Down-to-Earth String Theory

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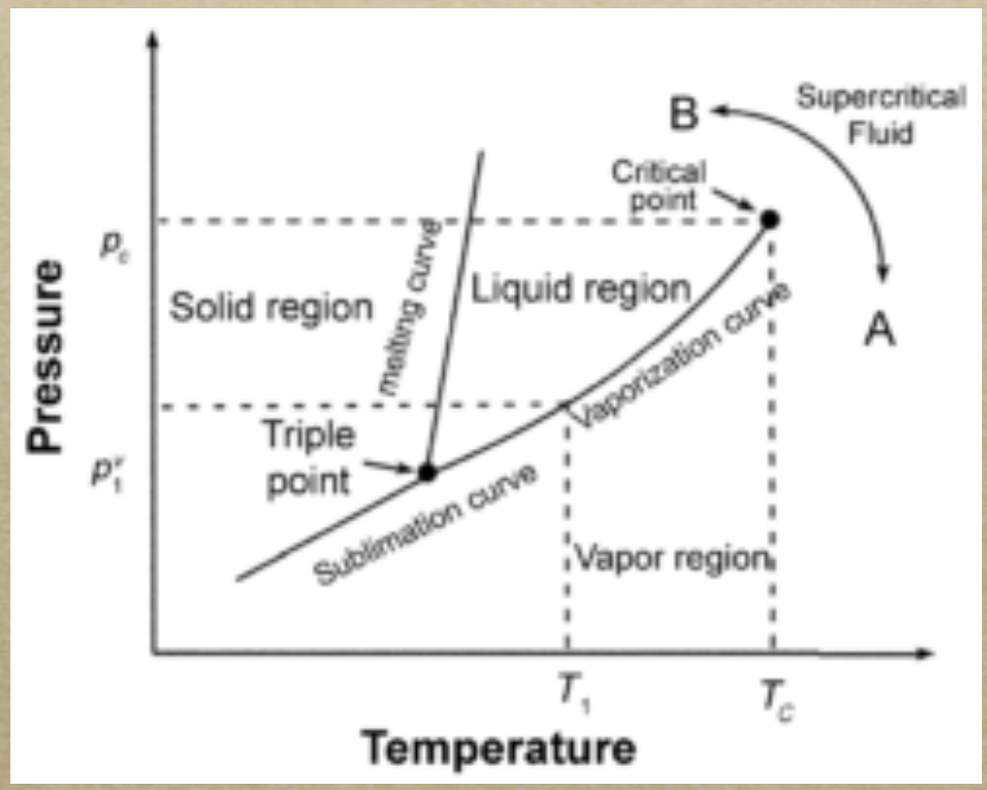
Down-to-Earth (more or less)

- *The critical point of liquid-vapour transition an archetype of a second order phase transition.*

- *Exhibits novel scaling behaviour.*

$$\xi = |T - T_c|^{-\nu}$$

$$G(x) \sim \frac{1}{|x|^{d-2+\eta}}$$



- *Universal class of systems including binary alloys, uniaxial magnetic materials - all captured by 3d Ising model.*

Universality

Universality Classes

Table 5.4.2. *Some critical exponents from theory and experiment.*

Exponent	α	β	γ	ν	η
Property	specific heat	order parameter	susceptibility	coherence length	correlation function
Definition	$C \sim t^{-\alpha}$	$\langle\phi\rangle \sim t^{\beta}$	$\chi \sim t^{-\gamma}$	$\xi \sim t^{-\nu}$	$G(q) \sim q^{-2+\eta}$
Mean-field	0	0.5	1	0.5	0
3D theory					
$n = 0$ (SAW)	0.24	0.30	1.16	0.59	
$n = 1$ (Ising)	0.11	0.32	1.24	0.63	0.04
$n = 2$ (xy)	−0.01	0.35	1.32	0.67	0.04
$n = 3$ (Heisenberg)	−0.12	0.36	1.39	0.71	0.04
Experiment					
3D $n = 1$	$0.11^{+.01}_{-.03}$	$0.32^{+.16}_{-.04}$	$1.24^{+.16}_{-.04}$	$0.63^{+.04}_{-.04}$	0.03 - 0.06
3D $n = 3$	$0.1^{+.05}_{-.04}$	$0.34^{-.04}_{-.04}$	$1.4^{+.07}_{-.07}$	$0.7^{+.03}_{-.03}$	
2D $n = 1$	$0.0^{+.01}_{-.003}$	$0.3^{+.04}_{-.04}$	$1.82^{+0.7}_{-.07}$	$1.02^{+.07}_{-.07}$	

Experiments on 3D $n = 1$ compiled from liquid-gas, binary fluid, ferromagnetic, and antiferromagnetic transitions.
Experiments on 3D $n = 3$ transitions compiled from some ferromagnetic and antiferromagnetic transitions.
Experiments on 2D $n = 1$ complied from some antiferromagnetic transitions.

From P. Chaikin and T Lubensky "Principles of Condensed Matter Physics"

Theory suggests that the class (i.e. set of exponents) depends on spatial dimensionality, symmetry of the order parameter and interaction (and range of the latter as well) but not on the detailed form or strength of the interactions

Universality and QFT

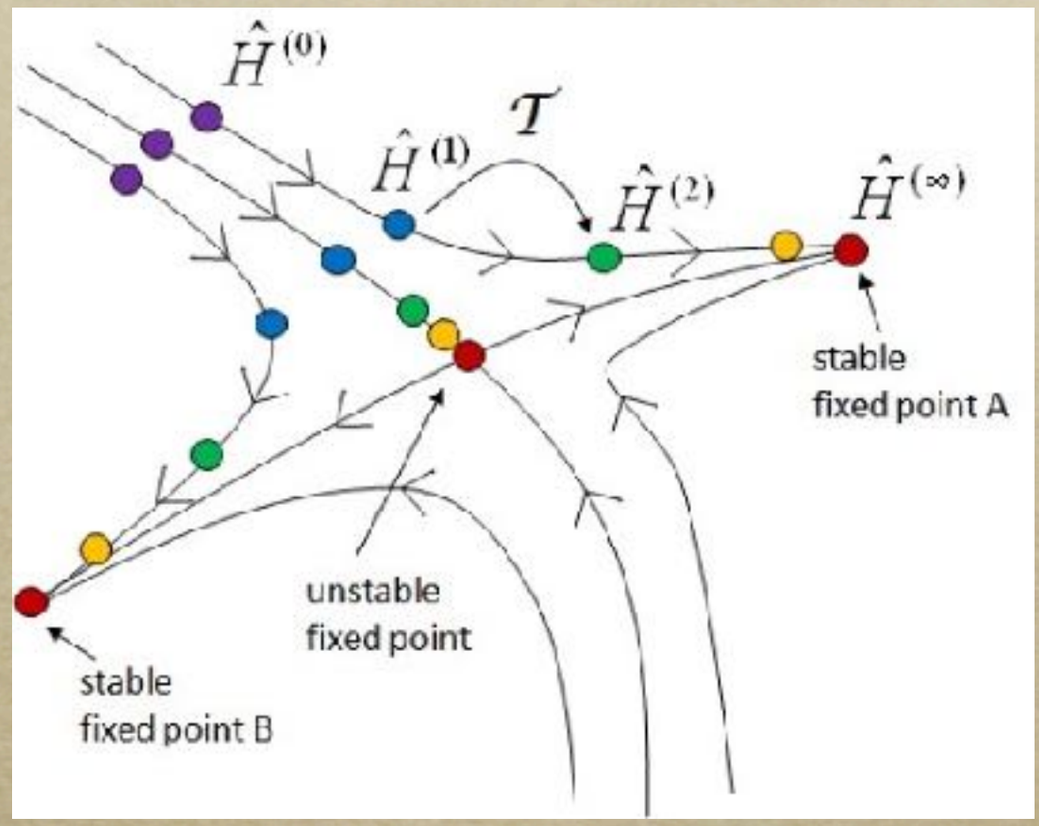
- *Universality understood from Wilson's deep insights into nature of QFT and the renormalisation group (RG).*
- *QFT describes quantum fluctuations of a continuum system. Equally well applies to statistical fluctuations.*

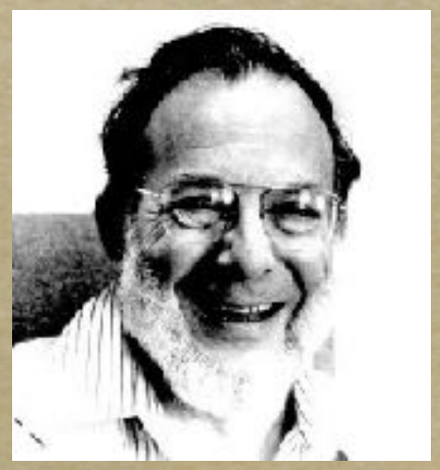
$$\int [\mathcal{D}\Phi] e^{\frac{i}{\hbar} S[\Phi]} \quad \longleftrightarrow \quad \int [\mathcal{D}\Phi] e^{-\beta \mathcal{H}[\Phi]}$$

- *Dependence of QFT on scale captured by a dynamical RG flow in the space of couplings/parameters.*
- *Critical points = fixed points of RG flow.*
- *Critical exponents determined by this fixed point.*

Fixed Points

- *Fixed points central to QFT.*
- *Fixed point theories are scale invt. or even conformally invt.*
- *Thus have more symmetry.*
- *Critical point of 3d Ising model universality class governed by the Wilson-Fisher fixed point.*
- *An interacting CFT (non-gaussian). Generalises to O(N).*





Conformal Field Theory

- *How do we try to understand the dynamics of non-gaussian CFTs? Very few techniques.*
- *Critical exponents (e.g. ν, η) captured by anomalous dimensions of operators (2-pt. fn.) at these fixed points.*
- *Other dynamical data is in 3-point functions*

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle$$

- *Conformal invariance constrains the form of these severely.*
- *Dynamical constraints (OPE) potentially fixes this.*

Enter String Theory

- *String Theory often perceived as a theoretical fantasy about 10 or 26 dimensions to do with quantum gravity.*
- *Actually, a broad and versatile framework.*
- *Gives new insights into QFT and links it in unexpected ways to GTR (General Theory of Relativity).*
- *Makes highly non-obvious, precise and falsifiable predictions for the behaviour of both QFT and GTR.*
- *Novel statements about two frameworks of 20th century physics which predate string theory.*

Holography and CFTs

- *Precise proposal by Maldacena equating dynamics of a large class of CFTs to that of gravity/strings in (asymptotically) Anti de-Sitter (AdS) spacetime.*

- *Holographic relation:* $CFT_d \leftrightarrow AdS_{d+1}$

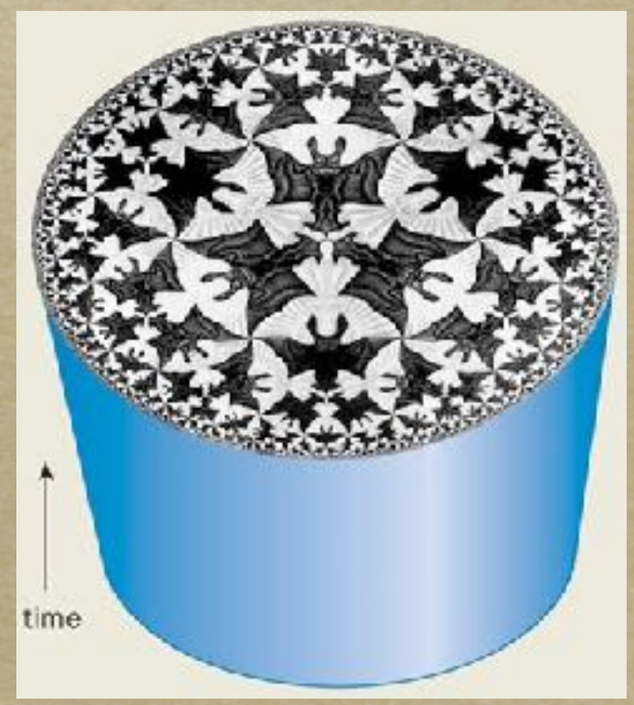
- *Dictionary between "boundary" and "bulk" observables.*

- *Anomalous dimensions* $\leftrightarrow$ *Energies*

- *Three point fns.* $\leftrightarrow$ *Cubic interactions*

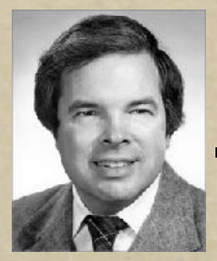
- *Questions in QFT* $\leftrightarrow$ *Gravity/Strings !!*

- *Large N limit* $\leftrightarrow$ *Classical limit.*



Minimal Holography

- *The O(N) Wilson-Fisher theory (for large N) has an* AdS_4 *dual. But the bulk description is unusual (Klebanov-Polyakov; Sezgin-Sundell).*
- *A classical theory of gravity in AdS with a single tower of interacting higher spin (s > 2) gauge fields (Vasiliev).*
- *Reproduces many known large N results in a completely novel way.*
- *A simpler analogue: large N family of interacting 2d CFTs - with an* AdS_3 *Vasiliev dual. (Gaberdiel-R.G.)*
- *Minimal model (coset) CFTs :* $\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}$
- *Generalisations of 2d Ising Model (N=2, k=1) - typically have (for N>2) higher spin conserved currents. Exactly solvable example of holography - detailed independent checks on both sides.*

Back to  - 

- *If we want to study the 3d Ising model (N=1), then large N limit not adequate for studying this CFT.*
- *Another expansion useful: Take ($d = 4 - \epsilon$) and expand in ϵ.*
- *The non-gaussian fixed point can now be studied in conventional QFT perturbation theory ($\lambda^\star = 16\pi^2 \frac{\epsilon}{3} + O(\epsilon^2)$).*
- *Through Feynman diagram techniques can compute anomalous dims. to low orders in ϵ, for O(N) theory. (Wilson-Kogut…..)*

$$\nu = \frac{6\epsilon}{N+8}[1 + \frac{3(3N+14)\epsilon}{(N+8)^2}] + O(\epsilon^3)$$

$$\eta = \frac{(N+2)\epsilon^2}{2(N+8)^2}[1 - \frac{(N^2-56N-272)\epsilon}{4(N+8)^2}] + O(\epsilon^4)$$

The Conformal Bootstrap

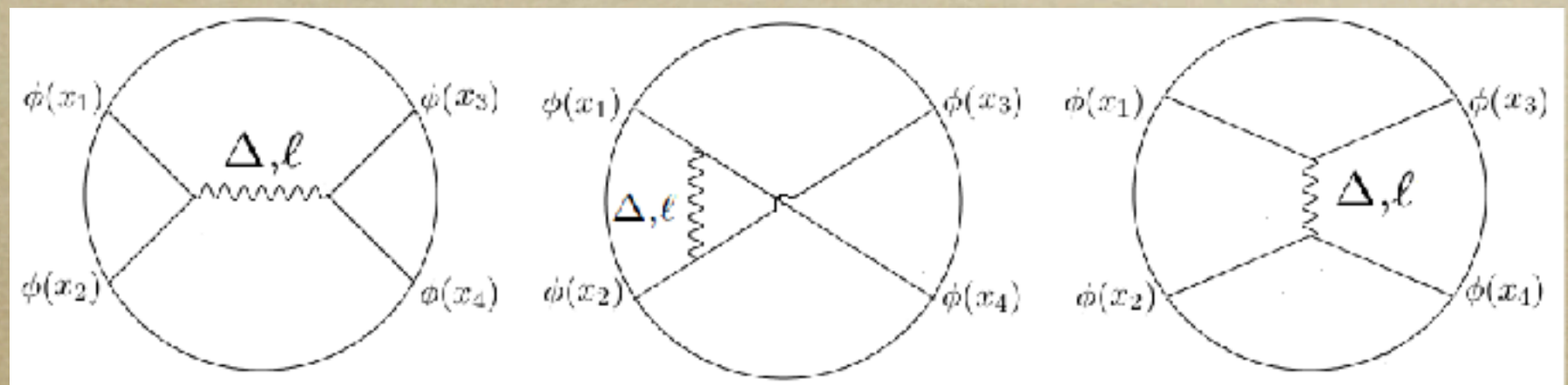
- *Proposed as an alternative to doing Feynman diagram computations. (Ferrara-Gatto-Grillo, Polyakov….)*
- *Applicable even when perturbation theory isn't.*
- *Uses the fact that a four point function may be viewed in two ways:*

- *Gives an infinite set of constraints on dims. and 3 pt. functions.*
- *Recently, implemented numerically by truncating and deriving inequalities - gives nontrivial bounds (Ratazzi-Rychkov-Tonni-Vichi).*
- *Gives the best numerical values for critical exponents in 3d !!*

Rebooting the Bootstrap

- *Very recently, proposed a new approach (R.G.-A.Kaviraj-K.Sen-A.Sinha).*
- *Instead write the four point function as a sum of AdS_{d+1} contributions.*

Inspired by AdS/CFT duality



- *Demand cancellations of spurious powers of positions.*
- *More efficient to translate to an analogue of momentum space - via a Mellin transform.*
- *Amplitudes have nice meromorphic properties.*
- *Condition is now cancellation of certain spurious poles.*

The Benefits of Bootstrapping

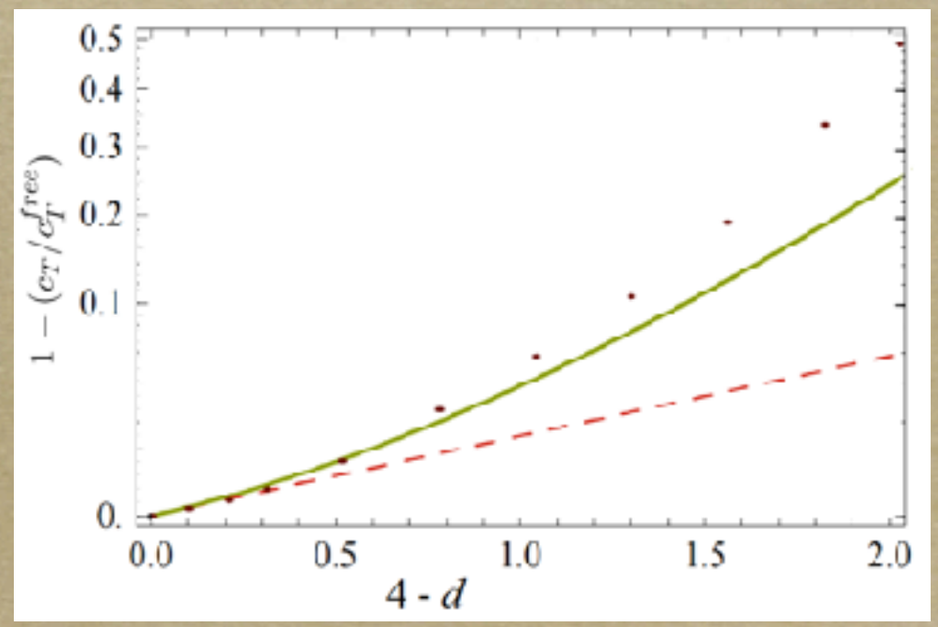
- *This new approach appears to be very effective.*
- *Easily reproduces the previously derived answers for (ν, η) !*
- *But can also go beyond and compute basic 3pt. fns. - very prohibitive to do with Feynman diagram techniques.*

- *For instance, "central charge" of the 3d Ising model is related to $C_{\ell=2} = C_{\phi\phi T}$ - 3 point function with energy-momentum tensor.*

NEW

$$\frac{c_T}{c_{free}} = 1 - \frac{5\epsilon^2}{324} - \frac{233\epsilon^3}{8748} + O(\epsilon^4)$$

String theory (methods) could help for studying properties of water !



A Sampling of Results

A. ϵ-expansion for Wilson-Fisher fixed point ($d = 4 - \epsilon$).

$$\Delta_\phi = 1 - \frac{\epsilon}{2} + \frac{1}{108}\epsilon^2 + \frac{109}{11664}\epsilon^3 + O(\epsilon^4) \quad ; \quad \Delta_{\phi^2} = 2 - \frac{2}{3}\epsilon + \frac{19}{162}\epsilon^2 + O(\epsilon^3) \quad \textit{[Wilson-Kogut]}$$

$$\gamma_\ell = \frac{\epsilon^2}{54}\left(1 - \frac{6}{\ell(\ell+1)}\right) + \epsilon^3\delta_\ell^{(3)} \qquad \left(\delta_\ell^{(3)} = \frac{[109\ell^3(\ell+2) + 373\ell^2 - 816\ell - 756] - 432\ell(\ell+1)H_{\ell-1}}{5832\ell^2(1+\ell)^2}\right)$$

$$C_{\phi\phi\phi^2} = 1 - \frac{1}{3}\epsilon - \frac{17}{81}\epsilon^2 + O(\epsilon^3)$$

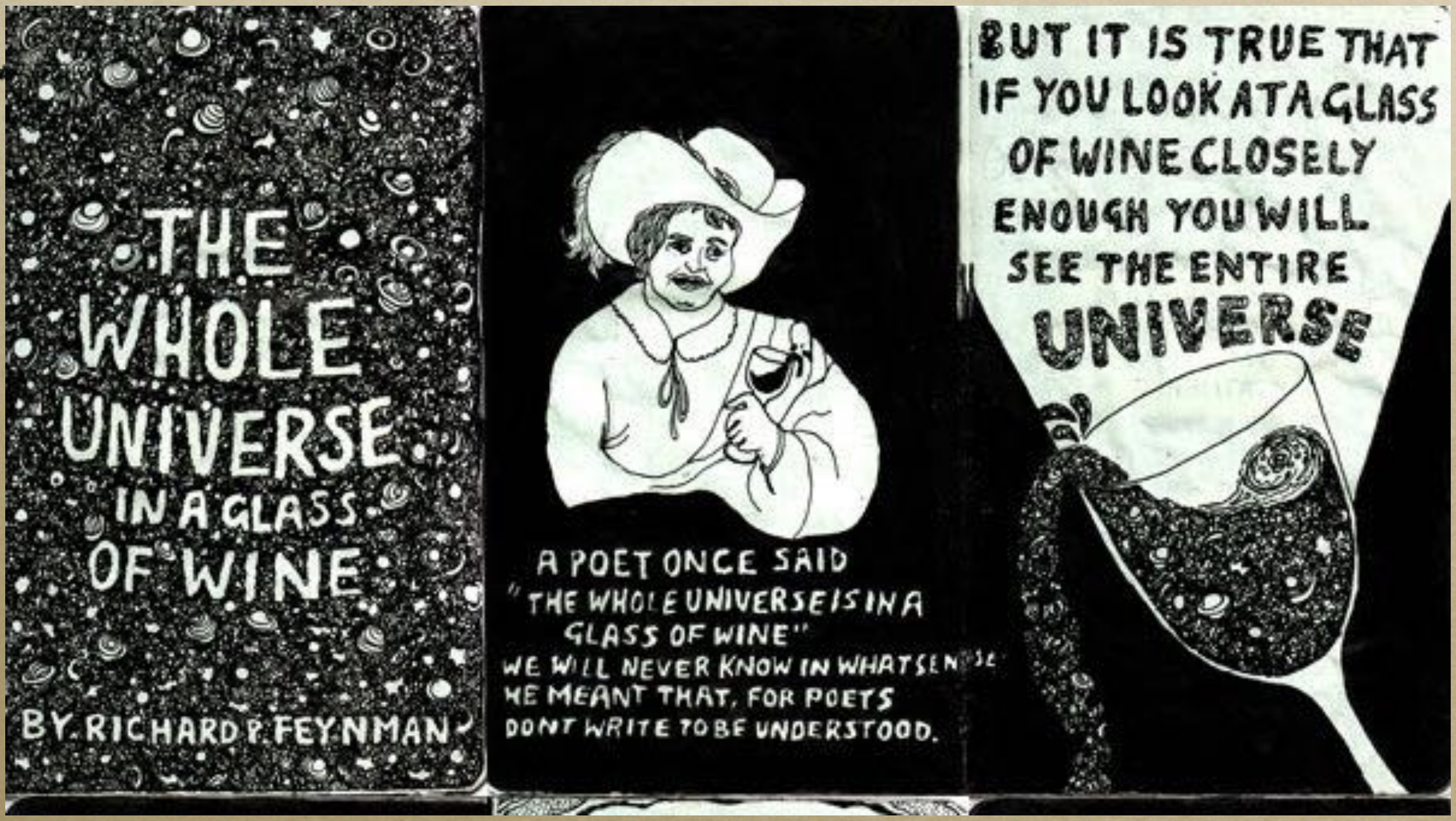
$$\frac{C_\ell}{C_\ell^{free}} = 1 + \frac{(\ell(\ell+1) - 1)\,(H_{2\ell} - H_{\ell-1})}{9\ell^2(1+\ell)^2}\epsilon^2 + C_\ell^{(3)}\epsilon^3$$

$$C_2^{(3)} = \frac{509}{17496} \qquad C_4^{(3)} = \frac{1019357}{114307200} \qquad C_6^{(3)} = \frac{3872826169}{871363785600}$$

$$C_8^{(3)} = \frac{54561737953}{20195722939392} \qquad C_{10}^{(3)} = \frac{58967348085478139}{3219027133886403840}\,.$$

To Sum Up

- *The framework of String Theory very powerful.*
- *Sheds new, unexpected light on QFT (& gravity).*
- *Through AdS dual formulation of CFTs.*
- *Can lead to down-to-earth consequences.*
- *Conceptually needs further unravelling.*
- *Need to understand underlying mechanism of AdS/CFT duality - the nuts and bolts.*



Source:http://nataliekaythatcher.com/The-whole-universe-in-a-glass-of-wine

A Toast to NCTS for it's Third Decade! Cheers!