λ-deformations

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Based on:

- Nucl. Phys. **B880** (2014) 225 (construction and integrability)

and with:

- D.C. Thompson and D.C. Thompson & S. Demulder
(supergravity embedding, 2014 & 15)
- G. Itsios and K. Siampos & K. Siampos (RG flows, 2014)
- D.C. Thompson and K. Siampos
(Relation to η-deformations via Poisson-Lie T-duality, 2015)
- G. Georgiou and K. Siampos (anomalous dimensions, 2015 & 16)

General settings and context – Motivation

Construction, integrability and exact results of deformed CFTs.

Deformations of Integrable models in gauge theories and AdS/CFT

- $\mathcal{N} = 4$ $SU(N)$ SYM for $N \gg 1$ exhibits integrability [Minahan-Zarembo 02]. Maximally supersymmetric and conformal. The $AdS_5 \times S^5$ dual backgrounds is also integrable [Bena-Polchinski-Roiban 03].
- Certain (marginal) deformations retain integrability and reduce susy, e.g. (γ-deformations) reduce to $\mathcal{N} = 1$ [Leigh-Strassler 95]. The gravity dual [Lunin-Maldacena 05] is integrable [Frolov 05].
- There are $\mathcal{N} = 1$ theories which have non-integrable supergravity duals,
  e.g. in $AdS_5 \times T_{1,1}$ motion of strings is even chaotic [Basu-Zayas 11]

Hence, susy does not imply integrability.

I will show that is it possible to have deformations which:

- Break supersymmetry completely.
- Preserve integrability.
  Recall even QCD exhibits some integrability in certain limits/high energy [Libatov 93].

Exact β-functions, anomalous dimensions, correlators

- The quantum behaviour of a field theory is encoded in the β-function eqs. and the anomalous dimensions of the operators.
  Traditionally these are determined perturbatively.
- Is it possible to compute the β-function and the anomalous dimensions exactly, i.e. all orders?
  One could discover new fixed point theories towards the IR.
- Exact correlation functions?
- Typically, these are very difficult tasks. In some cases if there is enough supersymmetry the 1-loop could be enough and the higher ones vanish, i.e. $\mathcal{N} = 2$ SYM in 4-dims.
- When these computations can be performed to all loops are rare and this is then very exciting.

I will show that these are possible.

The bosonized non-Abelian Thirring model

Uncover classical & quantum aspects of the action

$$\boxed{S_{k,\lambda}(g) = S_{\mathrm{WZW},k}(g) + \frac{k\lambda}{\pi}\int J_+^a J_-^a} ,$$

where $S_{\mathrm{WZW},k}(g)$ is the WZW action [Witten 83]

$$S_{\mathrm{WZW,k}}(g) = -\frac{k}{2\pi}\int \mathrm{Tr}(g^{-1}\partial_+ g g^{-1}\partial_- g) + \frac{ik}{6\pi}\int_B \mathrm{Tr}(g^{-1}dg)^3 , \quad g \in G .$$

- Related via bosonization to an action in 1+1 dims with fermions ψ and ψ^a in the quark rep of $SU(N)$, $a = 1, 2, \ldots N^2 - 1$ [Dashen-Frishman 73 & 75].

- When $\lambda = 0$ this action is Invariant under

$$g \to \Omega_-(\sigma_+)\ g\ \Omega_+(\sigma_-)\ .$$

There are two copies of a current algebra at level $k \in Z^+$ It is realized by

$$J_+^a = -i\mathrm{Tr}(t^a \partial_+ g g^{-1})\ , \qquad J_-^a = -i\mathrm{Tr}(t^a g^{-1} \partial_- g)\ .$$

The theory is a CFT.

- The extra term $J_+^a J_-^a$ breaks the symmetry to a global diagonal one

$$g \to \Lambda^{-1} g \Lambda\ , \qquad \Lambda \in G\ .$$

- Relevant perturbation [Kadanoff-Brown 79, Chaudhuri-Schwartz 89]. The parameter λ should run under the RG flow.

Derivation of the models [KS 13]

The starting point is the action

$$\boxed{S(g, \tilde{g}) = S_{WZW,k}(g) + S_{\mathrm{PCM}}(\tilde{g})} .$$

- $S_{\mathrm{WZW},k}(g)$ is the WZW action for $g \in G$

$$S_{\mathrm{WZW}}(g) = -\frac{k}{2\pi} \int \mathrm{Tr}(g^{-1}\partial_+ g g^{-1}\partial_- g) + \frac{ik}{6\pi} \int_B \mathrm{Tr}(g^{-1} dg)^3 ,$$

  This is a CFT; has a $G_{\mathrm{L,cur}} \times G_{\mathrm{R,cur}}$ current algebra symmetry.
- $S_{\mathrm{PCM}}(\tilde{g})$ is the PCM action for ($\tilde{g} \in G$) with coupling κ^2

$$S_{\mathrm{PCM}}(\tilde{g}) = -\frac{\kappa^2}{\pi} \int \mathrm{Tr}(\tilde{g}^{-1}\partial_+ \tilde{g} \tilde{g}^{-1}\partial_- \tilde{g}) .$$

  It is integrable with global $G_L \times G_R$ symmetry.

Derivation by gauging

We will gauge the group acting as

$$g \to \Lambda^{-1} g \Lambda \ , \qquad \tilde{g} \to \Lambda^{-1} \tilde{g} \ , \qquad \Lambda \in G \ .$$

Hence we consider the action

$$S_{k,\kappa^2}(g,\tilde{g}) = S_{\mathrm{g}WZW,k}(g, A_\pm) + S_{\mathrm{g}PCM}(\tilde{g}, A_\pm) \ ,$$

where

$$S_{\mathrm{gWZW},k}(g, A_\pm) = S_{WZW,k}(g)$$
$$+\frac{k}{\pi}\int \mathrm{Tr}\Big(A_-\partial_+ g g^{-1} - A_+ g^{-1}\partial_- g + A_- g A_+ g^{-1} - A_- A_+\Big) \ ,$$

and

$$S_{\mathrm{g}PCM}(\tilde{g}, A_\pm) = -\frac{1}{\pi}\int \mathrm{Tr}(\tilde{g}^{-1}\widetilde{D}_+\tilde{g}\tilde{g}^{-1}\widetilde{D}_-\tilde{g}) \ ,$$

with the covariant derivatives being $\widetilde{D}_\pm \tilde{g} = \partial_\pm \tilde{g} - A_\pm \tilde{g}$.

- We choose the gauge fixing

$$\tilde{g} = \mathbb{1} \ .$$

- Hence the gauged fixed action becomes

$$S_{k,\kappa^2}(g,\mathbb{1}) = S_{gWZW,k}(g,A_\pm) - \frac{\kappa^2}{\pi}\int \mathrm{Tr}(A_+A_-) \ .$$

- Integrating out the gauge fields we obtain the action

$$\boxed{S_{k,\lambda}(g) = S_{\mathrm{WZW},k}(g) + \frac{k}{\pi}\int J_+^a(\lambda^{-1}\mathbb{I} - D^T)^{-1}_{ab} J_-^b} \ ,$$

where

$$D_{ab} = \mathrm{Tr}(t_a g t_b g^{-1})$$

and

$$\lambda = \frac{k}{k+\kappa^2} \ .$$

is the deformation parameter.

- Generalization to a general matrix $\lambda\delta_{ab} \to \lambda_{ab}$ straightforward.

Basic properties

- For small λ it becomes the non-Abelian anisotropic (bosonised) Thirring model action

$$S_{k,\lambda}(g) = S_{\mathrm{WZW},k}(g) + \frac{k\lambda}{\pi} \int J_+^a J_-^a + \cdots \ .$$

  They share the same symmetries.
- The theory is driven away from the conformal point.
- Marginally relevant perturbation [Kadanoff-Brown 79, Chaudhuri-Schwartz 89]. The RG flow equation

$$\frac{d\lambda}{dt} = \cdots \ .$$

  to all-loops in λ, but to leading order in $1/k$.
- The model is integrable [KS 13].
- It has a Yangian symmetry [Itsios-KS-Siampos-Torrielli, 14].

A remarkable symmetry

The effective action has the symmetry [Itsios-KS-Siampos 14]

$$\boxed{S_{-k,\lambda^{-1}}(g^{-1}) = S_{k,\lambda}(g)} \ .$$

- A duality-type symmetry.
- It should be reflected as a symmetry in physical quantities and correlators.
- The renormalized currents $J^a_\pm$ are invariant under this transformation.

Example with $SU(2)$

Consider the case with

$$\lambda_{ab} = \lambda \delta_{ab} \ ,$$

let $G = SU(2)$. and parametrize

$$g = e^{i\alpha \hat{n}_i \sigma_i} \ , \qquad \hat{\mathbf{n}} = (-\sin\beta \sin\gamma, \sin\beta\cos\gamma, \cos\beta) \ ,$$

The corresponding σ-model has metric

$$ds^2 = k\left(\frac{1+\lambda}{1-\lambda} d\alpha^2 + \frac{1-\lambda^2}{\Delta(\alpha)} \sin^2\alpha \ ds^2(S^2)\right) \ ,$$

and antisymmetric tensor

$$B = k\left(-\alpha + \frac{(1-\lambda)^2}{\Delta(\alpha)} \cos\alpha \sin\alpha\right) \mathrm{Vol}(S^2) \ ,$$

where

$$\Delta(\alpha) = (1-\lambda)^2 \cos^2\alpha + (1+\lambda)^2 \sin^2\alpha \ .$$

Integrability and algebraic structure

Equations of motion

- Varying with respect to g we obtain that

$$D_-(D_+gg^{-1}) = F_{+-} \ , \qquad D_+(g^{-1}D_-g) = F_{+-} \ ,$$

which due to $[D_+, D_-]g = [g, F_{+-}]$, are equivalent

- Varying with respect to $A_\pm$'s

$$D_+gg^{-1} = (\lambda^{-1} - 1)A_+ \ , \qquad g^{-1}D_-g = -(\lambda^{-1} - 1)A_- \ ,$$

- The above can be cast as

$$\begin{aligned} \lambda\partial_+A_- - \partial_-A_+ &= [A_+, A_-] \ , \\ \partial_+A_- - \lambda\partial_-A_+ &= [A_+, A_-] \ . \end{aligned}$$

from which

$$\partial_\pm A_\mp = \pm\frac{1}{1+\lambda}[A_+, A_-] \ .$$

Integrability

- Assume the classical equations can be written as a Lax eq.

$$\mathrm{d}L = L \wedge L \qquad \text{or} \qquad \partial_+ L_- - \partial_- L_+ = [L_+, L_-] \,,$$

where $L_\pm = L_\pm(\tau, \sigma, \mu)$ and $\mu \in \mathbb{C}$ is the spectral parameter.

- Then, the classical monodromy matrix

$$M = P \exp \int_{-\infty}^{+\infty} \mathrm{d}\sigma \, L_1 \,, \qquad \partial_0 M = 0 \,.$$

gives rise to infinitely many conserved changes.

- In our case

$$L_\pm = \frac{2}{1+\lambda} \frac{\mu}{\mu \mp 1} A_\pm \,.$$

- Some integrable cases $\lambda_{ab} \neq \lambda \delta_{ab}$ [Thompson-Siampos-KS,15].

Extendable to semi-symmetric spaces ($\mathbb{Z}_4$ grading),
i.e. $PSU(2,2|4)/SO(1,4) \times SO(5)$, very important in AdS/CFT
[Hollowood-Miramontes-Schmidtt 14].

Algebraic properties

The Poisson brackets for $I_\pm \sim A_\pm$

$$\{I_\pm^a, I_\pm^b\} = e^2\, f_{abc}\, (I_\mp^c - (1+2x)I_\pm^c)\, \delta_{12} \pm 2e^2 \delta_{ab}\delta'_{12}\ ,$$
$$\{I_+^a, I_-^b\} = -e^2\, f_{abc}\, (I_+^c + I_-^c)\ \delta_{12}\ ,$$

where

$$e = \frac{2\lambda}{\sqrt{k(1-\lambda^2)}(1+\lambda)}\ , \qquad x = \frac{1+\lambda^2}{2\lambda} > 1\ .$$

- Algebra constructed in [Rajeev 89, Balog-Forgacs-Horvath-Palla 94]
- A deformation of the PCM algebra (for $x = 1$).
- There is a Yangian symmetry and provide non-trivial solutions to the Yang-Baxter equation via the Maillet brackets [Itsios-KS-Siampos-Torrielli, 14]. (Maillet brackets: Poisson brackets of the monodromy matrix; Jacobi identities implies the Yang–Baxter eq)

β-function and anomalous dimensions

Perturbative computations

The currents obey the OPEs

$$J^a(z)J^b(w) = \frac{\delta_{ab}}{(z-w)^2} + \frac{f_{abc}}{\sqrt{k}}\frac{J^c(w)}{z-w} + \cdots \ .$$

- Using these we may compute the 2-point functions

$$\langle J^a(x_1)J^b(x_2)\rangle_\lambda = \langle J^a(x_1)J^b(x_2)e^{-\frac{\lambda}{\pi}\int d^2z J^a(z)\bar{J}^a(\bar{z})}\rangle$$
$$\langle J^a(x_1)\bar{J}^b(x_2)\rangle_\lambda = \langle J^a(x_1)\bar{J}^b(x_2)e^{-\frac{\lambda}{\pi}\int d^2z J^a(z)\bar{J}^a(\bar{z})}\rangle$$

  perturbatively in λ by expanding the exponential.
- The basic correlators are

$$\langle J^a(x_1)J^b(x_2)\rangle = \frac{\delta_{ab}}{x_{12}^2}\ , \quad \langle J^a(x_1)J^b(x_2)J^c(x_3)\rangle = \frac{1}{\sqrt{k}}\frac{f_{abc}}{x_{12}x_{13}x_{23}}\ .$$

  and similarly for the $\bar{J}^a$'s. Mixed $J\bar{J}$ correlators vanish.
- For higher correlators use Ward dentities

Perturbative results; Renormalization

Relations between the bare and renormalized quantities

$$J_0^a = Z^{1/2} J^a \ , \qquad \bar{J}_0^a = Z^{1/2} \bar{J}^a \ , \qquad \lambda_0 = Z_1 \lambda \ ,$$

- The renormalized n-point functions are cutoff independent

$$\langle J^a(x_1) J^b(x_)\rangle_\lambda = Z^{-1} \langle J_0^a(x_1) J_0^b(x_)\rangle_{Z_1 \lambda}$$

- Up to three-loops this requires that

$$Z^{-1} = 1 + 2c_G \lambda^3 - \frac{c_G}{k}\left(\lambda^2 - 2\lambda^3 + \mathcal{O}(\lambda^4)\right) \ln(\varepsilon^2 \mu^2) \ ,$$
$$Z_1 = 1 + \frac{c_G}{k}\left(\frac{1}{2}\lambda - \lambda^2 + \mathcal{O}(\lambda^3)\right) \ln(\varepsilon^2 \mu^2) \ ,$$

  - Depends on the energy scale μ and a small distance cut-off.
  - c_G is the quadratic Casimir in the adjoint rep., i.e. $f_{acd} f_{bcd} = -c_G \delta_{ab}$.

The perturbative β-function and anomalous dimensions

- The β-function is by definition

$$\beta = \frac{1}{2}\mu\frac{d\lambda}{d\mu} = -\frac{c_G}{2k}\left(\lambda^2 - 2\lambda^3 + \mathcal{O}(\lambda^4)\right) ,$$

where the bare coupling λ_0 is kept fixed.

- The anomalous dimension of the currents is

$$\gamma^{(J)} = \mu\frac{d\ln Z^{1/2}}{d\mu} = \frac{c_G}{k}\left(\lambda^2 - 2\lambda^3 + \mathcal{O}(\lambda^4)\right) .$$

Is it possible to compute these exactly in λ?

Analyticity: λ-dependence of physical quantities

- Expand the action for $g = e^{ix^a t^a}$ around the identity

$$S_{k,\lambda} = \frac{k}{4\pi}\frac{1+\lambda}{1-\lambda}\int \partial_+ x^a \partial_- x^a + \cdots$$

- The β-function & anomalous dims may have poles at $\lambda = \pm 1$.
- The effective action has two well defined limits:
  - The non-Abelian T-duality limit

$$\lambda = 1 - \frac{\kappa^2}{k} \, , \qquad k \to \infty \, .$$

  - The pseudochiral model limit

$$\lambda = -1 + \frac{1}{b^{2/3} k^{1/3}} \, . \qquad k \to \infty \, .$$

The β-function & anomalous dims should have good limits.

- The β-function & anomalous dims should be invariant under

$$k \to -k \; , \qquad \lambda \to \frac{1}{\lambda} \; ,$$

  for $k \gg 1$.
- Some perturbative information and the above symmetry are enough to determine the β-function and the anomalous dimensions exactly in λ and to leading order in k.

The exact β-function and anomalous dimensions

The exact β-function and anomalous dimensions are of the form

$$\beta_\lambda = -\frac{c_G}{2k}\frac{f(\lambda)}{(1+\lambda)^2}\ , \qquad \gamma^{(J)} = \frac{c_G}{k}\frac{g(\lambda)}{(1-\lambda)(1+\lambda)^3}\ ,$$

where $f(\lambda)$ and $g(\lambda)$ are analytic in λ.

- They have a well defined non-Abelian and pseudodual limits.
- Due to the symmetry $(k,\lambda) \mapsto (-k,\lambda^{-1})$ we have that

$$\lambda^4 f(1/\lambda) = f(\lambda)\ , \qquad \lambda^4 g(1/\lambda) = g(\lambda)\ .$$

$f(\lambda)$ and $f(\lambda)$ are polynomials of, at most, degree four. They are fixed by the above symemtry and by the up to two-loops perturbative reult.

- The final result is

$$\boxed{\beta_\lambda = -\frac{c_G}{2k}\frac{\lambda^2}{(1+\lambda)^2} \leqslant 0}$$

and

$$\boxed{\gamma^{(J)} = \frac{c_G}{k}\frac{\lambda^2}{(1-\lambda)(1+\lambda)^3} \geqslant 0.}$$

Agree with perturbation theory to order checked, i.e. $\mathcal{O}(\lambda)^3$.

3-point functions of currents

With similar computations and arguments we compute:

$$\langle J^a(x_1)J^b(x_2)J^c(x_3)\rangle = \frac{1+\lambda+\lambda^2}{\sqrt{k(1-\lambda)(1+\lambda)^3}}\frac{f_{abc}}{x_{12}x_{13}x_{23}} \ .$$

and

$$\langle J^a(x_1)J^b(x_2)\bar{J}^c(\bar{x}_3)\rangle = \frac{\lambda}{\sqrt{k(1-\lambda)(1+\lambda)^3}}\frac{f_{abc}\bar{x}_{12}}{x_{12}^2\bar{x}_{13}\bar{x}_{23}} \ .$$

- These are leading order for $k \gg 1$ and respect the symmetry

$$k \to -k \ , \qquad \lambda \to \frac{1}{\lambda} \ .$$

- The other correlators follow from parity.
  Similarly one computes correlators involving primary fields.

A digression; Left-right assymetric deformations

Note: there is no new fixed point towards the IR.
This changes when two different levels k_L and k_R.

- Beta-function

$$\frac{d\lambda}{dt} = -\frac{c_G}{2\sqrt{k_L k_R}} \frac{\lambda^2(\lambda - \lambda_0)(\lambda - \lambda_0^{-1})}{(1-\lambda^2)^2} .$$

A new fixed point in the IR at $\lambda = \lambda_0 = \sqrt{\frac{k_L}{k_R}}$.

- Anomalous dimensions

$$\gamma_L = \frac{c_G}{k_R} \frac{\lambda^2(\lambda - \lambda_0^{-1})^2}{(1-\lambda^2)^3} , \quad \bar{\gamma}_R = \frac{c_G}{k_L} \frac{\lambda^2(\lambda - \lambda_0)^2}{(1-\lambda^2)^3} .$$

- Evidence of the RG flow to a diffenent CFT in the IR

$$G_{k_L} \times G_{k_R} \quad \overset{\text{IR}}{\Longrightarrow} \quad \frac{G_{k_L} \times G_{k_R - k_L}}{G_{k_R}} \times G_{k_R - k_L} .$$

For $G = SU(2)$ argued to describe a fermi liquid as the IR fixed point of interacting chiral fermions [Andrei-Douglas-Jerez 99]

Gravitational approach

Using the effective action

$$S_{k,\lambda}(g) = S_{\mathrm{WZW},k}(g) + \frac{k}{\pi}\int J_+^a (\lambda^{-1}\mathbb{I} - D^T)^{-1}_{ab} J_-^b$$

and the one-loop β-functions in $1/k$ [Ecker-Honerkamp 71, Friedan 80, Braaten-Curtright-Zachos 85, Fridling-van de Ven 86]

$$\frac{dG_{\mu\nu}}{dt} + \frac{dB_{\mu\nu}}{dt} = R^-_{\mu\nu} \ ,$$

same result for the beta-function [Itsios-KS-Siampos 14].

$$\beta_\lambda = -\frac{c_G}{2k}\frac{\lambda^2}{(1+\lambda)^2} \ .$$

By extracting the wave function renormalization it is possible to compute the exact anomalous dim from the eff.action [Georgiou-KS-Siampos 15].

Spacetimes - type-II Supergravity

We aim at using these σ-models as building blocks for constructing solutions of type-II Supergravity.

Need to:

- Decide which part of a 10-dim space to deform.
- Use for the NS-sector the σ-models field and for the dilaton

$$\Phi = -\frac{1}{2}\det(\lambda^{-1} - D^T) \ .$$

- Support these NS-sector with RR-fluxes.
- Find rules, if possible, for determining these RR fields, or compute them by brute force.

Several examples of λ-deformations [KS-Thompson 14, Demulder-KS-Thompson 15]. The RR-field rules are essentially the same as those for non-Abelian T-duality [KS-Thompson 10].

- $AdS_3 \times S^3 \times T^4$ using the $SU(2) \times SL(2,\mathbb{R})$ isometry.
- $AdS_2 \times S^2 \times T^6$ using the $SU(2) \times SL(2,\mathbb{R})$ isometry.
- $AdS_3 \times S^3 \times T^4$ using the $SU(2) \times SU(2) \times SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ isometry.
- $AdS_5 \times S^6$ using the $SO(6) \times SL(2,4)$ isometry.
- Supercoset embedding for $AdS_2 \times S^2$ [Borsato-Tseytlin-Wulff,16].

Explicit example: A new twist to the old black hole

The NS sector:
The metric is [KS 13]

$$
\begin{aligned}
ds^2 = k \left(\frac{1-\lambda}{1+\lambda} (-\coth^2 \rho dt^2 + d\rho^2) + \frac{4\lambda}{1-\lambda^2} (\cosh t d\rho + \sinh t \coth \rho dt)^2 \right) \\
+ k \left(\frac{1-\lambda}{1+\lambda} (d\omega^2 + \cot^2 \omega d\phi^2) + \frac{4\lambda}{1-\lambda^2} (\cos \phi d\omega + \sin \phi \cot \omega d\phi)^2 \right) \\
+ \sum_{i=4}^{9} dx_i^2 \ .
\end{aligned}
$$

1st line: A deformation of the $SL(2, \mathbb{R}) / U(1)$ exact CFT.
2nd line: A deformation of the $SU(2) / U(1)$ exact CFT.

In addition:

- The dilaton is
  $$e^{-2\Phi} = \sin^2 \omega \sinh^2 \rho \ .$$
- The antisymmetric tensor vanishes.

The RR-sector

- First define the frames

$$e^0 = \sqrt{k\frac{1-\lambda}{1+\lambda}}(\sinh t d\rho + \cosh t \coth \rho dt) \ , \quad e^1 = \cdots \ .$$

so that the metric is

$$ds^2 = \eta_{ab} e^a e^b \ , \qquad \eta^{ab} = \mathrm{diag}(-1, 1, \ldots, 1) \ .$$

- In $\mathbb{R}^6$ denote by

$$\begin{aligned} J_2 : &\quad \text{Kahler form} \ , \\ J_3 : &\quad \text{Real part of complex differential form of type } (3,0) \ . \end{aligned}$$

Then, there are two possibilities [KS-Thompson 14]:

- Type-IIB

$$F_5 = (1 + \star) f_5 \ ,$$

$$f_5 = \frac{1}{\sqrt{k}} \sqrt{\frac{4\lambda}{1-\lambda^2}} \sin\omega \sinh\rho \ e^0 \wedge e^3 \wedge J_3 \ .$$

- Type–IIA

$$F_2 = \frac{1}{\sqrt{k}} \sqrt{\frac{4\lambda}{1-\lambda^2}} \sin\omega \sinh\rho \ e^0 \wedge e^3 \ ,$$

$$F_4 = \frac{1}{\sqrt{k}} \sqrt{\frac{4\lambda}{1-\lambda^2}} \sin\omega \sinh\rho \ e^1 \wedge e^2 \wedge J_2 \ .$$

Deformation of the black hole found in [Witten 91].

Concluding remarks

- New integrable theories, as deformations of exact CFT WZW models
- The action can be thought of as the effective all-loop action for the non-Abelian (bosonized) Thirring model.
- It is possible to compute the exact β-function and anomalous dimensions and correlation functions of primary fields using the leading order perturbative result and symmetry and analyticity arguments.
- Coset versions G/H of these models can be embedded in type-II supergravity and represent deformations of SYM within the AdS/CFT correspondence.
- Intresting continuations:
  - Exact in both k and λ β-function and anomalous dims. First step large-N limit
  - Cases with anisotropy i.e. $\lambda_{ab} \neq \lambda\delta_{ab}$; some are integrable.