Pair Production in Near Extremal Charged Black Holes

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Outline

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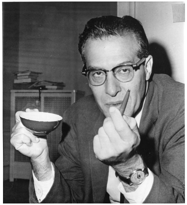
  - Motivation: Schwinger Mechanism and Hawking Radiation
- Scalar Production in Reissner-Nordström Black Holes
- Generalization: Spinor in RN and Scalar in Kerr-Newman Black Holes
- Thermal Interpretation (Scalar in KN)
- Dual CFT Description (Scalar in KN)
- Summary

Spontaneous Pair Production

- Quantum Vacuum Fluctuation: *virtual particles*
  - Heisenberg's uncertainty principle: $\Delta E \Delta t \geq \hbar/2$
  - creation of particle-antiparticle pairs (virtual particles)
- Spontaneous Pair Production: *from virtual to real*
  - Schwinger mechanism: electric field

    Schwinger, 1951
  - Hawking radiation: causal horizon (tunneling picture)

    Parikh, Wilczek, [hep-th/9907001]



Motivation

- Subject: particle creation in charged black holes
  - technical simplicity: constant electric field (exactly solvable)
  - holographic description: anti de Sitter
- Reissner-Nordström (RN) Black Holes: near extremal
  - near horizon region: where the pair production occurs

$$\mathrm{A}dS_2 \times S^2 + \text{constant electric field}$$

- (near) extremal RN black holes
  - (almost) vanishing HAWKING temperature: stable (thermal)
  - non-vanishing electric field: unstable (quantum)

CMC, S.P. Kim, Y.-J. Lin, J.-R. Sun, M.-F. Wu, arXiv:1202.3224 [hep-th]

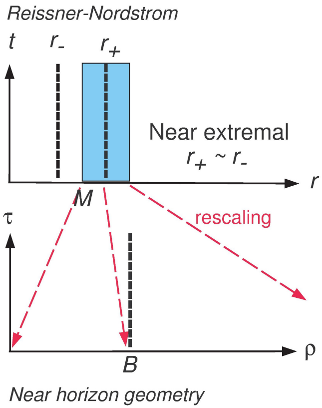
CMC, J.-R. Sun, F.-Y. Tang, P.-Y. Tsai, arXiv:1412.6876 [hep-th]

- (near) extremal KN black holes

CMC, S.P. Kim, J.-R. Sun, F.-Y. Tang, arXiv:1607.02610 [hep-th]

Near Horizon Geometry of RN Black Holes

- Near horizon geometry of near extremal Reissner-Nordström Black Holes:



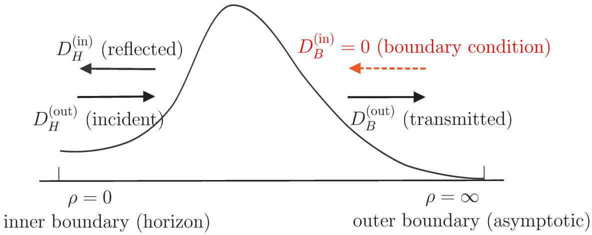
- Horizon radius:
$$r_\pm = M \pm \sqrt{M^2 - Q^2}$$
- Near extremal:
$$Q \to M \;\Rightarrow\; r_- \to r_+$$

Particle Creation: Boundary Conditions

- Pair Production: probe charged massive scalar/spinor field
  - The ratios of fluxes exhibit particle creation with suitable boundary conditions.

Kim, Page, [arXiv:hep-th/0005078]
Kim, Lee, Yoon, [arXiv:0910.3363 [hep-th]]

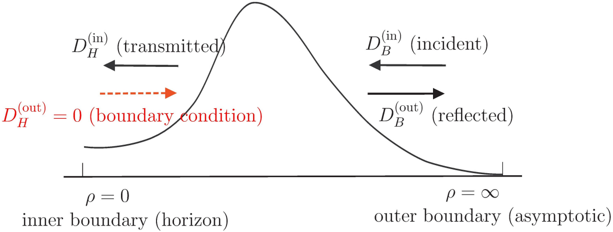
- Boundary condition I: particle view point



- incident: virtual particles
- reflected: re-annihilated
- transmitted: pair produced "particles"

Particle Creation: Boundary Conditions

- Boundary condition II: antiparticle view point



- incident: virtual particles
- reflected: re-annihilated
- transmitted: pair produced "antiparticles"

- Equivalence:
  - particles and antiparticles should always appear in pairs

  ** There is only ONE independent ratio. **

Particle Creation: Physical Quantities

- vacuum persistence amplitude: $|\alpha|^2$
  (⋆ probability for re-annihilation: $1/|\alpha|^2$ ⋆)

$$|\alpha|^2 \equiv \frac{D_{\rm incident}}{D_{\rm reflected}}, \qquad \frac{1}{|\alpha|^2} \equiv \frac{D_{\rm reflected}}{D_{\rm incident}}$$

- mean number of produced pairs: $|\beta|^2$

$$|\beta|^2 \equiv \frac{D_{\rm transmitted}}{D_{\rm reflected}}$$

- absorption cross section: (probability) $\sigma_{\rm abs}$

$$\sigma_{\rm abs} \equiv \frac{D_{\rm transmitted}}{D_{\rm incident}} = \frac{|\beta|^2}{|\alpha|^2}$$

- flux conservation and Bogoliubov relation

$$|D_{\rm incident}| = |D_{\rm reflected}| + |D_{\rm transmitted}| \quad \Longleftrightarrow \quad |\alpha|^2 - |\beta|^2 = 1$$

Scalar Production

CMC, S.P. Kim, Y.-J. Lin, J.-R. Sun, M.-F. Wu, arXiv:1202.3224 [hep-th]

- Near horizon geometry of near-extremal RN

$$\begin{aligned} ds^2 &= -\frac{\rho^2 - B^2}{Q^2} d\tau^2 + \frac{Q^2}{\rho^2 - B^2} d\rho^2 + Q^2 d\Omega_2^2 \\ A_{[1]} &= -\frac{\rho}{Q} d\tau; \qquad F_{[2]} = \frac{1}{Q} d\tau \wedge d\rho \end{aligned}$$

  - “rescaled” deviation from extremality: $\epsilon^2 B^2 = 2Q(M - Q)$
  - geometric structure: $\mathrm{A}dS_2 \times S^2$ (radius Q for both)
  - electric field: constant
- probe massive charged scalar: Φ

$$S_\Phi = \int d^4x \sqrt{-g} \left(-\frac{1}{2} D_\alpha \Phi^* D^\alpha \Phi - \frac{1}{2} m^2 \Phi^* \Phi \right)$$

  - $D_\alpha \equiv \nabla_\alpha - iqA_\alpha$
  - m and q are the mass and charge of Φ

Scalar Production

- Field equation: Klein-Gordon (KG) equation

$$(\nabla_\alpha - iqA_\alpha)(\nabla^\alpha - iqA^\alpha)\Phi - m^2\Phi = 0$$

- Flux:

$$D = i\sqrt{-g}g^{\rho\rho}(\Phi D_\rho \Phi^* - \Phi^* D_\rho \Phi)$$

- Ansatz:

$$\Phi(\tau, \rho, \theta, \phi) = \mathrm{e}^{-i\omega\tau + in\phi} R(\rho) S(\theta)$$

- separated field equations (exactly solvable !!!)

$$\partial_\rho \left[(\rho^2 - B^2)\partial_\rho R\right] + \left[\frac{(q\rho - \omega Q)^2 Q^2}{\rho^2 - B^2} - m^2 Q^2 - \lambda_l\right] R = 0$$

$$\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta S) - \left(\frac{n^2}{\sin^2\theta} - \lambda_l\right) S = 0$$

  - $S(\theta)$ is spherical harmonics with the eigenvalue $\lambda_l = l(l+1)$

Scalar Production

- Exact solution: in terms of hypergeometric functions
- Condition for Schwinger mechanism and/or Hawking radiation

$$(m^2 - q^2)Q^2 + (l + 1/2)^2 < 0$$

  - violation of Breitenlohner-Freedman (BF) bound in AdS_2
  - unstable mode
- Cosmic censorship: necessary condition

$$q^2 > m^2$$

  - avoiding naked singularity
- Schwinger mechanism: extremal to non-extremal

Scalar Production

- Bogoliubov coefficients

$$|\alpha|^2 = \frac{\cosh(\pi\kappa - \pi\mu)\cosh(\pi\tilde{\kappa} + \pi\mu)}{\cosh(\pi\kappa + \pi\mu)\cosh(\pi\tilde{\kappa} - \pi\mu)}$$

$$|\beta|^2 = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\kappa + \pi\mu)\cosh(\pi\tilde{\kappa} - \pi\mu)}$$

$$\kappa \equiv qQ, \quad \mu \equiv \sqrt{(q^2 - m^2)Q^2 - (l + 1/2)^2}, \quad \tilde{\kappa} \equiv \frac{\omega Q^2}{B}$$

- Absorption cross section:

$$\sigma_{\mathrm{abs}} = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\kappa - \pi\mu)\cosh(\pi\tilde{\kappa} + \pi\mu)}$$

- Leading term of $|\beta|^2$ ($\tilde{\kappa} \to \infty, q \gg m$) $\Rightarrow$ Schwinger formula

$$|\beta|^2 \approx e^{-\frac{\pi m^2 Q}{q}} \approx e^{-\frac{\pi m^2 r_H^2}{qQ}}$$

Generalizations

- Spinor Production in Reissner-Nordström black holes

  CMC, J.-R. Sun, F.-Y. Tang, P.-Y. Tsai, arXiv:1412.6876 [hep-th]

$$|\alpha|^2 = \frac{D_H^{(\mathrm{out})}}{D_H^{(\mathrm{in})}} = \frac{\sinh(\pi\mu - \pi\kappa)\cosh(\pi\mu + \pi\tilde{\kappa})}{\sinh(\pi\mu + \pi\kappa)\cosh(\pi\mu - \pi\tilde{\kappa})}$$

$$|\beta|^2 = \frac{D_\infty^{(\mathrm{out})}}{D_H^{(\mathrm{in})}} = \frac{\sinh(2\pi\mu)\cosh(\pi\tilde{\kappa} - \pi\kappa)}{\sinh(\pi\mu + \pi\kappa)\cosh(\pi\mu - \pi\tilde{\kappa})}$$

$$\sigma_{\mathrm{abs}} = \frac{D_\infty^{(\mathrm{out})}}{D_H^{(\mathrm{out})}} = \frac{\sinh(2\pi\mu)\cosh(\pi\tilde{\kappa} - \pi\kappa)}{\sinh(\pi\mu - \pi\kappa)\cosh(\pi\mu + \pi\tilde{\kappa})}$$

- Scalar Production in Kerr-Newman black holes

  CMC, S.P. Kim, J.-R. Sun, F.-Y. Tang, arXiv:1607.02610 [hep-th]

$$\tilde{\kappa} = \frac{\omega}{B}, \quad \kappa = \frac{qQ^3 - 2nar_0}{r_0^2 + a^2}, \quad \mu = \sqrt{\kappa^2 - m^2(r_0^2 + a^2) - \lambda - \frac{1}{4}}$$

Thermal Interpretation

- Hamilton-Jacobi action: $R(r) = \mathrm{e}^{iS(r)}$ (WKB)

$$S(r) = \int \frac{dr}{r^2 - B^2} \sqrt{\frac{[\omega(r_0^2 + a^2) - qQ^3 r + 2nar_0 r]^2}{(r_0^2 + a^2)^2} - \bar{m}^2(r_0^2 + a^2)(r^2 - B^2)}$$

$$\bar{m} = m\sqrt{1 + \frac{\lambda + 1/4}{m^2(r_0^2 + a^2)}}$$

- Residue contributions of the contour integrate at three simple poles: $r = \pm B$ and $r = \infty$

$$\begin{aligned} S_a &= S_- + S_+ = 2\pi \frac{qQ^3 - 2nar_0}{r_0^2 + a^2} = 2\pi\kappa \\ \tilde{S}_a &= S_- - S_+ = 2\pi \frac{\omega}{B} = 2\pi\tilde{\kappa} \\ S_b &= S_\infty = 2\pi\sqrt{\frac{(qQ^3 - 2nar_0)^2}{(r_0^2 + a^2)^2} - \bar{m}^2(r_0^2 + a^2)} = 2\pi\mu \end{aligned}$$

Thermal Interpretation

- Phase-integral formula:

$$N_S = \mathrm{e}^{-S_a + S_b} = \mathrm{e}^{-\frac{\bar{m}}{T_{\mathrm{KN}}}}$$

- Temperature for Schwinger effect:

$$T_{\mathrm{KN}} = T_U + \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}$$

R.-G. Cai, S. P. Kim, arXiv:1407.4569 [hep-th]

S. P. Kim, H. K. Lee, Y. Yoon, arXiv:1503.00218 [hep-th]

CMC, S. P. Kim, J.-R. Sun, F.-Y. Tang, arXiv:1607.026106 [hep-th]

- Unruh temperature and effective curvature:

$$T_U = \frac{qQ^3 - 2nar_0}{2\pi \bar{m}(r_0^2 + a^2)^2}, \qquad \mathcal{R} = -\frac{2}{r_0^2 + a^2}$$

Thermal Interpretation

- Mean number of produced pairs: extremal limit $\tilde{S}_a \to \infty$

$$|\beta|^2 = \left(\frac{\mathrm{e}^{-S_a+S_b} - \mathrm{e}^{-S_a-S_b}}{1+\mathrm{e}^{-S_a-S_b}}\right)\left(\frac{1-\mathrm{e}^{-\tilde{S}_a+S_a}}{1+\mathrm{e}^{-\tilde{S}_a+S_b}}\right)$$

- New parameters: $(\mathcal{R} \to 0 \ \Rightarrow \ T_{\mathrm{KN}} = 2T_U, \, \bar{T}_{\mathrm{KN}} = 0)$

$$\bar{T}_{\mathrm{KN}} = T_U - \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}$$

- Mean number of produced pairs: $(\mathrm{AdS}_2)\{\mathrm{Rindler}_2\}$

$$\mathcal{N} = \mathrm{e}^{\frac{\bar{m}}{T_{\mathrm{KN}}}} \left(\frac{\mathrm{e}^{-\frac{\bar{m}}{T_{\mathrm{KN}}}} - \mathrm{e}^{-\frac{\bar{m}}{\bar{T}_{\mathrm{KN}}}}}{1+\mathrm{e}^{-\frac{\bar{m}}{\bar{T}_{\mathrm{KN}}}}}\right) \left\{\frac{\mathrm{e}^{-\frac{\bar{m}}{\bar{T}_{\mathrm{KN}}}}\left(1-\mathrm{e}^{-\frac{\hat{\omega}-q\Phi_H-n\Omega_H}{T_H}}\right)}{1+\mathrm{e}^{-\frac{\hat{\omega}-q\Phi_H-n\Omega_H}{T_H}}\mathrm{e}^{-\frac{\bar{m}}{\bar{T}_{\mathrm{KN}}}}}\right\}$$

  - $\hat{\omega} = \varepsilon\omega$ (frequency in "original" coordinates), T_H (Hawking temperature), Φ_H (chemical potential), Ω_H (angular velocity)

Dual CFT Description

- Absorption cross section (pair production)

$$\sigma_{\text{abs}} = \frac{\sinh(2\pi\mu)}{\pi^2} \sinh(\pi\tilde{\kappa} - \pi\kappa) \left|\Gamma\left(\frac{1}{2} + i(\mu - \kappa)\right)\right|^2 \left|\Gamma\left(\frac{1}{2} + i(\mu + \tilde{\kappa})\right)\right|^2$$

- from 2-point correlator in 2D CFT

$$\sigma_{\text{abs}} \sim T_L^{2h_L - 1} T_R^{2h_R - 1} \sinh\left(\frac{\tilde{\omega}_L}{2T_L} + \frac{\tilde{\omega}_R}{2T_R}\right) \left|\Gamma\left(h_L + i\frac{\tilde{\omega}_L}{2\pi T_L}\right)\right|^2 \times \left|\Gamma\left(h_R + i\frac{\tilde{\omega}_R}{2\pi T_R}\right)\right|^2$$

- Conformal weight: $h_L = h_R = \frac{1}{2} + i\mu$

Dual CFT Description

- Twofold CFT descriptions for Kerr-Newman black holes

CMC, Huang, Sun, Wu, Zou, PRD 82 (2010) 066004 [arXiv:1006.4097 [hep-th]]

- J-picture: $c_L^J = c_R^J = 12J$ (central charges) temperatures

$$T_L^J = \frac{r_+^2 + r_-^2 + 2a^2}{4\pi a(r_+ + r_-)}, \quad T_R^J = \frac{r_+ - r_-}{4\pi a} \quad \Rightarrow \quad T_L^J \sim \frac{r_0^2 + a^2}{4\pi a r_0}, \quad T_R^J \sim \frac{B}{2\pi a}$$

- Q-picture: $c_L^Q = c_R^Q = \frac{6Q^3}{\ell}$ (central charges) temperatures

$$T_L^Q = \frac{(r_+^2 + r_-^2 + 2a^2)\ell}{4\pi Q(r_+ r_- - a^2)}, \quad T_R^Q = \frac{(r_+^2 - r_-^2)\ell}{4\pi Q(r_+ r_- - a^2)}$$

$$\Rightarrow \quad T_L^Q \sim \frac{(r_0^2 + a^2)\ell}{2\pi Q^3}, \quad T_R^Q \sim \frac{r_0 B \ell}{\pi Q^3}$$

- geometrical meaning of ℓ: radius of embedded extra circle

Dual CFT Description

- CFT entropy: (for both pictures)

$$S_{\text{CFT}} = \frac{\pi^2}{3}(c_L T_L + c_R T_R) \sim \pi(r_0^2 + a^2 + 2r_0 B)$$

- Black hole entropy and temperature

$$S_{\text{BH}} = \pi(r_+^2 + a^2) \quad \Rightarrow \quad S_{\text{BH}} \sim \pi(r_0^2 + a^2 + 2r_0 B)$$

$$\hat{T}_H = \frac{r_+ - r_-}{4\pi(r_+^2 + a^2)} \quad \Rightarrow \quad T_H \sim \frac{B}{2\pi}$$

- Identification via first law of thermodynamics ($\delta S_{\text{BH}} = \delta S_{\text{CFT}}$)

$$\frac{\delta M - \Omega_H \delta J - \Phi_H \delta Q}{T_H} = \frac{\tilde{\omega}_L}{T_L} + \frac{\tilde{\omega}_R}{T_R}$$

Dual CFT Description

- angular velocity and chemical potential (at $r = B$)

$$\Omega_H = \frac{2ar_0}{r_0^2 + a^2} B, \qquad \Phi_H = -\frac{Q^3 B}{r_0^2 + a^2}$$

- variation of parameters: $\delta M = \omega,\ \delta J = -n,\ \delta Q = -q$
- CFT "frequencies":

$$J\text{-picture}: \qquad \tilde{\omega}_R^J = \frac{\omega}{a}, \qquad \tilde{\omega}_L^J = -\frac{qQ^3 - 2nar_0}{2ar_0}$$

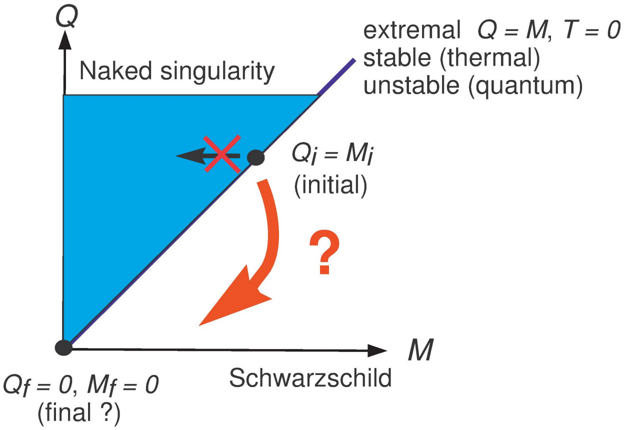
$$Q\text{-picture}: \qquad \tilde{\omega}_R^Q = \frac{2r_0 \ell \omega}{Q^3}, \qquad \tilde{\omega}_L^Q = -\frac{(qQ^3 - 2nar_0)\ell}{Q^3}$$

- for both pictures

$$\frac{\tilde{\omega}_L}{2T_L} = -\pi\kappa, \qquad \frac{\tilde{\omega}_R}{2T_R} = \pi\tilde{\kappa}$$

Pair Production

- Phase Diagram: for RN black holes



Summary

- Spontaneous pair production in near extremal charged black holes: exact Bogoliubov coefficients
- There is a remarkable thermal interpretation.
- The pair production (unstable mode) is holographically dual to an operator with complex conformal weight.
- Cosmic censorship is preserved.