Holographic Schwinger Effect and Chiral condensate in SYM Theory

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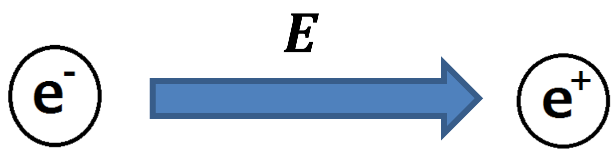
Schwinger effect

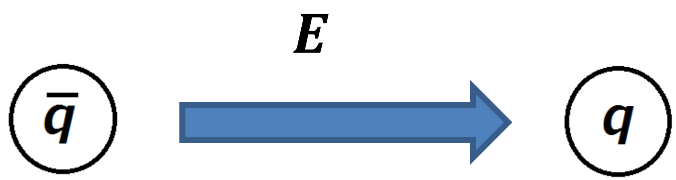
Production rate Γ of electron and positron pairs under the strong electric field $\boldsymbol{E}$

$$\Gamma = \frac{e^2 E^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} exp(-\frac{n\pi m^2}{eE})$$

J. Schwinger, 1951



By using **the holographic duality**, we consider **quark-antiquark pair production rate Γ**



- **Γ at finite temperature phase**
- **Γ at chiral symmetry breaking phase**

Contents

Quark-antiquark pair production rate Γ at finite temperature phase by holography

Effective quark mass and comparison with the NJL model

Quark-antiquark pair production rate Γ at chiral symmetry breaking phase by holography

Effective quark mass and comparison with the NJL model

Summary

Quark-antiquark pair production rate Γ at finite temperature phase by holography

Holographic dictionary

Quarks at finite temperature SU(N_c) gauge theory		**D7-branes in $AdS_5BH \times S^5$**
		A. Karch and E. Katz '02
Schwinger pair production rate		**Imaginary part of D7 Lagrangian**
Γ		$\frac{ImL_{D7}}{2\pi^2\tau_7}$ *K.Hashimoto and T. Oka '13*
Electric field E	**Holography** ⟺	Electric field E in D7-brane
current quark mass, chiral condensate	*J.M. Maldacena 1998*	D7-brane embedding
Temperature T		Black Hole horizon r_T
Color N_c		curvature radius $R \propto N_c^{\frac{1}{4}}$

D7-brane embedding

AdS_5 Black Hole $\times S^5$ metric

$$ds^2 = \frac{r^2}{R^2}\left(-f^2(r)dt^2 + \left(dx^i\right)^2\right) + \frac{1}{f^2(r)}\frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2 \qquad i = 1,2,3$$

R: AdS curvature radius

$$f(r) = \sqrt{1 - \left(\frac{r_T}{r}\right)^4} \qquad T = \frac{r_T}{\pi R^2}$$

D7-brane action in AdS_5 Black Hole $\times S^5$

$$S_{D7} = -\tau_7 \int d^8\xi \sqrt{-det(g_{ab})} = -2\pi^2\tau_7 \int d^4x d\rho \frac{R^2 r^2}{U^4}\rho^3 \sqrt{(1 + w'(\rho)^2)\left(\frac{r^4 f^2}{R^4}\right)}$$

D7-brane: $(t, x, y, z, X^4, X^5, X^6, X^7)$

$$\rho^2 \equiv \left(X^4\right)^2 + \left(X^5\right)^2 + \left(X^6\right)^2 + \left(X^7\right)^2$$

$$X^8 = w(\rho) \qquad X^9 = 0$$

$$U^2 \equiv \rho^2 + w(\rho)^2 = \frac{r^2\left(1 + f(r)\right)}{2}$$

Solution of D7-brane

D7-brane embedding solution $\mathrm{w}(\rho)$

At large ρ

$$w(\rho) = m_q + \frac{c}{\rho^2} + \cdots$$

Current quark mass m_q

VEV of chiral condensate $c \equiv -\langle\bar{\psi}\psi\rangle$

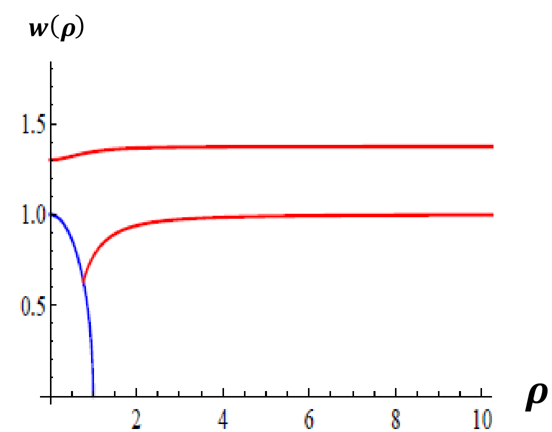
A. Karch and E. Katz 2002

Minkowski type: D7-brane is off the BH horizon

⟺ Stable mesons exist

Black Hole type: D7-brane attaches to the BH horizon

⟺ Mesons are unstable.



D. Mateos, R. C. Myers, R.M. Thomson, 2007

Pair creation rate Γ

Sudden application of an electric field

$$S_{D7} = -\tau_7 \int d^8\xi \sqrt{-det(g_{ab} + F_{ab})} \qquad A_x = -Et$$

The production rate Γ of quark-antiquark pair

K.Hashimoto and T. Oka, '13

$$\Gamma \equiv \frac{ImL_{D7}}{2\pi^2\tau_7} \qquad L_{D7} = \int d\rho \frac{R^2 r^2}{U^4} \rho^3 \sqrt{(1+w(\rho)^2)\left(\frac{r^4 f^2}{R^4} - E^2\right)}$$

$$S_{D7} = -2\pi^2\tau_7 \int d^4x L_{D7}$$

$w(\rho)$: D7-brane solution with $E = 0$

Γ for zero temperature ($AdS_5 \times S^5$)

At *T=0*, $AdS_5 BH \times S^5$ becomes $AdS_5 \times S^5$.

D7-brane solution : $\mathrm{w}(\boldsymbol{\rho}) = \boldsymbol{const} \equiv \boldsymbol{m_q}$

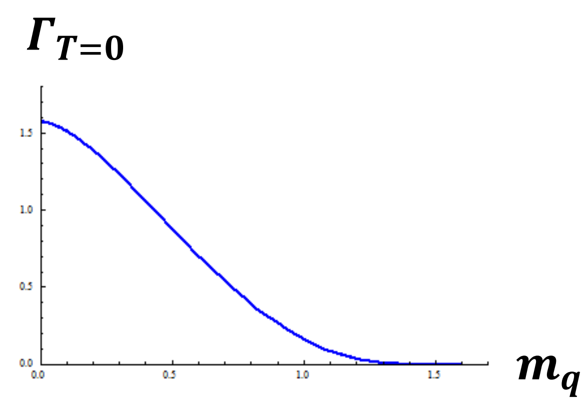
$\boldsymbol{m_q}$: current quark mass

Pair production rate ($\boldsymbol{\Gamma_{T=0}}$) is obtained analytically.

$$\Gamma_{T=0} = \int_0^{\rho_*} d\rho\, \rho^3 \sqrt{\frac{R^4E^2}{r^4} - 1} = \frac{R^4E^2}{2}\left(\frac{\pi}{4} - \theta_0 + \frac{1}{2}\sin\theta_0\left(\cos\theta_0 - \log\left(\frac{1+\cos\theta_0}{1-\cos\theta_0}\right)\right)\right)$$

$$\rho_* = \sqrt{R^2E^2 - m_q^2} \qquad r^2 = \rho^2 + m_q^2 \qquad \sin\theta_0 \equiv \frac{m_q^2}{R^2E}$$

- Quarks with small m_q is easy to be pair created
- At $m_q = \frac{\sqrt{E}}{R}$, $\Gamma_{T=0} = 0$



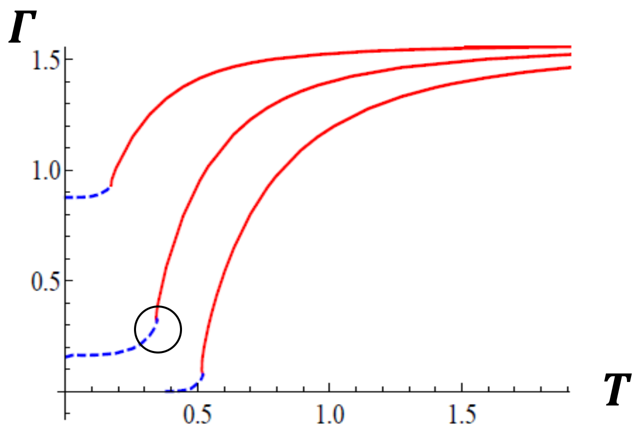
$E = 2$

Γ at finite temperature

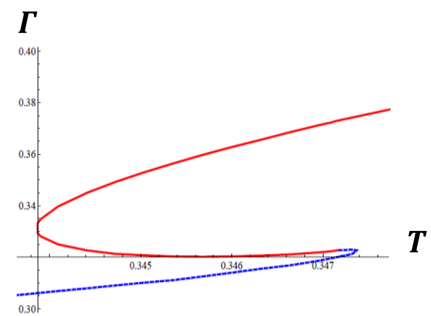
$$\Gamma_{T\neq 0} \equiv \frac{ImL_{D7}}{2\pi^2\tau_7}$$ is given numerically.

Red line: Black Hole Type of D7 brane

Blue line: Minkowski Type of of D7 brane



$\Gamma - T$ curve with $E = 2$ and $m_q = 0.5, 1, 1.5$ from the above



$\Gamma - T$ curve near T_c for $m_q = 1$

- **Γ increases with T ⟹ Effective quark mass decreases with T**

- Γ increases rapidly at the temperature between Minkowski type and BH type of D7-brane **with 1st order phase transition**

Effective quark mass

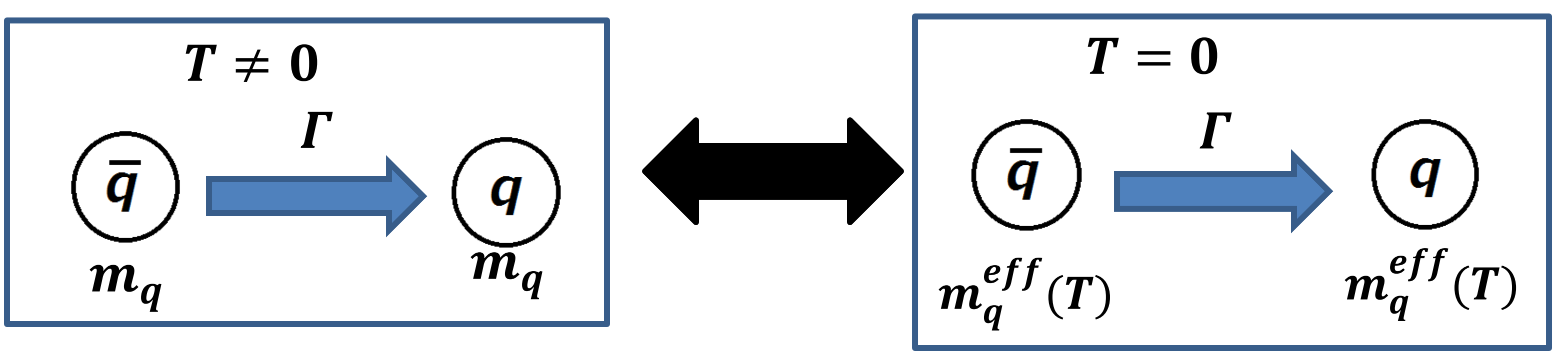
By comparing $\Gamma\,|_{T\neq0}$ with $\Gamma\,|_{T=0}$,
we derive effective quark mass m_q^{eff} at finite temperature T.

$$\Gamma_{T\neq0}(m_q) = \Gamma_{T=0}\left(m_q^{eff}(T)\right)$$

$$\Gamma_{T=0}(m_q^{eff}) = \frac{R^4E^2}{2}\left(\frac{\pi}{4} - \theta_{eff} + \frac{1}{2}\sin\theta_{eff}\left(\cos\theta_{eff} - \log\left(\frac{1+\cos\theta_{eff}}{1-\cos\theta_{eff}}\right)\right)\right)$$

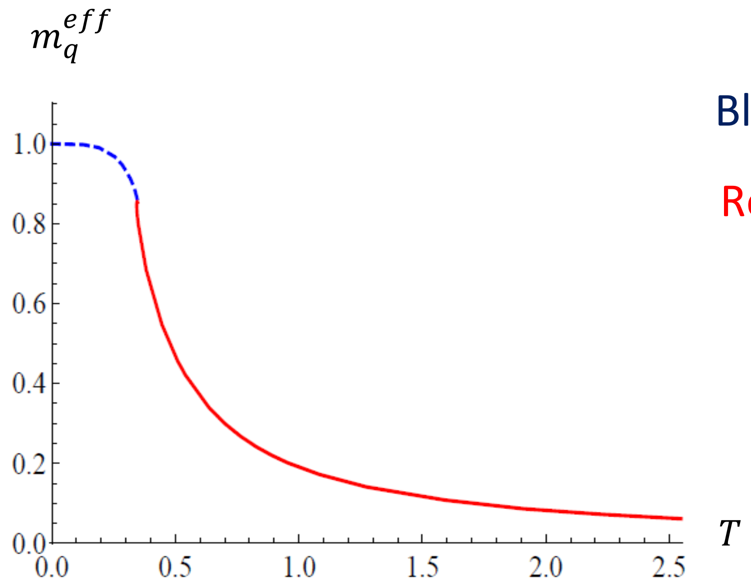
$$\sin\theta_{eff} \equiv \left(\frac{m_q^{eff}}{R}\right)^2 / E$$

The temperature effect is absorbed into the effective quark mass.



Effective quark mass

Temperature dependence of m_q^{eff}



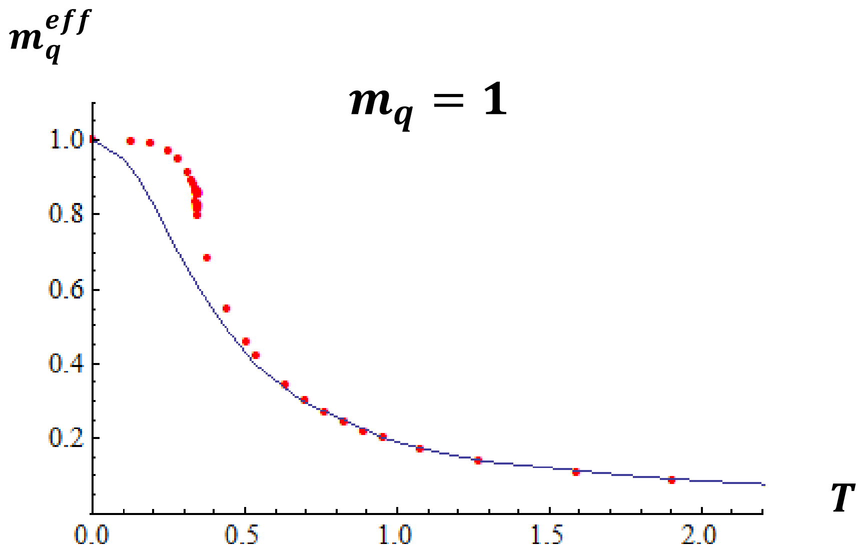
Blue line: Minkowski Type of D7 brane

Red line: BH Type of D7 brane

$E = 2\ ,\ m_q = 1$

m_q^{eff} decreases with temperature.

Comparing with NJL model



At high Temperature, holographic results (red dots) can be fitted with the effective mass of the NJL model at finite temperature (solid line)

$$m_q^{eff}(T) = m_q + 2G_s(T)N_c \frac{m_q^{eff}(T)}{\pi^2} \int_0^{\Lambda} dp \frac{p^2}{E_p} \tanh\left(\frac{E_p}{2T}\right)$$

He, Li, Shakin and Sun PRD 67, 114012 (2003)

$$G_S(T)N_c = \mathrm{g}(T_c^2 - T^2) \qquad T_c = 0, \quad \mathrm{g} = 6.5, \quad \Lambda = 3, \quad m_q = 1$$

Quark-antiquark pair production rate Γ at chiral symmetry breaking phase by holography
(zero temperature)

Quarks for the chiral symmetry breaking phase

10-dim gravity dual to the field theory at chiral symmetry breaking phase

A.Kehagias and K.Sfetsos '99

$$ds^2 = e^{\frac{\Phi}{2}}\left(\frac{r^2}{R^2}A^2(r)\left(-dt^2 + \left(dx^i\right)^2\right) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2\right)$$

$$e^{\Phi} = \left(\frac{\left(\frac{r}{r_0}\right)^4 + 1}{\left(\frac{r}{r_0}\right)^4 - 1}\right)^{\sqrt{3/2}} \qquad A(r) = \left(1 - \left(\frac{r_0}{r}\right)^8\right)^{\frac{1}{4}}$$

At $r_0 = 0$, the 10-dim metric becomes $AdS_5 \times S^5$

Quarks
with chiral symmetry breaking

⟷ Holography

D7-brane in
the 10D gravity background
with finite r_0

K. Ghoroku and M.Yahiro '04

D7 action

D7-brane action

$$S_{D7} = -\tau_7 \int d^8\xi \sqrt{-det(g_{ab})}$$

$$= -2\pi^2 \tau_7 \int d^4x d\rho A^2 e^{\frac{\Phi}{2}} \left(\frac{R}{r}\right)^2 \rho^3 \sqrt{(1 + w(\rho)^2)\left(A^4 e^{\Phi} \left(\frac{R}{r}\right)^4\right)}$$

$$e^{\Phi} = \left(\frac{\left(\frac{r}{r_0}\right)^4 + 1}{\left(\frac{r}{r_0}\right)^4 - 1}\right)^{\sqrt{3/2}} \qquad A = \left(1 - \left(\frac{r_0}{r}\right)^8\right)^{\frac{1}{4}}$$

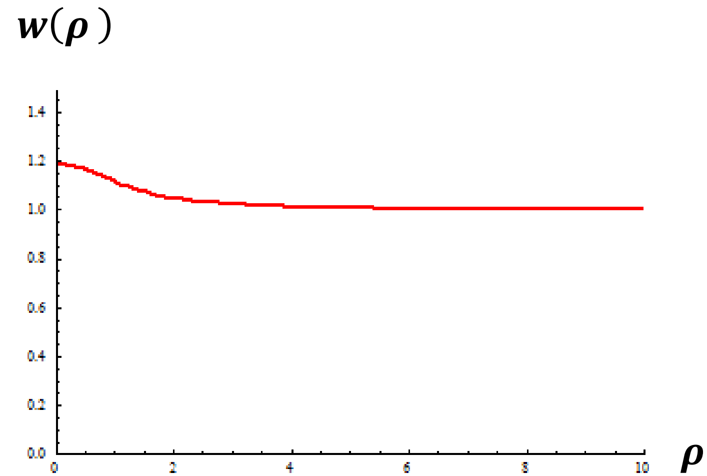
$$\rho^2 \equiv (X^4)^2 + (X^5)^2 + (X^6)^2 + (X^7)^2 \qquad (X^8, X^9) \equiv (w(\rho), 0)$$

$$r^2 = \rho^2 + w(\rho)^2$$

Effective quark mass

D7-brane embedding solution $\boldsymbol{w(\rho)}$

$$w(\rho) = m_q + \frac{c}{\rho^2} + \cdots$$



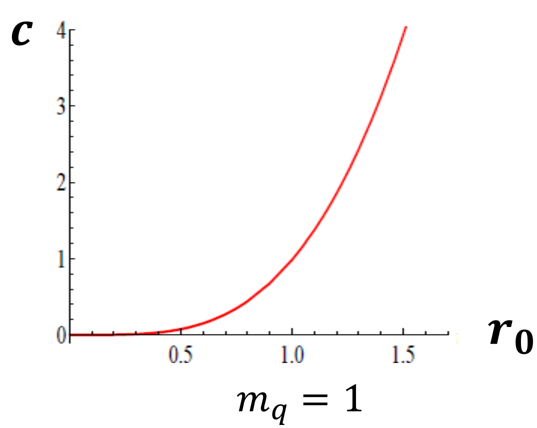
Current quark mass: m_q

VEV of chiral condensate: $c \equiv -\langle\bar{\psi}\psi\rangle > 0$

chiral symmetry breaking

The value of c depends on r_0

At $r_0 = 0, \quad c = 0$



Production rate and effective quark mass

Sudden application of an electric field

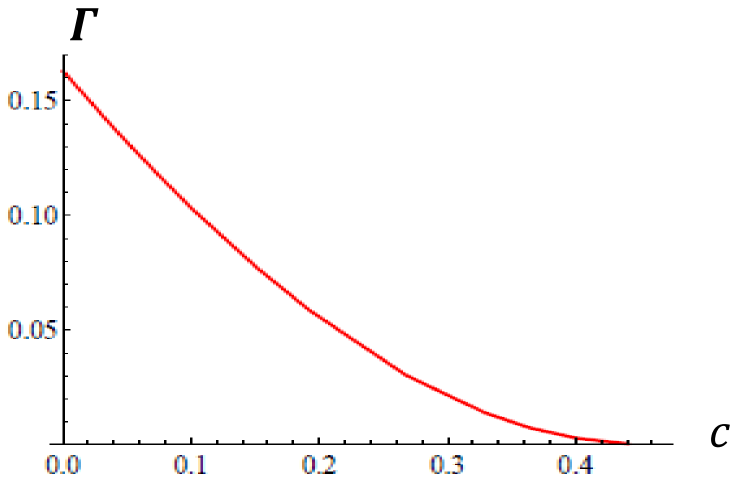
$$S_{D7} = -\tau_7 \int d^8\xi \sqrt{-det(g_{ab} + F_{ab})} \qquad A_x = -Et$$

The production rate Γ of quark-antiquark pair

$$\Gamma \equiv \frac{Im L_{D7}}{2\pi^2 \tau_7} \qquad L_{D7} = \int d\rho A^2 e^{\frac{\Phi}{2}} \left(\frac{R}{r}\right)^2 \rho^3 \sqrt{(1 + w(\rho)^2)\left(A^4 e^{\Phi}\left(\frac{R}{r}\right)^4 - E^2\right)}$$

$w(\rho)$: **D7-brane solution** with $E = 0$

Production rate



Relation between Γ and c with $m_q = 1$ and $E = 2$

Γ decreases with $\boldsymbol{c} \equiv -\langle \bar{\psi}\psi \rangle$

At large c, effective quark mass becomes large and it becomes difficult to be pair created.

Effective quark mass

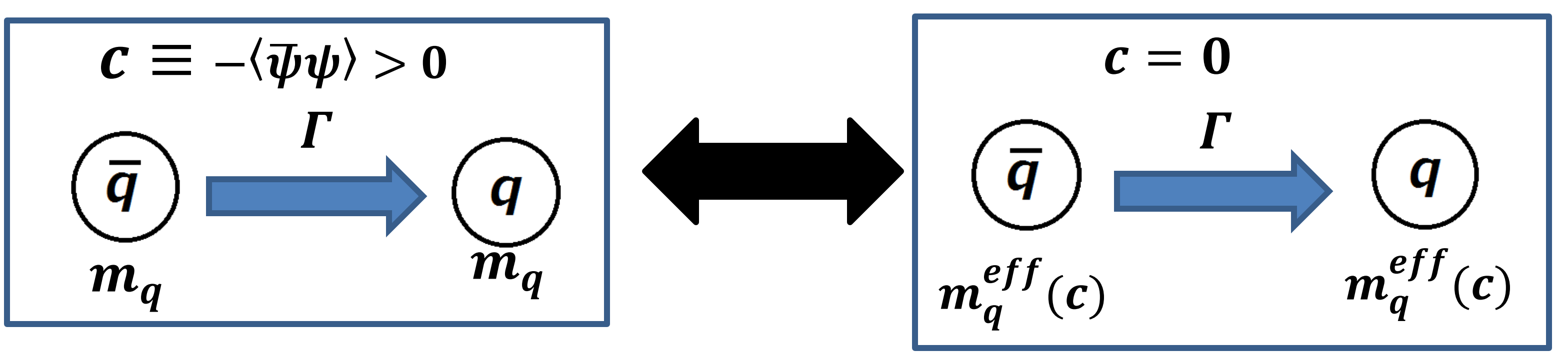
By comparing $\Gamma_{c=0}$ with $\Gamma_{c\neq0}$, we derive the effective quark mass for the chiral symmetry breaking phase.

$$\Gamma_{c\neq0}(m_q) = \Gamma_{c=0}\left(m_q^{eff}(c)\right)$$

$$\Gamma_{c=0}(m_q^{eff}) = \frac{R^4E^2}{2}\left(\frac{\pi}{4} - \theta_{eff} + \frac{1}{2}\sin\theta_{eff}\left(\cos\theta_{eff} - \log\left(\frac{1+\cos\theta_{eff}}{1-\cos\theta_{eff}}\right)\right)\right)$$

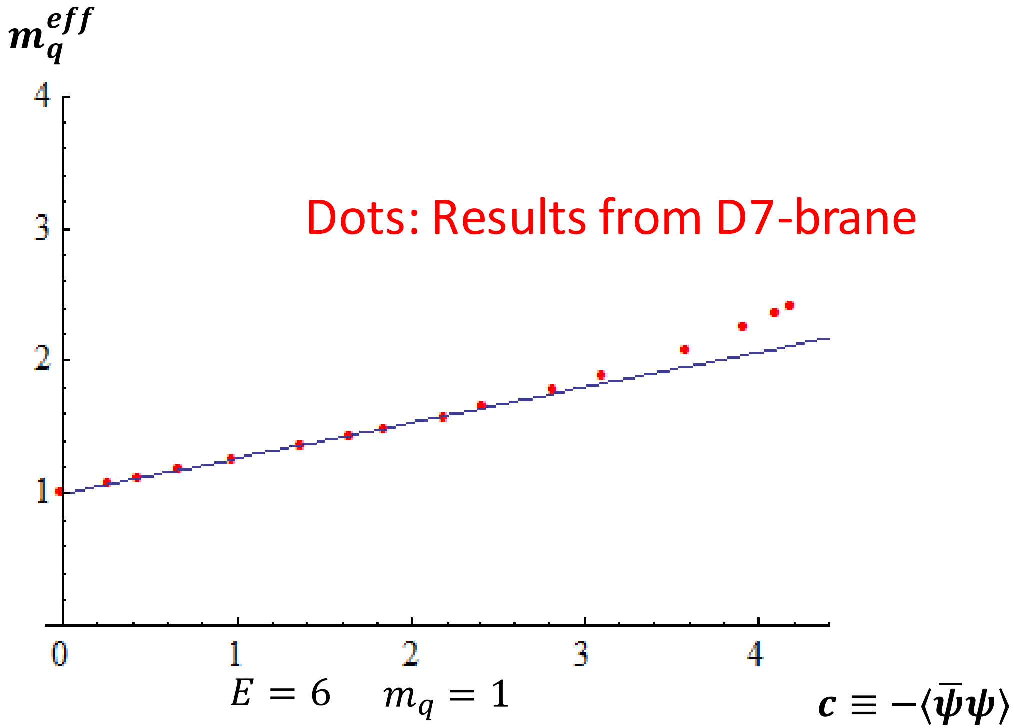
$$\sin\theta_{eff} \equiv \left(\frac{m_q^{eff}}{R}\right)^2 / E$$

The effect of chiral condensate is absorbed into the effective quark mass.



Effective quark mass

Numerical results for $m_q^{eff}(c)$



The effective quark mass agrees with NJL model.

$$\mathrm{m_q^{eff}} = m_q - 2g_s\langle\bar{\Psi}\Psi\rangle + \cdots,$$

$g_s = 0.74/E$

m_q: current quark mass

Summary

- We calculate quark-antiquark pair production rate $\boldsymbol{\Gamma}$ from the imaginary part of on-shell D7-brane Lagrangian

- For finite temperature phase, $\boldsymbol{\Gamma}$ increases rapidly at the temperature between Minkowski and Black Hole embedding. There is a 1st order phase transition.

- We derive the effective quark mass $\boldsymbol{m_q^{eff}}$ from $\boldsymbol{\Gamma}$, and it decreases with $\boldsymbol{T}$.

- We also calculate $\boldsymbol{\Gamma}$ at chiral symmetry breaking phase. $\boldsymbol{\Gamma}$ decreases with $\boldsymbol{c \equiv -\langle \bar{\psi}\psi \rangle}$. Γ

- The effective mass $\boldsymbol{m_q^{eff}}$ increases with $\boldsymbol{c}$ and it agrees with NJL model.

$$\boldsymbol{m_q^{eff} = m_q - 2g_s \langle \bar{\Psi}\Psi \rangle + \cdots} \qquad \boldsymbol{g_s = 0.74/E}$$

$\boldsymbol{m_q}$: current quark mass