N=1 Lagrangians for Argyres-Douglas theories

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Talk based on:
PA, Kazunobu Maruyoshi, Jaewon Song
"N=1 Deformations and RG Flows of N=2 SCFTs, Part II: Non-principal deformations",
1610.05311

Also see previous papers 1606.05632, 1607.04281 by Maruyoshi and Song

Outline

- Lightening fast introduction to Argyres-Douglas (AD) theories, their generalizations and recent progress in our understanding of them
- Introduction to our choice of $\mathcal{N} = 1$ deformations of $\mathcal{N} = 2$ superconformal field theories (SCFTs)
- Obtaining the IR SCFT data via a-maximization
- SUSY enhancement in the IR and the Superconformal Index (SCI) of (A_1, D_N) type Argyres-Douglas theories

Argyres-Douglas Theories

- 4d $\mathcal{N} = 2$ SCFTs living at special loci on the Coulomb branches of $\mathcal{N} = 2$ supersymmetric gauge theory. hep-th/9505062, hep-th/9511154
- At these special loci, particles with mutually non-local electromagnetic charges become massless. We can not write a Lagrangian describing this system. AD theories are inherently strongly coupled SCFTs
- Modern Approach: Wrap M5-branes on a sphere with one irregular and one regular puncture. 0907.3987, 1112.1691, 1203.1052, 1204.2270

- Central charges of AD theories were computed using holography and then via field theoretic methods in 0711.4532, 0804.1957, 1301.0210
- Their BPS particle spectrum in the Coulomb branch was studied in 9910182, 0907.3987, 1006.3435, 1103.5832, 1109.4941, 1112.3984, 1309.3050
- The Schur, Macdonald and Hall-Littlewood indices of AD theories were computed in 1506.00265, 1505.05884, 1509.05402, 1509.06730
- A general prescription to obtain their full superconformal index is not yet known

Central charges & operator dimensions of (A_1, A_k) type AD theories

- Obtained by going to the AD point on the Coulomb branch of $\mathcal{N} = 2$ $SU(k+1)$ pure SYM 9505062, 9603002

$$u_{(2)} = u_{(3)} = \cdots = u_{(k)} = 0, \; u_{(k+1)} = \Lambda^{2(k+1)}$$

- For $k = 2n$,

$$a = \frac{n(24n+19)}{24(2n+3)}, \; c = \frac{n(6n+5)}{6(2n+3)}$$

$$\Delta(\mathcal{O}_i) = \frac{2(2n+3-i)}{2n+3}, \; i = 2, 3, \ldots, n+1$$

- For $k = 2n + 1$,

$$a = \frac{12n^2 + 19n + 2}{24(n+2)}, \; c = \frac{3n^2 + 5n + 1}{6(n+2)}$$

$$\Delta(\mathcal{O}_i) = \frac{2n + 4 - i}{n + 2}, \; i = 2, 3, \ldots, n + 1$$

- For $k = 2n$, these theories do not have any global symmetry while for $k = 2n + 1$, these theories enjoy a $U(1)$ global symmetry. For $k = 3$, the $U(1)$ global symmetry gets enhanced to $SU(2)$
- The full superconformal index (SCI) of these theories was recently computed in 1606.05632, 1607.04281

Central charges & operator dimensions of (A_1, D_k) type AD theories

- Obtained by going to the maximally critical point of an $\mathcal{N} = 2$ $SU(k-1)$ gauge theory with two fundamental hypers 9511154, 9603002
- The Coulomb branch operators have dimensions

$$\Delta(\mathcal{O}_i) = 2 - \frac{2i}{k}, i = 1, 2, \ldots, [(k-1)/2]$$

- For $k = 2n + 1$

$$a = \frac{n(8n+3)}{8(2n+1)},\ c = \frac{n}{2}$$

- For $k = 2n + 2$,

$$a = \frac{n}{2} + \frac{1}{12},\ c = \frac{n}{2} + \frac{1}{6}$$

- When $k = 2n$, the (A_1, D_k) theories enjoy an $SU(2)$ global symmetry while for $k = 2n + 2$, the global symmetry is $SU(2) \times U(1)$. When $k = 4$, the $SU(2) \times U(1)$ global symmetry is enhanced to $SU(3)$
- The flavor central charge of the $SU(2)$ global symmetry is given by
$$k_{SU(2)} = \frac{4(k-1)}{k}$$
- The superconformal index of these theories will be the subject of this talk

A class of $\mathcal{N} = 1$ preserving deformations of $\mathcal{N} = 2$ SCFTs

- Consider an $\mathcal{N} = 2$ SCFT $\mathcal{T}_{UV}$ with flavor symmetry F
- Corresponding to F, there is a moment map operator μ. This is the scalar in the $\mathcal{N} = 2$ current multiplet for F
- Deform $\mathcal{T}_{UV}$ by coupling it to chiral gauge singlet M that transforms in the adjoint representation of F. This is through a superpotential

$$W = \mathrm{Tr} M\mu$$

- Give a nilpotent vev to M: $\langle M \rangle = \rho(\sigma^+)$
- $\rho(\sigma^+) : \mathfrak{su}(2) \hookrightarrow \mathfrak{f}$, where $\mathfrak{f}$ is the Lie algebra of F
- Flavor symmetry gets broken to commutant of $SU(2)$ in F

- Integrate out massive modes
- Of the fluctuations around $\langle M \rangle$, those corresponding to Goldstone modes will decouple in the IR. The supersymmetric version of this statement was provided in 1303.0836
- This will give rise to a “Fan” 1409.1908

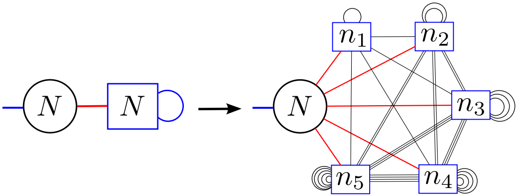

Figure 1: The Fan arising from $\langle M \rangle = \rho(\sigma^+)$, where $\rho(\sigma^+)$ corresponds to the partition of N given by $N = 1n_1 + 2n_2 + \ldots + 5n_5$. This breaks $SU(N)$ to $S[U(n_1) \times \ldots \times U(n_5)]$

- Generically, the above procedure is expected to give rise $\mathcal{N} = 1$ SCFTs in the IR
- The $\mathcal{N} = 1$ $U(1)_R$ is not preserved along the RG flow and the exact IR R-symmetry is determined using the principle of “a-maximization”. 0304128
- In certain cases, it might happen that the dimensions of certain gauge invariant operators in the chiral ring hit the unitarity bound ($\Delta \geq 1$) and become decoupled from the rest of the theory
- In such a scenario, the central charge, a, has to be appropriately corrected before maximizing it. 0308071

- Some simple examples of Lagrangian $\mathcal{N} = 2$ SCFTs : $SU(N)$ gauge theory coupled to $2N$ fundamental hypers, $Sp(N)$ gauge theories coupled to $4N + 4$ fundamental half-hypers and $SO(N)$ gauge theories coupled to $2N - 4$ half-hypers in the vector representation.
- Consider $\mathcal{N} = 1$ deformations of these theories as described above
- For some specific classes of $\langle M \rangle$, there is SUSY enhancement in the IR and the RG flow drives the theory into an $\mathcal{N} = 2$ fixed point corresponding to (A_1, A_N) and (A_1, D_N) AD theory

$SU(N)$ with $2N$ fundamental hypermultiplets

- F= $SU(2N) \times U(1)$
- Nilpotent embeddings of $SU(k)$ are in one-to-one correspondence with integer partitions of k. Here, we need $k = 2N$
- An $\mathcal{N} = 1$ deformation with $\langle M \rangle$ given by principal embedding of $SU(2N)$ triggers an RG flow, to an IR fixed with operator spectrum and central charges that match with those of the (A_1, A_{2N-1}) AD theory. 1607.04281
- When $\langle M \rangle$ is given by next-to-principal embedding, the data at the IR fixed point matches with that of (A_1, D_{2N}) gauge theory
- SUSY enhancement in the IR in both cases
- $U(1)^2$ enhances to $SU(2) \times U(1)$ in the second case

$Sp(N)$ with $4N + 4$ half-hypers

- F = $SO(4N + 4)$
- Nilpotent embeddings of $SO(k)$ are in one-to-one correspondence with those partitions of k, where even parts appear with even multiplicity (upto $\mathbb{Z}_2$ outer-automorphism of $SO(k)$, $k = 2m$)
- When $\langle M \rangle$ is given by principal embedding, the data at the IR fixed point matches with that of (A_1, A_{2N}) AD theory. 1607.04281
- When $\langle M \rangle$ corresponds to the nilpotent orbit of $SO(4N + 4)$ labeled by the partition $[4N + 1, 1^3]$, the data at the IR fixed point matches with that of (A_1, D_{2N+1}) theory
- SUSY enhancement in the IR in both cases

Salient features of the above RG flows

- After the deformation, the matter content of the theory consists of an adjoint chiral multiplet ϕ, some fundamental/anti-fundamental hypermultiplets q, $\tilde{q}$, and a number of gauge singlet fields M_j
- The gauge invariant chiral ring operators are given by $\mathrm{Tr}\phi^{\mathrm{k}}$, and M_j
- Along the RG flow, all of $\mathrm{Tr}\phi^{\mathrm{k}}$ and some M_j hit the unitarity bound and decouple from the rest of the theory
- The gauge singlet fields M_j that remain coupled to the interacting theory, are in one-to-one correspondence with the Coulomb branch operators of the corresponding AD theory (e.g. their operator dimensions match perfectly)

An explicit example: Lagrangian for (A_1, D_4) theory

- By deforming the $\mathcal{N} = 2$ $SU(2)$ SYM coupled to 4 hypermultiplets
- Introduce gauge singlets M transforming in the adjoint of the $SU(4) \times U(1) \subset SO(8)$ flavor symmetry and give it a vev corresponding to the $SU(4)$ nilpotent orbit labeled by the partition $[3, 1]$
- This leaves only a $U(1)^2 \subset SO(8)$ unbroken

- The matter content of the deformed theory is given by four chiral doublets of $SU(2)$, an adjoint chiral field ϕ and five chiral singlets labeled as M_i, $0 \leq i \leq 2$, $\widetilde{M}_1$ and $\widetilde{M'}_1$
- Superpotential:

$$W = \widetilde{q}_1 \phi q_1 + \widetilde{M}'_{N-1} q_1 \widetilde{q}_2 + \widetilde{M}_{N-1} \widetilde{q}_1 q_2 + q_1 \widetilde{q}_1 M_0 + q_2 \widetilde{q}_2 \phi^{2N-1} + \sum_{j=0}^{2} \sum_{l=0}^{2-j} q_2 (M_0)^l M_j \widetilde{q}_2 \phi^{2N-2-j-l} .$$

- Upon computing the R-charges and the operator dimension, via a-maximization, we find that the operator $\mathrm{Tr}\phi^2$ and fields M_0, M_1, $\widetilde{M}_1$ and $\widetilde{M'}_1$ decouple

- At the IR-fixed point the central charges of the interacting theory are given by

$$a = \frac{7}{12},\ c = \frac{2}{3}$$

- Also, $\Delta(M_2) = \frac{3}{2}$
- This matches exactly with the corresponding data for the (A_1, D_4) theory

Superconformal Indices

- The $\mathcal{N} = 1$ SCI of the above theories is of the form

$$\mathcal{I}_{\mathcal{N}=1}(p, q, \xi; \vec{a}) = \mathrm{Tr}(-1)^{\mathrm{F}} \mathrm{p}^{\mathrm{j_1+j_2+\frac{R}{2}}} \mathrm{q}^{\mathrm{j_2-j_1+\frac{R}{2}}} \xi^{\mathcal{F}} \prod_{\mathrm{i}} \mathrm{a}_{\mathrm{i}}^{\mathcal{F}_{\mathrm{i}}}$$

- ξ is the fugacity for the non-R, non-anomalous axial symmetry that is present in all the examples above
- From the point of view of an $\mathcal{N} = 1$ sub-algebra of an $\mathcal{N} = 2$ SUSY, this non-R axial symmetry must be identified with the unique axial flavor symmetry given by $I_3 - \frac{r}{2}$
- The general form of the $\mathcal{N} = 2$ superconformal index is given by

$$\mathcal{I}_{\mathcal{N}=2}(p, q, t) = \mathrm{Tr}(-1)^{\mathrm{F}} \mathrm{p}^{\mathrm{j_1+j_2+\frac{r}{2}}} \mathrm{q}^{\mathrm{j_2-j_1+\frac{r}{2}}} \mathrm{t}^{\mathrm{I_3-\frac{r}{2}}}$$

- The $\mathcal{N}=1$ SCI can be translated into the $\mathcal{N}=2$ SCI by redefining
$$\xi \to (t(pq)^{-\frac{2}{3}})^{\beta}$$
- $\beta : \mathcal{F} = \frac{1}{\beta}(I_3 - \frac{r}{2})$
- Using this, we computed the full superconformal index of (A_1, D_N) type AD theories and checked that they pass various non-trivial checks such as matching with the Schur/Macdonald/Hall-Littlewood limits proposed in 1506.00265, 1505.05884, 1509.05402, 1509.06730

The SCI of (A_1, D_4) AD theory

- It is easier to compute the explicit index by redefining the fugacities such that $p = \mathfrak{t}^3 y,\ q = \mathfrak{t}^3/y,\ t = \mathfrak{t}^4/v$
- Then,

$$\mathcal{I}_{\mathcal{N}=2}^{(A_1,D_4)} = 1 + \mathfrak{t}^3 v^{3/2} + \\ \mathfrak{t}^4 \left(v^{-1} \chi_{SU(3),\mathrm{adj}}(z_1, z_2) - \sqrt{v} \chi_{SU(2),f}(y) \right) + \dots$$

- The coefficient of $\mathfrak{t}^4 v^{-1}$ comes from the conserved current multiplet. 1110.3740
- The $U(1)^2$ fugacities (z_1, z_2) organize themselves into the adjoint representation of $SU(3)$ signaling the enhancement to $SU(3)$ global symmetry of (A_1, D_4)
- Various limits such as the Coulomb branch limit, Schur/Macdonald/Hall-Littlewood limits match with known results

Summary

- Argyres-Douglas theories are some of the believed to be some of the simplest known $\mathcal{N} = 2$ SCFTs
- Their spectrum consists of massless particles with mutually non-local electromagnetic charges, making them inherently “non-Lagrangian”
- We found $\mathcal{N} = 1$ Lagrangians that appear to experience SUSY enhancement along their RG-flow, such that their IR fixed point coincides with a certain category of Argyres-Douglas theory
- Used this to compute the full SCI of these AD theories
- The mechanics governing SUSY enhancement in these theories are still not clear
- Can we write other similar Lagrangians that reduce to other AD theories?

Thank you!