_{\mathbf{C}}{}^{\mathbf{M}} \hat{\chi}_{\mathbf{D}}{}^{\mathbf{N}}$

$$
\begin{aligned}
I_H \equiv \int [d\hat{x}]\chi \{ & \frac{1}{2}\bar{\Psi}\Gamma^{\mathbf{A}}\hat{\chi}_{\mathbf{A}}{}^{\mathbf{M}}(i\partial_{\mathbf{M}} + g_h \mathcal{A}_{\mathbf{M}})\Psi \\
& +\phi^{D_h-4}[-\frac{1}{4}(\tilde{\chi}_{\mathbf{ABA'B'}}^{\mathbf{MNM'N'}} \mathcal{F}_{\mathbf{MN}}^{\mathbf{AB}} \mathcal{F}_{\mathbf{M'N'}}^{\mathbf{A'B'}} + \mathcal{W}_{\mathbf{MN}}\mathcal{W}^{\mathbf{MN}}) \\
& +\alpha_E \phi^2 \frac{1}{4}\tilde{\chi}_{\mathbf{AA'}}^{\mathbf{MNM'N'}} \mathbf{G}_{\mathbf{MN}}^{\mathbf{A}} \mathbf{G}_{\mathbf{M'N'}}^{\mathbf{A'}} + \frac{1}{2}\hat{\chi}^{\mathbf{MN}} d_{\mathbf{M}}\phi d_{\mathbf{N}}\phi \\
& -\beta_E \phi^4]\} + 2\alpha_E g_h \partial_{\mathbf{M}}(\chi\phi^{D_h-2}\mathcal{A}_{\mathbf{N}}^{\mathbf{NM}}),
\end{aligned}
$$

$\tilde{\chi}_{\mathbf{ABA'B'}}^{\mathbf{MNM'N'}} \equiv \hat{\chi}_{\mathbf{C}}{}^{\mathbf{M}} \hat{\chi}_{\mathbf{D}}{}^{\mathbf{N}} \hat{\chi}_{\mathbf{C'}}{}^{\mathbf{M'}} \hat{\chi}_{\mathbf{D'}}{}^{\mathbf{N'}} \tilde{\eta}_{\mathbf{ABA'B'}}^{\mathbf{CDC'D'}}$

$\tilde{\chi}_{\mathbf{AA'}}^{\mathbf{MNM'N'}} \equiv \hat{\chi}_{\mathbf{C}}{}^{\mathbf{M}} \hat{\chi}_{\mathbf{D}}{}^{\mathbf{N}} \hat{\chi}_{\mathbf{C'}}{}^{\mathbf{M'}} \hat{\chi}_{\mathbf{D'}}{}^{\mathbf{N'}} \tilde{\eta}_{\mathbf{AA'}}^{\mathbf{CDC'D'}}$

$\mathbf{G}_{\mathbf{MN}}^{\mathbf{A}} = \hat{\partial}_{\mathbf{M}}\chi_{\mathbf{N}}^{\mathbf{A}} - \hat{\partial}_{\mathbf{N}}\chi_{\mathbf{M}}^{\mathbf{A}}$

Gauge gravitational interaction

Gauge gravity correspondence 规范引力对应!

Gravitational Origin of Gauge Symmetry

Decompose of hyper-spin gauge field

$$\mathcal{A}_{\mathrm{M}}^{\mathrm{AB}} \equiv \Omega_{\mathrm{M}}^{\mathrm{AB}} + \mathbf{A}_{\mathrm{M}}^{\mathrm{AB}}$$

Hyper-spin gravigauge field

$$\Omega_{\mathrm{M}}^{\mathrm{AB}} = \frac{1}{2}[\hat{\chi}^{\mathrm{AN}}\mathrm{G}_{\mathrm{MN}}^{\mathrm{B}} - \hat{\chi}^{\mathrm{BN}}\mathrm{G}_{\mathrm{MN}}^{\mathrm{A}} - \hat{\chi}^{\mathrm{AP}}\hat{\chi}^{\mathrm{BQ}}\mathrm{G}_{\mathrm{PQ}}^{\mathrm{C}}\chi_{\mathrm{MC}}]$$

$$\mathrm{G}_{\mathrm{MN}}^{\mathrm{A}} \equiv \partial_{\mathrm{M}}\chi_{\mathrm{N}}^{\mathrm{A}} - \partial_{\mathrm{N}}\chi_{\mathrm{M}}^{\mathrm{A}}$$

Inhomogeneous gauge transformation of hyper-spin gravigauge field

$$\Omega_{\mathrm{M}}^{\prime\mathrm{AB}} = \Lambda^{\mathrm{A}}_{\ \mathrm{C}}\Lambda^{\mathrm{B}}_{\ \mathrm{D}}\Omega_{\mathrm{M}}^{\mathrm{CD}} + \frac{i}{2}(\Lambda^{\mathrm{A}}_{\ \mathrm{C}}\partial_{\mathrm{M}}\Lambda^{\mathrm{BC}} - \Lambda^{\mathrm{B}}_{\ \mathrm{C}}\partial_{\mathrm{M}}\Lambda^{\mathrm{AC}})$$

$$\chi_{\mathrm{M}}^{\prime\,\mathrm{A}}(\hat{x}) = \Lambda^{\mathrm{A}}_{\ \mathrm{C}}(\hat{x})\chi_{\mathrm{M}}^{\mathrm{C}}(\hat{x})$$

$$\Lambda^{\mathrm{A}}_{\ \mathrm{C}}(\hat{x}) \in \mathrm{SP}(1,D_h\text{-}1)$$

Field strength of hyper-spin gravigauge field

$$\mathcal{R}_{\mathrm{MN}}^{\mathrm{AB}} = \partial_{\mathrm{M}}\Omega_{\mathrm{N}}^{\mathrm{AB}} - \partial_{\mathrm{N}}\Omega_{\mathrm{M}}^{\mathrm{AB}} + \Omega_{\mathrm{MC}}^{\mathrm{A}}\Omega_{\mathrm{N}}^{\mathrm{CB}} - \Omega_{\mathrm{NC}}^{\mathrm{A}}\Omega_{\mathrm{M}}^{\mathrm{CB}}$$

Field strength of hyper-spin homogauge field

$$\mathbf{F}_{\mathrm{MN}}^{\mathrm{AB}} = \mathcal{D}_{\mathrm{M}}\mathbf{A}_{\mathrm{N}}^{\mathrm{AB}} - \mathcal{D}_{\mathrm{N}}\mathbf{A}_{\mathrm{M}}^{\mathrm{AB}} + \mathbf{A}_{\mathrm{MC}}^{\mathrm{A}}\mathbf{A}_{\mathrm{N}}^{\mathrm{CB}} - \mathbf{A}_{\mathrm{NC}}^{\mathrm{A}}\mathbf{A}_{\mathrm{M}}^{\mathrm{CB}}$$

$$\mathcal{D}_{\mathrm{M}}\mathbf{A}_{\mathrm{N}}^{\mathrm{AB}} = \partial_{\mathrm{M}}\mathbf{A}_{\mathrm{N}}^{\mathrm{AB}} + \Omega_{\mathrm{MC}}^{\mathrm{A}}\mathbf{A}_{\mathrm{N}}^{\mathrm{CB}} - \Omega_{\mathrm{MC}}^{\mathrm{B}}\mathbf{A}_{\mathrm{N}}^{\mathrm{CA}}$$

Hyper-spacetime gravigauge & homogauge fields and field strengths

$$\Gamma_{MQ}^{P} \equiv \hat{\chi}_{A}{}^{P}\partial_{M}\chi_{Q}{}^{A} + \hat{\chi}_{A}{}^{P}\Omega_{MB}^{A}\chi_{Q}{}^{B}$$

$$A_{M}^{PQ} \equiv \hat{\chi}_{A}^{P}\hat{\chi}_{B}^{Q}A_{M}^{AB}$$

$$\mathcal{F}_{MN}^{AB} = \mathcal{R}_{MN}^{AB} + F_{MN}^{AB} \Longrightarrow \mathcal{F}_{MN}^{AB} = (\mathcal{R}_{MN}^{PQ} + F_{MN}^{PQ})\chi_{P}{}^{A}\chi_{Q}{}^{B}$$

Field strength of hyper-space gravigauge field

$$\mathcal{R}_{MNQ}^{P} = \partial_{M}\Gamma_{NQ}^{P} - \partial_{N}\Gamma_{MQ}^{P} + \Gamma_{ML}^{P}\Gamma_{NQ}^{L} - \Gamma_{NL}^{P}\Gamma_{MQ}^{L}$$

$$\Gamma_{MQ}^{P} = \frac{1}{2}\hat{\chi}^{PL}(\partial_{M}\chi_{QL} + \partial_{Q}\chi_{ML} - \partial_{L}\chi_{MQ})$$

Christofel symbol

$$\chi_{MN} = \chi_{M}^{A}\chi_{N}^{B}\eta_{AB}$$

hyper-gravimetric field

Field strength of hyper-space homogauge field

$$F_{MN}^{PQ} = \nabla_{M}A_{N}^{PQ} - \nabla_{N}A_{M}^{PQ} + A_{ML}^{P}A_{N}^{LQ} - A_{NL}^{P}A_{M}^{LQ}$$

$$\nabla_{M}A_{N}^{PQ} = \partial_{M}A_{N}^{PQ} + \Gamma_{ML}^{P}A_{N}^{LQ} + \Gamma_{ML}^{Q}A_{N}^{PL}$$

Unified Field Theory in Hidden Gauge Formalism

$$
\begin{aligned}
I_H = \int d\hat{x}\chi\{ & \frac{1}{2}\bar{\Psi}\Gamma^{\mathrm{M}}[i\partial_{\mathrm{M}} + (\Xi_{\mathrm{M}}^{\mathrm{PQ}} + g_h \mathbf{A}_{\mathrm{M}}^{\mathrm{PQ}})\frac{1}{2}\Sigma_{\mathrm{PQ}}]\Psi \\
& + \phi^{D_h-4}\{ -\frac{1}{4}(\tilde{\chi}_{\mathrm{PQP'Q'}}^{\mathrm{MNM'N'}} \mathbf{F}_{\mathrm{MN}}^{\mathrm{PQ}} \mathbf{F}_{\mathrm{M'N'}}^{\mathrm{P'Q'}} + \mathcal{W}_{\mathrm{MN}}\mathcal{W}^{\mathrm{MN}}) \\
& + \alpha_E[\phi^2\mathcal{R} - (D_h-1)(D_h-2)\partial_{\mathrm{M}}\phi\partial^{\mathrm{M}}\phi] - \beta_E\phi^4 \\
& + \frac{1}{2}\hat{\chi}^{\mathrm{MN}} d_{\mathrm{M}}\phi d_{\mathrm{N}}\phi\}\} + 2\alpha_E g_h \partial_{\mathrm{M}}(\chi\phi^{D_h-2}\mathbf{A}_{\mathrm{N}}^{\mathrm{NM}}),
\end{aligned}
$$

$$\Gamma^{\mathrm{M}} \equiv \hat{\chi}_{\mathrm{A}}^{\mathrm{M}}\Gamma^{\mathrm{A}}$$

$$\Xi_{\mathrm{M}}^{\mathrm{PQ}} \equiv \frac{1}{2}(\hat{\chi}_{\mathrm{C}}^{\mathrm{P}}\partial_{\mathrm{M}}\hat{\chi}^{\mathrm{QC}} - \hat{\chi}_{\mathrm{C}}^{\mathrm{Q}}\partial_{\mathrm{M}}\hat{\chi}^{\mathrm{PC}})$$

$$\tilde{\chi}_{\mathrm{PQP'Q'}}^{\mathrm{MNM'N'}} \equiv \chi_{\mathrm{P}}^{\mathrm{A}}\chi_{\mathrm{Q}}^{\mathrm{B}}\chi_{\mathrm{P'}}^{\mathrm{A'}}\chi_{\mathrm{Q'}}^{\mathrm{B'}}\hat{\chi}_{\mathrm{C}}^{\mathrm{M}}\hat{\chi}_{\mathrm{D}}^{\mathrm{N}}\hat{\chi}_{\mathrm{C'}}^{\mathrm{M'}}\hat{\chi}_{\mathrm{D'}}^{\mathrm{N'}}\tilde{\eta}_{\mathrm{ABA'B'}}^{\mathrm{CDC'D'}}$$

$$\mathcal{R} \equiv -\mathcal{R}_{\mathrm{MNQ}}^{\mathrm{P}}\eta_{\mathrm{P}}^{\mathrm{M}}\hat{\chi}^{\mathrm{NQ}}$$

Gravitational interaction is governed solely by the Einstein-Hilbert action term

Gravity geometry correspondence 引力几何对应！

Unified Field Theory in Two Equivalent Formalisms

$$I_H \equiv \int [d\hat{x}]\chi\,\{\frac{1}{2}\bar{\Psi}\Gamma^{\mathbf{A}}\hat{\chi}_{\mathbf{A}}^{\;\;\mathbf{M}}(i\partial_{\mathbf{M}} + g_h\mathcal{A}_{\mathbf{M}})\Psi$$
$$+\phi^{D_h-4}[-\frac{1}{4}(\tilde{\chi}_{\mathbf{ABA'B'}}^{\mathbf{MNM'N'}}\mathcal{F}_{\mathbf{MN}}^{\mathbf{AB}}\mathcal{F}_{\mathbf{M'N'}}^{\mathbf{A'B'}} + \mathcal{W}_{\mathbf{MN}}\mathcal{W}^{\mathbf{MN}})$$
$$+\alpha_E\phi^2\frac{1}{4}\tilde{\chi}_{\mathbf{AA'}}^{\mathbf{MNM'N'}}\mathbf{G}_{\mathbf{MN}}^{\mathbf{A}}\mathbf{G}_{\mathbf{M'N'}}^{\mathbf{A'}} + \frac{1}{2}\hat{\chi}^{\mathbf{MN}}d_{\mathbf{M}}\phi d_{\mathbf{N}}\phi$$
$$-\beta_E\phi^4]\} + 2\alpha_E g_h\partial_{\mathbf{M}}(\chi\phi^{D_h-2}\mathcal{A}_{\mathbf{N}}^{\mathbf{NM}}),$$

Gauge fixing SP(1,Dh-1) to a unitary gauge

$$\chi_{\mathbf{MA}} = \chi_{\mathbf{AM}}$$
$$\Downarrow$$
$$\chi_{\mathbf{MN}} = (\chi_{\mathbf{MA}})^2$$

$$I_H = \int d\hat{x}\chi\{\frac{1}{2}\bar{\Psi}\Gamma^{\mathbf{M}}[i\partial_{\mathbf{M}} + (\Xi_{\mathbf{M}}^{\mathbf{PQ}} + g_h\mathbf{A}_{\mathbf{M}}^{\mathbf{PQ}})\frac{1}{2}\Sigma_{\mathbf{PQ}}]\Psi$$
$$+\phi^{D_h-4}\{-\frac{1}{4}(\tilde{\chi}_{\mathbf{PQP'Q'}}^{\mathbf{MNM'N'}}\mathbf{F}_{\mathbf{MN}}^{\mathbf{PQ}}\mathbf{F}_{\mathbf{M'N'}}^{\mathbf{P'Q'}} + \mathcal{W}_{\mathbf{MN}}\mathcal{W}^{\mathbf{MN}})$$
$$+\alpha_E[\phi^2\mathcal{R} - (D_h-1)(D_h-2)\partial_{\mathbf{M}}\phi\partial^{\mathbf{M}}\phi] - \beta_E\phi^4$$
$$+\frac{1}{2}\hat{\chi}^{\mathbf{MN}}d_{\mathbf{M}}\phi d_{\mathbf{N}}\phi\}\} + 2\alpha_E g_h\partial_{\mathbf{M}}(\chi\phi^{D_h-2}\mathbf{A}_{\mathbf{N}}^{\mathbf{NM}}),$$

Equivalence of actions

Gravitational Gauge Geometry Duality 引力规范几何对偶

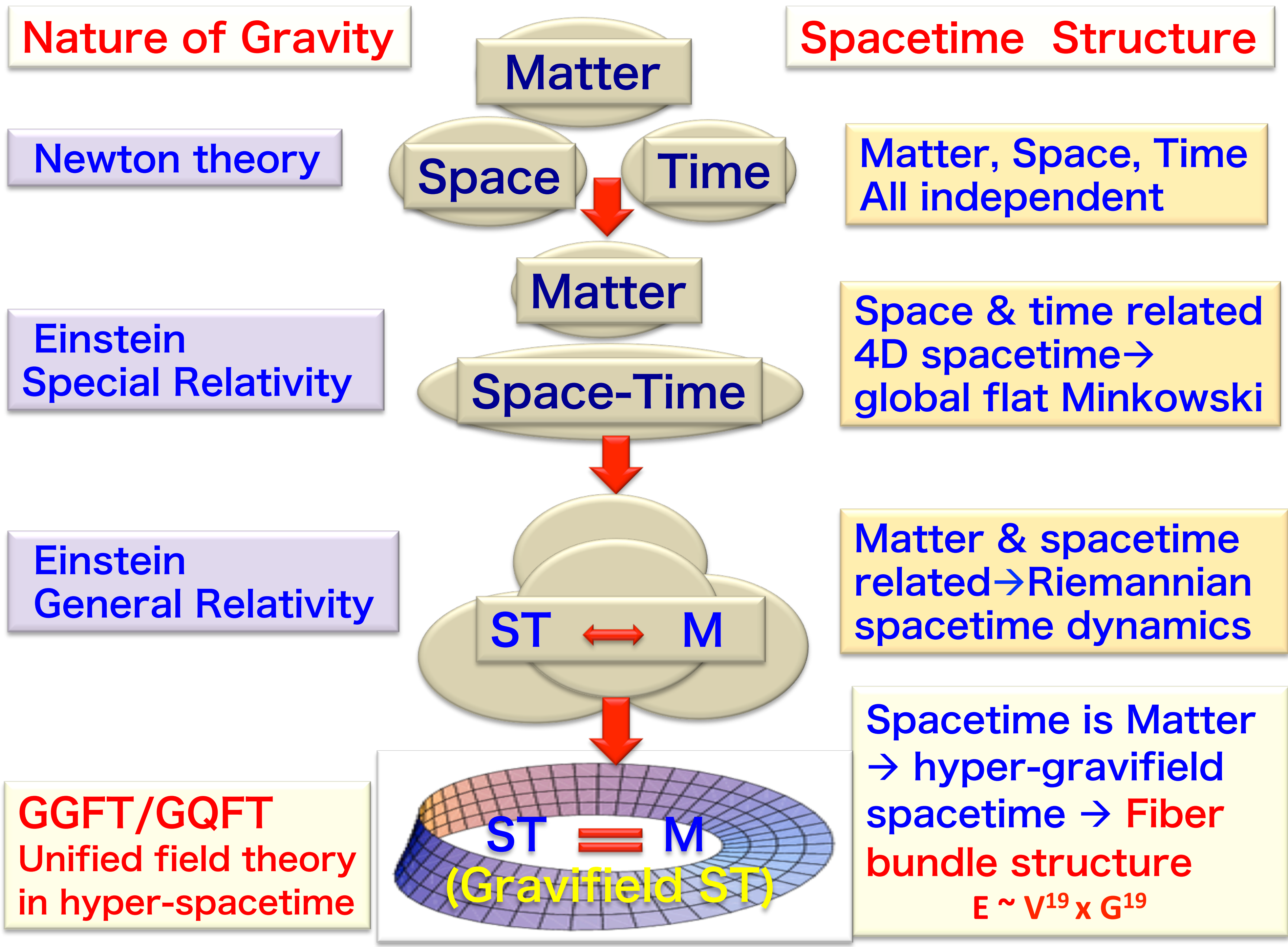
Conclusions & Remarks

The laws of nature is independent of any choice of coordinates: the postulate of gauge invariance and coordinate independence is more general and fundament than that of general coordinate invariance proposed by Einstein

Gauge symmetry has a gravitational origin: equivalence formulated in a locallly flat non-coordinate hyper-gravifield spacetime, in a globally flat coordinate Minkowski hyper-spacetime, in a hidden gauge formalism, gauge/gravity correspondence & gravity/geometry correspondence reveals a gravitational gauge geometry duality

All dimensions in the unified field theory have a physical origin due to their coherent relations to the basic quantum numbers of quarks and leptons. Unlike other extra dimensional theories including the 10D string theory and 11D M-theory.

$$\boxed{\begin{aligned} SP(1,18) \to SP(1,17) &\to SP(1,3) \times SO(10) \times SO(4) \\ &\to SP(1,3) \times SU(4) \times SU_L(2) \times SU_R(2) \\ &\to SP(1,3) \times SU(3) \times SU(2) \times U(1). \quad \text{SM} \end{aligned}}$$



Nature of Gravity
Spacetime Structure
Matter
Newton theory
Space
Time
Matter, Space, Time
All independent
Matter
Einstein
Special Relativity
Space-Time
Space & time related
4D spacetime→
global flat Minkowski
Einstein
General Relativity
ST ⟺ M
Matter & spacetime
related→Riemannian
spacetime dynamics
GGFT/GQFT
Unified field theory
in hyper-spacetime
ST = M
(Gravifield ST)
Spacetime is Matter
→ hyper-gravifield
spacetime → Fiber
bundle structure
E ~ V^{19} x G^{19}

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