On "Multiple Half-brane" Solutions in Modified Cubic String Field Theory

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with MIWA Akitsugu

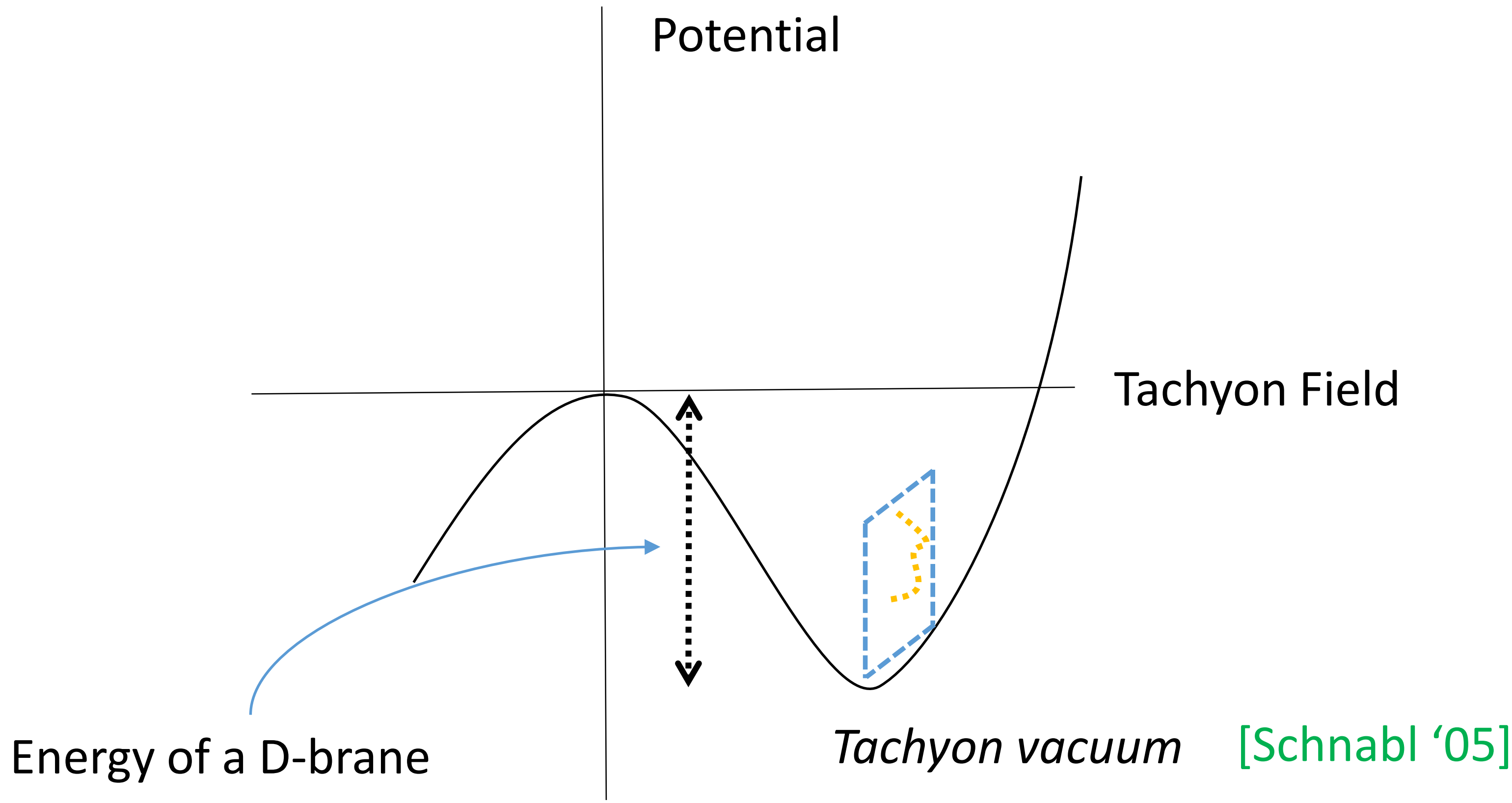
9th Taiwan String Workshop @新竹 2016/11/13

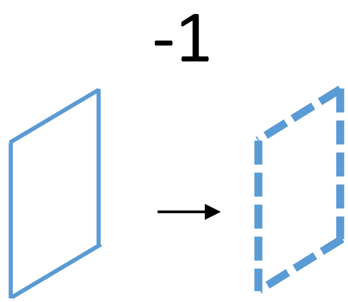
0. Introduction

String Field Theory :

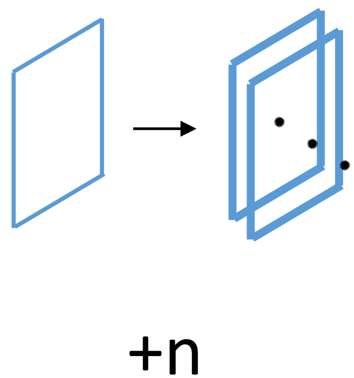
non-perturbative string theory

Tachyon condensation : Sen's conjecture





Tachyon vacuum solution



Multiple-brane solution
: Today's talk

Also non perturbative

1. Bosonic Multiple-brane Solution

Action and EOM

[Witten '86]

action $$S = -\mathrm{Tr}[\frac{1}{2}\Psi * Q\Psi + \frac{1}{3}\Psi * \Psi * \Psi]$$

EOM $$Q\Psi + \Psi * \Psi = 0$$

Ψ string field : gh#=1,

expansion in terms of string states

$$\Psi = \int \frac{d^{26}k}{(2\pi)^{26}} T(k)\hat{c}_1|0,k\rangle + A_\mu(k)\hat{\alpha}^\mu_{-1}\hat{c}_1|0,k\rangle + \frac{i}{\sqrt{2}}B(k)\hat{c}_0|0,k\rangle + \ldots$$

$T(k)$: scalar field

$A_\mu(k)$: gauge field

$B(k)$: aux. field

$\ldots$

Action and EOM

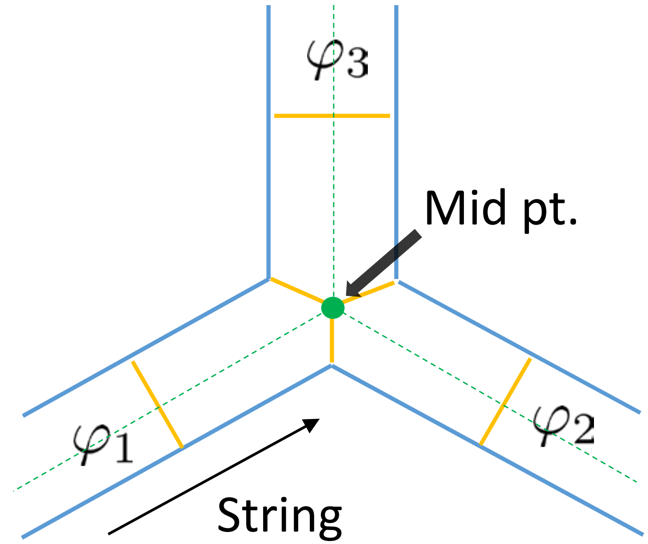
[Witten '86]

action $$S = -\mathrm{Tr}[\frac{1}{2}\Psi * Q\Psi + \frac{1}{3}\Psi * \Psi * \Psi]$$

EOM $$Q\Psi + \Psi * \Psi = 0$$

* Witten's star product : $\varphi_1 * \varphi_2 \to \varphi_3$ φ_i : string field

Mid pt. int.



Action and EOM

[Witten '86]

action $$S = -\mathrm{Tr}[\frac{1}{2}\Psi * Q\Psi + \frac{1}{3}\Psi * \Psi * \Psi]$$

EOM $$Q\Psi + \Psi * \Psi = 0$$

Q BRS operator : $Q\varphi_1 \to \varphi_2$ φ_i : string field

nilpotent $$Q^2 = 0,$$

derivative $$Q(\varphi_1 * \varphi_2) = (Q\varphi_1) * \varphi_2 + (-)^{\epsilon(\varphi_1)}\varphi_1 * (Q\varphi_2)$$

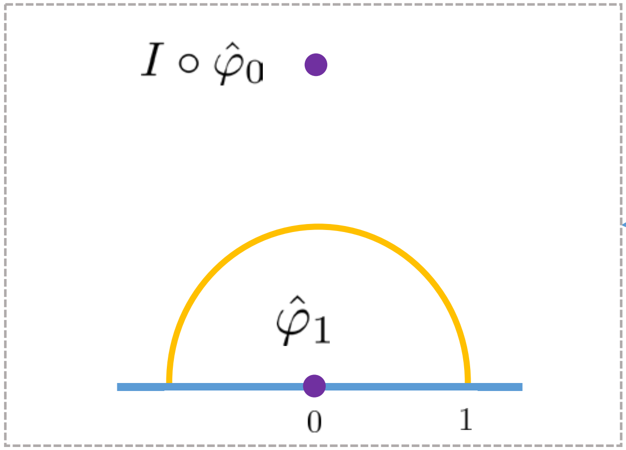
Action and EOM

[Witten '86]

action $$S = -\mathrm{Tr}[\frac{1}{2}\Psi * Q\Psi + \frac{1}{3}\Psi * \Psi * \Psi]$$

EOM $$Q\Psi + \Psi * \Psi = 0$$

Tr BPZ inner product: $\mathrm{Tr}[\varphi_0 * \varphi_1] = \langle I \circ \hat{\varphi}_0(0)\hat{\varphi}_1(0)\rangle_{\mathrm{UHP}}$



※ Non vanishing Tr → gh# = 3

Action and EOM

[Witten '86]

action $$S = -\mathrm{Tr}[\frac{1}{2}\Psi * Q\Psi + \frac{1}{3}\Psi * \Psi * \Psi]$$

EOM $$Q\Psi + \Psi * \Psi = 0$$

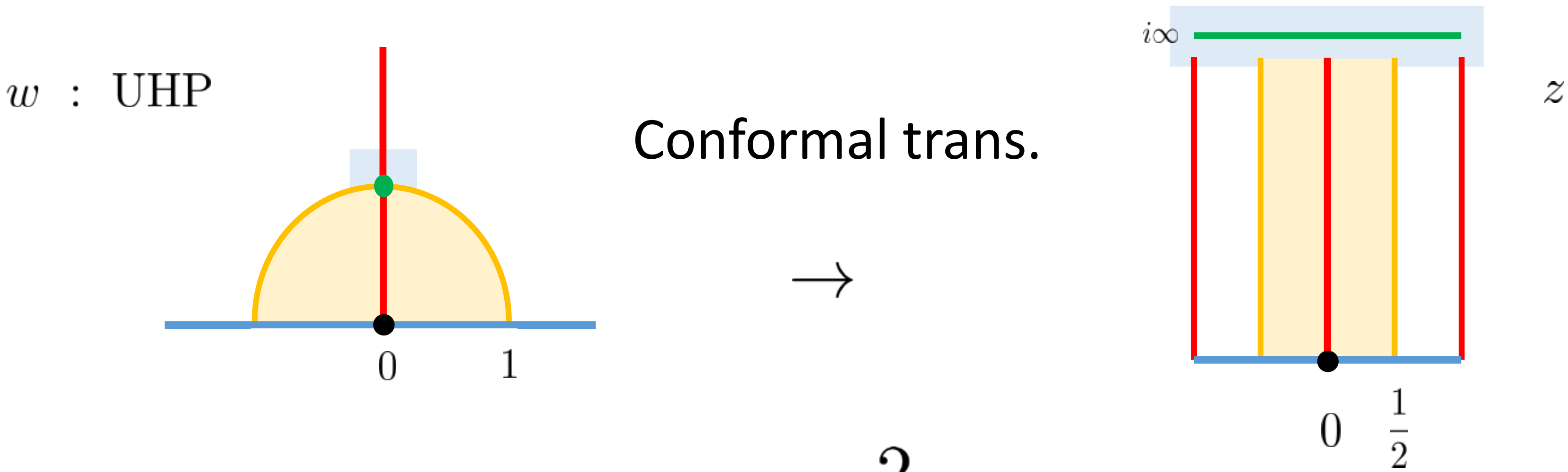
Gauge trans.

$$\Psi' = U^{-1}(Q + \Psi)U$$

U : gauge parameter

Sliver frame

[Rastelli-Zwiebach '01]



$$z = f_{\mathrm{s}}(w) = \frac{2}{\pi}\arctan w$$

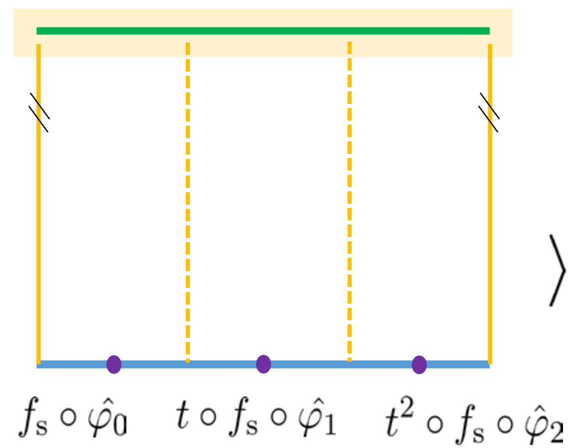
Sliver frame

[Rastelli-Zwiebach '01]

star product

$z \simeq z+3$

$$\mathrm{Tr}[\varphi_0 * \varphi_1 * \varphi_2] = \langle \quad \rangle_{\mathrm{sliver}}$$



φ_0 : test string field

$t : z \to z+1$

automatically

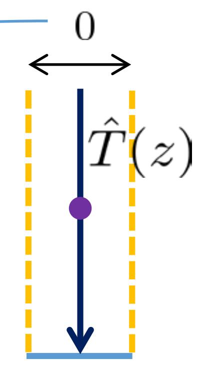
placing "sliver" side by side $\rightarrow$ mid pt. int.

String fields *K,B,c*

[Okawa '06]

K

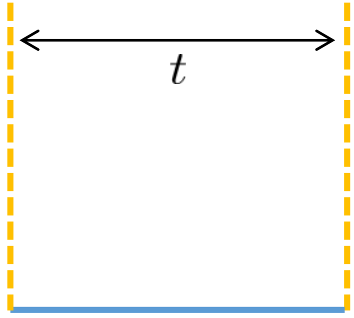
$$\mathrm{Tr}[\varphi_0 * K] = \langle f_{\mathrm{s}} \circ \hat{\varphi}_0(0) \int_{i\infty}^{-i\infty} \frac{dz}{2\pi i} \hat{T}(z) \rangle_{\mathrm{sliver}}$$



φ_0 : test string field

$\hat{T}$: energy momentum tensor

$e^{tK} \sim$

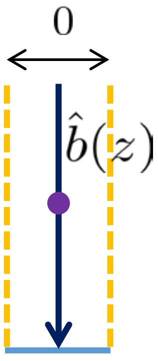


String fields *K,B,c*

[Okawa '06]

B

$$\mathrm{Tr}[\varphi_0 * B] = \langle f_{\mathrm{s}} \circ \hat{\varphi}_0(0) \int_{i\infty}^{-i\infty} \frac{dz}{2\pi i} \hat{b}(z) \rangle_{\mathrm{sliver}}$$



φ_0 : test string field

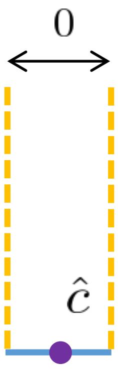
$\hat{b}$: bc gh.

String fields *K,B,c*

[Okawa '06]

c

$$\mathrm{Tr}[\varphi_0 * c] = \langle f_{\mathrm{s}} \circ \hat{\varphi}_0(0)\hat{c}(\frac{1}{2})\rangle_{\mathrm{sliver}}$$



φ_0 : test string field $\qquad$ $\hat{c}$: bc gh.

KBc alg.

[Okawa '06]

$$[K, B] = 0 \quad \{B, c\} = 1 \quad B * B = c * c = 0$$

$$QB = K \quad QK = 0 \quad Qc = c * K * c$$

$$(\mathrm{EOM}: \quad Q\Psi + \Psi * \Psi = 0)$$

↑ closed under Q and *

useful to construct analytic solutions

Pure-gauge-form solution

[Okawa '06]

gauge trans. : $\Psi' = U^{-1}(Q+\Psi)U$

↓

$\Psi = U^{-1} * QU$ is solution

$$U = B*c + c*B*g(K) \qquad (\because \ \mathrm{gh}\#(U)=0)$$

$(U^{-1} = B*c + c*B*g(K)^{-1})$

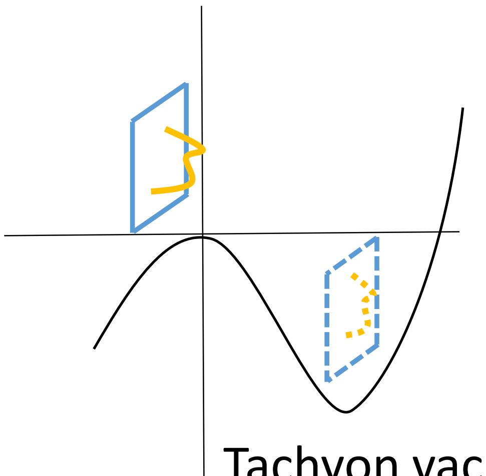
※ U: "singular" gauge trans. → non-trivial solution

Tachyon vacuum solution

[Schnabl ’05
Erler-Schnabl ‘09]

$$\Psi_{\rm tv} = U_1^{-1} Q U_1$$

$$= -(Q(cB) + c)\frac{1}{1-K}$$



Tachyon vacuum

$$U_1 = Bc + cB\left(\frac{-K}{1-K}\right)$$

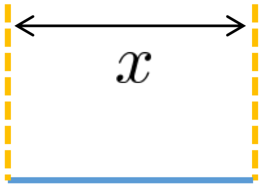
Change in # of D-brane

Energy of the tachyon vacuum solution

[Erler-Schnabl '09]

$$\frac{\pi^2}{3}\mathrm{Tr}[\Psi_{\mathrm{tv}} Q \Psi_{\mathrm{tv}}] = \frac{\pi^2}{3}\mathrm{Tr}[c\frac{1}{1-K}c\partial c\frac{1}{1-K}]$$

Schwinger parametrization: $\frac{1}{1-K} = \int_0^\infty dx e^{-x} e^{xK}$

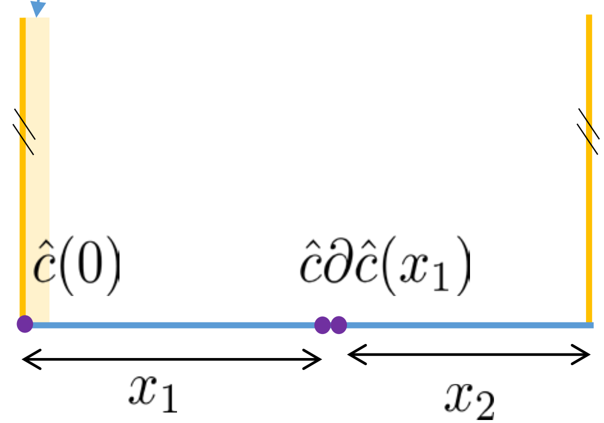
superposition of 

Energy of the tachyon vacuum solution

[Erler-Schnabl '09]

$$\frac{\pi^2}{3}\mathrm{Tr}[\Psi_{\mathrm{tv}} Q \Psi_{\mathrm{tv}}] = \frac{\pi^2}{3}\mathrm{Tr}[c\frac{1}{1-K}c\partial c\frac{1}{1-K}]$$

$$\int_0^\infty dx_1 e^{-x_1}\int_0^\infty dx_2 e^{-x_2}\langle \quad \rangle_{\mathrm{sliver}}$$



Energy of the tachyon vacuum solution

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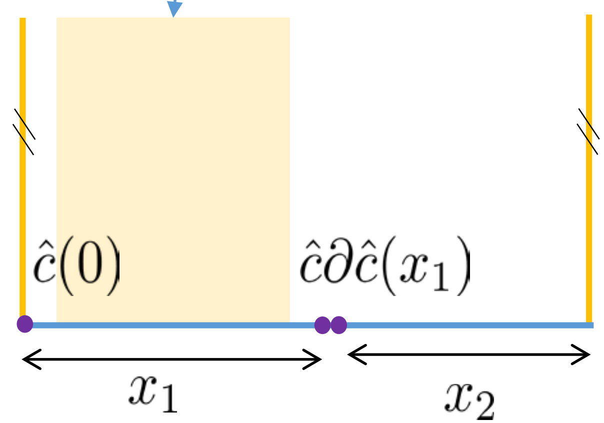
$$\int_0^\infty dx_1 e^{-x_1}\int_0^\infty dx_2 e^{-x_2}\langle$$



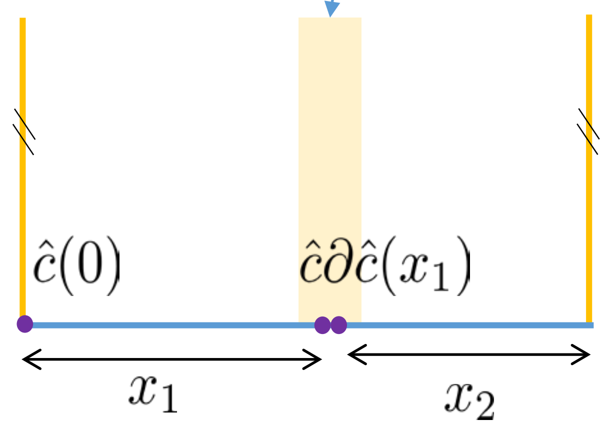
$$\rangle_{\mathrm{sliver}}$$

Energy of the tachyon vacuum solution

[Erler-Schnabl '09]

$$\frac{\pi^2}{3}\mathrm{Tr}[\Psi_{\mathrm{tv}} Q \Psi_{\mathrm{tv}}] = \frac{\pi^2}{3}\mathrm{Tr}[c\frac{1}{1-K}c\partial c\frac{1}{1-K}]$$

$$\int_0^\infty dx_1 e^{-x_1}\int_0^\infty dx_2 e^{-x_2}\langle \quad \rangle_{\mathrm{sliver}}$$



Energy of the tachyon vacuum solution

[Erler-Schnabl '09]

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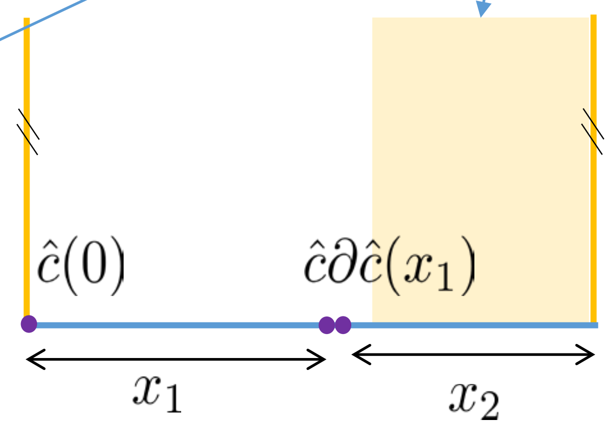
$$\int_0^\infty dx_1 e^{-x_1} \int_0^\infty dx_2 e^{-x_2} \langle \quad \cdots \quad \rangle_{\mathrm{sliver}}$$



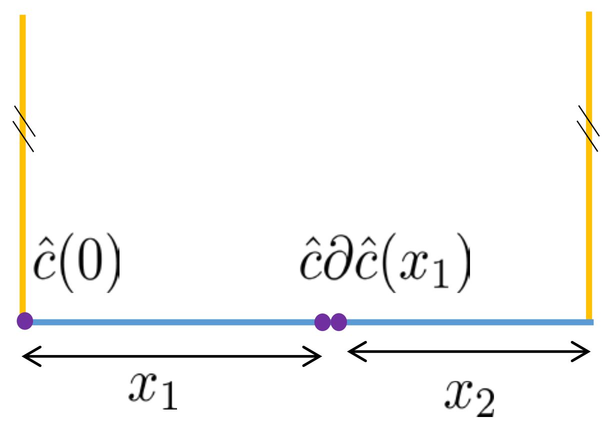
Energy of the tachyon vacuum solution

[Erler-Schnabl '09]

$$\frac{\pi^2}{3}\mathrm{Tr}[\Psi_{\mathrm{tv}}Q\Psi_{\mathrm{tv}}] = \frac{\pi^2}{3}\mathrm{Tr}[c\frac{1}{1-K}c\partial c\frac{1}{1-K}] = -1$$

※normalized by the energy of D25-brane

$$\int_0^\infty dx_1 e^{-x_1}\int_0^\infty dx_2 e^{-x_2}\langle$$



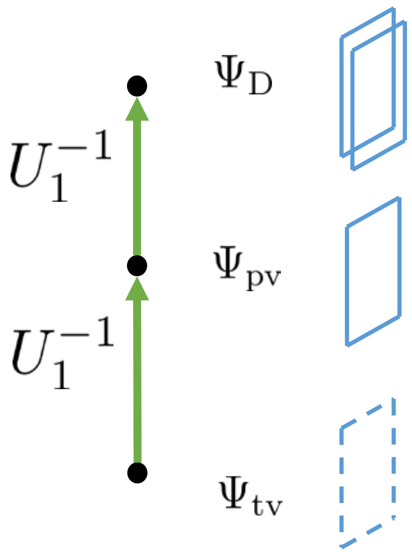
$$\rangle_{\mathrm{sliver}}$$

Double-brane Solution

[Murata-Schnabl '11
Hata-Kojita '12]

$$\Psi_{\mathrm{D}} = U_1 Q U_1^{-1} = -cB\frac{K^2}{1-K}c\frac{1}{K}$$

$$(\Psi_{\mathrm{tv}} = U_1^{-1} Q U_1)$$



$\frac{1}{K}$: *No* Schwinger parametrization

K_ϵ Regularization

[Hata-Kojita '12]

$$K_\epsilon \equiv K - \epsilon$$

$$\frac{1}{K} \to \frac{1}{K_\epsilon} = -\int_0^\infty dz \mathrm{e}^{-\epsilon z} \mathrm{e}^{zK}$$

The solution →

$$[\Psi_{\mathrm{D}}]_\epsilon = -cB \frac{K_\epsilon^2}{1-K_\epsilon} c \frac{1}{K_\epsilon}$$

EOM in the strong sense

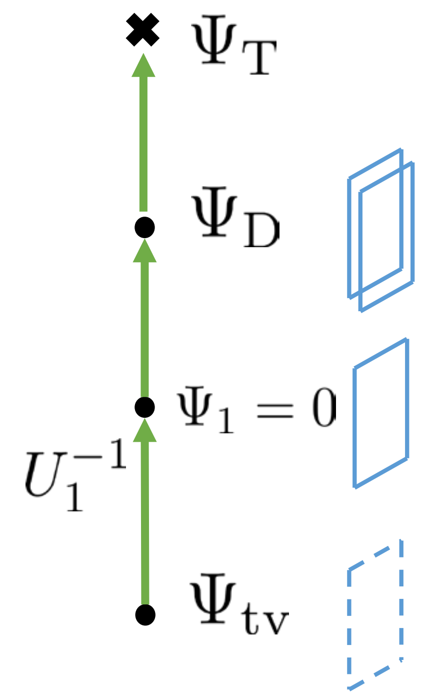
[Hata-Kojita '12]

$$\mathrm{EOMS}(\Psi) \equiv \mathrm{Tr}[\Psi(Q\Psi + \Psi^2)]$$

$$\lim_{\epsilon \to 0} \mathrm{EOMS}([\Psi_{\mathrm{D}}]_\epsilon) = 0$$

$$\lim_{\epsilon \to 0} \mathrm{EOMS}([\Psi_{\mathrm{T}}]_\epsilon) \neq 0$$

$$\Psi_{\mathrm{T}} = U_1^2 Q U_1^{-2}$$



2. Half-brane Solution

Modified cubic super SFT

[Preitschopf-Thorn-Yost
Arefeva-Medvedev-Zubarev '90]

action $$S = -\mathrm{Tr}_{Y_{-2}}[\frac{1}{2}\Psi Q\Psi + \frac{1}{3}\Psi^3]$$

EOM $$Q\Psi + \Psi^2 = 0$$

Ψ string field: NS sector, gh#=1, pic#=0,

$$\Psi = \begin{pmatrix} \mathrm{GSO}(+) & \mathrm{GSO}(-) \\ \mathrm{GSO}(-) & \mathrm{GSO}(+) \end{pmatrix}$$

[Aref'eva-Belov-Giryavets '02]

$$\text{string field}: \quad \tilde{\varphi} \otimes \sigma_\mu$$

ϵ	F	σ_μ
0	0	I_2
0	1	σ_2
1	0	σ_3
1	1	σ_1

Modified cubic super SFT

[Preitschopf-Thorn-Yost
Arefeva-Medvedev-Zubarev '90]

action $$S = -\mathrm{Tr}_{Y_{-2}}[\frac{1}{2}\Psi Q\Psi + \frac{1}{3}\Psi^3]$$

EOM $$Q\Psi + \Psi^2 = 0$$

$$\mathrm{Tr}_{Y_{-2}}$$

$$\mathrm{Tr}_{Y_{-2}}[\varphi_0\varphi_1] = (\frac{1}{2}\mathrm{Tr}[\sigma_\mu \ldots]) \cdot \langle \hat{Y}(i)\hat{Y}(-i) I \circ \hat{\varphi}_0(0)\hat{\varphi}_1(0)\rangle_{\mathrm{UHP}}$$

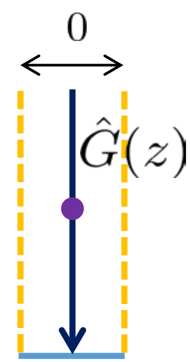
$\hat{Y}$ Inverse Picture Changing Operator

$KBcG\gamma$ alg.

[Erler '11]

G

$$\mathrm{Tr}[\varphi_0 * G] = \langle f_{\rm s} \circ \hat{\varphi}_0(0) \int_{i\infty}^{-i\infty} \frac{dz}{2\pi i} \hat{G}(z) \rangle_{\rm sliver}$$

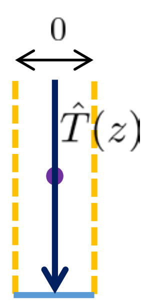


φ_0 : test string field

$\hat{G}$: super current

K

$$\mathrm{Tr}[\varphi_0 * K] = \langle f_{\rm s} \circ \hat{\varphi}_0(0) \int_{i\infty}^{-i\infty} \frac{dz}{2\pi i} \hat{T}(z) \rangle_{\rm sliver}$$



$\hat{T}(z)$: energy momentum tensor

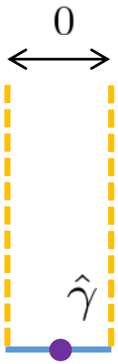
$KBcG\gamma$ alg.

[Erler '11]

γ

$$\mathrm{Tr}[\varphi_0 * \gamma] = \langle f_{\mathrm{s}} \circ \hat{\varphi}_0(0)\hat{\gamma}(\frac{1}{2})\rangle_{\mathrm{sliver}}$$

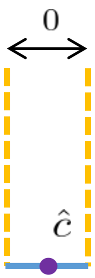
φ_0 : test string field



$\hat{\gamma}$: $\beta\gamma$ gh.

c

$$\mathrm{Tr}[\varphi_0 * c] = \langle f_{\mathrm{s}} \circ \hat{\varphi}_0(0)\hat{c}(\frac{1}{2})\rangle_{\mathrm{sliver}}$$



: bc gh.

$KBcG\gamma$ alg.

[Erler '11]

Same as *KBc* alg.

$$[K,B]=0,\ \{B,c\}=0,\{B,B\}=\{c,c\}=0,$$
$$QK=0,\ QB=K,$$

$$\{G,G\}=2K,\ [G,B]=0,\ [G,c]=2\gamma,\ \{G,\gamma\}=\frac{1}{2}\partial c$$
$$\{\gamma,B\}=\{\gamma,c\}=0,$$
$$QG=0,\ Qc=cKc+\gamma^2,\ Q\gamma=c\partial\gamma+\frac{1}{2}\gamma\partial c$$

$KBcG\gamma$ alg.

[Erler '11]

$$[K,B]=0,\ \{B,c\}=0, \{B,B\}=\{c,c\}=0,$$
$$QK=0,\ QB=K,$$
$$\{G,G\}=2K,\ [G,B]=0,\ [G,c]=2\gamma,\ \{G,\gamma\}=\frac{1}{2}\partial c$$
$$\{\gamma,B\}=\{\gamma,c\}=0,$$
$$QG=0,\ Qc=cKc+\gamma^2,\ Q\gamma=c\partial\gamma+\frac{1}{2}\gamma\partial c$$

super conformal trans.

Half-brane Solution

[Erler '11]

$$\Psi_{\rm H} = U_{\frac{1}{2}}^{-1} Q U_{\frac{1}{2}}$$
$$= -(Q(cB) + cBGc)\frac{1}{1-G}$$

$$U_{\frac{1}{2}} = Bc + cB\frac{-G}{1-G}$$

$$\Psi_{\rm tv} = U_1^{-1} Q U_1$$
$$= -(Q(cB) + c)\frac{1}{1-K}$$

$$U_1 = Bc + cB\frac{-K}{1-K}$$

Half-brane Solution

[Erler '11]

The energy of Ψ_{H}

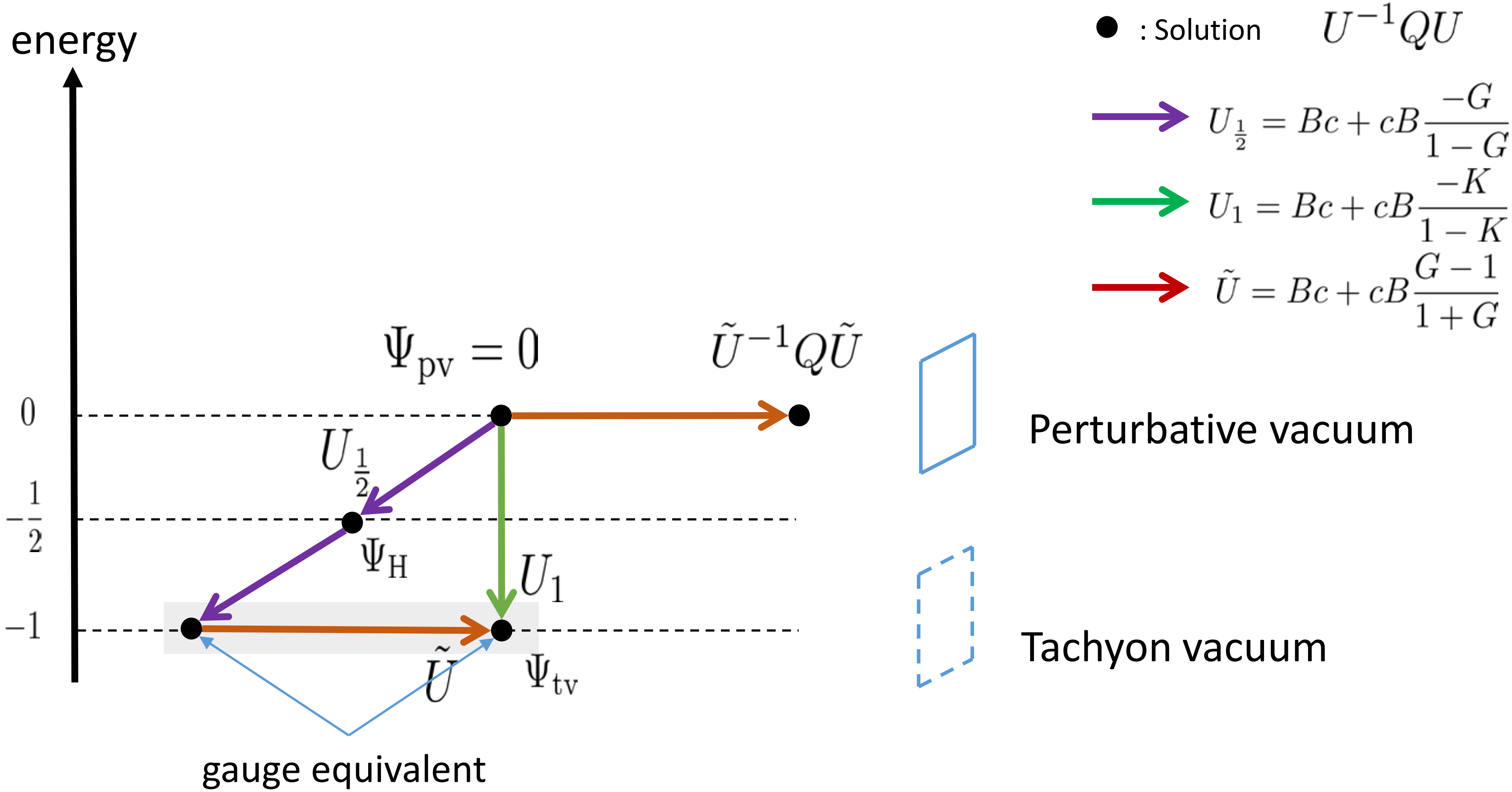
$\mathrm{energy}(\Psi_{\mathrm{tv}})$

$$\frac{\pi^2}{3}\mathrm{Tr}_{Y_{-2}}[\Psi_{\mathrm{H}}Q\Psi_{\mathrm{H}}] = -1 + \frac{1}{2}$$

※normalized by the energy of D9-brane

→ half the energy of the D9-brane

${U_{\frac{1}{2}}}^2$ and U_1



${U_{\frac{1}{2}}}^{-2}Q{U_{\frac{1}{2}}}^2$ and $U_1^{-1}QU_1 = \Psi_{\rm tv}$ are gauge equivalent.

3. “Multiple Half-brane” Solution

“Multiple Half-brane” Solution

$$\Psi_{?} = U_{\frac{1}{2}} Q U_{\frac{1}{2}}^{-1}$$

$$= -\left(-cBKc + B\gamma^2 + cBK\frac{1}{1-G}c\right)\frac{1}{G}$$

$$(\Psi_{\mathrm{H}} = U_{\frac{1}{2}}^{-1} Q U_{\frac{1}{2}})$$

$$U_{\frac{1}{2}} = Bc + cB\frac{-G}{1-G}$$

G_ϵ Regularization

$$G_\epsilon \equiv \tilde{G} \otimes \sigma_1 - \sqrt{-\epsilon} \otimes \sigma_3$$

$KBcG\gamma$ alg. involving G →

$$\{G_\epsilon, G_\epsilon\} = 2K_\epsilon,\ [G_\epsilon, B] = 0,\ [G_\epsilon, c] = 2\gamma,\ \{G_\epsilon, \gamma\} = \frac{1}{2}\partial c$$

For the solution

$$\Psi_? \to [\Psi_?]_\epsilon = -(-cBK_\epsilon c + B\gamma^2 + cBK_\epsilon \frac{1}{1-G_\epsilon} c)\frac{1}{G_\epsilon}$$

EOM in the strong sense

$$\mathrm{EOMS}(\Psi) \equiv \mathrm{Tr}_{Y_{-2}}[\Psi(Q\Psi + \Psi^2)]$$

$$\lim_{\epsilon\to 0} \mathrm{EOMS}([\Psi_?]_\epsilon) = 0$$

※ in the modified cubic super SFT $\lim_{\epsilon\to 0} \mathrm{EOMS}([\Psi_\mathrm{D}]_\epsilon) \neq 0$

$\leftrightarrow$ cubic *bosonic* SFT

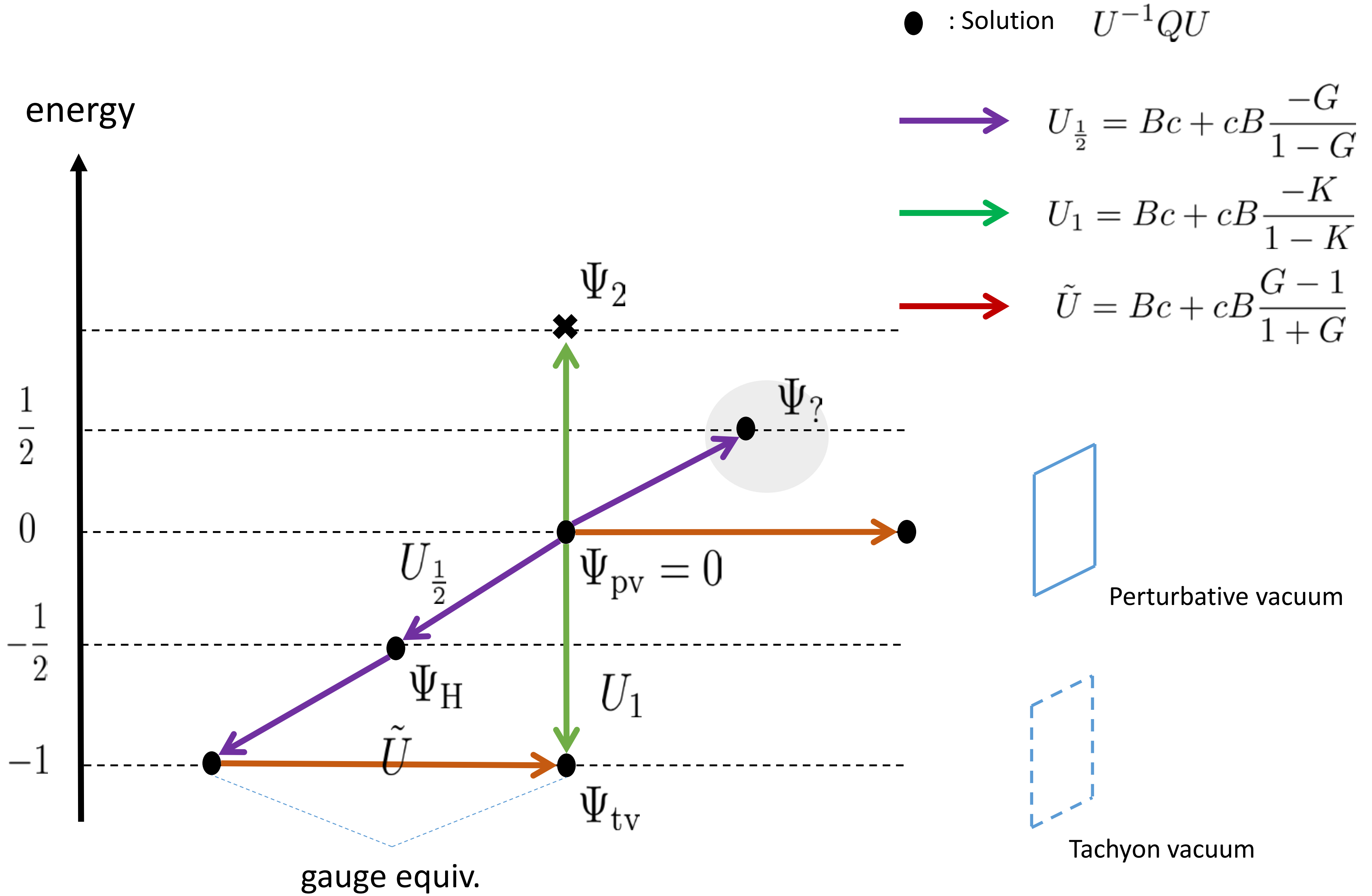
Energy

energy(Ψ_{tv})

$$\frac{\pi^2}{3}\mathrm{Tr}_{Y_{-2}}[[\Psi_?]_\epsilon Q[\Psi_?]_\epsilon] = -1 + \frac{3}{2}$$

※normalized by the energy of D9-brane

→ (3/2) × the energy of the D9-brane



: Solution $U^{-1}QU$
$U_{\frac{1}{2}} = Bc + cB\frac{-G}{1-G}$
$U_1 = Bc + cB\frac{-K}{1-K}$
$\tilde{U} = Bc + cB\frac{G-1}{1+G}$
energy
Ψ_2
$\Psi_?$
$\frac{1}{2}$
0
$U_{\frac{1}{2}}$
$\Psi_{\text{pv}} = 0$
$-\frac{1}{2}$
Ψ_{H}
U_1
-1
$\tilde{U}$
Ψ_{tv}
gauge equiv.
Perturbative vacuum
Tachyon vacuum

4. Summary

Summary

We present a new example of **a solution using *KBcGγ* alg.**
We introduce a ***Gε* regularization**.
The solution satisfy the **EOM in the strong sense**.
The energy of the solution is 3/2 times the **energy** of the D9-brane.

Future works

There is the generalization of this solution
to the Berkovits' WZW-like string field theory.
The Berkovits' SFT is more reliable to discuss physical meanings
of this solution.

Thank you!